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Topics in Axions, Supergravity, and the String Swampland

Author
Leedom, Jacob Michael

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Topics in Axions, Supergravity, and the String Swampland

by

Jacob Michael Leedom

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Abstract

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This dissertation is comprised of three parts, with each part focusing on topics in axions and dark matter, anomaly cancellation in supergravity theories, and the implications of the string swampland to cosmology, respectively. In Part I, we consider axions in particle extensions of the Standard Model and models of dark matter. We present a class of models where supersymmetry and the Peccei–Quinn symmetry are simultaneously broken and the messengers that mediate the effects of these symmetry breakings to the Standard Model are identical. We also describe a production scenario for QCD axion dark matter where the Peccei-Quinn phase transition occurs at a temperature far below the symmetry breaking scale. The produced axions tend to be warm. For a certain range of the decay constant, the effect of the predicted warmness on structure formation can be confirmed by future observations of 21 cm lines. Additionally, a portion of parameter space requires a mixing between the Peccei-Quinn symmetry breaking field and the Standard Model Higgs and thereby predicts an observable rate of rare Kaon decays. We also consider the late universe cosmology of ultralight axion dark matter models, and show that requiring the axion to have a matter-power spectrum that matches that of cold dark matter constrains the magnitude of the axion couplings to the visible sector. Comparing these limits to current and future experimental efforts, we find that many searches require axions with an abnormally large coupling to Standard Model fields, independently of how the axion was populated in the early Universe. We survey mechanisms that can alleviate the bounds, namely, the introduction of large charges, various forms of kinetic mixing, a clockwork structure, and imposing a discrete symmetry. We provide an explicit model for each case and explore their phenomenology and viability to produce detectable ultralight axion dark matter. In Part II, we use Pauli–Villars regularization to evaluate the conformal and chiral anomalies in the effective field theories from $\mathbb{Z}_3$ and $\mathbb{Z}_7$ compactifications of the heterotic string without Wilson lines and a Z_3 compactification of the heterotic string with two Wilson lines and an anomalous U(1). We show that parameters for Pauli–Villars chiral multiplets can be chosen in such a way that the anomaly is universal in the sense that its coefficient depends only on a single holomorphic function of the three

diagonal moduli. It is therefore possible to cancel the anomaly by a generalization of the four-dimensional Green–Schwarz mechanism. In particular, we are able to reproduce the results of a string calculation of the four-dimensional chiral anomaly for these models. In Part III, we discuss the relations between swampland conjectures and observational constraints on both inflation and dark energy. Using the requirement $|\nabla V| \geq cV$, with c as a universal constant whose value can be derived from inflation, there may be no observable distinction between constant and nonconstant models of dark energy. However, the latest modification of the above conjecture, which utilizes the second derivative of the potential, opens up the opportunity for observations to determine if the dark energy equation of state deviates from that of a cosmological constant. We also comment on the observability of tensor fluctuations despite the conjecture that field excursions are smaller than the Planck scale.

To My Parents

Contents

List of Figures

List of Tables

Acknowledgments

It is sometimes said that it takes a village to raise a child. I propose that a similar statement is true for dissertations - it takes a village, a university, and a host of companions to write a dissertation.

First, I would like to thank all of the incredible mentors and teachers that made me into the researcher I am today. The support, sage advice, and endless patience of Mary K. Gaillard has been invaluable throughout my time in graduate school. Similarly, I am very thankful to Hitoshi Murayama for his support, wise insights, and otherworldly ability to simply explain complex ideas. I also thank Ori Ganor very much for his support and willingness to listen to my half-formed ideas and provide guidance on how to develop them further. I am grateful to Keisuke Harigaya for his vast knowledge of particle physics and for working with me on my first published work. I also thank Jeff Dror for his amazingly keen physical intuition and valuable friendship. Lastly, I thank the professors that happily shared their time and knowledge with me - this includes Yasunori Nomura, Lawrence Hall, Raphael Bousso, and Petr Horava.

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Chapter 1

Introduction

The current state of particle physics is one of great uncertainty, but also excitement and opportunity. From observations of the cosmos, we know that the incredibly successful Standard Model of particle physics accounts for only 4.9% of the energy density of the Universe, with the remainder being made of dark matter and dark energy. Most features of both phenomena remain essentially unknown. Furthermore, there are a number open questions about the consistency and properties of the Standard Model. Several of these include the hierarchy problem, the Strong CP problem, and the quantum nature of gravity. These deficiencies provide incentive for innovation and discovery, but not all of the above gaps can be addressed in the same manner. For example, dark matter appears to require an extension to the particle content of the Standard Model. On the other hand, the complex nature of quantum gravity indicates that new, profound ideas are needed to bring our understanding of gravity to the level of the other fundamental forces. Thus progress towards a fundamental theory of interactions and matter depends on the interplay of experimental input, construction of new models, and advancement of our mathematical understanding of these models. This dissertation is broken into three parts, with each part describing progress made along various combinations of the above three core approaches.

In Part I, we will consider particle extensions of the Standard Model and their implications for particle and dark matter experiments. The primary focus will be on the class of hypothetical particles known as axions. Chapter 2 describes work on unifying models of supersymmetry and QCD axions. In chapter 3, we will will explore production methods for the QCD axion such that it is a viable candidate for dark matter. Both chapters are based on work done with Keisuke Harigaya [225, 226]. Chapter 5 will instead focus on constraints and models of ultralight axion-like particle (ALP) dark matter and is based on work completed with Jeff Dror [156].

In Part II, we will explore mathematical features of toy fundamental particle models. Chapters 5 and 6 will describe the process by which quantum anomalies are cancelled by the Green-Schwarz mechanism in supergravity models derived from various orbifold compactifications of the $E_8 \times E_8$ Heterotic string. Both chapters are based on work completed in collaboration with Mary K. Gaillard [185, 186].

Finally, Part III describes work that is a synthesis of theory, model building, and experimental prospects. The single chapter of the section discusses the application of the string swampland to the development of inflationary models and the potential observablility of quintessence models of dark energy. This singular chapter is based on work done with Chien-I Chiang and Hitoshi Murayama [110].

Part I

Axions in Particle Physics and Cosmology

Overview of Part I

In Part I, we consider the construction of models that ameliorate some of the deficiencies of the Standard Model mentioned above. We also describe the experimental prospects and signatures of these models. The primary issues we consider solving here are the Strong CP problem and the puzzle of dark matter. A viable method to address both problems is the introduction of the QCD axion into the Standard Model. The QCD axion is a viable candidate for dark matter and elegantly solves the Strong CP problem via the Peccei-Quinn mechanism [356]. Chapters 2 and 3 will be dedicated to particle and dark matter models of the QCD axion.

There are more general models where dark matter consists of axion-like particles. These particles have couplings to the Standard Model that resemble those of the QCD axion but they do not solve the Strong CP problem. While these models are less economical than the QCD axion, they are a distinct and viable alternative. Furthermore, axion-like particles are theoretically motivated as they appear to be generic features of string theory constructions. Chapter 4 will discuss constraints on theories of ultralight axion-light particles and methods to construct non-trivial models that partially evade these constraints.

