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Thermal sensation and comfort models for non-uniform and transient environments, part IV: Adaptive neutral setpoints and smoothed whole-body sensation model

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ABSTRACT

Models for body-segment-specific thermal sensation and comfort were put forward in 2010 in a three-part series in this journal. The models predict these subjective responses to the environment from thermophysiological measurements or simulations of skin and core temperatures, and apply to a range of environments: uniform and non-uniform, transient and stable. The models are based on unique experimental data, and formulated in a rational but piecewise structure that simplifies further validation and refinement. The models have received much attention and this experience has pointed out two issues needing improvement at the fundamental level. This paper presents solutions to these issues:

a) In the local sensation model, the neutral set-points for segment skin temperatures are sensitive to the distribution of clothing insulation provided by different clothing ensembles, and to metabolic rate. A new calculation sequence automatically creates accurate segment set-points for specific clothing and activity levels.
b) In the whole-body (overall) sensation model, the piecewise model construction produced unrealistic jumps in output at the transitions between pieces. A smoothing technique using the model's key organizational variables was developed and incorporated into the original model.

Several corrections and clarifications are listed in an Appendix.

1. Introduction

A three-part series in this journal has presented the development of models to predict, for different parts of human body, the local thermal sensation (Part I) [1], local thermal comfort (Part II) [2], and whole-body sensation and comfort for any combinations of these (Part III) [3]. The models were based on an extensive set of human subject tests conducted by the authors in 2001–2003, together with diverse findings from the literature. The subjects were engaged in sedentary activities in a range of environments, uniform or non-uniform, stable or transient.

Part I described local sensation models for 19 body parts. Human thermal sensation responses are determined by the character of thermoreceptors in the skin and core, which are sensitive to the rate of change of temperature. The models were formulated from such information together with transient tests on human subjects. The models are based on the logistic function, using as inputs: local skin temperature, mean skin (core) temperature, their derivatives, and each part's local set point, i.e. the part's local skin temperature when the sensation for the part is neutral. Impacts of whole-body thermal state, warm–cold asymmetry and dynamic responses in transient conditions are incorporated into the models. For the set point of each body part, separate 6-h-long tests under neutral conditions were performed to obtain segment-set-point temperatures.

In Part II, the local comfort models were developed from results in the literature and the authors' chamber experiments. The comfort models are based on local thermal sensation, and consider warm–cold asymmetry, the offset between comfort and neutrality which had been noticed by many researchers [4], and the effects of the overall thermal sensation on comfort magnitude [5]. Logistic-adapted linear models and exponential and quadratic equations were regressed to describe comfort for the different body parts.

Part III developed the models for both overall (whole-body) thermal sensation and for overall comfort. The overall thermal sensation model considers all the body segments' sensations by

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using a piecewise regression of experimental data including both uniform [6] and non-uniform tests [7]. All model coefficients were separately regressed and the results were satisfying.

The local thermal sensation models are the key to the entire series of models. During application in practice, two types of deficiencies were found in these models.

The first deficiency was in how neutral set-point was determined for calculating local sensation under different clothing levels and metabolic rates. The setpoints are the neutral skin temperatures for each body part. These temperatures depend on the local clothing and met rate of the body part, not just overall clothing insulation (clo units), as many combinations of local clo can produce the same overall clo value. The sensation model is sensitive to such differences in local clothing.

The other deficiency was caused by the discontinuity in the overall sensation model results. Across a range of environments, a sudden jump in overall sensation might appear even though the environment and local sensations was changing smoothly. This was caused by transitions in the piecewise formulation of the models described in papers Part I and Part III [1,3].

In this paper, we present improvements that deal with these two problems. The Appendix presents additional corrections and clarifications to the models that bring them up to date.

2. Set-point adaption

The neutral setpoints can be brought closer to reality through two steps: 1) determine the environmental temperature for a uniform environment that produces a neutral overall sensation for the clothing ensemble's overall clothing insulation. The PMV model [8] may be used for this. 2) Obtain the local skin temperatures that occur in that uniform neutral environment under the actual distribution of local clothing. A multi-segment physiological model is used for this. A description how to perform the two steps is provided below.

A given clothing ensemble will consist of several local insulation levels, which have been traditionally averaged into an overall clo value. Overall clo is used in models such as PMV and 2-node [9]. Table 1 contains the neutral-sensation air temperatures calculated by the PMV model for specific combinations of overall clo and met values. Intermediate values may be interpolated.

The multisegment physiological model then uses that neutral air temperature, and the ensemble's real distribution of segment-specific clothing values, runs for several hours to obtain the local neutral skin temperature for each body segment. The local sensation model uses these neutral skin temperatures as setpoints for predicting the local sensations for the given clothing ensemble under any combination of environmental conditions.

The above explanation is in terms of clo distribution. Metabolic heat generation will also not be distributed uniformly across the whole body and may differ by activity (e.g., walking liberates proportionally more heat in leg muscles than do sedentary activities). The structure of the sensation model as currently formulated may not account for activities that differ appreciably from sedentary. The empirical data upon which the model is based were obtained under sedentary test conditions, in which metabolic heat generation fits a certain pattern. For predicting local setpoints and thermal sensation under other exercise states, new empirical data will be needed [10].

Table 1
Ambient temperatures (°C) producing neutral sensation for given metabolic rates (met) and levels of overall clothing insulation (clo), calculated using the PMV model.

Met/clo	0.5	0.6	0.7	0.8	0.9	1
0.8	28.30	27.90	27.45	27.00	26.60	26.20
0.9	27.35	26.80	26.30	25.80	25.20	24.70
1.0	26.00	25.50	24.95	24.40	23.85	23.30
1.1	25.40	24.80	24.20	23.60	23.00	22.40
1.2	24.75	24.05	23.45	22.80	22.20	21.55
1.3	24.05	23.35	22.70	22.00	21.35	20.70

3. Smoothed overall sensation model

The overall sensation model consists of 7 sections (Table 2). During the development of the model, each section's parameters and accuracy were intensively validated. However, during simulations with the combined model, unrealistic jumps would appear in the output: when the environment is changing smoothly, the overall sensation might exhibit a sudden jump. In this section, we present a smoothing method to solve this problem without modifying the main parts of the original model.

