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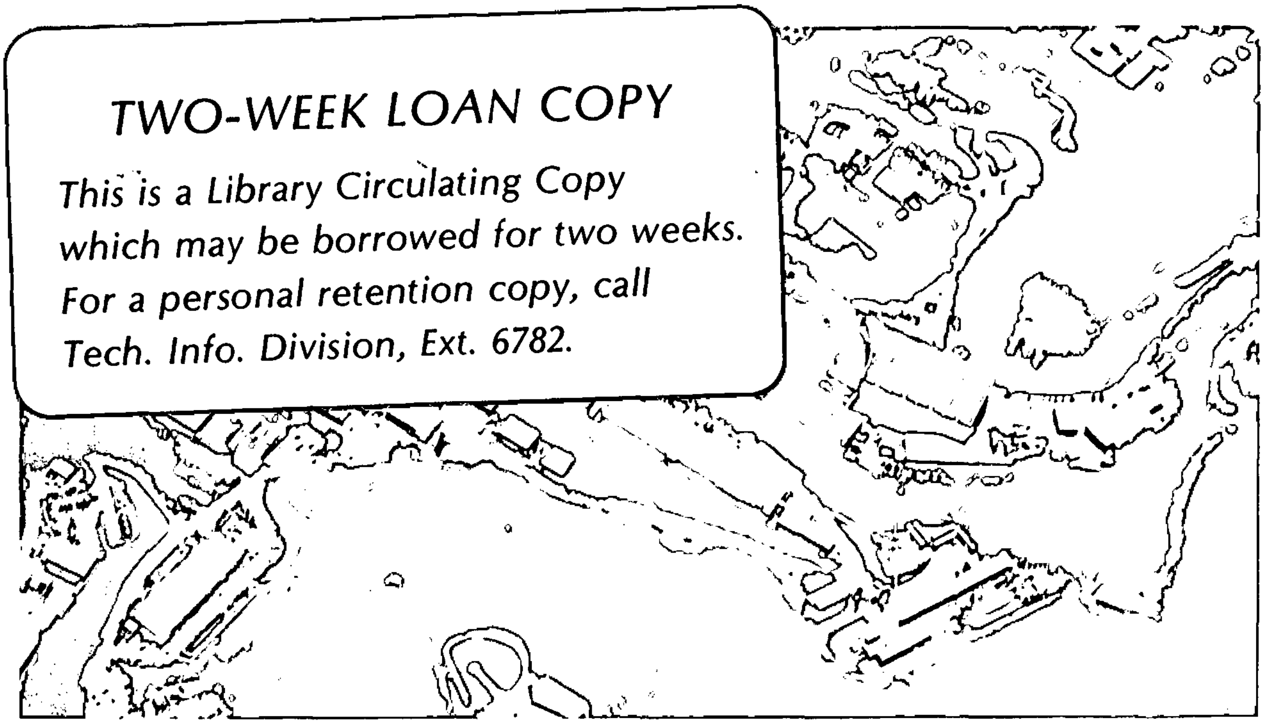

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BOUNDS ON THE MASSES OF NEUTRAL GENERATION-CHANGING GAUGE BOSONS

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ABSTRACT

The existence of several generations of quarks and leptons suggests the possibility of a gauge symmetry connecting the different generations. The neutral gauge bosons of such a scheme would mediate rare processes such as $K_L^0 \to \mu^{\pm} e^{\mp}$, $K^+ \to \pi^+ e^- \mu^+$, $\mu N \to eN$ and would contribute to $\Delta M(K_S^0 - K_L^0)$. We study these and other processes within a simple theoretical framework and derive bounds involving the masses and coupling constants of the generation-changing gauge bosons and various generation-mixing angles. The lower bounds for the relevant masses lie in the 10-100 TeV region. Various remarks concerning the relevance of these bounds to currently popular theoretical ideas and to future experiments are presented.

I. INTRODUCTION

An outstanding puzzle in particle physics is the apparent redundancy of quark and lepton flavors. There appear to exist at least three "generations" of "fundamental" fermions, each consisting of a tricolored $Q = +2/3$ quark, a tricolored $Q = -1/3$ quark, a neutral lepton and a $Q = -1$ lepton together with their anti-particles(1). The first generation consists of u, d, ν_e, e; the second of c, s, ν_μ, μ; the third of $t(?)$, b, ν_τ, τ. The evidence for this pattern is still incomplete, especially since there is no experimental proof for the existence of the t-quark. Nevertheless, alternative descriptions of the known fundamental fermions are, at best, complicated and unattractive. We shall, therefore, restrict our attention to the "standard" scheme of three (or more) generations following an identical pattern.

The concept of a "generation" is, at present, mostly intuitive. It is not well defined mathematically. The known Cabibbo mixing of quarks tells us that, even if we develop an exact meaning to the generation concept, we must encounter "generation mixing". If we define a generation as the set of quarks and leptons belonging to a representation of grand-unification algebra, such as SU(5) or SO(10), we find not only Cabibbo mixing but different generation-mixing for quarks on one hand and leptons on the other hand(2).

It is, therefore, clear that any hypothetical quantum number which we might use in order to label the generations cannot be an exactly conserved quantum number. At the same time, it is possible (and even likely) that all mixing angles are small. In the limit of no mixing, the generations could be well defined, and the realistic case of small mixing would then be roughly approximated by the ideal no-mixing assignments.

The generation pattern may or may not reflect further substructure beyond the level of quarks and leptons. However, independent of the possible existence of such a substructure, we might expect to find some kind of an underlying symmetry which relates the generations to each other or, perhaps, distinguishes among them.

Such a symmetry could be discrete or continuous. If it is continuous, it may or may not be a local gauge symmetry. If it is a local gauge symmetry, it may still appear in, at least, two algebraic forms:

(i) A complete "horizontal" gauge algebra H which commutes with all the gauge operators acting within a generation. This would lead to an overall gauge theory based on the group $G \otimes H$ where G is the (grand unification) algebra of one generation and H contains neutral gauge bosons which connect different generations[3]. The main advantage of such a scheme is the natural appearance of identical generations. The main disadvantage is the severe restrictions which are placed on the group H, making it very hard to develop a successful model.

(ii) An extended grand-unification scheme such as the ones based on SU(11)[4] or SO(14)[5] or SO(18),[6] in which several generations are assigned to one large representation, which we may call a "dynasty". In such a scheme there will again be gauge bosons connecting fermions of different generations. In some cases such a theory will actually have a $G \otimes H$ subgroup (for instance SO(14) has an $SU(5) \times SU(2)_H$ subgroup). However, in general, this is not

necessarily the case. In fact, inside the large group we may not always be able to find a closed "horizontal" algebra H, and we may find charged generation-changing gauge bosons.

