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ABSTRACT

This paper presents a methodology for the analysis of activity patterns based on a classification procedure in which the set of measurements that define human movement is represented by an Ndimensional pattern vector. Transformation techniques are then applied to the pattern vectors to develop a taxonomy for the pattern space. Subsequent inversion of the transformed patterns yields representative activity patterns and leads to attendent transformation of the results of analysis to the real world. Pattern recognition theory is demonstrated to be an effective means by which complex activity/travel patterns can be transformed into a structurally simpler space for purposes of analysis.

1. INTRODUCTION

The analysis of activity patterns is a classification problem in which the input is a set of measurements that define human movement, and the output is the classification of this movement into a finite set of either "natural" or predetermined categories. The time-geographic approach to human movement (Hägerstrand, 1974) has been adapted in the depiction of human activity as a continuous, piecewise smooth surface in the space/time continuum. Since the attendant dimensionality of such a space-time-activity representation is quite large, it becomes necessary to reduce the complexity of the measurement vector while maintaining the corresponding information content for pattern comparison.

Pattern recognition theory has been utilized in several fields as a method of image analysis and character recognition (Andrews, 1971). It is convenient to conceptualize the process in three successive stages--(1) pattern specification, (2) feature extraction, and (3) pattern classification. The three-dimensional representation of human activity is first decomposed into corresponding two-dimensional temporal patterns, then further reduced to dual pattern vectors through sampling the temporal variation. Various transformation techniques may then be utilized to construct a simpler feature space with reduced dimensionality, which enables the third stage to be performed in a more efficient, high information, reduced space. Subsequent inversion of the classification results leads to the identification of representative patterns.

2. THEORETICAL DEVELOPMENT

In this section some of the procedures of pattern recognition theory that appear to be of use in activity pattern analysis are outlined. A

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more detailed and formal treatment of these (and other) procedures is presented by Young and Calvert (1974), on which much of the material in this section is based. The set of measurements that define human movement is represented by an N-dimensional vector \underline{x} , which is labelled a "pattern vector." The components of \underline{x} are the N measurements and the taxonomy of activity patterns into "natural" categories depends on the vector \underline{x} , i.e.,

$$C = \delta(\underline{x}) \tag{1}$$

where C represents the category to which \underline{x} belongs, and $\delta(\underline{x})$ is the decision function. The set of all possible values that \underline{x} may assume is defined by $\Omega_{\underline{x}}$, the activity pattern space. For example, $\Omega_{\underline{x}}$ may comprise all points in time and space that an individual could reach from some arbitrary starting point during a continuous twenty-four hour period. Activity pattern analysis may then be described as finding a rule that divides the pattern space $\Omega_{\underline{x}}$ into a set of decision regions. Since the values of \underline{x} are determined by the set of N measurements that represent human movement, measurement selection defines the pattern space $\Omega_{\underline{y}}$.

The dimensionality of the measurement vector will, in general, be large (e.g., \underline{x} may consist of the spatial location and activity type participation of the individual at each 10-minute interval throughout a 24-hour period) and may span information superfluous to efficient classification of activity patterns. Consequently, it is advantageous to reduce the complexity of the measurement vector \underline{x} while retaining as much of the information content of \underline{x} relevant to its classification as possible. This may be accomplished by dividing activity pattern analysis into two sequential stages--feature extraction and classification. This process is shown schematically in Figure 1.

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FIGURE 1. ACTIVITY PATTERN CLASSIFICATION PROCESS

The feature extractor is a linear or nonlinear transformation that maps the N-dimensional vector \underline{x} in $\Omega_{\underline{x}}$ into an M-dimensional vector \underline{z} in $\Omega_{\underline{z}}$, i.e.,

$$\underline{z} = \delta(\underline{x}), \quad \underline{x} \in \Omega_{\mathbf{y}}, \quad \underline{z} \in \Omega_{\mathbf{z}}$$
(2)

where M < N and, hence $\delta^{-1}(\underline{z})$ is not unique, i.e., some information is lost in the feature extraction process. The selection of δ must be based on some combination of preserving the information content of \underline{x} while decreasing its dimensionality.