Note that the Appendix describes small corrections to the model that are incorporated in the discussion below. They should be noted when comparing against Part III [3] and Part I [1].

3.1. *"Jumps" in the model*

The sudden jumps are caused by the discontinuity of the model. The continuity of a function is defined as follows, in epsilon delta language:

Given a certain c in the function domain, for any number $\varepsilon > 0$, there always exists some number $\delta > 0$ such that for all x in the domain of f with $||x - c|| < \delta$, the value of f satisfies $||f(x) - f(c)|| \leq \varepsilon$.

The jump in the model means no matter how small $\delta > 0$ is, there is a scalar $M > 0$, such that $||f(x) - f(c)|| \geq M$. These cases generally exist when conditions change from one piece of a model to another, which will be analyzed below. In addition, we intend the proposed overall sensation model to be C-1 continuous, meaning the first derivatives of the model should also be continuous. This condition is stricter than the continuity defined above. Intuitively, C-1 continuity means the model is not only continuous but smooth. It is reasonable to expect that the overall thermal sensation should change smoothly with smooth changes in the local sensations.

We categorize the reasons of the sudden jump into two types: jumps **between** pieced models and jumps **within** a pieced model. The main idea in smoothing is to find key continuous variables and conditions that determine the jumps between pieces of model and to smooth the jumps by a smoothing function of these variables and conditions.

We first denote the model in a concise way, i.e. model index shown in Table 2, to reference them. The model 1–4 represent situations when all body parts feel warm or cool. They are presented in Fig. 1. Model 5–7 represent situations when the opposite sensations (warm and cool) for different body parts exist simultaneously. Term "dominant" or "dominate body parts" referred later indicate dominant body parts, i.e. chest, back, and pelvis. More detailed descriptions for these 7 models are presented in [3].

Table 2
Representation of the model.

Model index	Name of each pieced model
1	No-opposite-sensation high-level warm
2	No-opposite-sensation high-level cold
3	No-opposite-sensation low-level warm
4	No-opposite-sensation low-level cold
5	Opposite-dominated cold
6	Opposite warm
7	Opposite cold

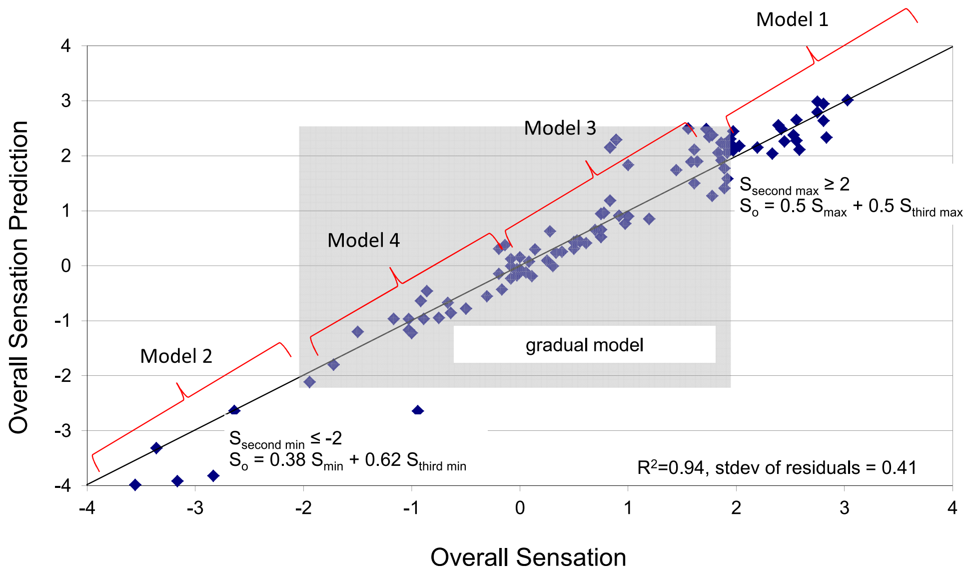

Fig. 1. Model indexes 1–4 in the overall sensation model.

3.2. Smoothing the jumps between pieced models

3.2.1. Jump analysis

Jumps between pieced models occur when local sensation values cross the boundary of any two adjacent pieced models. Two pieced models are adjacent if one of the models could change to the other by a small change in only one local sensation. For example, if current local sensations satisfied the condition of pieced model 1, i.e.

$$S_{overall} = 0.5S_{local,max} + 0.5S_{local,second,max} \tag{1}$$

then, when one of the non-dominant body part sensations decreases by a small value δ so that it is smaller than -1, which is the threshold, the overall sensation changes to model 6, i.e.

$$S_{overall}' = S_{overall,bigger-group} + \left[\max S_{overall,modifier} + 10\%\ \text{of second max}\ S_{overall,modifier}\right] \tag{2}$$

Generally, $S_{overall}' \neq S_{overall}$ and the gap is $\Delta S = S_{overall}' - S_{overall} \neq 0$ when $\delta \rightarrow 0$. Two or more concurrent changes of local sensations can be decomposed into a series of local sensation changes. Fig. 2 gives a detailed map of jumps between the adjacent pieced models in three parts. Each node in the figure represents a pieced model and the link between the nodes represents the neighborhood. There are total 18 neighbor relations, each of which is bidirectional. It can be easily seen that every pieced model has the potential to change to other pieced models by changing one of the local sensations by a small amount. Jumps may occur in all of the neighbor relations.

