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ABSTRACT

Dynamic traffic assignment (DTA) has been a topic of substantial research during the past decade. While DTA is gradually maturing, many aspects of DTA still need improvements, especially regarding its formulation and solution capabilities under the transportation environment impacted by the Advanced Transportation Management and Information Systems (ATMIS). It is necessary to develop a set of DTA models to acknowledge the fact that the traffic network itself is probabilistic and uncertain, and different classes of travelers respond differently under uncertain environment, given different levels of traffic information. This paper aims to advance the state-of-the-art in DTA modeling in the sense that the proposed model captures the travelers' decision making among discrete choices in a probabilistic and uncertain environment, in which both probabilistic travel times and random perception errors that are specific to individual travelers, are considered. Travelers' route choices are assumed to be made with the objective of minimizing perceived disutilities at each time. These perceived disutilities depend on the distribution of the variable route travel times, the distribution of individual perception errors and the individual traveler's risk taking nature at each time instant. We formulate the integrated DTA model through a variational inequality (VI) approach. Subsequently, we discuss the solution algorithm for the formulation. Experimental results are also given to verify the correctness of solutions obtained.

1. INTRODUCTION

Over the past several decades, various traffic assignment models have been developed based on whether network attributes are dynamic or time-dependent, whether network stochasticity is considered in the travelers' decision making, and whether travelers' route travel time perception errors are assumed. Following Chen's classification *(1)* and considering both static and dynamic cases in general, traffic assignment models can be categorized as in Table 1.

Table 1. Classification of Traffic Assignment Models

		Accurate Perception	Inaccurate Perception
Static	Deterministic Network	DN-UE	DN-SUE
	Stochastic Network	SN-UE	SN-SUE
Dynamic	Deterministic Network	DN-DUO	DN-SDUO
	Stochastic Network	SN-DUO	SN-SDUO

Where: DN = Deterministic Network
SN = Stochastic Network
UE = User Equilibrium
SUE = Stochastic User Equilibrium
DUO = Dynamic User Optimal
SDUO = Stochastic Dynamic User Optimal

Among those models in Table 1, the static user equilibrium (UE) models are usually used for the long-term planning purpose *(2)*. Most of them are formulated to be consistent with the Wardrop's first principle. This principle requires that, for used routes between a given origin-destination (OD) pair, the route cost equals the minimum route cost, and no used route has a lower cost. The basic assumption in the UE models is that over some period of time, each traveler learns and adapts to the transportation network conditions and the services available to him/her so that an equilibrium can be reached. In the DN-UE model, no network uncertainty is considered, i.e. link travel times are deterministic and each traveler is assumed to have perfect knowledge of the network travel times on all possible routes between his/her OD pair. To overcome the deficiencies of the deterministic model, DN-SUE model relaxes the assumption of travelers' perfect knowledge of network travel times, allowing travelers to select routes based on their perceived travel times. However, given that travel time uncertainty is one of the important factors in route choice as shown in a recent empirical study by Abdel-Aty et al. *(7)*, a realistic route choice model should capture the tradeoffs between expected travel time and travel time uncertainty in decision making. The SN-SUE model falls in this category.

On the other hand, with continuous information and advice from Advanced Transportation Management and Information System (ATMIS), static UE may not exist but rather the travelers choose routes and modes based on the current perceived condition of the traffic on the network *(3)*. Therefore, the dynamic generalization of the static UE concept is called the dynamic user optimal (DUO). One DUO traffic assignment problem is to determine vehicle flows at each instant of time on each link resulting from drivers using minimal-time routes. Thus, we are no longer considering day-to-day traffic equilibrium. Instead, we are trying to influence or control traffic and travel patterns of travelers optimally by providing accurate traffic information and effective traffic control measures. In a simple DTA model, all users of the network have perfect information on the travel times on the network and choose routes which minimize either their travel times or some generalized cost. Such a DTA model is typically deterministic. But in real life, since one might expect that network travel times for a given set of flows are stochastic in nature and also the travelers may not have perfect information about the network, the deterministic assumptions are questionable.

A large body of work is available on stochastic user equilibrium models. Dial was among the first to present the algorithm to assign link flows based on a logit model and later this stochastic user equilibrium model on deterministic networks (DN-SUE) was formulated as an optimization problem *(4)*. In the generalized SN-SUE model *(5)*, proposed by Mirchandani and Soroush, the travel time on each route is random and each traveler perceives inaccurately. Thus, the perceived distribution of travel times for each traveler is a function of the actual distribution of the network travel times as well as the distribution of the traveler's own perception error. Furthermore, because it is assumed that travelers are aware of the variable nature of travel times on the network, the decision making process involved in choosing a route is assumed to represent risk taking behavior under uncertainty. Since route choice decisions usually involve tradeoffs between expected travel time and travel time uncertainty, Tatineni indicated that it may be appropriate to model travelers as either risk averse, risk prone or risk neutral, and the risk in this case is the variability associated with route travel times *(6)*. Chen and Recker examined the effect of considering risk taking behavior in static route choice models and its impact on the estimation of travel time reliability of a road network subject to demand and supply variations *(1)*.

In order to realize the real-time traffic network monitoring and management function in ATMIS, a SN-SDUO model that captures travelers' route choice behavior in a dynamic and stochastic transportation network needs to be developed. In this model, it is essential to understand how drivers make route choices, especially in the light of the considerable information that the driver may receive within ITS environment, such as variable message signs (VMS), highway advisory radio (HAR), and in-vehicle navigation systems, etc. Along with the driver's prior knowledge of the traffic network, such a model, as shown in Figure 1, should replicate, to the extent possible, the driver's perception of available routes and his or her decision-making in selecting the routes. Based on different assumptions on the distributions of actual route travel time and perceived route travel time error, various DTA models can be obtained. Most of current stochastic DTA models consider either a stochastic route travel time without traveler perception error such as Boyce's model *(8)* or deterministic network with traveler perception error such as Ran's model *(9)*, but not both. To our knowledge, no SN-SDUO model has been proposed in the literature.

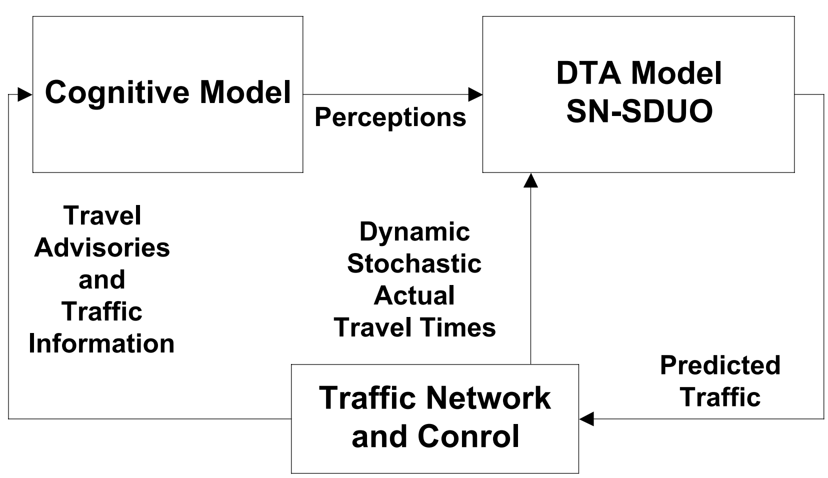


Figure 1. DTA Model with Probabilistic Travel Time and Perceptions

The objective of this paper is to propose a formulation and solution algorithm for the SN-SDUO model, which is a stochastic dynamic user optimal model based on stochastic dynamic network. This paper extends Mirchandani and Soroush's generalized traffic equilibrium *(5)* into a dynamic environment. The assumption is that route travel times are variable and perceived as such by travelers at each time instant. Each traveler uses a disutility function of travel time to evaluate each route and route choices are assumed to be made with the objective of minimizing perceived disutility at each time and no traveler can reduce his/her perceived expected disutility by changing to another route. These perceived disutilities depend on the distribution of the variable route travel times, the distribution of individual perception errors, and the individual traveler's risk taking nature at each time. By considering the traveler's risk-taking behavior in dynamic and probabilistic environment, the proposed model can capture the traveler's route choice characteristics such that a trade-off decision between a route with longer but reliable travel time versus another route with shorter but unreliable travel time.

