

# **A Bridge between Travel Demand Modeling and Activity-Based Travel Analysis**

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## **Abstract**

The focus of this paper is on the demonstration that some rather well-known network-based formulations in operations research, that have heretofore largely gone unnoticed in activity-based travel research, offer a potentially powerful technique for advancing the general development of the activity-based modeling approach. These formulations can provide an analytical framework that unifies the complex interactions among the resource allocation decisions made by households in conducting their daily affairs outside the home, while preserving the utility-maximizing principles presumed to guide such decisions. A mathematical programming formulation is developed and used to identify the similarities and differences between traditional trip-based modeling methodologies and those pertaining to an activity-based approach. It is demonstrated that the two approaches are directly related.

*Keywords:* Activity-based travel analysis, Travel demand modeling, Mathematical programming.

## **1. Introduction**

Proponents of activity-based approaches to analyzing travel behavior have argued that the conceptual clarity, theoretical consistency, and purported unmatched potential for policy application of such approaches have the potential to lead to substantially greater understanding and better prediction of travel behavior. Yet, the inherent complexities of an approach that is based on the spatio-temporal linkages among a whole collection of activities and their associated travel, rather than on characteristics of a single isolated trip, have proven to pose serious impediment to the development of the approach beyond either qualitative or rudimentary statistical analyses. The few exceptions to this generally fall into the categories of computational process models (Lenntorp, 1976; Jones *et al.*, 1983; McNally and Recker, 1986; Recker *et al.*, 1986; Gärling *et al.*, 1994; Golledge *et al.*, 1994; Ettema *et al.*, 1995b; Kitamura and Fujii, 1996),

simulation studies (Axhausen, 1990; Barnett, *et al*, 1995; RDC, Inc., 1995; Kitamura *et al*, 1996; Miller, 1996) or rule-based reconstruction protocols (Bhat and Koppelman, 1993; Ettema *et al*, 1995a; Pendyala *et al*, 1995).

Despite their obvious theoretical attractiveness, activity-based approaches to understanding and predicting travel behavior have suffered from the absence of an analytical framework that unifies the complex interactions among the resource allocation decisions made by households in conducting their daily affairs outside the home, while preserving the utility-maximizing principles presumed to guide such decisions. The formulation presented in this paper provides one approach toward removing this major obstacle to operationalizing activity-based behavioral travel analysis.

In the development of this particular framework, the focus is on the demonstration that some rather well-known network-based formulations in operations research that have heretofore largely gone unnoticed in activity-based travel research offer a potentially powerful technique for advancing the general development of this approach. Experience using generic solvers for solution of a set of examples that in the realm of activity-based research have been perceived to be at least practically intractable, demonstrates that such frameworks are not prohibitively computationally intensive (Recker, 1995; Recker and Parimi, 1999; Recker et al 2000); and, undoubtedly, the application of algorithms specifically tailored to the model formulation would be substantially more efficient.

## **2. Trip-Based Travel Demand Modeling as a Mathematical Program**

Prior to developing mathematical programming representations of Activity-Based modeling approaches, it is useful to first couch traditional trip-based approaches in a similar framework. Initially, we focus on the actual decision of the traveler rather than on the analyst's attempt to model the process and, for purposes of exposition, concentrate on the choice of trip destination. Suppose that there are  $n$  different substitutable destinations available to the traveler,

$A = \{1, 2, 3, \dots, w, \dots, n\}$ , which in the traveler's mind offer corresponding value or utility  $U = \{U_1, U_2, U_3, \dots, U_w, \dots, U_n\}$ . Under the usual assumption of utility maximizing principles governing the choice, it can be expected that the traveler's choice of destination,  $k$ , will be determined as:

$$k = j \ni U_j = \underset{\forall w \in A}{\text{Max}}(U_w) \quad (1)$$

Equation (1) has a simple mathematical programming equivalent:

$$\underset{X_{ow}}{\text{Max}} Z = \sum_{\forall w \in A} X_{ow} \cdot U_w \quad (2)$$

subject to:

$$\sum_{\forall w \in A} X_{ow} = 1 \quad (2a)$$

$$X_{ow} = \begin{cases} 0 \\ 1 \end{cases}, \forall w \in A. \quad (2b)$$

Equations (2) have the familiar representation of a simple integer programming network optimization formulation in which the  $X_{ow}$  decision variables signify the “flow” (values of 0,1 being “no flow”, “flow”, respectively) from some origin  $o$  to the destination  $w \in A$ . The constraint set, i.e., Equations (2a) and (2b), forces the restriction that flow occurs from the origin to one and only one destination; i.e., the destination that offers the greatest utility. Virtually any common trip-based travel choice can be represented in this fashion (travel mode and route choice the most obvious examples), although there is little practical value in doing so since the solutions to simple choices of the form of Equation (1) are obvious by inspection.

Continuing with our example, we note that, in the simplest case of linked travel, the trip to a particular destination  $k$  is inherently coupled to a return trip (at some later time) to the origin  $o$ . In cases where the travel times and relevant costs are equal for both the trip from the origin

and its return, we can simply solve Equation (1) and stipulate the return trip; in cases where they differ, we can simply include in the  $U_w$  the relevant contributions of the return trip (the return trip destination being fixed at  $o$ ). The equivalent mathematical programming formulation formally can be expressed as:

$$\underset{X_{ow}, X_{uo}}{\text{Max}} Z = \sum_{\forall w \in A} X_{ow} \cdot U_{ow} + \sum_{\forall u \in A} X_{uo} \cdot U_{uo} \quad (3)$$

subject to:

$$\begin{aligned} \sum_{\forall w \in A} X_{ow} &= 1 \\ \sum_{\forall u \in A} X_{uo} &= 1 \\ \sum_{\forall w \in A} (X_{ow} - X_{wo}) &= 0 \end{aligned} \quad (3a)$$

$$X_{ow}, X_{uo} = \begin{cases} 0 \\ 1 \end{cases}, \forall u, w \in A. \quad (3b)$$

Here, we have assumed that the  $U_w$  are broken down into their component parts,  $U_{ow} + U_{uo}$ . The first two constraints in Equations (3a) restrict flow to a single path both to and from the origin; the third constraint ensures that the choice of a particular destination is accompanied by a return trip from that and only that location. As we begin to assemble the rudiments of an activity pattern in Equation (3), we note that the principal complication to the trip-based formulation is expressed in a growing expansion in the dimensionality of the constraint space that is necessary to capture the linkages among activities and travel.

More complex travel choices that have common trip-based representations, such as the joint choice of travel mode and destination, extrapolate very easily to the type of mathematical programming formulation characterized by Equations (3). For example, suppose that the individual making the trip described by Equation (1) has available the set of travel modes

$V = \{1, 2, \dots, v, \dots, |V|\}$  for the trip. The trip-based representation for the joint choice of mode  $m$  and destination  $k$ , which we designate as  $(m, k)$ , is

$$(m, k) = (l, j) \ni U_j^l = \underset{\forall w \in A, v \in V}{Max} (U_w^v) \quad (4)$$

where  $U_w^v$  is the utility of any particular mode-destination combination. Based on Equation (3), Equation (4) has the obvious equivalent mathematical programming representation:

$$\underset{X_{ow}^v, X_{uo}^v}{Max} Z = \sum_{\forall v \in V} \sum_{\forall w \in A} X_{ow}^v \cdot U_{ow}^v + \sum_{\forall v \in V} \sum_{\forall u \in A} X_{uo}^v \cdot U_{uo}^v \quad (5)$$

subject to:

$$\begin{aligned} \sum_{\forall v \in V} \sum_{\forall w \in A} X_{ow}^v &= 1 \\ \sum_{\forall v \in V} \sum_{\forall u \in A} X_{uo}^v &= 1 \\ \sum_{\forall w \in A} (X_{ow}^v - X_{wo}^v) &= 0, \forall v \in V \end{aligned} \quad (5a)$$

$$X_{ow}^v, X_{uo}^v = \begin{cases} 0 \\ 1 \end{cases}, \forall u, w \in A, \forall v \in V. \quad (5b)$$

Solutions to Equation (5) can be represented as a pair of directed line segments in physical space forming a closed loop that connects the origin to the particular destination, via a particular mode, that maximizes the traveler's utility; i.e., the projection of the traveler's completed path through time and space from the origin to the destination and return to the origin (Figure 1). Nowhere captured in this solution is the unfolding of the spatial path along its temporal dimension – a limitation common to the formulations in both Equations (4) and (5). Indeed, we have reached the practical limitation to the information that can be readily gleaned from trip-based modeling

frameworks which, except for sweeping generalizations, assume the temporal dimension to be exogenous to the travel choice and irrelevant to the modeling process. This, of course, is not an unreasonable assumption in such rudimentary examples as those considered above (where we have isolated the choice situation from any linkages that it might have either to previous or to impending decisions) in which the utility of the choice is assumed to be invariant with time; it is completely unreasonable and unwarranted if there is any dependency among such choices.

### **3. Bringing Temporal Factors Into Trip-Based Travel Demand Applications Through Mathematical Programming**

The implicit incorporation of temporal considerations in the travel choice protocol marks the point of departure between conventional trip-based and activity-based approaches both for any practical application of a trip-based formulations (such as those in Equations 1 and 4) as well as the point at which mathematical programming formulations (such as those in Equations 3 and 5) begin to provide conceptual efficiencies. For example, suppose that the activity that is to be performed at the selected destination in the example above has an expected duration that is dependent on the location chosen; i.e., each destination  $i \in A$  is associated with an expected time needed to complete the activity that is equal to  $s_i$ . To complete the picture, we assume that the availability of the activity (e.g., the “open” hours of a shop) at each site  $w$  is bracketed by the two times  $[a_w, b_w]$ , and that the traveler’s availability for the activity is limited to the period defined by  $[a_o, b_{n+1}]$ . To these conditions, we add a travel time matrix with elements  $t_{uw}^v$  that represent the expected travel times between all locations  $u, w \in \{o, A\}$  by travel mode  $v \in V$ . Corresponding to these new temporal restrictions, we add to the list of decision variables in our mathematical programming formulation the optimal times  $T_o$  for leaving the origin and  $T_w$  for arriving at any particular location  $w \in A$ . With these additions, the mathematical program describing the choice can be written as:

$$\underset{X_{ow}^v, X_{uo}^v}{Max} Z = \sum_{\forall v \in V} \sum_{\forall w \in A} X_{ow}^v \cdot U_{ow}^v(T_w) + \sum_{\forall v \in V} \sum_{\forall u \in A} X_{uo}^v \cdot U_{uo}^v(T_u) \quad (6)$$

subject to:

$$\begin{aligned} \sum_{\forall v \in V} \sum_{\forall w \in A} X_{ow}^v &= 1 \\ \sum_{\forall v \in V} \sum_{\forall u \in A} X_{uo}^v &= 1 \\ \sum_{\forall w \in A} (X_{ow}^v - X_{wo}^v) &= 0, \forall v \in V \end{aligned} \quad (6a)$$

$$\begin{aligned} X_{ow}^v = 1 &\Rightarrow a_u \leq T_u \leq b_u - s_u, \forall u \in A, v \in V \\ X_{ow}^v = 1 &\Rightarrow T_o + s_w + t_{ow}^v \leq T_w, \forall w \in A, v \in V \\ X_{uo}^v = 1 &\Rightarrow T_u + s_u + t_{uo}^v \leq b_{n+1}, \forall u \in A, v \in V \\ T_o &\geq a_o \end{aligned} \quad (6b)$$

$$\begin{aligned} X_{ow}^v, X_{uo}^v &= \begin{cases} 0 \\ 1 \end{cases}, \forall u, w \in A, \forall v \in V \\ T_u &\geq 0, \forall u \in \{o, A\}. \end{aligned} \quad (6c)$$

