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THE COORDINATED VOLUNTEER'S DILEMMA[†]

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ABSTRACT. In the Volunteer's Dilemma, the effort of one player can benefit an entire group, but all who volunteer pay a cost. A well-known result due to Diekmann (1985) informs us that in equilibrium, larger groups are *less likely* to find even one volunteer in this setting, but players are equally well-off in any size group. We study a coordinated version of the Volunteer's Dilemma where one of the volunteers is randomly chosen to bear the cost. Although coordination increases both the likelihood of at least one volunteer and the welfare of players, the probability there is at least one volunteer still decreases in group size. However, in a coordinated Volunteer's Dilemma, players' welfare increases with group size.

Keywords: Volunteer's Dilemma; Social Dilemmas; Group Size

JEL Classification: C72, C70

"... every one is more negligent of what another is to see to, as well as himself, than of his own private business; as in a family one is often worse served by many servants than by a few." *Aristotle's Politics* Book II, Chapter 3

1. INTRODUCTION: THE VOLUNTEER'S DILEMMA

For many tasks, the efforts of a single individual suffice to serve an entire group. One sentinel can alert the entire community. One group member can investigate and report on the honesty of a vendor or the usefulness of a consumer good. Thus we might expect clustering into groups to be advantageous.

But, as the epigraph from Aristotle suggests, there is a countervailing force. The returns to scale offered by larger groups may be offset by free-rider problems that increase with group size. Aristotle makes two assertions, the first is that as group size increases, each member is less likely to act in the public interest. The second is the stronger claim that *total* effort decreases with group size.

The forces that underlie Aristotle's claims are nicely captured in a simple n -player game introduced by the sociologist, Andreas Diekmann, who called this game the *Volunteer's Dilemma* (Diekmann, 1985). In Diekmann's Volunteer's Dilemma game as originally presented, players have identical payoff functions and move simultaneously. Each player has the option of taking a costly action, *volunteer*, or of choosing to *not volunteer*. If at least one player chooses to volunteer, then all players receive a benefit b . Those who volunteer pay cost c and thus have net benefits $b - c > 0$ while those who choose to not volunteer pay nothing and receive net benefit b . If no player volunteers, then all get a payoff of 0.

In this game, with two or more players, there can not be a Nash equilibrium where everyone volunteers. If everyone volunteers, any player is better off not volunteering. Nor can there be an equilibrium where no one volunteers. If no one volunteers, a player is better off volunteering than

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not. Diekmann shows that for a group of size n , there is a unique symmetric mixed-strategy Nash equilibrium with the properties listed in Proposition 1.¹

Proposition 1 (Volunteer’s Dilemma). The uncoordinated Volunteer’s Dilemma has a unique symmetric Nash equilibrium. In this equilibrium, as the number of players increases:

- 1.1 The probability that each player volunteers decreases.
- 1.2 The probability that at least one player volunteers decreases.
- 1.3 Where $\rho = c/b$, the probability no player volunteers asymptotically approaches ρ .
- 1.4 The expected utility of each player remains constant and equal to $b - c$.

Diekmann’s result 1.1, that the probability that each player volunteers decreases with the number of players, echoes Aristotle’s assertion that “everyone is more negligent of what another is to see to, as well as himself...” Result 1.2, that the probability that no player volunteers increases with the number of players, affirms Aristotle’s stronger claim that “one is often worse served by many servants than a few.” Aristotle’s proclamations concerned the well-being of the served, but not that of the servants. Result 1.4 informs us that the servants are neither worse off nor better off as their number increases.

2. THE COORDINATED VOLUNTEER’S DILEMMA GAME

In Diekmann’s Volunteer’s Dilemma, every volunteer must bear the cost of volunteering, even though only one volunteer is needed. Sometimes the efforts of volunteers can be managed more efficiently. An organization may be able to solicit volunteers and then choose just one volunteer at random to perform the task. In this case, if there are k volunteers and c is the cost of performing the task, the expected cost to each volunteer is c/k and the total cost to all volunteers is c . A similar result is achieved when the work is shared among volunteers in such a way that with k volunteers, the total cost of the task is divided equally among them. In this case, the cost of volunteering when there are k volunteers is also c/k .²

The world has many examples of coordinated Volunteer’s Dilemmas, among humans and among other animals. An academic department that needs a single faculty member to serve on a university committee can select a single volunteer to perform that duty. The *World Marrow Donor Program* (WMDP) is a registry of persons who avow their willingness to donate stem cells to a leukemia patient. For patients with common immunity types, there are likely to be many eligible volunteers in the registry. The WMDP *chooses just one* of the matching registrants to make the donation (Bergstrom et al., 2009).

Archetti (2009) observed that some animal species living in groups manage to “efficiently coordinate their vigilance against predators into a sentinel system, alternating bouts of watchfulness with little overlap.” Among the species for which this coordination has been observed are Florida scrub jays (McGowan and Woolfenden, 1989), Arabian babblers (Wright et al., 2001), and meerkats (Clutton-Brock et al., 1999). These animal games are discussed in more detail in Section 4.

Projects for which each volunteer’s burden declines with the number of volunteers are also common. A recruiting committee may divide the applications to be reviewed among all the volunteers. A group of volunteers for a beach cleanup can share the task in such a way that the effort required by each volunteer is inversely proportional to the number of volunteers.

In this section we present a formal model of a coordinated Volunteer’s Dilemma and explore its properties. We define an n player *coordinated Volunteer’s Dilemma* as follows. There is a task to be performed and n players. Each player has two possible pure strategies: *Volunteer* and *Not Volunteer*. If no player volunteers, the task is not done, and all players receive a payoff of 0. If $k > 0$ players

¹We supply proofs of Diekmann’s propositions in the Appendix.