3.2.2. Smoothing method

To smooth the jump between pieced models, we first introduce the following key variables, which determine all the jumps between the adjacent pieced models.

$$x_1 = \min\left\{S_{chest}, S_{back}, S_{pelvis}\right\} \tag{3}$$

$$x_2 = \max\{S_{local}\} \tag{4}$$

$$x_3 = \min\{S_{local}\} \tag{5}$$

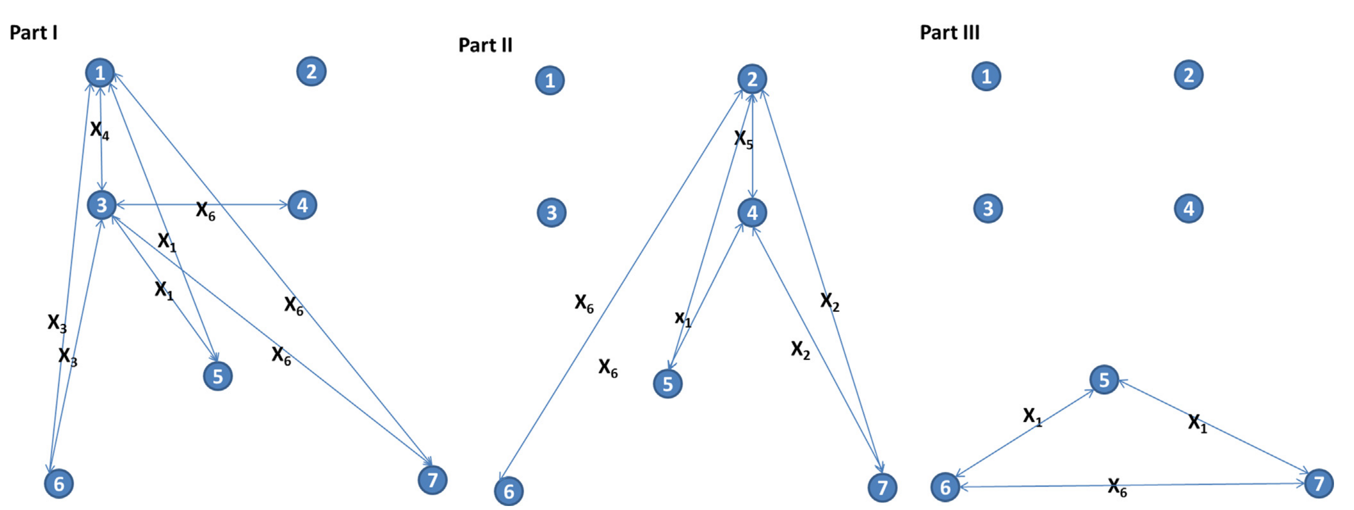

Fig. 2. The adjacency relationships between different pieced models. Each dot in the figure represents a pieced model. The two-way arrows shows the transitions between the pieced models. The key variables that determine the transitions between pieces are labeled, which are denoted in section 3.2.2.

$$x_4 = S_{\text{local},3,\max} \tag{6}$$

$$x_5 = S_{\text{local},3,\min} \tag{7}$$

$$x_6 = S_{\lceil N/2 \rceil,\max} \tag{8}$$

where $S_{\lceil N/2 \rceil,\max}$ means the $\lceil N/2 \rceil$ th maximum local sensation (rounded up to integer), N is the total number of segments (in our paper, $N = 16$).

These variables play important roles in representing model conditions and smoothing the jumps between pieced models. First, the conditions for the seven pieced models can be represented only using the key variables. For example: the condition for the pieced model 6 is$x_6 > 0$,$x_1 > -1$ and $x_3 < -1$. Second, all the possible jumps between pieced models can be associated with the key variables, which were labeled in Fig. 2. For example, the link between model 1 and 4 withx_4 labeled on it means the model 1 will change to model 4 if and only if $x_4 < 2$ while other sensations are not changed. Note that some links are associated with more than one key variable. Two reasons would account for this: first, the key variables are not independent, such asx_1 and x_3. Second, if some variables change across the threshold, several possible transitions might happen. Then the transit target is determined by the other key variables that divide one node into several inner states. For examples, for model 3, if x_6 changes to negative, the model can change to 4 or 7, depending whether x_2 is positive or not. We list the complete condition for every possible jump in Table 3. Furthermore, all the key variables are **continuous**, even some of them are the results of max/min operation. Then, if we find a smoothing function defined on those variables, we have the chance to construct a smoothed value when the model comes close to the boundary.

We chose a sigmoid function as the smoothing function, which is written as:

$$\text{sig}(x, \alpha, T) = \frac{1}{1 + e^{-\alpha(x-T)}} \tag{9}$$

where α is the scaling factor and T is the threshold. Sigmoid is a widely-used smoothing function in mathematical programming [11] because of its capability to approximate a step function to any level of accuracy without losing continuity.

The proposed sigmoid smoothing model in fact is also rational. We can interpret the original 7 pieces of the model as 7 sensation patterns the human will perceive. They all exist concurrently. However, at one time, only one sensation pattern dominates the others. When the environment changes gradually, the dominant sensation pattern will change gradually from one to the other. The gradual change influences the whole body change gradually by the sigmoid function and the transition conditions.

The sigmoid function approaches 1 when x is larger than T and approaches 0 otherwise. α controls the speed of the transient (Fig. 3). As analyzed above, if a certain key variable is crossing a threshold, the model will jump from one value to the other, producing a step output. If the step output was approximated using the sigmoid function defined on the key variable, the jump would disappear and the model would be C-1 continuous.

Mathematically, the smoothed model is written as

$$\hat{Y}_i = Y_i + \sum_{k \in N(i)} w_{ik} Y_{ik} \tag{10}$$

$$Y_{ik} = Y_k - Y_i \tag{11}$$

$$w_{ik} = \prod\nolimits_{t \in C_{ik}} \text{sig}(x_t, \alpha, T_i) \tag{12}$$

where Y_i was the ith original pieced model, $\hat{Y}_i$ was the smoothed ith model, $N(i)$ was the neighborhood sets of model i and C_{ik} was the set of key variables that determined the condition of the transition between model i and k.