The introduction of temporal considerations through the decision variables  $T_u$ , which are continuous in time, substantively alters the mathematical program by changing it from an integer to a mixed-integer programming problem; this change significantly increases the difficulties in obtaining its solution. The first three of the temporal constraints represented by Equations (6b) indicate that these constraints are enforced only for the selected alternative, i.e., only for the destination  $k$  for which the “flow” variables  $X_{ok}$  and  $X_{ko}$  are non-zero. In their present “conditional” form they are not amenable to standard formal solution techniques. However, they easily can be rewritten in an equivalent form:

$$\begin{aligned}
T_u - (b_u - s_u) &\leq M \cdot (1 - X_{ow}^v), \forall u \in A, v \in V \\
a_u - T_u &\leq M \cdot (1 - X_{ow}^v), \forall u \in A, v \in V \\
T_o + s_w + t_{ow}^v - T_w &\leq M \cdot (1 - X_{ow}^v), \forall w \in A, v \in V \\
T_u + s_u + t_{uo}^v - b_{n+1} &\leq M \cdot (1 - X_{uo}^v), \forall u \in A, v \in V,
\end{aligned} \tag{6b'}$$

where  $M$  is a large positive number; the conditions specified by Equations (6b') are practical constraints only when the value of the appropriate flow variable is unity, in which case the right-hand side of the expression takes on a value of zero. The solutions to Equations (6) are in the form of an annotated directed path through time and space that represents the unfolding of the traveler's motion to and from the selected destination (Figure 2); the projection of this path on the spatial dimension is the solution to Equations (5).

We note that, if the utility components in Equation (6) are independent of time, then the mathematical programming solution represents a formal specification of defining the choice set as including only those destinations that can be accessed from the origin (and the activity completed) during the "window" of time available to the traveler – a specification that is exogenous in trip-based methodologies. In such cases, a unique global optimum generally cannot be found, since any combination of values for  $T_o, T_k^m$  that satisfy Equations (6b) while ensuring  $(m, k)$  is the solution of Equation (4) will produce an identical maximum utility. In the more general case where  $U_{u,w}^v = U_{u,w}^v(T_w)$ , Equations (6) may produce a unique optimum; however, the solution procedure may be extremely complex because of the associated nonlinearities in the objective function, i.e., the product terms involving the  $X_{uw}^v$  and the  $U_{uw}^v$  as an implicit function of  $T_w$ . In either case, the solutions can be viewed as forming the essential "building blocks" of an activity pattern.

#### 4. The Bridge to Activity-Based Analysis

The destination choice example that we have been discussing provides a convenient kernel for developing a mathematical programming formulation of an activity-based travel choice



paradigm – one which may be viewed as a “stringing together” of multiple destination choice problems to form a continuous path in time and space that achieves some objective that is specified by the traveler. To keep matters simple for purposes of illustration, we first consider the case in which a single traveler is faced with travel choices involving the completion of two out-of-home activities,  $A = \{1, n\} = \{1, 2\}$ , with trip ends that we will designate by  $P^+ = \{1, 2\}$ , respectively. Also, we will assume that mode choice considerations are irrelevant to these choices, although we will later see that the incorporation of travel mode choice ranges from being trivial to challenging, depending on what is included in the choice set. Before introducing any of the temporal aspects of this problem, some discussion of its spatial character may prove helpful in understanding the notation that will be adopted in the more general cases that follow. The possible spatial projections of the activity pattern paths associated with this simple example are three in number, as shown in Figure 3; two of these (Alternative Paths b and c) are simple variants of each other, determined by trip-chaining sequence.

For the projection shown as “Alternative Path a” in Figure 3, an obvious notation can be adopted to identify the terminus (or trip end) of the “return-home” trip associated with each activity. Although both trips have the same physical location for their respective return-home trips, it is convenient to give a distinctive label to each of these trip ends that ties it uniquely to its corresponding out-of-home activity. We do this for the path shown as “Alternative Path a” in Figure 4, using the notation that the label identifying the terminus of the return-home trip be the label number of the activity plus the total number of activities in  $A$ . This same notation also can be applied to the two trip-chaining alternatives (Alternative Paths b and c), although perhaps not with so obvious reason – in these latter cases, we essentially construct a “shadow” path that does not include intermediate trip ends. Designate this set of return-home trip ends by  $P^- = \{n+1, n+2\} = \{3, 4\}$ . For convenience, to this list we add two other “home” nodes as “0” and “ $2n+1 = 5$ ”, designating the trip ends that mark the very first departure from the home and that signify the last node in the activity pattern from which no other trip can emanate, respectively. A word of explanation regarding the nature of this latter node – each out-of-home

activity is associated with its return-home trip end; the last of these return-home trip ends (either “3” or “4” in this example) is also associated with a link to trip end “5” (also at the physical home location) from which no further outgoing trips are possible.<sup>1</sup> Figure 5 shows this expanded labeling for the possible paths identified in Figure 4 .

We now begin to construct the mathematical program that we will use to describe the choice of the activity pattern (i.e., the selection of the traveler’s spatio-temporal path whose spatial projection is one of the three paths identified in Figure 5) for this example. Spatial connectivity requires that:

$$\sum_{w \in N} X_{uw} = 1, \quad \forall u \in P^+ \quad (7a_1)$$

$$\sum_{w \in N} X_{uw} - \sum_{w \in N} X_{wu} = 0, \quad \forall u \in P \quad (7a_2)$$

$$\sum_{w \in P^+} X_{ow} = 1 \quad (7a_3)$$

$$\sum_{u \in P^-} X_{u,2n+1} = 1$$

$$\sum_{w \in N} X_{wu} - \sum_{w \in N} X_{w,n+u} = 0, \quad \forall u \in P^+ \quad (7a_4)$$

$$\sum_{w \in P^-} X_{ow} = 0$$

$$\sum_{u \in N} X_{uo} = 0$$

$$\sum_{u \in P^+} X_{u,2n+1} = 0$$

$$\sum_{w \in N} X_{2n+1,w} = 0 \quad (7a_5)$$

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<sup>1</sup> In the parlance of the mathematical programming analog on which this formulation is based, trip end “5” represents the depot in a vehicle routing problem.

$$X_{n+u,u} = 0, \forall u \in P^+ \quad (7a_6)$$

where the notation  $P = P^+ \cup P^- = \{1,2,3,4\}$  and  $N = \{0,P,2n+1\} = \{0,1,2,3,4,5\}$  is used to designate appropriate sets of nodes. We have grouped these equations according to the types of conditions that they impose in order to assist in visualization of the relationships between the mathematical program and the requirements for spatial connectivity. Equations (7a<sub>1</sub>) impose the condition that there is one and only one path leading from any out-of-home activity. Equations (7a<sub>2</sub>) ensure that there is a connected path among the activities (and their return trips to home) and that no activity is revisited. Equations (7a<sub>3</sub>) enforce a restriction similar to that in Equations (7a<sub>1</sub>), but with reference to the paths leading from the origin and to the final termination (i.e., the depot). Equations (7a<sub>4</sub>) stipulate that the return-home trip be on the same path as it's associated out-of-home activity. Equations (7a<sub>5</sub>) rule out illogical flows. Finally, Equations (7a<sub>6</sub>) ensure that the path leading to a particular out-of-home activity precedes the return-home trip; these conditions will be subsumed by the temporal constraints that will be imposed later. Readers familiar with mathematical programming will recognize Equations (7a<sub>1</sub>) through (7a<sub>6</sub>) as the constraint equations in what is known as the “Pickup and Delivery Problem (PDP),” which itself is a derivative of the famous “Traveling Salesman Problem (TSP).”

The incorporation of temporal factors that restrict the feasibility of the three (in our simple example) spatial paths defined by Equations (7a<sub>1</sub>) through (7a<sub>6</sub>) and depicted in Figure 5 is relatively straightforward; the principal restrictions are simply generalizations of Equations 6b. First, there is the restriction that the commencement time of the activity associated with any trip end  $w$ , i.e.,  $T_w$ , requiring travel from another trip end  $u$  can occur no sooner than the termination time of the corresponding activity at  $u$  plus the travel time from the site of activity  $u$  to the site of activity  $w$ . This can be represented as:

$$X_{uw} = 1 \Rightarrow T_u + s_u + t_{uw} \leq T_w, \forall u, w \in P. \quad (7b_1)$$

or, equivalently,

$$T_u + s_u + t_{uw} - T_w \leq M \cdot (1 - X_{uw}), \forall u, w \in P \quad (7b'_1)$$

Similar restrictions hold for travel from the origin node,  $o$ , to any activity, as well as for travel from any activity to its “return home” activity:

$$\begin{aligned} X_{ow} = 1 &\Rightarrow T_o + t_{ow} \leq T_w, \forall w \in P^+ \\ X_{u,2n+1} = 1 &\Rightarrow T_u + s_u + t_{u,2n+1} \leq T_{2n+1}, \forall u \in P^-. \end{aligned} \quad (7b_2)$$

or, equivalently,

$$\begin{aligned} T_o + t_{ow} - T_w &\leq M \cdot (1 - X_{ow}), \forall w \in P^+ \\ T_u + s_u + t_{u,2n+1} - T_{2n+1} &\leq M \cdot (1 - X_{u,2n+1}), \forall u \in P^- \end{aligned} \quad (7b'_2)$$

To these conditions we add restrictions regarding the time windows available for activity completion:

$$\begin{aligned} a_u &\leq T_u \leq b_u - s_u, \forall u \in P \\ a_o &\leq T_o \leq b_o \\ a_{2n+1} &\leq T_{2n+1} \leq b_{2n+1}. \end{aligned} \quad (7b_3)$$