The simplest type of feature extractor is the linear transformation defined by the MxN matrix T,

$$z = Tx$$
 (3)

where T has rank M. The feature space $\Omega_{\rm Z}$ in this case is a subspace of $\Omega_{\rm \chi}.$

A special case of the linear feature extractor defined by Equation (3) is "feature selection," in which the M features selected are a subset of the N measurements. For this case each of the M rows of T consists of a non-zero element valued at one and (N - 1)

zero-elements, with the positions of the one-elements determined <u>a priori</u> by some criterion. A more systematic way of selecting T, given M, is to minimize the mean-square error in approximating the N-dimensional activity pattern vector \underline{x} by the set of M vectors that span Ω_z . This can be achieved by the expansion of \underline{x} in terms of a set of eigenvectors associated with the covariance matrix, known as the Karhunen-Loève expansion (see e.g. Young and Calvert (1974)). Then, the linear feature extractor is defined by

$$\underline{z}^{\mathsf{T}} = \underline{x}^{\mathsf{T}} \mathbf{T}^{\mathsf{T}}$$
(4)

with

$$\mathsf{T}^{\mathsf{T}} = [\underbrace{\mathsf{u}}_{1}, \underbrace{\mathsf{u}}_{2}, \dots, \underbrace{\mathsf{u}}_{\mathsf{M}}] \tag{5}$$

where $\underline{u}_1, \ldots, \underline{u}_M$ are the eigenvectors associated with the M largest eigenvalues of the covariance matrix.

While the linear extractor based on the Karhunen-Loève expansion is optimal in the sense of maintaining the information content of \underline{x} , implementation is hindered by the need to diagonalize rather large covariance matrices. Eigenvectors defined by other systems may be useful in defining a feature selection rotation matrix and are readily implementable. Two such transforms, Fourier and Walsh, are promising as feature extractors in activity pattern analysis for several reasons: 1) rapid implementation algorithms are available--Fast Fourier (Cooley and Tukey, 1956) and Fast Walsh (Whelchel and Guinn, 1968) Transform algorithms, 2) an information theoretic justification has been advanced (Pearl, 1971) based on rate distortion theory and 3) the transforms can be applied to continuous functions, x(t).

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The well-known Fourier sinusoidal transforms are based on transformation of x(t) in terms of an infinite series of orthogonal trigonometric functions. The corresponding feature vector \underline{z} in this case would have as components the first M coefficients a_n , b_n of the Fourier expansion.

Alternatively, the Walsh transform is based on binary functions (known as Walsh functions) which form a complete basis and are defined by:

$$\begin{array}{c} \text{cal} (\mathbf{i}, \theta) = \text{wal} (2\mathbf{i}, \theta) \\ \text{sal} (\mathbf{i}, \theta) = \text{wal} (2\mathbf{i} - 1, \theta) \\ \text{wal} (2\mathbf{i}, \theta) = \text{wal} (2\mathbf{i} - 1, \theta) = 0 , \qquad (6) \\ \theta < -1/2 \quad \text{or} \quad \theta \ge 1/2 \end{array}$$

where the difference equation defining wal(k, θ) is

wal
$$(2k + q, \theta) = (-1)^{2k+1}$$
 [wal $(k, 2 \theta + 1/2)$ (7)
+ $(-1)^{k+q}$ wal $(k, 2 \theta - 1/2)$

with

$$w(0, \theta) = \begin{cases} 1, & -1/2 \le \theta \le 1/2 \\ 0, & \theta \le 1/2, \theta \ge 1/2 \end{cases}$$
(8)

and

$$q = 0, 1; k = 0, 1, 2, \dots$$
 (9)

The corresponding Walsh-Fourier series expansion of x(t), defined over $-1/2 < t \le 1/2$ is

$$x(t) = a_0 wal(0,t) + \sum_{n=1}^{\infty} [a_c(n) cal(n,t) + a_s(n) sal(n,t)] (10)$$

where

$$a(0) = \int x(t) dt$$

$$-1/2$$
(11)

$$a_{c}(n) = \int x(t) cal(n,t) dt$$
 (12)
-1/2

$$a_{s}(n) = \int_{-1/2}^{1/2} x(t) \text{ sal } (n,t) \text{ dt}$$
 (13)

The corresponding feature vector \underline{z} would have as components the first M coefficients a(0), $a_c(n)$, $a_s(n)$.