This paper is organized as follows. The dynamic user-optimal route choice conditions are formulated and then a variational inequality (VI) is derived in Section 2. Traveler's perception under dynamic and stochastic network, the stratification of traveler's risk-taking behavior, and the route choice characteristics of these traveler classes will also be discussed in this section. In Section 3, we discuss an algorithm that can solve the VI formulation via a combination of relaxation technique, stochastic network loading, and Method of Successive Averages (MSA). Section 4 contains some computational results of applying the proposed approach to several simple scenarios, with the objective of verifying solution qualities. Finally, concluding remarks are discussed in Section 5.

2. THE VARIATIONAL INEQUALITY FORMULATION

2.1 Notation

In the following, superscript *"rs"* denotes origin-destination pair *rs,* subscript *"a"* (or *"b"*) denotes link a (or b), subscript "p" (or "$\tilde{p}$") denotes path p (or subpath $\tilde{p}$ between node j and destination s), and subscript "m" denotes traveler class m. All the variables used in the formulation are defined as follows:

$x_a(t)$ = number of vehicles on link a at time t (main problem variable)
$u_a(t)$ = inflow rate into link a at time t (main problem variable)
$v_a(t)$ = exit flow rate from link a at time t (main problem variable)
$y_a(k)$ = number of vehicles on link a at the beginning of time interval k (subproblem variable)
$\hat{y}_a^i(k)$ = number of vehicles on link a at the beginning of time interval k at iteration i (stochastic loading loop)
$p_a(k)$ = inflow into link a during interval k (subproblem variable)
$\hat{p}_a^i(k)$ = inflow into link a during interval k at iteration i (stochastic loading loop)
$q_a(k)$ = exit flow from link a during interval k (subproblem variable)
$\hat{q}_a^i(k)$ = exit flow from link a during interval k at iteration i (stochastic loading loop)
$f_m^{rs}(t)$ = class m departure flow rate from origin r to destination s at time t (given)
$f_{pm}^{rs}(t)$ = class m departure flow on path p from origin r to destination s at time t

$e^{rs}(t)$ = arrival flow rate from origin r toward destination s at time t

$E^{rs}(t)$ = cumulative number of vehicles arriving at destination s from origin r by time t (main problem variable)

$\overline{E}^{rs}(k)$ = cumulative number of vehicles arriving at destination s from origin r during interval k (subproblem variable)

$A(j)$ = set of links whose tail node is j (after j)

$B(j)$ = set of links whose head node is j (before j)

$P_p^{rs}(t)$ = proportion of flows between (r,s) that follow route p at time t

$\tau_a(t)$ = actual travel time over link a for flows entering link a at time t

$\overline{\tau}_a(t)$ = estimated mean actual travel time over link a for flows entering link a at time t

$T_a(t)$ = perceived travel time over link a for flows entering link a at time t

$\eta_p(t)$ = actual travel time for route p between (r, s) for flows departing origin r at time t

$\Omega_p(t)$ = perceived travel time for route p between (r, s) for flows departing origin r at time t

$\xi_p(t)$ = perceived travel time error for route p between (r, s) for flows departing origin r at time t

$R^{rs}(t)$ = the set of path between (r, s) at time t

$\pi^{rs}(t)$ = minimal disutility between (r, s) for flows departing origin r at time t

$DU_p^{rs}(t)$ = perceived disutility for route p between (r,s) for flows departing origin r at time t

2.2 Traveler's Perceptions Under Dynamic and Stochastic Network

Consider a traffic network represented by a directed graph consisting of a finite set of nodes and links. We assume that the travel times for traversing the links in the network are random variable and the probability density functions (PDF) of link travel times are dependent on the time when a link is entered. Therefore, the link travel time $\tau_a(t)$ can be modeled as a stochastic process. The stochasticity of the link travel time for a given set of flows on the traffic network could be resulted from different sources such as different weather conditions, different mix of vehicle types, and different delays experienced by different vehicles at intersections, etc. Incidents, such as vehicle breakdowns and signal failure, also contribute to the random effects of the traffic network. A network where the link travel time is modeled as a stochastic process is referred to as a dynamic and stochastic network *(10)*.

The accuracy of a traveler perception of the network depends on the traveler's pervious experience during similar network flow conditions and information available to the traveler from various sources such as travel time updates via radio, television, internet or advanced traveler information systems. In reality, travelers may have either or both imperfect information and different perceptions towards travel time rather than perfect information and homogeneous perceptions.

Let there be M different groups of travelers, where the travelers of each group have the same disutility function and same perception error distribution. In this paper, we assume that each traveler i from group m perceives the travel time on link a of route p at time t as a distribution comprising of the distribution of actual link travel time $\tau_a(t)$ and a perception error $\xi_a^{im}(t)$, whose distribution parameters are specific to traveler i.

$$T_a^{im}(t) = \tau_a(t) + \xi_a^{im}(t), \qquad a \in p, \quad p \in R^{rs}(t), \quad \forall r,s,i,m \tag{1}$$

Assume route p consists of nodes *(r, 1, 2, ...,s)*. Then, a recursive formula for the perceived route travel time $\Omega_p^{rs}(t)$ is:

$$\Omega_p^{r1}(t) = \tau^{r1}(t) + \xi^{r1}(t) = T^{r1}(t) \tag{2}$$

…

$$\Omega_p^{rj}(t) = \Omega_p^{r(j-1)}(t) + T_a(t + \Omega_p^{r(j-1)}(t)) \qquad \forall p,r,j;\ \mathrm{j} = 2,...,s; \tag{3}$$

where link $a = (j\text{-}1, j)$. Theoretically, we will have:

$$\Omega_p^{rs}(t) = \sum_{j=1}^{s} T_a(t + \Omega_p^{r(j-1)}(t)) = \sum_{j=1}^{s} (\tau_a(t + \Omega_p^{r(j-1)}(t)) + \xi_a(t + \Omega_p^{r(j-1)}(t))) \qquad \forall r,s,p \tag{4}$$

Equation (4) shows that the perceived route travel time is a random variable where the parameters of its associate probability density function (PDF) are dependent on the distribution of traveler's perception error as well as on the distribution of actual route travel time. Depending on the level of available travel time information (this includes traveler's previous knowledge) at different time period, the distribution of traveler perception error may vary dynamically. This in itself requires extensive empirical investigation; and the distribution may not be able to be represented by an algebraic function even after that. To make the model somewhat tractable, the following assumptions are made in this paper.

Assumption 1: At time instant t, the perceived error $\xi^{im}(t)$ of an individual i from group m for a segment of road with unit travel time has normal distribution $N(\mu^{im}(t), \theta^{im}(t))$. The parameters $\mu^{im}(t)$ and $\theta^{im}(t)$ are dependent on the level of travel time information available to the traveler at time t. Therefore, we also assume:

$$\mu^{im}(t) = \mu^{im} f(Info^{im}(t), t) \tag{5}$$

$$\theta^{im}(t) = \theta^{im} g(Info^{im}(t), t)\,. \tag{6}$$

Here, $Info^{im}(t)$ represents the available travel time information to the traveler i from group m at time t, f and g are functional relationships. The parameters μ^{im} and θ^{im} for a random individual i from group m have normal distribution $N(0, \tau^m)$ and gamma distribution $G(\alpha^m, \beta^m)$ over the population, respectively. Here, τ^m, α^m and β^m are some constant values which are specific to group m.