In the present paper we shall be concerned with the possibility that there exist heavy neutral gauge bosons, connecting fermions of different generations. These gauge bosons may mediate rare processes such as $\mu N \to eN$, $\mu \to 3e$, $K_L^o \to \mu^{\pm} e^{\mp}$, $K^+ \to \pi^+ e^- \mu^+$ etc. In some of these processes (e.g. $\mu N \to eN$) only one generation-changing vertex occurs (Fig. 1a). In other processes (such as $K^+ \to \pi^+ e^- \mu^+$) one fermion is "promoted" from the first to the second generation, while another is "demoted" Fig. 1b)

In the limit of vanishing "generation-mixing" angles we may tentatively define a "generation-number" G such that $G = G_1, G_2$ for the first two fermion generations. Arbitrarily normalizing $G_2 - G_1 = 1$, we may classify (still in the limit of no mixing!) all generation changing processes according to their ΔG values.

A neutral "horizontal" boson may connect to each other two neutral leptons (N) or charged leptons (L) or charge 2/3 quarks (U) or charge $-1/3$ quarks (D). If both ends of the "horizontal" boson couple to the same type of fermions (e.g. L and L in $\mu \to 3e$, Fig. 1c) we refer to the process as "diagonal". If the legs of the "horizontal" boson couple to two different types of fermions (e.g. L and D in $K_L^o \to \mu e$; Fig. 1d) we refer to the process as "non-diagonal".

In our classification of processes according to ΔG vlaues, it will prove useful to distinguish between diagonal and non-diagonal transitions. This is done in Table I. In the limit in which G is conserved and there is no mixing, $\Delta G \neq 0$ processes are forbidden. In the presence of generation mixing by small angles β_i, a $|\Delta G| = 1$ process is "first-forbidden" and its amplitude is proportional to some combination of the β_i. Similarly, $|\Delta G| = 2$ amplitudes are of order β^2, etc.

Within a well-defined theory, the present limit for the rate of a given rare process may lead to a bound on g_H^2/M^2. Here g_H is the gauge coupling of the "horizontal" generations-changing neutral boson H and M is either the mass of the "horizontal" boson or some parameter characterizing mass-differences among such bosons.

We do not have a well-founded theoretical prejudice for the expected magnitude of g_H or M. One guess may be $g_H \sim g_W$, where g_W is the standard electroweak coupling. In some technicolor schemes(7) we may actually expect $g_H > g_W$, since g_H is a "strong" coupling. In the same technicolor models, M_H may be expected to lie somewhere around 10-100 TeV, although it is not very hard to push it to higher values.

The bound obtained for each process depends on the specific model and, in $|\Delta G| \neq 0$ cases, on the values of generation-mixing angles. However, some of our results appear to be quite general and they provide useful restrictions on interesting classes of models. In particular, some of our bounds on g_H^2/M^2 are quite close to the range of values mentioned in the preceding paragraph. We, therefore, believe that a phenomenological analysis of these processes is worthwhile.

In the next section we specify the extremely simple theoretical framework within which we perform our analysis. We define the

the generation-mixing angles and distinguish among several schemes according to the "horizontal" boson mass spectrum.

Section III is devoted to an examination of six reactions which we find to be the most useful:

(i) $K_L^o \to \mu^\pm e^\mp$

(ii) $K^+ \to \pi^+ e^- \mu^+$

(iii) $\mu N \to eN$

(iv) $\mu^- \to e^- e^+ e^+$

(v) $\mu^- \to e^- \gamma$

(vi) $\Delta M(K_S^o - K_L^o)$

Finally, in Section IV we discuss the implications of our results.

II. FRAMEWORK

In order to study a specific situation we choose to consider a theory based on a large gauge group $G \otimes H$ where G acts within a generation and H is a horizontal gauge group containing neutral generation-changing gauge bosons. The group G presumably contains the standard electroweak algebra $SU(2)_W \times U(1)$. For simplicity, we shall assume that H is an SU(2) algebra(3) which we label $SU(2)_H$. For the present, we shall ignore the distinction between right-handed and left-handed fermions, and pretend that both are members of $SU(2)_W$ doublets. As an additional simplification, we shall limit ourselves, at first, to two generations. Later, we shall comment on a more realistic situation.

Since $SU(2)_W$ and $SU(2)_H$ commute, we can choose a basis of particle states which has simple transformation properties under both groups. Thus we indicate by L_1^o and L_2^o, the "primitive" electron and the "primitive" muon. Both are $T_3 = -1/2$ eigenstates. They are eigenstates of the $SU(2)_H$ generator, H_3, with eigenvalues $-1/2$ and $+1/2$ respectively, and $H_+ L_1^o = L_2^o$, etc. Similarly, we define the primitve states N_1^o, N_2^o; U_1^o, U_2^o; D_1^o, D_2^o for the neutral leptons and the quarks. All these states are eigenstates of T_3 and H_3. However, in general they are not mass eigenstates.

The mass eigenstates N, L, U, and D are related to the primitive states N^o, L^o, U^o, and D^o by a unitary transformation:

$$F \equiv \begin{bmatrix} N \\ L \\ U \\ D \end{bmatrix} = \begin{bmatrix} \mathcal{U}^N & & & \\ & \mathcal{U}^L & & \\ & & \mathcal{U}^U & \\ & & & \mathcal{U}^D \end{bmatrix} \begin{bmatrix} N^o \\ L^o \\ U^o \\ D^o \end{bmatrix} \equiv \mathcal{U} F^o , \tag{1}$$

where the unitary submatrix $\mathcal{U}^N$ acts only on the neutral lepton subspace, and so on.

In the primitive basis, the charged weak current is very simple:

$$J_+ = \bar{F}^o T_+ F^o = \overline{N^o L^o U^o D^o} \begin{bmatrix} 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} N^o \\ L^o \\ U^o \\ D^o \end{bmatrix} . \tag{2}$$

In the mass eigenstate basis we have:

$$J_+ = \bar{F} \mathcal{U} T_+ \mathcal{U}^\dagger F = \overline{N L U D} \begin{bmatrix} 0 & \mathcal{U}^N \mathcal{U}^{L\dagger} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{U}^U \mathcal{U}^{D\dagger} \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} N \\ L \\ U \\ D \end{bmatrix} . \tag{3}$$

The combination $\mathcal{U}^U \mathcal{U}^{D\dagger}$ is the usual Cabibbo mixing matrix. In the two-generation case it is a unitary 2×2 matrix. The overall phase of the matrix is uninteresting so we may consider the matrix simply to be a member of $SU(2)$. We may parameterize it with Euler angles:

$$\mathcal{U}^U \mathcal{U}^{D\dagger} = \exp(-i\alpha_{UD} H_3) \exp(-i\beta_{UD} H_2) \exp(-i\gamma_{UD} H_3) . \tag{4}$$

