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ABSTRACT

This report describes a new approach to the seismic analysis of multiply supported multi-degree-of-freedom secondary systems in power plant structures. It is intended to provide rational methods to estimate the seismic response of piping in nuclear plants that are at once computationally less expensive and more accurate than the current multiple-response spectrum methods. Two methods are developed which utilize the modal properties of the structure alone and the fixed base modal properties of the piping or secondary system. The first method can be used with time history if ground motion records are available or with ground response spectra. It provides proper combination rules which account for the effects of closely spaced modes. Also included is the proper combination rule for response spectra when the ground motion has multiple components. The second method uses input in the form of floor spectra at the support points of the piping system. However, unlike the conventional multiple-response spectrum method, the procedure developed here properly accounts for the correlation between motions at different support points.

I. INTRODUCTION

The current methods of seismic analysis of secondary systems such as piping in nuclear power plant structures are the single-response spectrum (SRS) method, multiple-response spectrum (MRS) method, and the time-history (TH) method. In all three methods, the analyst begins with a specification of the motion of the points of attachment of the piping system neglecting the interaction between the piping and the structure. In the SRS method, a single input floor response spectrum for the piping system is employed which is the envelop to the floor spectra at all attachment points. In the MRS method, the separate spectra at all attachment points are utilized. These floor spectra are usually generated through time-history analysis of the primary structure using an artificial ground motion history which is "compatible" with the design ground spectrum for the site. The response of the piping system in both SRS and MRS methods is obtained by combining modal components of piping response by approximate and ad hoc procedures.

Numerical studies comparing results for these approximate methods with results obtained by the TH method which, using complete time-history analysis of the piping system, is presumed to be accurate, have indicated that the SRS and MRS methods can be excessively conservative. On the other hand, the TH method requires a great deal of computation and is impractical for economic reasons. Furthermore, the validity of using a "spectrum-compatible" time history is very questionable in that there are no real earthquakes that have spectra that are compatible with a smooth design spectrum and, in addition, it is well known that there are severe problems of uniqueness with this approach, i.e., different time histories compatible with the same design spectrum may lead to very different estimates of the peak piping response. The concept of spectrum-compatible time histories is in contradiction with the basic methodology of random vibration theory and can not be justified on rational grounds.

The problems associated with the spectrum approaches can be summarized as follows:

1. Use of a single ground motion history which may not be characteristic of potential earthquakes at the site. This is true whether a real earthquake record or a spectrum compatible time history is employed.

2. Neglect of cross-correlations between components of ground motion when design is for multi-directional ground excitation.

3. Improper combination of contributions from closely spaced structural modes in the computation of the spectra at the piping support points, when a response spectrum or random vibration method is used.

4. Improper combination of contributions from closely spaced modes of the piping system in computing its peak dynamic response.

5. Neglect of cross-correlations between components of excitation at each support and between the supports.

6. Neglect of interaction between the primary structure and the piping, which can be important when some natural frequencies of the two subsystems are close to each other.

7. Neglect of the effect of nonproportional damping which can be significant in combined structure-piping system even though the structure alone and the piping alone are proportionally damped.

8. In addition, the fundamental question arises as to whether for a multi-degree-of-freedom system the response spectrum method can be used when the input is itself the response of a system, i.e., the primary system, which in general may not be a wide-band process.

In this report we address a number of these problems. Two methods are developed which utilize the modal properties of the structure alone and the fixed base modal properties of the piping or secondary system. The first method can be used with time history if ground motion records are available or with ground response spectra. It provides proper combination rules which account for the effects of closely spaced modes in the primary as well as in the secondary systems. Also included is the proper combination rule for response spectra when the ground motion has multiple components. It is a characteristic of the formulation of the problem in this method that it avoids the introduction of the pseudo-static displacement (which gives rise to the so called secondary stresses) and thus eliminates the unresolved question of the proper combination of secondary stresses with the primary or "dynamic" stresses. This method has the advantage that it bypasses the the need for generating floor spectra and the above mentioned problems associated with the standard MRS method.

The second method uses the formulation of the first method recast in the form of floor response spectra to provide an improved MRS method. It uses input in the form of floor spectra at the support points of the secondary system. However, unlike the conventional MRS method, the procedure developed here properly accounts for the correlation between motions at different support points. It should be clear that significant correlation between the support excitations exist, since the support point motions result from a single ground motion filtered through the structural system. The effect of closely spaced modes in each of the two subsystems is also included in this method.

Both methods developed herein can be used when the design input ground motion has multiple components. The interaction between the primary and secondary systems is not included in this analysis. However, this effect will be included in the continuation of this research work.

II. FORMULATION IN TERMS OF GROUND RESPONSE SPECTRUM

Consider a structure with N degrees of freedom subjected to a single ground motion time history $\ddot{u}_g(t)$, which may be one of three orthogonal components of the complete ground motion. Attached to this structure is an $n+m$ degree of freedom secondary system, which may represent an extended equipment item or a piping system. The attached degrees of freedom of the secondary system are identified by $i=1,2,...,m$ and the unattachment degrees of freedom are identified by $i=m+1,...,n+m$. In the structure, the degrees of freedom of the attachment points are identified by $I=N-m+1,...,N$, as shown in Fig. 1.

The structure is modeled by conventional mass, **M**, damping, **C**, and stiffness, **K**, matrices and modal damping is assumed. The secondary system is also modeled in this way again with modal damping assumed in the modes of the fixed base system.