As stated above, Equations (7b<sub>1</sub>), (7b<sub>2</sub>), and (7b<sub>3</sub>) are, expectedly, almost a direct translation of the temporal constraints enforced in the destination choice example described in Equations (6b). However, in that example, the nodes had no particular relation to each other, other than being alternative destinations for the same activity. In the current case, however, there is an implicit temporal relationship between the nodes in  $P^+$  and those in  $P^-$ ; those in the latter representing the “return home” trip ends of the former. As such, there is a restriction that the activity start

times for elements of  $P^+$  precede those of corresponding elements in  $P^-$ . This condition can be represented as:

$$T_u + s_u + t_{u,n+u} \leq T_{n+u}, \forall u \in P^+. \quad (7b_4)$$

To complete the current picture, we need to add non-negativity and integer constraints equivalent to those in Equations (6c):

$$\begin{aligned} X_{uw} &= \begin{cases} 0 \\ 1 \end{cases}, \forall u, w \in N \\ T_u &\geq 0, \forall u \in N. \end{aligned} \quad (7c)$$

Once again, those familiar with mathematical programming will recognize Equations (7a·), (7b·), and (7c) as the constraint equations in what is known as the ‘‘Pickup and Delivery Problem with Time Windows (PDPTW);’’ see, e.g., Solomon and Desrosiers, 1988 for details on this class of problem. In terms of the activity pattern problem that is being considered by this simple example, Equations (7b·) place additional constraints on the unfolding of the possible spatial paths shown in Figure 5 (and defined by Equations 7a·) in the temporal dimension. The effect of these constraints on the feasibility of the three paths is demonstrated by adding some temporal information to our example, corresponding to the various activity durations, time windows, and travel times:

$$[s_i] = \begin{bmatrix} 8 \\ 2 \end{bmatrix}, [a_i, b_i] = \begin{bmatrix} 8 & 8.5 \\ 10 & 20 \end{bmatrix}, [a_{n+i}, b_{n+i}] = \begin{bmatrix} 17 & 19.5 \\ 10 & 21 \end{bmatrix}, [a_0, b_0] = [6 \quad 20] \\ [a_{2n+1}, b_{2n+1}] = [6 \quad 21]$$

		$t_{uw}$						
		<b>TO</b>	0	1	2	3	4	5
<b>FROM</b>								
0		0.00	1.00	0.25	0.00	0.00	0.00	0.00
1		1.00	0.00	1.00	1.00	1.00	1.00	1.00
2		0.25	1.00	0.00	0.25	0.25	0.25	0.25
3		0.00	1.00	0.25	0.00	0.00	0.00	0.00
4		0.00	1.00	0.25	0.00	0.00	0.00	0.00
5		0.00	1.00	0.25	0.00	0.00	0.00	0.00

We note that the travel time matrix makes explicit that the physical locations of the “return home” activities is coincident with the home location “0”. Feasible space-time paths associated with spatial projections *a* and *b* are shown in Figures 6a and 6b, respectively. Under these conditions, all space-time paths associated with spatial projection *c* become infeasible (Figure 6c).

Introduction of some objective that is to be satisfied in the selection of an activity pattern from among those that are feasible completes the specification of the most rudimentary form of the Activity Pattern Problem (APP). For example, if the objective of the traveler in our current example is to minimize travel time, i.e.,

$$\text{Min Total Travel Time} = \sum_{\forall u \in N} \sum_{\forall w \in N} t_{uw} \cdot X_{uw}, \quad (7d)$$

then the optimal activity pattern is easily found to be that displayed in Figure 6b.

## 5. Some Examples of Easy Embellishments to the APP Kernel

Once couched in its most basic form, the activity pattern problem is easily extended to incorporate certain accepted aspects of travel decisions that are either difficult or impossible to account for using conventional trip-based formulations. For example, the notion that travel decisions are made within the context of both travel time as well as travel budget constraints is almost trivially included by adding to the base formulation the following constraints:

$$\begin{aligned} \sum_{\forall u \in N} \sum_{\forall w \in N} t_{uw} \cdot X_{uw} &\leq B_t \\ \sum_{\forall u \in N} \sum_{\forall w \in N} c_{uw} \cdot X_{uw} &\leq B_c \end{aligned} \quad (8)$$

where  $c_{uw}$  are the costs of travel between nodes, and  $B_t$  and  $B_c$  are the traveler's time and money budgets, respectively.

Other, less trivial but nonetheless straightforward, considerations include placing certain restrictions on the characteristics of the tours that comprise the activity pattern. For example, incorporation of considerations that are functions of the cumulative number of sojourns in a tour, e.g., the number of sojourns or the amount of time between stops at home, can be achieved by defining an ‘‘accumulator’’ variable, say  $d_w$ , that is incremented with each successive stop on a tour, and a register, say  $Y_w$ , that tracks the total accumulation of  $d_w$ , i.e.,

$$\begin{aligned} X_{uw} = 1 &\Rightarrow Y_u + d_w = Y_w, \forall u \in \{o, P\}, w \in P^+ \\ X_{uw} = 1 &\Rightarrow Y_u - d_{w-n} = Y_w, \forall u \in P, w \in P^- \\ Y_o &= 0. \end{aligned} \quad (9)$$

The first of these relations accumulates  $d_w$ , while the second releases the accumulation upon a ‘‘return home’’ activity; the third is simply an initial condition. Restriction of the value of the  $Y_w$  variable to some limiting value  $D$ , can then be used to capture a number of potentially limiting characteristics of multiple-sojourn tours. For example, if  $D$  is specified as:

$$D = \begin{cases} \text{maximum number of sojourns in any tour} \\ \text{or} \\ \text{maximum time spent away from home on any tour} \end{cases}$$

then the corresponding specification of the accumulator  $d_w$  would be

$$d_w = \begin{cases} 1 \\ \text{or} \\ s_w + t_{w'w}; w' = \text{stop on tour immediately preceding } w. \end{cases}$$

In addition to the minimization of total travel time (see, e.g., Equation 7d), a number of other components potentially important to the traveler's overall utility are easily incorporated. Examples of potential components of the disutility function of the household that may be easily specified in the objective function include:

$$\sum_{u \in N} \sum_{w \in N} c_{uw} \cdot X_{uw} \quad \text{total travel cost.}$$

$$\sum_{u \in P^+} (T_u - b_u) \quad \begin{array}{l} \text{a measure of the risk of the inability to complete activities} \\ \text{due to stochastic variations in travel times and/or} \\ \text{activity durations.} \end{array}$$

$$\sum_{u \in P^-} (T_u - b_u) \quad \begin{array}{l} \text{a measure of the risk of not returning home in time due} \\ \text{to stochastic variations in travel time or activity} \\ \text{participation.} \end{array}$$



$\sum_{u \in P^+} (T_{u+n} - T_u - s_u - t_{u,u+n})$  a measure of the delay in returning home incurred by trip chaining.

$(T_{2n+1} - T_o)$  the extent of the travel day.

We note that, with the exception of the first of these examples, such considerations can not be captured by trip-based methodologies.

## 6. Extension of the APP Kernel to the Basic Household Activity Pattern Problem (HAPP)

The mathematical programming approach outlined above has been extended by Recker (1995) to the general case in which the household activity pattern problem (HAPP) is posed as a network-based routing model incorporating vehicle assignment, ride-sharing behavior, activity assignment and scheduling, and time window constraints. The resulting HAPP formulation is in the form of a Mixed Integer Linear Programming (MILP) model. The equations describing the problem are contained in Recker (1995) and are not repeated here. However, the general form of the HAPP mathematical program formulation of the travel/activity decisions for a particular household, say  $i$ , during some time period is represented by:

$$\text{Minimize } Z(\mathbf{X}_i) = \mathbf{B}'_i \cdot \mathbf{X}_i$$

subject to : (10)

$$\mathbf{A}\mathbf{X}_i \leq \mathbf{0}$$

where

$$\mathbf{X}_i = \begin{bmatrix} \mathbf{X}^\square \\ \mathbf{H} \\ \mathbf{T} \end{bmatrix}, \quad \mathbf{X}^\square = \left[ X_{uw}^v = \begin{cases} 0 \\ 1 \end{cases} \right], \quad \mathbf{H} = \left[ H_{uw}^\alpha = \begin{cases} 0 \\ 1 \end{cases} \right], \quad \mathbf{T} = [T_u \geq 0],$$

The outputs  $\mathbf{X}_i$  of the optimization for each household  $i$  are specified by the following decision variables<sup>2</sup>:

$X_{uw}^v, u, w \in N, v \in V, u \neq w$  binary decision variable equal to unity if vehicle  $v$  travels from activity  $u$  to activity  $w$ , and zero otherwise.

$H_{uw}^\alpha, u, w \in N, \alpha \in \eta, u \neq w$  binary decision variable equal to unity if household member  $\alpha$  travels from activity  $u$  to activity  $w$ , and zero otherwise.

$T_u, u \in P$  the time at which participation in activity  $u$  begins.

$T_0^v, T_{2n+1}^v, v \in V$  the times at which vehicle  $v$  first departs from home and last returns to home, respectively.

$\bar{T}_0^\alpha, \bar{T}_{2n+1}^\alpha, \alpha \in \eta$  the times at which household member  $\alpha$  first departs from home and last returns to home, respectively.

The various sets referenced in the above are defined by the following notation:

$A = \{1, 2, \dots, j, \dots, n\}$  the set of out-of-home activities scheduled to be completed by travelers in the household.

$V = \{1, 2, \dots, v, \dots, |V|\}$  the set of vehicles used by travelers in the household to complete their scheduled activities.

$P^+ = \{1, 2, \dots, u, \dots, n\}$  the set designating location at which each activity is performed.

$P^- = \{n+1, n+2, \dots, n+j, \dots, 2n\}$  the set designating the ultimate destination of the "return to home" trip for each activity. (It is noted that the physical location of each element of  $P^-$  is "home".)

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<sup>2</sup> The notation used here is the same as that contained in Recker, 1995.

$P = P^+ \cup P^-$  the set of nodes comprising completion of the household's scheduled activities.

$N = \{0, P, 2n+1\}$  the set of all nodes, including those associated with the initial departure and final return to home.

$\mathbf{B}_i$  is a vector of coefficients that defines the relative contributions of each of the decision variables to the overall disutility of the travel regime to the household. Descriptively, the constraint sets  $\mathbf{A}\mathbf{X}_i \leq \mathbf{0}$  for this MILP are classified into six groups: (a) routing constraints that define the allowable spatial movement of vehicles and household members in completing the household's activity agenda; (b) scheduling constraints specify the relationship of arrival time, activity begin time, and waiting time, and continuity condition along the temporal dimension; (c) assignment constraints that are applied to match the relations between activity participation and vehicle usage as well as activity performers (household members); (d) time window constraints that are used to specify available schedules for activity participation; (e) coupling constraints that define the relations between vehicle-related variables and member-related variables; and (f) side constraints including budget, capacity, and rules for ride-sharing behavior. With the exception of the side constraints (i.e., classification "f" above), these constraints capture the physical conditions that ensure that each member of the household, as well as each vehicle used by the household, have a consistent, continuous, path through time-space that results in all of the activities on the household's agenda being successfully completed. The reader interested in a detailed derivation and explanation of these constraints is referred to the original work by Recker (*op cit*).