The first and last factors have no physical significance and can be absorbed by the phases of the basis states U nad D. Evaluating the remaining factor explicitly we see that

$$\beta_{UD} = 2\theta_{Cabibbo} \sim 26^o . \tag{5}$$

Of course, the contributions to the neutral weak current are not changed in form by the mass-diagonalizing matrix $\mathcal{U}$:

$$J_3 = \bar{F}^o T_3 F^o = \bar{F} \mathcal{U} T_3 \mathcal{U}^\dagger \bar{F} = \bar{F} T_3 F ,$$
$$J_o = \bar{F}^o F^o = \bar{F} F . \tag{6}$$

The horizontal currents again have an especially simple form in the primitive basis:

$$J_i^H = \bar{F}^o H_i F^o = \overline{N^o L^o U^o D^o} \begin{bmatrix} H_i & & & \\ & H_i & & \\ & & H_i & \\ & & & H_i \end{bmatrix} \begin{bmatrix} N^o \\ L^o \\ U^o \\ D^o \end{bmatrix} , \tag{7}$$

where, in the two-generation case:

$$H_i = \frac{1}{2} \sigma_i .$$

The horizontal bosons presumably obtain their masses by spontaneous symmetry breaking. In general, the three bosons associated with $SU(2)_H$ need not have the same mass. Indeed it is possible that all three masses are different. In general, then the horizontal bosons give rise to the following effective Lagrangian (we suppress the space-time structure):

$$\mathcal{L} = \frac{g_H^2}{2} \sum_{i=1}^{3} \bar{F}^o H_i F^o (M_i)^{-2} \bar{F}^o H_i F^o \ , \qquad (8)$$

where g_H is the gauge coupling constant, analogous to the electro-weak coupling g_W and M_i is the mass of the gauge boson H_i.

Consider first the possibility that all three horizontal bosons are degenerate. Then Eq. (8) becomes

$$\mathcal{L} = \frac{g_H^2}{2M_H^2} \bar{F}^o \underset{\sim}{H} F^o \cdot \bar{F}^o \underset{\sim}{H} F^o , \qquad (9)$$

or in terms of mass eigenstates

$$\mathcal{L} = \frac{g_H^2}{2M_H^2} \bar{F} \mathcal{U} \underset{\sim}{H} \mathcal{U}^\dagger F \quad \bar{F} \mathcal{U} \underset{\sim}{H} \mathcal{U}^\dagger F . \qquad (10)$$

It is apparent that the interaction, Eq. (10) conserves the primitive generation number, that is, there is a global $SU(2)_H$ symmetry. The conserved quantum number G is defined here for the primitive eigenstates and is equal to H_3. Conservation of G is broken because of mixing. For example, consider the $L - D$ term in Eq. (10):

$$\mathcal{L}_{LD} = \frac{g_H^2}{M_H^2} \bar{L} \mathcal{U}^L \underset{\sim}{H} \mathcal{U}^{L\dagger} L \cdot \bar{D} \mathcal{U}^D \underset{\sim}{H} \mathcal{U}^{D\dagger} D . \qquad (11)$$

This would contribute to processes like $K_L^o \to \mu^\pm e^\mp$. We may rewrite this as

$$\mathcal{L}_{LD} = \frac{g_H^2}{M^2} \bar{L} \underset{\sim}{H} L \cdot \bar{D} \mathcal{U}^D \mathcal{U}^{L\dagger} \underset{\sim}{H} \mathcal{U}^L \mathcal{U}^{D\dagger} D \qquad (12)$$

The matrix $\mathcal{U}^L \mathcal{U}^{D\dagger}$ is analogous to the Cabibbo matrix, but represents the relative orientation of L and D rather than U and D. It permits violation of G conservation. However, we now see that for diagonal terms like $L - L$ or $D - D$, G is exactly conserved. An especially important consequence is that if there is complete degeneracy of the horizontal bosons, there is no contribution to the $K_L - K_S$ mass difference (Fig. 1e) or to $\mu \to 3e$, (Fig. 1c) etc.

As in the case of the Cabibbo matrix, the overall phase of $\mathcal{U}^L \mathcal{U}^{D\dagger}$ is insignificant, so we can take the matrix to be an element of $SU(2)$, in the two-generations case:

$$\mathcal{U}^L \mathcal{U}^{D\dagger} = \exp(-i\alpha_{LD} H_3) \ \exp(-i\beta_{LD} H_2) \ \exp(-i\gamma_{LD} H_3) . \qquad (13)$$

The factors containing γ_{LD} and α_{LD} may be absorbed into redefinitions of the phases in D and L, respectively. An explicit evaluation of Eq. (12) yields

$$\bar{L} \underset{\sim}{H} L \ \bar{D} e^{i\beta_{LD} H_2} \underset{\sim}{H} e^{-i\beta_{LD} H_2} D =$$

$$\frac{1}{2} \cos^2 \frac{\beta_{LD}}{2} \left[\bar{L} H_+ L \bar{D} H_- D + \bar{L} H_- L \bar{D} H_+ D \right] + \cos \beta_{LD} \bar{L} H_3 L \ \bar{D} H_3 D$$

$$- \frac{1}{2} \sin^2 \frac{\beta_{LD}}{2} \left[\bar{L} H_- L \bar{D} H_+ D + \bar{L} H_- L \bar{D} H_- D \right]$$

$$+ \frac{1}{2} \sin \beta_{LD} \left[\bar{L} H_3 L \bar{D} (H_+ + H_-) D - \bar{L} (H_+ + H_-) L \bar{D} H_3 D \right] . \qquad (14)$$

As anticipated in our introduction, in the small β_{LD} limit, the $|\Delta G| = 1$ amplitudes are proportional to β_{LD} and the $|\Delta G| = 2$ amplitudes are proportional to β_{LD}^2.