The basic equation of motion of the structure is

$$\mathbf{M\ddot{U}} + \mathbf{C\dot{U}} + \mathbf{KU} = \mathbf{CR}\dot{u}_g + \mathbf{KR}u_g + \mathbf{F} \tag{1}$$

where **F** is the N-vector of interaction forces between the structure and the secondary system and **R** is the influence vector that couples the ground motion to the degrees of freedom of the structure. We note that **F** has the following form

$$\mathbf{F} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{f} \end{Bmatrix}$$

where **f** is an m-vector of interaction forces exerted by the secondary system on the primary system. The response N-vector, **U**, can be similarly partitioned in the form

$$\mathbf{U} = \begin{Bmatrix} \mathbf{U}_s \\ \mathbf{U}_c \end{Bmatrix}$$

The corresponding equation for the secondary system is

$$\begin{bmatrix} \mathbf{m}_m & \mathbf{0}^T \\ \mathbf{0} & \mathbf{m} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_c \\ \ddot{\mathbf{u}} \end{Bmatrix} + \begin{bmatrix} \mathbf{c}_m & \mathbf{c}_c^T \\ \mathbf{c}_c & \mathbf{c} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}}_c \\ \dot{\mathbf{u}} \end{Bmatrix} + \begin{bmatrix} \mathbf{k}_m & \mathbf{k}_c^T \\ \mathbf{k}_c & \mathbf{k} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_c \\ \mathbf{u} \end{Bmatrix} = -\begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \end{Bmatrix} \tag{2}$$

where **m**, **c**, and **k** are the fixed base properties of the secondary system, $\mathbf{c}_c$ and $\mathbf{k}_c$ are coupling matrices and $\mathbf{m}_m$, $\mathbf{c}_m$, and $\mathbf{k}_m$ are matrices associated with the attachment points of the secondary system. Note that the full matrices in Eq. 2 are the appropriate matrices for the response of the secondary system considered as a free system. Equation 2 can be separated into two equations:

$$\mathbf{m\ddot{u}} + \mathbf{c\dot{u}} + \mathbf{ku} = -\mathbf{c}_c\dot{\mathbf{u}}_c - \mathbf{k}_c\mathbf{u}_c \tag{3}$$

$$\mathbf{m}_m\ddot{\mathbf{u}}_c + \mathbf{c}_m\dot{\mathbf{u}}_c + \mathbf{c}_c^T\dot{\mathbf{u}} + \mathbf{k}_m\mathbf{u}_c + \mathbf{k}_c^T\mathbf{u} + \mathbf{f} = \mathbf{0} \tag{4}$$

The complete solution for the combined system involves a simultaneous solution of Eqs. 1,3, and 4 for the unknown vectors **U** and **u** in terms of the specified ground motion $\ddot{u}_g(t)$. This is a formidable undertaking for all but the simplest systems and has only been done correctly for the case of a single-degree-of-freedom secondary system attached to one point of a primary system, i.e. $n=m=1$, in Refs. [3]. The basic difficulty is the interaction between the systems and primarily the influence of **f** on the response of the primary system. The standard simplifying approach is to neglect the influence of **f** on **U** and this will be followed here, although a continuing aspect of this project is to include this interaction since it is clear that there are systems for which this interaction can not be neglected.

Neglecting this interaction, the equations to be solved are Eq. 1 with $\mathbf{F}=\mathbf{0}$ and Eq. 3. This means that the motion of the structure is determined as if the secondary system were absent and then the resulting motion at the attachment points is used as base input motion to the secondary system.

It is common practice in seismic analysis of structures and of secondary systems to use a modal approach, especially if a response spectrum method is to be used. Let $\mathbf{\Phi}=\left[\mathbf{\Phi}_1\ \mathbf{\Phi}_2\ \cdots\ \mathbf{\Phi}_N\right]$ be the modal matrix of the structure containing the modal vectors $\mathbf{\Phi}_I$ and $\mathbf{\Omega}$ be the diagonal matrix of the corresponding natural frequencies Ω_I. The fixed base modal matrix of the secondary system is denoted by $\boldsymbol{\phi}=\left[\boldsymbol{\phi}_1\ \boldsymbol{\phi}_2\ \cdots\ \boldsymbol{\phi}_n\right]$ and the diagonal matrix of the natural frequencies ω_i is denoted by $\boldsymbol{\omega}$.

The response of the structure is represented by

$$\mathbf{U}=\mathbf{\Phi}\mathbf{Q} \tag{5}$$

where $\mathbf{Q}$ is the N vector with components Q_I given by

$$\ddot{Q}_I+2B_I\Omega_I\dot{Q}_I+\Omega_I^2Q_I=\Gamma_I\left(2B_I\Omega_I\dot{u}_g+\Omega_I^2u_g\right)\ ,\quad I=1,2,\ldots,N \tag{6}$$

where B_I is the damping coefficient and Γ_I is the participation factor for the I-th mode of the structure, with

$$\Gamma_I=\frac{\mathbf{\Phi}_I^T\mathbf{M}\mathbf{R}}{\mathbf{\Phi}_I^T\mathbf{M}\mathbf{\Phi}_I} \tag{7}$$