²These games are formally equivalent under risk-neutrality of all players.

volunteer, the task is completed, and all players receive a benefit b . Each volunteer pays an expected cost of c/k .

Proposition 2 presents properties of the symmetric equilibrium of a coordinated Volunteer's Dilemma. These properties parallel those in Proposition 1 for an uncoordinated game.

Proposition 2 (Coordinated Volunteer's Dilemma). The coordinated Volunteer's Dilemma has a unique symmetric Nash equilibrium. In this equilibrium, as the number of players increases:

- 2.1 The probability that each player volunteers decreases.
- 2.2 The probability that at least one player volunteers decreases.
- 2.3 Where $\rho = c/b$, the probability that no player volunteers asymptotically approaches $\frac{-\rho}{W_{-1}(-\rho e^{-\rho})}$, where $W_{-1}(\cdot)$ is the lower branch of the Lambert W function.
- 2.4 The expected utility of each player increases.

Weesie and Franzen (1998) were the first to publish statements of Properties 2.1 and 2.2.³ The limiting value in Proposition 2.3 is stated in terms of the Lambert W function, which is the multi-valued inverse of the function $f(x) = xe^x$.⁴ Proposition 2.4 informs us that in coordinated Volunteer's Dilemma, expected utility of all players increases with the number of players. This result, as far as we know, has not been previously published. The proof of Proposition 2.4, which is found in the Appendix employs a remarkably useful result, first proved by Bennett and Jameson (2000), who showed that the average value of a convex function at n equally spaced points between two fixed endpoints decreases with n .

Proposition 2.1 informs us that Aristotle's remark that "every one is more negligent of what another is to see to (with) many servants rather than a few" applies to coordinated as well as uncoordinated Volunteer's Dilemmas. Perhaps surprisingly, according to Proposition 2.2, even when coordination allows selecting a single volunteer to complete the task, the symmetric equilibrium probability that the task will be performed decreases with the number of potential helpers. Thus Aristotle's dictum that "one is often worse served by many servants than by a few" applies although coordination prevents duplicative effort. On the other hand, we learn from Proposition 2.4, that in equilibrium for the coordinated Volunteer's Dilemma, the expected utility of all potential volunteers increases with their number. This contrasts with the result of Proposition 1.4, which tells us that in uncoordinated Volunteer's Dilemma the equilibrium expected utility of players is constant across group sizes.

The contrasting results of Propositions 2.2 and 2.4 raise the question *cui bono*. In the example of Aristotle and his servants, Aristotle finds that the task he wishes performed is less likely to be completed, the more servants that he has. On the other hand, the equilibrium utility of the servants increases with the number of servants. More generally, Proposition 2 suggests that, when faced with coordinated Volunteer's Dilemmas, employers and employees confront a tradeoff. Adding workers increases the utility of workers, but reduces the probability that the tasks which the employer wants performed get done.

With the penguins on an ice floe or the meerkats or birds using a sentry, the only agents whose utility is affected by actions of the potential volunteers are the potential volunteers themselves. In this case, Proposition 2.4 informs us that all relevant players in a coordinated Volunteer's Dilemma gain from belonging to a larger group despite the fact that the probability of no action increases with group size. Of course there is a trade-off between the economies of scale that appear in the coordinated Volunteer's Dilemma and other effects of group size, such as congestion or competition for limited resources.

³The journal's typesetters unfortunately failed to include mathematical notations in the printed proofs of these results, leaving them almost unintelligible (Jeroen Weesie, private correspondence). This paper offers proofs that take a different tack from theirs and which we hope will be easier to follow.

⁴Properties and a review of the Lambert W can be found in Corless et al. (1996). Weesie and Franzen state a limiting result in terms of the Riemann omega function, which is less easily interpreted.

3. COMPARING COORDINATED AND UNCOORDINATED VOLUNTEER'S DILEMMAS

Propositions 1 and 2 inform us about the effects of group size on the symmetric Nash equilibrium of the coordinated and uncoordinated Volunteer's Dilemmas respectively. Let us now consider the difference between the coordinated and uncoordinated games.

A mechanism that selects a single individual from among the volunteers yields social benefits for the obvious reason that it reduces unneeded duplication of costly efforts. If coordination does not diminish the equilibrium probability that there is no volunteer, then the coordinated game provides a higher expected utility to all players than the uncoordinated game. Proposition 3, which we prove in the Appendix, assures us that this is the case.⁵

Proposition 3. For any fixed $n > 1$ and $\rho = c/b$, in the symmetric Nash equilibria of the coordinated and uncoordinated Volunteer's Dilemmas:

- 3.1 The probability that each player volunteers is higher under coordination.
- 3.2 The probability that at least one player volunteers is higher under coordination.
- 3.3 The expected utility of each player is higher under coordination.

We can quantify the improvement by calculating the difference in expected utility of players in a coordinated and uncoordinated game with fixed parameter values. Figure I displays the improvement in per capita expected utility that arises from coordination in comparison to the uncoordinated Volunteer's Dilemma (in the symmetric Nash equilibria of both games) as a function of group size n and the ratio $\rho = c/b$ of cost to benefits.