A similar analysis can be obtained for the L-U term. However, if we consider both the L-D and L-U terms at once, as we must in the coherent capture of a μ by a nucleus with conversion to an electron, we cannot absorb both α_{LU} and α_{LD} into the phases of L. If we absorb α_{LD} into L, we have

$$\mathcal{L}_{LU+LD} = \frac{g_H^2}{M^2}\bar{L}\underset{\sim}{H}L\Big[\bar{D}e^{i\beta_{LD}H_2}\underset{\sim}{H}e^{-i\beta_{LD}H_2}D$$
$$+\,\bar{U}e^{i\beta_{LU}H_2}e^{i(\alpha_{LU}-\alpha_{LD})H_3}\underset{\sim}{H}e^{-i(\alpha_{LU}-\alpha_{LD})H_3}e^{-i\beta_{LU}H_2}U\Big]\,. \quad (15)$$

The explicit evaluation of this interaction yields Eq. (14) plus the L-U interaction

$$\mathcal{L}_{LU} = \frac{g_H^2}{M^2}\Big\{\frac{1}{2}\cos^2\frac{\beta_{LU}}{2}\left[e^{i\alpha}\bar{L}H_+L\,\bar{U}H_-U + e^{-i\alpha}\bar{L}H_-L\,\bar{U}H_+U\right]$$
$$+\cos\beta_{LU}\,\bar{L}H_3L\,\bar{U}H_3U$$
$$-\frac{1}{2}\sin^2\frac{\beta_{LU}}{2}\left[e^{i\alpha}\bar{L}H_+L\,\bar{U}H_+U + e^{-i\alpha}\bar{L}H_-L\,\bar{U}H_-U\right]$$
$$+\sin\beta_{LU}\left[\bar{L}H_3L\,\bar{U}(H_+ + H_-)U - e^{i\alpha}\bar{L}H_+L\,\bar{U}H_3U - e^{-i\alpha}\bar{L}H_-L\,\bar{U}H_3U\right]\Big\}. \quad (16)$$

Here $\alpha = \alpha_{LU} - \alpha_{DU}$. For a process involving only L-U interactions, the phases in Eq. (16) are unimportant, but for a process involving both L-U and L-D, α is physically significant.

To summarize the mixing angle arrangement for an $SU(2)_H$ model for two generations with three degenerate gauge bosons, we note that the diagonal interactions (L-L, D-D, etc.) conserve G so there is no contribution to the K_L-K_S mass difference nor is the decay $\mu \to 3e$ induced. The non-diagonal processes (L-D, L-U) violate G-conservation through mixing angles.

Let us turn now to the case in which two of the horizontal bosons are degenerate, with mass $\bar{M}$, and the third boson has a different mass, M_3. We choose the generator H_3 to go with the non-degenerate boson. Then the interaction is

$$\mathcal{L} = \frac{g_H^2}{2}\left[\bar{M}^{-2}\bar{F}^o\underset{\sim}{H}F^o\bar{F}^o\underset{\sim}{H}F^o + (M_3^{-2} - \bar{M}^{-2})\bar{F}^oH_3F^o\bar{F}^oH_3F^o\right]\,. \quad (17)$$

The second term induces violation of G-conservation even in diagonal interactions. We can make this explicit by writing

$$\bar{L}^oH_3L^o = \bar{L}\,\mathcal{U}^LH_3\mathcal{U}^{L+}L$$

where $\mathcal{U}^L$ can be expressed as

$$\mathcal{U}^L = \exp(-\,i\alpha_LH_3)\,\exp(-\,i\beta_LH_2)\,\exp(-\,i\gamma_LH_3) \quad (18)$$

Now the rotation by γ_L has no effect and the rotation by α_L may be absorbed into the definition of L. Thus the interaction is proportional to

$$(\bar{L}e^{-i\beta_L H_2} H_3 e^{i\beta_L H_2} L)^2 = \cos^2\beta_L \, \bar{L}H_3L \, \bar{L}H_3L$$

$$+ \frac{1}{2}\sin^2\beta_L \, \bar{L}H_+L \, \bar{L}H_-L + \sin\beta_L \cos\beta_L \, \bar{L}H_3L \, \bar{L}(H_+ + H_-)L$$

$$+ \frac{1}{4}\sin^2\beta_L (\bar{L}H_+L \quad \bar{L}H_+L + \bar{L}H_-L \, \bar{L}H_-L) \tag{19}$$

This would induce the decay $\mu \to 3e$ and the transition between muonium (μ^+e^-) and anti-muonium (μ^-e^+). An analogous term for D-D would contribute to the K_L-K_S mass difference. It is useful to define:

$$\Delta^{-2} = M_3^{-2} - \bar{M}^{-2} \tag{20}$$

The bounds on the "diagonal" processes then serve as bounds on $\frac{g_H^2}{\Delta^2}$ rather than on $\frac{g_H^2}{M_H^2}$.

The breaking of the degeneracy among the three horizontal bosons is enough to insure the existence of G-violating terms even in the diagonal interactions. It is possible to consider the case in which all three horizontal bosons are non-degenerate. Of course this will induce G violation in both the non-diagonal and diagonal terms. All three of the horizontal bosons would then be self-conjugate. While there would be more parameters characterizing the horizontal interactions, they would be qualitatively the same as for the case with two degenerate vectors.

Thus far we have ignored the chiral structure of the models. There is impressive evidence that the $SU(2)_W$ assignments of the

fundamental fermions are simply that all left-handed fermions are doublets and all right-handed ones are singlets, that is to say, the $SU(2)_W$ interactions are V - A. For our horizontal interactions we shall consider two possibilities. Intuitively, a pure V interaction seems more plausible, since the V - A structure might be expected to be unique to $SU(2)_W$, and we think of both the right and left handed pieces of the muon as being in the second generation. Nevertheless, we shall not stick only to this assignment, but shall consider V-A also, especially when it is easier to calculate. In most instances we expect the V-A and V theories to give similiar results (with an important exception: $K^o \to e\mu$).

The examination of $SU(2)_H$ above suggests some basic features which we should incorporate into our analysis in the following Section. Our intent is to avoid choosing a specific model (such as $SU(2)_H$) but we shall abstract from what we have learned above.

In particular, we shall adopt Eqs. (14) and (16) as describing the L-D and L-U interactions. As usual, a γ^μ is understood to lie between the spinors. If the interaction is V-A rather than V, each spinor is understood to have a $(1 - \gamma_5)/2$ attached to it.[8] We have in mind that the β_i are small and of roughly the same size as θ_c, as suggested in the Introduction.

For the diagonal interactions, there are two terms. The first is the G-conserving piece. The second arises from some explicit breaking of the global symmetry and violates G. We write the sum of the two as

$$\mathcal{L}_{L\text{-}L} = \frac{g_H^2}{2\bar{M}^2} \bar{L}\underset{\sim}{H}L \cdot \bar{L}\underset{\sim}{H}L + \frac{g_H^2}{2\Delta^2} [\cos^2\beta_L \, \bar{L}H_3L \, \bar{L}H_3L + \frac{1}{2}\sin^2\beta_L \, \bar{L}H_+L\bar{L}H_-L$$

$$+ \sin\beta_L \cos\beta_L \, \bar{L}H_3L \, \bar{L}(H_+ + H_-)L + \frac{1}{4}\sin^2\beta_L \quad (\bar{L}H_+L \, \bar{L}H_+L + \bar{L}H_-L \, \bar{L}H_-L)], \tag{21}$$

with similar expressions for the D-D and U-U interactions.