The term $2B_I\Omega_I\dot{u}_g$ is generally neglected in view of the small damping coefficient. The modal matrix $\mathbf{\Phi}$ is partitioned into

$$\mathbf{\Phi}=\begin{bmatrix}\mathbf{\Phi}_s\\ \mathbf{\Phi}_c\end{bmatrix} \tag{8}$$

in terms of which

$$\mathbf{U}_c = \mathbf{\Phi}_c\mathbf{Q} \tag{9}$$

For the secondary system the corresponding solution, dropping the small damping term on the right-hand side, is

$$\mathbf{u} = \boldsymbol{\phi}\mathbf{q} \tag{10}$$

where q_i is given by

$$\ddot{q}_i + 2\beta_i\omega_i\dot{q}_i + \omega_i^2 q_i = -\frac{\boldsymbol{\phi}_i^T\mathbf{k}_c\mathbf{u}_c}{m_i} \quad , \quad i = 1,2,\ldots,n \tag{11}$$

where $m_i = \boldsymbol{\phi}_i^T\mathbf{m}\boldsymbol{\phi}_i$ is the i-th modal mass of the fixed base secondary system. It is convenient to manipulate Eqs. 5-11 using Laplace transforms. Denoting the Laplace transform of a quantity by a superposed bar and p as the transform parameter, we have

$$\bar{\mathbf{u}} = \boldsymbol{\phi}\bar{\mathbf{q}} \tag{12}$$

$$\bar{\mathbf{U}}_c = \bar{\mathbf{u}}_c = \mathbf{\Phi}_c\bar{\mathbf{Q}} \tag{13}$$

with

$$\bar{q}_i = -\frac{\boldsymbol{\phi}_i^T\mathbf{k}_c\bar{\mathbf{u}}_c}{m_i\left(p^2 + 2\beta_i\omega_i p + \omega_i^2\right)} \tag{14}$$

and

$$\bar{Q}_I = \frac{\Gamma_I\Omega_I^2}{p^2 + 2B_I\Omega_I p + \Omega_I^2}\bar{u}_g \tag{15}$$

Substituting Eq. 13 in Eq. 14

$$\bar{q}_i = -\sum_{I=1}^{N}\frac{\boldsymbol{\phi}_i^T\mathbf{k}_c\mathbf{\Phi}_{cI}}{m_i\left(p^2 + 2\beta_i\omega_i p + \omega_i^2\right)}\bar{Q}_I$$

Using this expression together with Eq. 15 in Eq. 12,

$$\bar{\mathbf{u}} = -\sum_{i=1}^{n}\sum_{I=1}^{N}\boldsymbol{\phi}_i\gamma_{iI}\frac{\Gamma_I\Omega_I^2\omega_i^2}{\left(p^2 + 2\beta_i\omega_i p + \omega_i^2\right)\left(p^2 + 2B_I\Omega_I p + \Omega_I^2\right)}\bar{u}_g \tag{16}$$

The term $\gamma_{iI} = \dfrac{\boldsymbol{\phi}_i^T\mathbf{k}_c\mathbf{\Phi}_{cI}}{m_i\omega_i^2} = \dfrac{\boldsymbol{\phi}_i^T\mathbf{k}_c\mathbf{\Phi}_{cI}}{\boldsymbol{\phi}_i^T\mathbf{k}\boldsymbol{\phi}_i}$ is a mixed participation factor.

The absolute acceleration of the k-th nodal point of the secondary system is given by $\bar{\ddot{u}}_k$ where

$$\ddot{\bar{u}}_k = -\sum_{i=1}^{n}\sum_{I=1}^{N}\phi_{ki}\,\gamma_{iI}\,\Gamma_I \frac{\Omega_I^2\omega_i^2}{\left(p^2+2\beta_i\omega_i p+\omega_i^2\right)\left(p^2+2B_I\Omega_I p+\Omega_I^2\right)}\,\ddot{\bar{u}}_g$$

$$= -\sum_{i=1}^{n}\sum_{I=1}^{N}\Psi_{kiI}\,\frac{\Omega_I^2}{p^2+2\beta_i\omega_i p+\omega_i^2}\,\frac{\omega_i^2}{p^2+2B_I\Omega_I p+\Omega_I^2}\,\ddot{\bar{u}}_g\ , \quad k=m+1,\dots,m+n \qquad (17)$$

where Ψ_{kiI} is the effective participation factor for the absolute acceleration of the k-th node of the secondary system associated with modes i and I of the secondary and primary systems, respectively, given by

$$\Psi_{kiI} = \phi_{ki}\,\gamma_{iI}\,\Gamma_I \qquad (18)$$

Under the assumption of small damping the solution of Eq. 17 is the convolution of $\ddot{u}_g$ and the function obtained by inversion of the summation terms. This becomes

$$\ddot{u}_k(t) = -\left[\sum_{i=1}^{n}\sum_{I=1}^{N}\frac{\Psi_{kiI}}{1-\left(\dfrac{\omega_i}{\Omega_I}\right)^2}\,\omega_i\exp(-\beta_i\omega_i t)\sin\omega_i t + \sum_{I=1}^{N}\sum_{i=1}^{n}\frac{\Psi_{kiI}}{1-\left(\dfrac{\Omega_I}{\omega_i}\right)^2}\,\Omega_I\exp(-B_I\Omega_I t)\sin\Omega_I t\right] * \ddot{u}_g(t) \qquad (19)$$

where the * notation implies a convolution of two functions, i.e.,

$$f(t) * g(t) = \int_0^t f(t-\tau)g(\tau)\,d\tau = \int_0^t f(\tau)g(t-\tau)\,d\tau$$

In Eq. 19 there are $n+N$ modal contributions with the appropriate amplification factors for each contribution. It is clear from this result that if there are equal frequencies in the primary and secondary systems (i.e., tuning) the result in its present form is invalid. The reason for this is that in the analysis it was assumed that the contribution coming from the damping terms was small in comparison to those arising from differences in frequencies. When these differences become very small (i.e., tuning) this assumption is no longer valid and it becomes necessary to include the contributions from the damping terms. Although this contribution can be included, it complicates the interpretation of the results in terms of response spectra. This problem will be dealt with fully in a later report.