The solution vector,  $\mathbf{X}_n^*$ , to Equations (10) represents the household's utility maximizing behavior, relative to the prescribed objective  $Z(\mathbf{X}_i)$ <sup>3</sup>, with regard to completing its activity agenda. The solution patterns reveal personal travel behavior and activity participation within a

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<sup>3</sup> In this form, the specification of the objective function is prescribed by the analyst; e.g., the minimization of total travel time.

household context, while preserving the concept that the need for travel originates from participation in activities, that travel constitutes the linkage between activities, and in which all of the required components are contained in the activity scheduling problem. An application of the methodology in the estimation of the upper bounds of certain policy alternatives in reducing vehicle emissions can be found in Recker and Parimi (1999).

## **7. Activity-Based Demand Modeling**

In the examples considered in the previous sections to demonstrate the application of the mathematical framework, the specification of the objective function is known to both the decision maker and the analyst. The typical problem in demand modeling (of which the HAPP is a subset) is focused on inferring the relative weights associated with potential components of the utility function that are determinants to a population's revealed selection of the decision variables (in the model estimation phase) with subsequent forecasts made using these weights in conventional application of the model. In that sense, the modeling framework developed offers a real analytical option for estimating the relative importance of factors associated with the spatial and temporal interrelationships among the out-of-home activities that motivate a household's need or desire to travel. Such estimation can proceed in a manner similar to utility-maximizing estimation techniques used in conventional demand analysis (e.g., regression, logit and probit analyses) in which the choice situation is presumed to be unconstrained; the proposed framework provides both the necessary constraint considerations on the household's decision alternatives within a utility-maximizing structure as well as a convenient mechanism for generating the set of feasible alternatives that are likely to be considered.

### ***Discrete Choice Utility Maximization as a Mathematical Program***

As a means of better positioning the issues related to use of the HAPP mathematical programming formulation in travel demand analysis, we first examine traditional utility-maximizing discrete choice analysis from the perspective of mathematical programming. Stated

in simplified terms, the choice problem for a particular observation,  $n$ , is assumed to be: from a set of alternatives available to  $n$ , say  $Q_n = \{q_1, q_2, \dots, q_K\}$ , select the alternative  $q^*$  that maximizes the utility or, alternatively, minimizes the disutility  $Z_n$  that is derived from the choice. The disutility of any particular alternative  $q_i$ , say  $Z_n^i(q_i)$ , is presumed to be an implicit function of the vector of attributes,  $\mathbf{a}'_i = [a_{i1}, a_{i2}, \dots, a_{iJ}]$ , that define  $q_i$  relative to the choice situation; a common representation for  $Z_n^i$  is the linear function  $Z_n^i(\mathbf{a}_i) = \mathbf{b}'_n \cdot \mathbf{a}_i$ , where  $\mathbf{b}_n$  is a vector of coefficients (sometimes referred to as “utility weights”). The prime (‘) symbol is used here and throughout to designate the transpose of a vector or matrix.

For a mutually exclusive choice set, we define a vector of decision variables

$$\mathbf{X} = [x_i], x_i = \begin{cases} 0, & \text{if alternative } i \text{ is not chosen} \\ 1, & \text{if alternative } i \text{ is chosen} \end{cases},$$

and the choice problem defined above can be represented by the following mathematical program:

$$\text{Minimize } Z(\mathbf{X}_n) = \mathbf{B}'_n \cdot \mathbf{X}_n$$

$$\text{subject to :} \tag{11}$$

$$\mathbf{1}' \cdot \mathbf{X}_n = 1$$

where

$$\mathbf{1} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \quad \mathbf{B}_n = \begin{bmatrix} \mathbf{b}_n \cdot \mathbf{a}_1 \\ \mathbf{b}_n \cdot \mathbf{a}_2 \\ \vdots \\ \mathbf{b}_n \cdot \mathbf{a}_K \end{bmatrix}.$$

Equations (11) are, of course, an idealization of the choice process. The linearization of the utility function is at best a first order approximation to the mapping of attribute levels to utility, and the specification of  $\mathbf{X}_n$  undoubtedly will exclude some aspects (however minor) of the alternative that influence the decision. Inclusion of these, and other, unobservable effects on choice is the basis of random utility theory (Manski, 1977), which assumes that, while the decision maker selects the alternative with the greatest utility with certainty, the analyst is only able to specify utility to within some disturbance (or, random) term. The observed behavior, say  $\tilde{\mathbf{X}}_n$ , is related to the model as:

$$\begin{aligned} \text{Minimize } Z(\tilde{\mathbf{X}}_n) &= \mathbf{B}'_n \cdot \tilde{\mathbf{X}}_n + \xi_{\tilde{\mathbf{X}}_n} \\ \text{subject to :} & \\ 1' \cdot \tilde{\mathbf{X}}_n &= 1 \end{aligned} \tag{12}$$

where  $\xi_{\tilde{\mathbf{X}}_n}$  is a disturbance term that is specific to any particular candidate solution  $\tilde{\mathbf{X}}_n$  to Equations (12). Equations (12) are a non-deterministic system of equations. Because of the simplicity of the constraint set (a product of both the one-dimensional nature of the choice, and the mutual exclusivity property, as well as the countable choice set), the error term in Equations (12) can be incorporated directly into the utility weight matrix, i.e.,

$$\begin{aligned} \text{Minimize } Z(\tilde{\mathbf{X}}_n) &= \tilde{\mathbf{B}}'_n \cdot \tilde{\mathbf{X}}_n \\ \text{subject to :} & \\ 1' \cdot \tilde{\mathbf{X}}_n &= 1 \end{aligned}$$

where

$$\tilde{\mathbf{B}}_n = \begin{bmatrix} \mathbf{b}_n \cdot \mathbf{a}_1 + \xi_1 \\ \mathbf{b}_n \cdot \mathbf{a}_2 + \xi_2 \\ \vdots \\ \mathbf{b}_n \cdot \mathbf{a}_K + \xi_K \end{bmatrix},$$

and where the  $\xi_i$  are the disturbance terms. Under these conditions, the probability of any particular

$$\tilde{\mathbf{X}}_n = [\tilde{x}_i], \tilde{x}_i = \begin{cases} 0, & i \neq k \\ 1, & i = k \end{cases}$$

being a solution is obtained directly as  $\mathbf{Prob}(\mathbf{b}_n \cdot \mathbf{a}_k + \xi_k \geq \mathbf{b}_n \cdot \mathbf{a}_i + \xi_i, \forall i, i \neq k)$ ; the familiar result in the statement of the choice probability under random utility assumptions. Absent invoking this result, these same probabilities presumably would arise from an appropriate Monte Carlo simulation of the error terms in Equations (12).

### ***HAPP Utility Maximization***

We turn now to the HAPP activity-based modeling scheme. Recall that the HAPP mathematical programming formulation of the travel/activity decisions for a particular household, say  $n$ , during some time period is represented by Equations (10), above. The form of Equations (10) is generally similar to that of Equations (11), with some notable exceptions that greatly complicate its application in empirical demand analysis – 1) the set of feasible solutions (alternatives) in the system defined by Equations (10) is infinite, while that for Equations (12) is countable (and, usually small), 2) the solution vector of Equations (10) comprises continuous, as well as discrete, variables, 3) while the overall solution represents a mutually exclusive choice, the solution vector itself is composed of components that are not generally mutually exclusive, 4) the components of  $\mathbf{B}_n$  are not directly interpretable as utility weights of attributes, but rather are

related to these weights through a transformation matrix, and 5) the complexity of the constraint space of Equations (10) generally precludes the type of closed-form probability result noted above for Equations (11).

As in the discrete choice example, Equations (10) constitute a model of observed behavior, say  $\tilde{\mathbf{X}}_n$ . Owing to the specification of the model, the observed behavior is related to the model as:

$$\begin{aligned} \text{Minimize } Z(\tilde{\mathbf{X}}_n) &= \mathbf{B}'_n \cdot \tilde{\mathbf{X}}_n + \xi_{\tilde{\mathbf{X}}_n} \\ \text{subject to :} & \\ \mathbf{A} \cdot \tilde{\mathbf{X}}_n &\leq \mathbf{0} \end{aligned} \tag{13}$$

where  $\xi_{\tilde{\mathbf{X}}_n}$  is the disturbance term between the observed ( $\tilde{\mathbf{X}}_n$ ) and the modeled ( $\mathbf{X}_n$ ) behaviors. As was noted in 4) above, the  $\mathbf{B}_n$  in Equation (13) is related to the common concept of utility weights through a transformation matrix. For example, a commonly accepted component of travel disutility is the travel time. In the discrete choice formulation of Equation (11), this component for any particular alternative, say the  $k^{\text{th}}$ , would be represented by the product of the travel time associated with alternative  $k$ , say the  $j^{\text{th}}$  attribute  $a_{jk}$ , and the travel time utility weight  $b_{nj}$ , i.e.,  $a_{jk} \cdot b_{nj}$ . In the activity-based formulation of HAPP, the equivalent measure of disutility is the total travel time of all travel links that comprise the activity pattern:

$$\text{Total Travel Time} = \sum_{\forall v \in V} \sum_{\forall u \in N} \sum_{\forall w \in N} t_{uw}^v \cdot \tilde{X}_{uw}^v = [\mathbf{t}' \quad \mathbf{0} \quad \mathbf{0}]_n \cdot \begin{bmatrix} \tilde{\mathbf{X}}^v \\ \tilde{\mathbf{H}} \\ \tilde{\mathbf{T}} \end{bmatrix}_n = \mathbf{F}'_j \cdot \tilde{\mathbf{X}}_n$$



where  $\mathbf{t}$  is the vector of travel times that comprise the travel time matrix, ordered appropriately. The corresponding component of travel time disutility is then simply  $b_{nj} \cdot \mathbf{F}'_j \cdot \tilde{\mathbf{X}}_n$ . In general, the total disutility obtaining from a set of  $J$  factors (or attributes) would be represented by:

$$\text{Total Disutility} = \sum_{\forall j} b_{nj} \cdot \mathbf{F}'_j \cdot \tilde{\mathbf{X}}_n + \xi_{\tilde{\mathbf{X}}_n} \quad (14)$$

We note that Equation (14) can be rewritten as:

$$\text{Total Disutility} = \sum_{\forall j} b_{nj} \cdot \mathbf{F}'_j \cdot \tilde{\mathbf{X}}_n + \xi_{\tilde{\mathbf{X}}_n} = \mathbf{B}'_n \cdot \tilde{\mathbf{X}}_n + \xi_{\tilde{\mathbf{X}}_n}$$

where

$$\mathbf{B}'_n = \mathbf{b}'_n \cdot \mathbf{F}', \mathbf{b}_n = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_J \end{bmatrix}, \mathbf{F}' = \begin{bmatrix} \mathbf{F}'_1 \\ \mathbf{F}'_2 \\ \vdots \\ \mathbf{F}'_J \end{bmatrix}.$$

While it was possible to generate the probability distributions of optimal solutions to the standard discrete choice problem either directly or through simple Monte Carlo simulation of the error terms (coupled to any standard integer programming technique), in the case of HAPP this direct approach clearly is not possible. Rather, we are left with only approximate schemes to achieve this result. One procedure, for example, would be as follows:

1. Remove the random term from Equation (13) and then apply some sort of branch-and-bound algorithm to the resulting system.
2. At each stage of uncovering a feasible solution, attach to the value of its objective function the realization of a random draw from the presumed error distribution.
3. Continue this process to the solution.
4. Repeat the process.