	$n = 2$	$n = 3$	$n = 5$	$n = 10$	$n = 100$
$\rho = .10$	0.05	0.06	0.07	0.07	0.07
$\rho = .25$	0.11	0.13	0.14	0.15	0.15
$\rho = .50$	0.17	0.19	0.20	0.21	0.21
$\rho = .75$	0.15	0.16	0.17	0.17	0.17
$\rho = .90$	0.08	0.08	0.09	0.09	0.09

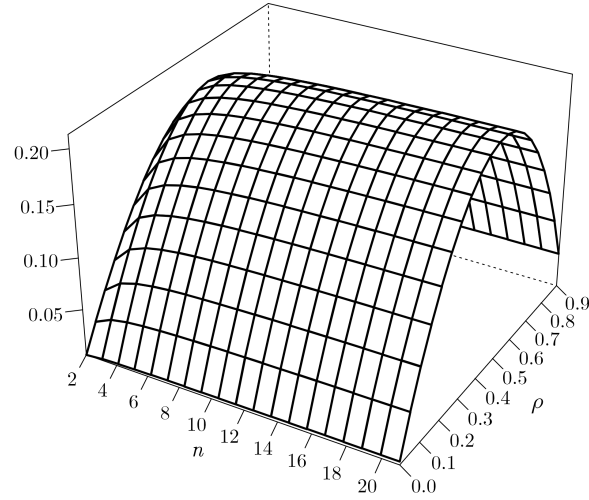


FIGURE I. Table and plot of expected difference in per capita net benefit (vertical axis on plot) between coordinated and uncoordinated Volunteer's Dilemma as a fraction of individual benefit b for alternative cost-benefit ratio values $\rho = c/b$ and group sizes n .

From this table, we note that the per capita gain as a fraction of the benefit b is increasing in n . This is expected since the expected utility in the uncoordinated game is invariant with n but is increasing in n with coordination (Proposition 2.4). While the effect of group size is relatively small, improvement offered by coordination is more dependent on the cost-benefit ratio ρ . Improvement is concave in ρ ; reaching a maximum around $\rho = 0.5$ and shrinking to zero at the endpoints. This is because as $\rho \rightarrow 1$, coordination does not provide any improvement since the probability that players

⁵Propositions 3.1 and 3.2 were previously stated by Weesie and Franzen (1998), though without formal proof.

volunteer converges to zero in both games. In this case, coordination reduces cost by a factor of n . As $\rho \rightarrow 0$ any cost savings become infinitesimally small relative to the benefit of having that task completed with certainty. Thus, the utility improvement as a fraction of the benefit b converges to zero there as well.

Since the improvement offered by coordination is increasing in n , we can bound the improvement by looking at the limit as $n \rightarrow \infty$. This maximum cannot be written in closed form but can be found numerically and occurs at $\rho^* \approx 0.532$ where the improvement is capped at approximately 0.216.⁶ This corresponds to an approximate 21.6% improvement in the chance the task is completed.

4. APPLICATIONS AND RELATED STUDIES OF VOLUNTARY BEHAVIOR

Eshel and Shapira (2001), study a class of Volunteer's Dilemmas in the animal kingdom, which they motivate by an example from "The Selfish Gene" by Richard Dawkins (1976).

"Perhaps we can sympathize more directly with the reported cowardly behaviour of emperor penguins in the Antarctic. They have been seen standing on the brink of the water hesitating before diving in, because of the danger of being eaten by seals. If only one of them would dive in, the rest would know whether there was a seal there or not. Naturally nobody wants to be the guinea-pig, so they wait, and sometimes even try to push each other in."

Since all of the penguins on an ice floe can observe whether someone has jumped in and can then readily determine whether a seal is present, they are participating in a coordinated Volunteer's Dilemma.

Meerkats are desert-adapted mongooses who live in arid areas of southern Africa in groups of 3 to 30 individuals. According to Clutton-Brock et al. (1999), they forage by digging in the ground. While digging, they are unable to detect predators. During much of the time when the group is foraging, a single meerkat stands upright on a raised surface, acting as a sentinel. If a predator appears, the sentinel alerts other group members by repeated alarm calls. Clutton-Brock and coauthors report that in most groups, all adults spend some time guarding, though individuals vary in their contributions. Evidence provided by Clutton-Brock and coauthors suggests that the "game" played by meerkat groups has much in common with the coordinated Volunteer's Dilemma. They found that when a second animal went on guard while another was already guarding, one of them usually stopped guarding immediately. They observed that the groups did not maintain a constant order of rotation. They report that "our results provide no indication that the alternation of raised guarding depends on social processes more complex than independent optimization of activity by individuals". Perhaps unsurprisingly, they find that the percentage of time that each individual spent as a raised guard decreases with group size. More curiously, and in line with Proposition 2.2, the percentage of time that *any* individual was on raised guard also decreased with group size.

Babcock et al. (2017) study the difference between the willingness of males and of females to volunteer to do "unpromotable tasks" in university and corporate environments. They support their arguments with an experiment in which subjects face a coordinated Volunteer's Dilemma. They

⁶According to Proposition 1.4, the expected utility in the uncoordinated game is always $b - c = b(1 - \rho)$. In equilibrium of the coordinated game, the expected utility must be equal to the utility of not volunteering which is the benefit b times the probability that at least one out of the $n - 1$ other players volunteers. As $n \rightarrow \infty$, this converges to the probability that there is at least one volunteer. Thus, using the limiting probability in Proposition 2.3, the limit of the expected utility in the coordinated game can be written as: $b \left(1 - \frac{-\rho}{W_{-1}(-\rho e^{-\rho})}\right)$. As a fraction of b , the limiting improvement from coordination can be written as $\rho - \frac{-\rho}{W_{-1}(-\rho e^{-\rho})}$. While the maximum of this function can be found analytically using the properties of the derivative of the Lambert-W function, the resulting solution cannot be written in closed form (even using Lambert-W itself).

conducted treatments with mixed gender, all-female, and all-male groups. They found that in mixed-gender groups, females volunteered significantly more frequently than males, while in single-sex groups, there was no significant difference between the proportions of men and women volunteering.