Assumption 2: An individual's perceived errors are independent for non-overlapping route segments and mutually independent over the population of travelers.

Using equation (5) and (6), an individual traveler's perception error under different ATMIS scenarios can be modeled, for example, different functional relatetionships *f* and *g* can be used for the link with or without VMS control. Equation (5) and (6) can aslo capture the temporal correlation in traveler's perception error since the same tripmaker is likely to perceive travel time in a similar way from one instant to the next.

As shown by Mirchandani and Soroush *(5)*, the moment generating function (MGF) of percived link travel time will be used in the following section to calculate the disutility functions. Based on Equation (1), the percieved link travel time equals the sum of actual link travel time and traveler's perception error, the MGF of the percieved link travel time can be expressed as follows:

$$M_{T_a^{im}(t)}(s) = M_{\tau_a(t)}[s(1+\mu^{im}(t)+s\theta^{im}(t)/2)] = M_{\tau_a(t)}[A] \tag{7}$$

Here, we set $A = s(1+\mu^{im}(t)+s\theta^{im}(t)/2)$. (8)

If we assume that the actual link travel time $\tau_a(t)$ is a non-negative continuous or discrete random variable with probability density function $f_{\tau_a(t)}$, and for the ease of representation, let _ be the possible value that can be attained for the link travel time, the MGF of the perceived link travel time in Equation (7) becomes:

$$M_{T_a^{im}(t)}(s) = M_{\tau_a(t)}[A] = \int_0^{+\infty} \exp(A\tau) f_{\tau_a(t)}(\tau)d\tau \tag{9}$$

$$M_{T_a^{im}(t)}(s) = M_{\tau_a(t)}[A] = \sum \exp(A\tau) f_{\tau_a(t)}(\tau) \tag{10}$$

2.3 Traveler's Risk Taking Behavior

In everyday life, we frequently acknowledge that people have differing attitudes to risk. The shape of a utility or disutility function models a decision maker's attitude to risk. To model different types of risk taking behavior of traveler, we take the stochasticity of route travel time as the risk associated when travelers choose routes. On the basis of the perceived distribution of network travel times, travelers are assumed to behave differently when choosing routes which are probabilistic. Some are risk averse, choosing routes with longer expected travel times but lower variations. Others, the risk takers, may choose routes with shorter expected travel times but higher variations in travel time reliability.

In this paper, travelers are stratified into three classes *(6)*, depending on their route choice behavior: (i) risk averse travelers; (ii) risk prone travelers; and (iii) risk neutral travelers. Different disutility functions are established for each class to reflect its risk-taking behavior and perceived disutilities from these functions are a function of the distribution of the variable route travel times, the distribution of individual perception errors, and the individual traveler's risk taking nature at each time. By the definitions of these disutility functions, risk averse traveler would tend to choose routes with low expected variance of travel time, and risk prone traveler would prefer routes with highly variable travel times in an effort to shorten the journey time.

Following the study by Tatineni et al. *(6)*, we also use the exponential disutility function, which is one of the most widely used disutility functions reported in the decision-making literature to model the different risk taking behaviors. The shapes of these different risk taking behaviors are provided in Figure 2.

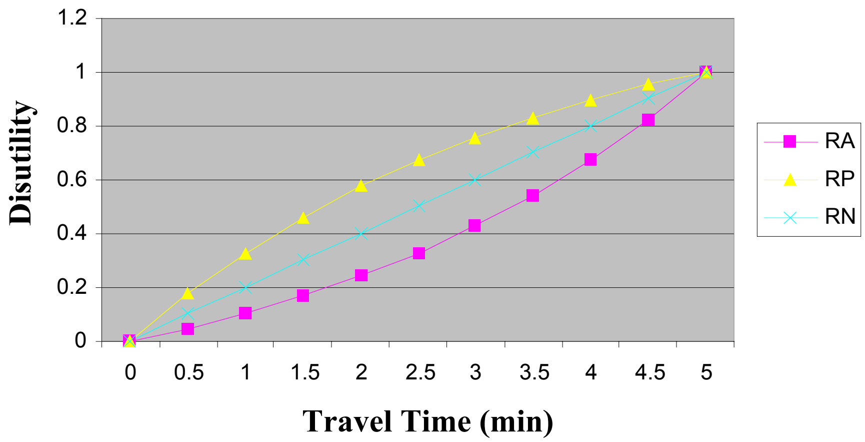


Figure 2. Disutility Functions for Different Risk-Taking Route Choice Models

To calculate the perceived expected disutility functions, the perceived route travel time is needed. A naive approach is to enumerate all paths, derive the PDF of the perceived travel time of these paths, and then compute the corresponding perceived expected disutilities. However, this could take considerable computational effort for even a small network. One important advantage of using the exponential function is that the disutility associated with a route can be estimated by summing the link disutilities on that route. This allows the classical Dijkstra-type shortest path algorithm to be used in finding the minimum expected disutility route. As discussed by Mirchandani and Soroush *(5)*, the MGF of perceived route travel time in Equation (7) and (8) could be used to calculate the expected disutility functions, without the requirement of path enumeration. We present the results in the following. In all cases, we assume that a route with 0 minutes travel time has a disutility of 0 and a route with 5 minutes travel time has a disutility of 1. Assume route p from origin r to destination s consists of j intermediate nodes, $j = (1, 2, \ldots, s)$, and link $a = (j-1, j)$ is on the route p.

For the risk averse case, the disutility function of a risk averse person takes the form of:

$$DU_p^{rs}(t) = a_1 \exp(\alpha \Omega_p^{rs}(t)) - a_2 \tag{11}$$

The perceived expected disutility function is:

$$E(DU_p^{rs}(t)) = a_1 \prod_{a \in p} M_{T_a(t+\Omega_p^{r(j-1)}(t))}(\alpha) - a_2 \tag{12}$$

With the boundary conditions and the risk averse assumption, the disutility function and the perceived expected disutility function finally have the forms of:

$$DU_p^{rs}(t) = 0.309(\exp(0.289\Omega_p^{rs}(t)) - 1) \tag{13}$$

$$E(DU_p^{rs}(t)) = 0.309(\prod_{a\in p} M_{T_a(t+\Omega_p^{r(j-1)}(t))}(0.289) - 1) \tag{14}$$

For the risk prone case, the disutility function of a risk prone person takes the form of:

$$DU_p^{rs}(t) = b_2 - b_1 \exp(-\beta\Omega_p^{rs}(t)) \tag{15}$$

The perceived expected disutility function is:

$$E(DU_p^{rs}(t)) = b_2 - b_1 \prod_{a\in p} M_{T_a(t+\Omega_p^{r(j-1)}(t))}(-\beta) \tag{16}$$

With the boundary conditions and the risk averse assumption, the disutility function and the perceived expected disutility function finally have the forms of:

$$DU_p^{rs}(t) = 1.309(1 - \exp(-0.289\Omega_p^{rs}(t))) \tag{17}$$

$$E(DU_p^{rs}(t)) = 1.309(1 - \prod_{a\in p} M_{T_a(t+\Omega_p^{r(j-1)}(t))}(-0.289)) \tag{18}$$

For the risk neutral case, the disutility function is a linear function with the expected perceived travel time. For the ease of modeling, we use exponential form to approximate linear disutility function. Therefore, the disutility function of a risk neutral person takes the form of:

$$DU_p^{rs}(t) \approx c_2 - c_1 \exp(-\gamma\Omega_p^{rs}(t)) \tag{19}$$

The perceived expected disutility function is:

$$E(DU_p^{rs}(t)) \approx c_2 - c_1 \prod_{a\in p} M_{T_a(t)}(-\gamma) \tag{20}$$

With the boundary conditions and the risk neutral assumption, the disutility function and the perceived expected disutility function finally have the forms of:

$$DU_p^{rs}(t) \approx 20.5(1 - \exp(-0.01\Omega_p^{rs}(t))) \tag{21}$$

$$E(DU_p^{rs}(t)) \approx 20.5(1 - \prod_{a\in p} M_{T_a(t)}(-0.01)) \tag{22}$$

For comparison purpose, note that risk neutral travelers make route choice decisions based on the mean perceived route travel times solely, regardless the variance of perceived route travel times. Essentially, risk neutral travelers consider the route travel time as deterministic in the sense that all routes have the mean travel times. So if we assume that all travelers are risk neutral, our SN-SDUO model becomes DN-SDUO.