The above procedure has the drawback that the result is biased by the particular branch-and-bound protocol (not all feasible solutions are identified) and branches typically are not revisited. However, a Monte Carlo simulation based on the procedure should produce acceptable results (akin to the k shortest path type analyses). A second, and perhaps thornier, problem is that the solution vector contains continuous (i.e., the starting times of activities), as well as discrete, variables (i.e., person and vehicle link assignments); hence, an infinity of alternatives exist. As such, the probability of any solution matching exactly the observed behavior is infinitesimally small. In dealing with continuous distributions, it is more proper to address the probability that an outcome lies within some  $\delta$  of a particular value. This approach could also be used in the HAPP case or, alternatively, the continuous variables that define the choice alternative could be specified in terms of a “band” around the actual value; e.g., an activity start time of  $t_j$  specified as a start time between  $t_j - \delta$  and  $t_j + \delta$ . We note that, since  $\tilde{\mathbf{X}}^v$  and  $\tilde{\mathbf{H}}$  are discrete, this latter procedure would only need to be applied to the  $\tilde{\mathbf{T}}$  component of  $\tilde{\mathbf{X}}$ .

## 8. Model Estimation Issues and Obstacles

Standard to demand analysis, the analyst can not directly observe  $\mathbf{B}_n$ ; rather, an estimate,  $\hat{\mathbf{B}}_n$ , is sought that can be inferred from the observed behavior,  $\tilde{\mathbf{X}}_n^*$ . The goal, then, is to find the  $\hat{\mathbf{B}}_n$  that minimizes some prescribed error  $\mathcal{E}_n$  between the solution vector  $\mathbf{X}_n^*$  and the observed behavior  $\tilde{\mathbf{X}}_n^*$ . The principal task of inferential statistics is to find such an estimate, using sample data, that can be extrapolated to the population by minimizing some aggregate form of this error over the entire sample. In the case of discrete choice analysis, this is usually accomplished with maximum likelihood estimation of  $\hat{\mathbf{B}}_n \equiv \hat{\mathbf{B}}, \forall n$ , i.e., assuming that the utility weights are common across observations, and assuming that the error terms  $\xi_i$  are independently identically distributed (IID). The standard application of maximum likelihood requires that the model choice probabilities be a differentiable function of the parameters contained in  $\mathbf{B}$ . Without such stipulation, the maximization process could be accomplished using some heuristic search. For

example, in the case of the discrete choice problem stated in Equations (12), one could start with an initial guess of  $\mathbf{B}$ , say  $\mathbf{B}^0$ , perform the Monte Carlo simulation described above to obtain the probabilities associated with the selected alternatives for the sample, compute the likelihood, and then invoke the heuristic to modify  $\mathbf{B}^0$  in a direction that is expected to increase the value of the likelihood function.

In the case of the HAPP model (i.e., Equations 13), the only option available is a heuristic. The form of the HAPP model may lend the accompanying maximum likelihood estimation problem to solution by genetic algorithm – we propose one possible procedure below.

The specification of  $\mathbf{X}_n$  in Equation (13) leads to:

$$P(\mathbf{X}_n = \tilde{\mathbf{X}}_n) = P(\mathbf{T}_n = \tilde{\mathbf{T}}_n | \tilde{\mathbf{X}}_n^v, \tilde{\mathbf{H}}_n) \cdot P(\mathbf{X}_n^v = \tilde{\mathbf{X}}_n^v | \tilde{\mathbf{H}}_n) \cdot P(\mathbf{H}_n = \tilde{\mathbf{H}}_n) \quad (15)$$

Were these events independent,

$$P(\mathbf{X}_n = \tilde{\mathbf{X}}_n) = P(\mathbf{T}_n = \tilde{\mathbf{T}}_n) \cdot P(\mathbf{X}_n^v = \tilde{\mathbf{X}}_n^v) \cdot P(\mathbf{H}_n = \tilde{\mathbf{H}}_n). \quad (15a)$$

The associated likelihood function for HAPP is given by

$$\mathbf{L}(\hat{\mathbf{B}}) = \prod_{\forall n} P(\mathbf{X}_n = \tilde{\mathbf{X}}_n) = \prod_{\forall n} P(\mathbf{T}_n = \tilde{\mathbf{T}}_n | \tilde{\mathbf{X}}_n^v, \tilde{\mathbf{H}}_n) \cdot P(\mathbf{X}_n^v = \tilde{\mathbf{X}}_n^v | \tilde{\mathbf{H}}_n) \cdot P(\mathbf{H}_n = \tilde{\mathbf{H}}_n) \quad (16)$$

and the log likelihood

$$\ln \mathbf{L}(\hat{\mathbf{B}}) = \sum_{\forall n} \ln P(\mathbf{T}_n = \tilde{\mathbf{T}}_n | \tilde{\mathbf{X}}_n^v, \tilde{\mathbf{H}}_n) + \sum_{\forall n} \ln P(\mathbf{X}_n^v = \tilde{\mathbf{X}}_n^v | \tilde{\mathbf{H}}_n) + \sum_{\forall n} \ln P(\mathbf{H}_n = \tilde{\mathbf{H}}_n)$$

The steps in the proposed genetic algorithm are as follows:

1. Define as a multi-objective function the three-tuple

$$\boldsymbol{\mu}(\hat{\mathbf{B}}_p) = \{\mu_1(\hat{\mathbf{B}}_p), \mu_2(\hat{\mathbf{B}}_p), \mu_3(\hat{\mathbf{B}}_p)\}$$

$$= \left\{ \sum_{\forall n} \ln P(\mathbf{T}_n(\hat{\mathbf{B}}_p) = \tilde{\mathbf{T}}_n | \tilde{\mathbf{X}}_n^v, \tilde{\mathbf{H}}_n), \sum_{\forall n} \ln P(\mathbf{X}_n^v(\hat{\mathbf{B}}_p) = \tilde{\mathbf{X}}_n^v | \tilde{\mathbf{H}}_n), \sum_{\forall n} \ln P(\mathbf{H}_n(\hat{\mathbf{B}}_p) = \tilde{\mathbf{H}}_n) \right\}$$

that indicates the fitness of any potential solution vector,  $\hat{\mathbf{B}}_p$ , in maximizing the log likelihood.

2. Generate a population of  $n_p$  candidate solutions  $\hat{\mathbf{B}}_p, p = 1, \dots, n_p$ . Since the signs of the coefficients of  $\hat{\mathbf{B}}$  typically are known, the utility components can always be specified in such a manner that the elements of  $\hat{\mathbf{B}}$  are strictly non-negative; and, since the solution is unaffected by the scaling of the utility coefficients, they can be scaled to be in the range  $[0,1]$ . Then, the candidate solutions are drawn from the range  $[0, 0, \dots, 0]$  to  $[1, 1, \dots, 1]$ . Each of these candidate solutions is termed a *chromosome*, and the elements of each chromosome are referred to as *genes*.
3. Each chromosome,  $\hat{\mathbf{B}}_p, p = 1, \dots, n_p$ , in the population is assigned a “fitness score,”  $\boldsymbol{\mu}(\hat{\mathbf{B}}_p) = \{\mu_1(\hat{\mathbf{B}}_p), \mu_2(\hat{\mathbf{B}}_p), \mu_3(\hat{\mathbf{B}}_p)\}$ .
4. Each chromosome is assigned a series of three “probability of reproduction” values, each of which is proportional to its fitness on the respective element  $\mu_i(\hat{\mathbf{B}}_p), i = 1, 2, 3$ , of  $\boldsymbol{\mu}(\hat{\mathbf{B}}_p)$  relative to the other chromosomes in the population. For example, a particular  $\hat{\mathbf{B}}_p$  that scored relatively high on  $\mu_3(\hat{\mathbf{B}}_p)$ , but low on  $\mu_1(\hat{\mathbf{B}}_p)$  and  $\mu_2(\hat{\mathbf{B}}_p)$ , would be given a corresponding set of one high and two low probabilities of reproduction.
5. According to the assigned probabilities of reproduction, a new population is generated by “mating” a selection drawn according to the probability weights for one of the three elements of  $\boldsymbol{\mu}(\hat{\mathbf{B}}_p)$  with one drawn by similar fashion from another (after removal of the first selected from the pool in the second).
6. The selected pairs of chromosomes generate offspring via the use of specific genetic operators, such as crossover and gene mutation. Crossover is applied to two chromosomes

(parents) and creates two new chromosomes (offspring) by selecting a random position along the coding and then splicing the section that appears before the selected position in the first string with the section that appears after selected position in the second string, and vice versa, e.g.,

$$\begin{array}{ccc}
 \hat{\mathbf{B}}_1 = \begin{bmatrix} B_{11} \\ B_{21} \\ B_{31} \\ B_{41} \\ \text{---} \\ B_{51} \\ B_{61} \end{bmatrix} & \Rightarrow & \hat{\mathbf{B}}_{1a} = \begin{bmatrix} B_{12} \\ B_{22} \\ B_{32} \\ B_{42} \\ \text{---} \\ B_{51} \\ B_{61} \end{bmatrix} \\
 \hat{\mathbf{B}}_2 = \begin{bmatrix} B_{12} \\ B_{22} \\ B_{32} \\ B_{42} \\ \text{---} \\ B_{52} \\ B_{62} \end{bmatrix} & & \hat{\mathbf{B}}_{2a} = \begin{bmatrix} B_{11} \\ B_{21} \\ B_{31} \\ B_{41} \\ \text{---} \\ B_{52} \\ B_{62} \end{bmatrix}
 \end{array}$$

7. The process is halted if a suitable solution is found. Otherwise, the process returns to step 3, where the new chromosomes are scored and the procedure iterates.

The procedure outlined above is suggested as one possible approach to a rather difficult problem, and has yet to be tested. It is advanced in the interest of demonstrating that examples exist that, in concept, complete the requisite components of an activity-based demand analysis procedure.

