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The Role of Reference-Dependent Preferences in Auctions and Negotiations

by

Antonio Rosato

A dissertation submitted in partial satisfaction of the
requirements for the degree of
Doctor of Philosophy

in

Economics

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Matthew Rabin, Chair
Professor Botond Kőszegi
Professor Benjamin E. Hermalin
Professor Edward Augenblick

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Abstract

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Doctor of Philosophy in Economics

University of California, Berkeley

Professor Matthew Rabin, Chair

This dissertation consists of three chapters exploring the role that reference-dependent preferences and loss aversion play in auctions and negotiations.

The first chapter characterizes the profit-maximizing pricing and product-availability strategies for a retailer selling two substitute goods to loss-averse consumers, showing that limited-availability sales can manipulate consumers into an ex-ante unfavorable purchase. When the products have similar social value, the seller maximizes profits by raising the consumers' reference point through a tempting discount on a good available only in limited supply (the bargain) and cashing in with a high price on the other good (the rip-off), which the consumers buy if the bargain is not available to minimize their disappointment. The price difference between the bargain and the rip-off is larger when the products are close substitutes than when they are distant substitutes; hence dispersion in prices and dispersion in consumers' valuations are inversely related. The seller might prefer to offer a deal on the more valuable product, using it as a bait, because consumers feel a larger loss, in terms of forgone consumption, if this item is not available and are hence willing to pay a larger premium to reduce the uncertainty in their consumption outcomes. I also show that the bargain item can be a loss leader, that the seller's product line is not welfare-maximizing and that she might supply a socially wasteful product.

The second chapter studies sequential first-price and second-price auctions when bidders are expectations-based loss-averse. A large body of empirical research in auctions documents that prices of identical products sold sequentially tend to decline across auctions (a phenomenon which has been dubbed "declining price anomaly" or "afternoon effect", as often later auctions take place in the afternoon whereas the first ones usually take place in the morning) . In this chapter I argue that expectations-based reference-dependent preferences and loss aversion provide an alternative, preference-based, explanation for the afternoon effect observed in sequential auctions. First, I show that when bidders have reference-dependent preferences, the equilibrium bidding functions are history-dependent, even if bidders have independent private values. The reason is that learning the type of the winner in the previous auction modifies a bidder's expectations about how likely he is to win in the current auction; and since expectations are the reference point, the optimal bid in each round is affected by this learning effect. More precisely, I identify what I call a "discouragement effect ": the higher the type of the winner in the first auction is, the less aggressively the

bidding behavior of the remaining bidders in the second auction. This discouragement effect in turn pushes bidders to bid more aggressively in the earlier auction. Moreover, the uncertainty about future own bids, due to the history-dependence of the equilibrium strategies, generates a precautionary bidding effect that pushes bidders to bid less aggressively in the first auction. The precautionary bidding effect and the anticipation of the discouragement effect go in opposite directions; when the latter effect is stronger, a declining price path arises in equilibrium.

The third chapter studies the role of expectations-based reference-dependent preferences and loss aversion in a sequential bargaining game with one-sided incomplete information between a seller who makes all the offers and a buyer. I show that loss aversion eases the rent-efficiency trade-off for the seller who can now serve a larger measuer of consumers at an earlier stage. Thus, in equilibrium the seller achieves higher profits and we have less delay with loss aversion than without it. Furthermore, I also show that, besides increasing the seller's profit and overall trade efficiency, loss aversion also reallocates surplus among consumers by increasing the equilibrium payoff of some low-valuation buyers and decreasing that of high-valuation ones.

To my parents, my sisters and my teachers

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Chapter 1

Selling Substitute Goods to Loss-Averse Consumers: Limited Availability, Bargains and Rip-offs

1.1 Introduction

Retailers frequently use low prices and offer deals to attract consumers. In many cases, these deals apply only to a subset of a store's product line and are often subject to "limited availability". Some shops, for example, offer deals that are valid only "while supplies last," or they might offer price reductions on sale items only to the very first customers of the day. Consider the two following examples:

Example 1 *A retailer in Berkeley California has offered the following:*
Converse All Star high-top in black for just $24.99 (offer valid while supplies last).
Any other color for $54.99.[1]

Example 2 *On Black Friday 2011, Best Buy offered, among other items, the following:*
Panasonic 50" Class / Plasma / 1080p / 600Hz / Smart HDTV for $599.99.
Panasonic 50" Class / Plasma / 720p / 600Hz / HDTV for $799.99.[2]

In the first example, the store is offering a deal on black shoes — $20 less than the regular price. There is, however, no deal on other colors; indeed their price is $10 higher than the regular price. The $30 difference between the price of black and non-black shoes is unlikely to be explained by differences in cost or demand. Furthermore, the deal on black

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[1]At the same retailer, Bancroft Clothing Co., the regular price during the "non-deal" weeks is $44.99, independent of color. The manufacturer online price is $50 plus shipping fees.

[2]Black Friday is the day following American Thanksgiving and traditionally marks the beginning of the Christmas shopping season. The 1080p TV first appeared at Best Buy on March 20, 2011 for $1,000 and its price has been constant until Thanksgiving Day of the same year. The 720p TV first appeared at Best Buy on March 28 for $719.99 and its price was reduced to $649.99 on August 9, 2011 and raised again up to $799.99 on November 10, 2011, two weeks before Thanksgiving. These data have been collected using camelbuy.com, a website that provides a price tracker and price history charts for products sold online at Amazon.com and Best Buy.com.

shoes is valid only while current supplies last and the price could well be higher once the store restocks. In the second example, the store is selling two very similar TVs for very different prices; moreover, somewhat puzzlingly, the TV with the higher-resolution screen, universally preferred, is offered at a lower price. The original Best Buy ad specified that the one on the superior TV was an online-only deal, that availability was "limited to warehouse quantity," and no rainchecks would be offered to consumers. Notice also that the goods in these examples are substitutes and consumers normally buy at most one unit. Why, then, do stores discount only a few items heavily, and why is there so much dispersion, within the same store, in the price of similar goods? How do stores select which products to offer for a discount?

Traditional search-theoretic models of sales based on costly information acquisition are not well-equipped to answer these questions, as they pertain mainly to retailers supplying only one product. Moreover, they are concerned with explaining price dispersion either across different stores (as in Salop and Stiglitz, 1977) or across different time periods (as in Varian, 1980), not with the issue of within-store price dispersion across similar items, nor they look at the role of product availability in retailing.[3]

In this paper, I propose a model of *bait-and-switch* where a retailer uses limited-availability bargain sales to exploit consumers' loss-aversion and prompt them to willfully engage in an ex-ante unfavorable purchase. I do so by introducing consumer loss aversion into an otherwise classical model of linear pricing: a risk-neutral profit-maximizing monopolist sells two substitutable goods to homogeneous consumers who demand at most one unit altogether and whose reference point for evaluating a purchase, following the model of Kőszegi and Rabin (2006), is given by their recent rational expectations about the purchase itself. With these preferences, a consumer's willingness to pay for a good is determined not only by his intrinsic value for it, but also endogenously by the market conditions and his own anticipated behavior. Moreover, the monopolist can directly affect consumers' expectations by making announcements regarding prices or availability. For example, if a consumer expects to buy with high probability, he experiences a loss if he fails to buy. This, in turn, increases his willingness to pay. On the other hand, compared to the possibility of getting a deal, paying a high price is assessed by the consumer as more of a loss, which in turn decreases his willingness to pay. Since expectations are the reference point and because expectations are (also) about own future behavior, the reference point is determined endogenously in the model by requiring that the (possibly stochastic) outcome implied by optimizing behavior conditional on expectations coincides with expectations.

The main result of the paper is that, when two goods have a similar social value, the profit-maximizing strategy for the monopolist is to offer a limited-availability deal on one of the goods and then cash in with a high price on the other. Consumers perceive this limited-availability sale as equivalent to a lottery on both which good they will end up with and how much they will pay. The price of the good on sale (the *bargain*) is chosen such that it

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[3]For an extensive survey of the search theory literature in IO, see Baye, Morgan and Scholten (2006). Rhodes (2012) and Zhou (2012) study multi-product search models with complements. A notable exception is provided by Konishi and Sandfort (2002). In their paper a multi-product store can increase its profits by discounting only some of its products, even when they are substitutes. However, consumers in this model shop for a "search good" and hence they learn their tastes only once they arrive at the store and discounts on few items are a way to increase store traffic. The logic in my model is quite different.

is not credible for the consumers to expect not to buy it. Thus, the limited-availability deal works as a bait in luring consumers into the store.[4] Then, because the consumers expect to make a purchase with positive probability and dislike the uncertainty in their consumption outcomes, in the event that the bargain is not available, they prefer to buy the substitute good, even at a higher price (the *rip-off*). In other words, consumers go to the store enticed by the possibility of the bargain, but if it is not there they buy a substitute good as a means of reducing their disappointment.[5]

I distinguish two cases depending on whether the two items are valued similarly by the consumers. If the goods are "close" substitutes, the seller chooses a price of the bargain and a price of the rip-off that are farther away than consumers' valuations. If instead the products are "distant" substitutes, the seller prices them closely or even equally. Hence, under limited availability and loss aversion, dispersion in prices and dispersion in valuations are inversely related. This provides a possible explanation for why relatively similar goods are often offered at different prices, like the shoes in Example 1 above and, at the same time, why different goods are sometimes priced equally, like different items in a restaurant's menu.

The limited-availability nature of the deal is critical for this strategy to work, and the degree of availability of each item is publicly announced by the seller. On the one hand, a high likelihood of availability for the bargain makes the consumers more attached to the idea of buying. This allows the seller to charge a higher price on the rip-off. On the other hand, a greater availability of the bargain necessarily means fewer sales of the rip-off. When choosing the supply level of the bargain item, the seller optimally trades off these two effects. I also show that if the bargain is the product with the smaller social surplus, its availability is bounded above by 50%, implying that less than half of the consumers actually end up buying the item on sale.[6]

According to the current FTC regulation, it is not a bait-and-switch if the store communicates up-front that availability is limited.[7] Nevertheless, the popular press and various consumers' associations seem to perceive limited-availability deals as being of an exploitative nature, as suggested by the following quotes:

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[4]There is a reason why in Black Friday jargon these deals are called "doorbusters."

[5]Because of loss aversion, consumers are willing to pay a premium in order to avoid the feeling of loss resulting from not getting the bargain. So, the seller is not exploiting a cognitive bias of the consumers. This is in contrast to several models with boundedly rational or naïve consumers, as in DellaVigna and Malmendier (2004), Eliaz and Spiegler (2006, 2008, 2011b), Gabaix and Laibson (2006), Grubb (2009), Rubinstein and Spiegler (2008), and Spiegler (2006). See Spiegler (2011) for a textbook treatment.

[6]Besides Black Friday, other examples of limited-availability sales that take place in the U.S. are: (i) Cyber Monday, the first Monday after Thanksgiving Day, which mainly pertains to online shopping; (ii) Back-to-School Sales taking place at the end of summer when most schools and colleges begin their school year; and (iii) the The Running of the Brides, which was a one-day sale of wedding gowns that used to take place in many Filene's Basement stores (in December 2011 Filene's Basement declared bankruptcy and went out of business). Moreover, many big national retailers, like Target and Toys R Us, have begun to hold Black Friday-style sales during the summer as well (see http://www.washingtonpost.com/wp-dyn/content/article/2010/07/22/AR2010072206101.html)

[7]The current FTC Guides Against Bait Advertising require retailers "to have available at all outlets listed in the advertisement a sufficient quantity of the advertised product to meet reasonably anticipated demands, unless the advertisement clearly and adequately discloses that supply is limited and/or the merchandise is available only at designated outlets" (16 C.F.R. Part 238.3).

> One of the biggest problems during significant sale days like Black Friday is the deceptive practice of offering a popular, expensive item for a great sale price, but only stocking a very limited number of these products. This is somewhat of a bait-and-switch because even if that product is unavailable, you are likely to stay at the store and take advantage of other, less valuable sales. (Denver Better Business Bureau, http://denver.bbb.org)

> Know why they call it "Black Friday?" It isn't because those sale items push retailers into the "black" (accounting speak for profitability). Those sale items are almost always loss leaders — items sold at a loss in order to lure you into the store in the hope you'll buy other, more profitable items. What really pushes retailers into the black are the profitable items you buy because you showed up at 4am and everything you hoped to buy was sold out and you HAD to buy SOMETHING. *(*http://www.thewisdomjournal.com/Blog/beware-of-black-friday-bait-and-switch/)

The above quotes seem to imply that among the consumers who go shopping during sales with the intention of getting a deal, some fulfill their goal and get a bargain; others, however, might not find what they were looking for and might end up buying a different and often not-on-sale item.[8] But, if they know in advance that the chance of getting a deal is small, why do consumers go shopping anyway?

Interestingly, by exploiting the time inconsistency of the consumers' preferences, with a limited-availability strategy the seller is able to push the consumers' reservation utility below zero. This is possible because with expectations-based reference-dependent preferences, the consumers' participation constraint is belief-dependent — and therefore endogenous — and the seller can manipulate the consumers' beliefs with her own strategy.[9] The intuition is as follows: if a consumer expects to find a product he likes available for a very low price, he will definitely plan to buy it. The attachment to the good induced by realizing that he will do so, however, changes his attitudes toward the purchasing decision. If the store runs out of the good on sale for a low price, but has a similar one available for a higher price, the consumer must now choose between a loss of money from paying a higher price and a loss of consumption from returning home empty-handed. While, in equilibrium, buying the expensive substitute is indeed the best response to his expectations, it is still worse than if he could have avoided the feeling of loss by avoiding the expectation of getting the bargain in the first place. More generally, because an expectations-based loss-averse consumer does not internalize the effect of his ex-post behavior on ex-ante expectations, the strategy that maximizes ex-ante expected utility is often not a credible plan. Moreover, consumers are hurt also by the uncertainty about which item they will get to consume and how much they will pay. Thus, despite the fact that, with some probability, they get a good deal, on average consumers are made worse off by this combination of limited availability, bargains,

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[8]Empirical studies in marketing and psychology reveal indeed that consumers are likely to buy substitute items when their preferred product is out of stock, and even more so if the product they were planning to buy was on sale or if the seller had announced up-front that quantities were limited. I review the evidence about consumers' response to stockouts in Section 2.

[9]Spiegler (2012b) studies the problem of incentivizing participation for agents with expectations-based reference-dependent preferences in more general environments.

and rip-offs. Hence, the current FTC Guides Against Bait Advertising, by allowing stores to credibly announce that they have limited supplies for bargain items, might have the perverse effect of reducing consumers' welfare.

Despite the products being substitutes, loss aversion creates positive demand spillovers between products so that the higher a consumer's intrinsic valuation for a product, the higher his willingness to pay is for a substitute of that product as well. When the goods are vertically differentiated, the seller tends to use the more valuable item as the bargain. This may, at first, seem odd, given that consumers are (intrinsically) willing to pay a higher price for the superior good. Yet, exactly because consumers value the superior item more, the possibility of a bargain causes them to feel a larger loss, in terms of forgone consumption, when this item is not available; hence, they are willing to pay an even bigger premium to reduce the uncertainty over their consumption outcome, which, in turn, allows the seller to charge an even higher price for the rip-off. So my model predicts that more valuable items should be more likely to be used as baits, as in Example 2 above.

A related implication is that the monopolist, in order to effectively induce uncertainty into the consumers' purchasing plans, might introduce a less socially desirable or, worse, socially wasteful product and the profit-maximizing product line could differ from the socially optimal one.[10] Although this implication appears also in models of second-degree price discrimination via quality distortion (i.e., Deneckere and McAfee, 1996), the motive in this case is not to screen the consumers, but rather to exploit the aforementioned positive spillover effect by selling a less valuable product at a higher price.

Furthermore, the bargain item can be a "loss leader" (i.e., being priced below cost). Traditional models of consumer behavior can explain the use of loss leaders for complementary goods (see Ambrus and Weinstein, 2008); my model instead can rationalize the use of loss leaders for substitutes. With classically assumed reference-free preferences, the scope for using loss leaders is to increase store traffic; however, for this increase in store traffic to be profitable, consumers must buy other items *in addition to* the loss leader. In my model, instead, loss leaders lure consumers into the store, but their profitability stems from the fact that, if the seller has run out of the loss-leading product, consumers will buy another item *instead of* the loss leader in order to minimize their disappointment. Moreover, while traditional models — like the one of Lal and Matutes (1994) — suggest that products with lower reservation prices are more natural candidates to be loss leaders, my model can explain the use of highly valuable products as loss leaders.[11] This is consistent with the observation that, on Black Friday, Best Buy offers a below-cost large-screen flat TV to the first ten people who buy one.

My paper is related to, and builds upon, the analysis in Heidhues and Kőszegi (forthcoming), which provides an explanation for why regular prices are sticky, but sales prices are variable, based on expectations-based loss aversion. In their model, a single-product monopolist maximizes profits by committing to a stochastic-price strategy made of low, variable

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[10]Klemperer and Padilla (1997) obtain a similar result in an oligopoly model where consumers have classical preferences and multi-unit demand. For this environment they show that a firm might want to introduce an additional, socially wastful variety, because of a profitable business stealing effect.

[11]Kamenica (2008) proposes a model of contextual inference from product lines where a firm may try to manipulate consumers' beliefs by introducing premium loss leaders — expensive goods of overly high quality that increase the demand for other goods.

sales prices and a high, sticky regular price. Their result and mine share a similar intuition: low prices work as baits to attract the consumers who, once in the store, are willing to pay a price even above their intrinsic valuation to avoid the loss resulting from going home empty-handed. The key-difference in my paper is that I consider a monopolist who sells two goods and uses one of them as a bait to attract the consumers and the other one to exploit them. My result on the optimality of limited-availability sales can be seen as a foundation as well as a more plausible re-interpretation of their result about the optimality of random-price sales.[12]

Katz and Nelson (1990) also study product availability and price dispersion for the case of a monopolist selling two substitutable goods to consumers with downward sloping and continuous multi-unit demand, who can choose whether to enter the market and have type-dependent outside options. They show that if the monopolist can credibly commit to have stockouts, there exists a two-price equilibrium in which the lower-price brand is understocked. However, they study only the case of perfect substitutes and their main result relies on the assumption that once a consumer enters the store, he forfeits his outside option and if faced with a stockout of the low-priced brand, he must buy the expensive one. In my model, instead, the consumers' behavior in the event of a stockout is not assumed, but it arises endogenously in equilibrium because consumers have expectations-based reference-dependent preferences and prefer to buy the expensive substitute instead of leaving the store empty-handed.

There are also a few papers focusing on the role of product availability as a strategic variable in various oligopoly settings (see Daughety and Reinganum, 1991; Chakravarty and Ghose, 1994; Balachander and Farquhar, 1994; Dana, 2001b; Watson, 2009). In these models, firms supply only one product and by competing (also) in availability, they are able to charge higher prices. However, how availability interplays with a firm's other strategic variables (quantity or price) varies considerably between the papers depending on the specific details of each model.

The remainder of this paper proceeds as follows. Section 2 briefly summarizes the key empirical evidence on sales and limited availability. Section 3 describes the baseline model with homogeneous consumers and the features of market demand when consumers have expectations-based reference-dependent preferences. Section 4 presents the main result about the seller's optimal pricing and availability with homogeneous consumers. Section 5 deals with three extensions of the baseline model: endogenous product lines, heterogeneous consumers' tastes and consumers' naïvete. Section 6 relates the paper to the literatures on firms' response to consumers' loss aversion, loss leaders, bait-and-switch, price discrimination, and other topics. Section 7 concludes by recapping the results of the model and pointing out some of its limitations as well as possible avenues for future research.

1.2   Evidence on Sales and Stock-Outs in Retailing

This section summarizes empirical evidence that points to three main facts: (1) sales are frequent but affect a small fraction of items, (2) products on sale are more likely to be out

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[12]I discuss in more detail the similarities and differences with Heidhues and Kőszegi (forthcoming) in Section 7.

of stock and (3) consumers are willing to buy substitute products when their preferred item is sold out. These facts frame the importance and relevance of the analysis of this paper in understanding why and how retailers use limited-availability sales, and how consumers react when facing alternatives for a product that is sold out.

Sales, in the sense of periodic, temporary price reductions, are a ubiquitous feature of retail pricing (see Hosken and Reiffen, 2004a and Nakamura and Steinsson, 2008).[13] However, among all the items supermarkets and other retailers carry, usually only a small fraction each week are offered at a low sale price and, within categories, retailers seem to systematically place some products on sale more often then others, with more popular items — those appealing to a wider range of customers — being more likely to go on sale (Hosken and Reiffen, 2004b). Relatedly, Nakamura (2008) finds that only a small fraction (19%) of price variation is common to all products in a category at a given retail store. According to a recent study by ShopAdvisor, a deferred shopping service used by independent websites and tablet magazines, in the 54 days from Nov. 1st through Dec. 24th 2011, the day with the lowest percentage (46%) of products on sale below their initial holiday season price was indeed Black Friday, Nov. 25th. In fact, Black Friday is also the day on which shoppers begin to see prices spike on selected items: on Black Friday itself, 24% of the toys on ShopAdvisor's list were priced above their initial holiday season price.[14] Strausz (2007) reports that the largest German discounters, Aldi and Lidl, weekly advertise limited-availability bargain sales on products that do not belong to their usual selling stock. Chevalier, Kashyap and Rossi (2003) find that the majority of sales are not caused by changes in wholesale pricing, implying therefore that sales are primarily due to changes in retailers' margins. Similarly, Anderson, Nakamura, Simester and Steinsson (2012) report that while regular prices react strongly to cost and wholesale price changes, the frequency and depth of sales is largely unresponsive.

While not as ubiquitous as sales, stockouts are also prevalent in retailing. Gruen, Corsten and Braradwaj (2002) report an 8.3% out-of-stock rate worldwide, rising to even 25% for some promoted items. Hess and Gerstner (1987) sampled two general merchandise stores and found that stockouts occurred more often for products on sale than for similar products not on sale. Using data from a supermarket chain in Spain, Aguirregabiria (2005) documents a significant amount of heterogeneity across items in the frequency of stockouts; most of this heterogeneity is within-product (i.e., among brands of the same product line) and not among products. Grant-Worley, Saltford and Zick (1982) surveyed five major non-food chains in Syracuse, New York and found that the average rate of unavailability for advertised products was 12%. Similarly, Taylor and Fawcett (2001) investigated availability of advertised products for three large national mass merchants, four category killers involved in the office supplies and electronics subcategories and three retail grocers in the Mid-West, and found that the stock-out ratio for advertised items was twice as high as that of comparable, non-advertised items. Bils (2004) presents evidence on temporary stockouts for durable consumer goods using data from the CPI *Commodities and Services Survey* and finds that from January 1988 to June 2004 the temporary stockout rate averaged between 8.8% and 9.2%.

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[13]Sales might also refer to systematic reductions in the price of fashion items; see Lazear (1986), Pashigian (1988) and Pashigian and Bowen (1991).

[14]See http://www.prnewswire.com/news-releases/shopadvisor-cautions-buyers-on-black-friday-not-worth-the-early-morning-rise-174618581.html

Several marketing and psychology studies on consumers' response to product unavailability (Emmelhainz, Stock and Emmelhainz, 1991; Anupindi, Dada and Gupta, 1998; Verbeke, Farris and Thurik, 1998; Fitzsimons, 2000; Campo, Gijsbrechts and Nisol, 2000, 2003; Zinn and Liu, 2001) show that consumers are often willing to buy substitute items when faced with stockouts: depending on the specific characteristics of the product and store under study, the percentage of consumers who is willing to buy a substitute — within the same store — ranges from 30% to 80%. Through a post-purchase questionnaire, Zinn and Liu (2001) find also that consumers are more likely to leave a store empty-handed if they are surprised by the stockout; this finding suggests that prior expectations of product availability may be an important predictor of out-of-stock response. Relatedly, Anderson, Fitzsimons and Simester (2006) and Ozcan (2008) find that consumers are more willing to buy a substitute if the stockout product was on sale or if limited supplies were announced up-front. Conlon and Mortimer (2011) conducted a field experiment by exogenously removing top-selling products from a set of vending machines and tracking subsequent consumer responses. Their results show that most consumers purchase another good when a top-selling product is removed. Moreover, some product removals increase the vendor's profits as consumers substitute toward products with higher margins. Ozcan (2008) ran a survey study in a grocery store where the manager had previously agreed to create stockouts artificially by removing some items entirely from the shelves. Of all the consumers who replied to the survey saying that they had experienced a stockout, 11% said they cancelled or postponed the purchase, 49% decided to switch store (there are two other supermarkets within a 4 minute walking distance from the treated store), and 40% said they bought a substitute item for the one that was not available.[15]

1.3 Model

In this section, I first introduce the consumers' preferences and outline the timing of the interaction between the monopolist and the consumers. Then, I describe the consumers' strategies and illustrate the logic behind the solution concepts. I end this section with a simple example that shows how the monopolist can achieve higher profits by strategically manipulating product availability.

1.3.1 Environment

There is a unit mass of identical consumers whose intrinsic valuation for good i is v_i, $i = 1, 2$. Assume $v_1 \geq v_2 > 0$. The goods are substitutes and each consumer is interested in buying at most one unit of one good. The goods could be two different brands of a consumer durable, such as a household appliance.[16]

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[15]Although product availability is probably more relevant for traditional brick and mortar stores than for online retailers, recent studies show that limited-availability sales and stockouts pertain to online shopping as well; see Breugelmans, Campo and Gijsbrechts, (2006), Jing and Lewis (2011) and Kim and Lennon (2011).

[16]Alternatively, this situation can be thought as one of vertical differentiation in which there are two versions of the same item, with good 2 being the "basic" version and good 1 being the "advanced" version. All consumers agree on the vertical ranking of the two goods.

Consumers have expectations-based reference-dependent preferences as formulated by Kőszegi and Rabin (2006). In this formulation, a consumer's (his) utility function has two components. First, when buying item i at price p_i, a consumer experiences *consumption utility* $v_i - p_i$. Consumption utility can be thought of as the classical notion of outcome-based utility. Second, a consumer also derives utility from the comparison of his actual consumption to a reference point given by his recent expectations (probabilistic beliefs).[17] For a riskless consumption outcome (v_i, p_i) and riskless expectations $(\widetilde{v}_i, \widetilde{p}_i)$, a consumer's total utility is given by

$$U\left[(v_i, p_i) \,|\, (\widetilde{v}_i, \widetilde{p}_i)\right] = v_i - p_i + \mu\left(v_i - \widetilde{v}_i\right) + \mu\left(\widetilde{p}_i - p_i\right) \qquad (1.1)$$

where

$$\mu\left(x\right) = \left\{ \begin{array}{lll} \eta x & \text{if} & x \geq 0 \\ \eta\lambda x & \text{if} & x < 0 \end{array} \right.$$

is *gain-loss utility.*

I assume $\eta > 0$ and $\lambda > 1$. By positing a constant marginal utility from gains and a constant, but larger marginal disutility from losses, this formulation captures prospect theory's (Kahneman and Tversky 1979, Tversky and Kahneman 1991) loss aversion, but without its diminishing sensitivity. The parameter η can be seen as the relative weight a consumer attaches to gain-loss utility, and λ can be seen as the coefficient of loss aversion.

According to (1.1), a consumer assesses gains and losses separately over product's quality and money.[18] For instance, if his reference point is that he will not get the product (and thus pay nothing), then he evaluates getting the product and paying for it as a gain in the item dimension and a loss in the money dimension rather than as a single gain or loss depending on total consumption utility relative to his reference point. This feature of the Kőszegi-Rabin's model is what allows the monopolist to extract more than the consumer's intrinsic valuation for the good.[19] Furthermore, this is consistent with much of the experimental evidence commonly interpreted in terms of loss aversion.[20]

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[17]Recent experimental evidence lends support to Kőszegi and Rabin's (2006, 2007) expectations-based model of reference-dependent preferences and loss aversion; see for instance Abeler, Falk, Goette, and Huffman (2011), Ericson and Fuster (2011), Gill and Prowse (2012), Karle, Kirchsteiger and Peitz (2012), Song (2012) and Sprenger (2011).

[18]The model of Kőszegi and Rabin (2006) assumes that the gain-loss utility function μ is the same across all dimensions. In principle, one could also allow for this function to differ across the item and the money dimension. For example, Novemsky and Kahneman (2005) and Kőszegi and Rabin (2009) argue that reference dependence and loss aversion are weaker in the money than in the item dimension.

[19]The other crucial feature of these preferences, which is that the reference point is determined by the decision maker's forward-looking expectations, is implicit in disappointment-aversion models of Bell (1985), Loomes and Sugden (1986), and Gul (1991). However, because in these models gains and losses are assessed along only one dimension, the consumer's intrinsic utility ($v_i - p_i$, in this paper), they are unable to predict the type of pricing schemes that is the subject of this paper.

[20]This feauture is able to predict the endowment effect observed in many laboratory experiments (see Kahneman, Knetsch, and Thaler 1990, 1991). The common explanation of the endowment effect is that owners feel giving up the object as a painful loss that counts more than money they receive in exchange,

Because in many situations expectations are stochastic, Kőszegi and Rabin (2006) extend the utility function in (1.1) to allow for the reference point to be a pair of probability distribution $F = (F^v, F^p)$ over the two dimensions of consumption utility. In this case a consumer's total utility from the outcome (v_i, p_i) can be written as

$$U\left[(v_i, p_i) \mid (F^v, F^p)\right] = v_i - p_i + \int_{\widetilde{v}_i} \mu\left(v_i - \widetilde{v}_i\right) dF^v\left(\widetilde{v}_i\right) + \int_{\widetilde{p}_i} \mu\left(\widetilde{p}_i - p_i\right) dF^p\left(\widetilde{p}_i\right) \qquad (1.2)$$

In words, when evaluating (v_i, p_i) a consumer compares it to each possible outcome in the reference lottery. For example, if he had been expecting to buy good 1 for \$15, then buying good 2 for \$10 feels like a loss of $v_1 - v_2$ on the quality dimension and a gain of \$5 on the money dimension.[21] Similarly, if a consumer had been expecting to buy good 1 for either \$10 or \$20, then paying \$15 for it feels like a loss of \$5 relative to the possibility of paying \$10, and like a gain of \$5 relative to the possibility of paying \$20. In addition, the weight on the loss (gain) in the overall experience is equal to the probability with which he had been expecting to pay \$10 (\$20).

To complete this theory of consumer behavior with the above belief-dependent preferences, Kőszegi and Rabin (2006) assume that beliefs must be consistent with rationality: a consumer correctly anticipates the implications of his plans, and makes the best plan he knows he will carry through. Notice that any plan of behavior — which in my setting amounts simply to a price-contingent strategy of which item to buy — induces some expectations. If, given these expectations, the consumer is not willing to follow the plan, then he could not have rationally formulated the plan in the first place. Hence, a credible plan must have the property that it is optimal given the expectations it generates. Following the original definitions in Kőszegi and Rabin (2006) and Kőszegi (2010), I call such a credible plan a *personal equilibrium* (PE). If there exist multiple credible plans, a rational consumer chooses the one that maximizes his expected utility from an ex-ante perspective. I call such a favorite credible plan a *preferred personal equilibrium* (PPE).[22]

The seller (she) is a monopolist supplying good 1 and good 2 at a unit cost of $c_1 \geq 0$ and $c_2 \geq 0$, respectively (these could be the wholesale prices). The seller does not experience economies of scale or scope in supplying these goods. For $i = 1, 2$, let q_i denote the amount or degree of availability of good i offered by the monopolist. If $q_i < 1$, then good i is subject to "limited availability" so that only a fraction q_i of the consumers can purchase it. I assume that, in the event of a stockout, rationing is proportional: each consumer has the same ex-ante probability of obtaining the good, which is allocated to consumers on a random first-come, first-serve basis.[23]

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so that they demand a lot of money for the object. But if gains and losses were defined over the value of the entire transaction, owners would not be more sensitive to giving up the object than to receiving money in exchange. Heffetz and List (2011), however, find no evidence that expectations alone play a part in the endowment effect.

[21]Therefore, the two goods are substitutes not only in the usual sense, but also in the sense of being evaluated along the same hedonic dimension.

[22]In the simple environment considered in this paper, a PPE always exists and is generically unique. Kőszegi (2010) discusses conditions for existence and uniqueness of PPE in more general environments.

[23]Gilbert and Klemperer (2000) show that rationing can be a profitable strategy if consumers must make sunk investements to enter the market, and Nocke and Peitz (2007) show that rationing across periods can be profitable in a model of intertemporal monopoly pricing under demand uncertainty.

The interaction between the monopolist and the consumers lasts two periods, 0 and 1. In period 0, the seller announces (and commits to) a price pair $(p_1, p_2) \in \mathbb{R}_+^2$ and a quantity pair $(q_1, q_2) \in [0,1]^2$; after observing the seller's choice of quantities and prices, consumers pick the plan that is consistent and that maximizes their expected utility (PPE). I assume also that consumers cannot commit ex-ante not to go to the store. In period 1, consumers execute their purchasing plans and payments are made. The assumption about the seller announcing both prices in period 0 is not very realistic because while stores frequently advertise their good deals, it is rather uncommon to see a store publicizing its high prices. However, in Appendix B I show that the main results of the paper are unchanged if the seller commits only to the price and availability of the bargain.[24] Finally, I assume that when indifferent between a plan that involves buying and another plan that involves not buying, consumers always break the indifference in favor of the first of these plans.

1.3.2 Consumers' Demand

Let $H \in \Delta\left([0,1]^2 \times \mathbb{R}_+^2\right)$ denote a consumer's expectations, induced by the seller's strategy, about the degree of availability and the prices he might face. For a given seller's choice of prices and degree of availability, a consumer chooses among five possible plans: (i) "never buy," (ii) "buy item 1 if available and don't buy otherwise," (iii) "buy item 2 if available and don't buy otherwise," (iv) "buy item 1 if available and otherwise buy item 2 if available" and (v) "buy item 2 if available and otherwise buy item 1 if available."[25] Let $\sigma \in \{\{\varnothing\}, \{1, \varnothing\}, \{2, \varnothing\}, \{1, 2\}, \{2, 1\}\}$ denote a consumer's plan and let $\Gamma_{H,\sigma}$ denote the distribution over final consumption outcomes induced jointly by H and σ. In a personal equilibrium the behavior generating expectations must be optimal given the expectations:

Definition 1 σ *is a Personal Equilibrium (PE) if*

$$U\left[\sigma | \Gamma_{H,\sigma}\right] \geq U\left[\sigma' | \Gamma_{H,\sigma}\right]$$

for any $\sigma' \neq \sigma$.

Utility maximization in period 0 implies that the consumer chooses the PE plan that maximizes his expected utility:

Definition 2 σ *is a Preferred Personal Equilibrium (PPE) if it is a PE and*

$$EU_{\Gamma_{H,\sigma}}\left[\sigma | \Gamma_{H,\sigma}\right] \geq EU_{\Gamma_{H,\sigma'}}\left[\sigma' | \Gamma_{H,\sigma'}\right]$$

for any σ' *such that* σ' *is a PE.*

In the remainder of this section, I analyze the conditions for when plans (i), (ii) and (iv) constitute a PE or a PPE.[26] This allows me to both illustrate the logic of PE and PPE,

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[24]Since consumers have rational expectations, they would correctly infer the price of the rip-off even if it was not publicly advertised.

[25]Mixing between plans on the consumers' side can easily be ruled out by the fact that the seller would never choose a price-pair inducing a buyer to buy with probability less than 1.

[26]The relevant conditions for plans (iii) and (v) are analogous to the ones for plans (ii) and (iv), respectively; hence, I do not show them here.

as well as to start developing the intuition for my main result on the optimality of limited-availability schemes. Specifically, a central element of the seller's strategy is to make sure that plan (i) is not a PE and I start by analyzing conditions for this.

Conditions for plan (i) to be a PE For never buying to be a PE, the consumer must expect not to buy. In this case his reference point is to consume nothing and pay nothing. Let the price of good 1 be p_1 and suppose the consumer sticks to his plan. Then, his overall utility is

$$U\left[(0,0)\,|\,\{\varnothing\}\right] = 0.$$

What if instead the consumer decides to deviate from his plan and buys item 1? In this case his overall utility is

$$U\left[(v_1, p_1)\,|\,\{\varnothing\}\right] = v_1 - p_1 + \eta v_1 - \eta \lambda p_1,$$

where $v_1 - p_1$ is his intrinsic consumption utility from buying item 1 at price p_1, ηv_1 is the gain he feels from consuming item 1 when he was expecting to consume nothing, and $-\eta \lambda p_1$ captures the loss he feels from paying p_1 when he was expecting to pay nothing. Thus, the consumer will not deviate in this way from the plan to never buy if

$$U\left[(0,0)\,|\,\{\varnothing\}\right] > U\left[(v_1, p_1)\,|\,\{\varnothing\}\right] \Leftrightarrow p_1 > \frac{1+\eta}{1+\eta\lambda}v_1.$$

A similar threshold can be derived for the case in which the consumer considers deviating from his original plan and buy item 2 at price p_2. Therefore, the plan to never buy is a PE if and only if $p_1 > \frac{1+\eta}{1+\eta\lambda}v_1 \equiv p_1^{\min}$ and $p_2 > \frac{1+\eta}{1+\eta\lambda}v_2 = p_2^{\min}$ because otherwise the consumers would not follow through their intended plan of not buying. The expected utility associated with the plan to never buy is

$$EU\left[\{\varnothing\}\,|\,\{\varnothing\}\right] = 0$$

as the expected utility from planning to consume nothing and pay nothing and expecting to follow this plan is of course zero.

Therefore, if either $p_1 \leq p_1^{\min}$ or $p_2 \leq p_2^{\min}$ plan (i) cannot be a PE and consumers must select a plan that involves buying at least one item with positive probability. As I will show in the next section, it turns out that (unsurprisingly) it is optimal for the seller to induce consumers to select plan (iv) and thus to expect to never leave the store empty-handed whenever an item is available; however, (less obviously) it is not optimal for that to be the only PE plan. Hence, the seller would like the consumer to prefer plan (iv) over plan (ii) ex-ante.

Conditions for plan (ii) to be a PE Suppose a buyer enters the store expecting to buy item 1 if available and not to buy otherwise. In this case his reference point on the product dimension is to enjoy v_1 with probability q_1 and to consume nothing with probability $1 - q_1$; similarly, on the price dimension he expects to pay p_1 with probability q_1 and to pay nothing with probability $1 - q_1$. If the consumer follows this plan his realized utility if item 1 is indeed available is

$$U\left[(v_1, p_1)\,|\,\{1, \varnothing\}\right] = v_1 - p_1 + \eta\left(1 - q_1\right)v_1 - \eta\lambda\left(1 - q_1\right)p_1,$$

where $v_1 - p_1$ is his intrinsic consumption utility from buying item 1 at price p_1, $\eta\left(1 - q_1\right)v_1$ is the gain he feels from consuming item 1 when he was expecting to consume nothing

with probability $1 - q_1$, and $-\eta\lambda\left(1 - q_1\right)p_1$ is the loss he feels from paying p_1 when he was expecting to pay nothing with probability $1 - q_1$. Suppose that item 1 is available but the buyer instead deviates and does not buy. In this case his overall utility is

$$U\left[(0,0)\,|\,\{1,\varnothing\}\right] = 0 - \eta\lambda q_1 v_1 + \eta q_1 p_1,$$

where 0 is his intrinsic consumption utility, $-\eta\lambda q_1 v_1$ is the loss he feels from consuming nothing when he was expecting to consume item 1 with probability q_1, and $\eta q_1 p_1$ is the gain from paying nothing instead of p_1 which he was expecting to pay with probability q_1. Thus, the consumer will not deviate in this way from his plan if

$$U\left[(v_1,p_1)\,|\,\{1,\varnothing\}\right] \geq U\left[(0,0)\,|\,\{1,\varnothing\}\right] \Leftrightarrow\ p_1 \leq \frac{1 + \eta\left(1 - q_1\right) + \eta\lambda q_1}{1 + \eta q_1 + \eta\lambda\left(1 - q_1\right)}v_1. \tag{1.3}$$

Next, consider the case in which item 1 is not available. If the buyer follows his plan, his overall utility is $U\left[(0,0)\,|\,\{1,\varnothing\}\right]$. If instead he deviates and buys item 2, for $p_1 \geq p_2$ his overall utility is

$$U\left[(v_2,p_2)\,|\,\{1,\varnothing\}\right] = v_2 - p_2 + \eta\left(1 - q_1\right)v_2 - \eta\lambda q_1\left(v_1 - v_2\right) + \eta q_1\left(p_1 - p_2\right) - \eta\lambda\left(1 - q_1\right)p_2,$$

where $v_2 - p_2$ is the intrinsic consumption utility from buying item 2 at price p_2, $\eta\left(1 - q_1\right)v_2$ is the gain he feels from consuming item 2 compared to the expectation of consuming nothing with probability $\left(1 - q_1\right)$, $-\eta\lambda q_1\left(v_1 - v_2\right)$ is the loss he feels from consuming item 2 instead of item 1 when he was expecting to consume item 1 with probability q_1 (recall that $v_1 \geq v_2$), $\eta q_1\left(p_1 - p_2\right)$ is the gain from paying p_2 instead of p_1 which he was expecting to pay with probability q_1, and $-\eta\lambda\left(1 - q_1\right)p_2$ is the loss from paying p_2 when he was expecting to pay nothing with probability $1 - q_1$. Thus, the consumer will not deviate in this way from his plan if

$$U\left[(0,0)\,|\,\{1,\varnothing\}\right] > U\left[(v_2,p_2)\,|\,\{1,\varnothing\}\right] \Leftrightarrow\ p_2 > \frac{1 + \eta\left(1 - q_1\right) + \eta\lambda q_1}{1 + \eta q_1 + \eta\lambda\left(1 - q_1\right)}v_2. \tag{1.4}$$

Notice that conditions (1.3) and (1.4) together imply that $U\left[(v_1,p_1)\,|\,\{1,\varnothing\}\right] > U\left[(v_2,p_2)\,|\,\{1,\varnothing\}\right]$, so that there is no need to check that a consumer does not want to deviate and buy item 2 when item 1 is available. Therefore, for $p_1 \geq p_2$, $\{1,\varnothing\}$ is a PE if and only if $p_2 > \frac{1 + \eta(1 - q) + \eta\lambda q}{1 + \eta q + \eta\lambda(1 - q)}v_2$ and $p_1 \leq \frac{1 + \eta(1 - q) + \eta\lambda q}{1 + \eta q + \eta\lambda(1 - q)}v_1$. Similarly, for $p_1 < p_2$, $\{1,\varnothing\}$ is a PE if and only if $p_1 < \frac{1 + \eta(1 - q_1) + \eta\lambda q_1}{1 + \eta q_1 + \eta\lambda(1 - q_1)}v_1$ and $p_2 > \frac{v_2[1 + \eta(1 - q_1) + \eta\lambda q_1] + q_1\eta(\lambda - 1)p_1}{1 + \eta\lambda}$. The expected utility associated with this plan is

$$EU\left[\{1,\varnothing\}\,|\,\{1,\varnothing\}\right] = q_1\left(v_1 - p_1\right) - q_1\left(1 - q_1\right)\eta\left(\lambda - 1\right)\left(v_1 + p_1\right). \tag{1.5}$$

The first term in (1.5), $q_1\left(v_1 - p_1\right)$, is standard expected consumption utility. The second term, $-q_1\left(1 - q_1\right)\eta\left(\lambda - 1\right)\left(v_1 + p_1\right)$, is expected gain-loss utility and it is derived as follows. On the product dimension, the consumer compares the outcome in which with probability q_1 he consumes item 1 and enjoys v_1 with the outcome in which with probability $1 - q_1$ he does not consume and gets 0. Similarly, on the price dimension he compares paying price p_1 with probability q_1 with paying 0 with probability $1 - q_1$. Notice that the expected gain-loss

utility is always negative as, since $\lambda > 1$, losses are felt more heavily than equal-size gains. Also, notice that uncertainty in the product and uncertainty in money are "added up" so that the expected gain-loss term is proportional to $v_1 + p_1$.