While these choices are based on the $SU(2)_H$ model, we expect they are appropriate to a broad class of models. With a sufficiently explicit model, we could hope to predict the various mixing angles which we have left as unknown parameters here.

The case of three or more generations is similar to the one discussed above except that we start with three or more "primitive" leptons and quarks, H_i is not given by σ_i but by a higher SU(2) representation, there are many more mixing angles and phases, etc. However, the main qualitative features remain unchanged, and the expressions derived for the case of two generations, probably still serve as a good approximation.

III. EXPERIMENTAL LIMITS ON GENERATION CHANGING CURRENTS

A. $\Delta G = 0$ Nondiagonal Processes: $K_L^o \to e\mu$, $K^+ \to \pi^+ e^- \mu^+$.

The decays $K_L^o \to e^{\pm}\mu^{\mp}$ and $K^+ \to \pi^+ e^- \mu^+$ violate the separate conservation of muon and electron number. Stringent bounds for the branching ratios for these decays have been established experimentally:[9,10]

$$BR(K_L^o \to e^{\pm}\mu^{\mp}) < 1.6 \times 10^{-9} , \tag{22}$$

$$BR(K^+ \to \pi^+ e^- \mu^+) < 4.8 \times 10^{-9} . \tag{23}$$

Horizontal bosons would be expected to induce such decays.

Consider first the decay $K^o \to e^-\mu^+$. Since the K^o is a pseudoscalar, this process would not be induced by a pure vector interaction coupling d to s and e to μ. It could be caused

by a pure vector interaction mediated by a charged "leptoquark" vector boson which coupled e to d and μ to s. We shall not consider such charged vectors here, assuming that they are extremely heavy as demanded by grand unification schemes. We therefore confine ourselves to the sort of vectors discussed in the preceeding Section (Fig. 1b, 1d). Let us suppose, however, that there is a V-A interaction

$$\mathcal{L} = \frac{g_H^2}{M_H^2}\,\frac{1}{2}\cos^2\beta_{LU}\;\bar{L}H_-L\,\bar{D}H_+D \to \frac{g_H^2}{2M_H^2}\;\cos^2\beta_{LU}(\bar{e}_L\gamma_\mu\mu_L)(\bar{s}_L\gamma^\mu d_L), \tag{24}$$

which may be compared with the usual charged weak current interaction which is responsible for the dominant decay $K^+ \to \mu^+\nu_\mu$:

$$\mathcal{L} = \frac{g_W^2}{2M_W^2}\;\sin\theta_c\;\bar{\nu}_L\gamma_\alpha\mu_L\bar{s}_L\gamma^\alpha u_L \tag{25}$$

We shall assume that $\cos\beta_{LU} \cong 1$, so we have

$$\frac{\Gamma(K_L^o \to e^-\mu^+)}{\Gamma(K^+ \to \mu^+\nu_\mu)} = \frac{\left(\frac{g_H^2}{2M_H^2}\right)^2}{\sin^2\theta_c\left(\frac{g_W^2}{2M_W^2}\right)^2} < 0.60 \times 10^{-9}$$

or

$$M_H > 36\ \text{TeV}\left|\frac{g_H}{g_W}\right| . \tag{26}$$

We have taken $M_W = 85$ GeV.

The decay $K^+ \to \pi^+ e^- \mu^+$ can occur through purely vector currents. Let us compare it to the conventional K_{e3} decay: $K^+ \to \pi^o\,{}^+\mu\nu_\mu$. We take as our horizontal interaction

$$\mathcal{L} = \frac{g_H^2}{2M_H^2} (\bar{e}\gamma_\alpha\mu)(\bar{s}\gamma^\alpha d) \tag{27}$$

while the weak interaction is again given by Eq. (25). Only the vector part of V-A contributes to the hadronic weak current, reducing by, $\frac{1}{4}$ the rate relative to a pure V interaction. The leptonic current factor in the rate is also smaller by $\frac{1}{2}$ for the V-A case than in the pure V case. An additional $\frac{1}{2}$ is generated by the requirement that the final state pseudoscalar be the π^o and not the η^o. The total of these effects is a reduction by 16 for the V-A case.

Altogether then, we have

$$\frac{\Gamma(K^+ \to \pi^+ e^- \mu^+)}{\Gamma(K^+ \to \pi^o \nu_\mu \mu^+)} = \left(\frac{g_H^2}{2M_H^2}\right)^2 16 \frac{1}{\sin^2\theta_c} \left(\frac{2M_W^2}{g_W^2}\right)^2$$

$$< 1.5 \times 10^{-7}, \tag{28}$$

or

$$M_H > 18 \text{ TeV} \left| \frac{g_H}{g_W} \right| . \tag{29}$$

B. $|\Delta G| = 1$ Nondiagonal Processes: $\mu N \to eN$

Muons which come to rest in matter either decay or are captured by nuclei. In the capture process, charge is transfered to the nucleus

so the process is incoherent to a very good approximation. Horizontal bosons can mediate the conversion of a muon to an electron in the presence of a nucleus (Fig. 1a). This neutral current process can occur coherently. Since the muon is decreased in generation number while the nucleus is unchanged, the process has $\Delta G = -1$. The natural basis for comparison is the relative rate of muon to electron conversion as measured against ordinary muon capture. Certain common factors, like the muon density at the nucleus, are cancelled in the ratio[11].

The effective Lagrangian for ordinary muon capture is

$$\mathcal{L} = \frac{g_W^2}{2M_W^2} \bar{\nu}_L\gamma^\alpha\mu_L \frac{1}{2} \bar{n}\gamma_\alpha(g_V - g_A\gamma_5)p. \tag{30}$$

For the horizontal interaction, let us consider a pure V theory (the axial current is not so enhanced by coherence in any event). From Eqs. (14) and (16) we have

$$\mathcal{L} = -\frac{g_W^2}{2M_H^2} \bar{e}\gamma_\lambda\mu \Big[\sin\beta_{LD}\bar{d}\,(-\tfrac{1}{2})\gamma^\lambda d$$

$$+ \sin\beta_{LU}e^{-i\alpha}\,\bar{u}(-\tfrac{1}{2})\gamma^\lambda u\Big] ,$$

$$\cong \frac{g_H^2}{2M_H^2} \frac{1}{2} \bar{e}\gamma_\lambda\mu \Big[\bar{p}\gamma^\lambda p\,(2\sin\beta_{LU}e^{-i\alpha} + \sin\beta_{LD})$$

$$+ \bar{n}\gamma^\lambda n(\sin\beta_{LU}e^{-i\alpha} + 2\sin\beta_{LD})\Big] . \tag{31}$$