We take model 3, low-level, no-opposite-sensation warm-side model, as an example. It can jump to model 1,5,6,7 and 4 as illustrated in Fig. 2. The key variables for these transitions are x_4,x_1,x_3 and x_1,x_6 and x_2,x_6 and x_2 respectively. Specifically, if x_4 is larger than 2, the model needs to transit from model 3 to model 1; if $x_3 < 1$ when $x_1 > -1$, the model will transit from model 3 to model 6, and vice versa for other transitions. So the modified model 3 is

$$\begin{aligned}\hat{Y}_3 = & Y_3 + \text{sig}(x_4, \alpha, 2)Y_{31} + \text{sig}(-x_1, \alpha, 1)Y_{35} \\ & + \text{sig}(-x_3, \alpha, 1)\text{sig}(x_1, \alpha, -1)Y_{36} \\ & + \text{sig}(-x_6, \alpha, 0)\text{sig}(x_2, \alpha, 1)Y_{37} \\ & + \text{sig}(-x_6, \alpha, 0)\text{sig}(-x_2, \alpha, -1)Y_{34}\end{aligned} \tag{13}$$

It means that if all the key variables fall into the condition of model 3, the modifier items (the items that constitute both sigmoid functions and Y_{ik}) all are approaching to 0, i.e $\hat{Y}_3 \approx Y_3$. If any of the possible transition conditions holds, the corresponding sigmoid function approaches 1. E.g., if $x_4 > 2$ and other transit conditions do not hold, then sig(x_4,α,2) $\rightarrow$ 1 and the other modifiers are

Table 3
Transitions and their corresponding conditions. Each line corresponds to the transition conditions from certain model to other models. E.g. the $x_4 < 2$ in the cross of line "From 1" and column "To 3"means when the overall sensation model is in model 1, if x_4, i.e. the 3rd maximum of local sensations becomes$x_4 < 2$, the overall model will change "from 1 to 3".

Model index	To 1	To 2	To 3	To 4	To 5	To 6	To 7
From 1			$x_4 < 2$		$x_1 < -1$	$x_1 > -1$ and $x_3 < -1$	$x_6 < 0$
From 2				$x_5 > -2$	$x_1 < -1$ and $x_2 > 0$	$x_6 > 0$ and $x_1 > -1$	$x_2 > 1$ and $x_1 > -1$
From 3	$x_4 > 2$			$x_6 < 0$ and $x_2 < 1$	$x_1 < -1$	$x_1 > -1$ and $x_3 < -1$	$x_6 < 0$ and $x_2 > 1$
From 4		$x_5 < -2$	$x_6 > 0$ and $x_3 > -1$		$x_1 < -1$ and $x_2 > 0$	$x_6 > 0$ and $x_1 > -1$ and $x_3 < -1$	$x_2 > 1$ and $x_1 > -1$
From 5	$x_6 > 0$ and $x_4 > 2$ and $x_3 > -1$	$x_6 < 0$ and $x_2 < 1$ and $x_5 < -2$ and $x_1 > -1$	$x_6 > 0$ and $x_4 < 2$ and $x_3 > -1$	$x_6 < 0$ and $x_2 < 1$ and $x_5 > -2$ and $x_1 > -1$		$x_6 > 0$ and $x_1 > -1$ and $x_3 < -1$	$x_6 < 0$ and $x_1 > -1$ and $x_2 > 1$
From 6	$x_4 > 2$ and $x_3 > -1$	$x_6 < 0$ and $x_2 < 1$ and $x_5 < -2$	$x_4 < 2$ and $x_3 > -1$	$x_6 < 0$ and $x_2 < 1$ and $x_5 > -2$	$x_1 < -1$		$x_6 < 0$ and $x_2 > 1$
From 7	$x_6 > 0$ and $x_4 > 2$ and $x_3 > -1$	$x_2 < 1$ and $x_5 < -2$	$x_6 > 0$ and $x_4 < 2$ and $x_3 > -1$	$x_2 < 1$ and $x_5 > -2$	$x_1 < -1$	$x_6 > 0$ and $x_3 < -1$	

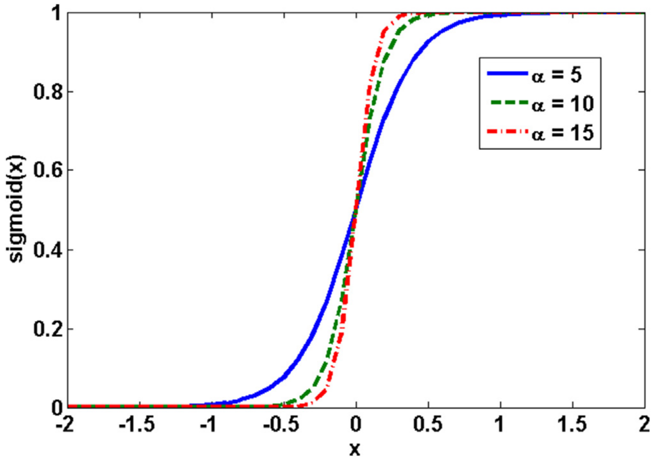

Fig. 3. Sigmoid function under different scaling factors α.

approaching 0, leading to the results $\widehat{Y}_3 \rightarrow Y_3 + Y_{31} = Y_1$. The jump is smoothed by the sigmoid function. For the rest of pieced models, smoothing can be done similarly based on Table 3.

Several points need to be considered in applying this smoothing method. The first is that there is a tradeoff between the smoothing effects and the difference before and after the smooth. The balance can be adjusted by changing scale factor α of the sigmoid function. When α is smaller, the smoothing effects are better, but there would be larger time duration within which the smoothed results fall between the two nearby model values. When $\alpha \rightarrow \infty$, the smoothed model would be the original model and the sudden jump would still exist. Second, the transition condition should be the sufficient and necessary condition under current pieced model conditions. For some transitions, more than one key variable are involved. We have listed all the transitions with their complete conditions in Table 3. Note that the conditions for each key variable in each possible jump should hold simultaneously, thus we use the product of the sigmoid functions.