Conditions for Plan (iv) to be a PE For the plan to buy item 1 if available and otherwise buy item 2, a consumer's reference point in the product dimension is to consume item 1 and enjoy v_1 with probability q_1, to consume item 2 and enjoy v_2 with probability q_2 and to consume nothing with probability $1 - q_1 - q_2$; similarly, in the price dimension, a consumer expects to pay p_1 with probability q_1, p_2 with probability q_2 and to pay nothing with probability $1 - q_1 - q_2$. Then, if he follows his plan and buys item 1, for $p_1 \geq p_2$, his realized utility is

$$U\left[(v_1, p_1) \,|\, \{1, 2\}\right] = v_1 - p_1 + \eta q_2 (v_1 - v_2) + \eta (1 - q_1 - q_2) v_1 - \eta \lambda q_2 (p_1 - p_2) - \eta \lambda (1 - q_1 - q_2) p_1.$$

If instead he deviates and buys item 2, his utility is

$$U\left[(v_2, p_2) \,|\, \{1, 2\}\right] = v_2 - p_2 - \eta \lambda q_1 (v_1 - v_2) + \eta (1 - q_1 - q_2) v_2 + \eta q_1 (p_1 - p_2) - \eta \lambda (1 - q_1 - q_2) p_2.$$

Thus, the consumer will not deviate in this way from his plan if

$$U\left[(v_1, p_1) \,|\, \{1, 2\}\right] \geq U\left[(v_2, p_2) \,|\, \{1, 2\}\right] \Leftrightarrow p_1 \leq p_2 + \frac{1 + \eta (1 - q_1) + \eta \lambda q_1}{1 + \eta q_1 + \eta \lambda (1 - q_1)} (v_1 - v_2). \quad (1.6)$$

Suppose now that once a consumer arrives at the store, item 2 is everything that is left. If he follows his plan and buys item 2 his overall utility is $U\left[(v_2, p_2) \,|\, \{1, 2\}\right]$. If instead he deviates and does not buy his utility is

$$U\left[(0, 0) \,|\, \{1, 2\}\right] = 0 - \eta \lambda q_1 v_1 - \eta \lambda q_2 v_2 + \eta q_1 p_1 + \eta q_2 p_2.$$

Thus, the consumer will not deviate in this way from his plan if

$$U\left[(v_2, p_2) \,|\, \{1, 2\}\right] \geq U\left[(0, 0) \,|\, \{1, 2\}\right] \Leftrightarrow p_2 \leq \frac{1 + \eta \lambda (q_1 + q_2) + \eta (1 - q_1 - q_2)}{1 + \eta (q_1 + q_2) + \eta \lambda (1 - q_1 - q_2)} v_2. \quad (1.7)$$

Notice that conditions (1.6) and (1.7) together imply that $U\left[(v_1, p_1) \,|\, \{1, 2\}\right] > U\left[(0, 0) \,|\, \{1, 2\}\right]$. Hence, for $p_1 \geq p_2$, $\{1, 2\}$ is a PE if and only if $p_1 \leq p_2 + \frac{1 + \eta (1 - q_1) + \eta \lambda q_1}{1 + \eta q_1 + \eta \lambda (1 - q_1)} (v_1 - v_2)$ and $p_2 \leq \frac{1 + \eta \lambda (q_1 + q_2) + \eta (1 - q_1 - q_2)}{1 + \eta (q_1 + q_2) + \eta \lambda (1 - q_1 - q_2)} v_2$. Similarly, for $p_1 < p_2$, $\{1, 2\}$ is a PE if and only if $p_2 \leq \frac{1 + \eta \lambda (q_1 + q_2) + \eta (1 - q_1 - q_2)}{1 + \eta q_2 + \eta \lambda (1 - q_2)} v_2 + \frac{\eta (\lambda - 1) q_1}{1 + \eta q_2 + \eta \lambda (1 - q_2)} p_1$. The expected utility associated with this plan is

$$
\begin{aligned}
EU\left[\{1, 2\} \,|\, \{1, 2\}\right] \;=\; & q_1 (v_1 - p_1) + q_2 (v_2 - p_2) && (1.8) \\
& - q_1 (1 - q_1 - q_2) \eta (\lambda - 1) (v_1 + p_1) \\
& - q_2 (1 - q_1 - q_2) \eta (\lambda - 1) (v_2 + p_2) \\
& - q_1 q_2 \eta (\lambda - 1) (v_1 - v_2) \\
& - q_1 q_2 \eta (\lambda - 1) (\max \{p_1, p_2\} - \min \{p_1, p_2\}).
\end{aligned}
$$

The first and second terms in (1.8), $q_1 (v_1 - p_1) + q_2 (v_2 - p_2)$, are the standard expected consumption utility terms. The third term, $q_1 (1 - q_1 - q_2) \eta (\lambda - 1) (v_1 + p_1)$, is always negative and captures expected gain-loss utility in both the product and the money dimensions from comparing the outcome in which the consumer buys item 1 and pays p_1 with

the outcome of returning home empty-handed. Similarly, the fourth term captures expected gain-loss utility in both dimensions from comparing the outcome of buying item 2 at price p_2 with the outcome of returning home empty-handed. The fifth term, $-q_1 q_2 \eta (\lambda - 1) (v_1 - v_2)$, captures expected gain-loss utility in the consumption dimension when comparing the two outcomes in which he buys something: with probability q_1 the consumer expects to buy good 1 and with probability q_2 he expects to buy good 2. Notice again that this term is negative, but it is proportional to $(v_1 - v_2)$. This is because with this plan, the consumer is "guaranteeing" himself to enjoy at least the item he values v_2 and the expected gain-loss utility is therefore related only to by how much more he would prefer to consume the other good (or, the degree of substitutability between the two goods). The sixth term, $-q_1 q_2 \eta (\lambda - 1) (\max \{p_1, p_2\} - \min \{p_1, p_2\})$, captures expected gain-loss utility in the money dimension when comparing the two outcomes in which he buys and can be explained in a similar fashion.

Conditions for Plan (iv) to be the PPE Suppose that $p_1 \geq p_2$. When both plan (ii) and (iv) are Personal Equilibria, a consumer will select plan (iv) rather than plan (ii) if and only if

$$EU \left[\{1, 2\} \,|\, \{1, 2\} \right] \geq EU \left[\{1, \varnothing\} \,|\, \{1, \varnothing\} \right] \Leftrightarrow v_2 - p_2 \geq \eta (\lambda - 1) (1 - 2q_1 - q_2) (v_2 + p_2). \tag{1.9}$$

Similarly, for $p_1 < p_2$ a consumer will select plan (iv) rather than plan (ii) if and only if

$$v_2 - p_2 \geq (1 - q_2) \eta (\lambda - 1) (v_2 + p_2) - 2q_1 \eta (\lambda - 1) (v_2 + p_1). \tag{1.10}$$

Notice, crucially, that conditions (1.9) and (1.10) might hold even if $p_2 > v_2$. Therefore, a consumer might prefer, from an ex-ante perspective, to plan to always buy even if $p_2 > v_2$. This happens because, by planning to always buy, the consumer is essentially insuring himself against extreme fluctuations in his consumption outcome.[27]

1.3.3 An Illustrative Example

Consider a monopolist supplying two goods, 1 and 2, to a unit mass of consumers who have expectations-based reference-dependent preferences with $\eta = 1$ and $\lambda = 3$. Let $v_1 = v$, $v_2 = \frac{2}{3}v$, $c_1 = \frac{3}{5}v$ and $c_2 = \frac{v}{3}$. If she had to provide full availability, the seller would supply only item 1 and price it at v, obtaining a profit of $\frac{2}{5}v$.

Consider instead the following limited-availability scheme: $q_1 = \frac{1}{4}$, $q_2 = \frac{3}{4}$, $p_1 = \frac{v}{2}$ and $p_2 = v$. Since $p_1 < p_1^{\min}$, it is not a PE for consumers to never buy: the price of item 1 is so low that if consumers had planned not to buy it, then if item 1 is indeed available, they would like to surprise themselves and buy it, and since the price is very low, the gain on the item dimension more than outweighs the loss on the money dimension.

The plan to buy item 1 if available and nothing otherwise is a PE because $p_1 < \frac{5}{7}v_1$ and $p_2 > \frac{5v_2 + p_1}{8}$. Intuitively, if consumers enter the store with the expectation of consuming item 1 with positive probability and item 1 is available, they are willing to follow their plan

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[27]More generally, as shown in Kőszegi and Rabin (2007), a decisionmaker with expectations-based loss aversion dislikes uncertainty in consumption utility because he dislikes the possibility of a resulting loss more than he likes the possibility of a resulting gain.

since the price of item 1 is relatively low compared to its intrinsic value; however, they are not willing to buy item 2 if they were not expecting to do so, since the price of item 2 is relatively high compared to its intrinsic value.

Similarly, the plan to buy item 1 if available and item 2 otherwise is a PE because $p_2 < \frac{8v_2+p_1}{5}$. The intuition is that, by planning to always buy something, consumers expect to enjoy at least v_2 for sure; because of this attachment effect, therefore, they are willing to buy item 2 if they were expecting to do so even if its price is relatively high. Furthermore, this plan is the PPE since

$$
\begin{aligned}
EU\left[\{1,2\}\,|\,\{1,2\}\right] &= \frac{1}{4}\left(v-\frac{v}{2}\right) + \frac{3}{4}\left(\frac{2}{3}v - v\right) - \frac{9}{16}\left(\frac{v}{3} + \frac{v}{2}\right)\\
&> \frac{1}{4}\left(v - \frac{v}{2}\right) - \frac{9}{16}\left(v + \frac{v}{2}\right) = EU\left[\{1,\varnothing\}\,|\,\{1,\varnothing\}\right].
\end{aligned}
$$

The reason why, from an ex-ante point of view, consumers prefer the plan to always buy is that this plan reduces the magnitude of the fluctuations of their consumption outcomes and, therefore, makes them subject to a smaller expected gain-loss disutility. Finally, notice that with this limited-availability scheme the seller's profit equals $\frac{19}{40}v$, which is higher than the profit under full availability.

This example illustrates already many of the key insights of the general model. First, with a limited-availability scheme the seller is able to obtain a higher profit than what she can obtain with perfect availability. The prices of the bargain and the rip-off are chosen by the seller in a way such that (i) not buying is not a PE for the consumers and (ii) planning to always buy is the consumers' PPE. Furthermore, the superior item is chosen as the bargain and it is priced below its marginal cost. The purpose of the next section is to formalize and generalize these insights.

1.4 Optimal Availability and Pricing

In this section I derive the seller's profit-maximizing schemes. I divide the analysis in two cases. In the first sub-section, I consider the case of close substitutes. I define two goods to be close substitutes if the two following conditions hold both:

(i) $v_2 > \left(\frac{1+\eta\lambda}{2+\eta\lambda+\eta}\right)v_1$;

(ii) $v_2 > \frac{\sqrt{v_1(1+\eta)[2(c_1-c_2)(\eta^2\lambda^2-\eta^2\lambda+2\eta\lambda-\eta+1)+v_1(1+\eta)]}-\eta(\lambda-1)(1+\eta)v_1}{\eta^2\lambda^2-\eta^2\lambda+2\eta\lambda-\eta+1}$.

The first condition ensures that the price of the superior product, when this is the rip-off, is higher under limited availability than under perfect availability. The second condition ensures that the price of the rip-off is higher than the price of the bargain, even when the inferior product is used as the rip-off. I show that when the products are close substitutes, the seller's profit-maximizing limited-availability scheme entails prices being farther away than valuations.

In the second sub-section, I consider the case of distant substitutes — when either condition (i) or (ii) fails — and I show that in this case the seller always uses the superior

product as a bargain. Moreover, when the products are distant substitutes, the seller's profit-maximizing limited-availability scheme encompasses prices being less dispersed than valuations.

1.4.1 Close Substitutes

For given prices (p_1, p_2) and "quantities" (q_1, q_2), the monopolist's profit is

$$\pi(p_1, p_2, q; c_1, c_2) = q_1(p_1 - c_1) + q_2(p_2 - c_2).$$

If consumers were not loss-averse, the profit-maximizing strategy for the seller would be to just set $p_i = v_i$, for $i = 1, 2$, and $q_1 = 1$ (resp. $q_2 = 1$) if $v_1 - c_1 \geq v_2 - c_2$ (resp. if $v_1 - c_1 < v_2 - c_2$). Consumers would get zero surplus and the seller's profit would be exactly $v_1 - c_1$ (resp. $v_2 - c_2$).

The first lemma of this section shows that with loss-averse consumers, if restricted to supply one good with certainty, the above mentioned strategy remains the monopolist's profit-maximizing one.[28]

Lemma 1 *With perfect availability the monopolist cannot extract more than v_1 from the consumers.*

In general, however, this strategy need not be the profit-maximizing one when consumers are loss-averse as the seller instead can achieve a higher profit by reducing the availability of some goods and thus inducing uncertainty into the buyers' plans.

The next lemma states that even though she might reduce the degree of availability of some goods, it is in the seller's best interest that all consumers get to buy a good for sure, and the uncertainty is only about which good they will buy.[29] The intuition for this result relies on the seller's intent to mitigate the "comparison effect" and simultaneously magnify the "attachment effect" for the consumers (Kőszegi and Rabin, 2006). An increase in the likelihood of buying increases a consumer's sense of loss if he does not buy, creating an "attachment effect" that increases his willingness to pay. On the other hand, for a fixed probability of buying, a decrease in the price a consumer expects to pay makes paying a higher price feel like more of a loss, creating a "comparison effect" that lowers his willingness to pay the high price.

Lemma 2 *The market is fully covered: $q_1 + q_2 = 1$.*

With $q_1 + q_2 = 1$, if a consumer plans to always buy, he is guaranteed to get at least the less preferred item (v_2) and thus he is not exposed anymore to the possibility of returning home empty-handed; this increases the consumer's willingness to pay through the attachment effect. At the same time, because the possibility of buying nothing has disappeared, the consumer expects to always spend some money; this also increases the consumer's willingness to pay through reducing the comparison effect.

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[28] All proofs are relegated to Appendix A.

[29] A similar result is provided by Pavlov (2011) and Balestrieri and Leao (2011) for the case of a monopolist selling substitutes to risk-neutral consumers.

Given Lemma 2, from this point forward I am going to use q and $1 - q$ to denote the quantities of good 1 and 2, respectively. The lemma below shows that with limited availability, the monopolist must offer at least one good at a discounted price.

Lemma 3 *If $q \in (0,1)$ then either $p_1 < v_1$ or $p_2 < v_2$.*

With limited availability, a consumer faces uncertainty about his consumption outcome before arriving at the store and because losses are felt more heavily than gains, if he expects to buy with positive probability, his expected gain-loss utility is negative. Therefore, for a consumer to be willing to plan to buy, the seller must guarantee him a strictly positive intrinsic surplus on at least one item, otherwise he would be better off by planning to not buy and this plan would be consistent for $p_1 \geq v_1$ and $p_2 \geq v_2$.

Having established that the monopolist can sell a strictly positive quantity of both goods only if one of them is priced at a discount, the next question is how big this discount must be. The next lemma states that the seller must offer a bargain on this good; in other words, its price must be so low that it is not credible for consumers to plan on not buying.

Lemma 4 *If $q \in (0,1)$ the seller chooses prices such that the plan to never buy is not a PE.*

Since, for a given product i, the highest price the seller can charge to make not buying a non credible plan is $p_i^{\min} \equiv \frac{1+\eta}{1+\eta\lambda} v_i$, then it must be that if the seller is producing both goods in strictly positive quantity, one of them is priced at this "forcing price."[30]

What about the price of the other item? If she produces a strictly positive quantity of both goods, the seller wants the buyers to plan to always buy. However, as the lemma below shows, it is not optimal for the seller to choose the other price such that always buying is the unique consistent plan. Instead, the optimal price pair is such that consumers are indifferent, ex-ante, between the plan of always buying and the plan of buying only the bargain item.

Lemma 5 *For $q \in (0,1)$, if the seller uses item 2 as the bargain (i.e., $p_2 = p_2^{\min}$), then the optimal price for item 1 is*

$$p_1^* = v_1 + \frac{2(1-q)\eta(\lambda-1)[v_2(2+\eta+\eta\lambda) - v_1(1+\eta\lambda)]}{(1+\eta\lambda)[1+\eta(\lambda-1)(1-q)]} > v_1.$$

If instead she uses item 1 as the bargain (i.e., $p_1 = p_1^{\min}$), then the optimal price for item 2 is

$$p_2^* = v_2 + \frac{2qv_1\eta(\lambda-1)(1+\eta)}{(1+\eta\lambda)[1+\eta(\lambda-1)q]} > v_2.$$

This last lemma implies that consumers are willing to pay a premium, in the form of a higher price on the item that is not on sale (and therefore in the form of a higher expected expenditure), to avoid ex-ante the disappointment of leaving the store empty-handed. Furthermore, p_i^* is the highest price such that consumers (weakly) prefer, from an ex-ante point of view, the plan of buying item j if available and item i otherwise to the plan of buying

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[30]This result is akin to the single-product one in Heidhues and Kőszegi (forthcoming) from whom I borrowed the term "forcing price."

item j if available and nothing otherwise, when item j is sold at its "forcing price." To gain intuition on why a consumer might find it optimal to plan to buy at $p_i^* > v_i$, suppose the seller uses item 1 as the bargain, by pricing it at $p_1^{\min}$. If a consumer plans to buy only item 1 and nothing otherwise his expected utility is equal to

$$q\left(v_1 - p_1^{\min}\right) - \eta\left(\lambda - 1\right)q\left(1 - q\right)\left(v_1 + p_1^{\min}\right).$$

While the term relating to consumption utility in the above expression is strictly positive, the expected gain-loss utility term is strictly negative. If instead the consumer plans to always buy, then his expected utility is

$$q\left(v_1 - p_1^{\min}\right) + \left(1 - q\right)\left(v_2 - p_2^*\right) - \eta\left(\lambda - 1\right)q\left(1 - q\right)\left(v_1 - v_2 + p_2^* - p_1^{\min}\right).$$

In the above expression the expected gain-loss utility is still negative, but now its magnitude is $\left(v_1 - v_2 + p_2^* - p_1^{\min}\right)$.[31] Therefore, as long as $p_2^* - v_2 < 2p_1^{\min}$, by planning to always buy a consumer is subject to a smaller expected gain-loss disutility and this allows the seller to raise p_2^* above v_2. Furthermore, the closer v_2 is to v_1, the more freedom the seller has in raising p_2^*, implying that dispersion in prices and dispersion in valuations are inversely related.

Both rip-off prices p_1^* and p_2^* are increasing in the degree of availability of their respective bargain item — $1 - q$ and q — implying that the attachment effect (see Kőszegi and Rabin, 2006 and Heidhues and Kőszegi, forthcoming) carries over to the case of multiple goods evaluated along the same hedonic dimension.

Similarly, notice that $\frac{\partial p_i^*}{\partial v_j} > 0$, for $i, j = 1, 2$, $i \neq j$. Thus, expectations-based loss aversion produces a kind of positive demand spillover across products, despite these being substitutes. Indeed, both p_1^* and p_2^* are written as the sum of two components: the direct effect, which simply equals the consumers' intrinsic valuation for the product, and the spillover effect due to loss aversion. Notice that while the spillover effect for p_2^* depends only on v_1 and is increasing in it, the spillover effect for p_1^* depends both on v_1 and v_2 and is increasing in the former and decreasing in the latter. Intuitively, increasing consumers' intrinsic value for item 1 makes item 2 is not such a good substitute for it. This, however, does not affect p_2^* because when item 1 is the bargain, a higher v_1 increases consumers' expected gain-loss disutility when planning to buy only the bargain and when planning to always buy by the same amount.

Having derived the optimal prices for the bargain and the rip-off, the next step for the seller is to choose the optimal degree of availability for each item. For example, consider the case in which the seller uses item 2 as the bargain. Then, she is going to choose the q that solves the following maximization problem:

$$\max_{q} q\left(p_1^* - c_1\right) + \left(1 - q\right)\left(p_2^{\min} - c_2\right).$$

The first-order condition yields

$$p_1^* - c_1 - \left(p_2^{\min} - c_2\right) + q\frac{\partial p_1^*}{\partial q} = 0. \tag{1.11}$$

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[31] The second condition about close substitutability ensures that $p_2^* - p_1^{\min} > 0$.

Notice that $q\frac{\partial p_1^*}{\partial q} < 0$ because of the attachment effect: the higher the degree of availability of the bargain, the more optimistic the consumers' beliefs about making a deal. This in turn, allows the seller to charge a higher mark-up on the rip-off. On the other hand, a greater availability of the bargain necessarily means fewer sales of the rip-off and hence reduces the seller's profits, as captured by $p_1^* - c_1 - \left(p_2^{\min} - c_2\right) > 0$. At the optimal degree of availability these two effects offset each other.

Lemma 6 *If the seller uses item 2 as the bargain, the optimal degree of availability of item 1 is $\bar{q} = \arg\max\limits_{q} \pi\left(p_1^*, p_2^{\min}, q; c_1, c_2\right)$, with $\bar{q} \in \left(\frac{1}{2}, 1\right)$. If instead she uses item 1 as the bargain, the optimal degree of availability of item 1 is $\underline{q} = \arg\max\limits_{q} \pi\left(p_1^{\min}, p_2^*, q; c_1, c_2\right)$, and $\underline{q} \in \left(0, \frac{1}{2}\right)$ if $v_2 - c_2 \geq v_1 - c_1$ or if $v_2 - c_2 < v_1 - c_1$ and $\eta \leq 1$. Furthermore, $\bar{q} > 1 - \underline{q}$.*

When the bargain is the product with the lower social surplus, the seller always supplies more units of the rip-off item than the bargain. So, even if a high degree of availability for the bargain allows her, via the attachment effect, to increase the price of the rip-off, the effect is not strong enough for the seller to be willing to sell the bargain more often than the rip-off. This can be seen most easily when the two items are perfect substitutes ($v_1 = v_2 = v$) and have zero costs. In this case, (1.11) reduces to:

$$1 + \frac{2\eta\left(\lambda - 1\right)\left(1 - q\right)}{1 + \eta\left(\lambda - 1\right)\left(1 - q\right)}\frac{1 + \eta}{1 + \eta\lambda} = \frac{1 + \eta}{1 + \eta\lambda} + \frac{2\eta\left(\lambda - 1\right)q}{\left[1 + \eta\left(\lambda - 1\right)\left(1 - q\right)\right]^2}\frac{1 + \eta}{1 + \eta\lambda}. \tag{1.12}$$

The left-hand-side of (1.12) captures the seller's marginal gain from an increase in q; similarly, the right-hand-side captures the seller's marginal loss. The following is necessary for (1.12) to hold:

$$\frac{2\eta\left(\lambda - 1\right)q}{\left[1 + \eta\left(\lambda - 1\right)\left(1 - q\right)\right]^2} > \frac{2\eta\left(\lambda - 1\right)\left(1 - q\right)}{1 + \eta\left(\lambda - 1\right)\left(1 - q\right)}$$

$$\Leftrightarrow \quad \frac{q}{1 - q} > 1 + \eta\left(\lambda - 1\right)\left(1 - q\right).$$

The above inequality can be satisfied only for $q > \frac{1}{2}$. Then, for $v_i - c_i > v_j - c_j$, $i, j \in \{1, 2\}$, $i \neq j$ if item j is the bargain it follows

$$p_i^* - c_i > v_i - c_i > v_j - c_j > p_j^{\min} - c_j,$$

so that the seller's margins on the two items are even further apart if the items are not perfect substitutes and have different costs. Hence, the seller wants to reduce the availability of the bargain below $\frac{1}{2}$ even more.

On the other hand, suppose that $v_2 - c_2 < v_1 - c_1$ but the seller uses item 1 as the bargain (as shown in the lemma below, this can be a profit-maximizing strategy for the seller). In this case we have that $p_2^* > v_2$ and $p_1^{\min} < v_1$, yet the difference $p_2^* - c_2 - \left(p_1^{\min} - c_1\right)$ could be relatively small. Then, as λ tends to 1 $p_1^{\min}$ approaches v_1 and for $\eta > 1$ the attachment effect could be strong enough for the seller to choose $\underline{q} > \frac{1}{2}$.

Furthermore, as $\bar{q} > 1 - \underline{q}$, the seller chooses a higher degree of availability for the bargain when this is the superior item. Intuitively, when the seller uses the superior item as

the bargain, some consumers will end up paying a very high price for the item they like the least; in order to convince them to do so, the seller must compensate the consumers with a higher ex-ante chance of making a deal.

The above analysis does not specify which item the seller would prefer to use as the bargain. To determine whether the seller would prefer to use item 1 or 2, we must compare her profits in the two cases. Unfortunately, these are complex non-linear functions of v_1 and v_2, which are difficult to sign even in the simplest cases and are intractable in general. To overcome this difficulty, I employ comparative statics techniques based on the envelope theorem; but the downside of this approach is that some of the results in the following lemma apply only for small changes in the relevant parameters.[32]

Lemma 7 *If the two goods are perfect substitutes (i.e., $v_1 = v_2$) the seller prefers to use as the bargain the one with the higher marginal cost and is indifferent if the two goods have the same marginal cost (i.e., $c_1 = c_2$). For $v_1 > v_2$, the seller uses item 2 as the bargain only if $v_1 - c_1 + c_2 > v_2 > \frac{2(1+\eta\lambda)(c_1-c_2)}{1+2\eta}$ and $v_1 \geq \widetilde{v}_1$, where $\widetilde{v}_1$ is implicitly defined by:*

$$\left[\frac{1 - \eta\,(\lambda - 1)\,(1 - \overline{q})}{1 + \eta\,(\lambda - 1)\,(1 - \overline{q})}\overline{q} - \frac{1+\eta}{1+\eta\lambda}\underline{q} - (1 - \underline{q})\,\frac{1+\eta}{1+\eta\lambda}\frac{2\underline{q}\eta\,(\lambda - 1)}{1 + \underline{q}\eta\,(\lambda - 1)} \right] (v_1 - v_2) \geq (\overline{q} - \underline{q})\,(c_1 - c_2)\,.$$

Otherwise, she prefers using item 1 as the bargain.

So, if $v_1 - c_1 \leq v_2 - c_2$, the seller always uses item 1 as the bargain. Arguably more interesting, however, is the fact that the seller might prefer to use item 1 as the bargain even when this is the item with the greater social surplus (i.e., $v_1 - c_1 > v_2 - c_2$). The intuition for this result can be seen in two steps. First, as $v_1 > v_2$ it follows that $p_1^{\min} > p_2^{\min}$ and this in turn implies that $p_2^*\left(\underline{q}\right) > p_1^*\left(\overline{q}\right)$ through both the attachment effect and the comparison effect. So both prices are higher when the seller uses item 1 as the bargain. However, from this we cannot yet conclude that the seller's revenue is higher when she supplies item 1 at a discount because the weights, $\overline{q}$ and $\underline{q}$, are different. Indeed, we know from lemma 6 that the seller supplies more units of the rip-off when this is the superior good. Nevertheless, for $v_1 - v_2$ small enough the difference in the weights is a second-order one and the seller prefers to use item 1 as the bargain even if $c_1 = c_2$. Second, if $c_2 < c_1$, by using the superior item as the bargain, the seller is able to reduce her average marginal cost by more, compared to the case in which she uses item 2 as the bargain.

Figure 1 shows how the profitability of different schemes changes with v_1 for the case in which $v_1 - c_1 > v_2 - c_2$ and the difference in marginal costs is small. The black line represents the seller's profits when supplying only item 1 at price $p_1 = v_1$, whereas the green and red curves depict the seller's profits with limited availability when either item 1 or 2 is used as the bargain item, respectively (notice that the seller's overall profit is given by the upper envelope of these three curves). Concerning the choice of the bargain item, in the graph we can distinguish three different regions, delimited by the two dashed vertical lines. For relatively low values of v_1, the profit-maximizing strategy for the seller is to use a limited-availability deal and use item 1 as the bargain. As v_1 increases, the difference

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[32]The results apply only for small changes because comparative statics techniques linearize profits around the maximum. Klemperer and Padilla (1997) use the same approach in a similar context.

between the green and the red curve becomes smaller and eventually the two cross. Then, for intermediate values of v_1, the seller maximizes profits by using item 2 as the bargain item. Finally, for high values of v_1 the seller prefers to supply just item 1 and price it at its intrinsic value.

Figure 1.1: Profits as a function of v_1, for $\eta = 1$, $\lambda = 3$, $v_2 = 80$, $c_1 = 12$, $c_2 = 10$.

When the difference in marginal costs is larger, however, the seller prefers to use item 1 as the bargain item for low as well as intermediate values of v_1. This is shown in Figure 2 where the green curve is always above the red one. In this case item 1 is more valuable to the consumer and it has a larger social surplus; yet it is never used as a rip-off item.
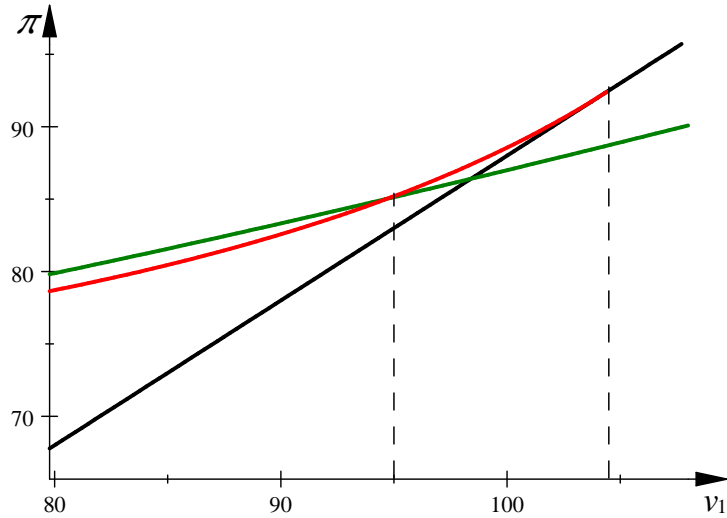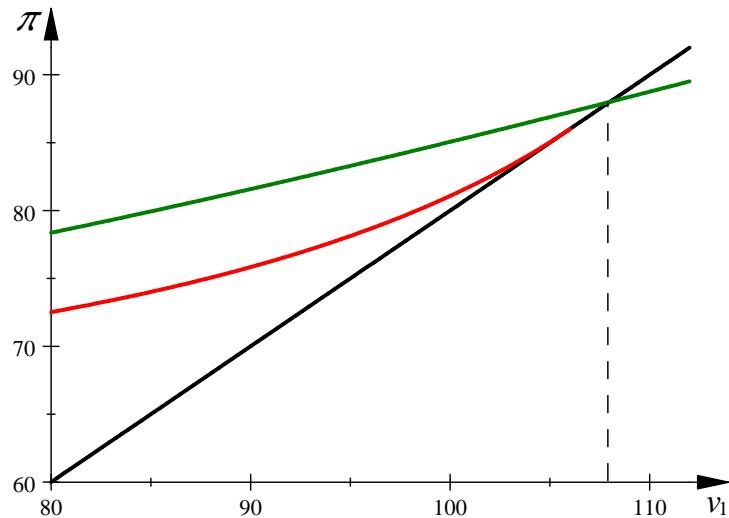

Figure 1.2: Profits as a function of v_1, for $\eta = 1$, $\lambda = 3$, $v_2 = 80$, $c_1 = 20$, $c_2 = 10$.

The following proposition, which constitutes the main result of this section, identifies the necessary and sufficient conditions for a limited-availability scheme to be profit-maximizing.

Proposition 1 *Fix any $\eta > 0$ and $\lambda > 1$. The seller's profit-maximizing strategy is as follows:*

(i) *for $v_1 \leq v_2 - c_2 + c_1$ there exists a $\alpha \left(v_2, c_1, c_2, \eta, \lambda\right)$ such that if $v_1 \geq \alpha$ the seller uses item 1 as the bargain and item 2 as the rip-off, and if $v_1 < \alpha$ she supplies only item 2;*

(ii) *for $\widetilde{v}_1 > v_1 > v_2 - c_2 + c_1$ there exists a $\beta \left(v_2, c_1, c_2, \eta, \lambda\right)$ such that if $v_1 \leq \beta$ the seller uses item 1 as the bargain and item 2 as the rip-off, and if $v_1 > \beta$ she supplies only item 1;*

(iii) *for $v_1 > \widetilde{v}_1$ there exists a $\gamma \left(v_1, c_1, c_2, \eta, \lambda\right)$ such that if $v_2 \geq \gamma$ the seller uses item 2 as the bargain and item 1 as the rip-off, and if $v_2 < \gamma$ she supplies only item 1.*

Furthermore, $\pi \left(p_1, p_2, q; c_1, c_2\right) \geq \max \left\{v_1 - c_1, v_2 - c_2\right\}$ and the inequality is strict if both items are supplied.

The exact expressions for α, β and γ are derived in the proof of the proposition in Appendix B. What they imply is that, if the two goods are close substitutes, the seller's profit-maximizing strategy consists of luring the consumers with a tempting discount on one good which is available only in limited supply ($p_i^{\min} < v_i$) and cashing in with a high price on the other ($p_j^* > v_j$). Moreover, by offering both products and inducing uncertainty into the buyers' plans through this type of limited-availability deals, the seller is able to achieve a profit higher than $\max \left\{v_1 - c_1, v_2 - c_2\right\}$.

The limited-availability scheme described in Proposition 10 cannot be rationalized by introducing a shopping (or search) cost into a model where consumers have traditionally assumed reference-free preferences. The reason is that, although shopping (or search) costs that are sunk once the consumers reach the store induce an ex-post boost in consumers' willingness to pay, this boost (i) is independent of a good's intrinsic consumption value, (ii) is always smaller than the intrinsic value itself — otherwise consumers would not go to the store, even if the price were to be zero — and, crucially, (iii) because randomization does not affect a risk-neutral consumer's reservation utility, any profit the seller can achieve with randomization could be also achieved with a single price.[33] Therefore, in this case the seller would simply supply the product with the larger social surplus and price it at its intrinsic value minus the shopping (or search) cost.

It is possible for the seller to find this limit-availability strategy profit-maximizing even if the bargain is a loss leader, as the following example shows.

Example 3 *(Loss Leader)* *Let $\eta = 1$, $\lambda = 3$, $v_1 = 60$, $v_2 = 40$, $c_1 = 35$ and $c_2 = 22$. For these parameters' values the seller profit-maximizing strategy is given by: $\underline{q} = \frac{3\sqrt{5}}{\sqrt{83}} - \frac{1}{2}$, $p_1^{\min} = 30$ and $p_2^* = \frac{200\underline{q}+40}{2\underline{q}+1} = 59.26$. Item 1 is used as a loss leader and the seller's profit is 27.27.*

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[33]If consumers are risk-averse in the sense of Expected Utility Theory, then randomization in prices yields always lower profits than committing to a single price since consumers must be compensated for the ex-ante risk they face about the price.

By combining the results in Proposition 10 with the condition for the bargain item to be a loss leader (i.e., $p_i^{\min} < c_i$) we immediately obtain the following result.

Corollary 1 *Item 1 is a loss leader if either $\frac{1+\eta\lambda}{1+\eta}c_1 > v_1 \geq \alpha$ or $v_1 < \min\left\{\left(\frac{1+\eta\lambda}{1+\eta}\right)c_1, \ \beta\right\}$. Similarly, item 2 is a loss leader if $\frac{1+\eta\lambda}{1+\eta}c_2 > v_2 \geq \gamma$.*

As shown by Ambrus and Weinstein (2008), classical models of consumers behavior can rationalize the use of loss leaders when the goods are complements but not when they are substitutes. The reason is that with classical preferences a store might benefit from using a loss-leading strategy only if consumers buy other items together with the loss leader. In my model, instead, the presence of loss leaders still attracts consumers into the store but, because the loss-leading product is in shortage, in equilibrium some consumers end up buying a different, more expensive product.

Despite the consumers being homogeneous in terms of tastes for both items, the bargains and rip-offs strategy described above endogenously separates them. Some consumers end up purchasing the good that is offered at a discount, making a bargain indeed. Others, instead, end up purchasing the other good and paying for it even more than their intrinsic valuation. The next result shows that in expectation consumers are hurt by this strategy.

Proposition 2 *For any $\eta > 0$ and $\lambda > 1$ a consumer's expected surplus is at most zero and therefore he would be better off if he could commit to a strategy of never buying rather than following through his actual equilibrium strategy of always buying.*

As with the similar result obtained in Heidhues and Kőszegi (forthcoming), Proposition 2 suggests that firms' sales are "manipulative" in the sense that they lead the consumers to go to the store even though ex-ante they would prefer not to. Consumers enter the store with the expectations — induced by the seller — of making a bargain by purchasing a good on sale and then might end up buying something else at an even higher price. Of course, this rather extreme result relies on the assumption that the seller knows the consumer's preferences perfectly. Nevertheless, Proposition 7 below shows that even with consumer heterogeneity, some consumers who buy would be better off making and following through a plan of never buying. Notice also that the assumption about the seller being able to credibly commit in advance to a given degree of availability is crucial. In fact, she has a strong incentive to always claim, ex-post, that the bargain item is sold-out and to try to sell only the rip-off. Having rational expectations, however, the consumers would correctly anticipate this and would never plan to buy to begin with and this plan would be consistent. Hence, the current FTC Guides Against Bait Advertising, by allowing to advertise limited-availability deals, provide the stores exactly with the commitment power they need to implement this exploitative scheme. Abolishing the role of limited-supply claims as a disclaimer for bait-and-switch or mandating retailers to issue rainchecks when advertised products are out of stock, would therefore improve consumers' welfare.

In addition to the consumers being worse off with limited availability, the monopolist's product line is sub-optimal:

Remark 1 *With limited availability, if $v_1 - c_1 \neq v_2 - c_2$, the monopolist's profit-maximizing product mix differs from the socially optimal one.*

Therefore, except for the non-generic case in which the two goods contribute equally to social surplus ($v_1 - c_1 = v_2 - c_2$), by employing a limited-availability strategy, the seller is reducing welfare compared to first-best, according to which only the item with the larger social value should be supplied. The monopolist, however, can make matters even worse and bring into the market a socially wasteful product, as the following examples show.

Example 4 (*Wasteful Product 1*) *Let* $\eta = 1$, $\lambda = 3$, $v_1 = 20$, $v_2 = 15$, $c_1 = 21$ *and* $c_2 = 10$. *For these parameters' values the seller profit-maximizing strategy is given by:* $\underline{q} = \frac{\sqrt{15}-3}{6}$, $p_1^{\min} = 10$ *and* $p_2^* = \frac{70\underline{q}+15}{2\underline{q}+1} = 35 - 4\sqrt{15}$ *for a total profit of* 6.52.

Example 5 (*Wasteful Product 2*) *Let* $\eta = 1$, $\lambda = 3$, $v_1 = 30$, $v_2 = 24$, $c_1 = 28$ *and* $c_2 = 25$. *For these parameters' values the seller profit-maximizing strategy is given by:* $\underline{q} = \frac{\sqrt{105}-7}{14}$, $p_1^{\min} = 15$ *and* $p_2^* = \frac{108\underline{q}+24}{2\underline{q}+1} = 54 - 2\sqrt{105}$ *for a total profit of* 3.52.

In fact, none of the results required $v_i > c_i$, $i \in \{1, 2\}$. The intuition in Example 4 is that, albeit socially wasteful, item 1 is highly valuable to the consumers and this makes it an ideal candidate for a bait. The intuition is somewhat different for Example 5 because the seller is now introducing an item that is socially wasteful as well as inferior for the consumers; the key here is that item 2 has a lower marginal cost than item 1 and therefore the seller can reduce her average marginal cost by introducing such a wasteful item. Average revenue also decreases, but as the example shows the cost-saving effect might outweigh the decrease in revenue. Furthermore, by comparing Example 4 with Example 5, we see also that the socially wasteful product can be either the bargain or the rip-off.

By combining the results in Proposition 10 with the condition for an item to be socially wasteful (i.e., $v_i < c_i$) we immediately obtain the following result.

Corollary 2 *The seller supplies a socially wasteful product only if item 1 is used as the bargain. She supplies a socially wasteful item 1 if and only if* $v_2 - c_2 \geq 0 > v_1 - c_1$ *and* $v_1 \geq \alpha$. *She supplies a socially wasteful item 2 if and only if* $v_1 - c_1 \geq 0 > v_2 - c_2$ *and* $\beta \geq v_1$.

Moreover, with limited availability the seller could even supply two socially wasteful products and still obtain strictly positive profits.[34]

Example 6 (*Two Wasteful Products*) *Let* $\eta = 1$, $\lambda = 3$, $v_1 = 20$, $v_2 = 9$, $c_1 = 21$ *and* $c_2 = 10$. *For these parameters' values the seller profit-maximizing strategy is given by:* $\underline{q} = \frac{\sqrt{2}-1}{2}$, $p_1^{\min} = 10$ *and* $p_2^* = \frac{58\underline{q}+9}{2\underline{q}+1} = 29 - 10\sqrt{2}$ *for a total profit of* 1.57.

Example 6 shows how the seller can simultaneously exploit the aforementioned effects and supply two socially wasteful products at the same time: item 1 is highly valuable and thus allows the seller to increase her revenue whereas item 2 has a strong cost-saving effect. Unlike other models where consumers buy socially wasteful products (i.e., Gabaix and Laibson, 2006

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[34] A similar implication arises also in the paper of Heidhues and Kőszegi (2012), where a single-product monopolist sells an item valued at $v > 0$ by the consumers. Because the monopolist is able to extract, in expectation, more than v from the consumer, she can still attain strictly positive profits for $c > v$.

and Heidhues, Kőszegi and Murooka, 2012), consumers are rational in my model and it is the combination of reference dependence and lack of ex-ante commitment that makes them buy socially wasteful products.

I end this section with the comparative statics with respect to the products' social value for the seller's profits under limited availability.

Proposition 3 *Let* $\pi_1 \equiv \pi\left(p_1^*, p_2^{\min}, \overline{q}; c_1, c_2\right)$ *and* $\pi_2 \equiv \pi\left(p_1^{\min}, p_2^*, \underline{q}; c_1, c_2\right)$ *and assume* $\eta \leq$ 1. *Then, we have:* $\frac{d\pi_1}{dv_2} > \left|\frac{d\pi_1}{dc_2}\right| > 0$, $\frac{d\pi_2}{dv_1} > \left|\frac{d\pi_2}{dc_1}\right| > 0$, $\frac{d\pi_2}{dv_2} = \left|\frac{d\pi_2}{dc_2}\right| > 0$ *and* $\left|\frac{d\pi_1}{dc_1}\right| > \frac{d\pi_1}{dv_1} > 0$.

When consumers have classically-assumed reference-free preferences, increasing their valuation for a product from v to $v+\varsigma$, with $\varsigma > 0$, by making it more appealing, or reducing the product's marginal cost by the same amount, would have the same effect on the seller's profit. Proposition 3 implies that this is no longer the case if consumers have reference-dependent preferences.

Intuitively, since the bargain item is a bait that lures consumers into the store and that the seller does not want to sell more often than necessary, her profits rise by more if this product is made more appealing than if its marginal cost is reduced. Indeed, as previously highlighted, expectations-based loss-averse preferences induce a positive demand spillover across products since the more valuable the bargain item is, the higher the price the seller can charge the consumers for the rip-off.

Things are different, however, for the rip-off. Since this is the item the monopolist sells more often, she has a bigger incentive to reduce its marginal cost. When item 2 is the rip-off, the two effects go in opposite directions, but have the same magnitude and end up offsetting each other. When instead item 1 is the rip-off, the gain from reducing its marginal cost is strictly larger than the one from increasing its appeal to consumers. In fact, if item 1 becomes more valuable by ς, consumers' ex-ante uncertainty in the product dimension also increases by ς so that the seller can raise p_1^* by less than ς. This can be easily seen by recalling that the spillover effect for p_1^* is decreasing in v_1.

1.4.2 Distant Substitutes

The conditions for the items being close substitutes pertain to the price of the rip-off item, not the price of the bargain. Therefore, the first four lemmas of the previous section apply also when the products are distant substitutes, since these lemmas do not rely on any assumption concerning the rip-off item.

Recall the first condition for close substitutability is

$$v_2 > \left(\frac{1+\eta\lambda}{2+\eta\lambda+\eta}\right)v_1 \Leftrightarrow p_2^{\min} > v_1 - v_2.$$

If this condition is violated, it is never profit-maximizing for the seller to use product 1 as the rip-off item, as shown in the following proposition.

Proposition 4 *Let* $p_2^{\min} \leq v_1 - v_2$. *Then, there does not exist a limited-availability scheme, where item 1 is used as a rip-off, yielding a higher profit than the perfect-availability scheme*

in which the seller supplies only the item with the larger social surplus and price it at its intrinsic value.

When the items are distant substitutes, and if they plan to always buy, consumers face a lot of ex-ante uncertainty in the product dimension; therefore, in order to reduce consumers' expected gain-loss disutility the seller must price the two goods quite closely. However, this cannot be done by using item 1 as the rip-off because such a scheme necessarily requires prices being further away than valuations; that is, $p_1 > v_1$ and $p_2 < v_2$.

The second condition for close substitutability, instead, pertains to the rip-off price of product 2:

$$v_2 > \frac{\sqrt{v_1 \left(1+\eta\right) \left[2 \left(c_1 - c_2\right) \left(\eta^2 \lambda^2 - \eta^2 \lambda + 2 \eta \lambda - \eta + 1\right) + v_1 \left(1+\eta\right)\right]} - \eta \left(\lambda - 1\right) \left(1+\eta\right) v_1}{\eta^2 \lambda^2 - \eta^2 \lambda + 2 \eta \lambda - \eta + 1}.$$

If the above condition is violated, then $p_2^* \leq p_1^{\min}$. In other words, the difference $v_1 - v_2$ is so large that it is impossible for the seller to price the inferior product higher than the superior one. Nevertheless, there exists a limited-availability scheme in which item 2 is used as the rip-off.

Lemma 8 *If the seller uses item 1 as a bargain (i.e., $p_1 = p_1^{\min}$), then its degree of availability is*

$$q^* = \frac{p_2 - v_2}{\eta \left(\lambda - 1\right) \left(p_2 + v_2\right)}$$

and the optimal price for item 2 is

$$p_2 = \min \left\{p_2^{\max}, \, p_1^{\min}\right\} > v_2$$

where $p_2^{\max} \equiv \frac{1+\eta\lambda}{1+\eta} v_2$. Furthermore, $q^* < \frac{1}{2}$.

Notice also that

$$p_2^{\max} \geq p_1^{\min} \Leftrightarrow \frac{v_2}{v_1} \geq \left(\frac{1+\eta}{1+\eta\lambda}\right)^2.$$

Thus, differently from the result in Lemma 6, when the products are not close substitutes the seller prices them close to one another, with $p_1 < v_1$ and $p_2 > v_2$, in order to mitigate consumer's expected gain-loss disutility. Furthermore, if $p_2^{\max} \geq p_1^{\min}$ the optimal limited-availability strategy entails flat pricing so that the consumers do not face any uncertainty in price. The following proposition delivers necessary and sufficient conditions for when such a limited-availability scheme is profit-maximizing.

Proposition 5 *Fix any $\eta > 0$ and $\lambda > 1$ and suppose $v_1 - c_1 \leq v_2 - c_2$:*

(i) *for $\frac{v_2}{v_1} < \left(\frac{1+\eta}{1+\eta\lambda}\right)^2$ a limited-availability scheme with $q = q^*$, $p_1 = p_1^{\min}$ and $p_2 = p_2^{\max}$ is profit-maximizing if and only if $v_1 \geq \frac{1+\eta\lambda}{1+\eta} \frac{\left[(1+\eta)^2 - \eta\lambda(1+\eta\lambda)\right]v_2 + (1+\eta)(c_1 - c_2)}{1+\eta}$;*

(ii) *for $\frac{v_2}{v_1} \geq \left(\frac{1+\eta}{1+\eta\lambda}\right)^2$ a limited-availability scheme with $q = q^*$, $p_1 = p_1^{\min}$ and $p_2 = p_1^{\min}$ is profit-maximizing if and only if $v_1 \geq \frac{1+\eta\lambda}{1+\eta} \left[\frac{c_1 - c_2}{\eta(\lambda-1)} - v_2 \right].$*

Similarly, fix any $\eta > 0$ and $\lambda > 1$ and suppose $v_1 - c_1 > v_2 - c_2$:

(iii) *for $\frac{v_2}{v_1} < \left(\frac{1+\eta}{1+\eta\lambda}\right)^2$ a limited-availability scheme with $q = q^*$, $p_1 = p_1^{\min}$ and $p_2 = p_2^{\max}$ is profit-maximizing if and only if $v_1 \leq \frac{1+\eta+2\eta\lambda+\eta^2\lambda+\eta^2\lambda^2}{1+3\eta\lambda+\eta^2\lambda+\eta^2\lambda^2} \left(c_1 - c_2 + \frac{1+\eta\lambda}{1+\eta} v_2 \right);$*

(iv) *for $\frac{v_2}{v_1} \geq \left(\frac{1+\eta}{1+\eta\lambda}\right)^2$ a limited-availability scheme with $q = q^*$, $p_1 = p_1^{\min}$ and $p_2 = p_1^{\min}$ is profit-maximizing if and only if $v_2 \geq \frac{(c_1-c_2)\left[1+\eta\left(\lambda\eta-\lambda^2\eta+1\right)\right]+\eta^2(\lambda-1)^2 v_1}{(c_1-c_2)[1-\eta(2\lambda-\lambda\eta+\lambda^2\eta+1)]-\eta^2(\lambda-1)^2 v_1} \frac{1+\eta}{1+\eta\lambda} v_1.$*

Furthermore, $\pi (p_1, p_2, q; c_1, c_2) \geq \max \{v_1 - c_1, v_2 - c_2\}$ and the inequality is strict if both items are supplied.