The capture rate for the ordinary process may be determined from the standard formula

$$\Gamma = \rho\sigma v = |\psi(0)|^2 \sigma v \tag{32}$$

where the product $\sigma \cdot v$ is evaluated for the scattering process in the limit of zero incident energy. For the charged current process we find

$$\sigma v \approx \frac{m_\mu^2}{2\pi} (g_V^2 + 3g_A^2)\left(\frac{\sqrt{2}\, g_W^2}{8M_W^2}\right)^2 . \tag{33}$$

Assuming a nucleus with equal numbers of protons and neutrons we find the coherent rate for muon to electron conversion by analogy:

$$(\sigma v)_{\mu N\to eN} \approx \frac{m_\mu^2}{2\pi} \left(\frac{\sqrt{2} g_H^2}{4M_H^2}\right)^2 Z^2 3^2 \left|\sin\beta_{LU} e^{-i\alpha} + \sin\beta_{LD}\right|^2 . \tag{34}$$

In fact, we have neglected an important correction for each of the processes. For the cohorent process we must include a nuclear form factor. For large Z, this reduces the rate by a factor of about $6/Z$ [11]. For the incoherent process, the rate is reduced by an exclusion prinicple factor: if the neutron which is converted into a proton is in a state already occupied by a proton, the conversion is prohibited. This reduced the charged current rate by a factor of about $1/8$[11]. Altogether, then we calculate a ratio

$$\frac{\Gamma(\mu N \to eN)}{\Gamma(\mu N \to \nu N)} \approx \left(\frac{6}{Z}\right) \; 8 \; \frac{9Z^2\, |\sin\beta_{LU} e^{-i\alpha} + \sin\beta_{LD}|^2}{Z\, \frac{1}{4}\,(1 + 3g_A^2)} \left(\frac{M_W^2}{M_H^2}\right)^2 \left(\frac{g_H^2}{g_W^2}\right)^2 . \tag{35}$$

The most recent experiment finds an upper limit of 1.5×10^{-10} for the relative rate.[12] Combining these numbers we find

$$M_H > 85 \text{ TeV} \left|\frac{g_H}{g_W}\right| |\sin\beta_{LU} e^{-i\alpha} + \sin\beta_{LD}|^{\frac{1}{2}} ,$$

$$> 85 \text{ TeV} \left|\frac{g_H}{g_W}\right| |\sin\beta_{LU} - \sin\beta_{LD}|^{\frac{1}{2}} . \tag{36}$$

C. $|\Delta G| = 1$ Diagonal Processes: $\mu \to 3e, \mu \to e\gamma$

We turn next to the $|\Delta G| = 1$ process $\mu \to 3e$. This is a diagonal process, involving only charged leptons. Thus it occurs only if the global symmetry is broken, say, by a mass splitting among the hroizontal bosons. Borrowing from our study of $SU(2)_H$, we write the interaction as

$$\mathcal{L} = \frac{g_H^2}{2} \Delta^{-2} \sin\beta_L \cos\beta_L\, \bar{\mu}_L \gamma_\alpha e_L \bar{e}_L \gamma^\alpha e_L . \tag{37}$$

We have taken a V-A interaction to facilitate comparison with the dominant decay, $\mu \to e\nu\bar{\nu}$ which is induced by

$$\mathcal{L} = \frac{g_W^2}{2M_W^2} \bar{\mu}_L \gamma_\alpha \nu_{\mu L} \bar{\nu}_{eL} \gamma^\alpha e_L \tag{38}$$

The limit on the branching ratio, then, is

$$\frac{\Gamma(\mu \to 3e)}{\Gamma(\mu \to e\nu\bar{\nu})} = \left(\frac{g_H^2}{g_W^2}\right)^2 [M_W^2 \Delta^{-2}]^2 \sin^2\beta_L \cos^2\beta_L < 1.9 \times 10^{-9}, \tag{39}$$

where the experimental limit is taken from Ref. 13. To express this in a form similar to our previous limits, we write

$$\Delta > 13 \text{ TeV} |\sin\beta_L \cos\beta_L|^{\frac{1}{2}} \left|\frac{g_H}{g_W}\right| . \tag{40}$$

The usefulness of this limit is reduced not just by our ignorance of the precise relation between, Δ and M_μ but also because β_L could be small or even vanishing!

A frequently discussed rare decay is $\mu \to e\gamma$. We calculate it from the diagrams in Fig. 2. The diagrams are separately divergent, but a GIM-like mechanism renders the sum finite. We calculate in a pure vector theory with a single vector boson which is supposed to mimic the effect of the symmetry breaking which permits the $|\Delta G| = 1$ process. We take the coupling of this single horizontal boson to be

$$g_H H_\alpha \bar{L}^o H_3 \gamma^\alpha L^o = g_H H_\alpha [\tfrac{1}{2}(\bar{\mu}\gamma^\alpha\mu - \bar{e}\gamma^\alpha e)\cos\beta_L + (\bar{\mu}\gamma^\alpha e + \bar{e}\gamma^\alpha\mu)\sin\beta_L] \quad (41)$$

In keeping with the spirit of our discussion of symmetry breaking, we use for its mass

$$M_H = \Delta \quad (42)$$

so that this interaction would generate the same effective four fermi expression as in Eq. (37). After a tedious calculation, we find[14]

$$\Gamma(\mu \to e\gamma) = \frac{\alpha}{256\pi^4}[g_H^2 \Delta^{-2}]^2 \cos^2\beta_L \sin^2\beta_L\, m_\mu^5 , \quad (43)$$

so that the branching ratio is[15]

$$BR(\mu \to e\gamma) = \frac{24\alpha}{\pi}\cos^2\beta_L \sin^2\beta_L \left(\frac{g_H}{g_W}\right)^4 < 1.9 \times 10^{-10}, \quad (44)$$

or

$$\Delta > 11 \text{ TeV } |\sin\beta_L \cos\beta_L|^{\frac{1}{2}} \left|\frac{g_H}{g_W}\right| . \quad (45)$$

D. $|\Delta G| = 2$ Diagonal Processes; $\Delta M(K_L^o - K_S^o)$.

The $K_L - K_S$ mass difference was used by Lee and Gaillard before the discovery of the ψ-particles to estimate the mass of the charmed quark[16]. Remarkably, they found a value of about 2 GeV. We use this success to argue that any additional contribution to the mass difference from the horizontal bosons (Fig. 1e) cannot be much bigger than the contribution of the charmed quark. Thus we compare the Gaillard-Lee Lagrangian[16] with our diagonal term D - D using a V-A interaction for convenience:

$$\mathcal{L}_{G-L} = -\frac{g_W^2}{8M_W^2}\left(\frac{\alpha}{4\pi}\right)\frac{M_c^2}{M_W^2 \sin^2\theta_W}\cos^2\theta_c \sin^2\theta_c [\bar{s}\gamma_\mu \tfrac{1}{2}(1-\gamma_5)d]^2 , \quad (46)$$

$$\mathcal{L}_{\text{Horizontal}} = \frac{g_H^2}{8}\Delta^{-2}\sin^2\beta_D [\bar{s}\gamma_\mu \tfrac{1}{2}(1-\gamma_5)d]^2 . \quad (47)$$

Thus we find, equating the strength of the two interactions,

$$\Delta > 400 \text{ TeV} \left|\frac{g_H}{g_W}\right| \sin\beta_D . \quad (48)$$

IV. DISCUSSION

The results of Section III are summarized in Tables II and III. In each case we used either V or V-A, and the choice was dictated by convenience rather than by any deep physical principle. However, except for the case of $K \to \mu e$, the bounds on M or Δ are modified by no more than a factor of two when we switch from V to V-A or back.