Chapter 2

Unified Models of the QCD Axion and Supersymmetry Breaking

2.1 Introduction

One of the most serious problems of the standard model, the so-called strong CP problem [8, 264, 94], is elegantly solved by the Peccei-Quinn (PQ) mechanism [356]. Another problem, the hierarchy problem, is considerably relaxed by low energy supersymmetry (SUSY) [301, 403, 417, 278]. The precise gauge coupling unification at a high energy scale also motivates low energy SUSY [161, 29, 200].

There are several hints for a potential connection between these two physical ideas. First, models of SUSY breaking often involve spontaneous breaking of global symmetry. In fact, it is one of the sufficient conditions for SUSY breaking [18]. It would be illuminating to identify this global symmetry with the PQ symmetry.

Second, if the PQ symmetry breaking field resides in the SUSY breaking sector, the super partners of the axion, namely the saxion and the axino, may obtain large masses [99, 100, 239, 227]. Such a model is free from the cosmological problems associated with light saxions and axinos (see [280] and references therein).

Finally, one realization of the PQ mechanism, the KSVZ model [289, 383], has the following superpotential,

$$W = ZQ\bar{Q}, \tag{2.1}$$

where Z is a PQ charged field with a non-zero vacuum expectation value (VEV), and Q and $\bar{Q}$ are PQ and standard model gauge (especially $SU(3)_c$) charged fields. If the chiral field Z also obtains a non-zero F term VEV, the SUSY breaking is mediated to super partners of standard model particles via the gauge interaction. This is nothing but the gauge mediation of SUSY breaking [153, 149, 151, 28, 343] with messenger fields Q and $\bar{Q}$.

Motivated by these hints, we propose a model where SUSY and the PQ symmetry are simultaneously broken, and the messenger fields that mediate SUSY breaking and the anomaly of the PQ symmetry are in fact the same. The model provides a unification for the physics of SUSY breaking and the PQ mechanism.

2.2 Unification of SUSY and PQ symmetry breaking

Simultaneous SUSY and PQ symmetry breaking in a single sector

We introduce chiral fields M_+ and M_-, whose $U(1)_{\rm PQ}$ charges are $+1$ and -1, respectively. The PQ symmetry is broken by introducing a chiral field X and a superpotential coupling,

$$W \supset \kappa X(M_+M_- - v^2), \tag{2.2}$$

where κ and v are constants. SUSY is broken by lifting the flat direction $M_+M_- = v^2$. To achieve this, we introduce chiral fields Z_+ and Z_-, and couple them to $M_\pm$ via mass terms. The superpotential of this minimal model is then given by

$$W = \kappa X(M_+M_- - v^2) + \lambda' r v Z_+ M_- + \frac{\lambda'}{r} v Z_- M_+, \tag{2.3}$$

where λ' and r are constants. By phase rotations of chiral fields, we take all constants in Eq. (2.3) to be real.

The simultaneous breaking of the PQ symmetry and SUSY via the superpotential in Eq. (2.3) is discussed in [99, 100]. As is shown in section 2.3, this model is the low energy effective theory of a dynamical SUSY breaking model with a deformed moduli constraint (the IYIT model) [263, 259], and is studied by [227, 168] in the context of the heavy scalar scenario [199, 414, 253, 213, 255]. Direct coupling between the SUSY and the PQ breaking sectors is also analysed in [239] using an effective field theory, while [48, 40, 170] connect the two sectors indirectly via quantum corrections involving messengers.

For $\lambda' < \kappa$, the VEVs of the fields are given by

$$\begin{aligned}
\langle M_+\rangle &= rv\sqrt{1-\frac{\lambda'^2}{\kappa^2}},\ \ \langle M_-\rangle = \frac{v}{r}\sqrt{1-\frac{\lambda'^2}{\kappa^2}},\\
\langle Z_+\rangle &= \langle Z_-\rangle \equiv z,\ \ \langle X\rangle = -\frac{\lambda' z}{\kappa\sqrt{1-\lambda'^2/\kappa^2}},
\end{aligned} \tag{2.4}$$

up to a $U(1)_{\rm PQ}$ rotation. The PQ symmetry is broken by the VEVs of the charged fields $M_\pm$ and $Z_\pm$, where z is undetermined at tree level. If $\lambda' > \kappa$, the VEVs of $M_\pm$ and $Z_\pm$ vanish, and the PQ symmetry is not broken. Thus we will adopt the above hierarchy and also assume that $\lambda' \ll \kappa$ for simplicity. SUSY is predominantly broken by the F terms of $Z_\pm$,

$$F_{Z_\pm} = -\lambda' v^2. \tag{2.5}$$

Mass spectrum

The chiral field X and a linear combination of $M_\pm$ obtain a large mass κv. We may integrate them out and parametrize $M_\pm$ as

$$
\begin{aligned}
M_+ &\to rv \times \exp(-\frac{A}{v\sqrt{r^2+1/r^2}}),\\
M_- &\to \frac{v}{r} \times \exp(\frac{A}{v\sqrt{r^2+1/r^2}}),
\end{aligned}
\tag{2.6}
$$

where A is a chiral field. The effective superpotential of $Z_\pm$ and A is then given by

$$
W_{\rm eff} = \lambda f^2 Z_+ {\rm exp}(\frac{A}{\sqrt{2}f}) + \lambda f^2 Z_- {\rm exp}(-\frac{A}{\sqrt{2}f}),
\tag{2.7}
$$

where $f \equiv v\sqrt{(r^2+1/r^2)/2}$ and $\lambda \equiv 2\lambda'/(r^2+1/r^2)$. We note that most of the following discussion relies only on this effective superpotential, and not on the UV completion in Eq. (2.3).

Let us first calculate the masses of scalar components of $Z_\pm$ and A. We decompose scalar components as

$$
\begin{aligned}
Z_\pm &\to \left(z + \frac{\pm\rho_H + \rho_L}{2}\right) \exp\left(i\frac{\pm\theta_H + \theta_L}{2z}\right),\\
A &\to \frac{s+i\phi}{\sqrt{2}}.
\end{aligned}
\tag{2.8}
$$

Expanding the scalar potential, we obtain the mass terms,

$$
\begin{aligned}
V_{\rm mass} =& \frac{1}{2}\lambda^2 f^2 \left(\theta_H + \frac{z}{f}\phi\right)^2\\
&+ \frac{1}{2}\lambda^2 f^2 \left(\rho_H + \frac{z}{f}s\right)^2 + \lambda^2 f^2 s^2.
\end{aligned}
\tag{2.9}
$$