If it is now assumed that the input data is a ground response spectrum $S(\omega,\beta)$, which represents the mean maximum absolute acceleration experienced by an SDOF oscillator with frequency ω and damping factor β subject to the ground motion $\ddot{u}_g$, then Eq. 19 can be used to obtain a prediction for the mean peak value of $\ddot{u}_k$. The mean peak value of the contribution from mode i is given in terms of $S(\omega_i,\beta_i)$. There are several approximate methods for combination of these contributions. If for example the natural frequencies of the two subsystems are well spaced for each subsystem and between the two subsystems (and the input is wide band), then the SRSS rule is acceptable and the result will take the form

$$E\left[\max|\ddot{u}_k(t)|\right] = \left\{\sum_{i=1}^{n}\left[\sum_{I=1}^{N}\frac{\Psi_{kiI}}{1-\left(\dfrac{\omega_i}{\Omega_I}\right)^2}\right]^2 S^2(\omega_i,\beta_i) + \sum_{I=1}^{N}\left[\sum_{i=1}^{n}\frac{\Psi_{kiI}}{1-\left(\dfrac{\Omega_I}{\omega_i}\right)^2}\right]^2 S^2(\Omega_I,B_I)\right\}^{1/2} \tag{20}$$

where $E[.]$ is the expectation operator. Alternatively, if closely spaced natural frequencies occur the appropriate summation rule should take into account the correlation between the modal responses. This can be done by use of the procedure outlined by Der Kiureghian [1,2] where the following expression for the modal correlation coefficient for wide-band inputs is introduced

$$\rho_{0,ij} = \frac{8\sqrt{\zeta_i\zeta_j\alpha_i\alpha_j}(\zeta_i\alpha_i+\zeta_j\alpha_j)\alpha_i\alpha_j}{(\alpha_i^2-\alpha_j^2)^2+4\zeta_i\zeta_j\alpha_i\alpha_j(\alpha_i^2+\alpha_j^2)+4(\zeta_i^2+\zeta_j^2)\alpha_i^2\alpha_j^2} \tag{21}$$

where α_i and α_j are modal frequencies (i.e., ω_i or Ω_I) and ζ_i and ζ_j are the corresponding damping factors (i.e., β_i or B_I). Following the procedure outlined in [2], the estimated peak value of $\ddot{u}_g$ is

$$E\left[\max|\ddot{u}_k(t)|\right] = \left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{0,ij}A_{ki}A_{kj}S(\omega_i,\beta_i)S(\omega_j,\beta_j) + 2\sum_{i=1}^{n}\sum_{I=1}^{N}\rho_{0,iI}A_{ki}A_{kI}S(\omega_i,\beta_i)S(\Omega_I,B_I) + \sum_{I=1}^{N}\sum_{J=1}^{N}\rho_{0,IJ}A_{kI}A_{kJ}S(\Omega_I,B_I)S(\Omega_J,B_J)\right\}^{1/2} \tag{22}$$

where the amplification factors A_{ki} and A_{kI} are given by

$$A_{kl} = \sum_{l=1}^{N} \frac{\Psi_{kil}}{1-\left(\frac{\omega_i}{\Omega_l}\right)^2} ; \quad A_{kl} = \sum_{i=1}^{n} \frac{\Psi_{kil}}{1-\left(\frac{\Omega_l}{\omega_i}\right)^2}$$

It is important to point out that this formulation avoids the artificial separation of the secondary system response into a quasi-static and a dynamic part. Thus no consideration need be given to the unresolved problem of properly combining these parts. Another advantage of the approach presented here is that the solution is expressed in terms of known properties of the basic subsystems; the modal properties of the combined system are not required. Furthermore, no time-history analyses using spectrum-compatible ground motion inputs to produce floor response spectra at the attachment points are required.

III. FORMULATION IN TERMS OF FLOOR RESPONSE SPECTRA

The solutions given in Eqs. 20 or 22 which provide the equipment or piping response directly in terms of the ground response spectrum can be cast in terms of floor response spectra. As pointed out previously, the MRS method has associated with it several difficulties. However, it has certain desirable features from a practical point of view. The most important of these is that once the floor spectra are given any number of equipment items or piping systems can be analyzed independently of the primary structure properties, provided proper account is taken of the cross-correlations between the motions at different support points. As mentioned before, the conventional MRS method neglects these correlations. In the method to be presented here, a procedure for including these correlations is developed based on random vibration techniques.

For the purpose of developing a multiple correlated response spectrum method, which for convenience will be referred to as the MCRS method, we introduce the specific response history function $\ddot{r}_K(\omega,\beta;t)$, which is the absolute acceleration time history of a hypothetical SDOF oscillator with frequency ω and damping factor β located at the K-th attachment point of the primary structure. For small damping, the Laplace transform of $\ddot{r}_K(\omega,\beta;t)$ is given by

$$\bar{\ddot{r}}_K(\omega,\beta;p) = \frac{\omega^2}{p^2+2\beta\omega p+\omega^2} \sum_{I=1}^{N} \frac{\Upsilon_{KI}\Omega_I^2}{p^2+2B_I\Omega_I p+\Omega_I^2} \bar{\ddot{u}}_g(p) \tag{23}$$

where

$$\Upsilon_{KI} = \Gamma_I \Phi_{KI} \tag{24}$$

in which Γ_I is given in Eq. 7 and Φ_{KI} is the K-th component of the I-th mode of the structure. We note that the floor acceleration response spectrum at the K-th attachment point, $S_K(\omega,\beta)$, is in fact the peak value of $\ddot{r}_K(\omega,\beta;t)$ for the duration of the disturbance.