2.4 The Dynamic Network Constraint Set

The constraint set for our DTA problem is summarized for each class of travelers.

Route Flow Assignment Constraints:

$$f_{pm}^{rs}(t) = f_m^{rs}(t)P_p^{rs}(t) \quad \textit{where} \quad f_m^{rs}(t) \quad \textit{is} \quad \textit{given}, \quad \forall r,s,p;m \tag{23}$$

$$f_{pm}^{rs}(t) = u_{apm}^{rs}(t) \qquad \forall r,s,p,m; a \in A(r); a \in p; \tag{24}$$

Other Constraints for all traveler classes:

Relationship between state and control variables:

$$\frac{dx_{apm}^{rs}}{dt} = u_{apm}^{rs}(t) - v_{apm}^{rs}(t) \quad \forall m,a,p,r,s \tag{25}$$

$$\frac{dE_{pm}^{rs}(t)}{dt} = e_{pm}^{rs}(t) \quad \forall p,m,r;s \neq r \tag{26}$$

Flow conservation constraints:

$$\sum_{a \in B(j)} v_{apm}^{rs}(t) = \sum_{a \in A(j)} u_{apm}^{rs}(t) \quad \forall j,p,m,r,s;j \neq r,s \tag{27}$$

$$\sum_{a \in B(s)} \sum_{p} v_{apm}^{rs}(t) = e_{pm}^{rs}(t) \quad \forall m,r,s;s \neq r \tag{28}$$

Flow propagation constraints:

$$x_{ap}^{rs}(t) = \sum_{b \in \tilde{p}} \{x_{bp}^{rs}[t + \tau_a(t)] - x_{bp}^{rs}(t)\} + \{E_p^{rs}[t + \tau_a(t)] - E_p^{rs}(t)\} \ \forall a \in B(j); j \neq r; p; r; s \tag{29}$$

Definitional constraints:

$$\sum_{rspm} u_{apm}^{rs}(t) = u_a(t), \ \sum_{rspm} v_{apm}^{rs}(t) = v_a(t), \ \sum_{rspm} x_{apm}^{rs}(t) = x_a(t), \ \forall a \tag{30}$$

Nonnegativity conditions:

$$x_{apm}^{rs}(t) \geq 0, \ u_{apm}^{rs}(t) \geq 0, \ v_{apm}^{rs}(t) \geq 0, \ \forall m,a,p,r,s \tag{31}$$

$$f_{pm}^{rs}(t) \geq 0, \ e_{pm}^{rs}(t) \geq 0, \ E_{pm}^{rs}(t) \geq 0, \ \forall p,m,r,s \tag{32}$$

Boundary conditions:

$$E_{pm}^{rs}(0) = 0, \ \forall p,m,r,s \tag{33}$$

$$x_{apm}^{rs(0)} = 0, \ \forall a,p,m,r,s\ . \tag{34}$$

For each class of travelers, the constraints expressed in (23) - (34), including flow propagation and conservation constraints, are applicable. These constraints are used to generate path and link flows when route departure flows are determined. The path departure flow $f_p^{rs}(t)$ is determined by the stochastic loading function. The link flow propagation constraints (29) are implemented for each link *a*, each route *p*, each O-D pair *rs,* and each time *t*, regardless of traveler classes. Therefore, the FIFO requirement can be ensured.

2.4 Link Travel Time and Delay Functions

Since a stochastic network is considered in this paper, variation of the link travel time should be required. Thus, in this paper, the actual link travel time $\tau_a(t)$ has two components: one is deterministic flow-dependent cruise time $c_a(t)$ and the other one is the stochastic delay $d_a(t)$. The stochastic delay may be caused by the traffic signal at the intersection for an arterial link or by the congestion for a freeway link.

There are various cruise time functions for different link types, such as freeway and arterial. To simplify the computation, it is assumed that the cruise time depends on the number of vehicles and the inflow rate. Equation (35) shows the link cruise time function chosen for the numerical results of this study:

$$c_a(t) = c_a[u_a(t), x_a(t)] = T_{a,f}[1 + \beta(\frac{u_a}{C_a} + \frac{x_a}{x_{a,\max}})^{\alpha}]\ \forall a \quad (35)$$

where α and β are coefficients, $T_{a,f}$ is the free flow travel time on link a, C_a is its capacity, and $x_{a,max}$ is its maximum holding capacity.

In this paper, to simplify our algorithm, the stochastic delay is modeled as a non-negative normal distribution, which relates directly to the cruise time of this link, as shown in Equation (36).

$$d_a = N(c_a\mu_a, c_a\sigma_a^2) \quad (36)$$

where μ_a is the mean parameter and σ_a^2 is the variance parameter.

2.5 The VI Formulation

Assume travelers are disutility minimizers. The probability that route p is chosen by an individual can be stated as follows:

$$P_p^{rs}(t) = \text{Prob}(E(DU_p^{rs}(t)) \le E(DU_q^{rs}(t))), \forall \text{ route } q \text{ between } r \text{ and } s, \forall r, s, p. \quad (37)$$

where Prob is the choice function representing the proportion of individuals who choose route p. The DUO route choice conditions are then defined as follows:

$$f_p^{rs}(t) - f^{rs}(t)P^{rs}(t) = 0 \quad \forall r, s, p \quad (38)$$

Note that the mean actual route travel time $\eta_p^{rs}(t)$ is increasing with path departure flow $f_p^{rs}(t)$, i.e., $\frac{\partial \eta_p^{rs}(t)}{\partial f_p^{rs}(t)} > 0 \quad \forall r, s, p$ (39)

For each path p and each O-D pair rs, define an auxiliary cost term as follows:

$$F_p^{rs}(t) = [f_p^{rs}(t) - f^{rs}(t)P_p^{rs}(t)]\frac{\partial \eta_p^{rs}(t)}{\partial f_p^{rs}(t)} = 0 \quad \forall r, s, p \quad (40)$$

It is obvious that the above equality states the DUO route choice conditions, since $\partial \eta_p^{rs}(t) / \partial f_p^{rs}(t) > 0$. As shown in *(11)*, the above system of equations is equivalent to the following variational inequality for each time instant $t \in [0, +\infty)$:

$$\sum_{rs}\sum_{p} F_p^{rs}(t)\{f_p^{rs}(t) - f_p^{rs*}(t)\} \ge 0 \quad (41)$$

where superscript * denotes that path departure flow f has an optimal value. Since $F_p^{rs}(t) = 0$, the above inequality is also equivalent to the integral form:

$$\int_0^T \sum_{rs}\sum_{p} F_p^{rs}(t)\{f_p^{rs}(t) - f_p^{rs*}(t)\}dt \ge 0 \quad (42)$$

3. THE SOLUTION ALGORITHM

To solve the VI problem, we need to convert our continuous time VI problem into a discrete time VI problem. The time period [$0,T$] is subdivided into K small time intervals. Each time interval is regarded as one unit of time. Then, $u_a(k)$ represents the inflow into link a during interval k, $v_a(k)$ represents the exit flow from link a during interval k, $x_a(k)$

represents the number of vehicles at the beginning of interval k, and $f_p(k)$ represents the departure flow from path p during interval k.