The mass eigenstates and eigenvalues are given by

$$
\begin{aligned}
a =& \frac{\phi - \epsilon\theta_H}{\sqrt{1+\epsilon^2}}, \quad b = \frac{\theta_H + \epsilon\phi}{\sqrt{1+\epsilon^2}}, \quad \epsilon \equiv \frac{z}{f}\\
\begin{pmatrix}\sigma_+\\ \sigma_-\end{pmatrix} =& \begin{pmatrix}{\rm cos}\alpha & -{\rm sin}\alpha\\ {\rm sin}\alpha & {\rm cos}\alpha\end{pmatrix}\begin{pmatrix} s\\ \rho_H\end{pmatrix},\\
{\rm tan}\alpha =& \frac{2\epsilon}{1+\epsilon^2+\sqrt{1+6\epsilon^2+\epsilon^4}},\\
m_a =& 0, \quad m_b = \lambda f\sqrt{1+\epsilon^2},\\
m_{\sigma\pm}^2 =& \frac{1}{2}\lambda^2 f^2\left[3+\epsilon^2 \pm \sqrt{1+6\epsilon^2+\epsilon^4}\right].
\end{aligned}
\tag{2.10}
$$

Scalar fields ρ_L and θ_L are massless at tree level but obtain masses through quantum corrections, as we will see later. The remaining massless field, a, is the axion.

Next we consider the masses of the fermionic components of $Z_\pm$ and A. The quadratic terms of $\delta Z_\pm \equiv Z_\pm - z$ and A in the superpotential in Eq. (2.3) are

$$W_{\text{eff,quad}} = \frac{1}{2}\lambda z A^2 + \lambda f A \frac{1}{\sqrt{2}}(\delta Z_+ - \delta Z_-). \tag{2.11}$$

The mass eigenstates $\psi_\pm$ and eigenvalues are

$$\begin{aligned} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} &= \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \psi_A \\ \psi_{Z_H} \end{pmatrix}, \quad \tan\beta = \frac{\sqrt{\epsilon^2+4}-\epsilon}{2}, \\ m_{\psi_\pm} &= \frac{1}{2}\lambda f \times \left[\sqrt{\epsilon^2+4} \pm \epsilon\right], \end{aligned} \tag{2.12}$$

where ψ_A and ψ_{Z_H} are the fermionic components of A and $Z_H \equiv (Z_+ - Z_-)/\sqrt{2}$, respectively. The fermionic component of $Z_L \equiv (Z_+ + Z_-)/\sqrt{2}$ is the goldstino and is eaten by the gravitino via the super Higgs mechanism.

The expressions for the mass eigenstates are simplified in the limit $\epsilon \ll 1$ or $\epsilon \gg 1$. In the limit $\epsilon \ll 1$, where the PQ symmetry is dominantly broken by the VEVs of $M_\pm$, we have

$$a = \phi,\ b = \theta_H,\ \sigma_+ = s,\ \sigma_- = \rho_H, \tag{2.13}$$

$$m_b = \lambda f,\ m_{\sigma_+} = \sqrt{2}\lambda f,\ m_{\sigma_-} = \lambda f, \tag{2.14}$$

$$\psi_\pm = \frac{1}{\sqrt{2}}\left(\psi_A \mp \psi_{Z_H}\right), \tag{2.15}$$

$$m_{\psi_\pm} = \lambda f. \tag{2.16}$$

In the limit $\epsilon \gg 1$, where the PQ symmetry is dominantly broken by the VEVs $\langle Z_\pm \rangle$, we obtain

$$a = -\theta_H,\ b = \phi,\ \sigma_+ = s,\ \sigma_- = \rho_H, \tag{2.17}$$

$$m_b = \lambda z,\ m_{\sigma_+} = \lambda z,\ m_{\sigma_-} = \frac{\sqrt{2}\lambda f^2}{z}, \tag{2.18}$$

$$\psi_+ = \psi_A,\ \psi_- = \psi_{Z_H}, \tag{2.19}$$

$$m_{\psi_+} = \lambda z,\ m_{\psi_-} = \frac{\lambda f^2}{z}, \tag{2.20}$$

where the masses of $\sigma_- = \rho_H$ and $\psi_- = \psi_{Z_H}$ are suppressed by the large Majorana masses λz of $\sigma_+ = s$ and $\psi_+ = \psi_A$.

Sgoldstino potential in the minimal model

As we have seen, the directions ρ_L and θ_L, which correspond to the sgoldstino components, are massless at tree level. Accordingly, z is undetermined at tree level. Here we discuss the stabilization of the sgoldstino in the mimimal model given by Eq. (2.3).

Quantum corrections generate a potential for the scalar component of $Z_L \equiv (Z_+ + Z_-)/\sqrt{2}$,

$$\begin{aligned}\Delta V_\pm(Z_L) = \frac{\lambda^4 f^4}{512\pi^2}\Big[&8(1+\epsilon^2)^2\ln(1+\epsilon^2)\\ +2(3+\epsilon^2+\sqrt{1+6\epsilon^2+\epsilon^4})^2\ln&\frac{3+\epsilon^2+\sqrt{1+6\epsilon^2+\epsilon^4}}{2}\\ +2(3+\epsilon^2-\sqrt{1+6\epsilon^2+\epsilon^4})^2\ln&\frac{3+\epsilon^2-\sqrt{1+6\epsilon^2+\epsilon^4}}{2}\\ -(\epsilon-\sqrt{4+\epsilon^2})^4\ln&\frac{(\epsilon-\sqrt{4+\epsilon^2})^2}{4}\\ -(\epsilon+\sqrt{4+\epsilon^2})^4\ln&\frac{(\epsilon+\sqrt{4+\epsilon^2})^2}{4}\Big]\\ \simeq \begin{cases} \frac{\lambda^4 f^2}{32\pi^2}(2\ln 2-1)|Z_L|^2 & : |Z_L| \lesssim f \\ \frac{\lambda^4 f^4}{16\pi^2}\ln\frac{|Z_L|}{f} & : |Z_L| \gtrsim f, \end{cases}\end{aligned} \tag{2.21}$$

where $\epsilon = |Z_+|/f$.

The supergravity effect induces a tadpole term for Z_L,

$$V(Z_L) = \Delta V_\pm(Z_L) + (-2\sqrt{2}\lambda f^2 m_{3/2} Z_L + \text{h.c.}), \tag{2.22}$$

where $m_{3/2}$ is the gravitino mass. We take $m_{3/2}$ to be real by a $U(1)_R$ rotation. The gravitino mass is related with the magnitude of the SUSY breaking by the (almost) vanishing cosmological constant condition

$$\sqrt{3}m_{3/2} = |F_{Z_L}|/M_{\rm Pl} = \sqrt{2}\lambda\frac{f^2}{M_{\rm Pl}}. \tag{2.23}$$

The tadpole term induces the VEV of Z_L and the messenger scale [291]. Assuming $|\langle Z_L\rangle| \lesssim f$, we obtain

$$\langle Z_L\rangle = \frac{64\sqrt{2}\pi^2}{2\ln 2-1}\frac{m_{3/2}}{\lambda^3} = \frac{128\pi^2}{\sqrt{3}(2\ln 2-1)\lambda^2}\frac{f^2}{M_{\rm Pl}}. \tag{2.24}$$

For small λ, the formula (2.24) yields $|\langle Z_L\rangle| > f$. In such a parameter region, the potential of Z_L given by the quantum correction is logarithmic, and cannot stabilize Z_L against the tadpole term. Instead Z_L is stabilized around $\langle Z_L\rangle \sim M_{\rm Pl}$ by the supergravity effect. Later, we couple $Z_\pm$ to the messenger field. If $\langle Z_L\rangle$ is as large as $M_{\rm Pl}$, the gauge mediated soft masses of supersymmetric standard model (SSM) particles are smaller than the gravitino mass. Thus, in the following, we concentrate on the parameter region where $\langle Z_L\rangle \ll M_{\rm Pl}$. Then in the minimal model, $\langle Z_L\rangle$ is at the most $\mathcal{O}(f)$.