All bounds are given in terms of $\frac{g_H}{g_W}$ which is likely to be of order one, or a bit larger. If we arbitrarily assume that $\frac{g_H}{g_W} = 1$

and all angle factors of order β are of order θ_c we find that all our lower limits are in the range 10-100 TeV. Needless to say, there are many other generation-changing processes $(\mu^- e^+ \to e^- \mu^+, \tau \to \mu\gamma, \Sigma \to pe\mu$, etc). However, all processes except for the six we discussed here, yield much poorer limits.

The relative significance of our limits depends on the values of the β-angles. The only mixing angle we know is the Cabibbo angle, but it does not appear in any of our bounds. In principle β_L or β_D (or both) could vanish. In such a case, some of our limits would become trivial. Consequently, we believe that the two $\Delta G = 0$ processes $K_L^0 \to \mu^\pm e^\mp$ and $K^+ \to \pi^+ e^- \mu^+$ provide us with the firmest and the least model-dependent bounds. While the $K_L^0 \to \mu^\pm e^\mp$ limit assumes that the horizontal interaction is not a pure vector interaction, the limit obtained from $K^+ \to \pi^+ e^- \mu^+$ is truly "safe". For both of these processes substantial experimental improvements are now possible. However, the bounds on M_H vary like the fourth root of the branching ratio. Thus a significant improvement of both branching ratios down to 10^{-11} would only improve the limits on M_H by a factor of 4 or so.

The limit for the $|\Delta G| = 1$ nondiagonal process $\mu N \to eN$ is also interesting. If $\beta_{LU} - \beta_{LD} \sim \theta_c$, we get the strongest limit on M_H from this process. While it is possible that β_{LU} and β_{LD} conspire to make the angle factor very small, it is likely that it is not. In this case, $\mu N \to eN$ may be our best process!

The limits obtained from the diagonal processes are, of course, totally meaningless if all horizontal masses are equal (i.e. $\Delta^{-2} = 0$). The $|\Delta G| = 1$ processes $\mu \to 3e$ and $\mu \to e\gamma$ provide us, in any case, with relatively low limits, considering the real possibility of a small β_L. However, if Δ happens to be of the same order of magnitude

as M_3 and $\bar{M}$ (e.g. $\Delta = M_3 = \sqrt{2}\bar{M}$), the limit obtained from the $K_S^0 - K_L^0$ mass difference is potentially very interesting. It depends, of course, also on β_D but for $\Delta \sim M$ and $\beta_D \sim \theta_c$ we get the strongest limit from this process. Here, however, no experimental improvements will help. All the uncertainties are purely theoretical, including the assumption that the horizontal boson contribution may be as large as that of the charmed quark contribution computed by Gaillard and Lee[16].

In view of the above remarks we conclude that improved measurements of $\mu N \to eN$, $K^+ \to \pi^+ e^- \mu^+$ and $K_L^0 \to \mu e$ may prove extremely interesting (especially if they reveal non-vanishing rates!) Even if no real effects are discovered, substantial improvements of the experimental limits will lead to an improved bounds for M_H. These bounds are already interesting because they penetrate the multi-TeV region where the next level of inter-generation physics may lie. Raising the limits on M_H beyond 100 TeV could prove to be a significant constraint on future explicit models.

Quite aside from our remarks, it is, of course, extremely important to push the experimental limits on all rare process, as hard as one can. The study of such processes, including our six reactions as well as proton decay and other esoteric processes, is almost the only experimental way of probing the physics which lies beyond the standard $SU(3)_c \times SU(2)_W \times U(1)$ gauge theory of the strong, electromagnetic and weak interactions.

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FIGURE CAPTIONS

Fig. 1. (a) Coherent μ to e conversion by H^o exchange with a nucleus. Only the leptonic vertex is generation-changing.

(b) The decay $K^+ \to \pi^+ \mu^+ e^-$ which conserves G.

(c) The decay $\mu^- \to e^- e^- e^+$ is $|\Delta G| = 1$. The process is diagonal in that the horizontal boson connects to the same horizontal doublet (L) at both ends.

(d) The non-diagonal, $|\Delta G| = 0$ decay $K^o \to \mu^+ e^-$.

(e) The $|\Delta G| = 2$ interaction which can contribute to the $K_L - K_S$ mass difference.

Fig. 2. The two diagrams which contribute to $\mu \to e\gamma$, according to Eq. (41).

TABLES

Table I: Classification of some rare generation-changing processes according to ΔG Diagonal and non-diagonal processes are distinguished. Diagonal $\Delta G = 0$ processes are allowed and are of no interest to us here. The amplitude for a given ΔG is of order $\beta^{|\Delta G|}$ where β is a small generation-mixing angle. We list only processes involving the first two generations.

	Diagonal	Non-Diagonal
$\Delta G = 0$	Allowed	$K_L^0 \to e^{\pm}\mu^{\mp}$; $K^+ \to \pi^+ e^- \mu^+$
$\lvert\Delta G\rvert = 1$	$\mu \to 3e$; $\mu \to e\gamma$	$\mu^- N \to e^- N$
$\lvert\Delta G\rvert = 2$	$\Delta M(K_S - K_L)$; $\mu^+ e^- \to \mu^- e^+$	$K^+ \to \pi^+ e^+ \mu^-$

Table II: Limits on horizontal boson masses for non-diagonal processes, assuming equal masses for horizontal bosons.