Battaglini and Palfrey (2025) study player welfare in an uncoordinated Volunteer’s Dilemma with private (random) costs. Recall that, by Proposition 1.4 the expected utility of players is constant in group size for the (traditional) uncoordinated Volunteer’s Dilemma. In contrast, they demonstrate that, with random costs, the expected utility of each player *increases* with the number of players regardless of individual cost. This complements our Proposition 2.4, which shows that expected utility is also increasing in the number of players for the coordinated Volunteer’s Dilemma.

Battaglini and Palfrey (2025) also show that the total inefficiency relative to first-best is unbounded in group size, driven by under-provision. This is consistent with our Proposition 2.3, which shows that the probability of non-provision has a non-zero limiting value in the coordinated Volunteer’s Dilemma, driving unbounded loss relative to the first-best of having at least one volunteer.

Leo and Pate (2025) study an extension of the uncoordinated Volunteer’s Dilemma where more than one volunteer is needed to provide a public good. They use this game to study a stylized model of an administrator who relies on multiple “complaints” to know what issues to address. They demonstrate that the most efficient complaint policies will keep the effort-cost of complaining low while requiring a large number of complaints before taking action. Although they do not study comparative statics of the game in terms of the number of players as in our Proposition 2.3, they also employ the Lambert W function to study the limiting properties of the game for a large number of players.

Bergstrom (2017b) examines a dynamic Volunteer’s Dilemma game, in which passing drivers observe a vehicle in distress by the roadside. In this case, it is relatively easy for the passers-by to determine whether someone else is already helping. The costs of stopping are assumed to differ among drivers. The benefit that a sympathetic driver confers by stopping to help is influenced by traffic density. The more frequent is traffic, the shorter the amount of time that the unfortunate motorist will have to wait until someone else helps. The model suggests that on less-traveled roads, passers-by are more likely to help, but since there is more time between potential helpers, the expected waiting time for a distressed motorist may be longer.

Bergstrom (2017a) considers the recommended behavior in an uncoordinated Volunteer’s Dilemma for someone guided by a Kantian ethic of the form: “Use a strategy that you would like best for everyone to employ.” Neither of the pure strategies, *Volunteer* nor *Don’t Volunteer* would satisfy this criterion. A Kantian solution is an outcome in which all participants use the symmetric mixed strategy that yields the highest payoff to each participant. Bergstrom (2017a) shows that in a Kantian ethical solution for an uncoordinated Volunteer’s Dilemma, with n players, where the ratio of costs to benefits is $\rho = c/b$, the probability that at least one player volunteers is $1 - \left(\frac{\rho}{n}\right)^{\frac{n}{n-1}}$. This is non-monotonic in group size, but eventually increases and converges to 1. This contrasts with Propositions 1.2 and 2.2 which show that, for self-interested players, the probability at least one player volunteers decreases with group size in both the uncoordinated and coordinated game.

Several papers study an alternative form of coordination that can be achieved when the Volunteer’s Dilemma is repeated. In these instances, players can take turns playing the efficient asymmetric equilibria where only one player volunteers. Riyanto and Roy (2019); Przepiorka et al. (2022); Kloosterman and Mago (2023) find that players are able to coordinate on turn-taking in repeated Volunteer’s Dilemma games in the lab primarily when costs are symmetric. Leo (2017) studies how a form of flexible turn-taking mechanism can improve efficiency over rote turn-taking in a repeated Volunteer’s Dilemma where costs are private and drawn independently across rounds.

Amir et al. (2026) study the equilibrium of the repeated coordinated Volunteer’s Dilemma and show that, while all pure-strategy equilibria in a single-shot game involve a single volunteer, equilibria with any number of volunteers can arise in repeated settings. Interestingly, the discount factor threshold needed to support an equilibrium is non-monotonic in the number of volunteers and coordination on the two-volunteer equilibrium requires the highest discount factor.

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APPENDIX A. MATHEMATICAL PROOFS

A.1. Proof of Proposition 1

Proposition 1 (Volunteer's Dilemma) The uncoordinated Volunteer's Dilemma has a unique symmetric Nash equilibrium. In this equilibrium, as the number of players increases:

- 1.1 The probability that each player volunteers decreases.
- 1.2 The probability that at least one player volunteers decreases.
- 1.3 Where $\rho = c/b$, the probability no player volunteers asymptotically approaches ρ .
- 1.4 The expected utility of each player remains constant and equal to $b - c$.

Proof. In a symmetric Nash equilibrium with n players, let q be the probability that a player does not volunteer. For any player, the probability that there are no other volunteers is $1 - q^{n-1}$. Therefore the payoff to a player who does not volunteer is therefore $b(1 - q^{n-1})$, while the payoff to a player who volunteers is $b - c$. Therefore in a symmetric mixed strategy Nash equilibrium, it must be that

$$b(1 - q^{n-1}) = b - c. \quad (1)$$

and hence

$$q = \left(\frac{c}{b}\right)^{\frac{1}{n-1}}. \quad (2)$$

The probability that no one volunteers is then

$$q^n = \left(\frac{c}{b}\right)^{\frac{n}{n-1}}. \quad (3)$$

Where $\frac{c}{b} < 1$ and $n > 1$, the expression on the right side of (2) is an increasing function of n . Hence q increases and the probability $p = 1 - q$ decreases as n increases (*Result 1.1*). The expression on the right side of (3) is also an increasing function of n . Hence the probability q^n that no one volunteers increases with n and so $1 - q^n$, the probability some player volunteers decreases with n (*Result 1.2*). It is straightforward to verify that $1 - q^n = 1 - \left(\frac{c}{b}\right)^{\frac{n}{n-1}}$ is decreasing and asymptotically approaches $1 - \frac{c}{b}$ from above (*Result 1.3*). In a symmetric equilibrium, all players must be indifferent between volunteering and not volunteering. Since the payoff to volunteering is $b - c$, the expected utility of each player is $b - c$ for all values of n (*Result 1.4*). □