It is easy to see that, as for the case of close substitutes, the bargain can be a loss leader and the seller's product line need not be welfare-maximizing. Finally, notice that, differently from the case of close substitutes, here $c_2 < c_1$ is a necessary condition for a limited-availability scheme to be profit-maximizing. Indeed, since the rip-off price of item 2 is at most equal to the bargain price of item 1 and because the latter is always below item 1's intrinsic value, it follows that the seller's revenue with limited availability is strictly less than v_1. Hence, she must also bring her marginal cost down at least by the same amount.

1.5 Extensions

In this section I analyze three extensions of the baseline model with close substitutes. In the first subsection, I consider the case in which the seller is able to create perfect substitutes of a given product through a cosmetic change at no additional cost. In this case, the profit-maximizing strategy is always a limited-availability one. Moreover, if item 2 is the socially superior item, the seller might want to introduce the socially inferior item 1, even if she can create a perfect substitute for item 2 at no additional cost.

In the second subsection, I consider a model in which consumers have heterogeneous tastes. I first analyze a case with single-dimension heterogeneity and I show that even in this more general case the seller's profit-maximizing strategy is to reduce availability and use a combination of bargains and rip-offs. Interestingly, with limited availability, the seller is able to serve a larger portion of the potential demand. Then, I look at a situation with multi-dimension heterogeneity and I show that the profit-maximizing scheme is a limited-availability one only if the seller serves all potential demand.

In the last subsection, I relax the assumption of rational expectations and derive the profit-maximizing strategy for a monopolist selling to overly optimistic loss-averse consumers. For moderate levels of optimism, the seller's profit-maximizing strategy is qualitatively similar to the one with rational consumers. However, when consumers are extremely optimistic,

there is no need for the seller to offer a tempting deal on one item to make not buying not a credible plan. Instead, she can simply induce the consumers to believe that they will find the bargain item available for sure at a price equal to its intrinsic value and then charge for the rip-off the highest price consumers are willing to pay *ex-post*.

1.5.1 Endogenous Product Line

In the model of the previous section, the seller was exogenously endowed with two differ-ent products that the consumers regarded as imperfect substitutes. However, retailers can often create almost-perfect substitutes of a given product through a small cosmetic change that does not affect consumers' valuations. For example, two TVs might share the same technology and have the same screen-size and number of pixels, thus providing consumers with the same picture quality, and just differ in their frame's color. An alternative interpre-tation is that the seller is able to charge different prices for some units of the same product. This happens, for example, when a retailer offers a price reduction on a particular product only for the first units sold on a day.

To formally model this idea, consider a situation in which the seller can create a perfect substitute for a product without incurring any additional cost and suppose she is allowed to price these *de facto* identical products differently. Therefore, the seller now has the choice between supplying two substitutable but distinct items or just supplying two slightly different versions of the same item. In either case, the seller has the option of reducing the availability of one of the items, just like in the model of the previous section.

Assume $v_1 > v_2$, $c_i \geq 0$, $i \in \{1, 2\}$ and, let $p_i^{\min}$, $\overline{q}(\eta, \lambda, v_1, v_2, c_1, c_2)$ and $\underline{q}(\eta, \lambda, v_1, v_2, c_1, c_2)$ be defined as in the previous section. Because now the seller can supply two different versions of the same product, let $p_{i,j}^*$ be the price of the rip-off item i, when item j is the bargain. The following proposition characterizes the seller's profit-maximizing strategy.

Proposition 6 *Fix any $\eta > 0$ and $\lambda > 1$. If $v_1 - c_1 > v_2 - c_2$, the seller maximizes profits by supplying two different versions of item 1: the bargain version is priced at $p_1^{\min}$, with degree of availability $1 - \overline{q}(\eta, \lambda, v_1, v_1, c_1, c_1)$ and the rip-off version is priced at $p_{1,1}^*$, with degree of availability $\overline{q}(\eta, \lambda, v_1, v_1, c_1, c_1)$. If $v_1 - c_1 \leq v_2 - c_2$, there exists a $\widetilde{v}_2 < v_1$ such that: (i) for $v_2 \leq \widetilde{v}_2$ the seller maximizes profits by using item 1 as a bargain, with price $p_1^{\min}$ and degree of availability $\underline{q}(\eta, \lambda, v_1, v_2, c_1, c_2)$ and item 2 as a rip-off, with price $p_{2,1}^*$ and degree of availability $1 - \underline{q}(\eta, \lambda, v_1, v_2, c_1, c_2)$; (ii) for $v_2 > \widetilde{v}_2$ the seller maximizes profits by supplying two different versions of item 2: the bargain version is priced at $p_2^{\min}$, with degree of availability $\underline{q}(\eta, \lambda, v_2, v_2, c_2, c_2)$ and the rip-off version is priced at $p_{2,2}^*$, with degree of availability $1 - \underline{q}(\eta, \lambda, v_2, v_2, c_2, c_2)$.*

Proposition 6 delivers several interesting results. First, if the seller can easily create per-fect substitutes of the same item that are valued equally by consumers, the profit-maximizing strategy is always a combination of limited availability, bargains and rip-offs.[35] This result can be interpreted as a foundation for the analysis in Heidhues and Kőszegi (forthcoming):

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[35]The results would be the same if the seller had to incur a positive cost k to create the artificial substitute, as long as k is not too large.

although it might not be possible for the seller to credibly commit to a stochastic pricing strategy, she could achieve the same goal by introducing many slightly different — but equivalent from the consumers' point of view — versions of the same product. Second, if the socially superior product is the most preferred by the consumers, the seller prefers to create perfect substitutes of this product instead of introducing another, inferior, one. On the other hand, if the socially superior item is the one consumers value the least, the seller might want to supply both products, even if she could create a perfect substitute for either product at no additional cost. The intuition is that, albeit socially inferior, item 1 is highly valuable to the consumers and this makes it an ideal candidate for a bait because it allows the seller to charge an even higher price for the rip-off, therefore increasing average revenue; although average cost also increases, the former effect might dominate. In this case the consumers' most preferred item is used as a bargain and the seller's product line is not welfare-maximizing. Finally, it is easy to see that the results from the previous section about loss leaders and socially wasteful products still apply in this context.

1.5.2   Heterogeneous Values

In the model analyzed in Section 4 the seller did not face any trade-off between margins and quantities due to the homogeneity assumption about the consumers' preferences. In this section, I consider a more general and realistic environment in which the monopolist faces a classical downward-sloping demand curve and I show that she can still make higher profits by using a limited-availability scheme with a bargain item and a rip-off item. The key insight for this result is that although the seller must choose between serving a large share of the demand with a low price or a small share of the demand with a high price, she can still extract from the marginal consumer more than his intrinsic value for the product.

Consider a seller supplying item 1 at a constant marginal cost $c_1 \geq 0$ to a unit mass of consumers who differ in their intrinsic value, v_1, for the seller' product. From the seller's point of view v_1 is a random variable with distribution F. Assume F is strictly increasing, weakly convex and differentiable, with positive density f everywhere on the support $\left[v_1^l, v_1^h\right]$ with $v_1^h > c_1 \geq v_1^l \geq 0$.[36]

Without loss aversion the seller would just choose the price $\widehat{p}_1$ that solves the following maximization problem:

$$\max_{p_1} \ (p_1 - c_1)\left[1 - F(p_1)\right].$$

Taking FOC and re-arranging yields

$$\widehat{p}_1 - c_1 = \frac{1 - F(\widehat{p}_1)}{f(\widehat{p}_1)}.$$

The consumer with value $v_1 = \widehat{p}_1$ is the "marginal" type; that is, the type who is exactly indifferent between buying or not. The seller's profit is equal to

$$(\widehat{p}_1 - c_1)\left[1 - F(\widehat{p}_1)\right]$$

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[36]The assumptions on F ensure that, for deterministic prices, the demand curve is decreasing and weakly concave (a property that is typically assumed in models of industrial organization).

and consumers' surplus is equal to

$$\int\limits_{\widehat{p}_1}^{v_1^h} (v_1 - \widehat{p}_1) \ dF(v_1).$$

As before, this perfect-availability strategy constitutes a feasible option for the seller also when consumers are expectations-based loss-averse. To see why, notice that, given the price announced by the seller, types below $\widehat{p}_1$ can just plan not to buy and this plan is not only consistent but it maximizes their expected utility; similarly, types above $\widehat{p}_1$ prefer the plan of buying for sure at price $\widehat{p}_1$. Since $q_1 = 1 - F(\widehat{p}_1)$, the measure of types who plan to buy coincides with the amount the seller is supplying and there is no uncertainty in the outcome that each type is expecting; therefore, gain-loss utility is zero in equilibrium. Yet, the seller can attain a higher profit through the introduction of a limited-availability deal. In this case the seller must induce some uncertainty in the buyers' plans otherwise, as argued above, gain-loss utility would be irrelevant.

Suppose that the seller can create an artificial perfect substitute for item 1 without incurring any additional cost and suppose she can price these *de facto* identical products differently. A type-v consumer will plan to buy with positive probability only if $p_1^{\min} \leq \frac{1+\eta}{1+\eta\lambda} v \equiv p_1^{\min}(v)$. From Section 4 we also know that, for given $1-q$ (the degree of availability of the bargain item), this consumer will be indifferent between the plan of buying only the bargain item and the plan of buying the bargain item if available and the rip-off item otherwise if and only if

$$p_1^* = v \left[1 + \frac{2\eta(\lambda-1)(1-q)}{1+\eta(\lambda-1)(1-q)} \frac{1+\eta}{1+\eta\lambda} \right] \equiv p_1^*(v).$$

In order to maximize how much surplus she can extract from this consumer, the monopolist chooses the following degree of availability:

$$\overline{q} = \arg\max_q \ q p_1^*(v) + (1-q) p_1^{\min}(v) - c_1.$$

Notice that $\overline{q}$ does not depend on either v or c_1 (see appendix B for the details).

With heterogeneous values there is an additional difficulty in characterizing the optimal limited-availability scheme because different types might select different PPEs. The lemma below describes the PPEs for all consumers' types.

Lemma 9 *Suppose the seller plays the limited-availability strategy that makes a type-v consumer indifferent between buying only the bargain item and always buying. Then, for types in $[v_1^l, v)$ the PPE plan is to never buy whereas for types in $[v, v_1^h]$ the PPE plan is to always buy. Furthermore, a consumer's equilibrium expected utility is weakly increasing in his type.*

In order to identify the profit-maximizing marginal type, the seller solves the following program:

$$\max_v \ \left[\overline{q} p_1^*(v) + (1-\overline{q}) p_1^{\min}(v) - c_1 \right] [1 - F(v)],$$

which can be re-written as

$$\max_{v} \ (\Phi v - c_1)\left[1 - F\left(v\right)\right],$$

where $\Phi \equiv \frac{4-2\eta^2+\eta^2\lambda^2+4\lambda\eta+\eta^2\lambda-2\sqrt{2(2+\eta+\eta\lambda)(1+\eta)(1+\eta\lambda-\eta)}}{\eta(\lambda-1)(1+\eta\lambda)} > 1$. Let $\widehat{v}_1$ be the solution to the above program. It is immediate to see that $\widehat{v}_1 < \widehat{p}_1$, implying that the seller serves a larger fraction of the consumers when using a limited-availability scheme. The following proposition characterizes the seller's profit-maximizing strategy.

Proposition 7 *For any $\eta > 0$ and $\lambda > 1$ the seller maximizes profits by supplying two different versions of item 1: the bargain version is priced at $p_1^{\min}\left(\widehat{v}_1\right)$, with degree of availability $1 - \overline{q}$ and the rip-off version is priced at $p_1^*\left(\widehat{v}_1\right)$, with degree of availability $\overline{q}$. The marginal type $\widehat{v}_1$ is implicitly defined by $\frac{1-F(\widehat{v}_1)}{f(\widehat{v}_1)} + \frac{c_1}{\Phi} = \widehat{v}_1$. Furthermore, consumers whose type is in $[\widehat{v}_1, v_1^s)$, where $v_1^s = \overline{q}p_1^*\left(\widehat{v}_1\right)\left[1 + (1 - \overline{q})\eta\left(\lambda - 1\right)\right] + (1 - \overline{q})p_1^{\min}\left(\widehat{v}_1\right)\left[1 - \overline{q}\eta\left(\lambda - 1\right)\right]$, get negative expected utility.*

Notice that in this case the overall welfare effect of limited availability is ambiguous, since with a limited-availability scheme the seller is serving a larger measure of consumers compared to the case of perfect availability. Nevertheless, some consumers, who would get a utility level of zero with perfect availability, are unambiguously worse off with this strategy.

The result in Proposition 7 can easily be extended to the case in which the seller's products are not perfect substitutes and have different marginal costs. Suppose the seller cannot create a perfect substitute for item 1, but she can supply item 2 at a constant marginal cost $c_2 = c_1 - k \geq 0$. Let v_2 denote consumers' taste for item 2 and assume $v_2 = v_1 - h$. To see the intuition, suppose $h = k$ so that with perfect availability the seller would be exactly indifferent between whether to supply item 1 or 2 and the marginal types would be $\widehat{p}_1$ and $\widehat{p}_2 = \widehat{p}_1 - k$, respectively.

With limited availability, we know from Lemma 7 that if $v_1 - c_1 = v_2 - c_2$ the seller maximizes profits by using item 1 as the bargain item. Therefore, she supplies $\underline{q}$ units of item 1 at price $p_1^{\min}\left(\widehat{v}_1\right)$ and $1 - \underline{q}$ units of item 2 at price $p_2^*\left(\widehat{v}_2\right)$, where $\widehat{v}_1 = \widehat{v}_2 + k$ and achieves higher profits than with perfect availability. Furthermore, $\widehat{v}_1 < \widehat{p}_1$ so that, also in this case, limited availability implies less exclusion than perfect availability.

Finally, let's consider a case with both horizontally and vertically differentiated tastes. Suppose each individual consumer is characterized by a pair of valuation (v_1, v_2) uniformly distributed on the square $[\underline{v}, \overline{v}]^2 \subset \mathbb{R}_2^+$. This distribution is common knowledge, whereas a consumer's individual valuations are his own private information. The goods are substitutes and consumers demand at most one unit of one good. Assume, for simplicity, that $c_1 = c_2 = c$. With perfect availability, the seller solves the following program:

$$\max_{p_1,p_2} \pi^{**} = (p_1 - c)\int_{p_1}^{\overline{v}}\int_{\underline{v}}^{v_1-p_1+p_2}\left(\frac{1}{(\overline{v} - \underline{v})^2}\right)dv_2 dv_1 + (p_2 - c)\int_{p_2}^{\overline{v}}\int_{\underline{v}}^{v_2-p_2+p_1}\left(\frac{1}{(\overline{v} - \underline{v})^2}\right)dv_1 dv_2.$$

Since the environment is symmetric, there is no loss of generality in restricting attention to a symmetric solution with $p_1 = p_2 = p$; the seller's problem then simplifies to

$$\max_{p} \frac{(p - c)(\overline{v} - p)(\overline{v} - 2\underline{v} + p)}{(\overline{v} - \underline{v})^2}.$$

Taking FOC and re-arranging yields:

$$p^{**} = \frac{2\underline{v} + c + \sqrt{3\overline{v}^2 - 6\overline{v}\underline{v} + 4\underline{v}^2 - 2\underline{v}c + c^2}}{3}. \tag{1.13}$$

Next, I turn to the analysis with limited availability. Suppose the seller employs the scheme that makes a (v, v)-type consumer exactly indifferent between planning to buy only the bargain and planning to always buy.[37] Let $p_1^{\min}(v) = \frac{1+\eta}{1+\eta\lambda}v$ and $\underline{q}(\eta, \lambda, v, v, c, c)$ be the price of the bargain and its degree of availability and $p_2^*(v)$ be the price of the rip-off.[38] Notice also that although the marginal type values the two items the same, the prices are not symmetric since $p_2^*(v) > v > p_1^{\min}(v)$. The lemma below describes the PPEs for all consumers' types.

Lemma 10 Let $\eta \leq 1$ and suppose the seller plays the limited-availability strategy that makes a type-(v, v) consumer indifferent between buying only the bargain item and always buying. Then, for types in $[v, \overline{v}]^2$ the PPE plan is to always buy; for types in $(v, \overline{v}] \times [\underline{v}, v)$ the PPE plan is to buy only the bargain if available and nothing otherwise; and for types in $[\underline{v}, v) \times (v, \overline{v}]$, there exist $a > 1$ and $b > 0$ such that consumers' PPE plan is to always buy if $v_2 \geq av - bv_1$ and to never buy otherwise. All other types plan to never buy.

Thus, the seller's program under limited availability can be written as

$$\max_v \pi_{LA}^{**} = \left[\underline{q}p_1^{\min}(v) + \left(1 - \underline{q}\right)p_2^*(v) - c\right]\left[\left(\frac{\overline{v} - v}{\overline{v} - \underline{v}}\right)^2 + \Omega(v)\right] + \underline{q}\left[p_1^{\min}(v) - c\right]\left(\frac{\overline{v} - v}{\overline{v} - \underline{v}}\right)\left(\frac{v - \underline{v}}{\overline{v} - \underline{v}}\right)$$

where

$$\Omega(v) = \begin{cases} \frac{\left(v - \frac{av - \overline{v}}{b}\right)(\overline{v} - av + bv)}{2} & \text{if } \frac{av - \overline{v}}{b} \geq \underline{v} \\ \frac{[(\overline{v} - av + bv) + (\overline{v} - av + b\underline{v})](v - \underline{v})}{2} & \text{if } \frac{av - \overline{v}}{b} < \underline{v} \end{cases}$$

denotes the area to the left of v and above $av - bv_1$.[39]

Let (v^{**}, v^{**}) denote the profit-maximizing marginal type. It is worth noticing that, for a fixed marginal type, under limited availability the seller is serving a smaller measure of consumers compared to the perfect-availability case. Indeed, while with perfect availability every consumer who values *at least* one item more than its price will buy something for sure, with limited availability instead a positive measure of the consumers who value item 1 more than its price prefer to stay out of the market and those consumers who value item 1 much more than item 2 prefer to plan to buy item 1 if available and nothing otherwise. Indeed,

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[37]It is easy to see that the marginal type must lie on the 45-degree line. Suppose by contradiction that the marginal type had valuations $(v + \varepsilon, v)$. The seller could then increase her profits by playing the limited-availability strategy that makes a type $\left(v + \frac{\varepsilon}{2}, v + \frac{\varepsilon}{2}\right)$ indifferent between buying only the bargain and always buying. In this way, the seller would be serving a larger measure of consumers at a higher average price.

[38]Given the symmetry assumptions about the values' distribution and the items' costs, and since the marginal type views the items as perfect substitutes, the seller is actually indifferent between which item to choose as the bargain.

[39]For $\frac{av - \overline{v}}{b} \geq \underline{v}$, $\Omega(v)$ is a right triangle with sides of length equal to $v - \frac{av - \overline{v}}{b}$ and $\overline{v} - (av - bv)$. Then, for $\frac{av - \overline{v}}{b} < \underline{v}$, $\Omega(v)$ becomes a right trapezoid of height $(v - \underline{v})$ and with sides equal to $\overline{v} - (av - bv)$ and $\overline{v} - (av - b\underline{v})$.

the seller's profit under limited availability might be strictly decreasing in v and, as shown in the following lemma, a limited-availability scheme can be profit-maximizing only if the monopolist serves all potential customers, so that $(v^{**}, v^{**}) = (\underline{v}, \underline{v})$.

Lemma 11 *If $\eta \leq 1$ and $(v^{**}, v^{**}) > (\underline{v}, \underline{v})$ there always exists a perfect-availability strategy that provides the seller with a higher profit than what she could achieve with any limited-availability scheme.*

The intuition for the above result relies on the fact that when the marginal type is in the interior of the support of the distribution, the heterogeneity in consumers' tastes, as captured by $\overline{v} - \underline{v}$, is so large that the seller cannot profitably exploit the attachment effect that a limited-availability deal creates for the consumers. This happens because, with multiple dimensions of heterogeneity, there exist "extreme types" who have a relatively high valuation for one of the goods but do not care much for the other. Consider consumers with valuations in $[\underline{v}, v^{**}) \times [\underline{v}, \overline{v}]$. For these consumers not buying is a credible plan since they do not value the bargain item very much; therefore they can "resist" going to the store and will plan to buy only if they expect to obtain a strictly positive surplus. On the other hand, consumers with valuations in $(v^{**}, \overline{v}] \times [\underline{v}, v^{**})$ do not value the rip-off item very much and, although they cannot avoid planning to buy with positive probability, they prefer to leave the store empty-handed if the bargain is not there. Nevertheless, a limited-availability strategy is profit-maximizing if the heterogeneity in consumers' tastes is not too large. The following proposition derives necessary and sufficient conditions for this.

Proposition 8 *Assume $\eta \leq 1$ and $\underline{v} > 0$. The seller's profit-maximizing strategy is a limited-availability one if*

$$\overline{v} \leq \underline{v} + \frac{\sqrt{2\underline{v}^2[9\Phi(3\Phi-4)+8]+2c[2\underline{v}(10-9\Phi)-c]-2(8\underline{v}+c-9\underline{v}\Phi)[\underline{v}^2\Phi(9\Phi-8)+2\underline{v}c(4-5\Phi)+c^2]^{\frac{1}{2}}}}{4} \quad (1.14)$$

where $\Phi \equiv \frac{4-2\eta^2+\lambda^2\eta^2+4\lambda\eta+\lambda\eta^2-2\sqrt{2(\eta+\lambda\eta+2)(\lambda\eta-\eta^2+\lambda\eta^2+1)}}{\eta(\lambda-1)(\lambda\eta+1)} > 1$. In this case, the seller uses item 1 as the bargain, with price $p_1^{\min}(\underline{v})$ and degree of availability $\underline{q}$ and item 2 as the rip-off, with price $p_2^*(\underline{v})$ and degree of availability $1-\underline{q}$, and all consumers plan to always buy, irrespective of their type. If condition (1.14) does not hold, the seller maximizes profits by employing a perfect-availability strategy where both items are priced at p^{**}.

Thus, when the distance between $\overline{v}$ and $\underline{v}$ is small, as defined by (1.14), the seller maximizes profits with a limited-availability strategy. Intuitively, if the degree of heterogeneity in consumers' tastes is small, under screening with perfect availability there is not going to be much exclusion of low types and hence, the valuation of the marginal type — which determines the price — is relatively low. In this case, then, a limited-availability scheme is superior because it extracts more than the marginal type's intrinsic valuation and it serves the entire market with no exclusion, as shown in the example below.

Example 7 (Two-dimension Heterogeneity) *Suppose $c = 0$, $\eta = 1$ and $\lambda = 3$. Then condition (1.14) reduces to*

$$\overline{v} \leq \underline{v} + \frac{v}{4}\sqrt{\frac{85}{2}\sqrt{\frac{1753}{16} - \frac{303}{4}\sqrt{2}} - 27\sqrt{2}\sqrt{\frac{1753}{16} - \frac{303}{4}\sqrt{2}} - \frac{837}{2}\sqrt{2} + \frac{4763}{8}} \simeq 1.8\underline{v}.$$

Let $(v_1, v_2) \overset{U}{\sim} [20, 30]$. The profit-maximizing price with perfect availability is $p^{**} = \frac{40+10\sqrt{7}}{3}$ for a profit of $\frac{200+140\sqrt{7}}{27}$. The profit-maximizing strategy with limited-availability is a to supply item 1 at price 10, with degree of availability $\frac{\sqrt{2}-1}{2}$ and item 2 at a price of $10\left(4 - \sqrt{2}\right)$ for a profit of $65 - 30\sqrt{2}$.

1.5.3   Optimistic Consumers

So far I have closely followed the model of Kőszegi and Rabin (2006) by assuming that consumers' beliefs must be consistent with rationality: a consumer correctly anticipates the implications of his period-0 plans, and makes the best plan she knows she will carry through. In this section I relax the assumption about rational expectations.

Suppose that when the seller announces a degree of availability q for a bargain, consumers are overly optimistic about their chance of getting a deal and when forming their purchasing plan, they think they will get the bargain with probability $\widetilde{q} = \min\{\chi q, 1\}$, where $\chi > 1$ parametrizes the degree of consumers' optimism. The seller knows χ, but cannot be held liable for the difference between perceived and actual availability; however, she cannot reduce product availability below the level q that she announces. On the other hand, after observing the seller's announcement of availability and prices, consumers still select a PPE purchasing plan, but they base their decisions and payoffs' comparison on the biased beliefs $\widetilde{q}$.

For simplicity, let's assume that the products are perfect substitutes ($v_1 = v_2 = v > 0$) and that marginal cost is zero for both of them, and as a normalization, let item 1 be the bargain item. Denote by $\widehat{q}$ the profit-maximizing degree of availability of item 1 when consumers have rational expectations ($\chi = 1$).

At first glance one could be tempted to guess that with naïve consumers, the seller would always choose a lower degree of availability for the bargain item, compared to the rational case. After all, the seller can just announce $q = \frac{\widehat{q}}{\chi}$, inducing the same attachment effect as with rational consumers but actually selling the bargain less often and hence making even higher profits. However, this intuition is incomplete. To see why, notice that for given q and p_1 that the seller announces for the bargain item, she can raise the price of the rip-off up to

$$p_2^*\left(q, p_1\right) = v + \left[\frac{2\eta\left(\lambda - 1\right)\chi q}{1 + \eta\left(\lambda - 1\right)\chi q}\right] p_1.$$

This means

$$\frac{\partial^2 p_2^*\left(q, p_1\right)}{\partial\chi\partial q} > 0 \Leftrightarrow 1 - q\chi\eta\left(\lambda - 1\right) > 0,$$

implying that if χ is small, the marginal gain from raising q is higher when consumers are optimistic.

The monopolist will then choose the degree of availability and price for item 1 that solves:

$$\max_{q, p_1} \widetilde{\pi} = qp_1 + \left(1 - q\right) p_2^*\left(q, p_1\right).$$

Let $q_\chi\left(p_1\right)$ be the solution to this maximization problem. The following proposition characterizes the seller's profit-maximizing strategy.

Proposition 9 *Fix any $\eta > 0$ and $\lambda > 1$. There exists a $\widetilde{\chi}$ such that the seller's profit-maximizing strategy is as follows:*

(i) *if $\chi < \widetilde{\chi}$, she announces a degree of availability for the bargain equal to $q_\chi \left(p_1^{\min} \right)$, and prices $p_1^{\min}$ and $p_2^* \left(q_\chi \left(p_1^{\min} \right), p_1^{\min} \right)$;*

(ii) *if $\chi \geq \widetilde{\chi}$, she announces a degree of availability for the bargain equal to $q_\chi = \frac{1}{\chi}$ and prices v and $p_2^* = v \left(1 + \frac{\eta(\lambda-1)}{1+\eta\lambda} \right)$.*

Furthermore, the seller's expected profit is strictly greater than v.

The first implication of Proposition 9 is that the monopolist profit displays a discontinuity at $\widetilde{\chi}$. The intuition is as follows. For moderate levels of consumers' optimism, the seller's profit-maximization problem is very similar to the one with rational consumers: she chooses the highest price for the bargain that makes not buying not a credible plan and the price of the rip-off is such that consumers ex-ante are (perceive to be) indifferent between planning to buy only the bargain and planning to always buy. Then, she announces a degree of availability for the bargain that trades off the gains from exploiting the attachment effect with those from selling the rip-off more often than the bargain. Hence, except for the fact that consumers believe to be more likely to make a deal than they actually are, the seller's profit-maximizing limited-availability scheme is qualitatively similar to the one derived in Section 4.

Things are different, however, when consumers are very optimistic. For $\chi = \widetilde{\chi}$ we have that:

$$\widetilde{q} \left(v - p_1^{\min} \right) - \widetilde{q} \left(1 - \widetilde{q} \right) \eta \left(\lambda - 1 \right) \left(v + p_1^{\min} \right) = 0,$$

where $\widetilde{q} = q_{\widetilde{\chi}} \left(p_1^{\min} \right)$. That is, $\widetilde{\chi}$ is the lowest degree of optimism for which, when the seller plays the scheme in part *(i)* of Proposition 9, consumers perceive their expected utility to be non-negative. In this case, there is no need for the seller to offer a tempting deal on item 1 to make not buying not credible; instead, she can just announce $q_\chi = \frac{1}{\chi}$, inducing consumers to believe that they will find item 1 available for sure, and price item 1 at its intrinsic value and item 2 at the highest price consumers are willing to pay *ex-post*. So at $\chi = \widetilde{\chi}$, the degree of availability of the bargain and the prices jump up and so does the seller's profit. Notice also that the optimal level of availability for the bargain is not monotone in the degree of optimism χ, as shown in Figure 3.

Clearly naïvete makes consumers worse off. However, notice that as χ tends to 1, the seller is choosing a higher degree of availability for item 1 compared to the case with rational consumers; hence, if χ is relatively small, although overly optimistic consumers on average are exploited even more than rational consumers, there is more of them that end up making a deal.
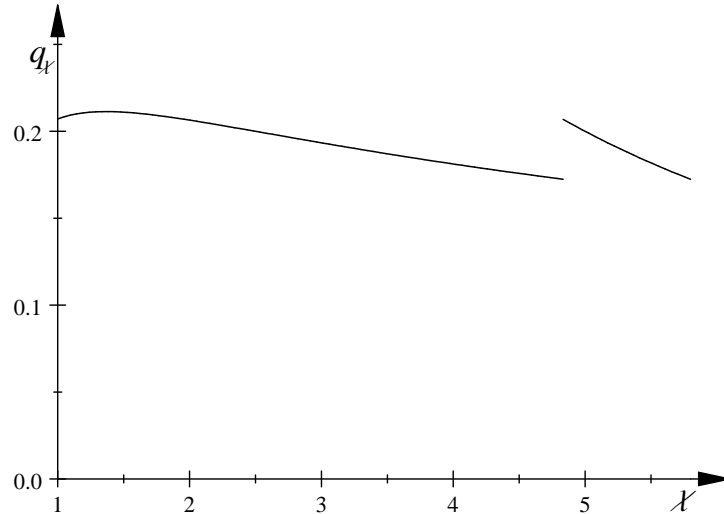
Figure 1.3: Level of availability with naïve consumers q_χ as a function of χ, for $\eta = 1$, $\lambda = 3$ and $v = 1$.

1.6   Related Literature

This paper belongs to a recent and growing literature on how firms respond to consumer loss aversion. Heidhues and Kőszegi (2008), Karle and Peitz (2012) and Zhou (2011) study the implications of reference-dependent preferences and loss aversion in an oligopolistic environment with differentiated goods. In a monopolistic-screening setting, Carbajal and Ely (2012), Hahn, Kim, Kim and Lee (2012) and Herweg and Mierendorff (forthcoming) analyze the implications of reference-dependent preferences and loss aversion for the design of profit-maximizing menus and tariffs. Karle (2012) studies the advertising strategy of a single–product monopolist when consumers are expectation–based loss-averse. He shows that the seller maximizes profits by releasing an advertising signal about the consumers' (unknown ex-ante) match-value for the product that, although informative, would be redundant if consumers had classical preferences; instead with loss-averse consumers this informative signal can have a persuasive effect and hence increase consumers' willingness to pay.[40]

As discussed in the Introduction, my paper is most related to Heidhues and Kőszegi (forthcoming), which provides an explanation for why regular prices are sticky, but sales prices are variable, based on expectations-based loss aversion. In their model, a monopolist sells only one good and maximizes profits by employing a stochastic-price strategy made of low, variable sales prices and a high, sticky regular price. My results share an intuition

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[40]For other applications of the Kőszegi-Rabin model of reference-dependent preferences outside the field of IO, see Aperjis and Balestrieri (2010) on advategeous selection in insurance markets, Crawford and Meng (2011) and Eliaz and Spiegler (2012) on labor supply, Herweg, Muller and Weinschenk (2010) and Macera (2012) on agency contracts, Lange and Ratan (2010) , Eisenhuth (2012) and Eisenhuth and Ewers (2012) on sealed-bid auctions and mechanism design, and Daido and Murooka (2012) on team incentives.

similar to theirs: low prices work as baits to lure consumers who, once in the store, are willing to pay a price even above their intrinsic valuation for the item. However, in my model the monopolist sells two goods and uses one of them as a bait to attract the consumers and the other to exploit them. Also, in Heidhues and Kőszegi (forthcoming) consumers face uncertainty about the price whereas in my case the uncertainty stems from the limited availability of the deal.[41] I consider my model to be an extension as well an improvement over theirs.[42] It is an extension because it shows that the intuition behind their main result holds also in the case of a multi-product monopolist and it is an improvement because I find my assumption about the seller endogenously choosing the degree of availability of a product more realistic than their assumption of the seller being able to credibly commit to an entire price distribution.[43] Moreover, by analyzing the case of a multi-product retailer, I can derive predictions about which products are more likely to be put on sale and I show that higher-value products are more likely to be used as baits.

Within the realm of industrial organization, this paper is also closely related to the literature on advertising, bait-and-switch and loss leaders. Lazear (1995) studies a duopoly with differentiated goods in which each firm produces only one good and consumers pay a search cost to visit a firm, and derives the conditions under which bait-and-switch is a profitable strategy. Although consumers have rational expectations and understand that a firm might engage in bait-and-switch, this strategy can be profitable if the goods sold by different firms are similar and if search is costly. However, bait-and-switch is a form of false advertising in which a firm claims to sell a different good than the one it actually produces. In my model, instead, the firm is not lying to the consumers but is using a truthful version of the bait and switch strategy through endogenously reducing the availability of the goods. Furthermore, in Lazear's model prices are exogenous whereas in mine they are optimally chosen by the seller. Gerstner and Hess (1990) present a model of bait-and-switch in which retailers advertise only selected brands, low-priced advertised brands are understocked and in-store promotions are biased towards more expensive substitute brands. In their model consumers are rational and foresee stock outages. However, the authors assume that in-store promotions can create a permanent utility increase for consumers and this is the reason why in equilibrium some consumers will switch to more expensive brands.

Ellison (2005) presents a model of competitive price discrimination with horizontal and vertical taste differences across consumers in which firms advertise a base price for a product and then try to sell "add-ons" or more sophisticated versions of the product for a higher price at the point of sale. Gabaix and Laibson (2006) study a model where firms benefit from

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[41]If consumers value the two goods equally and the goods have the same production cost, my model coincides with a special case of theirs in which the monopolist uses a two-price distribution. However, in my model the seller can credibly announce to the consumers that she is having a sale on some selected products — as stores often do indeed — whereas in their model the seller can only announce that she might have a sale.

[42]Spiegler (2012a) proposes another simplification and extension of Heidhues and Kőszegi (forthcoming).

[43]After entering a store that claims to use a stochastic-pricing strategy à la Heidhues and Kőszegi (forthcoming), and faced with a high-price draw, a consumer might reasonably doubt whether he was just unlucky or whether the seller was just pretending to randomize prices. In my model, instead, if a consumer does not find a bargain available, he has less of a reason to blame the seller because other consumers might have bought all the bargain items and he might even be mad at himself for not having gone to the store earlier. I thank Kfir Eliaz for suggesting this "shifting the blame" interpretation.

shrouding add-on prices if there myopic consumers who, mistakenly, do not consider the add-on price when forming their shopping plans. Apart from the result that the "basic" version of the product can be a loss leader, my model is different since I assume that all prices are known and that consumers correctly predict their own shopping behavior. Furthermore, the situation described in Example 2 in the Introduction, where the more sophisticated version of the product is offered at a lower price, can be rationalized by my model but not theirs. Eliaz and Spiegler (2011a) propose a model where stores compete for consumers' limited attention by expanding their product lines with "pure attention grabbers"; that is, products that have the sole purpose of attracting consumers' attention to the other products offered by the store. Once at the store, a consumer might realize that there exists another product that better suits his needs. Thus, differently from my model, a consumer might switch to another product even if the bargain item is available.

Hess and Gerstner (1987) develop a model in which multi-product firms might stock out of advertised products and offer rain checks to consumers, and Lal and Matutes (1994) consider multi-product firms competing for consumers who are initially unaware of prices. In both of these models firms might advertise loss leaders in order to increase store traffic. The profitability of this strategy, then, stems from the fact that once they arrive at the store, consumers will buy also other complementary items that are priced at a high mark-up; that is, each firm enjoys a form of monopoly on the other items once a consumer is attracted into the store by the loss leader.[44] My model is different as I consider a monopolist selling substitutable goods to consumers who demand at most one unit of one good and therefore loss-leading is not aimed at increasing store traffic in order to boost demand for complementary products. Furthermore, in these models the products with lower consumer value are the more natural candidates for loss-leading pricing; my model instead can also rationalize the use of more valuable or popular products as loss leaders.

Models of price dispersion under demand uncertainty (Dana, 1999, 2001a; Deneckere and Peck, 1995; Nocke and Peitz, 2007) and buying frenzies (De Graba 1995; Gilbert and Klemperer, 2000) also predict that rationing some consumers through voluntary stockouts can be a profit-maximizing strategy. However, these models apply mainly to new products that are launched on the market for the first time and for which either the seller or the consumers cannot predict what actual demand will turn out to be; or to industries with clear binding capacity constraints like airlines, hotels and restaurants. Yet, goods sold during bargain sales are usually not appearing on the market for the first time. Moreover, in these models, once the true demand-state is revealed, the scope for rationing disappears.

Finally, Thanassoulis (2004) studies the problem of a multi-product monopolist selling two substitute goods to risk-neutral consumers with unit demand, and derives conditions such that the optimal tariff includes lotteries.[45] In my model, when the seller endogenously

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[44]Related, but somewhat different explanations for the use of loss leaders are advanced by DeGraba (2006) and Chen and Rey (2012). DeGraba (2006) presents a multi-product pricing model in which the loss leaders are the goods purchased mainly by more profitable consumers — consumers who are more likely to buy larger quantities of other goods as well; hence, loss-leading is a way to price discriminate between differently profitable consumers. In Chen and Rey (forthcoming), a large retailer, competing with smaller stores offering a narrower range of products, can exercise market power by pricing below costs some of the products offered also by its rivals. Thus, loss-leading emerges as an exploitative device that allows the large retailer to discriminate multi-stop shoppers from one-stop shoppers.

[45]Pavlov (2011) solves for the optimal mechanism when selling two substitutable goods and generalizes

reduces the availability of the goods, from the consumers' point of view this is equivalent to taking a lottery on both which good they will end up with and how much they will have to pay.[46] Nevertheless, there are several differences between his model and mine. First, my result on the optimality of limited-availability deals holds also when consumers have homogeneous tastes, whereas his result on the optimality of lotteries does not. Second, in his lotteries there is uncertainty only on the item dimension but not on the price one, whereas in my case the uncertainty is on both dimensions. Last, in his model a lottery is offered in addition to each good being offered in isolation with its own posted price; in my model instead each good is offered in isolation with its own price, but because the items are in short supply, consumers are uncertain about their consumption outcomes.

1.7 Conclusion

Limited-availability sales are commonly employed by retailers selling durable consumer goods such as electronics, household appliances, or clothes. However, while limited-availability sales are familiar to consumers, economists have not devoted much attention to the importance of product availability in retailing.

In this paper, I have provided an explanation, based on consumer loss aversion, for why a monopolist selling substitute goods might find it profitable to use limited-availability sales. The optimal strategy for the monopolist resembles bait-and-switch: she lures the consumers with a limited-availability tempting deal on one good and cashes in with a high price on another one. The model also predicts that more valuable or popular items are more likely to be used as baits and that the bait can be a loss leader.

I conclude the paper by discussing some of the model's limitations, as well as some directions for future research.

An implicit premise of my model is that consumers cannot commit not to go shopping. Although this seems sometimes to be unrealistic, there exist some real-life situations in which this assumption is not that restrictive. For example, around Christmas many consumers "have to" go shopping in order to buy gifts for their friends and relatives. Furthermore, committing not to look at ads or not to learn about sales to avoid being manipulated by firms might require some costly effort on the part of the consumers. If this is the case, then the seller could easily "bribe" the consumers into visiting the store.[47]

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the analysis in Thanassoulis (2004). Balestrieri and Leao (2011) extend this result to an oligopoly setting where consumers have horizonally differentiated tastes. Fay and Xie (2008) show how lotteries can provide a buffer against a seller's own demand uncertainty and capacity constraints.

[46]Thanassoulis (2004) makes also the related point that capacity constraints, actual or alleged, are an indirect way to implement lotteries.

[47]In fact, introducing a small shopping cost into the model would not significantly affect the results. To see why, suppose that consumers must incur a positive shopping cost ϕ, with $0 < \phi < p_2^{\min}$, to go to the store and let the gain-loss utility in the shopping cost be evaluated separately from the product and money dimensions. Then, there exists a $\phi^* (\eta, \lambda, v_1, v_2, c_1, c_2)$ such that for $\phi \leq \phi^*$ the seller's profit-maximizing strategy is a limited-availability scheme with the only difference that now the price of the bargain must be reduced by $\frac{\phi}{\underline{q}}$ (or $\frac{\phi}{1-\overline{q}}$, depending on which item is the bargain) in order to make never buying non-credible for the consumers and therefore induce them to visit the store (the price of the rip-off should also be adjusted accordingly).

Another important assumption is that, from the consumers' perspective, the two products belong to the same hedonic dimension. This creates an insurance effect: by planning to always buy a consumer can reduce the uncertainty in his consumption compared to the plan of buying only the bargain item. The monopolist then, is able to exploit this insurance effect by charging a high price for the rip-off item. If the two goods were evaluated along different hedonic dimensions, the insurance effect would disappear, making the conditions for always buying to be the PPE more restrictive.

The analysis in this paper can be extended to the case of a monopolist carrying more than two goods. If the goods are perfect substitutes, or if the seller can endogenously fine tune their degree of differentiation, then she will always use as many products as possible and price them slightly differently to mitigate the consumers' comparison effect on the price dimension, implementing *de facto* the random-price strategy described in Heidhues and Kőszegi (forthcoming). However, if the products are not close enough in terms of substitutability, then the seller will supply only the most similar ones.

I have closely followed the model of Kőszegi and Rabin (2006) in specifying the reference point as the entire distribution of consumers' rational beliefs. However, the analysis would be the same if the reference point, for each hedonic dimension, was equal to the point expectation instead of the distribution. In fact, since all lotteries that consumers face in the model involve comparing only two possible outcomes, each realization is either a loss or a gain but not both, and the same would still be true if the reference point was a point expectation. On the other hand, the assumption that consumers assess gains and losses separately on each hedonic dimension of consumption utility is crucial for the results. If gain-loss utility were defined on the consumers' intrinsic surplus, $v - p$, then the seller could never raise p above v and the profit-maximizing scheme would be a perfect-availability one.

I have also assumed that all consumers show up at the store at the same time and are served randomly with equal probability. In reality, however, especially during popular promotions like Black Friday, consumers line up outside stores before they open. This suggests that consumers' heterogeneity in waiting costs is likely to play a role. Also, those consumers planning to go later in the day would most likely hold different beliefs about their chances of getting the bargain.

Since my model is one-shot, once a consumer arrives at the store and realizes there are no items left for a discounted price anymore, he has to choose between the feeling of loss on the item dimension by returning home empty-handed or the feeling of loss by paying a higher price for a substitute. In reality, the consumer could decide to wait and return to the store some time later. More generally, sales and promotions appear to be periodic and inter-temporal price discrimination on the part of firms is a big part of the story.

It would be interesting to study which results of this model, if any, continue to hold in a (possibly imperfect) competitive environment. Indeed, one of the most striking features of popular sales like Black Friday is that all retailers use limited-availability deals at the same time. At first glance, since Heidhues and Kőszegi (forthcoming) show that their result does not hold in an environment with two retailers selling a homogeneous product and competing à la Bertrand, one might think that also the results of this paper would not survive. However, given the multi-product framework that characterizes my model, firms would have a different strategy-space than in Heidhues and Kőszegi (forthcoming).

The interaction between the retailer and the manufacturing sector, not modeled in this

paper, could also modify the results. For example, if both goods are produced by the same upstream firm, then since the retailer is able to extract more surplus from the consumers through a limited-availability scheme, the firm could try to design a contractual agreement through which she extracts some of this extra surplus. On the other hand, if the goods are produced by two independent manufacturers, the firm producing the good used as a bargain would want to prohibit the retailer from using a limited-availability scheme, since this scheme shifts sales away from the bargain and towards the rip-off.

Chapter 2

Loss Aversion and the "Afternoon Effect"in Sequential Auctions

2.1 Introduction

Sequential auctions are a common practice for the sale of multiple lots of the same or similar goods. How one should expect the price to vary from one round to another? Weber (1983) and Milgrom and Weber (1982) showed that with symmetric, risk-neutral, unit-demand bidders having independent private values, the law of one price should hold and on average prices should be the same across different rounds.[1] Intuitively, if they were not, then demand from the rounds with a higher expected price should shift towards those rounds with a lower expected price, due to arbitrage opportunities. To see why, consider a two-round second-price auction. In the first round, it is optimal for bidders to shade their bids to account for the option value of participating in the subsequent second round. Bidders with a higher valuation also have a higher option value and, therefore, they shade their bids in the first round by a greater amount than do bidders with a lower valuation. As the auction proceeds, the number of bidders decreases. Over the sequence of auctions, the number of objects decreases as well. The first fact has a negative effect on the competition for an object and the second has a positive effect. Remarkably, the equilibrium is such that these two effects exactly offset each other and prices follow a martingale. As a result, all gains to waiting are arbitraged away and the expected prices in both rounds are the same. The latter result also holds for sequential auctions of more than two objects and does not depend on the specific type of auction.

However, this neat theoretical result does not seem to be supported by the data. Ashenfelter (1989), Ashenfelter and Genesove (1992), and McAfee and Vincent (1993) document a puzzling "declining price anomaly" (or "afternoon effect", reflecting that second round auctions often take place in the afternoon whereas the first round is in the morning). Declining price patterns have been also found in Beggs and Graddy (1997), Ginsburgh (1998), Vanderporten (1992a,b), and Van den Berg, Van Ours and Pradhan (2001).[2] Moreover, while

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[1]Technically, the price sequence of any standard auction is a martingale, so that the expected price in round $k + 1$, conditional on p_k, the price in round k, is equal to p_k.