Process	$\lvert\Delta G\rvert$	Lower limit for M_H	Interaction used for estimate
$K_L^0 \to \mu^{\mp} e^{\pm}$	0	36 TeV $\times (g_H/g_W)$	V - A
$K^+ \to \pi^+ e^- \mu^+$	0	18 TeV $\times (g_H/g_W)$	V
$\mu^- N \to e^- N$	1	85 TeV $\times (g_H/g_W) \lvert \sin\beta_{LU} - \sin\beta_{LD} \rvert^{\frac{1}{2}}$	V

Table III: Limits on the mass parameter Δ from $|\Delta G| \neq 0$ diagonal process. $\Delta^{-2} = M_3^{-2} - \bar{M}^{-2}$ in an $SU(2)_H$ model with $H_\pm$ and H_3 having masses $\bar{M}$ and M_3, respectively.

Process	$\lvert\Delta G\rvert$	Lower limit for Δ	Interaction used for estimate
$\mu \to 3e$	1	13 TeV $\times (g_H/g_W) \lvert \sin\beta_L \cos\beta_L \rvert^{\frac{1}{2}}$	V - A
$\mu \to e\gamma$	1	11 TeV $\times (g_H/g_W) \lvert \sin\beta_L \cos\beta_L \rvert^{\frac{1}{2}}$	V
$\Delta M(K_S^0 - K_L^0)$	2	400 TeV $\times (g_H/g_W) \sin\beta_D$	V - A

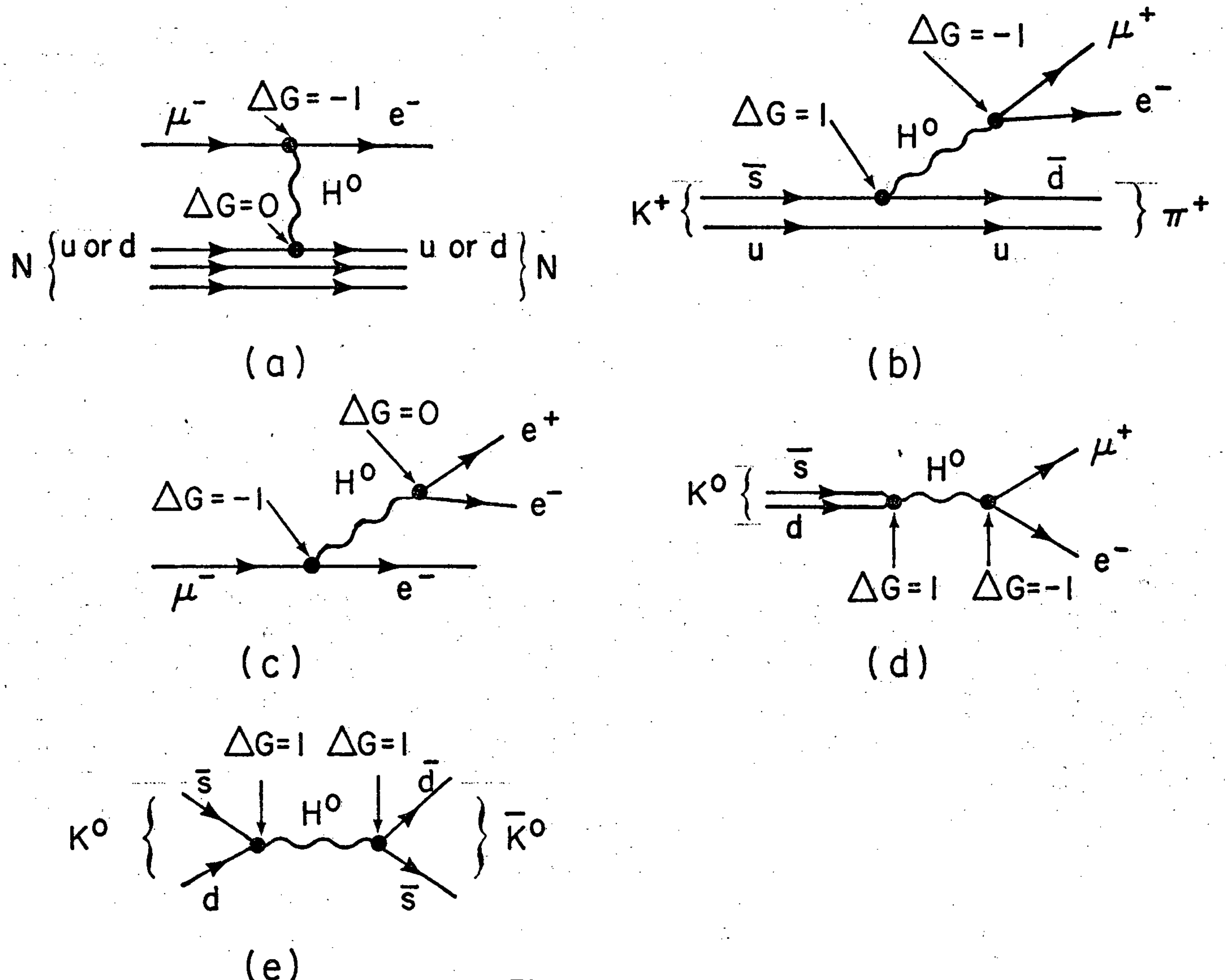

Figure 1

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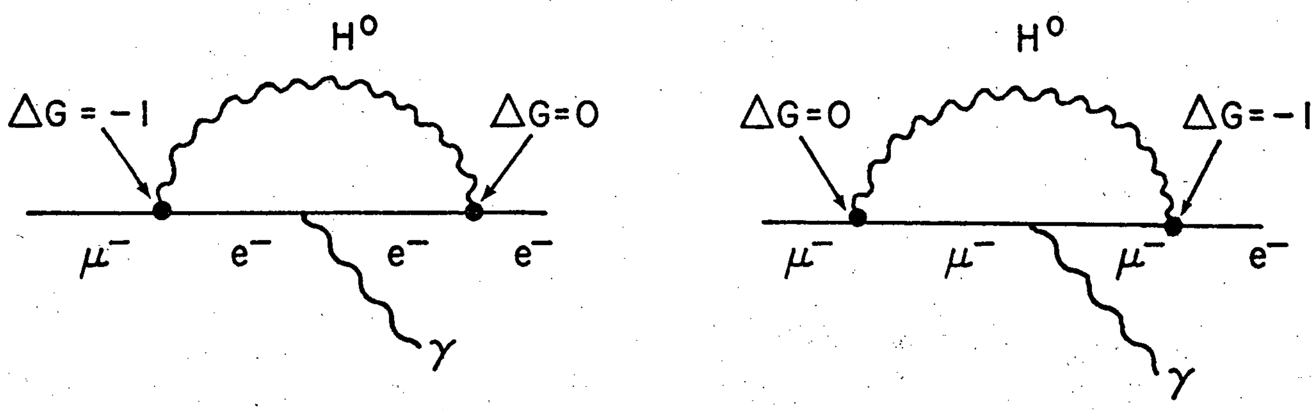

Figure 2

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