This discrete VI can be solved by using a combination of relaxation, stochastic network loading and Method of Successive Averages (MSA) techniques. In this combined algorithm, we define the travel time approximation procedure (relaxation) as the outer iteration and the MSA procedure as the inner iteration. For each relaxation (or diagonalization) iteration, we temporarily fix actual travel time $\tau_a(k)$ in link flow propagation constraints as $\overline{\tau}_a(k)$.

The algorithm for solving our proposed DTA model can be summarized as follows:

Step 0: Initialization. Initialize all link flows $\left\{x_{am}^{(0)}(k)\right\},\left\{u_{am}^{(0)}(k)\right\},\left\{v_{am}^{(0)}(k)\right\}$ to zero and calculate initial time estimates $\tau_a^{(1)}(k)$, regardless of traveler classes. Set the outer iteration counter l=1.

Step 1: Relaxation. Set the inner iteration counter n = 1. Find a new approximation of actual link travel times: $\overline{\tau}_a^{(n)}(k) = \tau\left(x_a^{(*)}(k)\right)$, where (*) denotes the final solution obtained from the most recent inner problem. Solve the route choice program for the main problem using stochastic network loading and method of successive averages.

[Step 1.1]: Subproblem - Stochastic Dynamic Network Loading. Perform Monte Carlo simulation by sampling random link travel times, calculate corresponding link disutilities. Compute minimal disutility paths and assign all departure flows $f^{rs}(k)$ to these routes during each Monte Carlo iteration. Let the temporary link flow vector resulted from the all-or-nothing loading be called $\left(\hat{p}^i, \hat{q}^i, \hat{y}^i\right)$ at Monte Carlo iteration i. Then, the stochastic dynamic network loading is solved by the following recursive equations:

$$p_a^i(k) = [(i-1)p_a^{(i-1)}(k) + \hat{p}_a^i(k)]/i \quad \forall a \tag{43}$$

$$q_a^i(k) = [(i-1)q_a^{(i-1)}(k) + \hat{q}_a^i(k)]/i \quad \forall a \tag{44}$$

$$y_a^i(k) = [(i-1)y_a^{(i-1)}(k) + \hat{y}_a^i(k)]/i \quad \forall a \tag{45}$$

Set $i = i + 1$. As i equals a prespecified number, stop. The vector (p^i, q^i, y^i) is used as the converged link flows at inner iteration n.

[Step 1.2]: Method of Successive Averages. Using the predetermined step size 1/n, yield a new MSA main problem solution through the following equations:

$$u_a^{n+1}(k) = u_a^n(k) + \frac{1}{n}[p_a^n(k) - u_a^n(k)] \quad \forall a \tag{46}$$

$$v_a^{n+1}(k) = v_a^n(k) + \frac{1}{n}[q_a^n(k) - v_a^n(k)] \quad \forall a \tag{47}$$

$$x_a^{n+1}(k) = x_a^n(k) + \frac{1}{n}[y_a^n(k) - x_a^n(k)] \quad \forall a \tag{48}$$

If n equals a prespecified number, go to step 2; otherwise $n = n+1$, and go to step 1.1.

Step 2: Convergence Test for the Outer Iterations. If $\left|\bar{\tau}_a^{(l)}(k) - \bar{\tau}_a^{(l-1)}(k)\right| <= \Delta$, stop. The current solution $\{u_a(k)\}, \{v_a(k)\}, \{x_a(k)\}$ is in a near optimal state; otherwise, set *l*=*l*+*1* and go to step 1. Δ is the pre-defined threshold.

The algorithm is shown in Figure 3. The number of inner iterations *n* and the number of outer iterations *l* are correlated. If we set *l* large, then *n* should be set small and vice versa. The computational convergence of this proposed solution algorithm deserves further study.

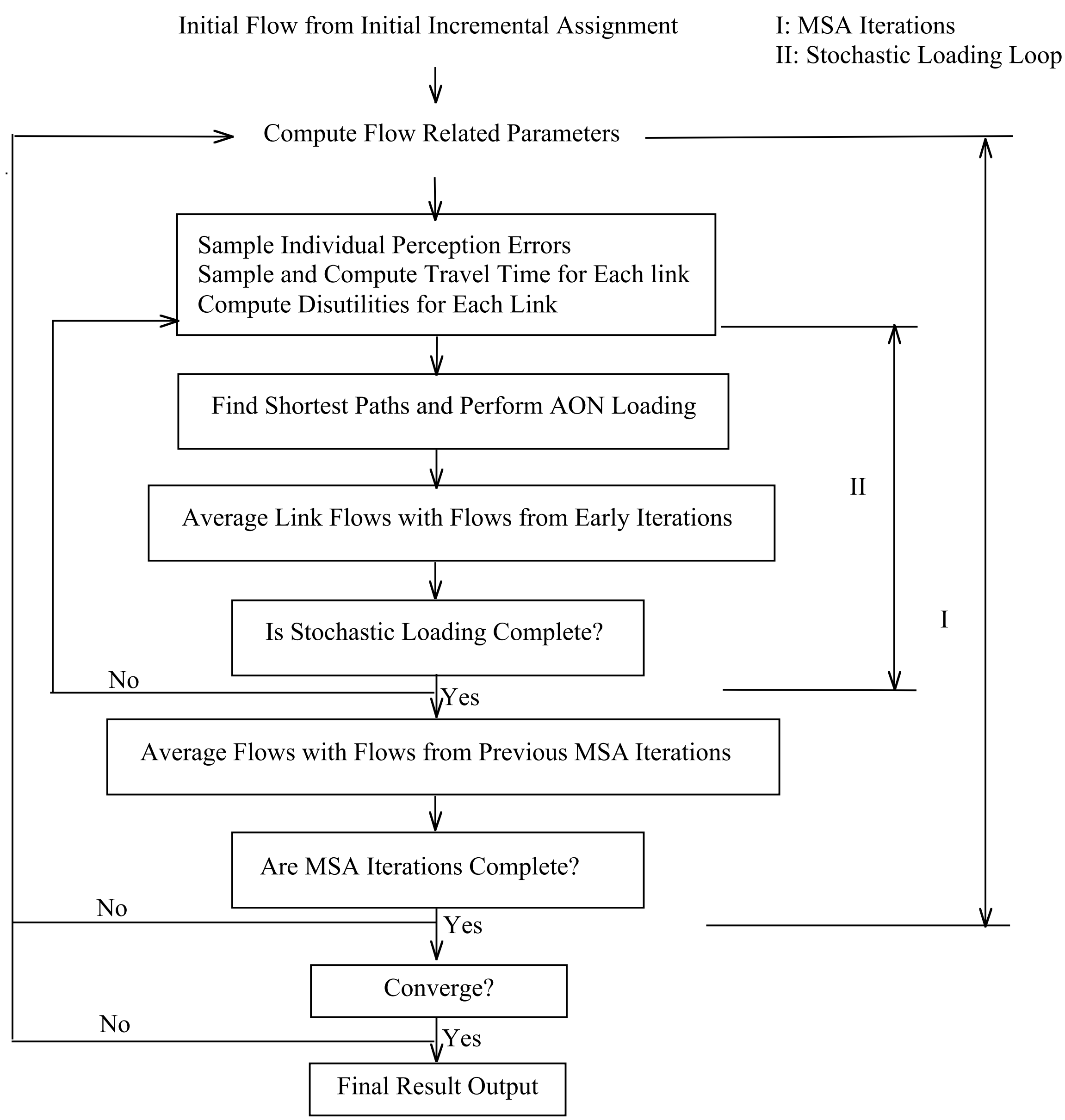
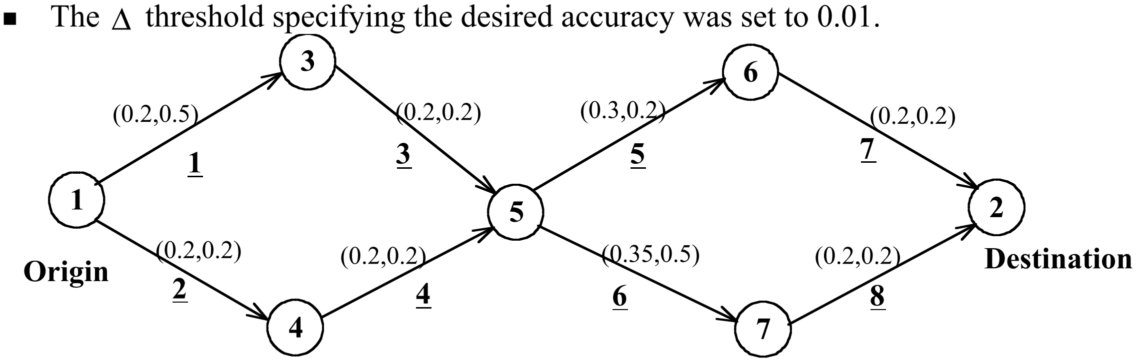