## 9. Concluding Remarks

The so-called activity-based approach to travel analysis has been on the scene for more than two decades (or considerably longer if it is dated back to the original works of Hagerstrand and his colleagues). Most transportation researchers probably agree with the basic tenets of the

approach – that the need or desire to participate in out-of-home activities drives the corresponding demand for travel, and that the linkages between sets of activities are potentially important to the characteristics of such demand. Yet, most probably because of the inherent overwhelming complexities of treating the “whole” of travel, the approach has not been embraced by the mainstream of transportation researchers as offering a viable practical paradigm for travel demand modeling. There may indeed be wisdom in this judgement in that, despite efforts over the past two decades, activity-based modeling advancement has been relegated largely to either descriptive or proscriptive study, falling well short of being able to be used in the context of actually forecasting changes in travel behavior.

In this paper, we have tried to couch the activity-based approach in terms that are amenable to its development as a framework for travel demand modeling. By showing that a particular mathematical programming paradigm can be used to describe the demand modeling processes both for conventional trip-based travel demand and for activity-based approaches it is hoped not only to facilitate the practicality of activity-based modeling approaches, but also to tap into the wealth of research that has guided mainstream travel demand analysis.

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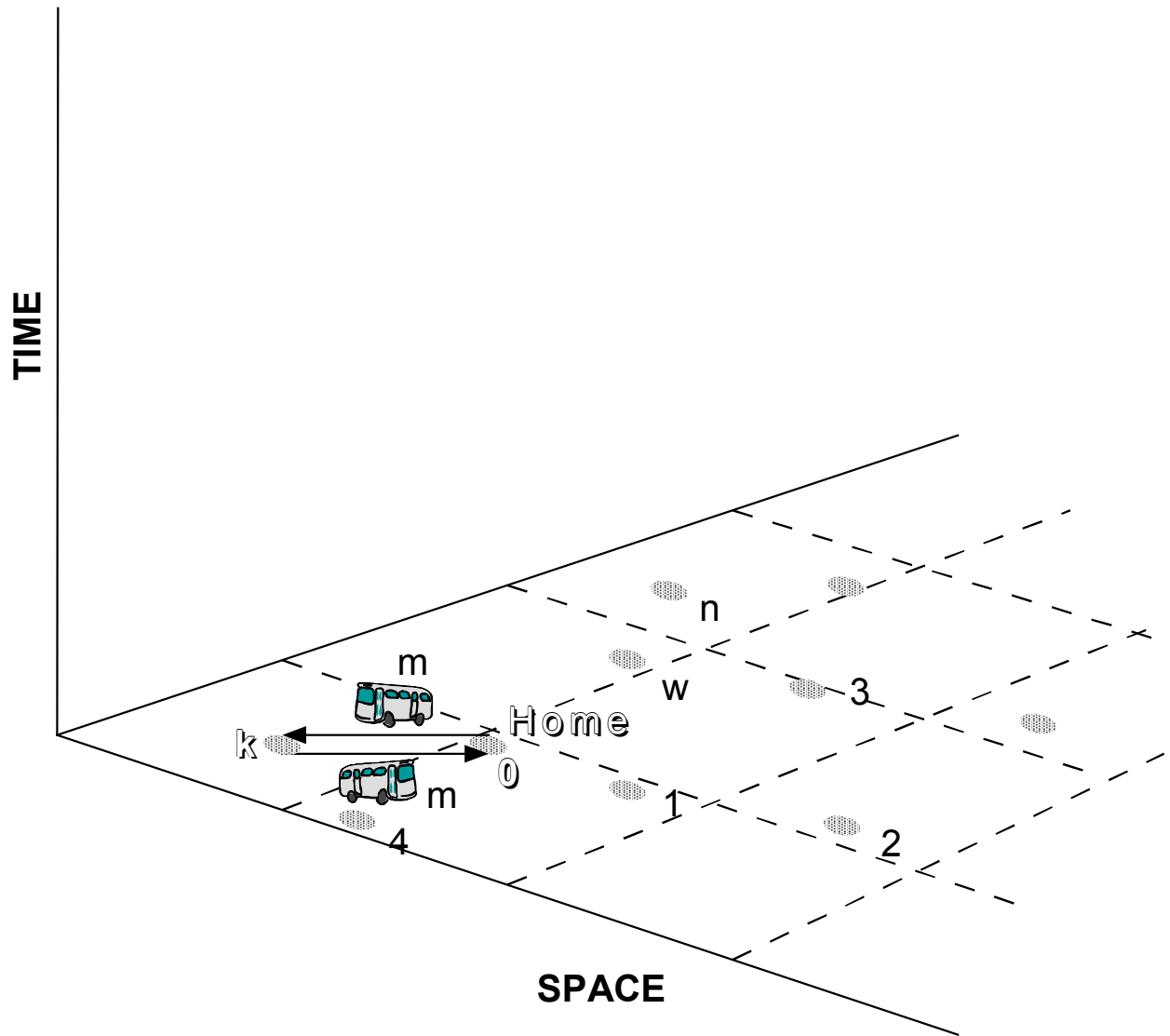