Simultaneous mediation of SUSY breaking and the anomaly of the PQ symmetry

The simplest possibility of the mediation is to introduce a pair of standard $SU(3)_c$ charged chiral fields Q and $\bar{Q}$ with the coupling,

$$W = yZ_+Q\bar{Q}. \tag{2.25}$$

The precise gauge coupling unification is maintained if Q and $\bar{Q}$ are complete multiplets of the $SU(5)$ GUT gauge group. Soft masses generated by loop corrections from the KSVZ axion sector is discussed in [13, 342, 223], while the PQ breaking field is not the dominant source of the SUSY breaking. Soft masses from the F term of the axion multiplet are analyzed in [56] using an effective field theory.

The mass terms of the scalar component of the messenger field are given by

$$V_{\rm mass} = \begin{pmatrix} Q^* & \bar{Q} \end{pmatrix} \begin{pmatrix} y^2 \langle Z_+ \rangle^2 & yF^*_{Z_+} \\ yF_{Z_+} & y^2 \langle Z_+ \rangle^2 \end{pmatrix} \begin{pmatrix} Q \\ \bar{Q}^* \end{pmatrix}. \tag{2.26}$$

To avoid tachyonic masses for the messenger fields, we require that

$$y > \frac{|F_{Z_+}|}{\langle Z_+ \rangle^2} = \frac{2\lambda f^2}{\langle Z_L \rangle^2}. \tag{2.27}$$

On the other hand, the quantum correction from the messenger loop generates a potential term for the SUSY breaking field,

$$\Delta V_{\rm mes} \simeq \frac{N_Q y^2}{32\pi^2} F^2_{Z_L} \ln\frac{|Z_L|^2}{\mu^2}, \tag{2.28}$$

where N_Q is the multiplicity of the messenger field. By requiring that this potential does not destabilize the SUSY breaking vacuum, we obtain

$$\frac{N_Q y^2 \lambda^2 f^4}{8\pi^2 |\langle Z_L \rangle|^2} < \frac{\partial^2 \Delta V}{\partial |\langle Z_L \rangle|^2} \equiv 2m_Z^2. \tag{2.29}$$

The bounds on y in Eqs. (2.27) and (2.29) are compatible if

$$N_Q < \frac{4\pi^2 \langle Z_L \rangle^6}{\lambda^4 f^8} m_Z^2. \tag{2.30}$$

In Fig. 2.1, we show the upper bound on N_Q as a function of Z_L. Here we have evaluated m_Z using $\Delta V_\pm$ in Eq. (2.21). It is evident that the upper bound is too severe and is inconsistent with $N_Q \gtrsim 3$, which leads us to extend the model to stabilize the sgoldstino.

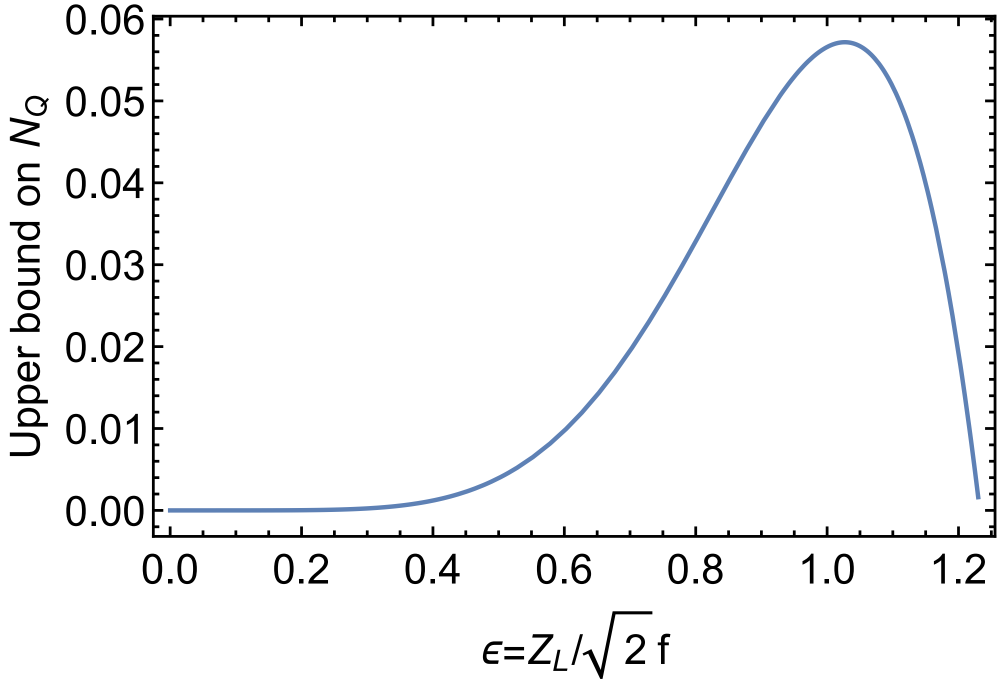

Figure 2.1: Upper bound on the multiplicity of the messenger N_Q for the minimal model.

Stabilization of the sgoldstino in extended models: model-independent analysis

By coupling the sgoldstino to other chiral multiplets, quantum corrections from these multiplets give additional contributions to the mass of the sgoldstino. Here we assume that a positive squared mass m_Z^2 is generated from a quantum correction. (For setups which generate a negative squared mass, see [256, 384, 258, 201, 164, 134].) Even in this generic situation, we show that there is a lower bound on the axion decay constant and the gravitino mass.