The three-dimensional (time-space-activity type) activity pattern can be depicted by two corresponding images in two-dimensional space. A space/time continua and an activity/time continua detail individual location and activity participation over time and become complementary pattern vectors for feature extraction.

An example of such a representation for a hypothetical activity pattern is shown in Figure 2 where the activity pattern is decomposed into its projections on the time/space and time/activity continua. It is noted that activity types are nominally scaled; no metric is implied. This nominal scale presents no analytic problem since the associated pattern recognition problem is basically one of label identification rather than measurement. The transformation decomposes each label into corresponding "transform building blocks."

Performance of an activity incurs no spatial displacement throughout its duration, thus the measurement vector x(t) is a constant. For the activity/time continua, x(t) is a pure "step-function," however non-zero slope segments link sequential activities in the space/time continua, corresponding to spatial displacement during travel. The nature of this decomposition provides a potential advantage of the Walsh over the

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Fourier transform, as the binary Walsh functions better represent the piece-wise constant characteristic of the patter vectors. Alternatively, with the Fourier sinusoidal transformation, transitions between travel and activity participation become less distinguishable in the feature space then in the pattern space.

If the temporal dimension is divided into R grid points with each point i assigned values X(i,1), X(i,2) corresponding to the individual's distance from home and activity participation at time i, respectively, the image depicted in Figure 2a can be represented as the R x 2 array [X(i,j)]. Generalizations to more complex representations of the activity pattern (e.g., the expansion of the spatial dimension to conventional two-dimensional coordinate values or inclusion of the number of other persons accompanying the individual) are easily effected by increasing the dimensionality of this array; i.e., by representation as the R x Q array [X(i,j)], where Q represents the number of salient characteristics associated with the individual's activity pattern. A general transformation of this R x Q image can be written as

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$$Z(k,1) = \frac{1}{\sqrt{RQ}} \sum_{i=0}^{R-1} \sum_{j=0}^{Q-1} X(i,j) W(i,j,k,1)$$
(14)

where W(i,j,k,1) is some weighting function. If the weighting function is assumed separable on the two axes, Equation (14) can be rewritten as

$$Z(k,1) = \frac{1}{\sqrt{RQ}} \sum_{i=0}^{R-1} \sum_{j=0}^{Q-1} b(k,i) x(i,j) a(j,1)$$
(15)

or, in matrix form

$$Z = \frac{1}{\sqrt{RQ}} B^{\mathsf{T}} X A \qquad (16)$$

A transformation based on the Walsh functions described previously is generated from a Hadamard matrix (Hadamard, 1893) and is known as the Walsh/Hadamard transformation. The Hadamard matrix is a square array of plus and minus ones whose rows and columns are Walsh functions which are orthogonal to each other. The simplest Hadamard matrix is

$$H = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$
(17)

and any Hadamard matrix G of order 2N can be constructed from a Hadamard matrix H of order N by

$$G = \begin{bmatrix} H & H \\ H & -H \end{bmatrix} .$$
(18)

The Walsh/Hadamard transform of an $N \times N$ image [x(ij)] is given by

$$[z(k,1)] = \frac{1}{N} [h(k,i)][x(i,j)][h(j,i)]$$
(19)

or, in matrix form,

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$$Z = \frac{1}{N} H X H \qquad (20)$$

A unique feature of the Walsh/Hadamard transformation is that the original image can be reconstructed from the transformed image as

$$X = \frac{1}{N} HZH \qquad (21)$$

If $N = 2^n$, the transform may be written in series form as

$$Z(k,1) = \frac{1}{N} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} x(k,j)(-1)^{p(i,j,k,1)}$$
(22)

where

$$p(i,j,k,1) = \sum_{s=0}^{n-1} (k_s i_s + l_s j_s)$$
(23)

in which k_s , i_s , l_s , and j_s denote the s-th bit in the binary representations of k, i, l, and j respectively, i.e.,

$$k = (k_{n-1}k_{n-2}k_{n-3} \dots k_1k_0)$$
, $k_s = 0$ or 1 . (24)