Figure 1. Optimal Space Path for Simple Destination/Mode Choice Example

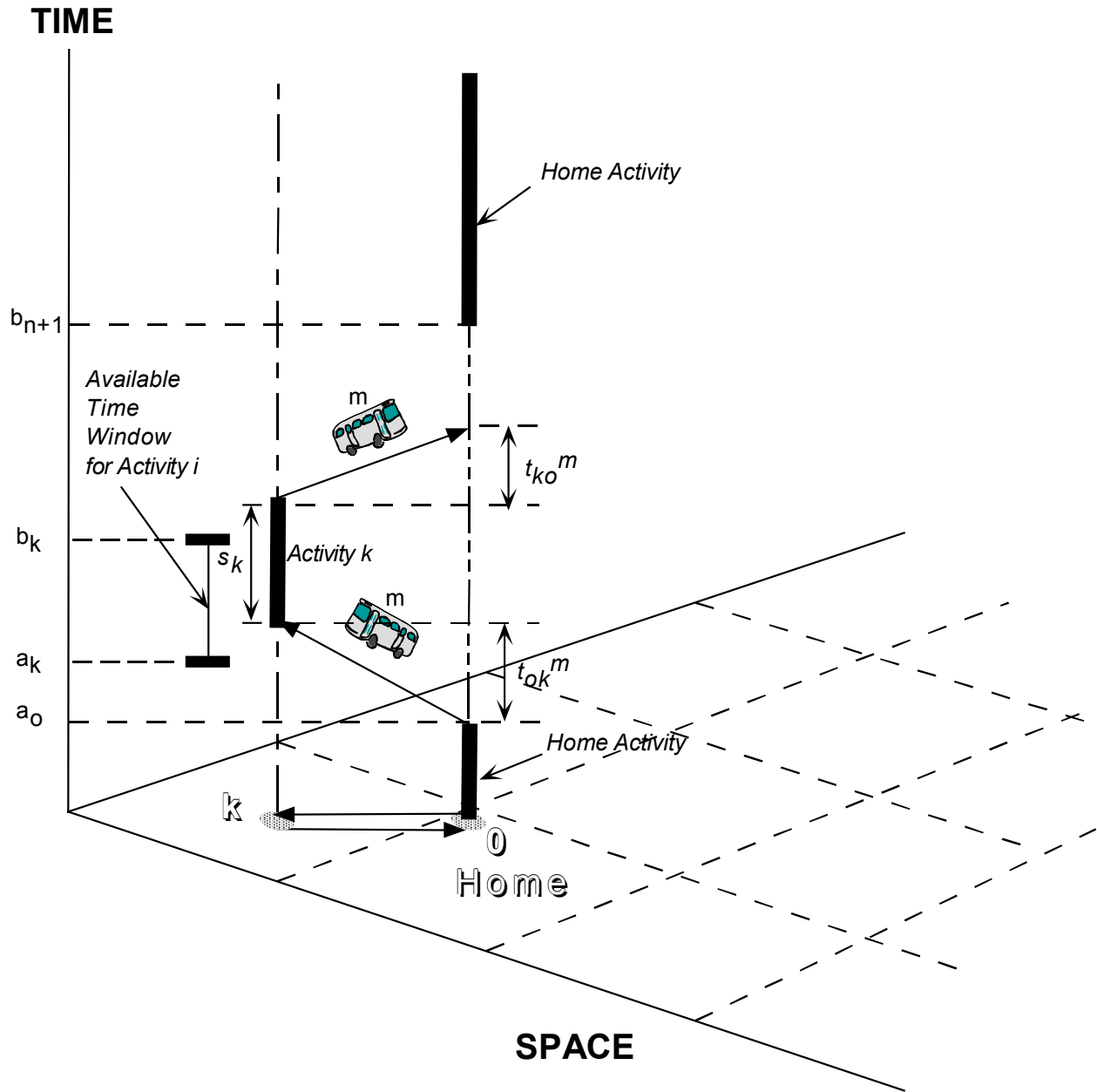
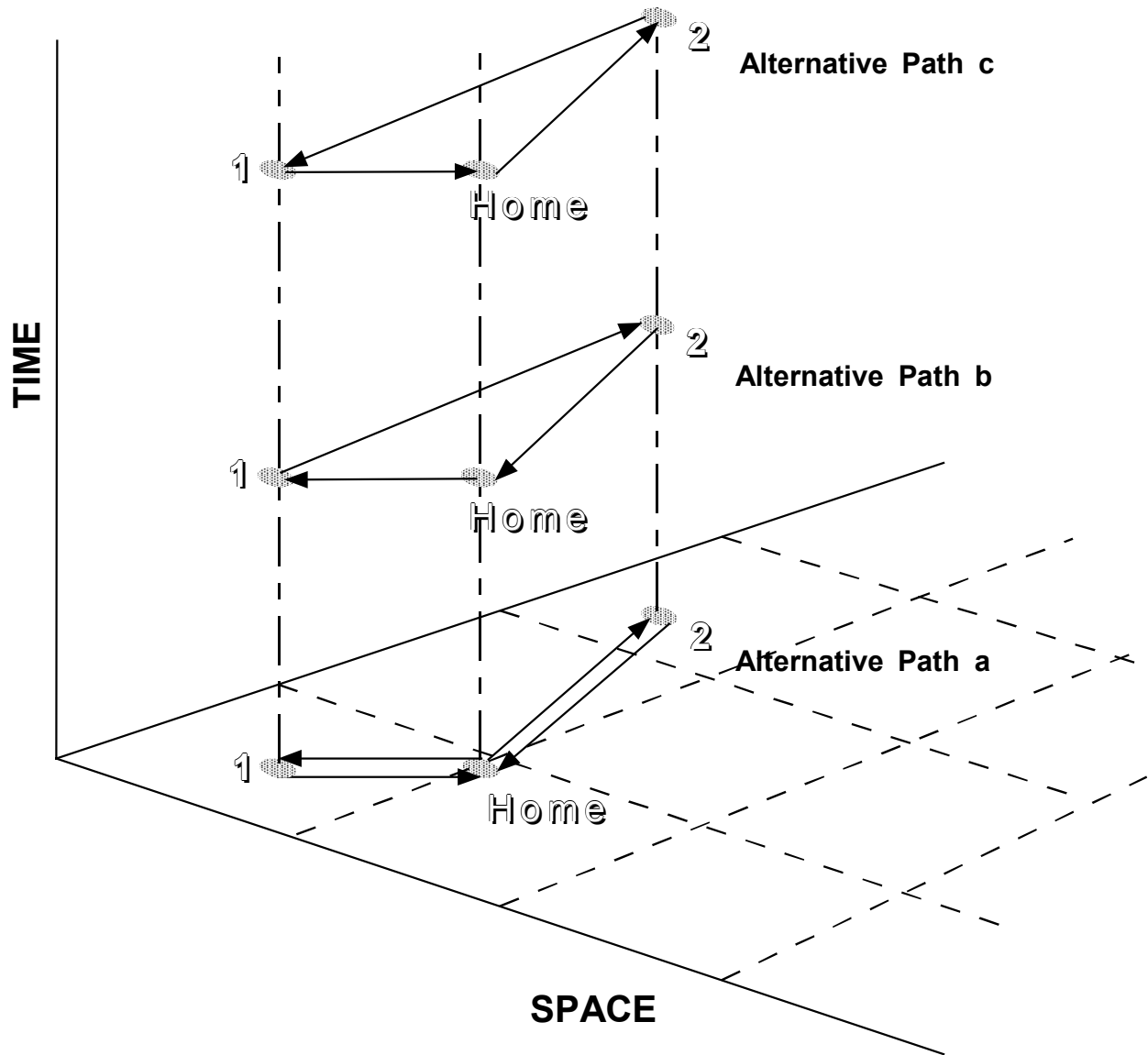
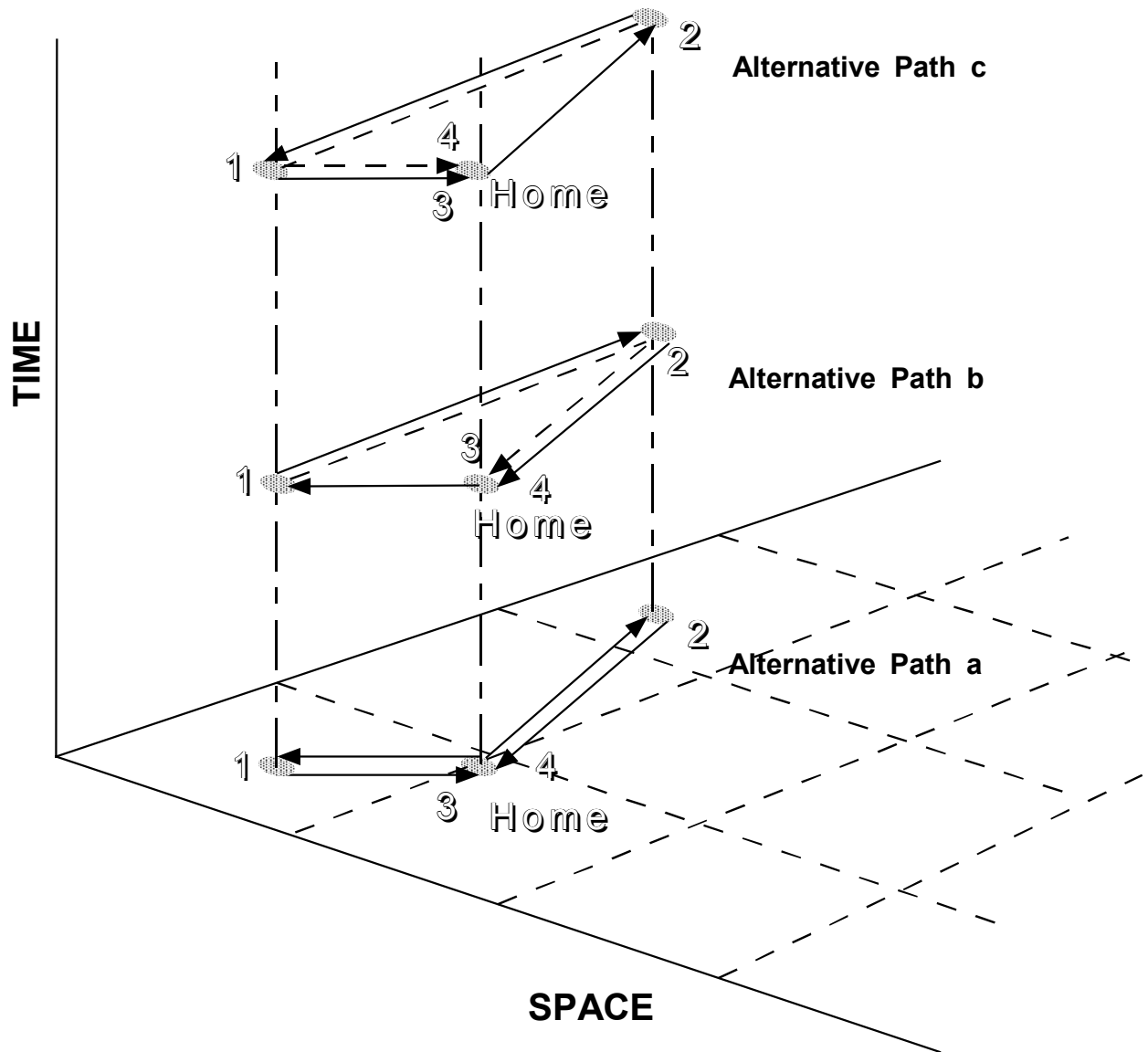


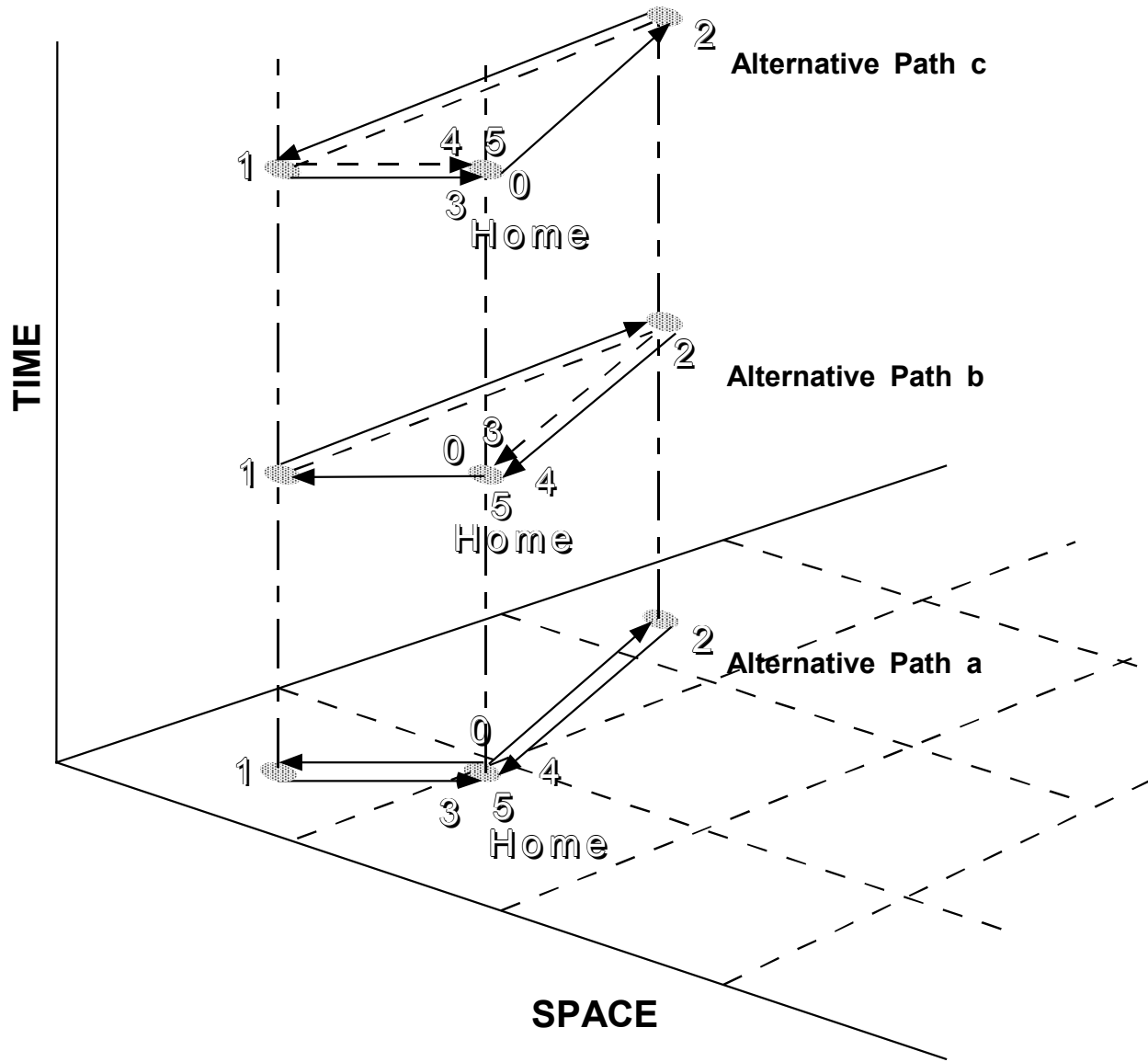
Figure 2. Optimal Space-Time Path for Simple Destination/Mode Choice Example