[2]Section 2 summarizes the empirical evidence on the price path in sequential auctions.

declining prices are more frequent, increasing prices have also been documented by Chanel, Gérard-Varet and Vincent (1996), Deltas and Kosmopoulou (2004), and Raviv (2006).[3]

Ashenfelter (1989) hypothesized risk aversion as a plausible explanation for the declining-price pattern. However, McAfee and Vincent (1993) argue that risk aversion does not give a convincing explanation. They studied two-round, private-value, first-price and second-price auctions, and showed that prices decline only if bidders display increasing absolute risk aversion. Under the more plausible assumption of decreasing absolute risk aversion, a monotone symmetric pure-strategy equilibrium fails to exist and prices need not decline.

Black and De Meza (1992) argue that declining prices are no anomaly if the winning bidder in one auction is allowed to purchase all subsequent lots at the same price (but Ashenfelter (1989) finds declining prices also for the case of bidders with unit-demand). Bernhardt and Scoones (1994) and Gale and Hausch (1994) consider sequential auctions of "stochastically equivalent" objects — that is, each bidders' valuations are identically distributed across the objects, but are not perfectly correlated — and show that in this case equilibrium bidding implies declining prices. Von der Fehr (1994) showed that even if bidders are risk-neutral, as long as they incur participation costs, a decline in the second stage selling price would be expected. Other studies have emphasized demand complementarities (Branco, 1997; Menezes and Monteiro, 2003), supply uncertainty (Jeitschko, 1999), and budget constraints (Pitchick and Schotter 1988) in accounting for the declining price anomaly.

In this paper, I argue that expectations-based reference-dependent preferences and loss aversion provide an alternative, preference-based, explanation for the "afternoon effect" observed in sequential auctions. Following the framework developed by Kőszegi and Rabin (2006, 2007), I assume that in addition to classical *consumption utility*, a bidder derives *gain-loss utility* from the comparison of his consumption utility in the product and money dimensions to a reference point equal to her lagged expectations regarding the same outcomes, with losses being more painful than equal-sized gains are pleasant. Moreover, I develop a dynamic extension of the Choice Acclimating Personal Equilibrium (*CPE)* introduced in Kőszegi and Rabin (2007) that I call Sequential Choice Acclimating Personal Equilibrium (S*CPE)*. In a SCPE, a decision maker correctly predicts his (possibly stochastic) strategy at each point in the future, folds-back the game tree using backward induction, and then applies the same (static) CPE as in Kőszegi and Rabin (2007) at every stage of the game.[4]

I show that when bidders have reference-dependent preferences, the equilibrium bidding functions are history-dependent, even if bidders have independent private values. The reason is that learning the type of the winner in the previous auction modifies a bidder's expectations about how likely he is to win in the current auction; and since expectations are the reference

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[3]Milgrom and Weber (1982) showed that if bidders' signals are affiliated and types are interdependent, then the equilibrium price sequence is a sub-martingale and the expected value of p_{k+1}, conditional on p_k, is higher than p_k. Mezzetti (2011) showed that affiliated types are not necessary to explain increasing-price sequences: interdependent types with informational externalities — that is, when a bidder's value is increasing in all bidders' private signals — even with independent signals, push prices to increase across rounds.

[4]Like the original notion of CPE in Kőszegi and Rabin (2007) is related to the models of "disappointment aversion" of Bell (1985), Loomes and Sugden (1986), and Gul (1991), where outcomes are also evaluated relative to a reference lottery that is identical to the chosen lottery, similarly the notion of SCPE is related to the notion of dynamic disappointment aversion proposed in Artstein-Avidan and Dillenberger (2011).

point, the optimal bid in each round is affected by this learning effect.[5] More precisely, I identify a *discouragement* effect: the higher the type of the winner in the previous auction is, the less aggressively the bidding behavior of the remaining bidders in the current auction. This is because from the point of view of a bidder who lost the auction in a previous round, the higher the type of the winner is, the less likely he feels to win in the current auction (conditioning on his own type); this in turn makes the bidder less attached to the idea of winning, therefore reducing his equilibrium bid. Notice also that this history-dependence is different than the one arising in a model with risk-neutral bidders and interdependent values. With interdependent values and informational externalities, since in equilibrium a bidder conditions on himself having the highest signal, if he loses the current auction, he learns that the winner had a higher signal than his; this in turn makes a losing bidder revise his estimate of the good upward and therefore he will bid more aggressively in subsequent auctions. The *discouragement* effect instead goes in the exact opposite direction by pushing bidders to bid more aggressively in the earlier auctions and creates an afternoon effect on the equilibrium price-path.

Furthermore, because the equilibrium bids are history-dependent, from the point of view of the current auction, a player's own bid in future auctions is a random variable , which depends not only on whether the player will actually get to participate in the next auction, but also on what will be the type of the winner in the current one. This uncertainty about future own bids generates a *precautionary bidding* effect that pushes bidders to bid more conservatively in earlier rounds. Therefore, the *precautionary bidding* effect and the *discouragement* effect go in opposite directions.

The seller can directly manipulate the discouragement and precautionary bidding effects, by choosing whether or not to disclose the winning bid in the first auction prior to the second one. When she decides not to disclose, the bidders remaining in the second auction do not receive any update on the valuations' distribution and hence there is no learning, implying that the equilibrium bidding strategies are not history-dependent anymore. In this case, the equilibrium price path is a martingale, like in the classical model with reference-independent preferences. However, I show that it is always in the seller's best interest to disclose the previous winning. Intuitively, if the seller does not, the bidders remaining in the second auction will bid as if the winner of the first auction were the highest possible type, which is equivalent to magnifying the discouragement effect. In this case, in both auctions the bidders behave less aggressively than they would if the seller were to disclose the winning bid in the previous round and, therefore, the seller's expected revenue decreases.

The three most recent papers that are related to mine are Eyster (2002), Kittsteiner, Nikutta and Winter (2004) and Mezzetti (2011). Eyster (2002) models the behavior of an agent who has a taste for rationalizing past actions by taking current actions for which those past actions were optimal; he shows that this taste for consistency gives rise to an "unsunk-

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[5]Jeitschko (1998) studies the role of information transmission and learning in a model of sequential second-price auction in which bidders have independent private values by assuming that the distribution of bidders' values is discrete. Since with a discrete distribution ties among bidders happen with strictly positive probability, a bidder can no longer condition his bid on him having the highest valuation; this in turn trigger the scope for the bidders to update their beliefs about their rivals based on the outcome of the first auction. However in his model the price of the second auction is still expected to be equal to the price of the first auction, regardless of the outcome of the first auction.

cost fallacy" that can rationalize declining prices in sequential second-price auctions.

Kittsteiner, Nikutta and Winter (2004) analyze an independent private values model where a number of objects are sold in sequential first and second-price auctions and where bidders have unit demand and their valuation for an object decreases in the rank number of the auction in which it is sold. They show that the sequence of prices constitutes a supermartingale and, after correcting for the decrease in valuations for objects sold in later auctions, conclude that average prices are declining.

More recently, Mezzetti (2011) introduced a special case of risk aversion, called "aversion to price risk" and showed that under this different notion a monotone pure-strategy equilibrium always exists in sequential auctions with independent private values and prices decline.[6] On the other hand, in the case of interdependent values with informational externalities and no aversion to price risk, prices increase along the equilibrium path of sequential auctions. Finally, when bidders are averse to price risk and values are interdependent, the equilibrium price path depends on which effect dominates.

The remainder of this paper proceeds as follows. Section 2 discusses the empirical literature on sequential auctions and the afternoon effect. Section 3 describes the model and the bidders' preferences and introduces the solution concept of Sequential CPE. Section 4 analyzes two-object sequential first-price auctions. Section 5 studies two-object sequential second-price auctions. Section 6 deals with information revelation. Section 7 concludes by recapping the results of the model and pointing out some of its limitations as well as possible avenues for future research.

2.2 Empirical Evidence

Ashenfelter (1989) analyzed sequential English (ascending) auctions for identical bottles of wine sold in same lot sizes at Sotheby's and Christie's in London, Christie's in Chicago and Butterfield's in San Francisco between August 1985 and December 1987. He found that prices are twice as likely to decrease as to increase. Moreover, for each series of auction he calculated the mean ratio of the price of the second auction to the price of the first one and showed that the ratio less than one and statistically significant (although the ratios vary between 0.99 and 0.96, so the effect is quite small).

Since Ashenfelter (1989), many empirical studies have found evidence of declining prices in auctions for wine. For example, the results reported by McAfee and Vincent (1993), Albert Di Vittorio and Ginsburgh (1994), Ginsburgh (1998) and Février, Roos and Visser (2005) are very similar to the ones in Ashenfelter (1989).[7]

Besides wine auctions, the declining price anomaly has been documented in a number of different types of auctions with different auction structures. Buccola (1982) found it occurring in livestock auctions; Milgrom and Weber (1982) for transponder leases; Thiel and

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[6]Mezzetti's notion of aversion to price risk assumes separability of a bidder's payoff between utility from winning the object and disutility from paying the price.

[7]However, Ginsburgh (1998) argues that the declining price anomaly is likely to be caused by the fact that most bids are entered by absentees, who use nonoptimal bidding strategies. Similarly, Février, Roos and Visser (2005) argue that the reason for the decling price is due to a buyer's option that gives the winner of an auction the right to purchase any number of units at the winning price.

Petry (1990) in stamp auctions; Engelbrecht-Wiggans and Kahn (1992) and Zulehner (2008) for dairy cattle; Lusht (1994) for commercial real estate; Chanel, Gerard-Varet, and Vincent (1996) for gold jewelry; Pesando and Shum (1996) and Beggs and Graddy (1997) for works of art; Pezanis-Christou (2001) for fish auctions; and Ginsburgh and Van Ours (2007) for Chinese porcelain recovered from shipwrecks.

All of the papers mentioned above study sequential English (ascending) or second-price auctions; however, the afternoon effect has been observed also for other auction formats. Ashenfelter and Genesove (1992) compared prices paid for identical condominium units in face-to- face bargaining with prices paid in a "pooled" or "right to choose" auction (a first-price auction in which the winner in each round can choose which good to pick) and found that the auction price was higher than the face-to-face price and depended on the order in which the units were auctioned. The winning bid declined by about 0.27% with each unit sold, and this drop did not seem to be attributable to quality differences between the units sold. Van den Berg, Van Ours, and Pradhan (2001) studied sequential Dutch flower auctions found evidence that prices decline throughout these sequential auctions. Moreover, even in long sequences of auctions, there is a declining price and, at any round, the decline is stronger if the number of remaining units is smaller.

Another explanation relates the price decline to the heterogeneity of objects. Beggs and Graddy (1997), for instance, found declining values and an "afternoon effect" in art auctions. Here winning bids have a tendency to decline relative to estimated market values as the auction progresses. They also showed that in an auction ordered by declining valuations, even with risk-neutral bidders, the price received relative to the estimate for later items in the auction should be less than the price relative to the estimate for earlier items. Ashenfelter and Graddy (2003) contains a general survey that focuses on art auctions.

There is also experimental evidence on declining prices in sequential auctions. Burns (1985) compared the bidding behavior of professional bidders with multi-unit demands in sequential English auction experiments to that of students and found that prices declined less severely in sequences involving students than in sequences involving professional bidders because the latter used heuristics they follow in real-world markets. Keser and Olson (1996) conducted a series of sequential first-price auction experiments. They report negative prices trends and a significant amount of overbidding, suggesting that both the bid and the price predictions of Milgrom and Weber (1982) are not supported by data. More recently, Neugebauer and Pezanis-Christou (2007) conducted a series of experiments in order to test the effects of an uncertain supply on bidding behavior and prices in sequential first-price auctions with independent private values and unit-demand bidders. They also observed significant overbidding in all but the last round, no matter whether supply is certain or not. Moreover, they find trend-free prices when supply is certain, and significant declining price trends when supply is uncertain. Février, Linnemer and Visser (2007) run an experiment on two-unit sequential auctions with and without a buyer's option (which allows the winner of the first auction to buy the second unit) and considered all the four main auction formats. They find that prices are declining when the buyer's option is available.

Several authors have also found increasing prices. Among them are Gandal (1997) for Israeli cable television licenses, and Raviv (2006) who looked at sequential English auction for used cars. Jones, Menezes, and Vella (1996) found that prices could increase or decrease in sequential auctions of wool, as did Chanel, Gerard-Varet, and Vincent (1996) for watches.

Deltas and Kosmopoulou (2004) find in a sale of library books that expected prices increase over the auction, but that probability of sale decreases. They attribute their findings to "catalogue" effects: it is important how and where and item appears in the pre-sale catalogue. Kells (2001) studied sequential English auctions for rare books in Australia and found that price-paths tended to be rising or falling (but not flat) between 1969 and 1978, and that they tended to be flat in the period 1983 to 1999.

Summing up, it is quite an interesting result that in a variety of different types of auctions, price direction throughout an auction can be predicted. Declining prices (on average) have been documented in more types of auctions than have rising prices. Declining prices do not occur in every auction, but they appear to be an important effect that the auction mechanism has on price.

2.3   Model

2.3.1   Auction Environment

Consider a situation in which K identical items are sold to N bidders, $N > K$, using a series of first-price sealed-bid auctions. More specifically, one of the items is sold using a first-price sealed-bid auction and the price at which it is sold is publicly announced. Then, a second item is sold again using a first-price sealed-bid auction and the price at which it is sold is announced. A third item is sold, and so on. Announcing the winning bid from former auctions prior to the current one is in accord with government procurement statutes and with actual practice in some auctions.

I restrict attention to the case in which each bidder wants at most one unit and bidders have independent private values. Each bidder's valuation θ_i, $i = 1, ..., N$, is drawn independently from the same distribution F with positive density f everywhere on the support $\left[0, \overline{\theta}\right]$. Throughout the paper, I restrict attention to symmetric equilibria in pure and monotone strategies.[8] It is convenient to think of the auctions as being held in different periods, however assume the auctions are held in a short enough time so that bidders do not discount payoffs from later auctions.

2.3.2   Preferences

Bidders have expectations-based reference-dependent preferences as formulated by Kőszegi and Rabin (2006). In this formulation, a bidder's utility function has two components. If he wins the auction at price p, a type-θ bidder experiences *consumption utility* $\theta - p$. Consumption utility can be thought of as the classical notion of outcome-based utility. Second, the bidder also derives utility from the comparison of his actual consumption to a reference point given by his recent expectations (probabilistic beliefs).[9] For a riskless consumption

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[8]In this case the items will be sold in order of decreasing values: the first item will go to the bidder with the highest value, the second to the bidder with the second-highest value and so on. In this way efficiency of the allocation is preserved.

[9]Recent experimental evidence lends support to Kőszegi and Rabin's (2006, 2007) expectations-based model of reference-dependent preferences and loss aversion; see for instance Abeler, Falk, Goette, and Huff-

outcome (θ, p) and riskless expectations (r^θ, r^p), the bidder's total utility is given by

$$U\left[(\theta, p) \mid (r^\theta, r^p)\right] = \theta - p + \mu^g\left(\theta - r^\theta\right) + \mu^m\left(r^p - p\right) \tag{2.1}$$

where

$$\mu^l(x) = \begin{cases} \eta^l x & \text{if } x \geq 0 \\ \eta^l \lambda^l x & \text{if } x < 0 \end{cases}$$

is *gain-loss utility,* for $l \in \{g, m\}$.

I assume $\eta^l > 0$ and $\lambda^l > 1$. By positing a constant marginal utility from gains and a constant, but larger marginal disutility from losses, this formulation captures prospect theory's (Kahneman and Tversky 1979, Tversky and Kahneman 1991) loss aversion, but without its diminishing sensitivity. The parameter η^l can be seen as the relative weight a consumer attaches to gain-loss utility, and λ^l can be seen as the coefficient of loss aversion.

According to (2.1), a bidder assesses gains and losses separately over consumption and money.[10] For instance, if his reference point is that he will not win the auction (and thus pay nothing), then he evaluates getting the product and paying for it as a gain in the item dimension and a loss in the money dimension rather than as a single gain or loss depending on total consumption utility relative to his reference point. This is a distinctive feature of the Kőszegi-Rabin's model of reference-dependent preferences.[11] Furthermore, this is consistent with much of the experimental evidence commonly interpreted in terms of loss aversion.[12]

Because in many situations expectations are stochastic, Kőszegi and Rabin (2006) extend the utility function in (2.1) to allow for the reference point to be a pair of probability distribution $H = \left(H^\theta, H^p\right)$ over the two dimensions of consumption utility. In this case a consumer's total utility from the outcome (θ, p) can be written asp

$$U\left[(\theta, p) \mid \left(H^\theta, H^p\right)\right] = \theta - p + \int_{r^\theta} \mu\left(\theta - r^\theta\right) dH^p\left(r^\theta\right) + \int_{r^p} \mu\left(r^p - p_k\right) dH^p\left(r^p\right) \tag{2.2}$$

In words, when evaluating (θ, p), a bidder compares it to each possible outcome in the reference lottery. For example, if he expected to win the auction with probability q and,

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man (2011), Ericson and Fuster (2011), Gill and Prowse (2012), Karle, Kirchsteiger and Peitz (2012), Song (2012), and Sprenger (2011).

[10] The model of Kőszegi and Rabin (2006) assumes that the gain-loss utility function μ is the same across all dimensions. In principle, one could also allow for this function to differ across the item and the money dimension. For example, Novemsky and Kahneman (2005) and Kőszegi and Rabin (2009) argue that reference dependence and loss aversion are weaker in the money than in the item dimension.

[11] The other crucial feature of these preferences, which is that the reference point is determined by the decision maker's forward-looking expectations, is implicit in disappointment-aversion models of Bell (1985), Loomes and Sugden (1986), and Gul (1991). However, because in these models gains and losses are assessed along only one dimension, the consumer's intrinsic utility ($\theta - p$, in this paper), they are unable to predict the type of pricing schemes that is the subject of this paper.

[12] This feauture is able to predict the endowment effect observed in many laboratory experiments (see Kahneman, Knetsch, and Thaler 1990, 1991). The common explanation of the endowment effect is that owners feel giving up the object as a painful loss that counts more than money they receive in exchange, so that they demand a lot of money for the object. But if gains and losses were defined over the value of the entire transaction, owners would not be more sensitive to giving up the object than to receiving money in exchange. Heffetz and List (2011), however, find no evidence that expectations alone play a part in the endowment effect.

conditioning on winning, to pay a price of \$15, then winning the auction feels like a gain of $\theta (1 - q)$, combined with a loss of $\$15 (1 - q)$; similarly, losing the auction results in a loss of θq and a gain of $\$15q$. Therefore, the weight on the loss (gain) in the overall experience is equal to the probability with which he had been expecting to win (lose) the auction.

2.3.3 Solution Concept

Each bidder learns his valuation before submitting his bids and therefore, maximizes his interim expected utility. Hence, if the distribution of the reference points is H and the distribution of consumption outcomes is G, a type-θ bidder's interim expected utility is given by

$$EU [G|H, \theta] = \int_{\{\theta, p\}} \int_{\{r^\theta, r^p\}} U \left[(\theta, p) \, | \, (r^\theta, r^p) \right] dH dG.$$

For each auction in which he participates, after placing a bid, a bidder basically faces a lottery between winning or losing the auction and the probabilities and potential payoffs depend on his own as well other players' bids. The final outcome is then evaluated with respect to any possible outcome from this lottery as a reference point. As laid out in Kőszegi and Rabin (2007), Choice Acclimating Personal Equilibrium (*CPE)* is the most appropriate solution concept for such decisions under risk when uncertainty is resolved after the decision is made so that the decision maker's strategy determines the distribution of the reference point as well as the distribution of final consumption outcomes.

A strategy for a bidder is a collection of bidding functions β_k, $k = 1, .., K$, one for each auction. Notice that in auction k, $1 < k \leq K$, the bidding function of each bidder depends not only on his private type θ but also on the public history of announced winning prices from the previous auctions.

Fixing all other bidders' behavior, a bidder's strategy $\beta_k (\theta, p_{k-1}, ..., p_k)$, induces a distribution, $G (\beta_k (\theta, p_{k-1}, ..., p_k), \beta_k (\theta_{-i}, p_{k-1}, ..., p_k))$ over the set of consumption outcomes $\mathcal{A} := \{0, 1\}^{NK} \times \mathbb{R}_+^{NK}$. Let EU_k denote a bidder expected life-time utility from auction k point of view if he plans to bid according to $\beta_k (\theta, p_{k-1}, ..., p_k)$ and expects his rivals to bid according to $\beta_k (\theta_{-i}, p_{k-1}, ..., p_k)$. To account for the intrinsic dynamics of sequential auctions, I introduce a slightly modified version of *CPE.*

Definition 3 *A sequence of bidding functions* β_k^* , $k = 1, .., K$ *constitutes an interim Sequential Choice Acclimating Personal Equilibrium (SCPE) if for all i, for all* θ_{-i} *and for all* k:

$$EU_k \left[G \left(\beta_k^* (\theta), \beta_k^* (\theta_{-i}) \right) | G \left(\beta_k^* (\theta), \beta_k^* (\theta_{-i}) \right) \right] \geq$$
$$EU_k \left[G \left(\widetilde{\beta}_k (\theta), \beta_k^* (\theta_{-i}) \right) | G \left(\widetilde{\beta}_k (\theta), \beta_k^* (\theta_{-i},) \right) \right]$$

for any $\widetilde{\beta}_k (\theta, p_{k-1}, ..., p_k) \neq \beta_k^* (\theta, p_{k-1}, ..., p_k)$.

The following assumption guarantees that all bidders participate in the auction for any realization of their own type, and that their equilibrium bidding functions derived below are strictly increasing:

Assumption 1 (*No dominance of gain-loss utility in the item dimension*) $\Lambda^g \leq 1$

This assumption places, for a given η^g (λ^g), an upper bound on λ^g (η^g) and ensures that an agent equilibrium expected utility is increasing in his type.[13]

For the remainder of the paper I also impose the following assumption:

Assumption 2 (*No gain-loss utility in the money dimension*) $\Lambda^m = 0$

In Appendix C, I analyze a two-round first-price auctions with $\Lambda^m > 0$ and discuss some of the intricacies that arise in this case.

2.4   Two Items

Consider a situation in which there are only two items to be sold ($K = 2$). In this case, a symmetric equilibrium consists of two bidding functions (β_1, β_2), one for each auction. I assume that both functions are strictly increasing and differentiable. The first-period bidding strategy is a function $\beta_1 : \left[0, \overline{\theta}\right] \to \mathbb{R}_+$ that depends only on the bidder's value. The bid in the second auction, instead, might depend also on the price paid in the first auction. Let $Y_1^{(N-1)} \equiv Y_1$ be the highest of $N - 1$ values, $Y_2^{(N-1)} \equiv Y_2$ be the second-highest and so on. Also, let F_1 and F_2 be the distributions of Y_1 and Y_2 respectively, with corresponding densities f_1 and f_2. Since the first-period bidding function β_1 is assumed to be invertible, after the first auction is over and its winning price is revealed the valuation of the winning bidder is commonly known to be just $y_1 = \beta_1^{-1}(p_1)$. Thus, the second-period strategy can be described as a function $\beta_2 : \left[0, \overline{\theta}\right] \times \left[0, \overline{\theta}\right] \to \mathbb{R}_+$ so that a bidder with value θ bids $\beta_2(\theta, y_1)$ if $Y_1 = y_1$.

To find equilibria that are sequentially rational, let's start by looking at the second period.

2.4.1   Second-period Strategy

Consider a bidder with type θ who plans to bid as if his type were $\widetilde{\theta} \neq \theta$ when all other $N - 2$ remaining bidders follow the equilibrium strategy $\beta_2(\cdot, y_1)$. His expected payoff is

$$
\begin{aligned}
EU_2 = \quad & F_2\left(\widetilde{\theta}|y_1\right)\left[\theta - \beta_2\left(\widetilde{\theta}, y_1\right)\right] \\
& -F_2\left(\widetilde{\theta}|y_1\right)\left[1 - F_2\left(\widetilde{\theta}|y_1\right)\right]\theta\Lambda^g
\end{aligned} \tag{2.3}
$$

where $F_2\left(\widetilde{\theta}|y_1\right)$ is the probability that the second highest valuation, among $N - 1$, is less than $\widetilde{\theta}$ conditional on $Y_1 = y_1$ being the highest and $\Lambda^g \equiv \eta^g(\lambda^g - 1)$ captures loss aversion in the item dimension. Taking FOC of (2.3) with respect to $\widetilde{\theta}$ yields

$$
f_2\left(\widetilde{\theta}|y_1\right)\left[1 - 2F_2\left(\widetilde{\theta}|y_1\right)\right]\theta\Lambda^g = f_2\left(\widetilde{\theta}|y_1\right)\left(\theta - \beta_2\left(\widetilde{\theta}, y_1\right)\right) - \beta_2'\left(\widetilde{\theta}, y_1\right)F_2\left(\widetilde{\theta}|y_1\right)
$$

where β_2' is the derivative of β_2 with respect to its first argument.

---

[13]Herweg, Muller, and Weinschenk (2010) first introduced Assumption 1 for the case with $\Lambda^g = \Lambda^m$ and referred to it as "no dominance of gain-loss utility". In the more general case with $\Lambda^g \neq \Lambda^m$, Assumption 1 has been used also by Lange and Ratan (2010, Eisenhuth (2012) and Eisenhuth and Ewers (2012).

Substituting $\theta = \widetilde{\theta}$ and re-arranging results in the following differential equation

$$\frac{\partial}{\partial \theta} \left\{ \beta_2 (\theta, y_1) F_2 (\theta|y_1) \right\} = f_2 (\theta|y_1) \theta \left\{ 1 - \Lambda^g [1 - 2F_2 (\theta|y_1)] \right\} \tag{2.4}$$

together with the boundary condition that $\beta_2 (0, y_1) = 0$.

Because the different values are drawn independently, we have that

$$F_2 (\theta|y_1) = \frac{F (\theta)^{N-2}}{F (y_1)^{N-2}}$$

and substituting into (2.4) yields

$$\beta_2^* (\theta, y_1) = \frac{\int_0^\theta x \left[1 - \Lambda^g \left(1 - 2 \left(\frac{F(x)}{F(y_1)} \right)^{N-2} \right) \right] f_2 (x) \, dx}{F (\theta)^{N-2}}.^{14}$$

The complete bidding strategy for the second round is to bid $\beta_2^* (\theta, y_1)$ if $\theta < y_1$ and to bid $\beta_2^* (y_1, y_1)$ if $\theta \geq y_1$. The latter might occur if a bidder of type $\theta \geq y_1$ underbid in the first period causing a lower type to win (of course this is an off-equilibrium event).

The first thing worth noticing is that even with independent private values, the optimal bidding strategy in the second period is history-dependent, as it is a function of y_1. With risk-neutral preferences this is not the case:

$$\beta_2^{RN} (\theta, y_1) = \frac{\int_0^\theta x f_2 (x) \, dx}{F (\theta)^{N-2}}.$$

Bidders bid their estimation of the highest valuation of their opponents, conditioning on their valuation being the highest. Because of this conditioning, bids are independent of the prior history of the game.

Furthermore, we have:

Lemma 12 *(Discouragement Effect) If $\Lambda^g \leq 1$, then $\frac{\partial \beta_2^*(\theta,y_1)}{\partial y_1} < 0 \; \forall \theta$.*

Proof.

$$\frac{\partial \beta_2^* (\theta, y_1; \Lambda^g, \Lambda^m)}{\partial y_1} = -\frac{2\Lambda^g f (y_1) (N-2)^2 \int_0^\theta F (x)^{2N-5} f (x) x dx}{F (y_1)^{N-1} F (\theta)^{N-2}}$$

$$< 0. \quad \blacksquare$$

According to the result in Lemma 12, the higher is the type of the winner in the first round, the less aggressively the remaining bidders will bid in the second round. The rationale for this negative effect, which I call the *discouragement* effect, is as follows. From the perspective of a bidder who lost the first auction, the higher the type of the winner, the less likely the bidder is to win in the second auction (conditioning on his own type); with

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[14]As shown in Eisenhuth and Ewers (2012), $\Lambda^g \leq 1$ is sufficient to ensure that $\frac{\partial \beta_2^*(\theta,y_1)}{\partial \theta} > 0 \; \forall \theta$.

expectations-based reference-dependent preferences a bidder who thinks that most likely he is not going to win does not feel a strong attachment effect to the idea of winning and this pushes him to bid more conservatively. Moreover, notice that although revealing the first-period winner's bid (and hence his type) creates an informational externality, the effect on the second-period bids is exactly the opposite of the one that we have with interdependent values. Indeed, with interdependent values the higher the signal of the first-period winner, the higher is the value of the object to all remaining bidders who in turn start bidding more aggressively. Thus, by looking at the distribution of bids in the second auction, one can use the discouragement effect to empirically test the implications of loss aversion against the implications of the classical model with either private values (history independence) or common values (the higher the winning price in the first auction, the more aggressively bidders behave in the second auction).

Furthermore, we have that

$$\frac{\partial \beta_2^* (\theta, y_1; \Lambda^g, \Lambda^m)}{\partial \Lambda^g} = \frac{\int_0^\theta x \left(2 \left(\frac{F(x)}{F(y_1)} \right)^{N-2} - 1 \right) dF(x)^{N-2}}{F(\theta)^{N-2} \left[1 + \Lambda^m \left(1 - \left(\frac{F(\theta)}{F(y_1)} \right)^{N-2} \right) \right]}$$

implying that there exists $0 < \widehat{\theta} < \overline{\theta}$, with $\left(\frac{F(\widehat{\theta})}{F(y_1)} \right)^{N-2} > \frac{1}{2}$, such that $\frac{\partial \beta_2^* (\theta, y_1; \Lambda^g, \Lambda^m)}{\partial \Lambda^g} > 0 \Leftrightarrow \theta > \widehat{\theta}$.

2.4.2 First-period Strategy

Let's look at a particular bidder with type θ who plans to bid as if his type were $\widetilde{\theta} > \theta$ when all other $N - 1$ bidders follow the equilibrium strategy β_1.[15] Further, suppose that all bidders expect to follow strategy the equilibrium $\beta_2^* (\theta, y_1)$ in the second auction, regardless of what happens in the first one (sequential rationality). His expected life-time utility is

$$
\begin{aligned}
EU_1 = \ & F_1 \left(\widetilde{\theta} \right) \left[\theta - \beta_1 \left(\widetilde{\theta} \right) \right] + \int_{\widetilde{\theta}}^{\overline{\theta}} F_2 (\theta | y_1) \left[\theta - \beta_2^* (\theta, y_1) \right] f_1 (y_1) \, dy_1 \\
& - \Lambda^g \theta \left[F_1 \left(\widetilde{\theta} \right) + \int_{\widetilde{\theta}}^{\overline{\theta}} F_2 (\theta | y_1) f_1 (y_1) \, dy_1 \right] \left[1 - F_1 \left(\widetilde{\theta} \right) - \int_{\widetilde{\theta}}^{\overline{\theta}} F_2 (\theta | y_1) f_1 (y_1) \, dy_1 \right]
\end{aligned}
\tag{2.5}
$$

where $F_1 \left(\widetilde{\theta} \right)$ is the probability that the highest valuation, among $N - 1$, is less than $\widetilde{\theta}$, and $F_2 (\theta | y_1)$ and Λ^g are defined as before.

Let's take a minute to describe the terms in (2.5) because this is where SCPE is playing a crucial role. The first line in (2.5) is the sum of expected consumption utilities in period 1 and 2. The second term captures expected gain-loss utility on the product dimension: $F_1 \left(\widetilde{\theta} \right) + \int_{\widetilde{\theta}}^{\overline{\theta}} F_2 (\theta | y_1) f_1 (y_1) \, dy_1$ is the sum of the probability with which a bidder of type

---

[15]The analysis is virtually identical for the case $\widetilde{\theta} < \theta$.

θ expects to win the first auction given that he pretends to be of type $\widetilde{\theta}$ and the period-1 expectation of the probability with which he expects to win in the second auction given that he he pretends to be of type $\widetilde{\theta}$ in the first auction but expects to behave as his real type in the second one.

Taking FOC of (2.5) with respect to $\widetilde{\theta}$ yields

$$
\begin{aligned}
0 = \quad & f_1\left(\widetilde{\theta}\right)\left[\theta - \beta_1\left(\widetilde{\theta}\right)\right] - \beta_1'\left(\widetilde{\theta}\right) F_1\left(\widetilde{\theta}\right) - F_2\left(\theta|\widetilde{\theta}\right)\left[\theta - \beta_2^*\left(\theta,\widetilde{\theta}\right)\right] f_1\left(\widetilde{\theta}\right) \\
& - \Lambda^g \theta \left[f_1\left(\widetilde{\theta}\right) - F_2\left(\theta|\widetilde{\theta}\right) f_1\left(\widetilde{\theta}\right) \right] \left[1 - F_1\left(\widetilde{\theta}\right) - \int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1 \right] \\
& - \Lambda^g \theta \left[F_1\left(\widetilde{\theta}\right) + \int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1 \right] \left[-f_1\left(\widetilde{\theta}\right) + F_2\left(\theta|\widetilde{\theta}\right) f_1\left(\widetilde{\theta}\right) \right]
\end{aligned}
$$

Notice that $f_1\left(\theta\right) = \left(N - 1\right) f\left(\theta\right) F\left(\theta\right)^{N-2}$. Then, substituting $\theta = \widetilde{\theta}$ and re-arranging results in the following differential equation

$$
\frac{d}{d\theta}\left\{\beta_1\left(\theta\right) F_1\left(\theta\right)\right\} = f_1\left(\theta\right) \beta_2^*\left(\theta,\theta\right)
$$

together with the boundary condition that $\beta_1\left(0\right) = 0$. Solving the differential equation yields

$$
\beta_1^*\left(\theta\right) = \frac{\int_0^\theta \beta_2^*\left(s,s\right) f_1\left(s\right) ds}{F_1\left(\theta\right)}. ^{16} \tag{2.6}
$$

The first thing worth noticing is that $\beta_1^*\left(\theta\right)$ depends on Λ^g only indirectly, through $\beta_2^*\left(s,s\right)$. This happens because, just like in the standard case with reference-free preferences, at the optimum a bidder conditions his bid on him having the highest type. Hence, a bidder expects that if he were to lose the current auction, he would win the next one for sure and this is why expected gain-loss utility on the item dimension does not directly appear into the first period bidding function.

It is easy to check that for $\Lambda^g = 0$ we get back to the risk-neutral benchmark:

$$
\beta_1^{RN}\left(\theta\right) = \frac{\int_0^\theta \beta_2^{RN}\left(s,s\right) f_1\left(s\right) ds}{F_1\left(\theta\right)}
$$

where $\beta_2^{RN}\left(s,s\right)$ does not depend of the type of the winner of the previous auction.

Let $y_1 = \beta_1^{-1}\left(p_1\right)$. Then we have that the expected equilibrium price in the second auction conditional on the type of the winner of the first auction is

$$
\mathbb{E}\left[p_2|p_1\right] = \mathbb{E}\left[p_2|\beta_1\left(y_1\right)\right] = \mathbb{E}\left[\beta_2^*\left(Y_1^{(N-1)}, y_1\right) | Y_1^{(N-1)} \le y_1\right] = \frac{\int_0^{y_1} \beta_2^*\left(\theta, y_1\right) f_1\left(\theta\right) d\theta}{F_1\left(y_1\right)}.
$$

The following proposition delivers the first main result of the paper:

---

[16]It is easy to verify that t $\frac{\partial \beta_1^*\left(\theta\right)}{\partial \theta} > 0 \; \forall \theta$.

Proposition 10 *(Afternoon Effect) Assume $\Lambda^m = 0$. The price sequence in a two-round first-price auction is a supermartingale and the afternoon effect arises in equilibrium. That is,*

$$p_1 = \beta_1^* (y_1) > \mathbb{E}\left[p_2 | \beta_1^* (y_1)\right] = \mathbb{E}\left[p_2 | p_1\right].$$

Proof. For $\Lambda^m = 0$, we immediately have that

$$
\begin{aligned}
\beta_1^* (y_1) &= \frac{\int_0^{y_1} \beta_2^* (\theta, \theta) f_1 (\theta) \, d\theta}{F_1 (y_1)} \\
&> \frac{\int_0^{y_1} \beta_2^* (\theta, y_1) f_1 (\theta) \, d\theta}{F_1 (y_1)} \\
&= \mathbb{E}\left[p_2 | p_1\right]. \qquad \blacksquare
\end{aligned}
$$

The intuition behind Proposition 10 is that, just like in the reference-independent case, in equilibrium bidders must be indifferent in expectation between winning in the first auction or in the second one. Hence, in the first auction a bidder bids the expectation of his own second-round bid conditional on having the highest type and being the price-setter, that is on being pivotal. However, by conditioning his first-period bid on being pivotal a bidder expects not to feel any discouragement effect in the second auction. And because the discouragement effect depresses bids in the second auction, he expects his own future bid to be higher than his (unconditional) expected bid.

It is interesting to compare the logic behind the discouragement effect with the one that arises in common value-auctions with informational externalities. In the symmetric equilibrium of a common-value auction, a bidder conditions his estimate of the value of the item on his rivals having the same signal as his. Hence, when bidding in the first round a bidder's expectation of his own future bid is lower than his (unconditional) expected bid. In this case, since a bidder revise his estimate of the value of the good upward when losing the first auction, the equilibrium price drifts upward. Conversely, with the informational externalities that arise in a private-value auction when bidders are expectations-based loss-averse, when losing the first auction a bidder becomes more pessimistic about how likely he is to win the second one (compared to his ex-ante expectations); this creates a discouragement effect that pushes bidders to behave less aggressively and, in turn, generates a declining price path in equilibrium.

2.5   Second-Price Auctions

In this section I assume that 2 identical items are sold using a series of second-price sealed-bid auctions. I will keep looking for symmetric equilibrium strategies that are increasing and suppose that, prior to bidding in a particular auction, the winning bids of all previous auctions are common knowledge.[17] Again, let's begin by looking at the second period.

---

[17]Notice that in a second-price sealed-bid auction the winning bid is not the price the winner actually ends up paying. This an important point because if the seller were to reveal the winning price of the first auction, then the bidders would infer the type of the highest remaining bidder and a symmetric equilibrium in monotone strategies would fail to exist.

2.5.1 Second-period Strategy

Fixing the bidding strategies of the other players, let $H(b_2|y_1)$ denote the probability with which a particular bidder expects to win with a bid equal to b_2 conditional on y_1 being the type of the first-round winner. The payment he has to make if he wins the auction is given by the second largest bid and follows the distribution $H(b|y_1)$. Then, the bidder's expected utility is

$$EU_2 = \int_0^{b_2} (\theta - p)\, dH(p|y_1) - H(b_2|y_1)[1 - H(b_2|y_1)]\,\theta\Lambda^g \tag{2.7}$$

Taking FOC of (2.7) with respect to b_2 yields

$$\theta - b_2 - \theta\Lambda^g[1 - 2H(b_2|y_1)] = 0.$$

In a symmetric equilibrium, $H(b_2|y_1) = F_2(\theta|y_1)$ and hence we obtain:

$$b_2^*(\theta, y_1) = \theta\left\{1 - \Lambda^g\left[1 - 2\frac{F(\theta)^{N-2}}{F(y_1)^{N-2}}\right]\right\}.$$

First of all, notice that while it is well known that without loss aversion ($\Lambda^g = 0$) equilibrium bidding is truthful, the above expression shows immediately that this is not the case with reference-dependent preferences.[18] Furthermore, we have: Furthermore, we have:

Lemma 13 *(Discouragement Effect) If $\Lambda^g \leq 1$, then $\frac{\partial b_2^*(\theta, y_1)}{\partial y_1} < 0\ \forall \theta$.*

Proof.

$$\frac{\partial b_2^*(\theta, y_1)}{\partial y_1} = -\frac{2\Lambda^g(N-2)\theta F(\theta)^{N-2}}{F(y_1)^{N-1}}$$
$$< 0. \ \blacksquare$$

The intuition behind Lemma 13 is the same as for Lemma 12: the higher the type of the winner in the first auction, the less likely the remaining bidders are to win the second auction and therefore they bid less aggressively.

2.5.2 First-period Strategy

As shown by Lange and Ratan (2010), if bidders are not averse to losses in the money dimension first-price and second-price auctions are revenue-equivalent. Hence, we can use the revenue equivalence principle to derive the first-round equilibrium.

In the first auction a type-θ bidder wins with probability $F_1(\theta)$ and, if he wins, the price he pays is $b_1^*(y_1)$, the bid of the highest rival. Thus, his expected payment in the first round is

$$F_1(\theta)\int_0^\theta b_1^*(y_1) f_1(y_1|\theta)\, dy_1.$$

---

[18] As for the first-price auction, $\Lambda^g \leq 1$ is a sufficient condition for $b_2^*(\theta, y_1)$ to be strictly increasing in θ.

In a first-price auction, instead, the winning bidder pays his own bid and therefore his expected payment in the first round is:

$$F_1(\theta)\beta_1^*(\theta) = F_1(\theta)\left[\frac{\int_0^\theta \beta_2^*(s,s)f_1(s)\,ds}{F_1(\theta)}\right]$$

where the equality follows from (2.6). From revenue equivalence it follows that

$$\int_0^\theta b_1^*(y_1)f_1(y_1|\theta)\,dy_1 = \frac{\int_0^\theta \beta_2^*(s,s)f_1(s)\,ds}{F_1(\theta)}$$

and differentiating both sides of the equality with respect to θ yields

$$b_1^*(\theta) = \beta_2^*(\theta,\theta).$$

Therefore, the equilibrium bid in the first of a two sequential second-price auctions is equal to the second round's bid of two sequential first-price auction where, in the latter, the bidder conditions on him having the highest type.

Finally, notice also that the afternoon effect arises in equilibrium since, by revenue equivalence, in each round the seller's expected revenue from a second-price auction is equal to the expected revenue from a first-price auction.

2.6 The Effect of Information on the Seller's Revenue

In the classical reference-free model, the optimal bidding strategy is memory-free since it does not depend on the (public) history of the winning prices. As shown in Section 4, this is no longer the case with expectations-based reference-dependent preferences. Hence, a question that arises naturally is: Would the seller be better off by not releasing any information about the winner of the previous auction? I answer this question in the context of sequential first-price auctions.

If at the end of the first auction the seller does not reveal the winning bid, then the remaining $N-2$ bidders face the following problem in the second auction:

$$\max_{\widetilde{\theta}} F_2\left(\widetilde{\theta}\right)[\theta - \beta_2(\theta)] - \Lambda^g\theta F_2\left(\widetilde{\theta}\right)\left[1 - F_2\left(\widetilde{\theta}\right)\right].$$

It is easy to see that the above problem does not depend on the outcome of the first auction. In this case, the equilibrium bidding strategy is simply

$$\overline{\beta}_2(\theta) = \frac{\int_0^\theta x\left[1 - \Lambda^g\left(1 - 2F(x)^{N-2}\right)\right]f_2(x)\,dx}{F(\theta)^{N-2}}$$

which coincides with the equilibrium bid derived in Section 4 for $F(y_1) = 1 \Leftrightarrow y_1 = \overline{\theta}$; that is, when the bidders do not get any information about where they are in the ranking of the remaining bidders' values. The following lemma shows that the remaining bidders in the second auction behave less aggressively in the second auction when they are not provided with any information about the winner of the first auction:

Lemma 14 *(Effect of information I) Equilibrium bidding in the second auction is less aggressive when the bidders participating in the second auction are not provided with any information concerning the winner of the first auction.*

Proof. By Lemma 12, $\frac{\partial \beta_2^*(\theta, y_1)}{\partial y_1} < 0$ and since $\overline{\beta}_2(\theta) = \beta_2^*(\theta, \overline{\theta})$, it follows that $\overline{\beta}_2(\theta) \leq \beta_2^*(\theta, y_1)$ and the inequality is strict for $y_1 < 1$ ∎

One way to interpret Lemma 14 is that by not revealing the winning bid of the first auction, the seller is maximizing the discouragement effect.

It is easy to see then that the equilibrium strategy in the first auction is given by

$$\overline{\beta}_1(\theta) = \frac{\int_0^\theta \overline{\beta}_2(s) f_1(s) \, ds}{F_1(\theta)}.$$

The following lemma shows that bidders behave less aggressively in the first auction as well.

Lemma 15 *(Effect of information II) Equilibrium bidding in the first auction is less aggressive when the bidders anticipated that in the second auction they will not be provided with any information concerning the winner of the first auction.*

Proof. We immediately have that

$$\begin{aligned}
\beta_1^*(\theta) &= \frac{\int_0^\theta \beta_2^*(s, s) f_1(s) \, ds}{F_1(\theta)} \\
&> \frac{\int_0^\theta \overline{\beta}_2(s) f_1(s) \, ds}{F_1(\theta)} \\
&= \overline{\beta}_1(\theta)
\end{aligned}$$

where the inequality follows from Lemma 14 ∎ Since bidders behave less aggressively in both auctions without information revelation, it immediately follows that the seller's revenue is lower in this case. Hence, she would always prefer to reveal the winning bid in each round. Moreover, it is also easy to see that without information revelation the equilibrium price path is a martingale so that $\mathbb{E}[p_2|p_1] = p_1$. Of course since the seller is not publicly announcing the winning price of the first auction, she would be the only one to know this.

2.7 Conclusions

In this paper I have studied sequential sealed bid auctions for two identical objects with symmetric bidders having independent and private values and I have proposed a novel, preference-based explanation, for the afternoon effect observed in sequential auctions by positing that bidders are expectations-based loss-averse.

Expectations-based reference-dependent preferences induce an informational externality between auctions so that the equilibrium strategies are history-dependent even when bidders have independent private values. Intuitively, learning the type of the winner in the previous

auction modifies a bidder's expectations about how likely he is to win in the current auction; and since expectations are the reference point, the optimal bid in each round is affected by this learning effect.

I identify what I call the discouragement effect: the higher is the type of the winner in the first auction, the less aggressively the remaining bidders will bid in the second one. Indeed, from the perspective of a bidder who lost the first auction, the higher the type of the winner, the less likely he is to win in the second auction (conditioning on his own type); with expectations-based reference- dependent preferences a bidder who thinks that most likely he is not going to win does not feel a strong attachment effect to the idea of winning and this pushes him to bid more conservatively. Moreover, although revealing the first auction winning bid creates an informational externality, the effect on the second auction bids is exactly the opposite of the one that arises in models with common values. Indeed, with common values the higher the signal of the first-period winner, the higher is the value of the object to all remaining bidders who in turn start bidding more aggressively. Thus, by looking at the distribution of bids in the second auction, one can use the discouragement effect to empirically test the implications of loss aversion against the implications of the classical model with either private values (history independence) or common values (the higher the winning price in the first auction, the more aggressively bidders behave in the second auction).