3.3. Smoothing the jumps within a pieced model

3.3.1. Jump analysis

Besides the jump between pieced models analyzed above, the structure of some pieced models leads to a jump even if the model has not transited from one to another. Recall the pieced model 3, i.e. low level, no-opposite-sensation warm-side model, which is rewritten as follows

$$Y = \frac{1}{\widehat{N}}\sum_{i=1}^{\widehat{N}} x_{(i)} \tag{14}$$

where $x_{(i)}$ is the i th maximum local sensation and $\widehat{N}$ is the smallest index of the ordered sensations that satisfied the condition $x_{(\widehat{N})} > T_{\widehat{N}} = 2 - 2(\widehat{N} - 2)/14$. We only take the warm-side case as example, vice versa for the case of cool-side.

The reason for the jump is the discontinuity of $\widehat{N}$. Suppose $x_{(k)} < T_{(k)}, k = 3, 4, \ldots, \widehat{N} - 1, x_{(\widehat{N})} > T_{(\widehat{N})}, x_{(\widehat{N}+1)} > T_{(\widehat{N}+1)}$, and there is a small change $\delta > 0$ in $x_{(\widehat{N})}$, such that $x_{(\widehat{N})}' = x_{(\widehat{N})} - \delta < T_{(\widehat{N})}$; then the model output would be

$$Y' = \frac{1}{\widehat{N}+1}\sum_{i=1}^{\widehat{N}+1} x_{(i)} - \frac{\delta}{\widehat{N}+1} \tag{15}$$

The difference of the outputs is

$$\Delta Y = \left(\frac{1}{\widehat{N}+1} - \frac{1}{\widehat{N}}\right)\sum_{i=1}^{\widehat{N}} x_{(i)} + \frac{1}{\widehat{N}+1}\left(x_{(\widehat{N}+1)} - \delta\right) \neq 0 \tag{16}$$

When $\delta \searrow 0$, then

$$\Delta Y \rightarrow \left(\frac{1}{\widehat{N}+1} - \frac{1}{\widehat{N}}\right)\sum_{i=1}^{\widehat{N}} x_{(i)} + \frac{1}{\widehat{N}+1} x_{(\widehat{N}+1)} \neq 0 \tag{17}$$

Because the opposite-sensation models also utilize the above formulation, they also are subject to the same problem.

3.3.2. Smoothing method

We follow the same logic used in smoothing jumps between pieced models. The smoothed model is as follows:

$$\widehat{Y} = \sum_{i=3}^{N} \beta_i x_{(1:i)} \tag{18}$$

where $x_{(1:i)} = 1/i\sum_{k=1}^{i} x_{(k)}$ and $\beta_i = \mathrm{sig}(x_{(i)}, \alpha, T_{(i)})\prod_{k=3}^{i-1} sig(-x_{(k)}, \alpha, -T_{(k)})$,$i$=3,4,…,$N$−1 and $\beta_N = 1 - \prod_{k=3}^{N-1}(1 - \beta_k)$.

The smoothed model can be interpreted as follows:

$x_{(1:i)}$,$i = 3,\ldots,N$ are the averages of local sensations for each number,β_i is the coefficient of $x_{(1:i)}$, which is between 0 and 1. When $x_{(i)} > T_{(i)}$ and $x_{(3,4,\ldots,i-1)} < T_{(3,4,\ldots,i-1)}$, the coefficient of β_i is approaching 1, otherwise it will be 0. Only one β_i is approaching 1 at a given time. Because the coefficient is smoothing, the overall model is smoothing. An important point is that the scale factor α should be larger than the value of α used for smoothing *between* pieced models, because the product of a series of sigmoid functions enlarges the smoothing effects.

However, for the opposite model, it is more complicated. When the manipulation is carried out in "bigger-group" (see "Part III"), the coefficient is

$$\beta_i = \left(\mathrm{sig}\left(x_{(i)}, \alpha, T_{(i)}\right) + \mathrm{sig}\left(-x_{(i)}, \alpha, T_{(i)}\right)\mathrm{sig}\left(x_{(i)}, \alpha, 0\right) \times \mathrm{sig}\left(-x_{(i+1)}, \alpha, 0\right)\right)\prod_{k=3}^{i-1}\mathrm{sig}\left(-x_{(k)}, \alpha, -T_{(k)}\right) \tag{19}$$

For the "extended-bigger group"(see Appendix), the coefficient is

$$\beta'_i = \left(\mathrm{sig}\left(x_{(i)}, \alpha, T_{(i)}\right) + \mathrm{sig}\left(-x_{(i)}, \alpha, T_{(i)}\right)\mathrm{sig}\left(x_{(i)}, \alpha, -1\right) \times \mathrm{sig}\left(-x_{(i+1)}, \alpha, 1\right)\right)\prod_{k=3}^{i-1}\mathrm{sig}\left(-x_{(k)}, \alpha, -T_{(k)}\right) \tag{20}$$

The differences are caused by the definitions of "bigger group" and "extended bigger group", i.e. whether the group threshold is (0 or −1). The Appendix describes the difference between these two definitions.