The VEV of Z_L is given by

$$\langle Z_L \rangle = \frac{4}{\sqrt{3}} \frac{\lambda^2 f^4}{M_{\rm Pl} m_Z^2}, \tag{2.31}$$

and the gauge mediated gluino mass is given by

$$m_{\tilde{g}} = \frac{\alpha_3}{4\pi} \frac{F_{Z_L}}{Z_L} = \frac{\alpha_3}{4\pi} \frac{\sqrt{6}}{4} \frac{m_Z^2 M_{\rm Pl}}{\lambda f^2}. \tag{2.32}$$

For given λ, f, and $m_{\tilde{g}}$, m_Z^2 is fixed,

$$m_Z^2 = \frac{8\pi}{\alpha_3}\sqrt{\frac{2}{3}}\frac{m_{\tilde{g}}\lambda f^2}{M_{\rm Pl}}. \tag{2.33}$$

There are two bounds that must be considered. One is Eq. (2.30),

$$N_Q < \frac{\alpha_3^5}{8\sqrt{6}\pi^3}\frac{\lambda^3 f^6}{m_{\tilde{g}}^5 M_{\rm Pl}}. \tag{2.34}$$

Another is

$$m_Z^2 > \frac{1}{2}\left|\frac{\partial^2 \Delta V_\pm}{\partial |\langle Z_L\rangle|^2}\right|. \tag{2.35}$$

Otherwise we need fine-tuning between $\Delta V_\pm$ and additional contributions to obtain a required value of m_Z^2. In Fig. 2.2, we show the constraints on (λ, f) as well as the contours of the axion decay constant f_a,

$$f_a = \sqrt{2\left(M_+^2 + M_-^2 + Z_+^2 + Z_-^2\right)}, \tag{2.36}$$

and the gravitino mass $m_{3/2}$. Here we assume that the messenger is in the **5** representation of the $SU(5)$ GUT group, so $N_Q = 5$. For the most part, the axion decay constant is dominated by the VEVs of $M_\pm$ in the left half of the parameter space and the VEVs of $Z_\pm$ in the right half. The blue shaded region is excluded as the messenger field becomes tachyonic. The region below a black dashed line calls for fine-tuning. We obtain lower bounds from them,

$$f_a \gtrsim 1.7\times 10^9 \left(\frac{m_{\tilde{g}}}{3\mathrm{TeV}}\right)^{2/3}\ \mathrm{GeV}, \tag{2.37}$$

$$m_{3/2} \gtrsim 0.2\times \left(\frac{m_{\tilde{g}}}{3\mathrm{TeV}}\right)^{5/3}\ \mathrm{MeV}. \tag{2.38}$$

A final issue to consider in this general approach is the tunneling rate per unit volume between the false and true vacua, $\Gamma/V = Ae^{-B}$ [127]. Using the result from [244, 159], we can estimate the bounce action with

$$B = 8\pi\left(\frac{\langle Z_L\rangle}{(\sqrt{2}\lambda f^2)^{1/2}}\right)^4$$

For the valid parameter space, $B > 10^9$, and so we expect our SUSY breaking vacuum to take much longer than the age of the Universe to decay.

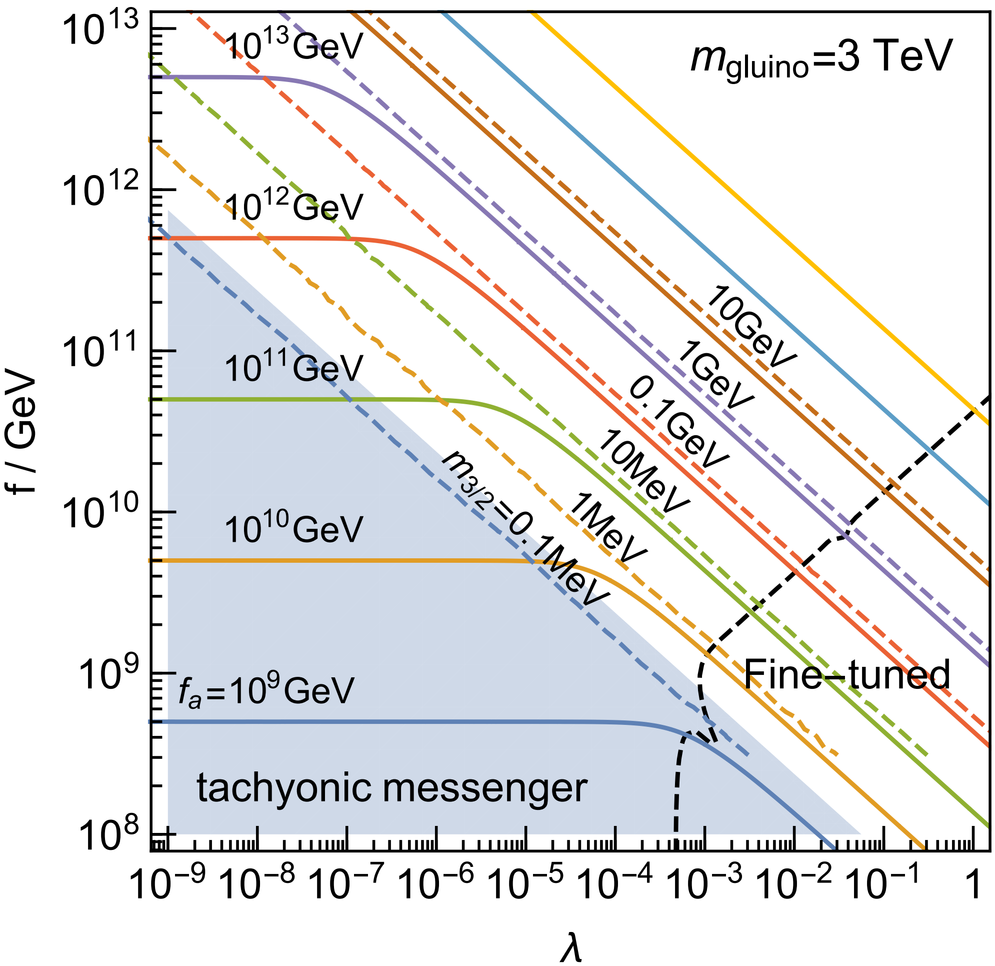

Figure 2.2: The model-independent bounds on (λ, f) and the contours of the axion decay constant f_a and the gravitino mass $m_{3/2}$. The blue shaded region is excluded as the messenger field is tachyonic. The region below the black dashed line requires fine-tuning.

Cosmology

We now address several cosmological topics that may affect the parameter space of our model.

Our model contains a SUSY preserving vacuum where the messengers obtain nonzero VEVs, so we must ensure that the SUSY breaking vacuum is selected during cosmological evolution. Following the discussion in [178], in the early Universe we assume that the SSM particles are in thermal equilibrium and therefore the sgoldstino field potential obtains finite temperature corrections from the messenger fields. We also take the sgoldstino field to be stabilized at the origin initially due to a positive Hubble-induced mass. The messenger masses become tachyonic about $\langle Z_L \rangle = 0$ as the universe cools, which in turn causes them to develop VEVs. To reach the SUSY-breaking vacuum, the sgoldstino field must obtain a sufficiently large VEV before this occurs. Since this condition references the masses of the messengers only about the origin, the model independent analysis performed in [178] should be applicable, and we obtain

$$\frac{y}{\sqrt{2}} < \left(\frac{3^{3/4}}{15} \frac{2g^2 + g'^2}{2} \right)^{2/5} \left(\frac{m_{3/2}}{M_{PL}} \right)^{1/5}. \tag{2.39}$$

Combining this with Eq. (2.35) and Eq. (2.27), we obtain

$$f_a \gtrsim 2.6 \times 10^{10} \left(\frac{m_{\tilde{g}}}{3\text{TeV}} \right)^{2/3} \text{ GeV}, \tag{2.40}$$

$$m_{3/2} \gtrsim 1.6 \times \left(\frac{m_{\tilde{g}}}{3\text{TeV}} \right)^{5/3} \text{ MeV}. \tag{2.41}$$

Hence vacuum selection raises the lower bounds by a factor $\mathcal{O}(10)$.