In this series representation the elements are not ordered in any useful manner as in the Fourier sinusoidal transform in which the elements are ordered according to increasing frequency. Walsh functions are ordered typically according to the number of zero crossings of the function over the period, called the sequency. The corresponding ordered form of the transformation in which the sequency of each row is greater than that of the previous row is given by:

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$$[z(k,1)] = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} x(i,j)(-1)^{q(i,j,k,1)}$$
(25)

where

$$q(i,j,k,1) = \sum_{s=0}^{N-1} [g_s(k)i_s + g_s(1)j_s]$$
(26)

in which

$$g_{0}(k) = k_{n-1}$$

$$g_{1}(k) = k_{n-1} + k_{n-2}$$

$$g_{2}(k) = k_{n-2} + k_{n-3}$$

$$\vdots$$

$$g_{n-1}(k) = k_{1} + k_{0}$$
(27)

As in the one-dimensional case discussed previously, the Walsh-Hadamard transformation is particularly appealing in activity pattern analysis because of its ability to maintain distinctions between classes of activity and travel.

Finally, a linear transform of potential use in activity pattern analysis is the Haar Transformation (Haar, 1910). Unlike the transforms considered previously, this transform provides a domain which is both locally and globally sensitive, that is, the functions sample the input image at progressively finer intervals, starting with the lowest resolution and increasing in powers of two. This feature is useful in the study of linkages between activities, such as chaining, and the transformation is easily obtained in a manner analogous to Eq. (19).

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3. APPLICATION

The theoretical concepts supporting the transformation from the pattern space, through the feature space, into the classification space are examined in application to actual activity patterns. Travel/activity diaries of 664 individuals from Orange County, California were randomly selected from the 1976 Southern California Association of Governments (SCAG) and California Department of Transportation (CALTRANS) Urban and Rural Travel Survey. The results of a simple linear feature extractor and of an application of the Walsh-Hadamard rotational transform are each utilized in a clustering classification algorithm to identify representative patterns.

3.1. FEATURE SELECTION

The simplest of the linear feature extractors, feature selection, involves the subjective, <u>a priori</u> choice of pattern features, divided among spatial and temporal aspects of the activity pattern (Table 1). A heuristic procedure was utilized to identify those attributes which best distinguished among sample patterns, and indices describing these attributes were constructed. The choice sample was cluster analyzed using a modified Ball and Hall algorithm (Ball and Hall, 1968) with standardized values of the salient aspects identified. A comparison of pseudo F-ratios revealed five distinct groupings; however, the absence of activity-specific aspects limit the interpretation of the results.

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Table 1

Identification of Activity Pattern Indices

<u>Spatial</u>	NEX:	Number of trips made by the individual
	TDIST:	Total distance traveled during these trips
	TDISTM:	Mean trip length (TDIST/NEX)
	TLAP:	Total length of activity pattern in space-time
	TEXRATIO:	Ratio of number of tours executed to most
		efficient activity pattern
	AVG:	Mean range of activity pattern
	STD:	Standard Deviation of mean range of activity
		pattern
	AREA:	Total area of the activity pattern
Temporal	TBAR:	Temporal centroid of the activity pattern
	SKEW:	Temporal skewness of the activity pattern
	PEAK:	Temporal peakedness of the activity pattern
	TRATIO:	Ratio of total time performing activities to
		total time spent traveling
	TNHRATIO:	Ratio of total time performing non-home
		activities to total time spent traveling
	TNHACTM:	Mean non-home activity duration

The five groups are characterized by distinct differences in activity pattern parameters. Group 1 consists of individuals who travel long distances to a fewer number of activities with realtively long durations, thus devoting a greater proportion of time accessing non-home activities. Activity patterns associated with individuals in Group 2 (44.2 percent of the sample) are characterized by a large number of efficiently organized short trips (i.e., chained trips) to activity sites of short duration. Group 3 is comprised of individuals who travel very short distances, earlier in the day, to one (or a few) activity sites with relatively long duration. The extreme negative value of the efficiency index, TEXRATIO, for this group is caused primarily by values of -1 asigned to this index for "single trip" activity patterns. Individuals in Group 4 exhibit characteristics similar to those of

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Group 1, but activities in general are closer to home. Patterns of individuals in Group 5 are characterized by a small number of very short trips, made later in the day, to activity sites with short duration. Figure 3 illustrates the feature profiles for each group, plotting deviation from the sample mean, for each of the indices as developed, for each representative profile. No representative activity pattern, <u>per se</u>, results from this technique.