A.2. Proof of Proposition 2

Proposition 2. (Coordinated Volunteer's Dilemma). The coordinated Volunteer's Dilemma has a unique symmetric Nash equilibrium. In this equilibrium, as the number of players increases:

- 2.1 The probability that each player volunteers decreases.
- 2.2 The probability that at least one player volunteers decreases.
- 2.3 Where $\rho = c/b$, the probability that no player volunteers asymptotically approaches $\frac{-\rho}{W_{-1}(-\rho e^{-\rho})}$, where $W_{-1}(\cdot)$ is the lower branch of the Lambert W function.
- 2.4 The expected utility of each player increases.

Our proof of the theorem employs the following definitions and lemmas.

Definition 1. Let $\rho = \frac{c}{b}$ denote the ratio of the cost c of doing the task to the benefit b that each player receives if the task is done.

Definition 2. Let $q(n, \rho)$ be the probability that a player will not volunteer in the symmetric Nash equilibrium when there are n players and the ratio of cost to benefits is ρ .

Definition 3. Let $Q(n, \rho) = q(n, \rho)^n$ denote the probability that nobody performs the task in symmetric equilibrium with n players and ratio ρ of cost to benefits.

Lemma 1. In a symmetric mixed-strategy Nash equilibrium for a coordinated volunteers' dilemma game with $\rho = \frac{c}{b}$, it must be that $q(n, \rho) = q$ where q satisfies the equation $\rho = H(n, q) \equiv q^{n-1} \frac{1}{A(n, q)}$ where $A(n, q) \equiv \frac{\sum_{r=0}^{n-1} q^r}{n}$.

Proof of Lemma 1. In a symmetric mixed-strategy Nash equilibrium, where players volunteer with probability $1 - q$, the expected payoff for this mixed strategy is the same as that for the pure strategy *Not Volunteer*. The probability that there is at least one volunteer is $1 - q^n$. Thus each player has expected benefit $(1 - q^n)b$. If there is at least one volunteer, the selected volunteer is equally likely to be any of the n players. The expected cost to a player who volunteers with probability $1 - q$ is $\frac{1}{n}(1 - q^n)c$. Therefore if each player volunteers with probability q , the expected payoff to each of them is

$$(1 - q^n)b - \frac{1 - q^n}{n}c. \quad (4)$$

If all other players volunteer with probability $1 - q$, then the expected payoff to *not Volunteer* is $(1 - q^{n-1})b$.

It follows that in a symmetric Nash equilibrium,

$$(1 - q^n)b - \frac{1 - q^n}{n}c = (1 - q^{n-1})b. \quad (5)$$

(5) implies that

$$(1 - q^n) - (1 - q^{n-1}) = \frac{1 - q^n}{n} \frac{c}{b}, \quad (6)$$

and hence

$$q^{n-1}(1 - q) = \frac{1 - q^n}{n} \frac{c}{b}. \quad (7)$$

Rearranging terms of (6), we see that (5) is equivalent to

$$nq^{n-1} \frac{1 - q}{1 - q^n} = \frac{c}{b}. \quad (8)$$

Since

$$\begin{aligned} \frac{1 - q^n}{1 - q} &= 1 + q + q^2 + \dots + q^{n-1} \\ &= \sum_{r=0}^{n-1} q^r, \end{aligned} \quad (9)$$

therefore (8) is equivalent to

$$\begin{aligned} \rho &= nq^{n-1} \frac{1 - q}{1 - q^n} \\ &= n(1 - q) \frac{q^{n-1}}{1 - q^n} \\ &= q^{n-1} \frac{n}{\sum_{r=0}^{n-1} q^r} \\ &= q^{n-1} \frac{1}{A(n, q)} \\ &= H(n, q). \end{aligned} \quad (10)$$

□

Lemma 2. The function $H(n, q)$ has the following properties:

- $H(n, q)$ is continuous and strictly increasing in q .
- $H(n, 0) = 0$ and $\lim_{q \rightarrow 1} H(n, q) = 1$.
- $H(n, q)$ is strictly decreasing in n .

Proof of Lemma 2. The function $H(n, q)$ is the ratio of a continuous function of q to a sum of continuous functions and is thus itself continuous. From (10)

$$\begin{aligned} H(n, q) &= \frac{nq^{n-1}}{1 + q + q^2 + \dots + q^{n-1}} \\ &= \frac{n}{q^{1-n} + q^{2-n} + \dots + 1}. \end{aligned} \quad (11)$$

The denominator of this expression is seen to be a decreasing function of q and thus $H(n, q)$ is an increasing function of q for all positive integers n .

It is straightforward to verify that $H(n, 0) = 0$ and $\lim_{q \rightarrow 1} H(n, q) = 1$.