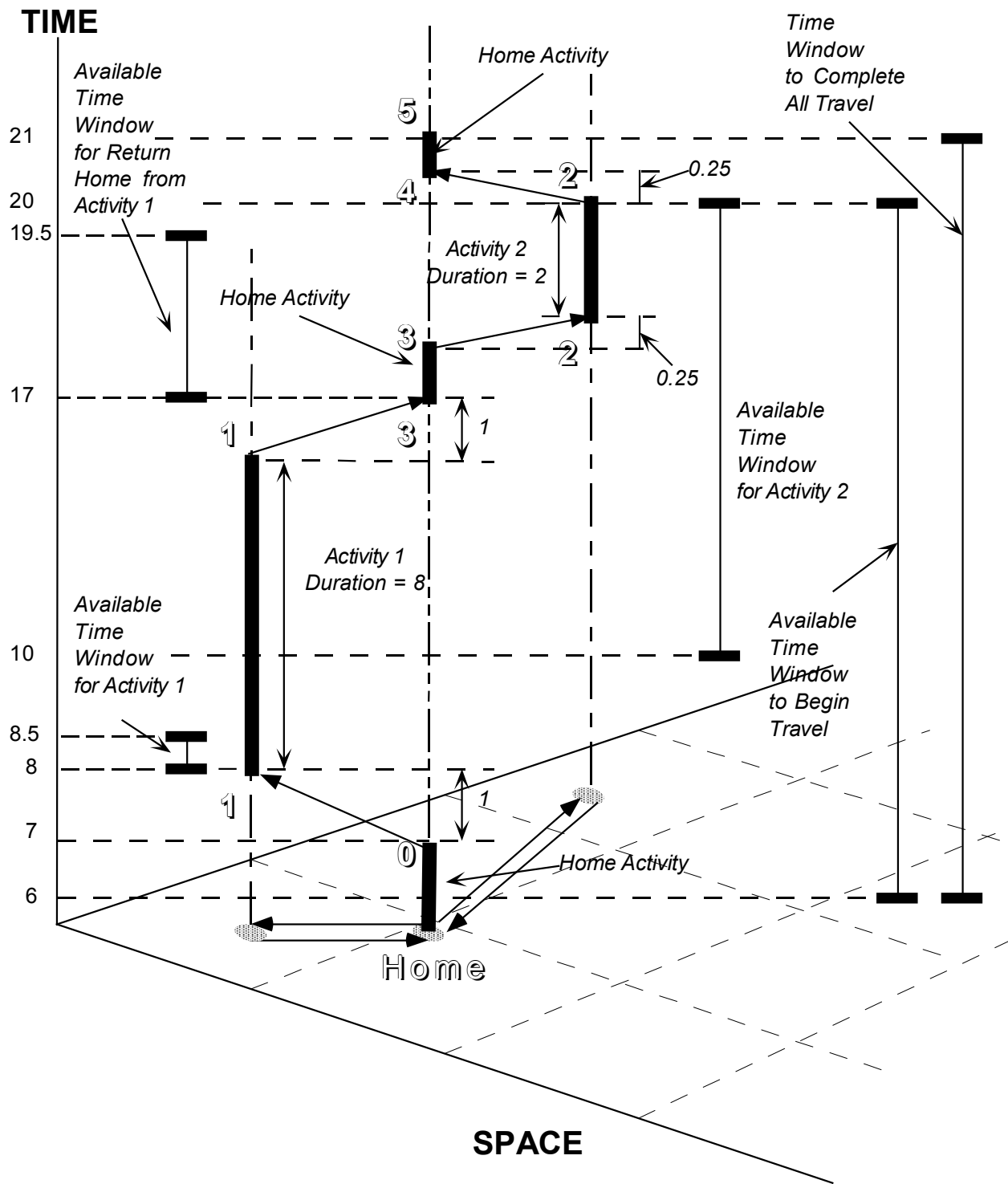
**Figure 3. Spatial Projection of Feasible Space-Time Paths**



**Figure 4. Terminus Labeling for Spatial Projection of Space-Time Paths**



**Figure 5. Expanded Labeling for Spatial Projection of Space-Time Paths**



**Figure 6a. Feasible Space-Time Path for Spatial Projection *a***

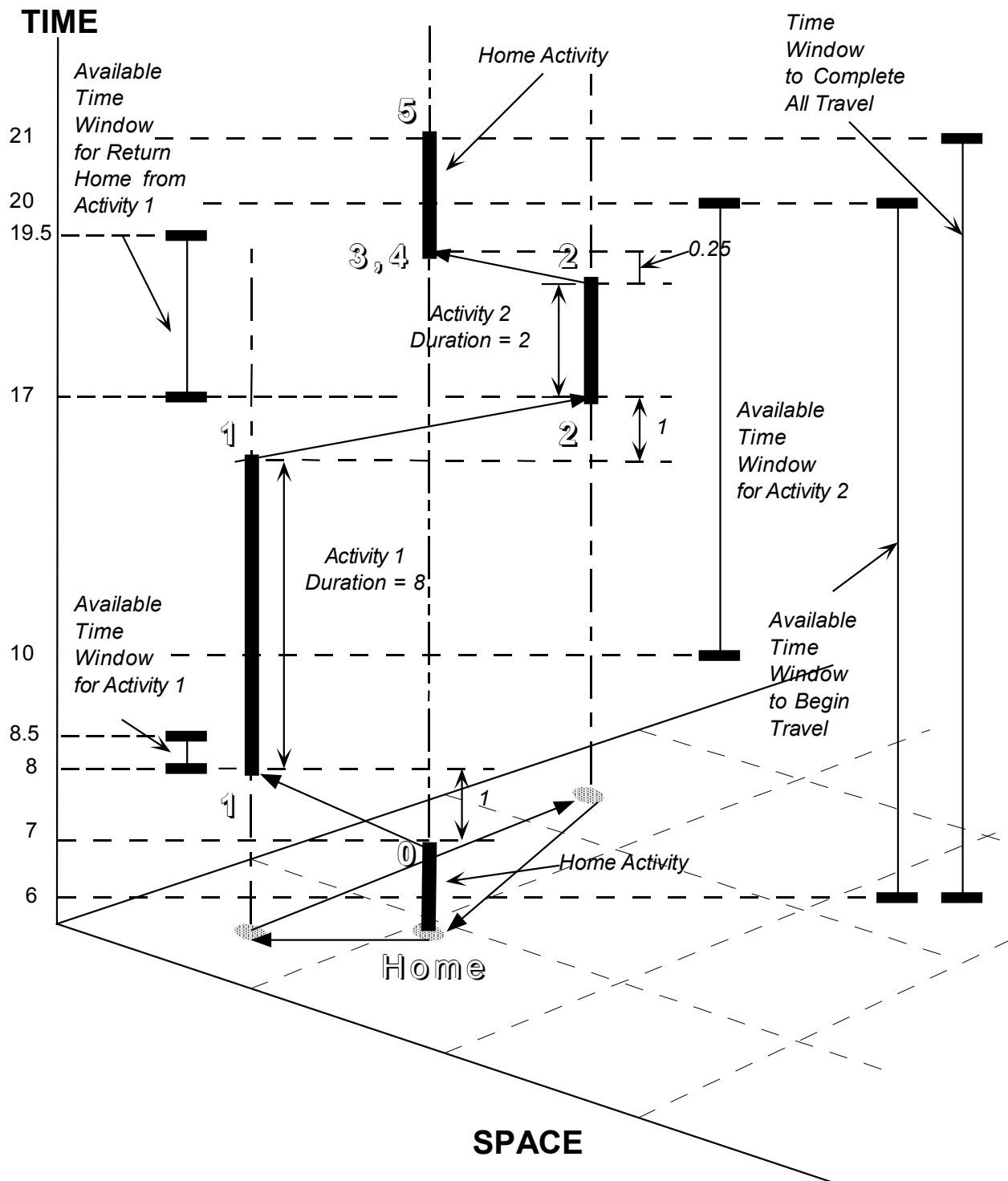


Figure 6b. Feasible Space-Time Path for Spatial Projection *b*



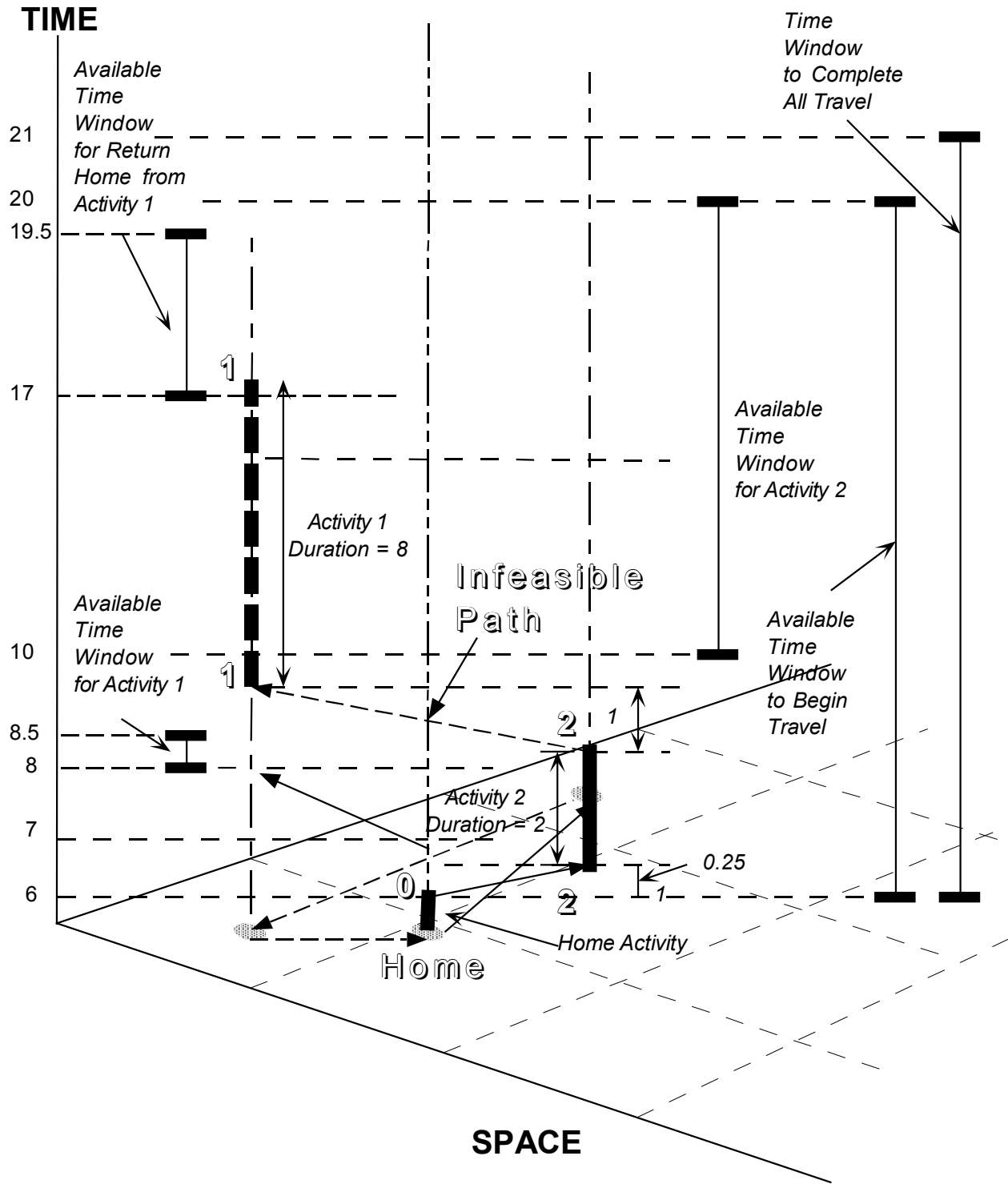


Figure 6c. Infeasible Space-Time Path for Spatial Projection c