The main result of the paper is to show that by pushing bidders to bid more aggressively in the earlier auction, the (anticipation of the) discouragement effect creates a declining price-path in equilibrium. So reference-dependent preferences with expectations as the reference point can rationalize the afternoon effect. Furthermore, I also show that revealing the winning bid in the first auction is necessary for the afternoon effect to arise in equilibrium because otherwise there would be no informational externalities, just like in the classical model with reference-independent preferences. However, it is in the seller's own interest to disclose the previous auction's winning bid as this increases her expected revenue.

Despite being able to rationalize the afternoon effect and to generate new testable predictions about the equilibrium distribution of bids, my model suffers from some limitations that I hope to address in future work. First, I have departed from the original model of expectations-based reference-dependent preferences proposed by Kőszegi and Rabin (2006, 2007) by assuming that bidders are loss-averse only over consumption, but not over money. This assumption considerably simplifies the analysis because the role of loss aversion in money is not clear-cut; for some details about the (more realistic) case in which bidders are loss-averse over both consumption and money, see Appendix C. Furthermore, with loss aversion over money, first and second-price auctions are not revenue equivalent anymore and the analysis of the second-price auction becomes much more intricate.

Ideally, one would like to characterize the equilibrium bidding strategies for general K-period auctions. However, since the bidders' optimization problem is not stationary, one cannot employ conventional recursive techniques. Indeed, augmenting the number of periods/items changes a bidder's reference point and makes the analysis much more intricate.

In some of the auctions discussed in the Introduction and Section 2 the goods up for sale are not sought after by the bidders for their consumption value, as much as for commercial purposes (i.e., a production or a resale motive). If this is the case, then what bidders care about is the monetary value of the good and therefore a model of reference-dependent

preferences in which gains and losses are evaluated with respect to the overall gains from trade ($\theta - p$) might be more appropriate than the one uses in this paper where gains and losses are evaluated separately over consumption value and money.

Finally, one could also compare expected revenues between sequential and simultaneous auctions. Since in simultaneous auctions there is no information revelation, I expect sequential auctions to be superior since, as shown in Section 6, the seller's revenue increases when she disclose information that allows the bidders to update their beliefs about how likely they are to win an object.

Chapter 3

Sequential Bargaining with Reference-Dependent Preferences

3.1 Introduction

Many factors might infuence the positions people take when negotiating and, in order to proceed, each side must adjust her position throughout the negotiation, ultimately arriving at either agreement or impasse. Most observed bargaining patterns typically involve gradual adjustments of positions. In this paper I analyze a two-period bargaining game between a seller and a buyer in which the latter has expectations-based reference-dependent preferences as proposed by Kőszegi and Rabin (2006). Since in bargaining negotiations no party can credibly commit in advance to a given strategy, the expectations about the possible outcomes of the bargaining process with which each party enters the negotiation play a crucial role in assessing the outcome of the negotiation itself. To what extent do the expectations with which a player enters a negotiation matter?

I analyze a two-period one-sided offer bargaining game with incomplete information on the seller's side in which at the end of each period there is an exogenous probabability of breakdow. The main result of the paper is that loss aversion eases the rent-efficiency trade-off for the seller who can now serve a larger number of consumers at an earlier stage. Intuitively, high-valuation loss-averse buyers are willing to pay a high price in the first period to avoid the risk of negotiations' breakdown. Thus, in equilibrium the seller achieves higher profits and we have less delay with loss aversion than without it. Furthermore, I also show that, besides increasing the seller's profit and overall trade efficiency, loss aversion also reallocates surplus among consumers by increasing the equilibrium payoff of some low-valuation buyers and decreasing that of high-valuation ones.

The paper proceeds as follows. Section 2 discusses the related literatures on Psychological Game Theory and Bargaining Theory. Section 3.1 introduces the model and describes the buyer's preferences. Section 3.2 derives the equilibrium of the model for the case in which the buyer's private information is described by a two-point distribution. Section 3.3 consider the case of continuously distributed values. Section 4 concludes.

3.2 Related Literature

This paper is related to three areas of research in economic theory: psychological games, non-cooperative models of bargaining with incomplete information and models that introduce loss aversion and reference-dependent preferences into strategic environments. In what follows, I briefly establish the connection to each of these separately.

As reference-dependent preferences à la Kőszegi and Rabin (2006) are an example of belief-based preferences, they cannot be extended to strategic environments using traditional game-theoretic models where utilities depend only on actions. Geanakoplos, Pearce and Stacchetti (1989) proposed the notion of a psychological game, which may be seen as a generalization of a traditional game. A psychological game differs from a traditional game in that utilities are defined on the players' beliefs as well as on the players' actions. Battigalli and Dufwenberg (2009) generalize the original notion of a psychological game by allowing a player's utility to depend also on updated higher-order beliefs, beliefs of other players and own plans of action.

With respect to non-cooperative models of bargaining with incomplete information, Fudenberg and Tirole (1983) studied a two-period bargaining model with the traders' private information described by a two-point distribution. The simple information structure and limited horizon enabled them to characterize the set of all sequential equilibria, and conclude that few (if any) comparative static results are true in all bargaining situations. Sobel and Takahashi (1983) analyzed an infinite-horizon model with one-sided uncertainty in which the uninformed bargainer makes all the offers. By having the uninformed trader make all the offers, Sobel and Takahashi are able to avoid the complications of strategic communication that arise when a player with private information makes offers.

Following Fudenberg and Tirole (1983) and Sobel and Takahashi (1983) a large number of models of this kind have been investigated in the literature. Fudenberg, Levine and Tirole (1985) study an infinite-horizon model with one-sided incomplete information whereas Cramton (1984) analyzes an infinite-horizon model with two-sided uncertainty. Admati and Perry (1987) analyze a bargaining model with one-sided incomplete information in which the time between offers is an endogenous strategic variable and find equilibria involving a delay to agreement that is due to the use of strategic time delay by bargainers to signal their relative strength. Cramton (1992) extends Admati and Perry's (1987) analysis of bargaining with one-sided uncertainty and two possible types to a setting of two-sided uncertainty and a continuum of types. Ausubel, Cramton and Deneckere (2001) provide an overview of the theoretical and empirical literature on bargaining with incomplete information.[1] However, as I am dealing mainly with a two-period model with one-sided uncertainty, my model and results should be compared with those of Fudenberg and Tirole (1983) and Sobel and Takahashi (1983).

Finally, two earlier papers that have introduced reference-dependent preferences into bargaining settings deserve also to be mentioned. Shalev (2002) extends Nash (1950)'s classical model of cooperative bargaining to incorporate loss aversion and finds that, as for risk aversion, increasing loss aversion for a player leads to worse outcomes for that player in

---

[1]There is a close connection between the literature on bargaining with incomplete information and the one on dynamic monopoly and the Coase conjecture. See Gul, Sonnenschein and Wilson (1986).

bargaining situations. This result emerges also in an alternating-offer game à la Rubinstein (1982) in which loss aversion modifies the notion of time preferences (loss aversion translates into higher impatience) and the reference point is the value of the outcome in the previous period.

More recently, Compte and Jehiel (2007) introduced reference-dependent preferences into a bargaining game with complete information à la Rubinstein (1982). In their model the reference point is determined by the positions adopted by the players in prior bargaining phases. Although these positions are endogenously determined in equilibrium, the function that specifies how the reference point depends on past positions is exogenous.

My analyisis differs from the ones cited above as (i) I consider only finite-horizon games in which one player, the seller, makes all the offers; (ii) I consider a game with one-sided incomplete information and (iii) I assume a player's reference point to be equal to the expectations with which the player enters the bargaining game

3.3   Model

Consider a risk-neutral seller (she) and a loss-averse buyer (he) bargaining over the trade of one unit of a good. The seller has a known production (or opportunity) cost of 0. The buyer has a private valuation for the good equal to v. The probability distribution $F(v)$, which is common knowledge, represents the beliefs that the seller has about the buyer's valuation.

The buyer is expectations-based loss-averse but here I depart from the original formulation of reference-dependent preferences in Koszegi and Rabin (2006) as I assume that the gain-loss component of the buyer's overall utility is defined over the overall gain from trade (and not on each dimension of consumption utility). For example, suppose the buyer expects to buy for sure at price p_0; then, her utility from buying at price p is given by

$$U(x|x_0) = \begin{cases} x + \eta(x - x_0) & \text{if} \quad x \geq x_0 \\ x + \eta\lambda(x - x_0) & \text{if} \quad x < x_0 \end{cases}$$

with $x = v - p$, $x_0 = v - p_0$, $\eta > 0$ and $\lambda > 1$.

The timing of the game is as follows. At $t = 0$, the buyer learns his valuation and forms his plans which will in turn determine his reference point in the subsequent periods. At $t = 1$, the seller sets a price p_1 for the current period. The buyer either buys in the first period, in which case the game ends, or rejects the offer. In the latter case, with probability $(1 - \delta)$ the game ends and with probability δ the buyer and the seller meet again at $t = 2$ when the seller sets another price p_2 and the buyer either buys or rejects the offer; in either case, the game then ends. I assume that the buyer's reference point does not change between periods. I also assume that neither the buyer nor the seller can commit in advance to a given strategy (i.e., the buyer cannot commit not to buy and the seller cannot commit to a given price nor to a price distribution).

In the remainder of this section I explain why re-defining gain-loss utility on the overall consumer's surplus enables the game to have a meaningful equilibrium. The next two sections deal with two particular information structures.

First of all, notice that with gain-loss utility defined on overall utility a buyer would never buy at a price above his intrinsic valuation for the item. Morevoer, a buyer planning not to buy will follow his plan only for prices above his intrinsic valuation (assuming that when exactly indifferent, the buyer always buys).

At time 0, the expected utility of a buyer who plans to buy for sure in period 2 at a price p_2 is given by

$$EU \text{ (buy at } t = 2|\text{buy at } t = 2) = \delta (v - p_2) [1 - (1 - \delta) \eta (\lambda - 1)].$$

So, planning to buy at $t = 2$ at $p_2 = v$ delivers zero expected utility (always) whereas planning to buy at $p_2 < v$ might deliver negative expected utility if $(1 - \delta) \eta (\lambda - 1) > 1$.

Since this possibility looks rather unattractive and also for reasons that will be come clearer later in the analysis, in what follows I shall assume that $\eta (\lambda - 1) < 1$ (notice this is a sufficient but not necessary condition to ensure that the expected utility from planning to buy in period 2 at $p_2 \leq v$ is non-negative).[2]

Assumption 1 *(No dominance of gain-loss utility)* $\eta (\lambda - 1) < 1$.

Finally, from period 0 planning to buy for sure in period 1 at price p_1 delivers higher expected utility than planning to buy for sure in period 2 at p_2 if and only if[3]

$$v - p_1 \geq \delta (v - p_2) [1 - (1 - \delta) \eta (\lambda - 1)]$$

$$\Longleftrightarrow v (1 - \delta) [1 + \delta \eta (\lambda - 1)] + p_2 \delta [1 - (1 - \delta) \eta (\lambda - 1)] \geq p_1. \tag{3.1}$$

However, once the buyer has planned to buy in period 1 at price p_1 she will not deviate in order to buy in period 2 at p_2 if and only if

$$v - p_1 \geq \delta (v - p_2) + \delta \eta (p_1 - p_2) - \eta \lambda (1 - \delta) (v - p_1)$$

$$\Longleftrightarrow \frac{v (1 - \delta) (1 + \eta \lambda) + p_2 \delta (1 + \eta)}{1 + \eta \delta + \eta \lambda (1 - \delta)} \geq p_1. \tag{3.2}$$

It is easy to verify that (3.2) implies (3.1) (whenever $v \geq p_2$), so that the "real constraint" is that the buyer does not want to deviate from his plan in period 1.[4]

Therefore, when gain-loss utility is defined on "overall gains from trade", $v - p$, and not separately for each dimension of consmption utility, the highest price at which the buyer is

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[2]Notice also that if we were to assume $\eta (\lambda - 1) > 1$ then the only optimal consistent plan (Personal Equilibrium) involving buying in period 2 would be that of buying at $p_2 = v$.

To see this, let $\eta = 1$, $\lambda = 3$ and $\delta = \frac{1}{4}$.

Then the plan to buy at $t = 2$ for sure at a price p_2 would provide the buyer with strictly negative expected utility $\forall p_2 < v$ (and exactly zero expected utility for $p_2 = v$). So the buyer would prefer to plan not to buy at $t = 2$ $\forall p_2 < v$. However, this plan is not consistent as when the buyer enters period 2 expecting not to buy she will instead buy whenever $p_2 \leq v$.

[3]If the seller is sequentially rational, it has to be that $p_1 \geq p_2$.

[4]This follows from the fact that the buyer is willing to pay more ex-ante (at $t = 0$) to avoid the risk of going to period 2 than he is ex-post (at $t = 1$). In other words, the buyer is more risk-averse ex-ante than ex-post (see Koszegi and Rabin, 2007).

willing to buy ex-post and the highest price for which he would plan to buy ex-ante coincide and are equal to v. This in turn eliminates the incentive for the seller to raise the price above v ex-post and, as shown in the next two sections, allows for the game to have a well defined equilibrium.

3.3.1 Two-type Case

The buyer's private valuation for the good is equal to v_H with probability π and to $v_L < v_H$ with probability $1 - \pi$.

3.3.1.1 Period 2

In equilibrium, the game can go to period 2 only if (at least) one type of buyer has planned to buy in this period with positive probability.

The highest price for which a buyer of type $\theta \in \{H, L\}$ would plan of buying in period 2 and follow his plan is $p_2 = v_\theta$.

If both types of buyers have planned to buy at $t = 2$, then the seller needs to choose the p_2 that maximizes her expected payoff.

The two relevant options are $p_2 = v_H$ in which case only a high type buyer would buy and $p_2 = v_L$ in which case both types of buyer would buy. Let $\mu \equiv \Pr(v = v_H | \text{history at } t = 2)$ denote the beliefs about the likelihood of facing a buyer of type H with which the seller enters period 2.

Then we have

$$p_2^* = \begin{cases} v_H & \text{if} \quad \mu \geq \frac{v_L}{v_H} \\ v_L & \text{if} \quad \mu < \frac{v_L}{v_H} \end{cases}.$$

3.3.1.2 Period 1

Let's distinguish two cases for the seller's prior beliefs, π.

If $\pi < \frac{v_H}{v_L}$ then the seller will play $p_2^* = v_L$ in the second period. Let this be the case of a "soft" seller. On the other hand, if $\pi \geq \frac{v_H}{v_L}$, then μ can be either above or below $\frac{v_H}{v_L}$ depending on the buyer's first period strategy. Let this be the case of a "tough" seller. In what follows I shall analyze the two cases separately, starting with the "soft" seller's one.

Soft Seller There are two possible sub-cases:

i) No-screening: The seller sells to both types at $t = 1$ and attains a profit of v_L.

ii) Screening: The seller sells to the high type at $t = 1$ and to the low type at $t = 2$, attaining profit of $\pi p_1^* + (1 - \pi) \delta p_2^*$.

For the Screening case, it must be that $p_2^* = v_L$ and

$$p_1^* = \frac{v_H (1 - \delta)(1 + \eta\lambda) + v_L \delta (1 + \eta)}{1 + \eta\delta + \eta\lambda (1 - \delta)}.$$

The Screening case provides the seller with a higher profit if and only if

$$\pi \left[\frac{v_H (1 - \delta)(1 + \eta\lambda) + v_L \delta (1 + \eta)}{1 + \eta\delta + \eta\lambda (1 - \delta)} \right] + (1 - \pi) \delta v_L \geq v_L$$

$$\Longleftrightarrow \pi \geq \frac{v_L \left[1 + \eta\lambda\left(1 - \delta\right) + \eta\delta\right]}{v_H\left(1 + \eta\lambda\right) - v_L\eta\delta\left(\lambda - 1\right)} \equiv \widehat{\pi}. \tag{3.3}$$

Notice that $\widehat{\pi} < \frac{v_L}{v_H}$.

Thus

$$(p_1^*, p_2^*) = \begin{cases} \left(\frac{v_H(1-\delta)(1+\eta\lambda)+v_L\delta(1+\eta)}{1+\eta\delta+\eta\lambda(1-\delta)}, v_L\right) & \text{if} \quad \pi \geq \widehat{\pi} \\ (v_L, v_L) & \text{if} \quad \pi < \widehat{\pi} \end{cases}.$$

In Fudenberg and Tirole (1983), when buyer and seller have the same discount factor, the No-screening case is always more profitable for a soft seller. Here, instead, the seller and the buyer share the same exogenous probability of breakdown δ, but the Screening case might be superior because due to the gain-loss component of the utility the seller can extract more rent from the high-type without reducing the rent she extracts from the low-type.

Tough Seller Now, I look at the case of a "tough" seller. There are two possible sub-cases:[5]

i) The seller can behave as if she were "soft".

ii) The seller and the high-type buyer both play a mixed-strategy.

Given that i) is identical to the analysis of a "soft" seller, let's look at sub-case ii).

Let

$$p_1^{\text{soft}} = p_1^* = \frac{v_H\left(1 - \delta\right)\left[1 + \eta\lambda\right] + v_L\delta\left[1 + \eta\right]}{1 + \eta\delta + \eta\lambda\left(1 - \delta\right)}.$$

Notice that p_1^{soft} is the highest price at which a buyer of type H would plan to buy in period 1 when he correctly anticipates that $p_2 = v_L$ and v_H is the highest price at which a buyer of type H would buy in period 1 when he correctly anticipates that $p_2 = v_H$.

Now consider the case $p_1 \in \left(p_1^{\text{soft}}, v_H\right]$.

In this case, a buyer of type H must mix in period 1.

To see this, suppose she was buying for sure at $p_1 \in \left(p_1^{\text{soft}}, v_H\right]$. Then, μ would be equal to zero and $p_2^* = v_L$. But then it would have been better for the buyer of type H not to buy in period 1.

On the other hand, suppose $p_1 \in \left(p_1^{\text{soft}}, v_H\right]$ and she does not buy in period 1. Then, $\mu = \pi \geq \frac{v_L}{v_H}$ so that $p_2^* = v_H$. But then it would have been better for the buyer of type H to buy in period 1.

So, a buyer of type H must in mix in period 1. And he has to do it in a way such that the seller in period 2 is exactly indifferent between charging $p_2^* = v_L$ and charging $p_2^* = v_H$. Let $\beta_{1,H}$ be the probability that a type-H buyer buys in the first period, then

$$\mu = \frac{\left[1 - \beta_{1,H}\right]\pi}{\left[1 - \beta_{1,H}\right]\pi + 1 - \pi} = \frac{v_L}{v_H} \Longrightarrow \beta_{1,H}^* = 1 - \frac{v_L\left(1 - \pi\right)}{\pi\left(v_H - v_L\right)}.^{6}$$

Notice that $\beta_{1,H}^*$ does not depend on p_1.

---

[5]There is actually a third possible sub-case in which the seller sells only to the high-type at $t = 2$ and attain an expected profits of $\delta\pi p_2$. However, it is easy to see that the profit in this case would be strictly less than what the seller could achieve by just playing "soft".

[6]It is easy to verify that $0 < \beta_{1,H}^* < 1$ whenever the seller is "tough".

However, for a buyer of type H to be willing to mix in period 0, she must be indifferent between planning to buy in period 1 and planning to buy in period 2.

Let α_2 be the probability that the seller charges v_H and $(1 - \alpha_2)$ be the probability that she charges v_L in period 2.

If he plans to buy in period 1 at $p_1 \in \left(p_1^{\text{soft}}, v_H\right]$ and follows his plan, a high-type buyer achieves a utility level of

$$v_H - p_1$$

whereas if he plans to buy in period 2 and follows his plan, he gets

$$(1 - \alpha_2)\,\delta\,(v_H - v_L)\left[1 - \eta\,(\lambda - 1)\,(\alpha_2 \delta + 1 - \delta)\right].$$

The equilibrium mixed strategy for the seller is the α_2^* for which the two payoffs above are equal.

Notice that, for the existence of a mixed strategy, we need $1 > \eta\,(\alpha_2\delta + 1 - \delta)\,(\lambda - 1)$ which is guaranteed by the no dominace of gain-loss utility assumption.

The solution is given by

$$\alpha_2^* = \frac{1 + \eta\,(\lambda - 1)\,(2\delta - 1)}{2\delta\eta\,(\lambda - 1)} +$$

$$-\frac{\sqrt{\left[1 + \eta^2\,(1 - \lambda)^2\right]\,(v_L - v_H)^2 + 4\eta p_1\,(1 - \lambda)\,(v_H - v_L) + 2\eta\,(\lambda - 1)\,(v_H - v_L)\,(v_H + v_L)}}{2\delta\eta\,(\lambda - 1)\,(v_H - v_L)}.$$

The seller then can either play as if she were soft or, given that $\beta_{1,H}^*$ is constant over $\left(p_1^{\text{soft}}, v_H\right]$, she could set $p_1 = v_H$ and randomize in the second period according to $\alpha_2^*\,(p_1)$ - notice that if $p_1 = v_H$, then $\alpha_2^* = 1$ - and attain an expected profits of

$$\left[1 - \frac{v_L\,(1 - \pi)}{\pi\,(v_H - v_L)}\right]\pi v_H + \delta\left[1 - \pi + \frac{v_L\,(1 - \pi)}{v_H - v_L}\right]v_L = \left[1 - \frac{v_L\,(1 - \pi)}{\pi\,(v_H - v_L)}\right]\pi v_H + \frac{\delta v_L\,(1 - \pi)\,v_H}{v_H - v_L}.$$

where the equality comes from the fact that, in equilibrium, at $t = 2$ the seller is indifferent between charging $p_2 = v_H$ or $p_2 = v_L$.

Now, a tough seller prefers this strategy to just behaving as a soft seller if

$$\left[1 - \frac{v_L\,(1 - \pi)}{\pi\,(v_H - v_L)}\right]\pi v_H + \frac{\delta v_L\,(1 - \pi)\,v_H}{v_H - v_L} > \pi\left[\frac{v_H\,(1 - \delta)\,[1 + \eta\lambda] + v_L\delta\,[1 + \eta]}{1 + \eta\delta + \eta\lambda\,(1 - \delta)}\right] + (1 - \pi)\,\delta v_L$$

$$\Leftrightarrow \pi \geq \frac{v_L\,(v_H - \delta v_L)\,[1 + \eta\delta + \eta\lambda\,(1 - \delta)]}{v_H\delta\,(1 + \eta)\,(v_H - v_L) + v_L\,(1 - \delta)\,[(1 + \eta\lambda)\,v_H - \delta\eta\,(\lambda - 1)\,v_L]} \equiv \widetilde{\pi}.^{[7]}$$

Thus,

---

[7]Notice that

$$\widetilde{\pi} = \frac{v_L}{v_H}\frac{v_H - \delta v_L}{\delta v_H + v_L\,(1 - 2\delta)}$$

if either $\eta = 0$ or $\lambda = 1$.

And that $\widetilde{\pi} = \frac{v_L}{v_H}$ and $\widetilde{\pi} = 1$ for $\delta = 1$ and $\delta = 0$, respectively.

$$(p_1^*, p_2^*) = \begin{cases} \left(\frac{v_H(1-\delta)[1+\eta\lambda]+v_L\delta[1+\eta]}{1+\eta\delta+\eta\lambda(1-\delta)}, v_L \right) & \text{if} \quad \pi < \widetilde{\pi} \\ (v_H, v_H) & \text{if} \quad \pi \geq \widetilde{\pi} \end{cases}.$$

Notice that

$$\widetilde{\pi} > \frac{v_L}{v_H} \frac{v_H - \delta v_L}{\delta v_H + v_L(1-2\delta)}$$

implying that the mixed-strategy is "less likely" to be most profitable when the buyer is loss-averse, compared to standard Fudenberg and Tirole (1983).

So we get the following result:

Proposition 11 *In the two-period one-sided incomplete information model with two types of buyers in which the seller makes all the offers there is a unique (perfect Bayesian) PPE:*

(i)   if $\pi \leq \widehat{\pi}$, each type of buyer plans to buy in the first period at p_1 and the seller chatges $p_1 = v_L = p_2$;

(ii) if $\widehat{\pi} < \pi \leq \widetilde{\pi}$ the high-type of buyer plans to buy in the first period, the low-type of buyer plans to buy in the second period and the seller charges $p_1 = \frac{v_H(1-\delta)[1+\eta\lambda]+v_L\delta[1+\eta]}{1+\eta\delta+\eta\lambda(1-\delta)}$ and $p_2 = v_L$;

(iii) if $\pi > \widetilde{\pi}$ the high-type of buyer mixes between the plan to buy at $t = 1$ and the plan to buy at $t = 2$ with probability $\beta_{1,H}^*$ and $1 - \beta_{1,H}^*$ respectively, the low-type buyer plans to never buy and the seller charges $p_1 = v_H = p_2$.

Proof. In the text. ■

Summing up, when the buyer is loss-averse, in a two-period model à la Fudenberg and Tirole (1983) in equilibrium:

1. A soft seller can extract more surplus from the high-type (Screening case for $\pi \geq \widehat{\pi}$).

2. A tough seller is more likely to act as if she were soft ($\widetilde{\pi}_{|\eta=0 \text{ or } \lambda=1} < \widetilde{\pi}$).

3. Loss-aversion increases the seller's expected profit.

4. There is an increase in efficiency, as the range of prior beliefs for which the seller prefers to sell only to the high-type buyer is reduced.

The following figures help to visualize these points.

3.3.1.3   Comparative Statics

Let

$$p_{1|\lambda=1}^* = v_H(1-\delta) + v_L\delta$$

be the first-period price in the screening case when the buyer is not loss-averse (recall, however, that screening is never optimal for a soft seller when the buyer is not loss-averse if the discount factor is the same for both the seller and the buyer).

$$p_1 = v_L \qquad\qquad p_1 = v_H\,(1-\delta) + \delta v_L \qquad\qquad p_1 = v_H$$
$$p_2 = v_L \qquad\qquad p_2 = v_L \qquad\qquad\qquad\qquad p_2 = v_H$$

$$0 \qquad\qquad\qquad \frac{v_L}{v_H} \qquad\qquad\qquad \frac{v_L}{v_H}\; \frac{v_H - \delta v_L}{\delta v_H + v_L(1-2\delta)} \qquad 1$$
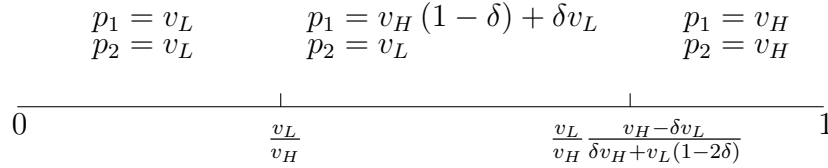
Figure 3.1: Equilibrium Prices in Fudenberg and Tirole (1983)

$$p_1 = v_L \qquad\qquad p_1 = \frac{v_H(1-\delta)(1+\eta\lambda) + v_L\delta(1+\eta)}{1+\eta\delta + \eta\lambda(1-\delta)} \qquad\qquad p_1 = v_H$$
$$p_2 = v_L \qquad\qquad p_2 = v_L \qquad\qquad\qquad\qquad\qquad\qquad p_2 = v_H$$

$$0 \qquad\qquad \widehat{\pi} \qquad \frac{v_L}{v_H} \qquad\qquad\qquad\qquad \widetilde{\pi} \qquad 1$$
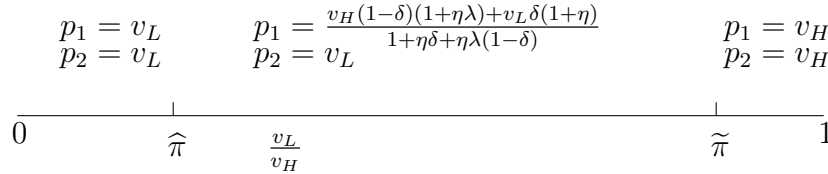
Figure 3.2: Equilibrium Prices with Loss Aversion

We have that

$$
\begin{aligned}
\frac{\partial p_1^*}{\partial v_H} &= \frac{(1-\delta)\,[1+\eta\lambda]}{1+\eta\delta + \eta\lambda\,(1-\delta)} \\
&> 1-\delta \\
&= \frac{\partial p_{1|\lambda=1}^*}{\partial v_H}
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial p_1^*}{\partial v_L} &= \frac{\delta\,[1+\eta]}{1+\eta\delta + \eta\lambda\,(1-\delta)} \\
&< \delta \\
&= \frac{\partial p_{1|\lambda=1}^*}{\partial v_L}.
\end{aligned}
$$

That is, when the valuations get more spread away the first-period price rises more when the buyer is loss-averse; conversely, as the valuations get closer with one another, the first-period price rises by less when the buyer is loss-averse.

What is the effect of δ ? We have

$$\frac{\partial p^*_{1|\lambda=1}}{\partial \delta} = v_L - v_H < 0$$

and

$$\frac{\partial p^*_1}{\partial \delta} = \frac{(v_L - v_H)(1+\eta)(1+\eta\lambda)}{[1+\eta\delta+\eta\lambda(1-\delta)]^2} < 0.$$

Notice also that

$$\frac{(1+\eta)(1+\eta\lambda)}{[1+\eta\delta+\eta\lambda(1-\delta)]^2} > 1$$

$$\Updownarrow$$

$$\delta > \frac{1+\eta\lambda - \sqrt{(1+\eta)(1+\eta\lambda)}}{\eta(\lambda-1)}.$$

So, the effect of δ is bigger (in absolute value) when the buyer is loss-averse than when the buyer is loss-neutral for δ is "high enough".

Furthermore,

$$\frac{\partial \widehat{\pi}}{\partial \lambda} = \frac{\delta v_L (1+\eta)(v_L - v_H)}{[v_H(1+\eta\lambda) - v_L\eta\delta(\lambda-1)]^2} < 0$$

and

$$\frac{\partial \widehat{\pi}}{\partial \delta} = \frac{(\lambda-1)\eta v_L \{v_L[1+\eta\lambda(1-\delta)+\eta\delta] - v_H(1+\eta\lambda) + v_L\eta\delta(\lambda-1)\}}{[v_H(1+\eta\lambda) - v_L\eta\delta(\lambda-1)]^2} < 0$$

implying that, as the buyer becomes more loss-averse and/or the probability of breakdown increases screening becomes more likely than just pricing at v_L in both periods. Notice that this last result about the effect of δ on the likelihood of the screening case is absent in Fudenberg and Tirole (1983) as their threshold is just $\frac{v_L}{v_H}$ (see Figure 1).

Finally,

$$\frac{\partial \widetilde{\pi}}{\partial \lambda} = \frac{\eta(1-\delta)v_L(v_H-\delta v_L)}{\{v_H\delta(1+\eta)(v_H-v_L) + v_L(1-\delta)[(1+\eta\lambda)v_H - \delta\eta(\lambda-1)v_L]\}^2} \times$$

$$\{v_H\delta(1+\eta)(v_H-v_L) + v_L(1-\delta)[(1+\eta\lambda)v_H - \delta\eta(\lambda-1)v_L] - v_L(v_H-\delta v_L)[1+\eta\delta+\eta\lambda(1-\delta)]$$

$$+ v_L(1-\delta)[(1+\eta\lambda)v_H - \delta\eta(\lambda-1)v_L] - v_L(v_H-\delta v_L)[1+\eta\delta+\eta\lambda(1-\delta)]\} > 0$$

and

$$\frac{\partial \widetilde{\pi}}{\partial \delta} = \frac{[v_L\delta^2\eta(\lambda-1) - v_H(1+\lambda\eta)](1+\eta)(v_H-v_L)^2 v_L}{[v_Lv_H(1+\eta\lambda) - 2v_Lv_H\delta - v_Lv_H\delta\eta(1+\lambda) + v_H^2\delta(1+\eta) + v_L^2\delta\eta(1-\lambda) - v_L^2\delta^2\eta(\lambda-1)]^2} < 0$$

implying that as the buyer becomes more loss-averse the screening case is more likely than pricing at v_H in both periods but as the the probability of breakdown increases screening becomes less likely than pricing at v_H in both periods (as in Fudenberg and Tirole, 1983)).

3.3.1.4   Loss Aversion or Risk Aversion?

Now I compare the predictions from the model with a loss-averse buyer to the ones that would arise if the buyer were risk-averse. In the latter case, let the buyer's utility be $U(v-p)$ with $U(0) = 0$, $U'(\cdot) > 0$ and $U''(\cdot) < 0$.

What is the price p_1^r at which a high-type buyer is indifferent between buying in period 1 or in period 2 when he correctly anticipates that $p_2 = v_L$?

We have

$$U(v_H - p_1) = \delta U(v_H - v_L)$$
$$\Longleftrightarrow p_1^r = v_H - U^{-1}(\delta U(v_H - v_L)).$$

This price is higher than the one at which a risk-neutral high-type buyer would be indifferent if and only if

$$U^{-1}(\delta U(v_H - v_L)) < \delta(v_H - v_L)$$
$$\Longleftrightarrow \delta U(v_H - v_L) < U(\delta(v_H - v_L))$$

which follows from Jensen's Inequality.

Furthermore, we know that with risk neutrality on the buyer's side a soft seller would never screen. What if the buyer is risk-averse ?

The seller will prefer the screening option iff

$$\left[v_H - U^{-1}(\delta U(v_H - v_L))\right]\pi + (1 - \pi)\delta v_L > v_L$$

$$\Longleftrightarrow \pi > \frac{v_L(1 - \delta)}{v_H - \delta v_L - U^{-1}(\delta U(v_H - v_L))} \equiv \widehat{\pi}^r.$$

It is easy to see that $\widehat{\pi}^r < \frac{v_L}{v_H}$, implying that screening might be optimal for a soft seller if the buyer is risk-averse. Thus loss aversion makes the same qualitative predictions as risk aversion.

3.3.2   Continuum of Types

Now suppose $v \overset{U}{\sim} [0, 1]$. Let $\mathbb{E}[p_2|p_1]$ be the price a buyer expects to face in period 2 when offered p_1 in period 1. Since the seller's pricing strategy must be sequentially rational, it follows that $\mathbb{E}[p_2|p_1] < p_1$.

So the "marginal" buyer in period 1 is the one for which

$$v - p_1 = \delta(v - \mathbb{E}[p_2|p_1]) + \delta\eta(p_1 - \mathbb{E}[p_2|p_1]) - \eta\lambda(1 - \delta)(v - p_1)$$

$$\Longleftrightarrow v = \frac{p_1[1 + \eta\delta + \eta\lambda(1 - \delta)] - \delta\mathbb{E}[p_2|p_1](1 + \eta)}{(1 - \delta)(1 + \eta\lambda)} \equiv a(p_1).$$

Hence, if the buyer does not buy in period 1 at p_1, the seller's posterior beliefs in period 2 will be that $v \overset{U}{\sim} [0, a(p_1)]$ in which case she will set the price in period 2 equal to

$$p_2^*(p_1) = \frac{a(p_1)}{2}.$$

By substituting $\frac{a(p_1)}{2} = \mathbb{E}[p_2|p_1]$ into the definition of $a(p_1)$ and re-arranging

$$a(p_1) = \frac{2p_1[1 + \eta\delta + \eta\lambda(1 - \delta)]}{2(1 - \delta)(1 + \eta\lambda) + \delta(1 + \eta)}.$$

The seller then will pick p_1 to maximize her profits:

$$U_S(p_1) = \Pr[v \geq a(p_1)]p_1 + \delta\Pr[p_2 \leq v < a(p_1)]p_2^*(p_1)$$

$$= \left(1 - \frac{2p_1[1 + \eta\delta + \eta\lambda(1 - \delta)]}{2(1 - \delta)(1 + \eta\lambda) + \delta(1 + \eta)}\right)p_1 + \frac{\delta}{4}\left(\frac{2p_1[1 + \eta\delta + \eta\lambda(1 - \delta)]}{2(1 - \delta)(1 + \eta\lambda) + \delta(1 + \eta)}\right)^2.$$

The FOC yields

$$p_1^* = \frac{[2(1 - \delta)(1 + \eta\lambda) + \delta(1 + \eta)]^2}{2[1 + \eta\delta + \eta\lambda(1 - \delta)]\{4(1 - \delta)(1 + \eta\lambda) + 2\delta(1 + \eta) - \delta[1 + \eta\delta + \eta\lambda(1 - \delta)]\}}.$$

By substituting it follows that

$$a(p_1^*) = \frac{2(1 - \delta)(1 + \eta\lambda) + \delta(1 + \eta)}{4(1 - \delta)(1 + \eta\lambda) + 2\delta(1 + \eta) - \delta[1 + \eta\delta + \eta\lambda(1 - \delta)]}$$

and

$$p_2^* = \frac{2(1 - \delta)(1 + \eta\lambda) + \delta(1 + \eta)}{8(1 - \delta)(1 + \eta\lambda) + 4\delta(1 + \eta) - 2\delta[1 + \eta\delta + \eta\lambda(1 - \delta)]}.$$

So we get the following result:

Proposition 12 *In the two-period one-sided incomplete information model with buyer's types continuously and uniformly distributed on $[0, 1]$ and in which the seller makes all the offer there is a unique Sequential PPE in which:*
(i) buyers with $v \in [a(p_1^*), 1]$ plan to buy in the first period at

$$p_1^* = \frac{[2(1 - \delta)(1 + \eta\lambda) + \delta(1 + \eta)]^2}{2[1 + \eta\delta + \eta\lambda(1 - \delta)]\{4(1 - \delta)(1 + \eta\lambda) + 2\delta(1 + \eta) - \delta[1 + \eta\delta + \eta\lambda(1 - \delta)]\}};$$

(ii) buyers with $v \in [p_2^*, a(p_1^*)]$ plan to buy in the second period at

$$p_2^* = \frac{2(1 - \delta)(1 + \eta\lambda) + \delta(1 + \eta)}{8(1 - \delta)(1 + \eta\lambda) + 4\delta(1 + \eta) - 2\delta[1 + \eta\delta + \eta\lambda(1 - \delta)]};$$

(iii) and buyers with $v \in [0, p_2^*)$ plan to never buy.

Proof. In the text. ■

For the sake of comparison, notice that

$$p_{1|\lambda=1}^* = \frac{\left(1 - \frac{\delta}{2}\right)^2}{2\left(1 - \frac{3}{4}\delta\right)},$$

$$a\left(p^*_{1|\ \lambda=1}\right) = \frac{1 - \frac{\delta}{2}}{2\left(1 - \frac{3}{4}\delta\right)}$$

and

$$p^*_{2|\ \lambda=1} = \frac{1 - \frac{\delta}{2}}{4\left(1 - \frac{3}{4}\delta\right)}.$$

It is easy to see then that

$$a\left(p^*_1\right) < a\left(p^*_{1|\ \lambda=1}\right)$$

and thus

$$p^*_2 < p^*_{2|\ \lambda=1}.$$

Comparing first-period prices we get

$$p^*_1 > p^*_{1|\lambda=1}$$

$$\Updownarrow$$

$$\lambda > \frac{26\delta - 16 - 13\delta^2 + \eta\delta\left(12\delta - 8 - 7\delta^2 + 2\delta^3\right)}{\eta\left(1 - \delta\right)\left(16 - 18\delta + 7\delta^2 - 2\delta^3\right)}$$

which is true $\forall \lambda > 1$.
For the seller's overall profits we get

$$U_S = \left(\frac{H}{2H - \delta K}\right)^2 \left(\frac{\delta}{4} + \frac{H - \delta K}{2K}\right)$$

and

$$U_S|_{|\lambda=1} = \left(\frac{2 - \delta}{4 - 3\delta}\right)^2 \left(\frac{\delta}{4} + 1 - \delta\right)$$

where $H \equiv 2\left(1 - \delta\right)\left(1 + \eta\lambda\right) + \delta\left(1 + \eta\right)$ and $K \equiv 1 + \eta\delta + \eta\lambda\left(1 - \delta\right)$.
Thus,

$$U_S > U_S|_{|\lambda=1}$$

$$\Longleftrightarrow 2\left(1 - \delta\right)\left(1 + \eta\lambda\right) + \delta\left(1 + \eta\right) > \left[1 + \eta\delta + \eta\lambda\left(1 - \delta\right)\right]\left(2 - \delta\right) \Leftrightarrow \lambda > 1.$$

Summing up, when the buyer is loss-averse (and gain-loss utility is defined on the overall gain from trade) in a two-period model with types uniformly distributed on $[0, 1]$ in equilibrium:

1. The seller's profits are higher than in the case without loss aversion.

2. The first-period price is higher than when the buyer is not loss-averse; nonetheless, more types are buying in the first period with loss aversion because $a\left(p^*_1\right) < a\left(p^*_{1|\lambda=1}\right)$.

3. The second-period profits go down because $a\left(p^*_1\right) < a\left(p^*_{1|\lambda=1}\right)$ and $p^*_2 = \frac{a\left(p^*_1\right)}{2} < \frac{a\left(p^*_{1|\lambda=1}\right)}{2} = p^*_{2|\lambda=1}$.

4. The "Skimming Property" holds.

5. There is definitely an increase in efficiency as more types are buying with loss aversion than without.

6. Some types who do not buy in the second period without loss-aversion do buy here, so there is not only an increase in efficiency but also some types who were getting zero now are getting some stirctly positive surplus. Moreover, some types who would buy in the second period with and without loss aversion are paying a lower price with loss aversion. So loss aversion "benefits" buyers with lower types.

7. Some types who would buy in the second period in the non loss-averse case, do buy in the first period with loss aversion - but whether they are better off or worse off depends on the discount factor and on the difference between the first period prices with and without loss aversion.

The following figures help to visualize these points.

$$0 \qquad p^*_{2|\lambda=1} \quad p^*_{1|\lambda=1} \qquad a\left(p^*_{1|\lambda=1}\right) \qquad 1$$
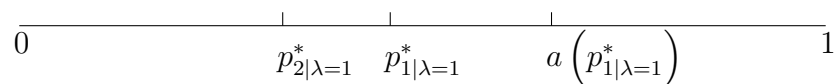
Figure 3.3: Equilibrium Prices in Sobel and Takahashi (1983)

$$0 \qquad p^*_2 \qquad p^*_1 \quad a\left(p^*_1\right) \qquad 1$$
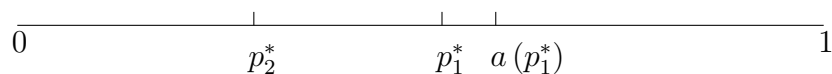
Figure 3.4: Equilibrium Prices with Loss Aversion

3.3.2.1   Comparative Statics

What is the effect of δ on the first-period equilibrium price and on the cut-off valuation of the marginal type?

We have

$$\frac{\partial a\left(p_1^*\right)}{\partial \delta} = \frac{2 + \eta\lambda\left(4 - \delta^2\right) + 2\eta^2\lambda^2\left(1 - \delta^2\right) + \eta\delta^2\left(3\eta\lambda + 1 - \eta\right)}{\left\{4\left(1 - \delta\right)\left(1 + \eta\lambda\right) + 2\delta\left(1 + \eta\right) - \delta\left[1 + \eta\delta + \eta\lambda\left(1 - \delta\right)\right]\right\}^2} > 0$$

and

$$\frac{\partial p_1^*}{\partial \delta} > 0$$

$$\Updownarrow$$

$$4\eta - 3\delta + 2\lambda\eta - 7\delta\eta - 2\lambda\delta\eta + 8\lambda\eta^2 + 2\delta\eta^2 - 18\lambda\delta\eta^2 + 2\lambda\delta\eta^3 - 2\lambda^2\eta^2 + 4\lambda^2\eta^3 - 2\lambda^3\eta^3 - 6\delta^2\eta^2 + \delta^3\eta^2 - \delta^3\eta^3 +$$

$$+ 12\lambda\delta^2\eta^2 + 7\lambda^2\delta\eta^2 - 6\lambda\delta^2\eta^3 - 2\lambda\delta^3\eta^2 - 11\lambda^2\delta\eta^3 + 4\lambda\delta^3\eta^3 + 6\lambda^3\delta\eta^3 - 6\lambda^2\delta^2\eta^2 +$$

$$+ 12\lambda^2\delta^2\eta^3 + \lambda^2\delta^3\eta^2 - 5\lambda^2\delta^3\eta^3 - 6\lambda^3\delta^2\eta^3 + 2\lambda^3\delta^3\eta^3 + 2 < 0$$

$$\Updownarrow$$

$$\delta > \overline{\delta}\left(\eta, \lambda\right).$$

Notice that the sign of the coparative statics are the same for the case without loss-aversion ($\overline{\delta}_{|\lambda=1} = \frac{2}{3}$ whereas $\overline{\delta}\left(\eta, \lambda\right) > \frac{2}{3}$.).

3.4   Conclusion

In this paper I have analyzed the role of expectations-based loss aversion and reference-dependent preferences in bargaining negotiations. Loss aversion on the buyer's side appears to have two main consequences for the overall efficiency of the negotiations and the division of surplus. First, the seller's equilbrium payoff increases in the buyer's degree of loss aversion. This happens because the more loss-averse the buyer is, the more he is willing to pay at early stages to avoid the risk of negotiations' breakdown. More precisely, the seller is able to more effectively screen high-valuation buyers with a higher price because these are the ones that have the biggest incentive to buy early to avoide the risk of breakdown. Second, there is also an increase in the overall trade efficiency. This should not come as a surprise since loss aversion eases the rent-efficiency trade-off for the seller who can now serve a larger number of consumers at an earlier stage.

When consumers' valuations are continuously distributed, there is an additional effect: loss aversion benefits low-type buyers who buy in the second period as they do so at a price lower than the one they would face with classical preferences. That is, not only loss aversion shifts rent from high-type buyers to the seller, but it also increases the equilibirum payoff of some low-type buyers.

Bibliography

[1]   ABELER, J., A. FALK, L. GOETTE and D. HUFFMAN (2011), "Reference Points and Effort Provision" *American Economic Review*, 101(2), 470–492.

[2]   ADMATI, A. and M. PERRY (1987), "Strategic Delay in Bargaining." *Review f Economic Studies*, 54(3), 345-364.

[3]   AGUIRREGABIRIA, V. (2005), "Retail Stockouts and Manufacturer Brand Competition" *Mimeo*.

[4]   AMBRUS, A. and J. WEINSTEIN (2008), "Price Dispersion and Loss Leaders" *Theoretical Economics*, 3(4), 525-537.

[5]   ANDERSON, E., G.J., FITZSIMONS, and D. SIMESTER (2006), "Measuring and Mitigating the Cost of Stockouts" *Management Science*, 52(11), 1751–1763.

[6]   ANDERSON, E., E. NAKAMURA, D. SIMESTER and J. STEINSSON (2012), "Temporary Sales: On Autopilot and Funded by Frequent Flyer Accounts" *Mimeo*.

[7]   ANUPINDI, R., M. DADA and S. GUPTA (1998), "Estimation of Consumers Demand with Stock-Out Based Substitution: An Application to Vending Machine Products" *Marketing Science*, 17 (4) 406-423.

[8]   APERJIS, C. and F. BALESTRIERI (2010), "Loss Aversion Leading to Advantageous Selection" *Mimeo*.