Another potential concern is that the sgoldstino, which may be produced in the early universe by thermal or nonthermal processes, might affect Big Bang Nucleosynthesis (BBN). The relevant decay modes of the sgoldstino are $Z_L \to aa$ and $Z_L \to gg$ with decay rates

$$\Gamma_{Z_L \to aa} = \frac{m_Z^3}{128\pi} \left(\frac{\langle Z_L \rangle}{2f^2 + \langle Z_L \rangle^2} \right)^2, \tag{2.42}$$

$$\Gamma_{Z_L \to gg} = \frac{\alpha_3^2}{128\pi^3} \frac{m_z^3}{z^2}, \tag{2.43}$$

respectively. Looking to the parameter space in Fig. 2.2, the former decay dominates in most of the area where $\langle Z_\pm \rangle$ controls the axion decay constant, while the latter decay dominates for a majority of the remaining allowed parameter space. Sgoldstino decay into gravitinos dominates in the upper right portion of the parameter space but the gravitino is heavy in this region and so it is not favored. In both of the relevant regions, the decay time is short enough that the sgoldstino does not affect BBN.

It should also be noted that the super partners of the axion obtain large masses. This is a merit of the setup described above [99, 100, 239, 227]. In general, the super partners of the axion obtain only small masses, typically smaller than the masses of SSM particles. Since they couple to SSM particles very weakly while being light, they cause various cosmological problems (see [280] and references therein). These problems are particularly serious in gauge mediation, where the SUSY breaking scale is small and the super partners of the axion are light. In our setup, since the axion multiplet resides in the SUSY breaking sector, the super partners of the axion can be much heavier than SSM particles and do not cause cosmological problems.

The only light particle that could affect cosmology is the gravitino due to either demanding a low reheating temperature [285, 162, 331] or overclosing the Universe [350, 411]. The latter issue could be resolved by diluting the gravitino abundance through large entropy production. Possible example mechanisms include using the sgoldstino [216, 178, 254], saxion [288, 317, 279, 120], messenger fields [175] or hidden sector fields [256]. Note that the sgoldstino and saxion masses are determined by the parameters of our model and the condition for successful entropy dilution could pin down the viable parameter space. For example, entropy dilution via sgoldstino is possible for a gravitino mass of $\mathcal{O}(1\ \mathrm{GeV})$ [178]. We leave the discussion about the saxion for future work.

Alignment of CP phases

An interesting feature of our model is that the phases of the gravitino mass and the gaugino masses are aligned with each other. This is because the phase of the VEV of $\langle Z_\pm \rangle$, which generates the messenger scale, is aligned with the gravitino mass in a phase convention where the F term of the SUSY breaking field Z_L is real. Thus, the CP phase of the $B\mu$ term (in a convention where the μ term is real) due to the supergravity effect [334] is absent in our model. This feature would be advantageous if one requires that SUSY particles are light (e.g. to explain the experimental anomaly of the muon anomalous magnetic moment [67, 212, 142] by SUSY particles [313, 109, 332]) while the gravitino mass is large (e.g. to be consistent with a large reheating temperature.)

2.3 Example of an extended model:low energy theory of the IYIT model

Effective theory of the IYIT model

Let us consider a vector-like SUSY breaking sector based on $SU(2)$ hidden strong gauge dynamics [263, 259]. We introduce four chiral fields which are in the fundamental representation of $SU(2)$, q_i (i = 1-4), and six singlet chiral fields, Z_+, Z_-, $Z_{0,a}$ (a = 1-4). We assume $U(1)_{\mathrm{PQ}}$ charges shown in Table 2.1, and consider the following superpotential,

$$\begin{aligned} W =& \lambda_+ Z_+ q_1 q_2 + \lambda_- Z_- q_3 q_4 \\ &+ Z_{0,a} \left(\lambda_a^{13} q_1 q_3 + \lambda_a^{14} q_1 q_4 + \lambda_a^{23} q_2 q_3 + \lambda_a^{24} q_2 q_4 \right), \end{aligned} \tag{2.44}$$

where the λ's are constants, and summation over a is assumed. The genericity of the superpotential can be guaranteed by symmetries. One concrete example of $U(1)_R$ and Z_4 charges is shown in Table 2.1.

Below the dynamical scale of the hidden $SU(2)$, Λ, the theory is described by meson fields $M_{ij} \simeq q_i q_j / \eta\Lambda$ with the deformed moduli constraints, $\mathrm{Pf} M_{ij} = \Lambda^2/\eta^2$ [381]. Here and hereafter, we assume the naive dimensional analysis to count factors of $\eta \sim 4\pi$ [315, 126]. The deformed moduli constraint may be expressed by introducing a Lagrange multiplier field

Table 2.1: Charge assignment of chiral fields

	q_1	q_2	q_3	q_4	Z_+	Z_-	$Z_{0,a}$	$Q\bar{Q}$
$U(1)_R$	0	0	0	0	2	2	2	0
$U(1)_{\mathrm{PQ}}$	$-1/2$	$-1/2$	$+1/2$	$+1/2$	1	-1	0	-1
Z_4	1	1	1	1	2	2	2	2

X,

$$W_{\mathrm{eff}} = \kappa X \left(M_{12}M_{34} + M_{13}M_{24} + M_{14}M_{23} - \frac{\Lambda^2}{\eta^2} \right). \tag{2.45}$$

The tree-level superpotential in Eq. (2.44) becomes

$$\begin{aligned} W_{\mathrm{tree}} =& \lambda_+ \frac{\Lambda}{\eta} Z_+ M_{12} + \lambda_- \frac{\Lambda}{\eta} Z_- M_{34} \\ &+ \frac{\Lambda}{\eta} Z_{0,a} \left(\lambda_a^{13} M_{13} + \lambda_a^{14} M_{14} + \lambda_a^{23} M_{23} + \lambda_a^{24} M_{24} \right). \end{aligned} \tag{2.46}$$