3.4. Summary and implementations

Now, we summarize the smoothed overall thermal sensation model as follows

$$\widehat{Y}_i = \tilde{Y}_i + \sum_{k \in N(i)} w_{ik} Y_{ik} \tag{21}$$

where $\tilde{Y}_{ik} = \tilde{Y}_k - \tilde{Y}_i$ and $w_{ik} = \prod_{t \in C_{ik}} \mathrm{sig}(x_t, \alpha, T_i)$

With

$$\tilde{Y}_i = \sum_{t=3}^{N} \beta_t x_{(1:t)}, \text{for } i = 3, 4, 6, 7 \tag{22}$$

$$\tilde{Y}_i = Y_i, \text{otherwise} \tag{23}$$

where $x_{(1:t)} = 1/t\sum_{k=1}^{t} x_{(k)}$ and
when i=3,4

$$\beta_t = \text{sig}\left(x_{(t)}, \alpha, T_{(t)}\right)\prod_{k=3}^{t-1}\text{sig}\left(-x_{(k)}, \alpha, -T_{(k)}\right) \text{ for } t = 3,4,\ldots,N-1 \ \beta_N = 1-\prod_{k=3}^{N-1}\left(1-\beta_k\right) \tag{24}$$

And when $i = 6,7$

$$\tilde{Y}_i = S_{\text{overall,ex-bigger-group}} + [\text{combined force}] \tag{25}$$

$$\text{delta}S_{\text{local}} = S_{\text{local}} - S_{\text{overall,bigger-group}} \tag{26}$$

where $S_{\text{overall,bigger-group}} = \sum_{t=3}^{N}\beta_t^{'}x_{(1:t)}$, $S_{\text{overall,ex-bigger-group}} = \sum_{t=3}^{N}\beta_t x_{(1:t)}, x_{(1:t)} = 1/t\sum_{k=1}^{t}x_{(k)}$
for the warm side:

$$\beta'_i = \left(\text{sig}\left(x_{(i)}, \alpha, T_{(i)}\right) + \text{sig}\left(-x_{(i)}, \alpha, T_{(i)}\right)\text{sig}\left(x_{(i)}, \alpha, 0\right) \times \text{sig}\left(-x_{(i+1)}, \alpha, 0\right)\right)\prod_{k=3}^{i-1}\text{sig}\left(-x_{(k)}, \alpha, -T_{(k)}\right) \tag{27}$$

$$\beta_i = \left(\text{sig}\left(x_{(i)}, \alpha, T_{(i)}\right) + \text{sig}\left(-x_{(i)}, \alpha, T_{(i)}\right)\text{sig}\left(x_{(i)}, \alpha, -1\right) \times \text{sig}\left(-x_{(i+1)}, \alpha, 1\right)\right)\prod_{k=3}^{i-1}\text{sig}\left(-x_{(k)}, \alpha, -T_{(k)}\right) \tag{28}$$

and for the cold side

$$\beta'_i = \left(\text{sig}\left(-x_{(i)}, \alpha, -T_{(i)}\right) + \text{sig}\left(x_{(i)}, \alpha, T_{(i)}\right)\text{sig}\left(-x_{(i)}, \alpha, 0\right) \times \text{sig}\left(x_{(i+1)}, \alpha, 0\right)\right)\prod_{k=3}^{i-1}\text{sig}\left(x_{(k)}, \alpha, T_{(k)}\right) \tag{29}$$

$$\beta_i = \left(\text{sig}\left(-x_{(i)}, \alpha, -T_{(i)}\right) + \text{sig}\left(x_{(i)}, \alpha, T_{(i)}\right)\text{sig}\left(-x_{(i)}, \alpha, 1\right) \times \text{sig}\left(x_{(i+1)}, \alpha, -1\right)\right)\prod_{k=3}^{i-1}\text{sig}\left(x_{(k)}, \alpha, T_{(k)}\right) \tag{30}$$

The implementation of the smoothed model can be easily incorporated into the original one, without changing much. First, we should calculate all the results of the seven pieced models, i.e.$\tilde{Y}_i, i = 1,2,\ldots,7$, no matter which piece it should fall into. Then the original flow of the model is applied. When the model piece is determined, the smoothed overall sensation is calculated using the corresponding equations as above.

3.5. Numerical illustrations

3.5.1. Illustrations of overall smoothed effect

We here present smoothed results for sudden jumps between pieced models (Fig. 4) and sudden jumps within a model (Fig. 5). Results from the original form of the model are also presented for comparison. We randomly generated local sensations sequences so that the model could fall into different pieced models. Fig. 4 is a section of the sequence. The upper subfigure shows the overall sensation results, including the original one and the smoothed one. The middle subfigure gives the local sensation and the bottom subfigure shows the model index, describing which piece of the overall sensation model underlies the current local sensations shown in the middle figure. The first jump happens at around about time 38, where the model jumps from model 7 to model 6. We can see that the hands are locally cooled gradually and overall sensation is decreasing. However, at the same time, one of the body parts, for example, the thigh, feels a little bit warm, just across zero, and it happens to be the key variable that determines the boundary between model 7 and 6. Then the jump happens. Our proposed method adjusts the overall sensation before the jump happens and the overall sensation is thereby more smoothed. Another jump is in one of the dominant