[9]   ARTSTEIN-AVIDAN, S. and D. DILLENBERGER (2011), "Dynamic Disappointment Aversion" *Mimeo*.

[10] ASHENFELTER, O. (1989), "How Auctions Work for Wine and Art" *Journal of Economic Perspective*, 3(3), 23-36.

[11] ASHENFELTER, O. AND D. GENESOVE (1992), "Testing for Price Anomalies in Real-Estate Auctions" *American Economic Review*, 82(2), 501-505.

[12] ASHENFELTER, O. and K. GRADDY (2003), "Auctions and the Price of Art" *Journal of Economic Literature*, 41(3), 763–787.

[13] AUSUBEL, L., P. CRAMTON, and R. DENECKERE, "Bargaining with Incomplete Information." Handbook of Game Theory. Ed. Robert J. Aumann and Sergiu Hart. Amsterdam: Elsevier Science B.V., 2001.

[14] BALACHANDER, S. and P.H. FARQUHAR (1994), "Gaining More by Stocking Less: A Competitive Analysis of Product Availability" *Marketing Science*, 13 (1) 3-22.

[15] BALESTRIERI, F. and J. LEAO (2011), "The Market for Surprises: Selling Substitute Goods through Lotteries" *Mimeo*.

[16] BATTIGALLI, P. and M. DUFWENBERG (2009), "Dynamic psychological games." *Journal of Economic Theory*, 144(1), 1-35.

[17] BAYE, M.R., J. MORGAN AND P. SCHOLTEN (2006) "Information, Search, and Price Dispersion," Chapter 6 in Handbook in Economics and Information Systems Volume 1 (T. Hendershott, Ed.), Amsterdam: Elsevier.

[18] BEGGS, A. and GRADDY, K. (1997) "Declining Values and the Afternoon Effect: Evidence from Art Auctions", *Rand Journal of Economics*, 28(3), 544–565.

[19] BELL, D. (1985), "Disappointment in Decision Making under Uncertainty" *Operations Research*, 33(1), 1-27.

[20] BERNHARDT, D. AND D. SCOONES (1994), "A Note on Sequential Auctions" *American Economic Review*, 84(3), 501-505.

[21] BILS, M. (2004), "Studying Price Markups from Stockout Behavior" *Mimeo*.

[22] BLACK, J. AND D. DE MEZA (1992), "Systematic Price Differences between Successive Auctions are no Anomaly" *Journal of Economics and Management Strategy*, 1(4), 607-628.

[23] BREUGELMANS, E., K. CAMPO and P. NISOL (2006), "Opportunities for Active Stock-Out Management in Online Stores: The Impact of the Stock-Out Policy on Online Stock-Out Reactions" *Journal of Retailing*, 82(3), 215-228.

[24] BUCCOLA, S. (1982), "Price Trends at Livestock Auctions" *American Journal of Agricoltural Economics*, 64(1), 63–69.

[25] BURNS, P. (1985), "Experience and Decision Making: A Comparison of Students and Businessmen in a Simulated Progressive Auction" in (V. Smith, ed.), *Research in Experimental Economics: A Research Annual*, 139–157, Vol. 3, Greenwich, Connecticut: JAI.

[26] CAMPO, K., E. GIJSBRECHTS and P. NISOL (2000), "Towards Understanding Consumer Response to Stock-Outs" *Journal of Retailing*, 76(2), 219-242.

[27] CAMPO, K., E. GIJSBRECHTS and P. NISOL (2003), "The Impact of Retailer Stock-outs on Whether, How Much and What to Buy" *International Journal of Research in Marketing*, 20(3), 273-286.

[28] CARBAJAL, J.C. and J. ELY (2012), "Optimal Contracts for Loss Averse Consumers" *Mimeo*.

[29] CHAKRAVARTY, A.K AND S. GHOSE (1994), "Strategic Underproduction with Product Switching" *Computers and Operations Research*, 21(1), 39-56.

[30] CHANEL, O., L.A. GERARD-VARET and S. VINCENT (1996), "Auction Theory and Practice: Evidence from the Market for Jewelry" in (V. Ginsburgh and P.M. Menger, eds.) *Economics of the Arts; Selected Essays*, 135-149, Amsterdam: Elsevier.

[31] CHEN, Z. AND P. REY (2012), "Loss Leading as an Exploitative Practice" *American Economic Review,* 102(7), 3462–3482.

[32] CHEVALIER, J.A., A.K. KASHYAP and P.E. ROSSI (2003), "Why Don't Prices Rise During Periods of Peak Demand? Evidence from Scanner Data" *American Economic Review*, 93(1), 15-37.

[33] COMPTE, O. and P. JEHIEL (2007), "Bargaining with Reference Dependent Preferences." *Mimeo.*

[34] CONLON, C.T. and J.H. MORTIMER (2011), "Effects of Product Availability: Experimental Evidence" *Mimeo.*

[35] CRAMTON, P. (1984), "Bargaining with Incomplete Information: An Infinite-Horizon Model with Two-Sided Uncertainty." *Review f Economic Studies*, 51(4), 579-593.

[36] CRAMTON, P. (1992), "Strategic Delay in Bargaining." *Review f Economic Studies*, 59(1), 205-225.

[37] CRAWFORD, V. and J.J. MENG (2011), "New York City Cab Drivers' Labor Supply Revisited: Reference-Dependent Preferences with Rational-Expectations Targets for Hours and Income" *American Economic Review*, 101(5), 1912–1932.

[38] DAIDO, K. and T. MUROOKA (2012), "Team Incentives and Reference-Dependent Preferences" *Mimeo.*

[39] DANA, J.D., JR. (1999), "Equilibrium Price Dispersion under Demand Uncertainty: The Roles of Costly Capacity and Market Structure" *Rand Journal of Economics,* 30(4), 632-660.

[40] DANA, J.D., JR. (2001a), "Monopoly Price Dispersion under Demand Uncertainty" *International Economic Review,* 42(3), 649-670.

[41] DANA, J.D., JR. (2001b), "Competition in Price and Availability when Availability is Unobservable" *Rand Journal of Economics,* 32(4), 497-513.

[42] DAUGHETY, A.F. and J.F. REINGANUM (1991), "Endogenous Availability in Search Equilibrium" *Rand Journal of Economics*, 22(2), 287–306.

[43] DEGRABA, P. (1995), "Buying Frenzies and Seller-Induced Excess Demand" *Rand Journal of Economics,* 26(2), 331-342.

[44] DeGraba, P. (2006), "The Loss Leader is a Turkey: Targeted Discounts from Multi-Product Competitors" *International Journal of Infustrial Organization,* 24(3), 613-628.

[45] DellaVigna, S. and U. Malmendier (2004), "Contract Design and Self-Control: Theory and Evidence" *Quarterly Journal of Economics*, 119(2), 353–402.

[46] Deltas, G. and G. Kosmopoulou (2004), "Bidding in Sequential Auctions: "Catalogue" vs. "Order-of-Sale" Effects" *Economic Journal,* 114(492), 28-54.

[47] Deneckere, R. and P. McAfee (1996), "Damaged Goods" *Journal of Economics and Management Strategy*, 5(2), 149-174.

[48] Deneckere, R. and J. Peck (1995), "Competition over Price and Service Rate when Demand is Stochastic: A Strategic Analysis" *Rand Journal of Economics*, 26(1), 148-162.

[49] Di Vittorio, A. and V. Ginsburgh (1994) "Pricing Red Wines of Médoc Vintages from 1949 to 1989 at Christie's Auctions," *Mimeo.*

[50] Eisenhuth, R. (2012), "Reference Dependent Mechanism Design" *Mimeo.*

[51] Eisenhuth, R. and M. Ewers (2012), "Auctions with Loss Averse Bidders" *Mimeo.*

[52] Eliaz, K. and R. Spiegler (2006), "Contracting with Diversely Naïve Agents" *Review of Economic Studies*, 73(3), 689–714.

[53] Eliaz, K. and R. Spiegler (2008), "Consumer Optimism and Price Discrimination" *Theoretical Economics*, 3(4), 459–497.

[54] Eliaz, K. and R. Spiegler (2011a), "On the Strategic Use of Attention Grabbers" *Theoretical Economics*, 6(1), 127–155.

[55] Eliaz, K. and R. Spiegler (2011b), "Consideration Sets and Competitive Marketing" *Review of Economic Studies*, 78 (1), 235-262.

[56] Eliaz, K. and R. Spiegler (2012), "Reference Dependence and Labor-Market Fluctuations" *Mimeo.*

[57] Ellison, G. (2005), "A Model of Add-on Pricing" *Quarterly Journal of Economics*, 120(2), 585-637.

[58] Emmelhainz, M.A., J.R. Stock and L.W. Emmelhainz (1991), "Consumer Responses to Retail Stock-outs" *Journal of Retailing,* 67(2), 139-147.

[59] Engelbrecht-Wiggans, R. and Kahn, C. M. (1999), "Calibration of a Model of Declining Prices in Cattle Auctions" *Quarterly Review of Economics and Finance*, 391(), 113–128.

[60] Ericson Marzilli, K.M. and A. Fuster (2011), "Expectations as Endowments: Evidence on Reference-Dependent Preferences from Exchange and Valuation Experiments" *Quarterly Journal of Economics*, 126(4), 1879-1907.

[61] Eyster, E. (2002), "Rationalizing the Past: A Taste for Consistency" *Mimeo.*

[62] Fay, S. and J. Xie (2008), "Probabilistic Goods: A Creative Way of Selling Products and Services" *Marketing Science*, 27(4), 674-690.

[63] Février, P., W. Roos and M. Visser (2005), "The Buyer's Option in Multi-Unit Ascending Auctions: The Case of Wine Auctions at Drouot" *Journal of Economics & Management Strategy*, 14(4), 813-847.

[64] Février, P., L. Linnemer and M. Visser (2007), "Buy or Wait, That is the Option: The Buyer's Option in Sequential Laboratory Auctions" *Rand Journal of Economics*, 38(1), 98-118.

[65] Fitzsimons, G.J. (2000), "Consumer Response to Stockouts" *Journal of Consumer Research*, 27(2), 249-266.

[66] Fudenberg, D. and J. Tirole (1983), "Sequential Bargaining with Incomplete Information." *Review of Economic Studies*, 50(2), 221-247.

[67] Fudenberg, D., D. Levine and J. Tirole (1985), "Infinite-Horizon Models of Bargaining with One-Sided Incomplete Information." in Alvin Roth (ed), Game Theoretic Models of Bargaining, Cambridge, England: Cambridge University Press.

[68] Gabaix, X. and D. Laibson (2006), "Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets" *Quarterly Journal of Economics*, 121(2), 505-540.

[69] Gale, I. and D. Hausch (1994), "Bottom-Fishing and Declining Prices in Sequential Auctions" *Games and Economic Behavior*, 7(3), 318-331.

[70] Gandal N. (1997), "Sequential Auctions of Interdependent Objects: Israeli Cable Television Licenses" *Journal of Industrial Economics*, 45(3), 227-244.

[71] Geanakoplos, J., D. Pearce and E. Stacchetti (1989), "Psychological games and sequential rationality." *Games and Economic Behavior*, 1(1), 60-79.

[72] Gerstner, E. and J. Hess (1990), "Can Bait and Switch Benefit Consumers?" *Marketing Science*, 9(2), 114-124.

[73] Gilbert, R. and P. Klemperer (2000), "An Equilibrium Theory of Rationing" *Rand Journal of Economics*, 31(1), 1-21.

[74] Gill, D. and V. Prowse (2012), "A Structural Analysis of Disappointment Aversion in a Real Effort Competition" *American Economic Review*, 102(1), 469-503.

[75] Ginsburgh, V. (1998) "Absentee Bidders and the Declining Price Anomaly in Wine Auctions" *Journal of Political Economy*, 106 (6), 1302–1319.

[76] Ginsburgh, V. and J.C. Van Ours (2007), "On Organizing a Sequential Auction: Results from a Natural Experiment by Christie's" *Oxford Economic Papers*, 59(1), 1–15.

[77] GRANT-WORLEY, J., N.C. SALTFORD and C. ZICK (1982), "Unavailability of Advertised Products in Selected Non-Food Stores" *Journal of Consumer Studies and Home Economcis*, 6(3), 237-249.

[78] GRUEN, T., D. CORSTEN and S. BRARADWAJ (2002), "Retail Out of Stocks: A Worldwide Examination of Causes, Rates and Consumer Responses" *Grocery Manufacturers of America*, Washington, D.C.

[79] GRUBB, M. (2009), "Selling to Overconfident Consumers" *American Economic Review*, 99(5), 1770–1807.

[80] GUL, F. (1991) "A Theory of Disappointment Aversion" *Econometrica*, 59(3), 667–686.

[81] GUL, F., H. SONNENSCHEIN and R. WILSON (1986), "Foundations of Dynamic Monopoly and the Coase Conjecture." *Journal of Economic Theory*, 39(1), 155-190.

[82] HAHN, J., J. KIM, S. KIM and J. LEE (2012), "Screening Loss Averse Consumers" *Mimeo*.

[83] HEFFETZ, O. and J. LIST (2011), "Is the Endowment Effect a Reference Effect?" NBER Working Paper No. w16715.

[84] HEIDHUES, P. and B. KŐSZEGI (2008), "Competition and Price Variation when Consumers are Loss Averse" *American Economic Review*, 98(4), 1245–1268.

[85] HEIDHUES, P. and B. KŐSZEGI (forthcoming), "Regular Prices and Sales" *Theoretical Economics*.

[86] HEIDHUES, P., B. KŐSZEGI and T. MUROOKA (2012), "The Market for Deceptive Products" *Mimeo*.

[87] HERWEG, F. and K. MIERENDORFF (forthcoming), "Uncertain Demand, Consumer Loss Aversion, and Flat-Rate Tariffs" *Journal of the European Economic Association.*

[88] HERWEG, F., D. MÜLLER and P. WEINSCHENK (2010), "Binary Payment Schemes: Moral Hazard and Loss Aversion" *American Economic Review*, 100(5), 2451-2477.

[89] HESS, J. and E. GERSTENR (1987), "Loss Leader Pricing and Rain Check Policy" *Marketing Science*, 6 (4), 358-374.

[90] HOSKEN, D. and D. REIFFEN (2004a), "Patterns of Retail Price Variation" *Rand Journal of Economics*, 35 (1), 128-146.

[91] HOSKEN, D. and D. REIFFEN (2004b), "How Retailers Determine which Products Should Go on Sale" *Journal of Consumer Policy*, 27 (2), 141-177.

[92] JEITSCHKO, T. (1998), "Learning in Sequential Auctions" *Southern Economic Journal*, 65(1), 98-112.

[93] JEITSCHKO, T. (1999), "Equilibrium Price Paths in Sequential Auctions with Stochastic Supply" *Economics Letters*, 64(1), 67–72.

[94] JING, X. and M. LEWIS (2011), "Stockouts in Online Retailing" *Journal of Marketing Research*, 48 (2), 342-354.

[95] JONES, C., F. MENEZES and F. VELLA (2004), "Auction Price Anomalies: Evidence from Wool Auctions in Australia" *Economic Record*, 80(250), 271–288.

[96] KAHNEMAN, D. and A. TVERSKY (1979), "Prospect Theory: An Analysis of Decision under Risk" *Econometrica*, 47 (2), 263-291.

[97] KAHNEMAN, D., J.L. KNETSCH and R. THALER (1990), "Experimental Tests of the Endowment Effect and the Coase Theorem" *Journal of Political Economy*, 98(6), 1325-1348.

[98] KAHNEMAN, D., J.L. KNETSCH and R. THALER (1991), "Anomalies: The Endowment Effect, Loss Aversion, and Status Quo Bias" *Journal of Economic Perspectives*, 5(1), 193-206.

[99] KAMENICA, E. (2008), "Contextual Inference in Markets: On the Informational Content of Product Lines" *American Economic Review*, 98(5), 2127–2149.

[100] KARLE, H. (2012), "Creating Attachment through Advertising: Loss Aversion and Pre–Purchase Information" *Mimeo*.

[101] KARLE, H.,and M. PEITZ (2012), "Competition under Consumer Loss Aversion" *Mimeo*.

[102] KARLE, H., G. KIRCHSTEIGER and M. PEITZ (2012), "Loss Aversion and Consumption Choice: Theory and Experimental Evidence" *Mimeo*.

[103] KATZ, B.G. and J. NELSON (1990), "Product Availability as a Strategic Variable: The Implications of Regulating Retailer Stockouts" *Journal of Regulatory Economics*, 2(4), 379-395.

[104] KELLS, S. (2001), "Price paths in sequential auctions: Preliminary evidence from Australian rare book auctions" *Mimeo*.

[105] KESER, C. and M. OLSON (1996). Experimental examination of the declining price anomaly , in (V. Ginsburgh and P.M. Menger, eds.), *Economics of the Arts: Selected Essays*, 151–76, Amsterdam: Elsevier.

[106] KIM, M. and S.J. LENNON (2011), "Consumer Response to Online Apparel Stockouts" *Psychology and Marketing*, 28(2), 115-144.

[107] KITTSTEINER, T., J. NIKUTTA and E. WINTER (2004), "Declining Valuations in Sequential Auctions" *International Journal of Game Theory*, 33(1), 89-106.

[108] KLEMPERER, P. and A.J. PADILLA (1997), "Do Firms' Product Lines Include too Many Varieties?" *Rand Journal of Economics*, 28 (3), 472-488.

[109] KONISHI, H. and M. SANDFORT (2002), "Expanding Demand through Price Advertisement" *International Journal of Industrial Organization*, 20(7), 965-994.

[110] KŐSZEGI, B. (2010), "Utility from Anticipation and Personal Equilibrium" *Economic Theory*, 44(3), 415-444.

[111] KŐSZEGI, B. and M. RABIN (2006), "A Model of Reference-Dependent Preferences" *Quarterly Journal of Economics*, 121(4), 1133-1165.

[112] KŐSZEGI, B. and M. RABIN (2007), "Reference-Dependent Risk Attitudes" *American Economic Review*, 97(4), 1047-1073.

[113] KŐSZEGI, B. and M. RABIN (2009), "Reference-Dependent Consumption Plans" *American Economic Review*, 99(3), 909-936.

[114] LAL, R. and C. MATUTES (1994), "Retail Pricing and Advertising Strategies" *Journal of Business*, 67 (3), 345-370.

[115] LANGE, A. and A. RATAN (2010), "Multi-dimensional Reference-dependent Preferences in Sealed-bid auctions: How (most) Laboratory Experiments Differ from the Field" *Games and Economic Behavior*, 68 (2), 634-645.

[116] LAZEAR, E. (1986), "Retail Pricing and Clearance Sales" *American Economic Review*, 76(1), 14-32.

[117] LAZEAR, E. (1995), "Bait and Switch" *Journal of Political Economy*, 103(4), 813-830.

[118] LOOMES, G. and R. SUDGEN (1986), "Disappointment and Dynamic Consistency in Choice under Uncertainty" *Review of Economic Studies*, 53 (2), 271-282.

[119] LUSHT, K. (1994), "Order and Price in a Sequential Auction," Journal of Real Estate Finance and Economics, 8(3), 259–266.

[120] MACERA, R. (2012), "Intertemporal Incentives Under Loss Aversion" *Mimeo*.

[121] MCAFEE, R.P. AND D. VINCENT (1993), "The Declining Price Anomaly" *Journal of Economic Theory*, 60(1), 191212.

[122] MENEZES, F.M. and P.K. MONTEIRO (2003), "Synergies andPrice Trends in Sequential Auctions" *Review of Economic Design*, 8(1), 85–98.

[123] MEZZETTI, C. (2011), "Sequential Auctions with Informational Externalities and Aversion to Price Risk: Decreasing and Increasing Price Sequences" *Economic Journal*, 121(555), 990-1016.

[124] MILGROM, P. AND R. WEBER (1982), "A Theory of Auctions and Competitive Bidding, II" Working Paper, Stanford University. Published with new foreword in (P. Klemperer, ed.) (2000). *The Economic Theory of Auctions*, 179-194, Vol. 1, Cheltenham: Edward Edgar.

[125] NAKAMURA, E. (2008), "Pass-Through in Retail and Wholesale" *American Economic Review P&P*, 98(2), 430-437.

[126] NAKAMURA, E. and J. STEINSSON (2008), "Five Facts about Prices: Reevaluation of Menu Cost Models" *Quarterly Journal of Economics*, 123 (4), 1415-1464.

[127] NASH, J.F. (1950), "The Bargaining Problem." *Econometrica*, 18(2), 155-162.

[128] NEUGEBAUER, T. and P. PEZANIS-CHRISTOU (2007), "Bidding Behavior at Sequential First-price Auctions with(out) Supply Uncertainty: A Laboratory Analysis" *Journal of Economic Behavior & Organization*, 63 (1), 55–72.

[129] NOCKE, V. and M. PEITZ (2007), "A Theory of Clearance Sales" *Economic Journal*, 117 (7), 964-990.

[130] NOVEMSKY, N. and D. KAHNEMAN (2005), "The Boundaries of Loss Aversion" *Journal of Marketing Research*, 42 (2), 119-128.

[131] OZCAN, T. (2008), "Involuntary Switching Behavior in Restricted Decision Environments: Consumer Responses to Stockouts" PhD Dissertation, University of Rhode Island.

[132] PASHIGIAN, P.B. (1988), "Demand Uncertainty and Sales: A Study of Fashion and Markdown Pricing" *American Economic Review*, 78(5), 936-953.

[133] PASHIGIAN, P.B and B. BOWEN (1991), "Why are Products Sold on Sales? Explanations of Pricing Regularities" *Quarterly Journal of Economics*, 106 (4), 1015-1038.

[134] PAVLOV, G. (2011), "Optimal Mechanism for Selling Two Goods" *B.E. Journal of Theoretical Economics*, 11(1).

[135] PESANDO, J. E. and P. SHUM (1996), "Price Anomalies at Auction: Evidence from the Market for Modern Prints" in (V. Ginsburgh and P.M. Menger, eds.) *Economics of the Arts; Selected Essays*, 113-134, Amsterdam: Elsevier.

[136] PEZANIS-CHRISTOU, P. (2001), "Sequential Auctions with Asymetric Buyers: Evidence from a Fish Market" *Mimeo.*

[137] PITCHIK, C. and A. SCHOTTER (1988), "Perfect Equilibria in Budget-constrained Sequential Auctions: An Experimental Study" *Rand Journal of Economics*, 19(3), 363–388.

[138] RAVIV, Y. (2006), "New Evidence on Price Anomalies in Sequential Auctions" *Journal of Business & Economic Statistics* , 24(3), 301-312.

[139] RHODES, A. (2011), "Multiproduct Retailing" *Mimeo.*

[140] RUBINSTEIN, A. (1982), "Perfect Equilibrium in a Bargaining Model." *Econometrica*, 50(1), 97–109.

[141] RUBINSTEIN, A. and R. SPIEGLER (2008), "Money Pumps in the Market" *Journal of the European Economic Association*, 6(1), 237–253.

[142] SALOP, S. and J. STIGLITZ (1977), "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion" *Review of Economic Studies*, 44 (3), 493-510.

[143] SHALEV, J. (2000), "Loss Aversion Equilibrium." *International Journal of Game Theory*, 29(2), 269–287.

[144] SHALEV, J. (2002), "Loss Aversion and Bargaining." *Theory and Decision*, 52(3), 201–232.

[145] SOBEL, J. and I. TAKAHASHI (1983), "A Multistage Model of Bargaining." *Review of Economic Studies*, 50(3), 411-426.

[146] SONG, C. (2012), "An Experiment on Reference Points and Expectations" *Mimeo.*

[147] SPIEGLER, R. (2006), "Competition over Agents with Boundedly Rational Expectations" *Theoretical Economics*, 1(2), 207-231.

[148] SPIEGLER, R. (2011), Bounded Rationality and Industrial Organization, Oxford University Press, New York.

[149] SPIEGLER, R. (2012a), "Monopoly Pricing when Consumers are Antagonized by Unexpected Price Increases: A "Cover Version"of the Heidhues–Kőszegi–Rabin Model" *Economic Theory*, 51 (3), 695-711.

[150] SPIEGLER, R. (2012b), "Incentivizing Participation under Reference-Dependent Preferences: A Note" *Mimeo.*

[151] SPRENGER, C. (2011), "An Endowment Effect for Risk: Experimental Tests of Stochastic Reference Points" *Mimeo.*

[152] STRAUSZ, R. (2007), "Regulating Availability with Demand Uncertainty" *German Economic Review*, 8(2), 107-121.

[153] TAYLOR, J.C. and S.E. FAWCETT (2001), "Retail On-Shelf Performance of Advertised Items: An Assessment of Supply Chain Effectiveness at the Point of Purchase" *Journal of Business Logistics*, 22 (1), 73-89.

[154] THIEL, S. and G. PETRY (1995), "Bidding Behaviour in Second-Price Auctions: Rare Stamp Sales, 1923–1937" *Applied Economics* 27(1), 11–16.

[155] TVERSKY, A. and D. KAHNEMAN (1991), "Loss Aversion in Riskless Choice: A Reference-Dependent Model" *Quarterly Journal of Economics*, 106 (4), 1039-1061.

[156] THANASSOULIS, J. (2004), "Haggling over Substitutes" *Journal of Economic Theory*, 117(2), 217-245.

[157] VAN DEN BERG, G., VAN OURS, J.C., and PRADHAN, M. (2001) "The Declining Price Anomaly in Dutch Dutch Rose Auctions" *American Economic Review*, 91(4), 1055–62.

[158] VANDERPORTEN, B. (1992a) "Strategic Behavior in Pooled Condominium Auctions" *Journal of Urban Economics* 3(1), 123–137.

[159] VANDERPORTEN, B. (1992b) "Timing of Bids at Pooled Real Estate Auctions" *Journal of Real Estate Finance and Economics* 5(3), 255–267.

[160] VARIAN, H. (1980), "A Model of Sales" *American Economic Review*, 70(4), 651-659.

[161] VERBEKE, W., P. FARRIS and R. THURIK (1998), "Consumer Response to the Preferred Brand Out-of-Stock Situation" *European Journal of Marketing*, 32(11), 1008-1028.

[162] VON DER FEHR, N. (1994) "Predatory Bidding in Sequential Auctions" *Oxford Economic Papers* ,46(3), 345–356.

[163] WATSON, R. (2009), "Search and Availability in Monopolistic Competition" *Mimeo*.

[164] WEBER, R. (1983), "Multi-Object Auctions" in (R. Engelbrecht-Wiggans, M. Shubik and R.M. Stark, eds.), *Auctions, Bidding and Contracting: Uses and Theory*, 165-194, New York: New York University Press.

[165] ZHOU, J. (2011), "Reference Dependence and Market Competition" *Journal of Economics and Management Strategy*, 20(4), 1073-1097.

[166] ZHOU, J. (2012), "Multiproduct Search" *Mimeo*.

[167] ZINN, W. and P.C. LIU (2001), "Consumer Response to Retail Stockouts" *Journal of Business Logistics*, 22 (1), 49-71.

[168] ZULEHNER, C. (2009), "Bidding Behavior in Sequential Cattle Auctions" *International Journal of Industrial Organization*, 27(1), 33-42.

Appendix A

Chapter 1 Proofs

Proof of Lemma 12: As shown in Kőszegi and Rabin (2006), the plan of buying good $i = 1, 2$ is a PE if and only if $p_i \leq \frac{1+\eta\lambda}{1+\eta} v_i \equiv p_i^{\max}$ and the plan of not buying good i is a PE if and only if $p_i > \frac{1+\eta}{1+\eta\lambda} v_i \equiv p_i^{\min}$. Therefore, for $p_i \in \left(p_i^{\min}, p_i^{\max}\right]$ both plans are consistent. However, the plan of buying good i at p_i is the PPE if and only if

$$EU\left[\{i\} \,|\, \{i\}\right] \geq EU\left[\{\varnothing\} \,|\, \{\varnothing\}\right]$$

$$\Leftrightarrow \quad v_i - p_i \geq 0$$

and this proves the statement. ∎

Proof of Lemma 2: The result holds trivially for the case of perfect availability. Then, let $q_1 > 0$, $q_2 > 0$ with $q_1 + q_2 < 1$ and suppose the seller charges p_1 for item 1 and p_2 for item 2, with $p_2 \geq p_1$. The highest price the seller can charge for item 2 is the one that makes the following inequality bind:

$$EU\left[\{1, 2\} \,|\, \{1, 2\}\right] \geq EU\left[\{\varnothing\} \,|\, \{\varnothing\}\right]. \tag{A.1}$$

Substituting and re-arranging yields

$$p_2 \leq \frac{v_2 \left[1 + \eta\left(\lambda - 1\right) q_1 - \eta\left(\lambda - 1\right)\left(1 - q_1 - q_2\right)\right] - 2\eta\left(\lambda - 1\right) q_1}{1 + \eta\left(\lambda - 1\right)\left(1 - q_2\right)}.$$

It is easy to see that the right-hand-side of the above inequality is increasing in q_2. Therefore, the seller can raise q_2 up to $1 - q_1$ and increase her profits without violating condition (A.1). A similar analysis applies if $p_2 < p_1$. ∎

Proof of Lemma 17: I prove the result by contradiction. Suppose that $q \in (0, 1)$ and $p_i = v_i$ for $i = 1, 2$ and that $v_1 > 2v_2$; then we have that

$$
\begin{aligned}
EU\left[\{\varnothing\} \,|\, \{\varnothing\}\right] \;=\; & 0 \\
> \;& -2\eta\left(\lambda - 1\right) q\left(1 - q\right) v_2 = EU\left[\{2, \varnothing\} \,|\, \{2, \varnothing\}\right] \\
> \;& -2\eta\left(\lambda - 1\right) q\left(1 - q\right)\left(v_1 - v_2\right) = EU\left[\{1, 2\} \,|\, \{1, 2\}\right] \\
> \;& -2\eta\left(\lambda - 1\right) q\left(1 - q\right) v_1 = EU\left[\{1, \varnothing\} \,|\, \{1, \varnothing\}\right].
\end{aligned}
$$

Furthermore, we know that not buying is a PE when $p_i = v_i$. Therefore, for this quantity vector and this price vector the buyers would strictly prefer the plan of not buying . The seller would then do better by setting $p_i = p_i^{\min}$ for at least one good and thus force the consumers to buy it. The same argument applies to the case in which $v_1 \leq 2v_2$ (just switch the first and second inequalities). ∎

Proof of Lemma 13: I prove the result by contradiction. Suppose that $q \in (0, 1)$ and $p_i > p_i^{\min}$ for $i = 1, 2$ and that $v_1 - c_1 \geq v_2 - c_2$. By producing a strictly positive quantity of both goods, the seller wants the buyers to choose the plan to always buy; however, for this plan to be the PPE it must be that

$$EU\left[\{1, 2\} \,|\, \{1, 2\}\right] \geq EU\left[\{\varnothing\} \,|\, \{\varnothing\}\right]$$

$$\Rightarrow \quad q\left(v_1 - p_1\right) + \left(1 - q\right)\left(v_2 - p_2\right) > 0$$

$$\Leftrightarrow \quad qp_1 + \left(1 - q\right)p_2 < qv_1 + \left(1 - q\right)v_2$$

$$\Rightarrow \quad q\left(p_1 - c_1\right) + \left(1 - q\right)\left(p_2 - c_2\right) < q\left(v_1 - c_1\right) + \left(1 - q\right)\left(v_2 - c_2\right) \leq v_1 - c_1.$$

But then the seller would prefer to set $q = 1$ and $p_1 = v_1$ and this contradicts the assumption that seller produces a strictly positive quantity of both goods. The same argument applies to the case in which $v_1 - c_1 < v_2 - c_2$. ∎

Proof of Lemma 16: Let $q \in (0, 1)$. From Lemma 13 we know that $p_i = p_i^{\min}$ for at least one good; let this be good 2. I now show that it is not profitable for the seller to choose p_1 such that the plan to always buy is the unique credible plan for the consumers. First, we have that, for $p_2 = p_2^{\min}$, the highest price the seller can use, in order to make the plan to buy only good 2 not credible, is

$$p_1 \leq \frac{\left(1 + \eta\right)v_1 + \eta\left(\lambda - 1\right)\left(1 - q\right)v_2\left(1 + \frac{1+\eta}{1+\eta\lambda}\right)}{1 + \eta\lambda} \equiv \widetilde{p}_1\left(q\right)$$

Then, we have that, for $p_2 = p_2^{\min}$, the plan to always buy is a PE if and only if

$$p_1 \leq \frac{\left[1 + \eta\left(1 - q\right) + \eta\lambda q\right]v_1 + \eta\left(\lambda - 1\right)\left(1 - q\right)v_2\left(1 + \frac{1+\eta}{1+\eta\lambda}\right)}{1 + \eta q + \eta\lambda\left(1 - q\right)} \equiv \overline{p}_1\left(q\right).$$

It is readily verified that $\overline{p}_1\left(q\right) > \widetilde{p}_1\left(q\right) \Leftrightarrow q > 0$. However, for $\overline{p}_1\left(q\right) \geq p_1 > \widetilde{p}_1\left(q\right)$ both the plan to always buy and the plan to buy only item 2 are personal equilibria; but the plan of always buying is the PPE if and only if

$$p_1 \leq v_1 + \frac{2\left(1 - q\right)\eta\left(\lambda - 1\right)\left[v_2\left(2 + \eta + \eta\lambda\right) - v_1\left(1 + \eta\lambda\right)\right]}{\left(1 + \eta\lambda\right)\left[1 + \eta\left(\lambda - 1\right)\left(1 - q\right)\right]} \equiv \widehat{p}_1\left(q\right).$$

It is easy to see that $\widehat{p}_1\left(q\right) > \widetilde{p}_1\left(q\right)$. Therefore, the highest price p_1^* at which a buyer prefers the plan to always buy is given by

$$p_1^* = \min\left\{\overline{p}_1\left(q\right), \widehat{p}_1\left(q\right)\right\}$$

and this proves that it is not profit-maximizing for the seller to make always buying the unique consistent plan.

Then, in order to prove that $p_1^* = \widehat{p}_1(q)$, notice that

$$\overline{p}_1(q) < \widehat{p}_1(q)$$

$$\Leftrightarrow q < \frac{v_2(1+2\eta\lambda)(2+\eta+\eta\lambda) - \eta v_1(1+\lambda)(1+\eta\lambda) - \sqrt{A^2 v_1^2 - 2B v_1 v_2 + C^2 v_2^2}}{2v_2\eta(\lambda-1)(2+\eta+\eta\lambda)}$$

where $A \equiv \eta(1+\eta\lambda)(1+\lambda)$, $B \equiv \eta(1+\eta\lambda)(2+\eta+\eta\lambda)[3+2\eta+2\eta(\lambda-1)]$ and $C \equiv (1+2\eta)(2+\eta+\eta\lambda)$. It is also easy to verify that

$$\frac{v_2(1+2\eta\lambda)(2+\eta+\eta\lambda) - \eta v_1(1+\lambda)(1+\eta\lambda) - \sqrt{A^2 v_1^2 - 2B v_1 v_2 + C^2 v_2^2}}{2v_2\eta(\lambda-1)(2+\eta+\eta\lambda)} < 1.$$

However, it is in the seller's interest to select the p_1^* that maximizes $q p_1^*$ and since

$$v_2(2+\eta+\eta\lambda) - v_1(1+\eta\lambda) > 0 \Rightarrow \frac{\partial[q\widehat{p}_1(q)]}{\partial q} > 0,$$

it follows that $p_1^* = \widehat{p}_1(q)$. The same argument applies if the seller uses item 1 as the bargain (i.e., $p_1 = p_1^{\min}$). ■

Proof of Lemma 6: Suppose the seller uses item 2 as the bargain and thus prices it at $p_2^{\min}$. Then, by Lemma 16 we know that the optimal price for item 1 is

$$p_1^* = v_1 + \frac{2(1-q)\eta(\lambda-1)[v_2(2+\eta+\eta\lambda) - v_1(1+\eta\lambda)]}{(1+\eta\lambda)[1+\eta(\lambda-1)(1-q)]}.$$

This pair of prices provides the seller with profits equal to

$$q\left\{\frac{v_1[1-\eta(\lambda-1)(1-q)] + 2\eta(\lambda-1)(1-q)\left(1+\frac{1+\eta}{1+\eta\lambda}\right)v_2}{1+\eta(\lambda-1)(1-q)} - c_1\right\}$$

$$+ (1-q)\left(p_2^{\min} - c_2\right).$$

The above expression is maximized at

$$q = \frac{1+\eta\lambda-\eta}{\eta(\lambda-1)} - \frac{\sqrt{2}}{\eta(\lambda-1)}\frac{\sqrt{(1+\eta\lambda-\eta)(-v_1+2v_2+\eta v_2 - \lambda\eta v_1 + \lambda\eta v_2)}}{\sqrt{(-c_1+c_2-v_1)(1+\eta\lambda) + v_2(3+\eta+2\eta\lambda)}}$$

$$\equiv \overline{q}(\eta,\lambda,v_1,v_2,c_1,c_2).$$

Notice that for the above expression to be well-defined, it must be that

$$(-c_1+c_2-v_1)(1+\eta\lambda) + v_2(3+\eta+2\eta\lambda) > 0$$

since we know that $(2+\eta+\eta\lambda)v_2 > (1+\eta\lambda)v_1$ for p_1^* to be greater than v_1. It is easy to see that $\overline{q} > 0$. Furthermore, we have that

$$\overline{q} < 1 \Leftrightarrow v_1\left[1+3\eta\lambda - 2\eta + 2\eta^2\lambda(\lambda-1)\right] < (c_1-c_2)(1+\eta\lambda) +$$
$$v_2\left[1+4\eta\lambda - 3\eta + 2\eta^2(\lambda-1)(\lambda+1)\right].$$

Notice that

$$\bar{q}\left(\eta,\lambda,v_1,v_2,c_1,c_2\right) > \frac{1}{2}$$

since

$$\bar{q}\left(\eta,\lambda,v_1,v_2,c_1,c_2\right) > \bar{q}\left(\eta,\lambda,v,v,c,c\right)$$

$$\Leftrightarrow \quad \frac{1+\eta\lambda-\eta}{\eta\left(\lambda-1\right)} - \frac{\sqrt{2}}{\eta\left(\lambda-1\right)}\frac{\sqrt{\left(1+\eta\lambda-\eta\right)\left(-v_1+2v_2+\eta v_2-\eta\lambda v_1+\eta\lambda v_2\right)}}{\sqrt{\left(-c_1+c_2-v_1\right)\left(1+\eta\lambda\right)+v_2\left(3+\eta+2\eta\lambda\right)}} >$$

$$\frac{1+\eta\lambda-\eta}{\eta\left(\lambda-1\right)} - \frac{\sqrt{2}}{\eta\left(\lambda-1\right)}\sqrt{\frac{\left(1+\eta\right)\left(\eta\lambda-\eta+1\right)}{\left(\eta+\eta\lambda+2\right)}}$$

$$\Leftrightarrow \left(1+\eta\lambda\right)\left(1+\eta\lambda-\eta\right)\left[\left(c_1-c_2\right)\left(1+\eta\right)+\left(v_2-v_1\right)\left(1+\eta\lambda\right)\right] < 0$$

which is true for any $\eta > 0$ and $\lambda > 1$ provided that $v_1 - c_1 > v_2 - c_2$ (which, as shown below, is a necessary condition for the seller to use item 2 as the bargain); and

$$\bar{q}\left(\eta,\lambda,v,v,c,c\right) > \frac{1}{2}$$

$$\Leftrightarrow \quad \frac{1}{\eta\left(\lambda-1\right)}\left(\eta\lambda-\eta+1\right) - \frac{1}{\eta\left(\lambda-1\right)}\sqrt{\frac{2\left(1+\eta\right)\left(\eta\lambda-\eta+1\right)}{\eta+\eta\lambda+2}} > \frac{1}{2}$$

$$\Leftrightarrow \quad \frac{\eta\left(\lambda-1\right)\left(\eta^2\lambda^2+6\eta\lambda-\eta^2-6\eta+4\right)}{\eta+\eta\lambda+2} > 0$$

which is true for any $\eta > 0$ and $\lambda > 1$.

If instead the seller uses item 1 as the bargain, then by Lemma 16 we know that the optimal price for item 2 is

$$p_2^* = v_2 + \frac{2qv_1\eta\left(\lambda-1\right)\left(1+\eta\right)}{\left(1+\eta\lambda\right)\left[1+\eta\left(\lambda-1\right)q\right]}.$$

This pair of prices provides the seller with profits equal to

$$q\left(p_1^{\min}-c_1\right)+\left(1-q\right)\left\{\frac{v_2\left[1+\eta\left(\lambda-1\right)q\right]+2\eta\left(\lambda-1\right)q\left(\frac{1+\eta}{1+\eta\lambda}\right)v_1}{1+\eta\left(\lambda-1\right)q}-c_2\right\}.$$

The above expression is maximized at

$$q = \frac{\sqrt{2}}{\eta\left(\lambda-1\right)}\frac{\sqrt{v_1\left(1+\eta\right)\left(1+\eta\lambda-\eta\right)}}{\sqrt{\left(c_1-c_2+v_2\right)\left(1+\eta\lambda\right)+v_1\left(1+\eta\right)}} - \frac{1}{\eta\left(\lambda-1\right)}$$

$$\equiv \underline{q}\left(\eta,\lambda,v_1,v_2,c_1,c_2\right).$$

We have that

$$\underline{q} < 1 \Leftrightarrow v_1\left(1+\eta\right)\left(1+\eta-\eta\lambda\right) < \left(1-\eta+\eta\lambda\right)\left(v_2-c_2+c_1\right)\left(1+\eta\lambda\right).$$

Similarly, we also have

$$\underline{q} > 0 \Leftrightarrow v_1 (1 + \eta) (1 + 2\eta\lambda - 2\eta) > (v_2 - c_2 + c_1) (1 + \eta\lambda).$$

Notice that

$$\underline{q} (\eta, \lambda, v_1, v_2, c_1, c_2) < \frac{1}{2}$$

$$\Leftrightarrow \quad 2\sqrt{\frac{2v_1 (1 + \eta) (1 + \eta\lambda - \eta)}{(c_1 - c_2 + v_1 + v_2 + \eta v_1 + \lambda\eta c_1 - \lambda\eta c_2 + \lambda\eta v_2)}} < \eta (\lambda - 1) + 2$$

$$\Leftrightarrow \quad \left\{ 8 (1 + \eta\lambda - \eta) - [\eta (\lambda - 1) + 2]^2 \right\} (1 + \eta) v_1 < (v_2 - c_2 + c_1) (1 + \eta\lambda) [\eta (\lambda - 1) + 2]^2.$$
$$\text{(A.2)}$$

Condition (A.2) is trivially satisfied for any $\eta > 0$ and $\lambda > 1$ if $v_2 - c_2 \geq v_1 - c_1$ since $[\eta (\lambda - 1) + 2]^2 - \left\{ 8 (1 + \eta\lambda - \eta) - [\eta (\lambda - 1) + 2]^2 \right\} = 2\eta^2 (\lambda - 1)^2 > 0$. Condition (A.2) holds also for $v_2 - c_2 < v_1 - c_1$ if $\eta \leq 1$ since, as shown below in the proof of proposition 1, if the seller prefers to use item 1 as the bargain when this is the item with the larger social surplus then $v_1 < (v_2 - c_2 + c_1) \left(\frac{1 + \eta\lambda}{1 + \eta} \right)$.

Finally, we have that

$$\overline{q} > 1 - \underline{q}$$

$$\Leftrightarrow \quad \frac{v_2 (2 + \eta + \eta\lambda) - v_1 (1 + \eta\lambda)}{(-c_1 + c_2 - v_1) (1 + \eta\lambda) + v_2 (3 + \eta + 2\eta\lambda)} < \frac{v_1 (1 + \eta)}{(c_1 - c_2 + v_2) (1 + \eta\lambda) + v_1 (1 + \eta)}$$

$$\Leftrightarrow \quad v_2 (2 + \eta\lambda + \eta) (v_2 - v_1 + c_1 - c_2) - \eta (\lambda - 1) v_1 (c_1 - c_2) < 0$$

which holds for any $\eta > 0$ and $\lambda > 1$ given we know that $v_2 (2 + \eta\lambda + \eta) > v_1 (1 + \eta\lambda)$ from Lemma 16 and provided that $v_1 - c_1 > v_2 - c_2$ which, as shown below, is the only case in which $\overline{q}$ and $\underline{q}$ are comparable. ∎

Proof of Lemma 7: Define $\pi_1 \equiv \pi \left(p_1^*, p_2^{\min}, \overline{q}; c_1, c_2 \right)$ and $\pi_2 \equiv \pi \left(p_1^{\min}, p_2^*, \underline{q}; c_1, c_2 \right)$ and recall that $\overline{q} = \arg\max_q \pi \left(p_1^*, p_2^{\min}, q; c_1, c_2 \right)$ and $\underline{q} = \arg\max_q \pi \left(p_1^{\min}, p_2^*, q; c_1, c_2 \right)$.

First, consider the special case with $v_1 = v_2$ and $c_1 = c_2$. It is easy to see that in this case $p_1^{\min} = p_2^{\min}$, $p_1^* = p_2^*$, $\overline{q} = 1 - \underline{q}$ so that $\pi_1 = \pi_2$. Therefore the seller is indifferent between which item to use as the bargain. Furthermore, by the envelope theorem we have that $\frac{d\pi_1}{dc_1} = -\overline{q}$, $\frac{d\pi_1}{dc_2} = -(1 - \overline{q})$, $\frac{d\pi_2}{dc_1} = -(1 - \underline{q})$ and $\frac{d\pi_2}{dc_2} = -\underline{q}$. By lemma 6 we know that $\overline{q} > 1 - \underline{q}$ and therefore it follows that when the two goods are perfect substitutes, the seller maximizes profits by using the more expensive one as the bargain.

Next, suppose to change v_1 by dv_1 and c_1 by dc_1 with $dv_1 = dc_1 = \delta > 0$ so that $v_1 > v_2$ but $v_1 - v_2 = c_1 - c_2$.