Figure 3. Solution Algorithm Flow Chart

4. EXPERIMENTAL RESULTS

In this section, we present some numerical results from our experiments for a small test network using the proposed SN-SDUO model. The objective here is not to illustrate or discuss the network performance as a function of mixed vehicle class, but merely to demonstrate solution quality. The test network is indicated in Figure 4 with seven nodes and eight links. The length of each link is 2.5 miles. Detailed link characteristics are shown in Table 2. Four scenarios, as listed in Table 3, are designed to demonstrate that the algorithm produces results that are consistent with the definitions of SN-SDUO route choices. Especially, the results from the 100% risk neutral travelers should be same with that from the DN-SDUO model. The scenarios are deliberately chosen to be simple so that one can verify the results easily. These four scenarios share the following common input characteristics:

- Origin is node 1 and destination is node 2.
- The O-D flows are 15 vehicles for each of the five 60-second periods (equivalent to a flow of 900 vehicles per hour). The total flows from Origin to Destination for the whole analysis period is 75.
- Free flow speed is 50 miles per hour.
- The delay distribution for link a is $N(c_a\mu, c_a\sigma^2)$, where c_a is the deterministic flow-dependent cruise time for link a. μ and σ^2 for each link are listed in Table 4 and shown in Figure 3.
- $\tau^m = 0.012,\ \alpha^m = 0.5,\ \beta^m = 0.01, \qquad m = 1,2,3$
- The Δ threshold specifying the desired accuracy was set to 0.01.



(The underlined number is the link number)

Figure 4. Experimental Network

Table 2. Link Information

Link Number	Start Node	End Node	Length (miles)	Capacity (# of Vehi.)	# of Lane
1	1	3	2.5	2200	1
2	1	4	2.5	2200	1
3	3	5	2.5	2200	1
4	4	5	2.5	2200	1
5	5	6	2.5	2200	1
6	5	7	2.5	2200	1
7	6	2	2.5	2200	1
8	7	2	2.5	2200	1

Table 3. Distinctive Features of Four Scenarios

Scenarios	Distinctive features
1	100% Risk Averse Traveler
2	100% Risk Prone Traveler
3	100% Risk Neutral Traveler
4	1/3 for each group

Table 4. Parameters of Intersection Delay for Each Link

Link Number	Mean(μ)	Variance(σ^2)
1	0.2	0.5
2	0.2	0.2
3	0.2	0.2
4	0.2	0.2
5	0.3	0.2
6	0.35	0.5
7	0.2	0.2
8	0.2	0.2

To better present the results, we accumulate the number of vehicles passing through each link for the entire analysis period as shown in Figure 5 to Figure 8. These numbers could be verified by the time-dependent results for each link at every time interval shown as Table 5 to Table 8 in the appendix. Since the flow from origin 1 will go to node 5 and then reach destination 2, we can divide the network into two parts at node 5 and analyze them separately. Thus, links 1, 2, 3, 4 compose a sub-network and links 5, 6, 7, 8 compose another sub-network. Here we call them L-Network and R-Network, respectively.

In scenario #1, we have one group of travelers who are risk averse and their disutility function is expressed as formulation (11). For the L-Network, the intersection delay for each of the links follows normal distribution $N(0.2c_a, 0.2c_a)$ except link 1 which has a larger variance (0.5). Since the risk-averse travelers prefer route with smaller travel time variance if the means are identical, the number of travelers choosing route 1->4->5 should be more than those choosing route 1->3->5. Consequently, our algorithm assigned 58.1% (43.58 out of 75) and 41.9% (31.42 out of 75) of the total flows to these two routes, respectively. The reason that nearly 42% percent travelers still chose route 1->4->5 is because of the perception errors. For the R-Network, the intersection delay for each of the links follows normal distribution $N(0.2c_a, 0.2c_a)$ except links 5 and 6. As we have expected, 63.5% (47.64 out of 75) of the total travelers chose route 5->6->2 due to the smaller mean and variance of the intersection delay for link 5 (0.3 and 0.2, respectively) compared with link 6 (0.35 and 0.5, respectively).

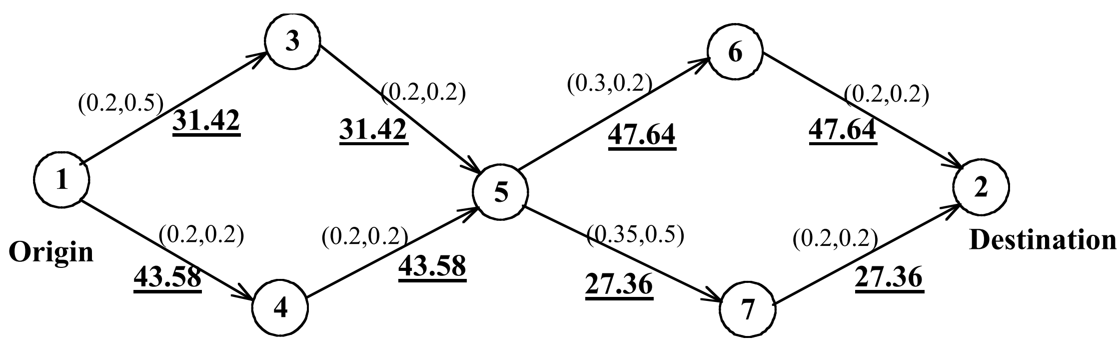
In scenario #2, suppose that we have only risk-prone travelers who will prefer route with larger disutility variance given that the means are the same. Our algorithm assigned 60.7% of the travelers to route 1->3->5 and 39.3% to route 1->4->5 for the L-Network. At the same time, for the R-Network, it assigned 47.5% of the total travelers to route 5-

>6->2 and 52.5% to route 5->7->2. Note that, although link 6 has a larger mean (0.35) than link 5 (0.3), more travelers still chose route 5->7->2 because of the much larger variance that link 6 (0.5) has when compared with link 5 (0.2).

We consider the risk-neutral travelers in scenario #3. They will choose route mainly based on the mean of the route disutility. Therefore, our algorithm assigned the flows almost evenly to route 1->3->5(50.1%) and route 1->4->5(49.9%) because each of the four links (1,2,3,4) has the same mean (0.2). Meanwhile, for the R-Network, the mean of link 5 is smaller than that of link 6, more travelers should choose route 5->6->2 instead of route 5->7->2. This is exactly what our algorithm indicates: it assigned 55.8% of the total travelers to route 5->6->2 and 44.2% to route 5->7->2.