Using Eqs. 17 and 23, and rearranging orders of summation, the equipment absolute acceleration at the k-th degree of freedom, $\ddot{u}_k$, can be written in terms of functions $\ddot{r}_K$ in Laplace transform space in the form

$$\begin{aligned}
\overline{\ddot{u}}_k(p) &= -\sum_{i=1}^{n}\sum_{I=1}^{N}\phi_{ki}\frac{\boldsymbol{\phi}_i^T\mathbf{k}_c\boldsymbol{\Phi}_{cI}}{m_i\omega_i^2}\frac{\Gamma_I\Omega_I^2\omega_i^2}{\left(p^2+2\beta_i\omega_i p+\omega_i^2\right)\left(p^2+2B_I\Omega_I p+\Omega_I^2\right)}\overline{\ddot{u}}_g(p) \\
&= -\sum_{i=1}^{n}\frac{\phi_{ki}}{m_i\omega_i^2}\sum_{I=1}^{N}\left(\sum_{K=1}^{m}\sum_{l=1}^{n}\phi_{li}k_{clK}\Phi_{KI}\right)\frac{\Gamma_I\Omega_I^2\omega_i^2}{\left(p^2+2\beta_i\omega_i p+\omega_i^2\right)\left(p^2+2B_I\Omega_I p+\Omega_I^2\right)}\overline{\ddot{u}}_g(p) \\
&= -\sum_{i=1}^{n}\frac{\phi_{ki}}{m_i\omega_i^2}\sum_{K=1}^{m}\left(\sum_{l=1}^{n}\phi_{li}k_{clK}\right)\overline{\ddot{r}}_K(\omega_i,\beta_i;p) \\
&= -\sum_{i=1}^{n}\chi_{ki}\sum_{K=1}^{m}\kappa_{iK}\overline{\ddot{r}}_K(\omega_i,\beta_i;p)
\end{aligned} \tag{25}$$

where

$$\chi_{ki} = \frac{\phi_{ki}}{m_i\omega_i^2} \tag{26}$$

$$\kappa_{iK} = \sum_{l=1}^{n}\phi_{li}k_{clK} \tag{27}$$

In the above expression, ϕ_{ki} represents the k-th component of the i-th modal vector of the fixed base equipment and k_{clK} is the element in the l-th row and K-th column of the coupling stiffness matrix $\mathbf{k}_c$.

To develop the MCRS method the power spectral density of the response, $G_{\ddot{u}_k\ddot{u}_k}(\omega)$, is utilized. From Eq. 25, this can be written as

$$G_{\ddot{u}_k\ddot{u}_k}(\omega) = \sum_{i=1}^{n}\sum_{j=1}^{n}\chi_{ki}\chi_{kj}\sum_{K=1}^{m}\sum_{L=1}^{m}\kappa_{iK}\kappa_{jL}G_{\ddot{r}_{Ki}\ddot{r}_{Lj}}(\omega) \tag{28}$$

where $G_{\ddot{r}_{Ki}\ddot{r}_{Lj}}(\omega)$ is the cross-power spectral density of $\ddot{r}_K(\omega_i,\beta_i;t)$ and $\ddot{r}_L(\omega_j,\beta_j;t)$ and ω is the Fourier transform parameter. Using Eq. 23, the cross power spectral density is

$$G_{\ddot{r}_{Ki}\ddot{r}_{Lj}}(\omega) = \omega_i^2 \omega_j^2 h_i(\omega)\, h_j^*(\omega)\, G_{\ddot{U}_K \ddot{U}_L}(\omega) \tag{29}$$

where $G_{\ddot{U}_K \ddot{U}_L}(\omega)$ is the cross-power spectral density of the acceleration responses of the structure at support points K and L and is given by

$$G_{\ddot{U}_K \ddot{U}_L}(\omega) = \sum_{I=1}^{N} \sum_{J=1}^{N} \Omega_I^2 \Omega_J^2 \Upsilon_{KI} \Upsilon_{LJ}\, H_I(\omega)\, H_J^*(\omega)\, G_{\ddot{u}_g \ddot{u}_g}(\omega) \tag{30}$$

in which $G_{\ddot{u}_g \ddot{u}_g}(\omega)$ is the power spectral density of the input ground motion. The functions $h_i(\omega)$ and $H_I(\omega)$ in the preceding equations are oscillator frequency response functions given by

$$h_i(\omega) = \frac{1}{\omega_i^2 - \omega^2 + 2 i \beta_i \omega_i \omega} \tag{31}$$

$$H_I(\omega) = \frac{1}{\Omega_I^2 - \omega^2 + 2 i B_I \Omega_I \omega} \tag{32}$$

and the superposed asterisks denote complex conjugates.

The most important statistical response quantity of practical interest is the mean square, which is equal to the area underneath the power spectral density function. Considering one-sided power spectral densities, the mean-square of $\ddot{u}_k$, denoted by $\lambda_{0,kk}$, from Eq. 28 is

$$\lambda_{0,kk} = \sum_{i=1}^{n} \sum_{j=1}^{n} \chi_{ki} \chi_{kj} \sum_{K=1}^{m} \sum_{L=1}^{m} \kappa_{iK} \kappa_{jL} \lambda_{0,KiLj} \tag{33}$$

where

$$\lambda_{0,KiLj} = \int_0^\infty G_{\ddot{r}_{Ki}\ddot{r}_{Lj}}(\omega)\, d\omega$$

$$= \omega_i^2 \omega_j^2 \int_0^\infty h_i(\omega)\, h_j^*(\omega)\, G_{\ddot{U}_K \ddot{U}_L}(\omega)\, d\omega \tag{34}$$

In the special case when the modal frequencies of the secondary system are well spaced, it is anticipated that the modal cross terms in Eqs. 28 and 33 (i.e., those for $i \neq j$) will be negligible in comparison to the diagonal terms (i.e., those for $i = j$). It can be shown that for terms with $i = j$ only the real part of $G_{\ddot{U}_K \ddot{U}_L}(\omega)$ makes a contribution. The typical term of interest in Eq. 33 then takes the form

$$\lambda_{0,KiLi} = \omega_i^4 \int_0^\infty |h_i(\omega)|^2 \mathrm{Re}\left[G_{\ddot{U}_K \ddot{U}_L}(\omega) \right] d\omega \tag{35}$$