We define

$$\begin{aligned} M_- &\equiv M_{12}, \;\; M_+ \equiv M_{34}, \\ M_{0,1} &\equiv \frac{1}{\sqrt{2}} \left(M_{13} + iM_{24} \right), M_{0,2} \equiv \frac{1}{\sqrt{2}} \left(M_{13} - iM_{24} \right), \\ M_{0,3} &\equiv \frac{1}{\sqrt{2}} \left(M_{14} + iM_{23} \right), M_{0,4} \equiv \frac{1}{\sqrt{2}} \left(M_{14} - iM_{23} \right). \end{aligned} \tag{2.47}$$

Then the effective superpotential in Eq. (2.45) is given by

$$W_{\mathrm{eff}} = \kappa X \left(M_+ M_- + \frac{1}{2} M_{0,a}^2 - \frac{\Lambda^2}{\eta^2} \right). \tag{2.48}$$

By $SU(4)$ rotations of $M_{0,a}$ and $Z_{0,a}$, the total superpotential can be simplified as

$$\begin{aligned} W =& \kappa X \left(M_+ M_- + \frac{1}{2} c_{ab} M_{0,a} M_{0,b} - \frac{\Lambda^2}{\eta^2} \right) \\ &+ \lambda_+ \frac{\Lambda}{\eta} Z_+ M_- + \lambda_- \frac{\Lambda}{\eta} Z_- M_+ + \lambda_{0,a} \frac{\Lambda}{\eta} Z_{0,a} M_{0,a} \end{aligned} \tag{2.49}$$

with c_{ab} as a unitary matrix. We will work with only one pair of neutral fields (Z_0, M_0), which corresponds to the generic case that there exists a mild hierarchy in the neutral coupling constants so that the effect of only one neutral field dominates. Therefore, after a redefinition of constants, we have the effective superpotential

$$\begin{aligned} W =&\kappa X(M_+M_- + \frac{c}{2}M_0^2 - v^2) \\ &+ \lambda' r v Z_+ M_- + \lambda' \frac{1}{r} Z_- M_+ + \lambda_0' v Z_0 M_0. \end{aligned} \tag{2.50}$$

The coupling constant κ originates from strong dynamics and is expected to be large. The absolute value of the coupling constant c is at maximum unity. To maximize the quantum correction, we assume $|c| = 1$ in the following. We also assume that $\lambda_0' v^2 > \lambda f^2$, since otherwise M_0 obtains a VEV instead of $M_\pm$. The vacuum is then given by

$$\begin{aligned} &\langle M_+ \rangle = rv, \;\; \langle M_- \rangle = \frac{1}{r}v, \;\; \langle Z_+ \rangle = \langle Z_- \rangle = z, \\ &\langle M_0 \rangle = \langle Z_0 \rangle = 0. \end{aligned} \tag{2.51}$$

Stabilization of the sgoldstino by neutral fields in the IYIT model

To estimate the quantum correction from Z_0 and M_0, we use the parametrization [106]

$$M_+ \to r\sqrt{v^2 - M_0^2/2}, M_- \to \frac{1}{r}\sqrt{v^2 - M_0^2/2}. \tag{2.52}$$

Here we have neglected the dependence on A, which is irrelevant for the quantum correction from Z_0 and M_0 to Z_L. The effective superpotential of Z_L and Z_0, M_0 is given by

$$\begin{aligned} W_{\rm eff} \simeq&\lambda f^2 (Z_+ + Z_-)\sqrt{1 - \frac{M_0^2}{2v^2}} + \lambda_0' v Z_0 M_0 \\ \simeq&\sqrt{2}\lambda f^2 Z_L - \frac{\sqrt{2}}{4}R^2 \lambda Z_L M_0^2 + \lambda_0 f Z_0 M_0, \\ R \equiv&\frac{f}{v} > 1, \lambda_0 \equiv \frac{1}{R}\lambda_0'. \end{aligned} \tag{2.53}$$

The quantum correction to the potential of Z_L from Z_0 and M_0 is given by

$$\begin{aligned}
\Delta V_0 =& \frac{\lambda^4 R^4 f^4}{64\pi^2} f\left(\frac{\lambda R^2 z}{\lambda_0 f}\right) \left(1 + O\left((\lambda R/\lambda_0)^4\right)\right) \\
\simeq & \begin{cases} \frac{\lambda^4 R^8 f^2}{96\pi^2}\left(\frac{\lambda}{\lambda_0}\right)^2 |Z_L|^2 & : \lambda R^2 |Z_L| \lesssim \lambda_0 f \\ \frac{\lambda^4 R^4 f^4}{16\pi^2} \ln\frac{\lambda R^2 |Z_L|}{\lambda_0 f} & : \lambda R^2 |Z_L| \gtrsim \lambda_0 f, \end{cases} \\
f(x) =& (4+x^2)^{-2}\Big[32 + 20x^2 + 3x^4 \\
& + \left(16 - 4\sqrt{1+4/x^2} + 8x^2 + x^4 - 6x\sqrt{4+x^2}\right. \\
& \left. - x^3\sqrt{4+x^2}\right) \ln\left(1 + \frac{x^2}{2} - x\sqrt{1+x^2/4}\right) \\
& + \left(16 + 4\sqrt{1+4/x^2} + 8x^2 + x^4 + 6x\sqrt{4+x^2}\right. \\
& \left. + x^3\sqrt{4+x^2}\right) \ln\left(1 + \frac{x^2}{2} + x\sqrt{1+x^2/4}\right)\Big].
\end{aligned} \tag{2.54}$$

In this model, m_Z^2 is given by

$$m_Z^2 = \frac{\lambda^4 R^8 f^2}{96\pi^2}\left(\frac{\lambda}{\lambda_0}\right)^2 + \frac{1}{2}\frac{\partial^2 \Delta V_\pm}{\partial |\langle Z_L \rangle|^2}. \tag{2.55}$$

Parameter window of the IYIT model

Let us now discuss constraints on the parameter space. The constraint from the stability of the vacuum, $\lambda_0' v^2 > \lambda f^2$, is

$$\lambda R < \lambda_0. \tag{2.56}$$

Constants $\lambda' r$, λ'/r and λ_0' are dimensionless coupling constants in the IYIT model, and are at the most $O(1)$. This gives upper bounds on λ_0 and R,

$$\lambda R^3 < 1, \tag{2.57}$$

$$\lambda_0 R < 1. \tag{2.58}$$

Finally, the potential of Z_L becomes logarithmic for $\lambda R^2 Z_L > \lambda_0 f$, and cannot stabilize the sgoldstino against the tadpole term, so

$$\lambda R < \sqrt{\frac{4\pi}{\sqrt{2}\alpha_3}\frac{m_{\tilde{g}}}{f}}\lambda_0^{1/2}. \tag{2.59}$$

By combining the bounds in Eqs. (2.56-2.59), we obtain upper bounds on R^4/λ_0,

$$\frac{R^4}{\lambda_0} < \begin{cases} \lambda^{-2} & \text{Eqs. } (2.56), (2.57) \\ \lambda^{-8/3} h^2 & \text{Eqs. } (2.57), (2.59) \\ \lambda^{-10/3} h^{10/3} & \text{Eqs. } (2.58), (2.59) \end{cases} , \quad h \equiv \sqrt{\frac{4\pi}{\sqrt{2}\alpha_3} \frac{m_{\tilde{g}}}{f}} \tag{2.60}$$

These give upper bounds on m_Z^2.