FIGURE 3. INDEX CLASSIFICATION ON THE BASIS OF FIVE GROUPS

3.2. ROTATIONAL TRANSFORMATION

A modified Walsh-Hadamard transformation algorithm was developed and applied to a dual pattern vector representation of the sample activity patterns. The three dimensional (time-space-activity) image characterizing each individual's behavior was split into two interrelated, two-dimensional images--the temporal variations of distance from home and of activity participation. The dual pattern vectors were constructed by sampling the temporal variation in patterns over the

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19-hour analysis day (5:30 AM to 12:30 AM) at approximately 9-minute intervals, a value determined by the transformation algorithm's restriction of 2^{N} sample points and computational efficiency (yielding 128 sample points at 8.9 minute intervals).

Activities were classified into five major categories (including travel) of which two were further divided into subcategories of similar activities (Table 2). The categories were ordinally ranked (subjectively) according to assumed temporal/spatial characteristics, with the total distance between subcategories equal to half that between categories. This apparent interval scaling constitutes an attempt to minimize classification errors after transformation. The purpose of the pattern recognition formulation is simply to distinguish among labels and nor further meaning in scale is implied. Travel is positioned at the extreme opposite of the scale to emphasize its dissimilar nature.

Category	Activity	Scale Value
 1.	Home	9.0
2a.	Work	7.0
2b.	Work related	6.5
2c.	Education	6.0
3.	Shopping	4.0
4a.	Social/entertainment	2.0
4b.	Recreation	1.5
4c.	Other	1.0
5.	Travel	-9.0

TABLE 2. ACTIVITY CLASSIFICATION

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As a preliminary, a random sample of images was selected for transformation into Walsh-Hadamard space. These transformed images were subsequently inverted, retaining only a subset of the complete set of transform coefficients, and the mean square error between the inverted image and the original image was calculated. The mean of the mean square errors for random samples of various numbers of coefficients retained is shown in Figure 4 for both the "distance" and "activity" images. Results indicate that with 30 to 50 coefficients, the original image can be reconstructed to within limits of error which are tolerable. On the basis of these experimental findings 50 transform coefficients were retained for each image. This reduced the analysis problem of 256 bits of information in real space to an equivalent problem of 100 bits in transformed space (a reduction of over 60 percent).



FIGURE 4. IMAGE REGENERATION ERRORS

The dual feature vectors, composed of the first 50 coefficients of each of the pattern vectors, were cluster analyzed in Walsh-Hadamard transformation space. The results (pseudo F-ratio) indicated that the

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respondents were best classified into 9 distinct groups in transformed space.

The transformed images associated with the coefficients centroids were inverted by Walsh-Hadamard inversion formulae to reconstruct the actual activity patterns that are representative of the travel/activity behavior of individuals in each group. The resulting activity patterns represent distinct sets of behavior by which the study population can be classified. However, because these patterns are <u>aggregated mean</u> <u>responses</u>, the definition of the representative patterns is somewhat less than that of the original individual patterns. Correspondingly, there is some latitude in the interpretation of the results. Results are presented in the form of the temporal distributions of the group members' distance from home and activity participation during the 19-hour analysis day. These two distributions were then combined to produce the representative activity pattern of the group.

For example, figures 5 through 8 illustrate the results for two of the cluster groupings. The representative pattern associated with Group B (Figure 5) is indicative of 8.4 percent of the respondents. Characterized by a traditional work activity approximately 7 miles from home and evening shopping activity within 3 miles of home, respondents in Group B are primarily employed male household heads (Figure 6).