A careful examination of the integrand in the preceding equation reveals a behavior which permits a simple approximation to be made under a broad set of conditions. Observe that for a lightly damped secondary system, the term $|h_i(\omega)|^2$ is very sharply peaked at $\omega = \omega_i$. On the other hand, the term $\mathrm{Re}\left[G_{\ddot{U}_K \ddot{U}_L}(\omega)\right]$, which as shown in Eq. 30 is made up of contributions from all modes of the primary structure, is relatively slowly varying in ω. Because of this, most of the contribution to $\lambda_{0,KiLi}$ comes from the neighborhood of ω_i, which permits the integral to be approximated by

$$\int_0^\infty |h_i(\omega)|^2 \mathrm{Re}\left[G_{\ddot{U}_K \ddot{U}_L}(\omega)\right] d\omega \approx \mathrm{Re}\left[G_{\ddot{U}_K \ddot{U}_L}(\omega_i)\right] \int_0^\infty |h_i(\omega)|^2 d\omega \tag{36}$$

Using this approximation and introducing the coefficient

$$\eta_{KL}(\omega_i) = \frac{\mathrm{Re}\left[G_{\ddot{U}_K \ddot{U}_L}(\omega_i)\right]}{\left[G_{\ddot{U}_K \ddot{U}_K}(\omega_i) G_{\ddot{U}_L \ddot{U}_L}(\omega_i)\right]^{1/2}} \tag{37}$$

$\lambda_{0,KiLi}$ in Eq. 35 can be written as

$$\begin{aligned}
\lambda_{0,KiLi} &\approx \omega_i^4 \mathrm{Re}\left[G_{\ddot{U}_K \ddot{U}_L}(\omega_i)\right] \int_0^\infty |h_i(\omega)|^2 d\omega \\
&= \omega_i^4 \eta_{KL}(\omega_i) \left[G_{\ddot{U}_K \ddot{U}_K}(\omega_i)\, G_{\ddot{U}_L \ddot{U}_L}(\omega_i)\right]^{1/2} \int_0^\infty |h_i(\omega)|^2 d\omega \\
&= \eta_{KL}(\omega_i) \left[\omega_i^4 G_{\ddot{U}_K \ddot{U}_K}(\omega_i) \int_0^\infty |h_i(\omega)|^2 d\omega\right]^{1/2} \left[\omega_i^4 G_{\ddot{U}_L \ddot{U}_L}(\omega_i) \int_0^\infty |h_i(\omega)|^2 d\omega\right]^{1/2}
\end{aligned} \tag{38}$$

Now using the approximation expressed in Eq. 36 in the opposite sense, $G_{\ddot{U}_K \ddot{U}_K}(\omega_i)$ and $G_{\ddot{U}_L \ddot{U}_L}(\omega_i)$ in Eq. 38 can be taken inside the integral and their arguments, ω_i, replaced by ω. This results in

$$\lambda_{0,KiLi} = \eta_{KL}(\omega_i) \sqrt{\lambda_{0,KiKi} \lambda_{0,LiLi}} \tag{39}$$

The next step is to relate the mean square values to the floor response spectrum ordinates. It is well known that the mean peak response is related to the root-mean-square of the response through a peak factor which varies slowly with the average frequency of the response and depends weakly on the shape of the power spectral density function [2]. This relationship leads to the following expressions for the response quantities of interest in this study:

$$\sqrt{\lambda_{0,KiKi}} = \frac{1}{p_i} E\left[\max |\ddot{r}_K(\omega_i,\beta_i;t)|\right] = \frac{1}{p_i} S_K(\omega_i,\beta_i) \tag{40}$$

$$E\left[\max |\ddot{u}_k(t)|\right] = p\sqrt{\lambda_{0,kk}} \tag{41}$$

in which p_{iK} and p are peak factors for the processes $\ddot{r}_K$ and $\ddot{u}_k$, respectively. Using Eq. 40 in Eq. 39, one obtains

$$\lambda_{0,KiLi} = \eta_{KL}(\omega_i) \frac{1}{p_{iK}p_{iL}} S_K(\omega_i,\beta_i) S_L(\omega_i,\beta_i) \tag{42}$$

Substituting this result in Eq. 33, and recalling that only terms $i=j$ are included in this approximation, the mean square of the equipment response is obtained as

$$\lambda_{0,kk} = \sum_{i=1}^{n} \chi_{ki}^2 \sum_{K=1}^{m} \sum_{L=1}^{m} \kappa_{iK}\kappa_{iL}\eta_{KL}(\omega_i) \frac{1}{p_{iK}p_{iL}} S_K(\omega_i,\beta_i) S_L(\omega_i,\beta_i) \tag{43}$$

Finally, using Eq. 41, the mean of the peak acceleration response of the secondary system at the k-th degree of freedom is obtained as

$$E\left[\max |\ddot{u}_k(t)|\right] = \left[\sum_{i=1}^{n} \chi_{ki}^2 \sum_{K=1}^{m} \sum_{L=1}^{m} \kappa_{iK}\kappa_{iL}\eta_{KL}(\omega_i) \frac{p^2}{p_{iK}p_{iL}} S_K(\omega_i,\beta_i) S_L(\omega_i,\beta_i)\right]^{1/2} \tag{44}$$