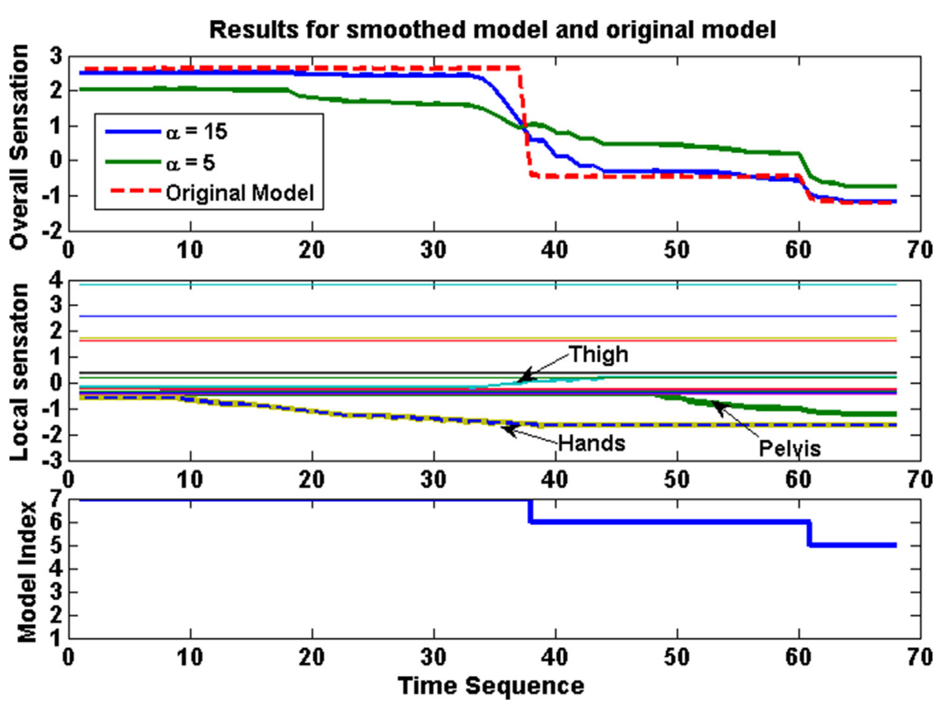

Fig. 4. Smooth effects for the jumps *between* models. In the upper subfigure, the scaling factors in the smoothed function for this inter-jump are 5 and 15 respectively. The middle figure shows local sensations, from top to bottom: head, face, breath, chest, back, upper arm, lower arm, thigh, neck, leg, foot, pelvis and hands. The lower subfigure shows the model index.

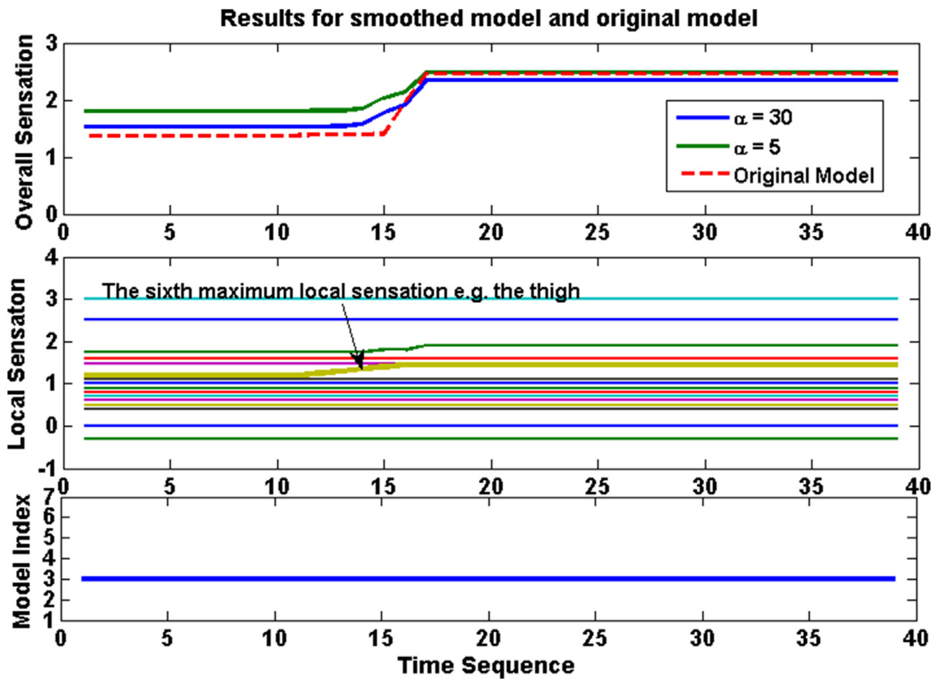

Fig. 5. Smoothing effects for a jump *inside* the model. In upper subfigure, the scaling factors in the smoothed function for this intra-jump are 15 and 30 respectively. The upper subfigure shows the overall sensations with scaling factors as 5 and 30 respectively. The middle figure shows local sensations from top to bottom: head, face, breath, neck, chest, thigh, back, pelvis, upper arm, lower arm, hand, leg and foot.

body parts, e.g., the pelvis cooling below −1. The original model jumps from model 6 to model 5. Our proposed method also has a more smoothed result. For Fig. 5, the overall sensation falls into model 3 in the whole sequence. The 6th maximum local sensation, i.e. the thigh in this case, increases gradually and at time 18 it goes over a threshold (2-(6-2)*2/14 = 1.43) that triggers a jump within the model causing a jump (in the upper subfigure). From both figures, we can see that jumps exist in the original model for continuous local sensation changes, and that the proposed modified model has smoothed the jumps. Note that a smaller scale parameter α results in a more substantial smoothing effect. The scale factor for the inter-model jump smoothing is shown for 5 and 15, while for the intra-model jump it is 5 and 30.

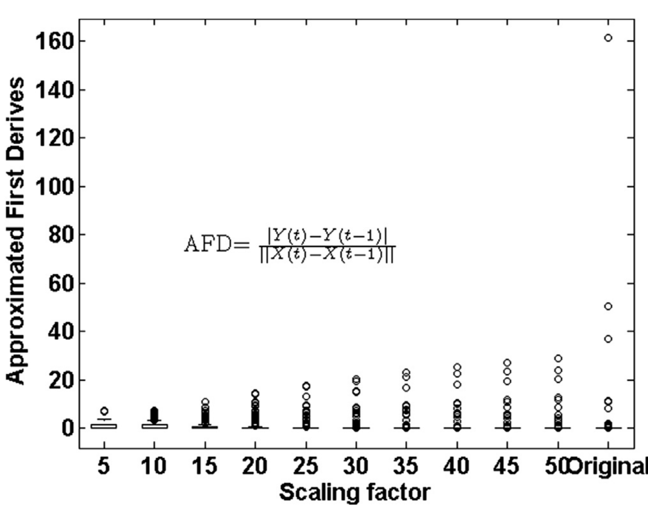

Fig. 6. Smoothness evaluations under different smoothing factors using Approximated First Derivatives.