The corresponding patterns for Group C (Figure 7), representing 12.5 percent of the respondents, differs significantly from Group B. This group is evenly distributed by sex and consists primarily of school-aged children and spouses of household heads (Figure 8). The representative pattern reflects the proximity to home of both schools and the employed spouse's work place. An evening sojourn to social/entertainment and/or recreational activities completes the representative pattern.

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The results for each representative pattern are summarized in Table 3. In addition to travel/activity characteristics and predominant group socio-economic attributes, indicators of urban form resulting from a multiple discriminant analysis are given.

REPRESENTATIVE ACTIVITY PATTERN (RAP) Number (%) Socio-Economic Characteristics Urban Form Travel/Activity Characteristics Single work trip of about 25 miles Predominantly employed, male Low density/ A 32 (4.8) No evening travel household heads high income Age (25-34) 97% Drivers Single work trip of about 7 miles Predominantly employed, male Low density/ B 56 (8.4) Evening shooping trip household heads high income Age (35-44) 93% Drivers Work/school activity within 3 miles Non-employed spouses and High density/ children, even sex and age distributions 57% Drivers C 83 (12.5) of home, evening social/recreation low income activity Multiple non-work sojourns Predominantly female non-Low density/ 62 (9.3) within 5 miles of home, employed Age (> 25) high income D no evening travel 71% Drivers Single work trip of about 15 miles Predominantly employed male Low density/ Evening work/school activity within 2 miles E 47 (7.1) household heads Age (25-54) 96% Drivers high income Single work trip of about 2 miles NA NΑ 6 (0.9) Multiple non-work evening sojourns (no return trip home before 12:00 A.M.) Single school/work trip of about Predominantly female 50% High density/ G 306 (46.1) 1 mile, no evening travel employed adults 50% school low income aged children 47% Drivers Single work trip of about 7 miles Predominantly employed even High density/ H 66 (9.9) No evening travel sex distribution low income Age (25-54) 76% Drivers Extremely long travel NA NA 6 (0.9) (not identified) I

TABLE 3. SUMMARY MEASURES OF REPRESENTATIVE ACTIVITY PATTERNS

Whereas the linear feature extractor was restricted to spatial and temporal attributes, the Walsh-Hadamard transformation explicitly incorporates the activity dimension. A cross-classification of cluster results depicted in Figure 9 shows how the transformation further

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discriminates the representative patterns based on differences in activity type. For example, over 80 percent of groups C and D are categorized as having the index profile 2, but the transformation results further incorporate the multiple sojourn aspects of Group D, and the evening travel of Group C.



4. CONCLUSIONS

Travel behavior research has entered the "third generation" of transportation demand analysis, characterized by the integration of the full individual activity pattern into the decision-making process. The transforms discussed and their application to behavioral research represent a significant departure from conventional methods of transportation analysis.

The techniques derived and examined in this study form an initial framework for the quantitative analysis of complex travel behavior in the form of individual activity patterns. No attempt was made to synthesize a definitive theory of movement behavior. The intricate mathematical

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requirements of such an initial examination of complex behavior are not yet warranted. The goal of this study was to develop alternate techniques to quantitatively depict individual activity patterns facilitating classification of pattern profiles within the population as a means of identifying common travel behavior. The techniques presented should not be considered an alternate theoretical formulation of travel behavior but, rather, tools to describe and explain complex movement.

The trade-offs between resolution of representative patterns and complexity of analysis must be examined in detail. An increase in information efficiency in image construction would allow the incorporation of additional image characteristic vectors without added computational requirements in addition to reducing the loss of information involving short duration activities during transformation. This latter improvement is also associated with the alternate usage of the Haar transform, which favors lost activities and trips in its treatment of local and regional correlations in the activity pattern image.

The prospect for utilizing transform elements (Hadamard or Haar) as building blocks for activity patterns in transform space is promising. This extension enables the transition of the graphical representation of activity patterns to functional analysis, greatly simplifying analysis of complex behavior.

Theoretical models of pattern responses are dependent on significant advances in the descriptive and explanatory power of pattern recognition, classification, and analysis techniques. Those proposed herein represent a starting point for such further research.

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