It has been shown [2] that peak factor ratios of the form $\frac{p}{p_{iK}}$ are close to unity and for practical applications can be set equal to 1. Thus, Eq. 44 simplifies to

$$E\left[\max |\ddot{u}_k(t)|\right] = \left[\sum_{i=1}^{n} \chi_{ki}^2 \sum_{K=1}^{m} \sum_{L=1}^{m} \kappa_{iK}\kappa_{iL}\eta_{KL}(\omega_i) S_K(\omega_i,\beta_i) S_L(\omega_i,\beta_i)\right]^{1/2} \tag{45}$$

The only term in this expression which remains to be determined is $\eta_{KL}(\omega_i)$. This coefficient, which plays the role of the correlation coefficient between the responses $\ddot{r}_K(\omega_i,\beta_i;t)$ and $\ddot{r}_L(\omega_i,\beta_i;t)$, can be obtained, using Eqs. 37 and 30, as

$$\eta_{KL}(\omega_i) = \frac{\sum_{I=1}^{N}\sum_{J=1}^{N} \Omega_I^2 \Omega_J^2 \Upsilon_{KI}\Upsilon_{LJ} H_{IJ}(\omega_i)}{\left[\sum_{I=1}^{N}\sum_{J=1}^{N} \Omega_I^2 \Omega_J^2 \Upsilon_{KI}\Upsilon_{KJ} H_{IJ}(\omega_i)\right]^{1/2} \left[\sum_{I=1}^{N}\sum_{J=1}^{N} \Omega_I^2 \Omega_J^2 \Upsilon_{LI}\Upsilon_{LJ} H_{IJ}(\omega_i)\right]^{1/2}} \tag{46}$$

where

$$H_{IJ}(\omega_i) = \mathrm{Re}\left[H_I(\omega_i) H_J^*(\omega_i)\right] \tag{47}$$

It should be pointed out that $\eta_{KL}(\omega_i)$ as defined in Eq. 37 is not formally a correlation coefficient, although it serves the purpose of one in this approach.

Equation 45 represents a modal combination rule for the MCRS method which explicitly accounts for the correlation between floor spectra. Note, however, that that equation does not include the contributions that may arise from correlation between modal responses of the secondary system. Cross terms are needed to account for these contributions. Although the support excitations are not wide-band processes, it is anticipated that the expression for correlation between modal responses in Eq. 21, which is based on response to a white-band input, will provide a reasonable approximation. The bases for this expectation are that in general the cross terms are important only when the modal frequencies are very close to each other, and that for such cases, especially in the presence of small dampings of the secondary system, the modal correlation coefficient is influenced only by the shape of the input power spectral density function in the immediate neighborhood of the two frequencies. Note that in the limit as the frequencies and damping of the two modes become equal, the expression in Eq. 21 yields a value of unity for $\rho_{0,ij}$, as it should. Because of this property, the expression for $\rho_{0,ij}$ provides good approximation in the region of importance where the correlation is strong, i.e., for frequency ratios close to unity. Thus, using Eq. 45 in conjunction with Eq. 21 and employing the procedure in Ref. [2],

$$E\left[\max|\ddot{u}_k(t)|\right] = \left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\chi_{ki}\chi_{kj}\rho_{0,ij}\left[\sum_{K=1}^{m}\sum_{L=1}^{m}\kappa_{iK}\kappa_{iL}\eta_{KL}(\omega_i)S_K(\omega_i,\beta_i)S_L(\omega_i,\beta_i)\right]^{1/2}\left[\sum_{K=1}^{m}\sum_{L=1}^{m}\kappa_{jK}\kappa_{jL}\eta_{KL}(\omega_j)S_K(\omega_j,\beta_j)S_L(\omega_j,\beta_j)\right]^{1/2}\right\}^{1/2} \qquad (48)$$

IV. DISCUSSION

The main results of this study are Eqs. 19, 22, and 48. These equations provide the response of a multiply attached secondary system, such as a piping system in a power plant, in terms of the properties of the primary system alone and the properties of the secondary system alone. The basic difference between the three equations is the form used to describe the input excitation.

Equation 19 is for the case when the input excitation is given in the form of a deterministic ground acceleration time history. The system parameters required for the evaluation of that equation are the modal frequencies and damping ratios of the primary system alone and of the fixed base secondary system, and the coefficients Ψ_{kil}, which can be interpreted as mixed effective participation factors. The latter, defined in Eq. 18, are expressed in terms of the modal vectors and participation factors of the primary system, the modal vectors of the fixed base secondary system, and the stiffness matrix of the unattached secondary system. As stated before, this result ignores the effect of interaction between the primary and secondary systems and is valid only when there is no tuning or near tuning of the two subsystems. The principal advantage achieved is circumventing the necessity of solving the eigenvalue problem for the combined system. It is pointed out that although this formulation may be useful in analyzing the response of a secondary system for a recorded earthquake ground motion, it is not useful for design purposes. As has already been noted, since future ground motions are unknown, the proper procedure is to use a stochastic approach. In earthquake engineering, this can be accomplished through the use of the response spectrum, as is done in Eqs. 22 and 48.

Equation 22 gives the mean peak response of the secondary system directly in terms of the ground mean response spectrum. The system parameters involved are the same as those mentioned in the preceding paragraph. The limitations of the formulation are also the same, i.e., interaction is ignored and tuning between the two subsystems is not allowed. Note that closely spaced modes in each of the subsystems are permitted because the significant correlation between such modes is included through the cross terms. The major advantage of this formulation is that one avoids introducing the floor spectra as an intermediary device and, hence, avoids all the problems associated with their use. Another advantage is that the response spectrum method is more accurate in this formulation than it is for the formulation employing the floor spectra. The reason for this is that generally the ground motion is a broader band excitation than are the floor motions, which leads to greater accuracy in the modal combination rules. A potential disadvantage of this formulation from a practical standpoint is that the analyst of the secondary system must use the modal properties of the primary system explicitly. This might be inconvenient in situations where for one reason or another the analyst of the secondary system does not wish to deal with the properties of the primary system.