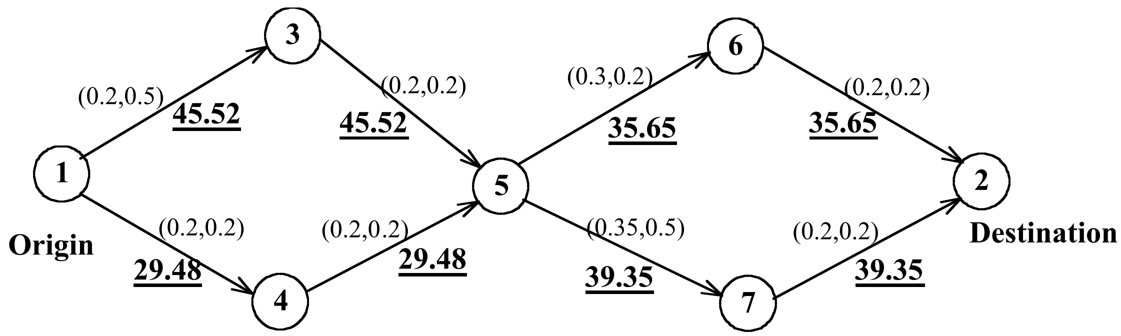
In scenario #4, travelers are consisted of all the three kinds of trip-makers evenly. Because the number of risk-averse travelers is just the same as that of risk-prone travelers, the risk-taking behavior of these two groups will counteract with each other to a great extent. Thus, the aggregated route-choice behavior of all the travelers in this scenario should be similar with scenario 3 in which all travelers are risk-neutral. As expected in this scenario, 50.6% and 49.4% travelers are assigned to route 1->3->5 and route 1->4->5, respectively for the L-Network, due to identical disutility mean of the four links (0.2). Whereas, a little bit more travelers (58.0%) are assigned to route 5->6->2 for the R-Network due to the smaller mean of link 5 than that of link 6.

The above analysis of experimental results from four distinctive scenarios demonstrates that our proposed analytical DTA model can fulfill the objectives of SN-SDUO, and produce realistic and reasonable dynamic traffic flow assignment for travelers traversing on a dynamic and stochastic network with different risk-taking behavior.



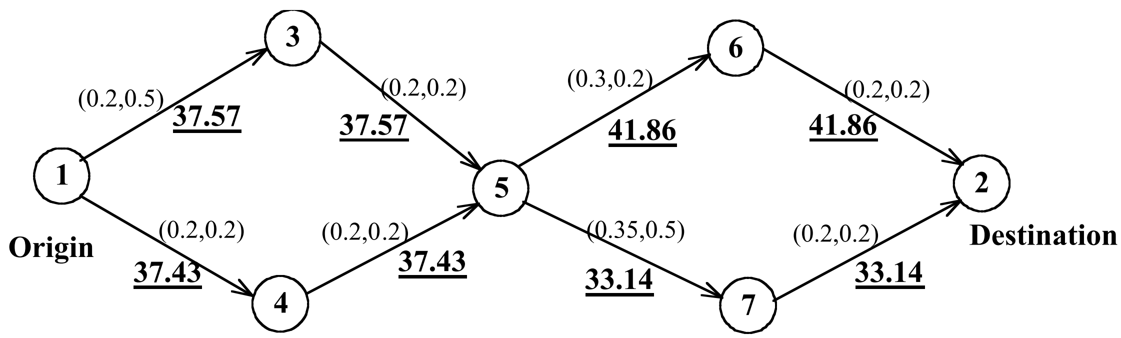
(The underlined number is the total inflow of the link)

Figure 5. Results from Scenario #1



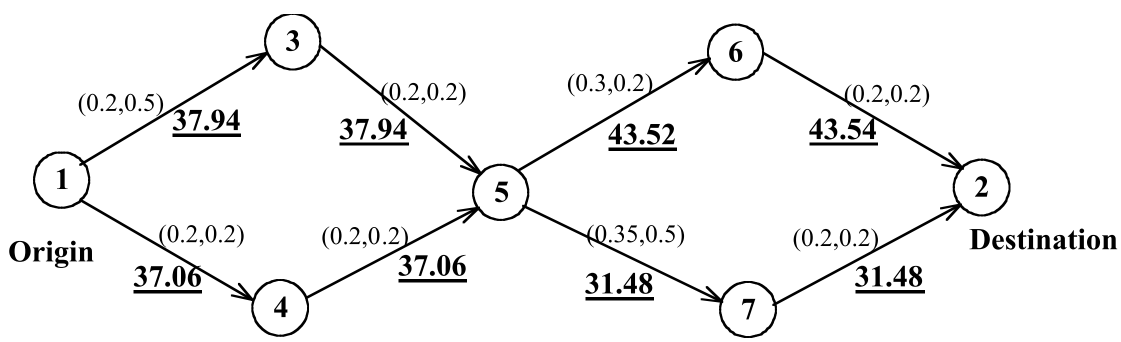
(The underlined number is the total inflow of the link)

Figure 6. Results from Scenario #2



(The underlined number is the total inflow of the link)

Figure 7. Results from Scenario #3



(The underlined number is the total inflow of the link)

Figure 8. Results from Scenario #4

5. CONCLUDING REMARKS

In this paper, we presented an analytical approach to formulate a dynamic traffic assignment model, which capture travelers' route choice behavior in a dynamic and stochastic network. Each traveler chooses a "perceived optimal route" which minimizes the perceived expected disutility of travel time from his origin node to his destination node. Our proposed model incorporates travelers' risk-taking behavior since the traffic network under consideration is stochastic. We take the stochasticity of route travel time as the risk associated when travelers choose routes. In this DTA model, three kinds of risk-taking behavior are taken into consideration: (i) risk averse case, (ii) risk prone case, and (iii) rise neutral case. Through a variational inequality formulation, they are integrated into one modeling framework. We proposed a solution algorithm by combining a relaxation approach, stochastic network loading and method of successive averages. Four scenarios were tested to gain some computational experiences of the algorithm and to verify the solutions obtained.

The eventual goal of this effort is to develop an analytical model that can be used to examine issues and evaluate various strategies in ATMIS. A larger network consisting of both freeway links and arterial links will be used to test the model and the correctness of results will be verified in the subsequent papers. Since our model allows for the possibility that travelers may have either or both imperfect information and different perception towards probabilistic and uncertain travel time, it can be used to model the driver's perception of available routes and his or her decision-making in selecting routes under dynamic and stochastic environment, especially in the light of the considerable information that the driver may receive within ITS environment, such as variable message signs, highway advisory radio, in-vehicle navigation systems and other telematics devices. Since different information devices may have different coverage of traffic information and therefore may have different impact on traveler's route choice decision process, we will investigate and incorporate different information devices in our modeling framework for future research.

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Appendix:

Table 5. Time-Dependent Flow for Scenario #1 (100% Risk Averse)

Time Interval	Link	Flow
1	1 -> 3	5.9
1	1 -> 4	9.1
2	1 -> 3	11.84
2	1 -> 4	18.16
3	1 -> 3	17.37
3	1 -> 4	27.63
4	1 -> 3	18.87
4	1 -> 4	26.13
4	3 -> 5	5.9
4	4 -> 5	9.1
5	1 -> 3	19.58
5	1 -> 4	25.42
5	3 -> 5	11.84
5	4 -> 5	18.16
6	1 -> 3	14.05
6	1 -> 4	15.95
6	3 -> 5	17.37
6	4 -> 5	27.63
7	1 -> 3	6.64
7	1 -> 4	8.36
7	3 -> 5	18.87
7	4 -> 5	26.13
7	5 -> 6	9.23
7	5 -> 7	5.77
8	3 -> 5	19.58
8	4 -> 5	25.42
8	5 -> 6	18.83
8	5 -> 7	11.17
9	3 -> 5	14.05
9	4 -> 5	15.95
9	5 -> 6	28.81
9	5 -> 7	16.19
10	3 -> 5	6.64
10	4 -> 5	8.36
10	5 -> 6	29.11
10	5 -> 7	15.89
10	6 -> 2	9.23
10	7 -> 2	5.77
11	5 -> 6	28.81
11	5 -> 7	16.19
11	6 -> 2	18.83
11	7 -> 2	11.17
12	5 -> 6	18.83
12	5 -> 7	11.17
12	6 -> 2	28.81
12	7 -> 2	16.19
13	5 -> 6	9.3
13	5 -> 7	5.7
13	6 -> 2	29.11
13	7 -> 2	15.89
14	6 -> 2	28.81
14	7 -> 2	16.19
15	6 -> 2	18.83
15	7 -> 2	11.17
16	6 -> 2	9.3
16	7 -> 2	5.7