To establish that $H(n, q)$ is strictly decreasing in n , note that (11) implies that

$$\frac{n}{H(n, q)} = q^{1-n} + q^{2-n} + \dots + 1, \quad (12)$$

and

$$\frac{n+1}{H(n+1, q)} = q^{-n} + q^{1-n} + \dots + 1. \quad (13)$$

It follows from (12) and (13) that

$$\frac{n+1}{H(n+1, q)} = \frac{n}{H(n, q)} + q^{-n}. \quad (14)$$

(14) implies that

$$\begin{aligned} n \left(\frac{1}{H(n+1, q)} - \frac{1}{H(n, q)} \right) &= q^{-n} - \frac{1}{H(n+1, q)} \\ &= q^{-n} - \frac{q^{-n} + q^{1-n} + q^{2-n} + \dots + 1}{n} \\ &> 0. \end{aligned} \quad (15)$$

where the final inequality in (15) follows from the fact that when $0 < q < 1$, it must be that $q^{-n} > q^{i-n}$ for all $i > 0$. The inequality in (15) implies that $H(n+1, q) < H(n, q)$. Thus, $H(n, q)$ is strictly decreasing in n . $\square$

Lemma 3, which is used to prove Proposition 2.4, is a slight modification of a result found by Bennett and Jameson (2000).⁷

Lemma 3. Where $f(\cdot)$ is a strictly convex function from the real numbers to the reals and n is an integer, define

$$B_n(f) = \frac{1}{n+1} \sum_{r=0}^n f\left(\frac{r}{n}\right).$$

The function $B_n(f)$ is a strictly decreasing function of n for $n \geq 2$.

Proof. Where $0 \leq r \leq n$, we have

$$\begin{aligned} \frac{r}{n} &= \frac{r}{n} \frac{r-1}{n-1} + \frac{r}{n} \frac{n-r}{n-1} \\ &= \frac{r}{n} \frac{r-1}{n-1} + \frac{n-r}{n} \frac{r}{n-r}. \end{aligned} \quad (16)$$

Since $f(\cdot)$ is a strictly convex function, it must be that

$$f\left(\frac{r}{n}\right) \leq \frac{r}{n} f\left(\frac{r-1}{n-1}\right) + \frac{n-r}{n} f\left(\frac{r}{n-r}\right), \quad (17)$$

with strict inequality where $0 < r < n$.

⁷Bennett and Jameson (2000) contains several previously unfamiliar results on properties of convex and concave functions that we suspect economists will find useful.

It follows from (17) that for $n \geq 2$,

$$\begin{aligned} \sum_{r=0}^n f\left(\frac{r}{n}\right) &< \frac{n}{n}f(0) + \frac{1}{n}f(0) + \frac{n-1}{n}f\left(\frac{1}{n-1}\right) + \frac{2}{n}f\left(\frac{1}{n-1}\right) + \cdots + \frac{1}{n}f(1) + \frac{n}{n}f(1) \\ &= \frac{n+1}{n} \sum_{r=0}^{n-1} f\left(\frac{r}{n-1}\right). \end{aligned} \quad (18)$$

(18) is equivalent to

$$\frac{1}{n+1} \sum_{r=0}^n f\left(\frac{r}{n}\right) < \frac{1}{n} \sum_{r=0}^{n-1} f\left(\frac{r}{n-1}\right). \quad (19)$$

(19) is equivalent to the inequality $B_n(f) < B_{n-1}(f)$. Thus it must be that for $n \geq 2$, $B_n(f)$ is decreasing in n . □

A.2.1. Proof of Proposition 2.1

Proposition 2.1 *In the symmetric Nash equilibrium for the coordinated Volunteer's Dilemma, as the number of players increases, the probability that each player volunteers decreases.*

Proof. Lemma 2 informs us that for all n , $H(n, q)$ is a continuous, increasing function of q on the interval $[0, 1)$, with $H(n, 0) = 0$ and $\lim_{q \rightarrow 1} H(n, q) = 1$. It follows that for any $\rho \in [0, 1)$ and any n , there is a unique q , such that $H(n, q) = \rho$. Thus the function $q(n, \rho)$ is well-defined.

To see that $q(n, \rho)$ is increasing in n , suppose that $n' > n$ and $q' = q(n', \rho)$. Then

$$\begin{aligned} 0 &= H(n, q) - H(n', q') \\ &= H(n, q) - H(n, q') + H(n, q') - H(n', q'). \end{aligned} \quad (20)$$

According to Lemma 2, $H(n, q)$ is a decreasing function of n . Therefore $H(n, q') - H(n', q') > 0$. It then follows from (20) that $H(n, q) - H(n, q') < 0$. According to Lemma 2, $H(n, q)$ is an increasing function of q . Therefore, $H(n, q) - H(n, q') < 0$ implies that $q' > q$. Since $q' = q(n', \rho)$, and $q = q(n, \rho)$, it follows that $q(n, \rho)$ is an increasing function of n . Hence, the larger the population, the lower the probability that each person volunteers. □

A.2.2. Proof of Proposition 2.2

Proposition 2.2 *In the symmetric Nash equilibrium for the coordinated Volunteer's Dilemma, as the number of players increases, the probability that at least one player volunteers decreases.*

Proof. The result is equivalent to $1 - Q_n > 1 - Q_{n+1}$ or $Q_{n+1} > Q_n$. We prove this result by contradiction. Suppose that $Q_{n+1} \leq Q_n$. Then $q_{n+1}^{n+1} \leq q_n^n$, and $q_{n+1} \leq q_n^{\frac{n}{n+1}}$. According to Lemma 1, $\rho = H(n, q_n) = H(n+1, q_{n+1})$, Lemma 2 implies that $H(n+1, q)$ is an increasing function of q . Therefore if $Q_{n+1} \leq Q_n$, then

$$H(n, q_n) \leq H(n+1, q_n^{\frac{n}{n+1}}). \quad (21)$$

Recall from (10) that $H(n, q) = n(1-q)^{\frac{q^{n-1}}{1-q^n}}$. Using this, (21) is equivalent to

$$n(1-q_n) \frac{q_n^{n-1}}{1-q_n^n} \leq (n+1) \left(1 - q_n^{\frac{n}{n+1}}\right) \frac{\left(q_n^{\frac{n}{n+1}}\right)^n}{1 - \left(q_n^{\frac{n}{n+1}}\right)^{n+1}}. \quad (22)$$