By the envelope theorem the effect of these changes on profits are

$$d\pi_1 \simeq \frac{\partial \pi_1}{\partial v_1} dv_1 + \frac{\partial \pi_1}{\partial c_1} dc_1 = \left(\overline{q} \frac{\partial p_1^*}{\partial v_1} - \overline{q} \right) \delta$$

and

$$d\pi_2 \simeq \frac{\partial \pi_2}{\partial v_1} dv_1 + \frac{\partial \pi_2}{\partial c_1} dc_1 = \left[\underline{q} \frac{1 + \eta}{1 + \eta\lambda} + (1 - \underline{q}) \frac{\partial p_2^*}{\partial v_1} - \underline{q} \right] \delta.$$

By substituting and re-arranging, it follows that $d\pi_2 > d\pi_1$ if and only if

$$\underline{q}\frac{\eta(\lambda-1)}{1+\eta\lambda}\left[\frac{2\left(1-\underline{q}\right)(1+\eta)-\underline{q}\eta(\lambda-1)-1}{\underline{q}\eta(\lambda-1)+1}\right] > \bar{q}\left[\frac{1-\eta(\lambda-1)(1-\bar{q})}{1+\eta(\lambda-1)(1-\bar{q})}-1\right]. \quad \text{(A.3)}$$

As the expression on the right-hand-side of (A.3) is negative, it suffices to show that

$$2\left(1-\underline{q}\right)(1+\eta)-\underline{q}\eta(\lambda-1)-1 > 0 \Leftrightarrow \frac{1+2\eta}{2+\eta+\eta\lambda} > \underline{q}.$$

Substituting $v_1 = v_2$ and $c_1 = c_2$ into the expression for $\underline{q}$ yields

$$\frac{2+\eta+\eta\lambda+\eta(\lambda-1)(1+2\eta)}{2+\eta+\eta\lambda} > \sqrt{\frac{2+\eta+\eta\lambda+\eta(\lambda-1)(1+2\eta)}{2+\eta+\eta\lambda}}$$

which is of course true for any $\eta > 0$ and $\lambda > 1$. Thus, the seller maximizes profits by using item 1 as the bargain if $v_1 > v_2$ and $v_1 - v_2 = c_1 - c_2$. Furthermore, it is easy to see that the same result holds also if $dc_1 > dv_1 > 0$ so that $v_1 - c_1 < v_2 - c_2$. Therefore, we have that $\pi_2 \geq \pi_1$ for $v_1 - c_1 \leq v_2 - c_2$.

Finally, consider the case in which $v_1 - c_1 > v_2 - c_2$. Again, let's start with $v_1 = v_2$ and $c_1 = c_2$ so that $\pi_1 = \pi_2$ and suppose to change v_1 by dv_1 and c_1 by dc_1 with either $dv_1 > dc_1 \geq 0$ or $dv_1 \geq 0 > dc_1$. By the envelope theorem the effect of these changes on profits are

$$d\pi_1 \simeq \frac{\partial\pi_1}{\partial v_1}dv_1 + \frac{\partial\pi_1}{\partial c_1}dc_1 = \bar{q}\frac{\partial p_1^*}{\partial v_1}dv_1 - \bar{q}dc_1$$

and

$$d\pi_2 \simeq \frac{\partial\pi_2}{\partial v_1}dv_1 + \frac{\partial\pi_2}{\partial c_1}dc_1 = \left[\underline{q}\frac{1+\eta}{1+\eta\lambda} + (1-\underline{q})\frac{\partial p_2^*}{\partial v_1}\right]dv_1 - \underline{q}dc_1.$$

By substituting and re-arranging, it follows that $d\pi_1 \geq d\pi_2$ if and only if

$$\left[\frac{1-\eta(\lambda-1)(1-\bar{q})}{1+\eta(\lambda-1)(1-\bar{q})}\bar{q} - \frac{1+\eta}{1+\eta\lambda}\underline{q} - (1-\underline{q})\frac{1+\eta}{1+\eta\lambda}\frac{2\underline{q}\eta(\lambda-1)}{1+\underline{q}\eta(\lambda-1)}\right]dv_1 \geq \left(\bar{q}-\underline{q}\right)dc_1. \quad \text{(A.4)}$$

We know that for $dv_1 = dc_1 > 0$ condition (A.4) is violated; but for either $dv_1 > dc_1 \geq 0$ or $dv_1 \geq 0 > dc_1$ it can hold (for example, it is readily satisfied for $dv_1 = 0$ and $dc_1 < 0$). Then, let $\widetilde{v}_1$ be the value of v_1 for which (A.4) binds; if such a value exists then it is unique because the term on the left-hand-side of (A.4) is continuous and increasing in dv_1. Notice also that $\widetilde{v}_1$ increases with $c_1 - c_2$.

However, from lemma 6 we know that

$$\bar{q} < 1 \Leftrightarrow v_1 < \frac{(c_1-c_2)(1+\eta\lambda)+v_2(1+4\eta\lambda-3\eta)+2\eta^2v_2(\lambda-1)(\lambda+1)}{1+3\eta\lambda-2\eta+2\eta^2\lambda(\lambda-1)}.$$

Therefore, a necessary condition for the seller to use item 2 as the bargain when $v_1 - c_1 > v_2 - c_2$ is that

$$\frac{(c_1-c_2)(1+\eta\lambda)+v_2(1+4\eta\lambda-3\eta)+2\eta^2v_2(\lambda-1)(\lambda+1)}{1+3\eta\lambda-2\eta+2\eta^2\lambda(\lambda-1)} > v_2 - c_2 + c_1$$

$$\Leftrightarrow \ \eta \left(\lambda - 1 \right) \left[2 \left(1 + \eta \lambda \right) \left(c_2 - c_1 \right) + v_2 \left(1 + 2\eta \right) \right] > 0$$

$$\Leftrightarrow \ 2 \left(1 + \eta \lambda \right) \left(c_2 - c_1 \right) + v_2 \left(1 + 2\eta \right) > 0$$

$$\Leftrightarrow \ v_2 > \frac{2 \left(1 + \eta \lambda \right) \left(c_1 - c_2 \right)}{1 + 2\eta}.$$

However, the above condition is not sufficient as it could still be that

$$\widetilde{v}_1 > \frac{\left(c_1 - c_2 \right) \left(1 + \eta \lambda \right) + v_2 \left(1 + 4\eta \lambda - 3\eta \right) + 2\eta^2 v_2 \left(\lambda - 1 \right) \left(\lambda + 1 \right)}{1 + 3\eta \lambda - 2\eta + 2\eta^2 \lambda \left(\lambda - 1 \right)}. \ \blacksquare$$

Proof of Proposition 10: For an arbitrary price-pair (p_1, p_2) and an arbitrary quantity-pair $(q, 1 - q)$ the monopolist's profit is

$$\pi \left(p_1, p_2, q; c_1, c_2 \right) = q \left(p_1 - c_1 \right) + \left(1 - q \right) \left(p_2 - c_2 \right).$$

By Lemma 12 we know that if the seller produces only one good, then she will price it at its intrinsic value.

By Lemma 17 and Lemma 13 we know that if the seller produces a strictly positive quantity of both goods then one of them, say good i, must be priced at the discounted price $p_i^{\min}$. By Lemma 16 we also know that in this case the seller will price good j at p_j^*. Therefore, the seller has three options:

i) Set $p_2 = p_2^{\min}$, $p_1 = p_1^*$ and $q = \overline{q}$. In this case the seller's profit is

$$\overline{q} \left(p_1^* - c_1 \right) + \left(1 - \overline{q} \right) \left(p_2^{\min} - c_2 \right) \equiv \pi_1.$$

ii) Set $p_1 = p_1^{\min}$, $p_2 = p_2^*$ and $q = \underline{q}$. In this case the seller's profit is

$$\underline{q} \left(p_1^{\min} - c_1 \right) + \left(1 - \underline{q} \right) \left(p_2^* - c_2 \right) \equiv \pi_2.$$

iii) Set $p_i = v_i$ for $i = 1, 2$. This pair of prices provides the seller with profits equal to

$$q \left(v_1 - c_1 \right) + \left(1 - q \right) \left(v_2 - c_2 \right).$$

The above expression is maximized at $q = 1$ (resp. $q = 0$) if $v_1 - c_1 > v_2 - c_2$ (resp. if $v_1 - c_1 \leq v_2 - c_2$).

Depending on the degree of substitutability between the two goods, their marginal costs and the degree of loss aversion, the seller will choose the option that will give her the highest profit. Suppose first that $v_1 - c_1 \leq v_2 - c_2$. By Lemma 7 we know that if she were to produce both goods, the seller would prefer to use item 1 as the bargain Then,

$$\pi \left(p_1^{\min}, p_2^*, \underline{q}; c_1, c_2 \right) \geq v_2 - c_2$$

$$\Leftrightarrow \ v_1 \geq \frac{v_2 - c_2 + c_1}{1 + 2\eta \left(\lambda - 1 \right)} \frac{1 + \eta \lambda}{1 + \eta} \equiv \alpha \left(v_2, c_1, c_2, \eta, \lambda \right).$$

Now suppose that $\widetilde{v}_1 > v_1 > v_2 - c_2 + c_1$. By Lemma 7 we know that if she were to produce both goods, the seller would again prefer to use item 1 as the bargain. Therefore,

$$\pi\left(p_1^{\min}, p_2^*, \underline{q}; c_1, c_2\right) \geq v_1 - c_1$$

$$\Leftrightarrow \quad v_1 \leq (v_2 - c_2 + c_1)\left(\frac{1 + \eta\lambda}{1 + \eta}\right) \Xi(\eta, \lambda) \equiv \beta(v_2, c_1, c_2, \eta, \lambda).$$

where

$$\Xi(\eta, \lambda) \equiv [1 + \eta(\lambda - 1)] \times$$
$$\left[\frac{3\eta + 4\eta^2 + 2\eta^3 + \eta^2\lambda^2(1+\eta) - \eta\lambda(1 + 3\eta^2 + 4\eta) - 2\eta(\lambda - 1)\sqrt{2(1+\eta)^3} + 1}{4\eta(1 + \eta^3) + \eta^4\lambda^4 - 2\eta^3\lambda^3(1 + 3\eta) + \eta^2\lambda^2(13\eta^2 + 2\eta - 5) - 2\eta\lambda(6\eta^3 - 3\eta + 1) + 1}\right].$$

Furthermore, since $\Xi(\eta, \lambda) < 1$ for $\eta \leq 1$, we have that

$$\eta \leq 1 \Rightarrow \beta(v_2, c_1, c_2, \eta, \lambda) < (v_2 - c_2 + c_1)\left(\frac{1 + \eta\lambda}{1 + \eta}\right).$$

Finally, if $v_1 \geq \widetilde{v}_1$ then by Lemma 7 the seller prefers to use item 2 as the bargain and we have

$$\pi\left(p_1^*, p_2^{\min}, \overline{q}; c_1, c_2\right) \geq v_1 - c_1 \Leftrightarrow v_2 \geq \frac{v_1 - c_1 + c_2 + 2\eta(\lambda - 1)v_1}{1 + \eta(\lambda - 1)\left(\frac{3 + 2\eta\lambda + 2\eta}{1 + \eta\lambda}\right)} \equiv \gamma(v_1, c_1, c_2, \eta, \lambda).$$

To conclude the proof, notice that the seller's profits, if she chooses to produce only one good, are equal to $\max\{v_1 - c_1, v_2 - c_2\}$. Since she would choose a different option only if this provides her with at least as much, it thus follows that $\pi \geq \max\{v_1 - c_1, v_2 - c_2\}$, and the inequality is strict when either option i) or ii) is profit-maximizing. ∎

Proof of Proposition 2: Suppose the seller uses item 2 as the bargain. We have:

$$\overline{q}\left(p_1^* - c_1\right) + (1 - \overline{q})\left(p_2^{\min} - c_2\right) > \max\{v_1 - c_1, v_2 - c_2\}$$
$$\geq \overline{q}(v_1 - c_1) + (1 - \overline{q})(v_2 - c_2)$$

$$\Rightarrow \overline{q}p_1^* + (1 - \overline{q})p_2^{\min} > \overline{q}v_1 + (1 - \overline{q})v_2.$$

In this case, therefore, a consumer expects to buy with probability one at an expected price strictly greater than his expected valuation. Hence, his consumption utility is negative. Furthermore, in any PE expected gain-loss utility is non-positive. If instead he could commit to the plan of never buying, both his consumption utility and his gain-loss utility would be zero. The same argument applies for the case in which the seller uses item 1 as the bargain. ∎

Proof of Proposition 3: First, consider the seller's profits when item 1 is used as the bargain. We have:

$$\pi_2 = \pi\left(p_1^{\min}, p_2^*, \underline{q}; c_1, c_2\right) = \underline{q}\left(p_1^{\min} - c_1\right) + (1 - \underline{q})(p_2^* - c_2).$$

By the envelope theorem, we have that:

$$
\begin{aligned}
\frac{d\pi_2}{dv_1} &= \underline{q}\frac{\partial p_1^{\min}}{\partial v_1} + (1-\underline{q})\frac{\partial p_2^*}{\partial v_1} \\
&= \frac{1+\eta}{1+\eta\lambda}\underline{q} + (1-\underline{q})\frac{1+\eta}{1+\eta\lambda}\frac{2\eta(\lambda-1)\underline{q}}{1+\eta(\lambda-1)\underline{q}} \\
&= \frac{1+\eta}{1+\eta\lambda}\underline{q}\left[1 + \frac{2\eta(\lambda-1)(1-\underline{q})}{1+\eta(\lambda-1)\underline{q}}\right] \\
&> \underline{q} \\
&= \left|\frac{d\pi_2}{dc_1}\right|
\end{aligned}
$$

where the inequality follows from

$$
1 + \frac{2\eta(\lambda-1)(1-\underline{q})}{1+\eta(\lambda-1)\underline{q}} > \frac{1+\eta\lambda}{1+\eta}
$$

$$
\begin{aligned}
&\Leftrightarrow & 2(1-\underline{q})(1+\eta) &> 1+\eta(\lambda-1)\underline{q} \\
&\Leftrightarrow & \frac{1+2\eta}{2+\eta\lambda+\eta} &> \underline{q} \\
&\Leftrightarrow & v_1 &< \frac{2(\eta+1)(\eta^2\lambda^2-\eta^2\lambda+2\eta\lambda-\eta+1)(c_1-c_2+v_2)}{2\eta+3\eta^2+2\eta^3+\eta^2\lambda^2+2\eta\lambda-2\eta^2\lambda-2\eta^3\lambda+2}
\end{aligned}
$$

and

$$
\frac{2(\eta+1)(\eta^2\lambda^2-\eta^2\lambda+2\eta\lambda-\eta+1)(c_1-c_2+v_2)}{2\eta+3\eta^2+2\eta^3+\eta^2\lambda^2+2\eta\lambda-2\eta^2\lambda-2\eta^3\lambda+2} > \beta(v_2,c_1,c_2,\eta,\lambda)
$$

for $\eta \leq 1$.

Similarly, we also have that

$$
\begin{aligned}
\frac{d\pi_2}{dv_2} &= (1-\underline{q})\frac{\partial p_2^*}{\partial v_1} \\
&= (1-\underline{q}) \\
&= \left|\frac{d\pi_2}{dc_2}\right|.
\end{aligned}
$$

Next, consider the seller's profits when item 2 is used as the bargain. We have:

$$
\pi_1 = \pi\left(p_1^*, p_2^{\min}, \overline{q}; c_1, c_2\right) = \overline{q}(p_1^* - c_1) + (1-\overline{q})\left(p_2^{\min} - c_2\right).
$$

Then, we have that

$$
\begin{aligned}
\frac{d\pi_1}{dv_1} &= \overline{q}\frac{\partial p_1^*}{\partial v_1} \\
&= \overline{q}\left[\frac{1-\eta(\lambda-1)(1-\overline{q})}{1+\eta(\lambda-1)(1-\overline{q})}\right] \\
&< \overline{q} \\
&= \left|\frac{d\pi_1}{dc_1}\right|.
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
\frac{d\pi_1}{dv_2} &= \overline{q}\frac{\partial p_1^*}{\partial v_2} + (1-\overline{q})\frac{\partial p_2^{\min}}{\partial v_2} \\
&= \frac{2\eta(\lambda-1)(2+\eta\lambda+\eta)\overline{q}(1-\overline{q})}{(1+\eta\lambda)[1+\eta(\lambda-1)(1-\overline{q})]} + (1-\overline{q})\frac{1+\eta}{1+\eta\lambda} \\
&= (1-\overline{q})\left[\frac{2\eta(\lambda-1)(2+\eta\lambda+\eta)\overline{q}}{(1+\eta\lambda)[1+\eta(\lambda-1)(1-\overline{q})]} + \frac{1+\eta}{1+\eta\lambda}\right] \\
&> (1-\overline{q}) \\
&= \left|\frac{d\pi_1}{dc_2}\right|
\end{aligned}
$$

where the inequality follows from

$$
\begin{aligned}
\frac{2\eta(\lambda-1)(2+\eta\lambda+\eta)\overline{q}}{(1+\eta\lambda)[1+\eta(\lambda-1)(1-\overline{q})]} + \frac{1+\eta}{1+\eta\lambda} &> 1 \\
\Leftrightarrow \quad 2(2+\eta\lambda+\eta)\overline{q} &> 1+\eta(\lambda-1)(1-\overline{q}) \\
\Leftrightarrow \quad (4+3\eta\lambda+\eta)\overline{q} &> 1+\eta(\lambda-1) \\
\Leftarrow \quad \overline{q} &> \frac{1}{2}
\end{aligned}
$$

and this concludes the proof. ∎

Proof of Proposition 4: If the seller uses item 1 as the rip-off, item 2 must be priced at $p_2^{\min}$. Let $p_2^{\min} \le v_1 - v_2$. First, notice that if the seller uses item 1 as the rip-off, then it must be that $p_1 > v_1$. To see why, suppose, by contradiction, that $p_1 \le v_1$. The seller's profit is

$$
q(p_1 - c_1) + (1-q)\left(p_2^{\min} - c_2\right).
$$

We have that

$$
p_1 \le v_1 \Rightarrow q(p_1 - c_1) + (1-q)\left(p_2^{\min} - c_2\right) < q(v_1 - c_1) + (1-q)(v_2 - c_2) < \max\{v_1 - c_1, v_2 - c_2\}.
$$

But then the seller would prefer to choose either $q = 1$ or $q = 0$, contradicting the hypothesis that she is producing a strictly positive quantity of both goods.

Next, recall that the seller's scheme must make the consumers indifferent between planning to buy only the bargain (item 2 in this case) and planning to always buy:

$$
\begin{aligned}
(1-q)\left(v_2 - p_2^{\min}\right) - q(1-q)\eta(\lambda-1)\left(v_2 + p_2^{\min}\right) &= q(v_1 - p_1) + (1-q)\left(v_2 - p_2^{\min}\right) \\
&\quad - q(1-q)\eta(\lambda-1)\left(v_1 - v_2 + p_1 - p_2^{\min}\right)
\end{aligned}
$$

$$
\Leftrightarrow (1-q)\eta(\lambda-1)\left[v_1 + p_1 - 2\left(v_2 + p_2^{\min}\right)\right] = v_1 - p_1. \tag{A.5}
$$

Since the right-hand-side of (A.5) is negative, it follows that

$$
v_1 + p_1 - 2\left(v_2 + p_2^{\min}\right) < 0
$$

$$
\Leftrightarrow \frac{v_1 + p_1}{2} - v_2 < p_2^{\min}. \tag{A.6}
$$

Condition(A.6) and the assumption that $p_2^{\min} \leq v_1 - v_2$ combined together imply

$$\frac{v_1 + p_1}{2} - v_2 < v_1 - v_2 \Leftrightarrow p_1 < v_1.$$

The result then follows by *reductio ad absurdum*. ∎

Proof of Proposition 14: Let $p_1 = p_1^{\min}$ and suppose $p_2 < p_1$. Let $q \in (0,1)$ be the degree of availability of the bargain item, and suppose consumers plan to always buy. If item 2 is the only product left in the store, a consumer will follow his plan and buy if

$$U\left[(v_2, p_2) \mid \{1,2\}\right] \geq U\left[(0,0) \mid \{1,2\}\right]$$

$$\Leftrightarrow v_2 - p_2 - q\eta\lambda(v_1 - v_2) + q\eta\left(p_1^{\min} - p_2\right) \geq -q\eta\lambda v_1 - (1-q)\eta\lambda v_2 + q\eta p_1^{\min} + (1-q)\eta p_2$$

$$\Leftrightarrow p_2 \leq \frac{1 + \eta\lambda}{1 + \eta} v_2 \equiv p_2^{\max}.$$

However, we assumed that the price of good 2 must be lower than the price of good 1. Hence, if

$$p_2^{\max} > p_1^{\min} \Leftrightarrow v_2 > \left(\frac{1+\eta}{1+\eta\lambda}\right)^2 v_1$$

then the highest price that the seller could charge for good 2 is $p_1^{\min}$.

Given that the seller is charging the highest price for good 2 that consumers are willing to pay ex-post, she must select a degree of availability for good 1 that makes consumers ex-ante indifferent between planning to buy only the bargain item and planning to always buy. Suppose first that $p_2 = p_2^{\max}$. Then we have

$$q\left(v_1 - p_1^{\min}\right) - q(1-q)\eta(\lambda-1)\left(v_1 + p_1^{\min}\right) = q\left(v_1 - p_1^{\min}\right) + (1-q)\left(v_2 - p_2^{\max}\right)$$
$$-q(1-q)\eta(\lambda-1)\left(v_1 - v_2 + p_1^{\min} - p_2^{\max}\right)$$

$$\Leftrightarrow q = \frac{p_2^{\max} - v_2}{\eta(\lambda-1)\left(p_2^{\max} + v_2\right)}.$$

Next, suppose that $p_2 = p_1^{\min}$. Then we have

$$q\left(v_1 - p_1^{\min}\right) - q(1-q)\eta(\lambda-1)\left(v_1 + p_1^{\min}\right) = q\left(v_1 - p_1^{\min}\right) + (1-q)\left(v_2 - p_1^{\min}\right)$$
$$-q(1-q)\eta(\lambda-1)(v_1 - v_2)$$

$$\Leftrightarrow q = \frac{p_1^{\min} - v_2}{\eta(\lambda-1)\left(p_1^{\min} + v_2\right)}.$$

Therefore, in both cases we have that

$$q = \frac{p_2 - v_2}{\eta(\lambda-1)(p_2 + v_2)} \equiv q^*.$$

Finally, notice that

$$\frac{p_2^{\max} - v_2}{\eta(\lambda-1)\left(p_2^{\max} + v_2\right)} > \frac{p_1^{\min} - v_2}{\eta(\lambda-1)\left(p_1^{\min} + v_2\right)} \Leftrightarrow p_2^{\max} > p_1^{\min}$$

and

$$\frac{p_2^{\max} - v_2}{\eta\left(\lambda - 1\right)\left(p_2^{\max} + v_2\right)} = \frac{\frac{1+\eta\lambda}{1+\eta}v_2 - v_2}{\eta\left(\lambda - 1\right)\left(\frac{1+\eta\lambda}{1+\eta}v_2 + v_2\right)}$$

$$= \frac{1}{2 + \eta + \eta\lambda}$$

$$< \frac{1}{2}$$

for any $\eta > 0$ and $\lambda > 1$. Hence, $q^* < \frac{1}{2}$. This concludes the proof of the lemma. $\blacksquare$

Proof of Proposition 5: Suppose $v_2 - c_2 \geq v_1 - c_1$ and that $\frac{v_2}{v_1} < \left(\frac{1+\eta}{1+\eta\lambda}\right)^2$. Hence, the rip-off price for product 2 is equal to $p_2^{\max}$ and the seller will prefer to use a limited-availability scheme if and only if

$$q^*\left(p_1^{\min} - c_1\right) + \left(1 - q^*\right)\left(p_2^{\max} - c_2\right) \geq v_2 - c_2$$

Solving the above condition for v_1 yields

$$v_1 \geq \frac{1 + \eta\lambda}{1 + \eta}\frac{\left[\left(1 + \eta\right)^2 - \eta\lambda\left(1 + \eta\lambda\right)\right]v_2 + \left(1 + \eta\right)\left(c_1 - c_2\right)}{1 + \eta}.$$

On the other hand, if $\frac{v_2}{v_1} \geq \left(\frac{1+\eta}{1+\eta\lambda}\right)^2$ the rip-off price for product 2 is equal to $p_1^{\min}$ and the seller will prefer to use a limited-availability scheme if and only if

$$q^*\left(p_1^{\min} - c_1\right) + \left(1 - q^*\right)\left(p_1^{\min} - c_2\right) \geq v_2 - c_2 \Leftrightarrow$$

Solving the above condition for v_1 yields

$$v_1 \geq \frac{1 + \eta\lambda}{1 + \eta}\left[\frac{c_1 - c_2}{\eta\left(\lambda - 1\right)} - v_2\right].$$

The conditions for when $v_2 - c_2 < v_1 - c_1$ can be derived in a similar fashion. $\blacksquare$

Proof of Proposition 6: First, we prove that if the seller can create artificial substitutes, a combination limited availability, bargains and rip-offs always yields higher profits than perfect availability. Let $v_1 - c_1 > v_2 - c_2$ so that the maximum level of profits the seller can achieve with perfect availability is $v_1 - c_1$. If the seller can create perfect substitutes for item 1, then her profits are equal to

$$\widehat{q}\left(p_{1,1}^* - c_1\right) + \left(1 - \widehat{q}\right)\left(p_1^{\min} - c_1\right)$$

where $\widehat{q} = \overline{q}\left(\eta, \lambda, v, v, c, c\right)$. Then, it suffices to show that

$$\widehat{q}\left(1 + \frac{2\left(1 - \widehat{q}\right)\eta\left(\lambda - 1\right)}{1 + \left(1 - \widehat{q}\right)\eta\left(\lambda - 1\right)}\frac{1 + \eta}{1 + \eta\lambda}\right) + \frac{1 + \eta}{1 + \eta\lambda}\left(1 - \widehat{q}\right) > 1$$

$$\Leftrightarrow \ \widehat{q} > \frac{1 + \eta\,(\lambda - 1)}{\eta + \lambda\eta + 2}.$$

Substituting for $\widehat{q}$ yields

$$\frac{2 + 2\lambda\eta - 2\eta^2 + 2\lambda\eta^2 - \sqrt{2\,(\eta + 1)\,(\lambda\eta - \eta + 1)}\sqrt{\eta + \lambda\eta + 2}}{\eta\,(\lambda - 1)\,(\eta + \lambda\eta + 2)} > 0$$

$$\Leftrightarrow \ 2\eta\,(\lambda - 1)\,(\lambda\eta - \eta + 1)\,\left(2\eta^2 + 3\eta + 1\right) > 0$$

which is of course true for any $\eta > 0$ and $\lambda > 1$. A similar argument applies if $v_1 - c_1 \leq v_2 - c_2$.

Next, we prove the first part of the proposition. Define $\pi_{1,2} \equiv \pi\left(p_{1,2}^*, p_2^{\min}, \overline{q}; c_1, c_2\right)$, $\pi_{2,1} \equiv \pi\left(p_1^{\min}, p_{2,1}^*, \underline{q}; c_1, c_2\right)$, $\pi_{1,1} \equiv \pi\left(p_{1,1}^*, p_1^{\min}, \widehat{q}; c_1, c_1\right)$ and $\pi_{2,2} \equiv \pi\left(p_{2,2}^*, p_2^{\min}, \widehat{q}; c_2, c_2\right)$. Recall that $\overline{q} = \arg\max\limits_{q} \pi\left(p_1^*, p_2^{\min}, q; c_1, c_2\right)$, $\underline{q} = \arg\max\limits_{q} \pi\left(p_1^{\min}, p_2^*, q; c_1, c_2\right)$ and let $\widehat{q} = \arg\max\limits_{q} \pi\left(p_i^{\min}, p_{i,i}^*, q; c_i, c_i\right)$, for $i \in \{1, 2\}$. If $v_1 = v_2$ and $c_1 = c_2$, then $p_1^{\min} = p_2^{\min}$, $p_{1,2}^* = p_{1,1}^* = p_{2,2}^* = p_{2,1}^*$ and $\overline{q} = 1 - \underline{q} = \widehat{q}$ so that $\pi_{1,1} = \pi_{1,2} = \pi_{2,1} = \pi_{2,2}$.

Suppose to change v_1 by dv_1 and c_1 by dc_1 with either $dv_1 > dc_1 \geq 0$ or $dv_1 \geq 0 > dc_1$. By the envelope theorem the effect of these changes on profits are

$$d\pi_{1,2} \simeq \frac{\partial\pi_{1,2}}{\partial v_1}dv_1 + \frac{\partial\pi_{1,2}}{\partial c_1}dc_1 = \overline{q}\frac{\partial p_{1,2}^*}{\partial v_1}dv_1 - \overline{q}dc_1$$

$$d\pi_{2,1} \simeq \frac{\partial\pi_{2,1}}{\partial v_1}dv_1 + \frac{\partial\pi_{2,1}}{\partial c_1}dc_1 = \left[\underline{q}\frac{1 + \eta}{1 + \eta\lambda} + (1 - \underline{q})\frac{\partial p_{2,1}^*}{\partial v_1}\right]dv_1 - \underline{q}dc_1$$

$$d\pi_{1,1} \simeq \frac{\partial\pi_{1,1}}{\partial v_1}dv_1 + \frac{\partial\pi_{1,1}}{\partial c_1}dc_1 = \widehat{q}\left[\frac{\partial p_{1,1}^*}{\partial v_1}dv_1 - dc_1\right] + (1 - \widehat{q})\left[\frac{1 + \eta}{1 + \eta\lambda}dv_1 - dc_1\right]$$

and

$$d\pi_{2,2} = 0.$$

By substituting and re-arranging, we have that $d\pi_{1,1} > d\pi_{2,1}$ since

$$\left[\widehat{q} + \frac{2\,(1 - \widehat{q})\,\eta\,(\lambda - 1)\,\widehat{q}}{1 + \eta\,(\lambda - 1)\,(1 - \widehat{q})}\frac{1 + \eta}{1 + \eta\lambda} + (1 - \widehat{q} - \underline{q})\frac{1 + \eta}{1 + \eta\lambda} - \frac{2\underline{q}\eta\,(\lambda - 1)\,(1 - \underline{q})}{1 + \eta\,(\lambda - 1)\,\underline{q}}\frac{1 + \eta}{1 + \eta\lambda}\right]dv_1 > (1 - \underline{q})\,dc_1$$

$$\Leftrightarrow \ dv_1 > dc_1$$

where the last inequality follows from $1 - \underline{q} = \widehat{q}$. Similarly, $d\pi_{1,1} > d\pi_{1,2}$ since

$$\left[\widehat{q} + \frac{2\,(1 - \widehat{q})\,\eta\,(\lambda - 1)\,\widehat{q}}{1 + \eta\,(\lambda - 1)\,(1 - \widehat{q})}\frac{1 + \eta}{1 + \eta\lambda} + (1 - \widehat{q})\frac{1 + \eta}{1 + \eta\lambda} - \overline{q} + \frac{2\,(1 - \overline{q})\,\eta\,(\lambda - 1)\,\overline{q}}{1 + \eta\,(\lambda - 1)\,(1 - \overline{q})}\right]dv_1 > (1 - \overline{q})\,dc_1$$

$$\Leftrightarrow \ \left[\frac{2\eta\,(\lambda - 1)\,\overline{q}}{1 + \eta\,(\lambda - 1)\,(1 - \overline{q})}\left(\frac{2 + \eta\lambda + \eta}{1 + \eta\lambda}\right) + \frac{1 + \eta}{1 + \eta\lambda}\right]dv_1 > dc_1$$

where the last inequality follows from $\overline{q} = \widehat{q} > \frac{1}{2}$ and $dv_1 > dc_1$.

Finally, consider the case in which $v_1 - c_1 \leq v_2 - c_2$. Again, let's start with $v_1 = v_2$ and $c_1 = c_2$ and suppose to change v_2 by dv_2 and c_2 by dc_2 with $dc_2 \leq dv_2 < 0$ so that $v_1 - c_1 \leq v_2 - c_2$. By the envelope theorem the effect of these changes on profits are

$$d\pi_{1,2} \simeq \frac{\partial \pi_{1,2}}{\partial v_2} dv_2 + \frac{\partial \pi_{1,2}}{\partial c_2} dc_2 = \left[\overline{q} \frac{\partial p_{1,2}^*}{\partial v_2} + (1 - \overline{q}) \frac{1 + \eta}{1 + \eta \lambda} \right] dv_2 - (1 - \overline{q}) dc_2$$

$$d\pi_{2,1} \simeq \frac{\partial \pi_{2,1}}{\partial v_2} dv_2 + \frac{\partial \pi_{2,1}}{\partial c_2} dc_2 = \left(1 - \underline{q} \right) \frac{\partial p_{2,1}^*}{\partial v_2} dv_2 - \left(1 - \underline{q} \right) dc_2$$

$$d\pi_{1,1} = 0$$

and

$$d\pi_{2,2} \simeq \frac{\partial \pi_{2,2}}{\partial v_2} dv_2 + \frac{\partial \pi_{2,2}}{\partial c_2} dc_2 = \left[\widehat{q} \frac{\partial p_{2,2}^*}{\partial v_2} + (1 - \widehat{q}) \frac{1 + \eta}{1 + \eta \lambda} \right] dv_2 - dc_2.$$

By substituting and re-arranging, we have that $d\pi_{2,1} \geq d\pi_{1,2}$ since

$$\left(1 - \underline{q} \right) (dv_2 - dc_2) \geq (1 - \overline{q}) \left\{ \left[\frac{2\eta (\lambda - 1) \overline{q}}{1 + \eta (\lambda - 1)(1 - \overline{q})} \frac{2 + \eta \lambda + \eta}{1 + \eta \lambda} + \frac{1 + \eta}{1 + \eta \lambda} \right] dv_2 - dc_2 \right\}$$

$$\Leftarrow \quad \frac{2\eta (\lambda - 1) \overline{q}}{1 + \eta (\lambda - 1)(1 - \overline{q})} \frac{2 + \eta \lambda + \eta}{1 + \eta \lambda} + \frac{1 + \eta}{1 + \eta \lambda} \geq 1$$

where the last inequality follows from $\overline{q} = 1 - \underline{q} > \frac{1}{2}$ and $0 > dv_2 \geq dc_2$.

Finally, we have that $d\pi_{2,1} \geq d\pi_{2,2}$ if and only if

$$\left(1 - \underline{q} \right) (dv_2 - dc_2) \geq \widehat{q} \left[1 + \frac{2 (1 - \widehat{q}) \eta (\lambda - 1)}{1 + \eta (\lambda - 1)(1 - \widehat{q})} \frac{1 + \eta}{1 + \eta \lambda} \right] dv_2 + (1 - \widehat{q}) \frac{1 + \eta}{1 + \eta \lambda} dv_2 - dc_2$$

$$\Leftrightarrow \quad 0 \geq \frac{1 + \eta}{1 + \eta \lambda} \left[\frac{2\eta (\lambda - 1) \widehat{q}}{1 + \eta (\lambda - 1)(1 - \widehat{q})} + 1 \right] dv_2 - dc_2 \qquad (A.7)$$

where the last inequality follows from $\widehat{q} = 1 - \underline{q}$. Notice that, although $dv_2 - dc_2 > 0$, condition (A.7) might hold. Therefore, let $\widetilde{v}_2$ be the value of v_2 for which condition (A.7) binds. This completes the proof of the proposition. ∎

Proof of Lemma 9: We already know that if a consumer of type v is indifferent between the plan of buying only the bargain and the plan of always buying, then his equilibrium expected utility must be negative since he is paying a price above his valuation and, moreover, he is facing uncertainty over the price. Next, consider the equilibrium expected utility for a consumer with type $\overline{v} \in (v, v_1^h]$. If he plans to buy only the bargain item, his expected utility in equilibrium equals

$$(1 - \overline{q}) \left[\overline{v} - p_1^{\min} (v) \right] - \eta (\lambda - 1) \overline{q} (1 - \overline{q}) \left(\overline{v} + p_1^{\min} (v) \right). \qquad (A.8)$$

Differentiating (A.8) with respect to $\overline{v}$ yields $(1 - \overline{q}) [1 - \eta (\lambda - 1) \overline{q}]$. On the other hand, if he plans to always buy, his expected utility in equilibrium is

$$\overline{v} - (1 - \overline{q}) p_1^{\min} (v) - \overline{q} p_1^* (v) - \eta (\lambda - 1) \overline{q} (1 - \overline{q}) \left[p_1^* (v) - p_1^{\min} (v) \right]. \qquad (A.9)$$

Differentiating (A.9) with respect to $\overline{v}$ yields 1. Therefore, all consumers with type $\overline{v} \in \left(v, v_1^h\right]$ prefer the plan to always buy to the plan to buy only the bargain item.

Next, consider the plan of buying only the rip-off item and nothing otherwise. In this case the consumers' equilibrium expected-utility is

$$\overline{q}\left[\overline{v} - p_1^*\left(v\right)\right] - \eta\left(\lambda - 1\right)\overline{q}\left(1 - \overline{q}\right)\left(\left[\overline{v} + p_1^*\left(v\right)\right]\right). \tag{A.10}$$

It is easy to see that (A.9) is always larger than (A.10) since

$$\overline{v}\left[1 + \eta\left(\lambda - 1\right)\overline{q}\right] > p_1^{\min}\left(v\right)\left[1 - \eta\left(\lambda - 1\right)\overline{q}\right]$$

and therefore we have proved that all consumers with type $\overline{v} \in \left(v, v_1^h\right]$ prefer to always buy.

Last, consider the consumers with type $\underline{v} \in \left[v_1^l, v\right)$. For these types, not buying is a credible plan since $p_1^{\min}\left(v\right) > p_1^{\min}\left(\underline{v}\right)$. Therefore, they are going to plan to buy with positive probability only if they can make (weakly) positive utility in expectation. From (A.8) we have that a consumer's expected utility when planning to buy the bargain item and nothing otherwise is non-decreasing in his own type if and only if $1 - \eta\left(\lambda - 1\right)\overline{q} \geq 0$. If this condition holds, then since a type-v consumer gets strictly negative utility in equilibrium so would a a type-$\underline{v}$ if he were to plan to buy; therefore, the latter would prefer planning not to buy. This argument does not work when $1 - \eta\left(\lambda - 1\right)\overline{q} < 0$ because in this case a consumer's expected utility is decreasing with his type when he plans to buy only the bargain. However, the utility of a type-$\underline{v}$ consumer when planning to buy only the bargain is equal to

$$\begin{aligned}&\left(1 - \overline{q}\right)\left[\underline{v} - p_1^{\min}\left(v\right)\right] - \eta\left(\lambda - 1\right)\overline{q}\left(1 - \overline{q}\right)\left[\underline{v} + p_1^{\min}\left(v\right)\right]\\ =\ &\left(1 - \overline{q}\right)\left\{\underline{v} - p_1^{\min}\left(v\right) - \eta\left(\lambda - 1\right)\overline{q}\left[\underline{v} + p_1^{\min}\left(v\right)\right]\right\}\end{aligned}$$

which is negative for $1 - \eta\left(\lambda - 1\right)\overline{q} < 0$. Therefore, also in this case consumers prefer not to buy. By the same argument, it is easy to see that these consumers would never plan to buy only the rip-off either and this concludes the proof. ∎

Proof of Proposition 7: From Lemma 9 we know that for a given marginal type v, types above v plan to always buy and types below v plan to never buy. Then, the problem reduces to a standard monopoly-pricing one where the seller charges an expected price equal to

$$\overline{q}p_1^*\left(v\right) + \left(1 - \overline{q}\right)p_1^{\min}\left(v\right) = \Phi v$$

where $\Phi \equiv \frac{4 - 2\eta^2 + \eta^2\lambda^2 + 4\lambda\eta + \eta^2\lambda - 2\sqrt{2(2 + \eta + \eta\lambda)(1 + \eta)(1 + \eta\lambda - \eta)}}{\eta(\lambda - 1)(1 + \eta\lambda)} > 1$. Let $\widehat{v}_1$ be the profit-maximizing marginal type. In equilibrium a consumer of type-v attains a positive expected utility if and only if

$$v \geq \left(1 - \overline{q}\right)p_1^{\min}\left(\widehat{v}_1\right) + \overline{q}p_1^*\left(\widehat{v}_1\right) + \eta\left(\lambda - 1\right)\overline{q}\left(1 - \overline{q}\right)\left[p_1^*\left(\widehat{v}_1\right) - p_1^{\min}\left(\widehat{v}_1\right)\right] \equiv v_1^s$$

and this concludes the proof. ∎

Proof of Lemma 10: Suppose the seller plays the limited-availability strategy that makes a (v, v)-type consumer indifferent between planning to buy only the bargain item and planning to always buy; that is:

$$\underline{q}\left[v - p_1^{\min}\left(v\right)\right] - \underline{q}\left(1 - \underline{q}\right)\eta\left(\lambda - 1\right)\left[v + p_1^{\min}\left(v\right)\right] =$$

$$\underline{q}\left[v - p_1^{\min}(v)\right] + \left(1 - \underline{q}\right)\left[v - p_2^*(v)\right] - \underline{q}\left(1 - \underline{q}\right)\eta\left(\lambda - 1\right)\left[p_2^*(v) - p_1^{\min}(v)\right]$$

$$\Leftrightarrow \; v - p_2^*(v) = \underline{q}\eta\left(\lambda - 1\right)\left[p_2^*(v) - v - 2p_1^{\min}(v)\right]. \tag{A.11}$$

It is easy to see that consumers whose values lie in $\left[\underline{v}, p_1^{\min}(v)\right] \times \left[\underline{v}, p_2^*(v)\right]$ will plan to never buy and this plan is consistent for them. For consumers in $\left[p_1^{\min}(v), v\right) \times \left[\underline{v}, v\right)$ not buying is consistent as well; hence, they would choose a different plan only if it provides them with non-negative expected utility. Planning to buy item 2 yields negative expected utility; similarly planning to buy item 1 if available and item 2 otherwise, also yields negative expected utility as $\underline{q}p_1^{\min}(v) + \left(1 - \underline{q}\right)p_2^*(v) > v$. Thus, we only need to check whether the consumers would prefer to plan to buy item 1 if available:

$$\underline{q}\left(v_1 - p_1^{\min}(v)\right) - \underline{q}\left(1 - \underline{q}\right)\eta\left(\lambda - 1\right)\left(v_1 + p_1^{\min}(v)\right) > 0$$

$$\Leftrightarrow \; v_1 > \frac{1 + \eta\left(\lambda - 1\right)\left(1 - \underline{q}\right)}{1 - \eta\left(\lambda - 1\right)\left(1 - \underline{q}\right)}\frac{1 + \eta}{1 + \eta\lambda}v$$

where the second inequality follows from the fact that $p_1^{\min}(v) = \frac{1+\eta}{1+\eta\lambda}v$. However, we have

$$\frac{1 + \eta\left(\lambda - 1\right)\left(1 - \underline{q}\right)}{1 - \eta\left(\lambda - 1\right)\left(1 - \underline{q}\right)}\frac{1 + \eta}{1 + \eta\lambda} > 1$$

$$\Leftrightarrow \; \eta\lambda\left(1 - \underline{q}\right) + 1 > \eta\left(1 - \underline{q}\right) + 2\underline{q}.$$

Substituting for $\underline{q}$ yields

$$\eta\left(\lambda - 1\right)\left(1 + 2\eta\right) > 0.$$

Therefore,

$$v \geq v_1 > \frac{1 + \eta\left(\lambda - 1\right)\left(1 - \underline{q}\right)}{1 - \eta\left(\lambda - 1\right)\left(1 - \underline{q}\right)}\frac{1 + \eta}{1 + \eta\lambda}v > v$$

yielding a contradiction.

For consumers with values in $[v, \overline{v}] \times [\underline{v}, v]$, neither planning of never buying nor planning to buy item 2 is credible. They prefer the plan to buy item 1 if available to the plan of always buying if and only if

$$v_2 - p_2^*(v) \leq \underline{q}\eta\left(\lambda - 1\right)\left[p_2^*(v) - v_2 - 2p_1^{\min}(v)\right].$$

Since $v_2 \leq v$, the result follows.

Next, consider those consumers whose type is in $[v, \overline{v}] \times [v, \overline{v}]$. For these consumers, planning to never buy is not a credible option. Suppose first that $v_1 > v_2$. In this case planning to buy item 2 is not a PE, as if a consumer were to find item 1 available, he would prefer to deviate and buy it. Then, consumers prefer the plan of always buying to the plan of buying only item 1 if and only if

$$v_2 - p_2^*(v) \geq \underline{q}\eta\left(\lambda - 1\right)\left[p_2^*(v) - v_2 - 2p_1^{\min}(v)\right].$$

Since $v_2 \geq v$, the result follows. Now, suppose instead that $v_1 \leq v_2$. Consumers prefer always buying to buying only item 2 if and only if

$$v_1 - p_1^{\min}(v) \geq (1 - \underline{q}) \underline{q} \eta (\lambda - 1) \left[-v_1 - p_1^{\min}(v) \right]$$

which is of course true; and they prefer always buying to buying only item 1 if and only if

$$v_2 - p_2^*(v) \geq \underline{q} \eta (\lambda - 1) \left[v_2 + p_2^*(v) - 2v_1 - 2p_1^{\min}(v) \right]$$

$$\Leftrightarrow v_2 \left[1 - \underline{q} \eta (\lambda - 1) \right] \geq p_2^*(v) \left[1 + \underline{q} \eta (\lambda - 1) \right] - 2\underline{q} \eta (\lambda - 1) \left[v_1 + p_1^{\min}(v) \right]$$

$$\Leftrightarrow v_2 \left[1 - \underline{q} \eta (\lambda - 1) \right] \geq v \left[1 + \underline{q} \eta (\lambda - 1) \right] - 2\underline{q} \eta (\lambda - 1) v_1$$

where the last inequality follows from (A.11). If $1 - \underline{q} \eta (\lambda - 1) > 0$, then the result follows since $v < v_1 < v_2$. Notice that

$$1 - \underline{q} \eta (\lambda - 1) > 0 \Leftrightarrow 3 + 2\eta + \eta\lambda + \eta^2 - \eta^2\lambda > 0$$

and, because we assumed $\eta \leq 1$, the result follows.