Table 6. Time-Dependent Flow for Scenario #2 (100% Risk Prone)

Time Interval	Link	Flow
1	1 -> 3	9.63
1	1 -> 4	5.37
2	1 -> 3	19.24
2	1 -> 4	10.76
3	1 -> 3	27.39
3	1 -> 4	17.61
4	1 -> 3	26.59
4	1 -> 4	18.41
4	3 -> 5	9.63
4	4 -> 5	5.37
5	1 -> 3	26.28
5	1 -> 4	18.72
5	3 -> 5	19.24
5	4 -> 5	10.76
6	1 -> 3	18.13
6	1 -> 4	11.87
6	3 -> 5	27.39
6	4 -> 5	17.61
7	1 -> 3	9.3
7	1 -> 4	5.7
7	3 -> 5	26.59
7	4 -> 5	18.41
7	5 -> 6	6.98
7	5 -> 7	8.02
8	3 -> 5	26.28
8	4 -> 5	18.72
8	5 -> 6	13.78
8	5 -> 7	16.22
9	3 -> 5	18.13
9	4 -> 5	11.87
9	5 -> 6	20.23
9	5 -> 7	24.77
10	3 -> 5	9.3
10	4 -> 5	5.7
10	5 -> 6	21.52
10	5 -> 7	23.48
10	6 -> 2	6.98
10	7 -> 2	8.02
11	5 -> 6	21.87
11	5 -> 7	23.13
11	6 -> 2	13.78
11	7 -> 2	16.22
12	5 -> 6	15.42
12	5 -> 7	14.58
12	6 -> 2	20.23
12	7 -> 2	24.77
13	5 -> 6	7.16
13	5 -> 7	7.84
13	6 -> 2	21.52
13	7 -> 2	23.48
14	6 -> 2	21.87
14	7 -> 2	23.13
15	6 -> 2	15.42
15	7 -> 2	14.58
16	6 -> 2	7.16
16	7 -> 2	7.84

Table 7. Time-Dependent Flow for Scenario #3 (100% Risk Neutral)

Time Interval	Link	Flow
1	1 -> 3	7.78
1	1 -> 4	7.22
2	1 -> 3	15.71
2	1 -> 4	14.29
3	1 -> 3	23.05
3	1 -> 4	21.95
4	1 -> 3	22.48
4	1 -> 4	22.52
4	3 -> 5	7.78
4	4 -> 5	7.22
5	1 -> 3	21.87
5	1 -> 4	23.13
5	3 -> 5	15.71
5	4 -> 5	14.29
6	1 -> 3	14.52
6	1 -> 4	15.48
6	3 -> 5	23.05
6	4 -> 5	21.95
7	1 -> 3	7.32
7	1 -> 4	7.68
7	3 -> 5	22.48
7	4 -> 5	22.52
7	5 -> 6	8.31
7	5 -> 7	6.69
8	3 -> 5	21.87
8	4 -> 5	23.13
8	5 -> 6	16.34
8	5 -> 7	13.66
9	3 -> 5	14.52
9	4 -> 5	15.48
9	5 -> 6	24.76
9	5 -> 7	20.24
10	3 -> 5	7.32
10	4 -> 5	7.68
10	5 -> 6	24.9
10	5 -> 7	20.1
10	6 -> 2	8.31
10	7 -> 2	6.69
11	5 -> 6	25.52
11	5 -> 7	19.48
11	6 -> 2	16.34
11	7 -> 2	13.66
12	5 -> 6	17.1
12	5 -> 7	12.9
12	6 -> 2	24.76
12	7 -> 2	20.24
13	5 -> 6	8.65
13	5 -> 7	6.35
13	6 -> 2	24.9
13	7 -> 2	20.1
14	6 -> 2	25.52
14	7 -> 2	19.48
15	6 -> 2	17.1
15	7 -> 2	12.9
16	6 -> 2	8.65
16	7 -> 2	6.35

Table 8. Time-Dependent Flow for Scenario #4 (1/3 for each group)

Time Interval	Link	Flow (Averse)	Flow (Neutral)	Flow (Prone)	Flow (Total)
1	1 -> 3	2.12	2.6	3.1	7.82
1	1 -> 4	2.88	2.4	1.9	7.18
2	1 -> 3	3.91	5.02	6.07	15.01
2	1 -> 4	6.09	4.98	3.93	14.99
3	1 -> 3	5.96	7.45	9.39	22.81
3	1 -> 4	9.04	7.55	5.61	22.19
4	1 -> 3	5.92	7.44	9.32	22.68
4	1 -> 4	9.08	7.56	5.68	22.32
4	3 -> 5	2.12	2.6	3.1	7.82
4	4 -> 5	2.88	2.4	1.9	7.18
5	1 -> 3	6.2	7.2	9.54	22.93
5	1 -> 4	8.8	7.8	5.46	22.07
5	3 -> 5	3.91	5.02	6.07	15.01
5	4 -> 5	6.09	4.98	3.93	14.99
6	1 -> 3	4.15	4.76	6.22	15.13
6	1 -> 4	5.85	5.24	3.78	14.87
6	3 -> 5	5.96	7.45	9.39	22.81
6	4 -> 5	9.04	7.55	5.61	22.19
7	1 -> 3	2.08	2.17	3.19	7.44
7	1 -> 4	2.92	2.83	1.81	7.56
7	3 -> 5	5.92	7.44	9.32	22.68
7	4 -> 5	9.08	7.56	5.68	22.32
7	5 -> 6	3.48	3.01	2.7	9.2
7	5 -> 7	1.52	1.99	2.3	5.8
8	3 -> 5	6.2	7.2	9.54	22.93
8	4 -> 5	8.8	7.8	5.46	22.07
8	5 -> 6	6.58	6.16	5.22	17.96
8	5 -> 7	3.42	3.84	4.78	12.04
9	3 -> 5	4.15	4.76	6.22	15.13
9	4 -> 5	5.85	5.24	3.78	14.87
9	5 -> 6	10.11	8.88	7.47	26.46
9	5 -> 7	4.89	6.12	7.53	18.54
10	3 -> 5	2.08	2.17	3.19	7.44
10	4 -> 5	2.92	2.83	1.81	7.56
10	5 -> 6	10.17	8.58	7.28	26.02
10	5 -> 7	4.83	6.42	7.72	18.98
10	6 -> 2	3.48	3.01	2.7	9.2
10	7 -> 2	1.52	1.99	2.3	5.8
11	5 -> 6	10.31	8.06	7.19	25.55
11	5 -> 7	4.69	6.94	7.81	19.45
11	6 -> 2	6.58	6.16	5.22	17.96
11	7 -> 2	3.42	3.84	4.78	12.04
12	5 -> 6	6.78	5.33	4.94	17.05
12	5 -> 7	3.22	4.67	5.06	12.95
12	6 -> 2	10.11	8.88	7.47	26.46
12	7 -> 2	4.89	6.12	7.53	18.54
13	5 -> 6	3.24	2.63	2.43	8.3
13	5 -> 7	1.76	2.37	2.57	6.7
13	6 -> 2	10.17	8.58	7.28	26.02
13	7 -> 2	4.83	6.42	7.72	18.98
14	6 -> 2	10.31	8.06	7.19	25.55
14	7 -> 2	4.69	6.94	7.81	19.45
15	6 -> 2	6.78	5.33	4.94	17.05
15	7 -> 2	3.22	4.67	5.06	12.95
16	6 -> 2	3.24	2.63	2.43	8.3
16	7 -> 2	1.76	2.37	2.57	6.7