Let $z = q_n^{\frac{1}{n+1}}$. (22) is equivalent to

$$n(1-z^{n+1}) \frac{z^{n^2-1}}{1-z^{n(n+1)}} \leq (n+1)(1-z^n) \frac{z^{n^2}}{1-z^{n(n+1)}}. \quad (23)$$

Dividing both sides of (23) by $\frac{z^{n^2}}{1-z^{n(n+1)}} \frac{1}{z}$ yields

$$n(1-z^{n+1}) \leq (n+1)(z-z^{n+1}). \quad (24)$$

(24) simplifies to

$$n \leq nz + z(1-z^n), \quad (25)$$

which is equivalent to

$$n \leq z \frac{1-z^n}{1-z}. \quad (26)$$

Since $\frac{1-z^n}{1-z} = 1 + z + z^2 + \dots + z^{n-1}$, (26) is equivalent to

$$n \leq z + z^2 + \dots + z^n. \quad (27)$$

Each of the n terms on the right side of (27) is strictly less than 1, their sum must be less than n . Thus the assumption that $Q_{n+1} \leq Q_n$ leads to a contradiction. Therefore it must be that $Q_{n+1} > Q_n$. $\square$

A.2.3. Proof of Proposition 2.3

Proposition 2.3 *In the symmetric Nash equilibrium of the coordinated Volunteer's Dilemma, as the number of players n approaches infinity, the probability $Q(n, \rho)$ that no player volunteers asymptotically approaches $\frac{-\rho}{W_{-1}(-\rho e^{-\rho})}$, where $W_{-1}(\cdot)$ is the lower branch of the Lambert W function.*

Proof. Since $Q(n, \rho) = q(n, \rho)^n$, the equilibrium condition $\rho = H(n, q)$ can be written as

$$\rho = \frac{n \left(1 - Q(n, \rho)^{\frac{1}{n}}\right) Q(n, \rho)^{\frac{n-1}{n}}}{1 - Q(n, \rho)}, \quad (28)$$

which implies that

$$(1 - Q(n, \rho)) \rho = n \left(Q(n, \rho)^{1-\frac{1}{n}} - Q(n, \rho) \right). \quad (29)$$

Where $t = 1/n$, (29) can be written as

$$\left(1 - Q\left(\frac{1}{t}, \rho\right)\right) \rho = \frac{1}{t} \left(Q\left(\frac{1}{t}, \rho\right)^{1-t} - Q\left(\frac{1}{t}, \rho\right) \right). \quad (30)$$

Since $t = \frac{1}{n}$, we have $\tilde{Q}(\rho) \equiv \lim_{n \rightarrow \infty} Q(n, \rho) = \lim_{t \rightarrow 0} Q\left(\frac{1}{t}, \rho\right)$. It follows from (30) that

$$(1 - \tilde{Q}(\rho)) \rho = \lim_{t \rightarrow 0} \frac{\tilde{Q}(\rho)^{1-t} - \tilde{Q}(\rho)}{t}. \quad (31)$$

Where we define the function $g(x) = Q^x$, we have

$$\begin{aligned} \lim_{t \rightarrow 0} \frac{\tilde{Q}(\rho)^{1-t} - \tilde{Q}(\rho)}{t} &= \lim_{t \rightarrow 0} \frac{g(1-t) - g(1)}{t} \\ &= -\lim_{t \rightarrow 0} \frac{g(1-t) - g(1)}{-t} \\ &= -g'(1) \\ &= -\tilde{Q}(\rho) \ln \tilde{Q}(\rho). \end{aligned} \quad (32)$$

From Equations 31 and 32 it follows that

$$\rho = \frac{-\tilde{Q}(\rho)}{1 - \tilde{Q}(\rho)} \ln \tilde{Q}(\rho). \quad (33)$$

The function $\tilde{Q}(\rho)$ can not in general be expressed in terms of standard elementary functions. However, it can be inverted using Lambert W function.

Note that (33) can be rewritten as:

$$-\frac{\rho}{Q}e^{-\frac{\rho}{Q}} = -\rho e^{-\rho}. \quad (34)$$

This is of the form: $ye^y = x$ which can be inverted using Lambert W function and has solution:

$$-\frac{\rho}{Q} = W(-\rho e^{-\rho}). \quad (35)$$

We note that $-\frac{1}{e} < -\rho e^{-\rho} < 0$ for $0 < \rho < 1$. W is double-valued in the negative domain with two “branches”. However, the principal branch (denoted W_0) obeys the identity $W_0(-\rho e^{-\rho}) = -\rho$. This implies $Q = 1$ for all $\rho \in (0, 1)$, which is inconsistent with Nash equilibrium. Thus, only the lower branch (denoted W_{-1}) is solution associated with the family of symmetric Nash equilibria.