Finally, consider those consumers whose type is in $[\underline{v}, v] \times [v, \overline{v}]$, with $v_i > p_i$ for $i = 1, 2$. Again, for these consumers, planning not to buy is credible and yields zero. Planning to buy item 1 yields negative utility (the proof is the same as for consumers whose type is in $\left[p_1^{\min}(v), v \right] \times [\underline{v}, v]$) and planning to always buy is preferred to planning to buy only item 2 (the proof is the same as for those consumers whose type is in $[v, \overline{v}] \times [v, \overline{v}]$ and $v_2 \geq v_1$). Planning to always buy is preferred to never buying if and only if

$$\underline{q} \left[v_1 - p_1^{\min}(v) \right] + (1 - \underline{q}) \left[v_2 - p_2^*(v) \right] - \underline{q} (1 - \underline{q}) \eta (\lambda - 1) \left[v_2 - v_1 + p_2^*(v) - p_1^{\min}(v) \right] \geq 0. \tag{A.12}$$

Since $\eta \leq 1$ implies that $1 - \underline{q} \eta (\lambda - 1) > 0$, condition (A.12) can be re-written as

$$v_2 \geq av - bv_1$$

where

$$
\begin{aligned}
a &= \frac{\underline{q}}{1 - \underline{q}} \frac{1 + \eta}{1 + \eta\lambda} \frac{1 - (1 - \underline{q}) \eta (\lambda - 1)}{1 - \underline{q} \eta (\lambda - 1)} + \\
&\quad \frac{1 + \underline{q} \eta (\lambda - 1)}{1 - \underline{q} \eta (\lambda - 1)} \frac{\sqrt{2 (\eta + 1) (\eta\lambda - \eta + 1)} (2\eta + \lambda\eta + 3) - 2 (1 + \eta) \sqrt{\eta + \eta\lambda + 2}}{(1 + \eta\lambda) \sqrt{2 (1 + \eta) (-\eta + \eta\lambda + 1)}} \\
&> 1
\end{aligned}
$$

and

$$b = \frac{\underline{q}}{1 - \underline{q}} \frac{1 + (1 - \underline{q}) \eta (\lambda - 1)}{1 - \underline{q} \eta (\lambda - 1)} > 0.$$

This concludes the proof. ∎

Proof of Lemma 11: Under limited availability the seller solves the following program:

$$\max_v \pi_{LA}^{**} = [\Phi v - c] \left[\left(\frac{\overline{v} - v}{\overline{v} - \underline{v}} \right)^2 + \Omega(v) \right] + [\Psi v - \underline{q}c] \left(\frac{\overline{v} - v}{\overline{v} - \underline{v}} \right) \left(\frac{v - \underline{v}}{\overline{v} - \underline{v}} \right) \tag{A.13}$$

where $\Phi \equiv \frac{4-2\eta^2+\eta^2\lambda^2+4\lambda\eta+\eta^2\lambda-2\sqrt{2(2+\eta+\eta\lambda)(1+\eta)(1+\eta\lambda-\eta)}}{\eta(\lambda-1)(1+\eta\lambda)} > 1$ and $\Psi \equiv \frac{1+\eta}{1+\eta\lambda} \frac{\sqrt{2\left(\frac{\eta+1}{\eta+\lambda\eta+2}\right)(\lambda\eta-\eta+1)}-1}{\eta(\lambda-1)} <$ 1. Since $\Omega(v) \leq \left(\frac{\overline{v}-v}{\overline{v}-\underline{v}}\right)\left(\frac{v-\underline{v}}{\overline{v}-\underline{v}}\right)$, the value of expression (A.13) is bounded above by

$$\max_{v} \widetilde{\pi}_{LA}^{**} = [\Phi v - c]\left[\left(\frac{\overline{v}-v}{\overline{v}-\underline{v}}\right)^2 + \left(\frac{\overline{v}-v}{\overline{v}-\underline{v}}\right)\left(\frac{v-\underline{v}}{\overline{v}-\underline{v}}\right)\right] + [\Psi v - \underline{q}c]\left(\frac{\overline{v}-v}{\overline{v}-\underline{v}}\right)\left(\frac{v-\underline{v}}{\overline{v}-\underline{v}}\right)$$

(A.14)

Now I show that if the maximization problem (A.14) has an interior solution at some value $v^{**} \in (\underline{v}, \overline{v})$, then the seller could achieve higher profits than in (A.14) with a perfect-availability strategy.

Let $c = 0$ (since $\Psi < \underline{q}$ if the result holds for $c = 0$, then it must hold *a fortiori* for $c > 0$). We have an interior solution for the program in (A.14) if and only if $\frac{\partial \widetilde{\pi}_{LA}^{**}}{\partial v}$, evaluated at $v = \underline{v}$ is positive:

$$\Phi(\overline{v} - \underline{v}) - 2\Phi\underline{v} + (\Phi + \Psi)\underline{v} > 0$$

$$\Leftrightarrow \overline{v} > \frac{2\Phi - \Psi}{\Phi}\underline{v}. \tag{A.15}$$

Since any uniform distribution can be translated, with appropriate re-normalization, into the $[0, 1]$ interval, for simplicity let $\overline{v} = 1$ and $\underline{v} = 0$ (notice that condition (A.15) is trivially satisfied in this case). Hence, the maximization program in (A.14) can be re-written as:

$$\max_{v} \widetilde{\pi}_{LA}^{**} = \Phi v(1 - \overline{v})^2 + (\Phi + \Psi)v(1 - v)v.$$

Taking FOC and re-arranging yields:

$$v^{**} = \frac{1}{3\Psi}\left(\Psi - \Phi + \sqrt{\Psi^2 + \Psi\Phi + \Phi^2}\right)$$

for a profit of

$$\frac{(\Psi - \Phi)(2\Psi + \Phi)(\Psi + 2\Phi) + (2\Psi^2 + 2\Phi^2 + 2\Psi\Phi)\sqrt{\Psi^2 + \Psi\Phi + \Phi^2}}{27\Psi^2}.$$

Recall that with perfect availability the seller maximizes profits by selling both items at price $p^{**} = \frac{1}{\sqrt{3}}$ and obtains profits equal to $\frac{2}{9}\sqrt{3}$. Therefore,

$$\frac{2}{9}\sqrt{3} > \frac{(\Psi - \Phi)(2\Psi + \Phi)(\Psi + 2\Phi) + (2\Psi^2 + 2\Phi^2 + 2\Psi\Phi)\sqrt{\Psi^2 + \Psi\Phi + \Phi^2}}{27\Psi^2}$$

$$\Leftrightarrow -3\Psi^2\left(8\sqrt{3}\Psi^3 + 9\Psi^2\Phi^2 + 12\sqrt{3}\Psi^2\Phi - 36\Psi^2 + 18\Psi\Phi^3 - 12\sqrt{3}\Psi\Phi^2 + 9\Phi^4 - 8\sqrt{3}\Phi^3\right) > 0$$

$$\Leftrightarrow \frac{\frac{8}{9}\sqrt{3}\Psi^3 + \Phi^4 + \Psi^2\Phi^2 + 2\Psi\Phi^3 + \frac{4}{3}\sqrt{3}\Psi^2\Phi}{\frac{2}{3}\sqrt{3}\Phi^3 + 3\Psi^2 + \sqrt{3}\Psi\Phi^2} < \frac{4}{3}. \tag{A.16}$$

Notice first that Φ is increasing in η since

$$\frac{\partial \Phi\left(\eta, \lambda\right)}{\partial \eta} > 0 \Leftrightarrow \sqrt{2}\left(\eta^3 + 3\lambda\eta^2 - 4\lambda\eta^3 - \lambda\eta^4 + 9\lambda^2\eta^2 + 5\lambda^2\eta^3 + 2\lambda^3\eta^3 + \lambda^3\eta^4 + 11\lambda\eta + \eta + 4\right) >$$

$$\sqrt{\left(\eta + 1\right)\left(\eta + \lambda\eta + 2\right)\left(\lambda\eta - \eta + 1\right)}\left(3\lambda^2\eta^2 - \lambda\eta^2 + 8\lambda\eta + 2\eta^2 + 4\right)$$

$$\Leftrightarrow \lambda^4\eta^4\left(\eta - 1\right) + \lambda^3\eta^3\left(11\eta + 2\eta^2 - 3\right) + \lambda^2\eta^3\left(10\eta + \eta^2 + 35\right) +$$

$$4\lambda\eta\left(9\eta + 2\eta^2 + 1\right) + 2\eta\left(2\eta + 2\eta^2 + \eta^3 + 6\right) + 2 > 0.$$

Then, we have that

$$\Phi\left(1, \lambda\right) = \frac{5\lambda + \lambda^2 - 4\sqrt{\lambda\left(\lambda + 3\right)} + 2}{\lambda^2 - 1}.$$

The above function reaches its maximum for $\lambda^* \approx 2.88$, and $\Phi\left(1, \lambda^*\right) \approx 1.13$.[1] This bound implies that with limited availability the seller can extract 13% more profits than with perfect availability, at the most.

Similar, but much more tedious algebra shows that Ψ is increasing in η as well. So we have that

$$\Psi\left(1, \lambda\right) = \frac{2}{\left(\lambda - 1\right)\left(\lambda + 1\right)}\left(2\frac{\sqrt{\lambda}}{\sqrt{\lambda + 3}} - 1\right).$$

The above function is strictly decreasing in λ, and is therefore bounded above by $\lim_{\lambda \to 1} \Psi\left(1, \lambda\right) = \frac{3}{8}$. It is easy to see that $\Phi\left(\eta, \lambda\right) + \Psi\left(\eta, \lambda\right)$ is increasing in η. Moreover, $\Phi\left(1, \lambda\right) + \Psi\left(1, \lambda\right)$ is strictly decreasing in λ and bounded above by $\lim_{\lambda \to 1}\left(\Phi\left(1, \lambda\right) + \Psi\left(1, \lambda\right)\right) = \frac{11}{8}$. Since in the function to be maximized in (A.14) Φ carries a bigger weight than Ψ, suppose $\lambda = 2.88$ so that Φ is at its maximum, 1.13 and Ψ is therefore equal to 0.11. Arithmetic shows that for these values condition (A.16) holds. Since the expression on the left-hand-side of (A.16) is increasing in both Φ and Ψ, then the condition is always satisfied. Finally, since the function to be maximized in (A.14) is an upper bound for the one in (A.13), the result easily follows. ∎

Proof of Proposition 8: We know by Lemma 11 that if the profit-maximizing marginal type were in the interior of the valuations' support, the seller would never use a limited-availability scheme. Hence, her profits must be strictly decreasing in v. A necessary condition for this is $\overline{v} < 3\underline{v}$.

If the seller employs the limited-availability strategy that makes a type-$(\underline{v}, \underline{v})$ consumer exactly indifferent between buying only the bargain and always buying, then for all consumers not buying is not a PE. It is easy to see that for all types on the 45-degree line, with $v_1 = v_2 = v$, the PPE is to always buy since for these types, like for the marginal one, expected gain-loss utility in the item is zero and

$$\frac{dEU\left[\{1, 2\} \mid \{1, 2\}\right]}{dv} = 1.$$

To see that all consumers prefer to always buy, irrespective of their type, it suffices to show that types at the corners $(\overline{v}, \underline{v})$ and $(\underline{v}, \overline{v})$ prefer to always buy. Consider type $(\overline{v}, \underline{v})$ first. Planning to buy item 2 if available and nothing otherwise is not a PE because $v_1 > v_2$ and $p_1 < p_2$. Furthermore, this consumer is indifferent between planning to buy only item 1 and planning to always buy since, for $v_1 > v_2$:

$$\frac{dEU\left[\{1, \varnothing\} \mid \{1, \varnothing\}\right]}{dv_1} = \underline{q} - \underline{q}\left(1 - \underline{q}\right)\eta\left(\lambda - 1\right) = \frac{dEU\left[\{1, 2\} \mid \{1, 2\}\right]}{dv_1}.$$

Next, consider type $(\underline{v}, \overline{v})$. If he plans to buy only item 1, this consumer's expected utility level is the same as that of the marginal type $(\underline{v}, \underline{v})$, since $EU\left[\{1, \varnothing\} \mid \{1, \varnothing\}\right]$ does not depend on v_2. Furthermore, this consumer is indifferent between planning to buy only item 2 and planning to always buy since, for $v_1 < v_2$:

$$\frac{dEU\left[\{2, \varnothing\} \mid \{2, \varnothing\}\right]}{dv_2} = \left(1 - \underline{q}\right) - \underline{q}\left(1 - \underline{q}\right)\eta\left(\lambda - 1\right) = \frac{dEU\left[\{1, 2\} \mid \{1, 2\}\right]}{dv_2}.$$

Notice that

$$\left(1 - \underline{q}\right) - \underline{q}\left(1 - \underline{q}\right)\eta\left(\lambda - 1\right) > 0$$
$$\Leftrightarrow \ 1 - \underline{q}\eta\left(\lambda - 1\right) > 0$$

which is always true for $\eta \leq 1$. Therefore, this consumer's PPE is to always buy.

Since the PPE plan for types $(\overline{v}, \underline{v})$ and $(\underline{v}, \overline{v})$ is to always buy and these are the types with the most asymmetric preferences, then it follows that all consumers with $(v_1, v_2) \in (\underline{v}, \underline{v}) \times (\overline{v}, \overline{v})$ will also prefer to always buy.

For this limited-availability scheme, the seller's profit is equal to $\pi_{LA}^{**}(\underline{v}) = \Phi\underline{v} - c$. This scheme is profit-maximizing if and only if

$$\pi_{LA}^{**}(\underline{v}) > \pi^{**}(v^{**})$$

$$\Leftrightarrow \ \Phi\underline{v} - c > \frac{\left(3\overline{v} - 2\underline{v} - c - \sqrt{-6\overline{v}\underline{v} - 2\underline{v}c + 3\overline{v}^2 + 4\underline{v}^2 + c^2}\right)}{27\left(\overline{v} - \underline{v}\right)^2} \times \tag{A.17}$$
$$\left(2\underline{v} - 2c + \sqrt{-6\overline{v}\underline{v} - 2\underline{v}c + 3\overline{v}^2 + 4\underline{v}^2 + c^2}\right)\left(3\overline{v} - 4\underline{v} + c + \sqrt{-6\overline{v}\underline{v} - 2\underline{v}c + 3\overline{v}^2 + 4\underline{v}^2 + c^2}\right).$$

Solving (A.17) for $\overline{v}$, re-arranging and simplifying yield the desired result. ∎

Proof of Proposition 9: Suppose $p_1 = p_1^{\min} \equiv \frac{1+\eta}{1+\eta\lambda}v$. Then, not buying is not a credible plan for the consumers and for a given q their perceived expected utility when planning to buy item 1 if available and nothing otherwise is

$$EU\left[\{1, \varnothing\} \mid \{1, \varnothing\}\right] = \widetilde{q}\left(v - p_1^{\min}\right) - \widetilde{q}\left(1 - \widetilde{q}\right)\eta\left(\lambda - 1\right)\left(v + p_1^{\min}\right) \tag{A.18}$$

where $\widetilde{q} = \chi q > q$. Consumers will be indifferent between the above plan and the plan to always if and only if

$$p_2 \leq v + \left[\frac{2\eta\left(\lambda - 1\right)\widetilde{q}}{1 + \eta\left(\lambda - 1\right)\widetilde{q}}\right]p_1^{\min} \equiv p_2^*.$$

This pair of prices provides the seller with profits equal to

$$qp_1^{\min} + (1-q)\, p_2^*.$$

The above expression is maximized at

$$q_\chi = \frac{\sqrt{2\,(\eta+1)\,(\eta+\lambda\eta+2)\,(-\chi\eta+\lambda\chi\eta+1)}}{\chi\eta\,(\lambda-1)\,(\eta+\lambda\eta+2)} - \frac{1}{\chi\eta\,(\lambda-1)}.$$

Next, notice that expression (A.18) is a continuous function of $\widetilde{q}$, and its value is 0 for $\widetilde{q} = 0$ and $v - p_1^{\min} > 0$ for $\widetilde{q} = 1$. Furthermore, its derivative evaluated at $\widetilde{q} = 0$ is equal to

$$v - p_1^{\min} - \eta\,(\lambda-1)\left(v + p_1^{\min}\right) = -\frac{\eta\,(\lambda-1)}{1+\eta\lambda}\,(1+\eta+\eta\lambda)\,v < 0$$

and therefore it must have another zero for $\widetilde{q} \in (0,1)$; it follows that

$$
\begin{aligned}
0 &= \widetilde{q}\left(v - p_1^{\min}\right) - \widetilde{q}\,(1-\widetilde{q})\,\eta\,(\lambda-1)\left(v + p_1^{\min}\right)\\
\Leftrightarrow \quad \chi q_\chi &= \frac{1+\eta+\eta\lambda}{2+\eta+\eta\lambda}\\
\Leftrightarrow \quad \chi &= \frac{\eta^3\lambda^3 + \eta^3\lambda^2 + 4\eta^2\lambda^2 - \eta^3\lambda + 4\eta^2\lambda + 8\eta\lambda - \eta^3 + 6\eta + 6}{2\,(1+\eta)\,(\eta+\eta\lambda+2)} \equiv \widetilde{\chi}.
\end{aligned}
$$

Therefore, for $\chi \geq \widetilde{\chi}$ consumers' perceived expected utility is zero. But then, the seller can set $q = \frac{1}{\chi}$ and $p_1 = v$ without affecting consumers' perceived expected utility. In this case, since consumers believe they will consume item 1 at price v for sure, the highest price they are willing to pay for item 2 if they do not find item 1 available is

$$p_2 = v\left(1 + \frac{\eta\,(\lambda-1)}{1+\eta\lambda}\right)$$

and it is easy to see that this scheme provides the seller with higher profits since consumers' realized consumption utility is at most zero in any contingency. ∎

Appendix B

Partial Commitment

While retailers frequently advertise their good deals, it is rather uncommon to see a store publicizing its high prices. Therefore, consistently with this observation about stores' advertising patterns, in this section I assume that in period 0 the seller commits only to the price of the bargain $p_i^{\min}$, $i = 1, 2$, and its degree of availability. In this case, consumers form rational expectations about the price of the item that is not publicly advertised.

Suppose that the products are close substitutes and the seller uses item 1 as the bargain by announcing that she has q units of it available for sale at price $p_1^{\min}$. Once at the store, a buyer who had planned to buy item 1 if available and item 2 otherwise will follow his plan and buy item 2 when this is the only item left in the store if

$$U\left[(v_2, p_2) \,|\, \{1, 2\}\right] \geq U\left[(0, 0) \,|\, \{1, 2\}\right]$$

$$\Leftrightarrow \quad p_2 \leq \frac{(1 + \eta\lambda) v_2 + \eta (\lambda - 1) q \left(\frac{1+\eta}{1+\eta\lambda}\right) v_1}{1 + \eta\lambda q + \eta (1 - q)}. \tag{B.1}$$

Notice that this price is higher than the one we found under full commitment because now the price of the rip-off is the highest price consumers are willing to pay *ex-post*. However, for the consumers to be willing to make the plan of always buying to begin with, the seller's announced degree of availability for the bargain must be such that

$$EU\left[\{1, 2\} \,|\, \{1, 2\}\right] \geq EU\left[\{2, \varnothing\} \,|\, \{2, \varnothing\}\right]. \tag{B.2}$$

To have an optimum for the seller both conditions (B.1) and (B.2) have to bind, defining a system of two non-linear equations in q and p_2. The relevant solution is

$$p_2^* = \frac{v_1 (1 + \eta) (1 + 2\eta) + v_2 (1 + \eta\lambda) (1 + \eta + \eta\lambda) - \sqrt{Y}}{2\eta (1 + \eta\lambda)}$$

$$\underline{q} = \frac{v_2 \lambda (1 + \eta\lambda) - \frac{1+\eta\lambda}{2\eta(1+\eta\lambda)} \left[v_1 (1 + \eta) (1 + 2\eta) + v_2 (1 + \eta\lambda) (1 + \eta + \eta\lambda) - \sqrt{Y}\right]}{v_1 (1 + \eta) (\lambda - 1) + v_2 (1 + \eta\lambda) (\lambda - 1)}$$

where

$$\begin{aligned} Y \equiv\ & v_1^2 (1 + \eta)^2 (2\eta + 1)^2 + v_2^2 (1 - \eta + \eta\lambda)^2 (1 + \eta\lambda)^2 \\ & - 2 v_1 v_2 (1 + \eta) (1 + \eta\lambda) \left(-\eta - 2\eta^2 - \lambda\eta + 2\eta^2\lambda - 1\right). \end{aligned}$$

Similarly, if the goods are close substitutes and the seller uses item 2 as the bargain, degree of availability of item 1 and its price are

$$p_1^* = \frac{v_1\eta(\lambda-1)(1+\eta\lambda) + v_2(1+2\eta)(2+\eta+\lambda\eta) - \sqrt{Z}}{2\eta(1+\eta\lambda)}$$

$$\bar{q} = \frac{v_2(2\lambda-\eta-2\eta^2-\eta\lambda-2\eta^2\lambda+\eta\lambda^2-2) - v_1\lambda(\eta-\eta^2+\eta\lambda+\eta^2\lambda+1)}{v_2(\lambda-1)(\eta+\lambda\eta+2)}$$

$$+ \frac{\frac{1+\eta\lambda}{2\eta(1+\eta\lambda)}\left[v_1\eta(\lambda-1)(1+\eta\lambda) + v_2(1+2\eta)(2+\eta+\eta\lambda) - \sqrt{Z}\right]}{v_2(\lambda-1)(\eta+\eta\lambda+2)}$$

where

$$\begin{aligned} Z \equiv{}& v_1^2\eta^2(1+\lambda)^2(1+\eta\lambda)^2 + v_2^2(1+2\eta)^2(2+\eta+\lambda\eta)^2 \\ & -2\eta v_1 v_2(-\lambda+2\eta+2\eta\lambda+3)(1+\eta\lambda)(2+\eta+\eta\lambda). \end{aligned}$$

Compared to the situation where she is able to commit in advance to both prices, now the price of the rip-off is higher but the degree of availability of the bargain is higher as well. Intuitively, since the seller is charging a higher price for the rip-off, and the consumers anticipate this, she must compensate them with a higher ex-ante chance of making a deal otherwise they would not plan to always buy. Thus, given both prices, the seller is not choosing the degree of availability that maximizes her profits. This is because by not committing in advance to the price of the rip-off, the seller must use the degree of availability of the bargain to induce the consumers to select the to plan to always buy. Furthermore, the optimal degree of availability with full commitment takes into account also the difference in the marginal costs of the two items, whereas with partial commitment it does not. Therefore, the seller's profits are lower when she cannot commit to both prices.

Unfortunately, in this case it is hard to obtain a full characterization, like the one in proposition 10, for when the seller would find it profitable to use a limited-availability strategy made of bargains and rip-offs. Nevertheless, a combination of bargains and rip-offs might be profit-maximizing as the following example shows.

Example 8 *Let $v_1 = 250$, $v_2 = 230$, $c_1 = 20$ and $c_2 = 10$. If the seller produces only one good, then she would produce item 1 and price it at $p_1 = 250$, obtaining a profit of 230. Let $\eta = 1$ and $\lambda = 2$ and suppose the seller uses item 1 as a bargain by pricing it at $p_1^{\min} = \frac{500}{3}$. In this case the seller will also commit to sell $\underline{q} = \frac{2}{119}\sqrt{3459} - \frac{75}{119}$ units of item 1 and will price item 2 at $p_2^* = 710 - \frac{20}{3}\sqrt{3459}$, obtaining a profit of 250.15.*

Moreover, example 8 shows that also in this case of partial commitment the seller might prefer to use the superior item as the bargain, exactly for the same reason as in the analysis with full commitment.[1]

---

[1] For the parameters in example 8, if the seller were to use item 2 as the bait by pricing it at $p_2^{\min} = \frac{460}{3}$ then the optimal degree of availability of the bait would be $1 - \bar{q} = \frac{1}{23}\sqrt{489} - \frac{12}{23}$ and the price of item 1 would be $p_1^* = 700 - \frac{50}{3}\sqrt{489}$ for a total profit of 237.52. Less than what the seller can obtain by using item 1 as the bait, but still better than what she would make by selling only item 1.

Appendix C

Two-period case with $\Lambda^m > 0$

Consider a situation in which there are only two items to be sold ($K = 2$). In this case, a symmetric equilibrium consists of two bidding functions (β_1, β_2), one for each auction. I assume that both functions are strictly increasing and differentiable. The first-period bidding strategy is a function $\beta_1 : [0, \overline{\theta}] \to \mathbb{R}_+$ that depends only on the bidder's value. The bid in the second auction, instead, might depend also on the price paid in the first auction. Let $Y_1^{(N-1)} \equiv Y_1$ be the highest of $N - 1$ values, $Y_2^{(N-1)} \equiv Y_2$ be the second-highest and so on. Also, let F_1 and F_2 be the distributions of Y_1 and Y_2 respectively, with corresponding densities f_1 and f_2. Since the first-period bidding function β_1 is assumed to be invertible, after the first auction is over and its winning price is revealed the valuation of the winning bidder is commonly known to be just $y_1 = \beta_1^{-1}(p_1)$. Thus, the second-period strategy can be described as a function $\beta_2 : [0, \overline{\theta}] \times [0, \overline{\theta}] \to \mathbb{R}_+$ so that a bidder with value θ bids $\beta_2(\theta, y_1)$ if $Y_1 = y_1$. To find equilibria that are sequentially rational, let's start by looking at the second period.

Consider a bidder with type θ who plans to bid as if his type were $\widetilde{\theta} \neq \theta$ when all other $N - 2$ remaining bidders follow the equilibrium strategy $\beta_2(\cdot, y_1)$. His expected payoff is

$$
\begin{aligned}
EU_2 \quad = \quad & F_2\left(\widetilde{\theta}|y_1\right)\left[\theta - \beta_2\left(\widetilde{\theta}, y_1\right)\right] \\
& -F_2\left(\widetilde{\theta}|y_1\right)\left[1 - F_2\left(\widetilde{\theta}|y_1\right)\right]\theta\Lambda^g \\
& -F_2\left(\widetilde{\theta}|y_1\right)\left[1 - F_2\left(\widetilde{\theta}|y_1\right)\right]\beta_2\left(\widetilde{\theta}, y_1\right)\Lambda^m
\end{aligned}
\tag{C.1}
$$

where $F_2\left(\widetilde{\theta}|y_1\right)$ is the probability that the second highest valuation, among $N - 1$, is less than $\widetilde{\theta}$ conditional on $Y_1 = y_1$ being the highest and $\Lambda^l \equiv \eta^l\left(\lambda^l - 1\right)$ for $l \in \{g, m\}$ captures loss aversion in the item and money dimensions, respectively.

Taking FOC of (C.1) with respect to $\widetilde{\theta}$ yields

$$
\begin{aligned}
0 = \ & f_2\left(\widetilde{\theta}|y_1\right)\left(\theta - \beta_2\left(\widetilde{\theta}, y_1\right)\right) - \beta_2'\left(\widetilde{\theta}, y_1\right) F_2\left(\widetilde{\theta}|y_1\right) \\
& - f_2\left(\widetilde{\theta}|y_1\right)\left[1 - 2F_2\left(\widetilde{\theta}|y_1\right)\right]\theta\Lambda^g \\
& - f_2\left(\widetilde{\theta}|y_1\right)\left[1 - 2F_2\left(\widetilde{\theta}|y_1\right)\right]\beta_2\left(\widetilde{\theta}, y_1\right)\Lambda^m \\
& - F_2\left(\widetilde{\theta}|y_1\right)\left[1 - F_2\left(\widetilde{\theta}|y_1\right)\right]\beta_2'\left(\widetilde{\theta}, y_1\right)\Lambda^m
\end{aligned}
$$

where β_2' is the derivative of β_2 with respect to its first argument.

Substituting $\theta = \widetilde{\theta}$ and re-arranging results in the following differential equation

$$
\frac{\partial}{\partial\theta}\left\{\beta_2\left(\theta, y_1\right) F_2\left(\theta|y_1\right)\left[1 + \Lambda^m\left(1 - F_2\left(\theta|y_1\right)\right)\right]\right\} = f_2\left(\theta|y_1\right)\theta\left\{1 - \Lambda^g\left[1 - 2F_2\left(\theta|y_1\right)\right]\right\} \quad \text{(C.2)}
$$

together with the boundary condition that $\beta_2\left(0, y_1\right) = 0$.

Because the different values are drawn independently, we have that

$$
F_2\left(\theta|y_1\right) = \frac{F\left(\theta\right)^{N-2}}{F\left(y_1\right)^{N-2}}
$$

and substituting into (C.2) yields

$$
\beta_2^*\left(\theta, y_1; \Lambda^g, \Lambda^m\right) = \frac{\int_0^\theta x\left[1 - \Lambda^g\left(1 - 2\left(\frac{F(x)}{F(y_1)}\right)^{N-2}\right)\right]dF\left(x\right)^{N-2}}{F\left(\theta\right)^{N-2}\left[1 + \Lambda^m\left(1 - \left(\frac{F(\theta)}{F(y_1)}\right)^{N-2}\right)\right]}.^1
$$

The complete bidding strategy for the second round is to bid $\beta_2^*\left(\theta, y_1; \Lambda^g, \Lambda^m\right)$ if $\theta < y_1$ and to bid $\beta_2^*\left(y_1, y_1; \Lambda^g, \Lambda^m\right)$ if $\theta \geq y_1$. The latter might occur if a bidder of type $\theta \geq y_1$ underbid in the first period causing a lower type to win (of course this is an off-equilibrium event).

The first thing worth noticing is that even with independent private values, the optimal bidding strategy in the second period is history-dependent, as it is a function of y_1. With risk-neutral preferences this is not the case:

$$
\beta_2^*\left(\theta, y_1; 0, 0\right) = \frac{\int_0^\theta x\, dF\left(x\right)^{N-2}}{F\left(\theta\right)^{N-2}}.
$$

Bidders bid their estimation of the highest valuation of their opponents, conditioning on their valuation being the highest. Because of this conditioning, bids are independent of the prior history of the game.

Furthermore, we have:

Lemma 16 *(Discouragement Effect) If $\Lambda^g \leq 1$, then $\frac{\partial\beta_2^*(\theta, y_1; \Lambda^g, \Lambda^m)}{\partial y_1} < 0\ \forall\theta$.*

---

[1] As shown in Eisenhuth and Ewers (2012), $\Lambda^g \leq 1$ is sufficient to ensure that $\frac{\partial\beta_2^*(\theta, y_1; \Lambda^g, \Lambda^m)}{\partial\theta} > 0$.

Proof.

$$\frac{\partial \beta_2^* \left(\theta, y_1; \Lambda^g, \Lambda^m\right)}{\partial y_1} = \frac{-\left[\Lambda^m \int_0^\theta F\left(x\right)^{N-3} f\left(x\right) x \left(1 - \Lambda^g + 2\Lambda^g \left(\frac{F(x)}{F(y_1)}\right)^{N-2}\right) dx\right] f\left(y_1\right)\left(N-2\right)^2}{F\left(y_1\right)^{N-1} \left[1 + \Lambda^m \left(1 - \left(\frac{F(\theta)}{F(y_1)}\right)^{N-2}\right)\right]^2} +$$

$$\frac{-\left[2\Lambda^g \left(1 + \Lambda^m - \Lambda^m \left(\frac{F(\theta)}{F(y_1)}\right)^{N-2}\right) \int_0^\theta F\left(x\right)^{2N-5} f\left(x\right) x dx\right] f\left(y_1\right)\left(N-2\right)^2}{F\left(y_1\right)^{N-1} F\left(\theta\right)^{N-2} \left[1 + \Lambda^m \left(1 - \left(\frac{F(\theta)}{F(y_1)}\right)^{N-2}\right)\right]^2}$$

$$< \quad 0$$

According to the result in Lemma 16, the higher is the type of the winner in the first round, the less aggressively the remaining bidders will bid in the second round. The rationale for this negative effect, which I call the *discouragement* effect, is as follows. From the perspective of a bidder who lost the first auction, the higher the type of the winner, the less likely the bidder is to win in the second auction (conditioning on his own type); with expectations-based reference-dependent preferences a bidder who thinks that most likely he is not going to win does not feel a strong attachment effect to the idea of winning and this pushes him to bid more conservatively. Moreover, notice that although revealing the first-period winner's bid (and hence his type) creates an informational externality, the effect on the second-period bids is exactly the opposite of the one that we have with interdependent values. Indeed, with interdependent values the higher the signal of the first-period winner, the higher is the value of the object to all remaining bidders who in turn start bidding more aggressively.

It is also easy to see that $\frac{\partial \beta_2^*(\theta, y_1; \Lambda^g, \Lambda^m)}{\partial \Lambda^m} < 0$, implying that loss aversion on the money dimension pushes bidders to behave less aggressively compared to the risk-neutral case. On the other hand, we have that

$$\frac{\partial \beta_2^* \left(\theta, y_1; \Lambda^g, \Lambda^m\right)}{\partial \Lambda^g} = \frac{\int_0^\theta x \left(2 \left(\frac{F(x)}{F(y_1)}\right)^{N-2} - 1\right) dF\left(x\right)^{N-2}}{F\left(\theta\right)^{N-2} \left[1 + \Lambda^m \left(1 - \left(\frac{F(\theta)}{F(y_1)}\right)^{N-2}\right)\right]}$$

implying that there exists $0 < \widehat{\theta} < \overline{\theta}$, with $\left(\frac{F(\widehat{\theta})}{F(y_1)}\right)^{N-2} > \frac{1}{2}$, such that $\frac{\partial \beta_2^*(\theta, y_1; \Lambda^g, \Lambda^m)}{\partial \Lambda^g} > 0 \Leftrightarrow \theta > \widehat{\theta}$.

Let's look at a particular bidder with type θ who plans to bid as if his type were $\widetilde{\theta} > \theta$ when all other $N - 1$ bidders follow the equilibrium strategy β_1.[2] Further, suppose that all bidders expect to follow strategy the equilibrium $\beta_2^* \left(\theta, y_1\right)$ in the second auction, regardless of what happens in the first one (sequential rationality). His expected life-time utility is

---

[2]The analysis is virtually identical for the case $\widetilde{\theta} < \theta$.

$$
\begin{aligned}
EU_1 \;=\;& F_1\left(\widetilde{\theta}\right)\left[\theta - \beta_1\left(\widetilde{\theta}\right)\right] + \int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right)\left[\theta - \beta_2^*\left(\theta, y_1\right)\right] f_1\left(y_1\right) dy_1 \qquad\qquad \text{(C.3)} \\[2mm]
& -\Lambda^g \theta \left[F_1\left(\widetilde{\theta}\right) + \int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1 \right]\left[1 - F_1\left(\widetilde{\theta}\right) - \int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1 \right] \\[2mm]
& -\Lambda^m \beta_1\left(\widetilde{\theta}\right) F_1\left(\widetilde{\theta}\right)\left[1 - F_1\left(\widetilde{\theta}\right) - \int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1 \right] \\[2mm]
& -\Lambda^m \int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) \beta_2^*\left(\theta, y_1\right) f_1\left(y_1\right) dy_1 \left[1 - F_1\left(\widetilde{\theta}\right) - \int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1 \right] \\[2mm]
& -\Lambda^m F_1\left(\widetilde{\theta}\right) \int_{\widetilde{\theta}}^{\overline{\theta}} \left[\beta_1\left(\widetilde{\theta}\right) - \beta_2^*\left(\theta, y_1\right) \right] F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1
\end{aligned}
$$

where $F_1\left(\widetilde{\theta}\right)$ is the probability that the highest valuation, among $N-1$, is less than $\widetilde{\theta}$, and $F_2\left(\theta|y_1\right)$, Λ^g and Λ^m are defined as before.

Let's take a minute to describe the terms in (C.3) because this is where SCPE is playing a crucial role. The first line in (C.3) is the sum of expected consumption utilities in period 1 and 2. The second term captures expected gain-loss utility on the product dimension: $F_1\left(\widetilde{\theta}\right) + \int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1$ is the sum of the probability with which a bidder of type θ expects to win the first auction given that he pretends to be of type $\widetilde{\theta}$ and the period-1 expectation of the probability with which he expects to win in the second auction given that he he pretends to be of type $\widetilde{\theta}$ in the first auction but expects to behave as his real type in the second one. Similarly, the third and fourth terms in (C.3) are expected gain-loss utility on the payment dimension: he expects to pay $\beta_1\left(\widetilde{\theta}\right)$ with probability $F_1\left(\widetilde{\theta}\right)$ (that is, if he wins the first auction), to pay $\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) \beta_2^*\left(\theta, y_1\right) f_1\left(y_1\right) dy_1$ if he wins the second auction and to pay nothing otherwise. Finally the fifth term captures the comparison between winning the first auction at price $\beta_1\left(\widetilde{\theta}\right)$ and expecting to win the second auction at price $\beta_2^*\left(\theta, y_1\right)$.[3]

---

[3]Notice also that the way the last term is written in (2.5) embeds the implicit assumption that

$$
\int_{\widetilde{\theta}}^{\overline{\theta}} \left[\beta_1\left(\widetilde{\theta}\right) - \beta_2^*\left(\theta, y_1\right) \right] F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1 > 0;
$$

that is,in expectation all bidders behave more aggressively in the first auction than in the second. We will need to check that the candidate solution β_1^* does not violate this assumption.

Taking FOC of (C.3) with respect to $\widetilde{\theta}$ yields

$$
\begin{aligned}
0 = \; & f_1\left(\widetilde{\theta}\right)\left[\theta - \beta_1\left(\widetilde{\theta}\right)\right] - \beta_1'\left(\widetilde{\theta}\right)F_1\left(\widetilde{\theta}\right) - F_2\left(\theta|\widetilde{\theta}\right)\left[\theta - \beta_2^*\left(\theta,\widetilde{\theta}\right)\right]f_1\left(\widetilde{\theta}\right) \\
& -\Lambda^g\theta\left[f_1\left(\widetilde{\theta}\right) - F_2\left(\theta|\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right)\right]\left[1 - F_1\left(\widetilde{\theta}\right) - \int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1\right] \\
& -\Lambda^g\theta\left[F_1\left(\widetilde{\theta}\right) + \int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1\right]\left[-f_1\left(\widetilde{\theta}\right) + F_2\left(\theta|\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right)\right] \\
& -\Lambda^m\beta_1'\left(\widetilde{\theta}\right)F_1\left(\widetilde{\theta}\right)\left[1 - F_1\left(\widetilde{\theta}\right) - \int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1\right] \\
& -\Lambda^m\beta_1\left(\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right)\left[1 - F_1\left(\widetilde{\theta}\right) - \int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1\right] \\
& -\Lambda^m\beta_1\left(\widetilde{\theta}\right)F_1\left(\widetilde{\theta}\right)\left[-f_1\left(\widetilde{\theta}\right) + F_2\left(\theta|\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right)\right] \\
& +\Lambda^m F_2\left(\theta|\widetilde{\theta}\right)\beta_2\left(\theta,\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right)\left[1 - F_1\left(\widetilde{\theta}\right) - \int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1\right] \\
& -\Lambda^m\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right)\beta_2\left(\theta,y_1\right)f_1\left(y_1\right)dy_1\left[-f_1\left(\widetilde{\theta}\right) + F_2\left(\theta|\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right)\right] \\
& -\Lambda^m f_1\left(\widetilde{\theta}\right)\int_{\widetilde{\theta}}^{\overline{\theta}}\left[\beta_1\left(\widetilde{\theta}\right) - \beta_2\left(\theta,y_1\right)\right]F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1 \\
& -\Lambda^m F_1\left(\widetilde{\theta}\right)\left\{-\left[\beta_1\left(\widetilde{\theta}\right) - \beta_2\left(\theta,\widetilde{\theta}\right)\right]F_2\left(\theta|\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right) + \int_{\widetilde{\theta}}^{\overline{\theta}}\beta_1'\left(\widetilde{\theta}\right)F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1\right\}
\end{aligned}
$$

Notice that $f_1\left(\theta\right) = \left(N-1\right)f\left(\theta\right)F\left(\theta\right)^{N-2}$. Then, substituting $\theta = \widetilde{\theta}$ and re-arranging results in the following differential equation

$$
\begin{aligned}
\frac{d}{d\theta}\left\{\beta_1\left(\theta\right)F_1\left(\theta\right)\left[1 + \Lambda^m\left(1 - F_1\left(\theta\right)\right)\right]\right\} = \; & f_1\left(\theta\right)\beta_2\left(\theta,\theta\right) \\
& +\Lambda^m\beta_2\left(\theta,\theta\right)f_1\left(\theta\right)\left[1 - 2F_1\left(\theta\right) - \int_{\theta}^{\overline{\theta}} F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1\right] \\
& +\Lambda^m f_1\left(\theta\right)\int_{\theta}^{\overline{\theta}}\beta_2\left(\theta,y_1\right)F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1
\end{aligned}
$$

together with the boundary condition that $\beta_1\left(0\right) = 0$. Solving the differential equation yields

$$
\begin{aligned}
\beta_1^*\left(\theta;\Lambda^g,\Lambda^m\right) = \; & \frac{\int_0^\theta \beta_2^*\left(s,s\right)\left\{1 + \Lambda^m\left[1 - 2F_1\left(s\right)\right]\right\}f_1\left(s\right)ds}{F_1\left(\theta\right)\left[1 + \Lambda^m\left(1 - F_1\left(\theta\right)\right)\right]} \\
& +\frac{\Lambda^m\int_0^\theta\Psi\left(s\right)f_1\left(s\right)ds}{F_1\left(\theta\right)\left[1 + \Lambda^m\left(1 - F_1\left(\theta\right)\right)\right]}
\end{aligned} \tag{C.4}
$$

where

$$\Psi\left(s\right)=\int_{s}^{\bar{\theta}}\beta_{2}^{*}\left(s,y_{1}\right)F_{2}\left(s|y_{1}\right)f_{1}\left(y_{1}\right)dy_{1}-\beta_{2}^{*}\left(s,s\right)\int_{s}^{\bar{\theta}}F_{2}\left(s|y_{1}\right)f_{1}\left(y_{1}\right)dy_{1}.^{4}$$

The first thing worth noticing is that β_{1}^{*} depends on Λ^{g} only indirectly, through $\beta_{2}\left(s,s\right)$. This happens because, just like in the standard case with reference-free preferences, at the optimum a bidder conditions his bid on him having the highest type. Hence, a bidder expects that if he were to lose the current auction, he would win the next one for sure and this is why expected gain-loss utility on the item dimension does not directly appear into the first period bidding function.

Again, it is easy to check that for $\Lambda^{g}=\Lambda^{m}=0$ we get back to the risk-neutral benchmark:

$$\frac{\int_{0}^{\theta}\beta_{2}\left(s,s\right)f_{1}\left(s\right)ds}{F_{1}\left(\theta\right)}$$

where $\beta_{2}\left(s,s\right)=\beta_{2}\left(s\right)$ and is independent of the type of the winner of the previous auction.

The term $\Psi\left(s\right)$ in expression above is negative and captures the *precautionary bidding* effect. In fact, if the second-auction bid were not history-dependent, this term would be equal to zero. Furthermore, we have

Lemma 17 *(Precautionary Bidding Effect) If* $\Lambda^{g}\leq1$*, then* $\Psi\left(s\right)<0$*.*

Proof. $\Lambda^{g}\leq1\underset{\text{(by Lemma 12)}}{\Rightarrow}\frac{\partial\beta_{2}(s,y_{1})}{\partial y_{1}}<0$. Hence,

$$\int_{s}^{\bar{\theta}}\beta_{2}\left(s,y_{1}\right)F_{2}\left(s|y_{1}\right)f_{1}\left(y_{1}\right)dy_{1}<\beta_{2}\left(s,s\right)\int_{s}^{\bar{\theta}}F_{2}\left(s|y_{1}\right)f_{1}\left(y_{1}\right)dy_{1}. \qquad \blacksquare$$

Let $y_{1}=\beta_{1}^{-1}\left(p_{1}\right)$. Then we have that the expected equilibrium price in the second auction conditional on the type of the winner of the first auction is

$$\mathbb{E}\left[p_{2}|p_{1}\right]=\mathbb{E}\left[p_{2}|\beta_{1}\left(y_{1}\right)\right]=\mathbb{E}\left[\beta_{2}^{*}\left(Y_{1}^{(N-1)},y_{1}\right)|Y_{1}^{(N-1)}\leq y_{1}\right]=\frac{\int_{0}^{y_{1}}\beta_{2}^{*}\left(\theta,y_{1}\right)f_{1}\left(\theta\right)d\theta}{F_{1}\left(y_{1}\right)}.$$

In Section 4 I have shown that for $\Lambda^{g}\leq1$ and $\Lambda^{m}=0$, prices always decline on the equilibrium path. However, the afternoon effect can arise also for $\Lambda^{m}>0$ as shown in the following examples.

---

[4]I have not been able to show that β_{1}^{*} is strictly increasing in θ. It certainly is for $\Lambda^{m}=0$ (see Section 4). I conjecture that $\Lambda^{m}\leq1$ is a sufficient condition for a symmetric equilibrium in increasing strategies to exist.

Let $\theta \overset{U}{\sim} [0,1]$ and suppose $N = 4$. The following figure shows the equilibrium bidding function in the first auction (solid) and the expected price of the second auction conditional on the type of the winner of the first one (dashed), for three different cases:

i) $\Lambda^g = 1$ and $\Lambda^m = 0$ (red)

ii) $\Lambda^g = 0$ and $\Lambda^m = 1$ (green)

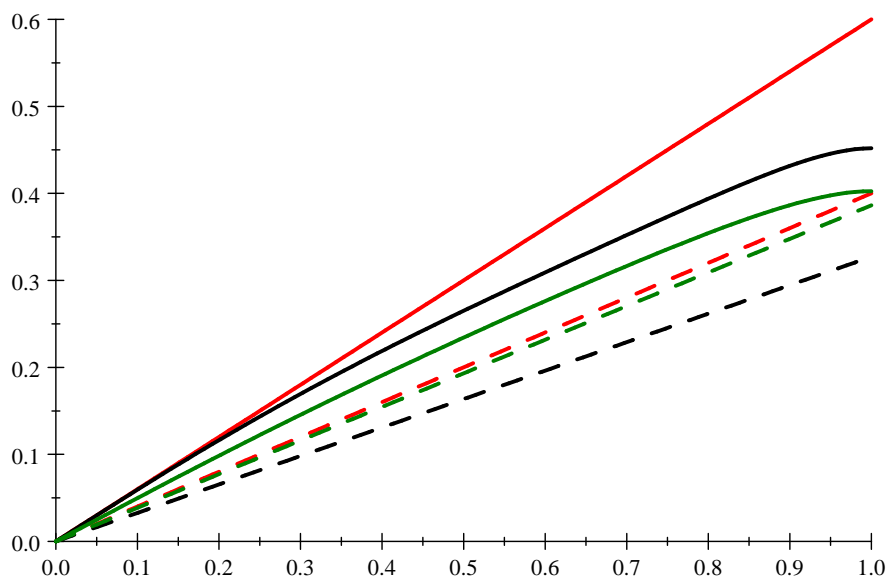iii) $\Lambda^g = 1$ and $\Lambda^m = 1$ (black)

Figure C.1: Afternoon Effect

In all three cases we have that the expected price of the second auction is smaller than the price of the first one; that is, the afternoon effect arises in equilibrium. It is also easy to see that the difference between the price of the first auction price and the expected price of the second one is largest when $\Lambda^g = 1$ and $\Lambda^m = 0$ and smallest when $\Lambda^g = 0$ and $\Lambda^m = 1$, with the case $\Lambda^g = 1$ and $\Lambda^m = 1$ falling in between. I conjecture that as Λ^m increases, an increasing price path could arise.[5]

---

[5]However, Novemsky and Kahneman (2005) and Kőszegi and Rabin (2009) argue that reference dependence and loss aversion are weaker in the money than in the product dimension.