3.5.2. Numerical comparisons before and after applying the smoothed models

This section presents quantitative results comparing the smoothed model and the original model. We use the comparisons of the first derivatives of the models to evaluate their smoothness. Although the absolute first derivatives bear the information of model structure, their relative magnitude can partly reflect the smoothness of the function. The larger of the absolute first derivatives, the more sharply the function will change, with less smoothing of the function. We approximate it by using

$$\mathrm{AFD} = \frac{|y(t) - y(t-1)|}{||X(t) - X(t-1)||} \tag{31}$$

where we use 2-norm to evaluate the distance of local sensation vectors. Fig. 6 illustrates the different approximated first derivatives under different scaling factors, compared with the original model. The results were obtained using several simulation sequences of local sensations. We can see that when the scale factor becomes larger, the dispersion of the derivatives is larger, indicating a smaller smoothing effect. For the original model, the derivatives reached as high as 160, indicating a jump had occurred. The smoothing effects are good for small values of scaling factor.

At the same time, we also evaluated the variations under different scaling factors. We randomly generated local sensations so that the model could fall into different pieced models. For the models 1–5 (see Table 2), structures were not changed, so we compare the differences between the smoothed and the original models. Fig. 7 shows the results under about 1000 stochastic evaluations. As we can see, when the scale factor becomes larger, the differences between the smoothed and the original models are smaller. Overall, when the scaling factor is larger than 15, the overall sensation difference is less than 5%.

Combining the results of Figs. 6 and 7, we recommend a default value of 15 for the scale factor because it combines good

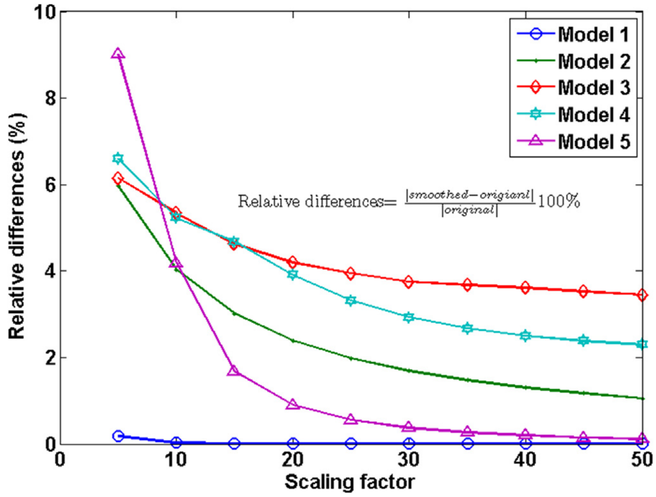

Fig. 7. Relative difference between the smoothed and original model versus scaling factor.

smoothness and small differences with the original models without the smoothed functions.

4. Conclusions

In this paper, we have improved the local and overall sensation models proposed in the previous three-part papers on sensation and comfort models. The set point adaption and the jump problems have been investigated and solutions provided. The set points adaption accounts for human adaptation feature. People can feel neutral under different combinations of clothing and metabolic levels, result in different local skin temperatures, which should be used as the setpoints to calculate local thermal sensations. Solution to smooth overall sensation models is provided using the sigmoid function. The sigmoid function in fact is rational. Instead of a local dominant sensation to have a sudden influence on overall sensation, the sigmoid function allows the influences to be added gradually. The scale parameter of the smoothing factor can be specified more accurately as more data available in the future about transient sensations. We have implemented the modified model in software, which users can access at: www.cbe.berkeley.edu.

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Appendix. Additional corrections to the sensation and comfort model

Several modifications have been made to the Overall Sensation Model described in Part III [3]. Some of these correct typos and others are small changes that make the model more rational.

A. Corrections in typos

Equation (1) in "part III", is now:

$$S_{\text{overall}} = 0.5S_{\text{local,max}} + 0.5S_{\text{local,second,max}} \quad \text{(A. 1)}$$

and equation (2) in "part III" is now:

$$S_{\text{overall}} = 0.38S_{\text{local,min}} + 0.62S_{\text{local,second,min}} \quad \text{(A. 2)}$$

In section 2.3.2.1, $i = 0,1,2\ldots,n^+$ should be$i = 1,2,\ldots,N-1$.

In section 2.3.1.2.2, n^- should also be $N-1$. $S_{\text{local},I,\text{min}}$ should be $S_{\text{local},i,\text{min}}$.

B. Corrections in model structure

We have newly defined an "extended-bigger-group" that extends beyond the range of the existing "bigger-group" in section 2.3.2 of "part III". It includes sensations that are higher than −1 for the warm side and lower than 1 for the cold side. We keep the previous "bigger" group definition unchanged as sensations higher or lower than 0 for warm or cold side separately.

Then equation (7) in "part III" is changed to:

$$S_{\text{overall}} = S_{\text{overall,ex-bigger-group}} + [\text{combined force}] \quad \text{(B.1)}$$

deltaS_{local} is not changed, i.e.

$$\text{delta}S_{\text{local}} = S_{\text{local}} - S_{\text{overall,bigger-group}} \quad \text{(B.2)}$$

This modification is based on the following: For the warm-side model, the "opposite model" used the average operation defined in "Part III" on sensations of the larger group plus the individual force. This led to the situation where the sensations falling between [0, −1] were ignored and did not contribute to the overall sensation. The extended bigger group incorporates the contribution of the [0, −1] sensations. It also holds for the cold-side model for sensations falling between [0, 1].

C. Corrections in model coefficients

The following are some corrections in the coefficients for sensation and comfort models based on our experiment and simulation results.

In Table 1 of "part I", regression coefficients for Eq. (3) – steady-state model of local sensation in asymmetrical environments, the update of the coefficients is as following:

C1: ≥0,head:3.96, chest:1, back 1. Thigh: 0.29; foot: 0.26
C1: <0, head: 0.38; upper arm: 0.29;
K1: ≥0: head: 0.18; thigh: 0.11.
K1: <0: head: 0.18; thigh 0.11

In Table 1 of "part II", regression coefficients for local thermal comfort models, the following coefficients are changed.

C31, foot = −3.5, pelvis −0.59, upper arm −0.3.

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