This can then be rearranged to get:

$$\tilde{Q} = \frac{\rho}{-W_{-1}(-\rho e^{-\rho})}. \quad (36)$$

□

A.2.4. Proof of Proposition 2.4

Proposition 2.4 *In the symmetric Nash equilibrium of the coordinated Volunteer's Dilemma, as the number of players increases, the expected utility of each player increases.*

Proof. We prove this result by contradiction. Suppose that for some $n > 1$, the expected utility of each player in a symmetric equilibrium for a volunteer's dilemma with $n + 1$ players is no larger than that with n players. Since the symmetric equilibrium is a mixed strategy equilibrium, the equilibrium expected utility is equal to the expected utility of not volunteering. Thus, under this assumption $b(1 - q_{n+1}^n) \leq b(1 - q_n^{n-1})$ and $q_n^{\frac{n-1}{n}} \leq q_{n+1}$. According to Lemma 1, $\rho = H(n, q_n) = H(n + 1, q_{n+1})$, Lemma 2 implies that $H(n + 1, q)$ is an increasing function of q . Therefore if $b(1 - q_{n+1}^n) \leq b(1 - q_n^{n-1})$, then

$$H(n, q_n) \geq H\left(n + 1, q_n^{\frac{n-1}{n}}\right). \quad (37)$$

Recall from (10) that $H(n, q) = nq^{n-1} \left(\frac{1-q}{1-q^n} \right)$. Using this, (37) is equivalent to

$$nq_n^{n-1} \frac{n(1-q_n)}{1-q_n^n} \geq (n+1) \left(q_n^{\frac{n-1}{n}} \right)^n \frac{1 - q_n^{\frac{n-1}{n}}}{1 - \left(q_n^{\frac{n-1}{n}} \right)^{n+1}} \quad (38)$$

Since $\left(q_n^{\frac{n-1}{n}} \right)^n = q_n^{n-1}$, (38) is equivalent to

$$\frac{1}{n+1} \frac{1 - \left(q_n^{\frac{n-1}{n}} \right)^{n+1}}{1 - q_n^{\frac{n-1}{n}}} \geq \frac{1}{n} \frac{1 - q_n^n}{1 - q_n}. \quad (39)$$

Let $z = q_n^{\frac{1}{n}}$. (39) is equivalent to

$$\frac{1}{n+1} \frac{1 - z^{(n+1)(n-1)}}{1 - z^{n-1}} \geq \frac{1}{n} \frac{1 - z^{n^2}}{1 - z^n}. \quad (40)$$

Since for $x \in (0, 1)$ and $n > 1$, $\frac{1-x^n}{1-x} = 1 + x + x^2 + \dots + x^{n-1}$, (40) is equivalent to

$$\frac{1 + z^{n-1} + z^{2(n-1)} + \dots + z^{n(n-1)}}{n+1} \geq \frac{1 + z^n + z^{2n} + \dots + z^{(n-1)n}}{n}. \quad (41)$$

We will use Lemma 3 to show that (41) cannot be satisfied. This will establish our counterexample.

Define the function $f(y) = z^{n(n-1)y}$. Since $f''(y) > 0$, the function f is strictly convex. The expression on the left side of (41) is equal to

$$\frac{1}{n+1} \sum_{r=0}^n f\left(\frac{r}{n}\right) \equiv B_n(f). \quad (42)$$

The expression on the right side of (41) is equal to

$$\frac{1}{n} \sum_{r=0}^{n-1} f\left(\frac{r}{n-1}\right) \equiv B_{n-1}(f). \quad (43)$$

Thus (41) is equivalent to $B_n(f) \geq B_{n-1}(f)$. But according to Lemma 3, $B_n(f) < B_{n-1}(f)$. Thus the assumption that utility does not rise with n leads to a contradiction. $\square$

A.3. Proof of Proposition 3

Proposition 3. *For any fixed $n > 1$ and $\rho = c/b$, in the symmetric Nash equilibria of the coordinated and uncoordinated Volunteer's Dilemmas:*

- 3.1 *The probability that each player volunteers is higher under coordination.*
- 3.2 *The probability that at least one player volunteers is higher under coordination.*
- 3.3 *The expected utility of each player is higher under coordination.*

Proof. For a fixed n and $\rho = \frac{c}{b}$, let q_c be the symmetric Nash equilibrium probability that a player does not volunteer in a coordinated game and q_u be the symmetric Nash equilibrium probability that a player does not volunteer in an uncoordinated game. Proposition 3.1 asserts that $1 - q_u < 1 - q_c$, while proposition 3.2 asserts that $1 - q_u^n < 1 - q_c^n$. Since the expected utility of a player in the symmetric (mixed strategy) equilibrium must be equal to the utility of not volunteering, Proposition 3.3 can be written as $b(1 - q_c^{n-1}) > b(1 - q_u^{n-1})$. All of these statements are equivalent to $q_u > q_c$. According to equations (2) and (8),

$$q_u = \rho^{\frac{1}{n-1}} \text{ and } nq_c^{n-1} \frac{1 - q_c}{1 - q_c^n} = \rho. \quad (44)$$

From Equations (44) it follows that

$$n^{\frac{1}{n-1}} q_c \left(\frac{1 - q_c}{1 - q_c^n} \right)^{\frac{1}{n-1}} = \rho^{\frac{1}{n-1}}. \quad (45)$$

From (44) and (45)

$$q_u = q_c n^{\frac{1}{n-1}} \left(\frac{1 - q_c}{1 - q_c^n} \right)^{\frac{1}{n-1}}. \quad (46)$$

From (46), $q_u > q_c$ iff $n^{\frac{1}{n-1}} \left(\frac{1 - q_c}{1 - q_c^n} \right)^{\frac{1}{n-1}} > 1$, which simplifies to

$$n > \frac{1 - q_c^n}{1 - q_c}. \quad (47)$$

The right side of Equation (47) is a partial geometric series with n terms

$$n > 1 + q_c + q_c^2 + \dots + q_c^{n-1}. \quad (48)$$

Since $q_c < 1$, (48) is true for any $n > 1$. Therefore $q_u > q_c$, which establishes Propositions 3.1, 3.2, and 3.3. $\square$