Equation 48 gives the mean peak response of the secondary system in terms of floor spectra and $\eta_{KL}(\omega)$, a matrix function the elements of which are normalized, dimensionless cross-floor coefficients. These coefficients, which depend only on the properties of the primary system, play the role of correlation coefficients between the floor motions. The expression includes the effect of correlation between modal responses of the secondary system. The floor spectra can be developed using the method of Ref. [3] directly in terms of the ground spectra. Note that the method in that reference can deal with the problem of tuning between modes of the primary and secondary systems and with closely spaced modes in the primary system. Thus, the method represented by Eq. 48 is valid for any distribution of the frequencies of the primary and secondary systems. Besides this important advantage, this approach is convenient from a practical point of view because once the floor spectra and the coefficient functions $\eta_{KL}(\omega)$ have been supplied, the secondary system can be analyzed without knowledge of the properties of the primary system. This can be observed in Eqs. 26 and 27 where coefficients χ_{ki} and κ_{iK}, which appear in Eq. 48, are seen to be functions only of the secondary system properties. The disadvantage of this approach is the approximations employed in Eq. 36 and in passing from Eq. 45 to Eq. 48. Finally, one potential advantage of this approach is that the effect of interaction between the primary and secondary systems may be included with some modification of Eq. 48 employing the interaction floor spectra in Ref. [3]. Numerical studies are in progress to evaluate the accuracy of this method.

V. REFERENCES

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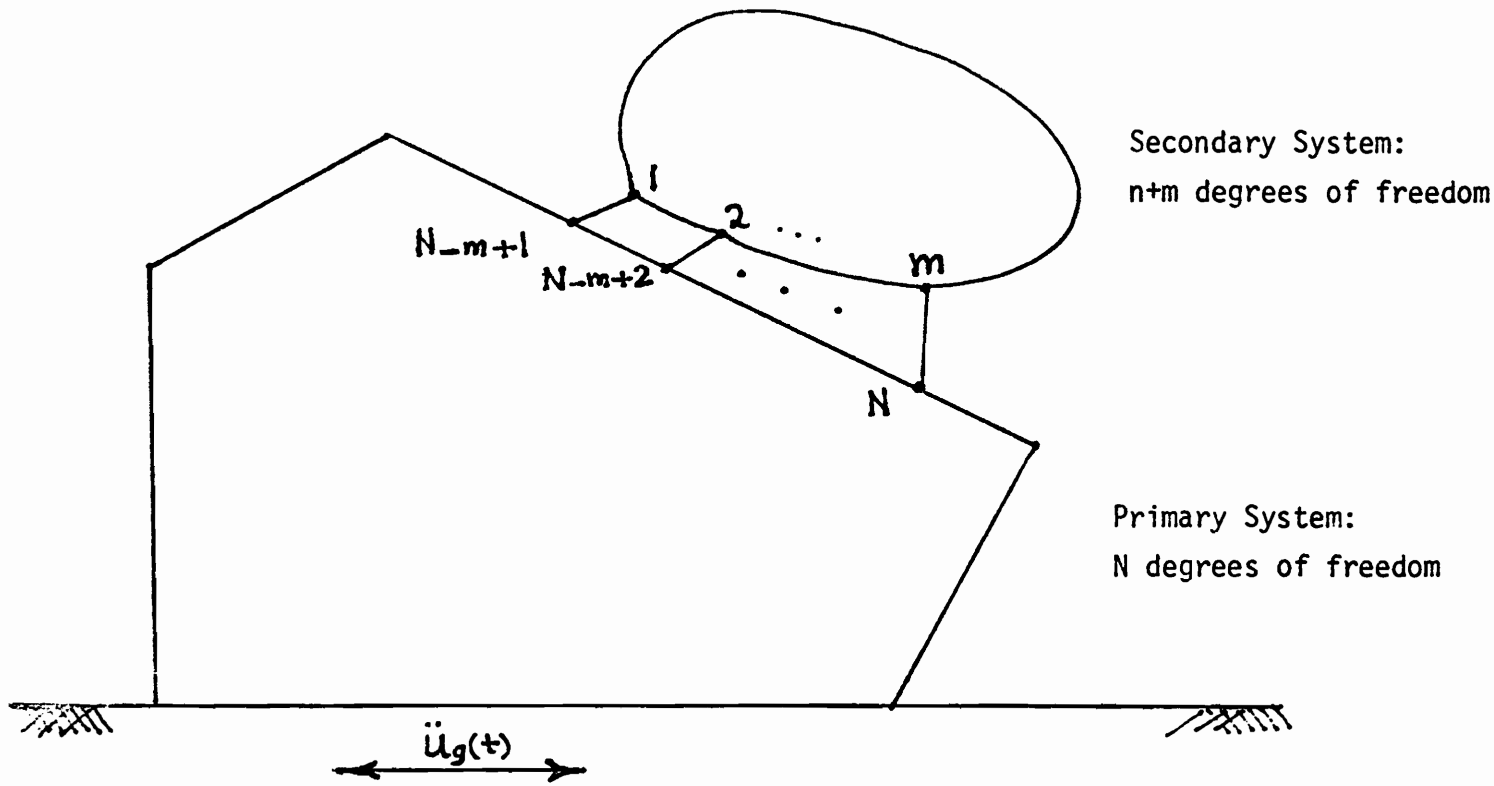

Figure 1. Idealization of Primary and Secondary Systems