In Fig. 2.3, we show the constraints on (λ, f). The meaning of the blue shaded region and the black dashed line are the same as in Fig. 2.2. In the green shaded region, the bound on m_Z^2 from Eqs. (2.55) and (2.60) is inconsistent with the required value of m_Z^2 shown in Eq. (2.33). We obtain the bounds on the axion decay constant f_a and the gravitino mass $m_{3/2}$

$$10^9 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}, \tag{2.61}$$

$$0.1 \text{ MeV} \lesssim m_{3/2} \lesssim 10 \text{ MeV}, \tag{2.62}$$

for a gluino mass $\mathcal{O}(\text{TeV})$. It is interesting that the allowed range of f_a is consistent with the axion dark matter scenario [363, 11, 152, 143].

2.4 Conclusion

In this chapter, we have presented a model that tackles several outstanding issues in the Standard Model and its supersymmetric extension.

We have examined a minimal hidden sector that consists of a superpotential with a $U(1)$ symmetry, which we identify with the PQ symmetry, and messenger quarks that carry $SU(3)_c$ charges.

Supersymmetry and this PQ symmetry are spontaneously broken while lowest order supergravity effects create the messenger scale. Quantum effects generate a potential for the sgoldstino and force constraints on model parameters to ensure the stability of the SUSY-breaking vacuum. These constraints proved to be too stringent and required that we supplement the minimal model with extra matter fields. We have shown that classes of models that share features with ours, such as a quantum mechanically induced sgoldstino mass and a minimal messenger sector, automatically obtain lower bounds on the axion decay constant and gravitino mass. This fact encouraged us to supplement our minimal model in the hopes that such attractive features could be preserved and expanded upon in a stable extended model.

An IYIT model with $SU(2)$ gauge dynamics is a natural candidate for such an extended model since the minimal model is easily embedded in the $U(1)$ charged subsector of this larger model. Combining the inequalities from vacuum stability and IYIT coupling constants, upper bounds for the sgoldstino mass were derived. The resulting window in parameter space was found to restrict the gravitino mass to lie between $0.1 \text{ MeV} \lesssim m_{3/2} \lesssim 10$ MeV and the axion decay constant to $10^9 \text{ GeV} \lesssim f_a \lesssim 10^{12}$ GeV, which is the suitable range for invisible axion dark matter.

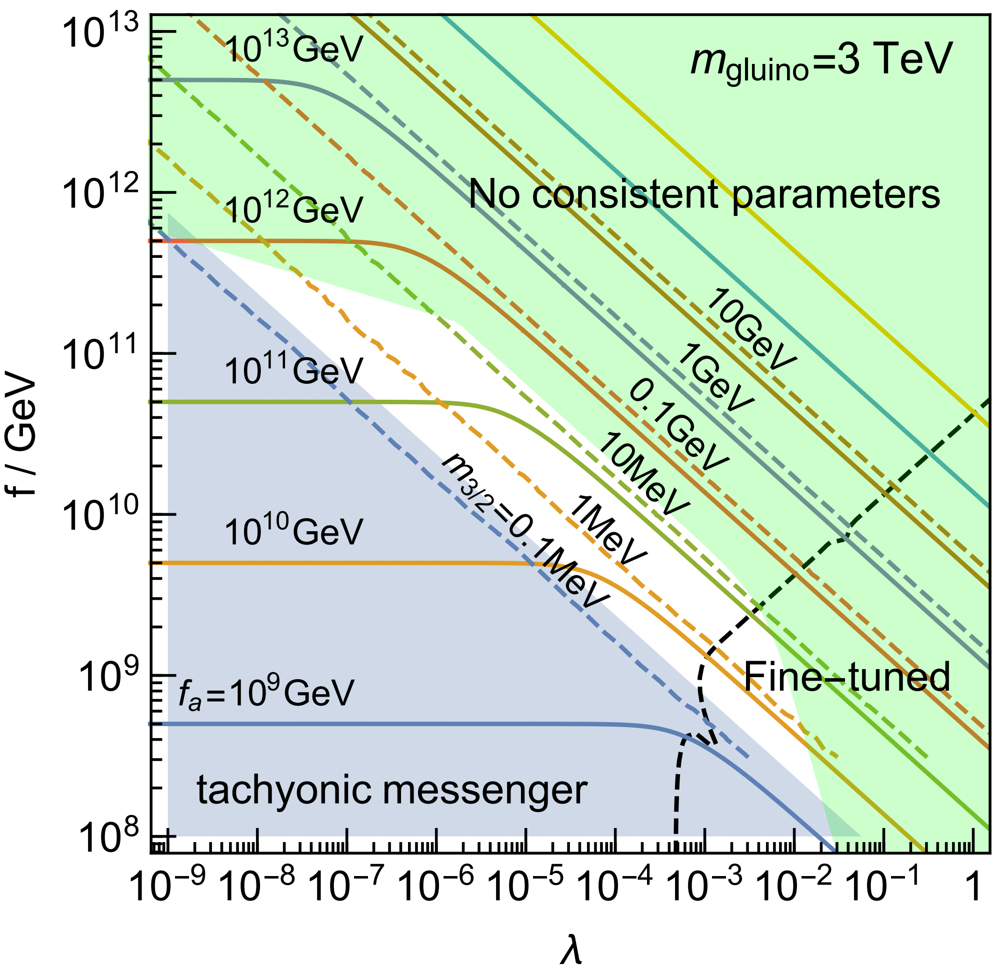

Figure 2.3: The bounds on (λ, f) for the IYIT model and the contours of the axion decay constant f_a and the gravitino mass $m_{3/2}$. The blue shaded region is excluded as the messenger field is tachyonic. The region below the black dashed line requires fine-tuning. There is no consistent parameter (λ_0, R) to yield the green shaded region.

Chapter 3

QCD Axion Dark Matter from a Late Time Phase Transition

3.1 Introduction

The CP violation in QCD [8], expressed by the so-called θ parameter, is extremely small - $\theta < 10^{-10}$ [53]. The smallness of the CP violation is elegantly explained by the Peccei-Quinn (PQ) mechanism [356, 355]. One introduces a spontaneously broken global symmetry which is explicitly broken by the QCD anomaly, and predicts a pseudo-Nambu-Goldstone boson called an axion [409, 416]. For a large enough symmetry breaking scale, the axion is stable and a dark matter candidate [363, 11, 152].

Two production mechanisms of axion dark matter in the early universe are widely recognized. One is the misalignment mechanism [363, 11, 152], where the displacement of the axion field from the vacuum turns into oscillations which behave as dark matter. Another is the emission of axions from the string-domain wall network produced after the spontaneous breaking of the PQ symmetry [143, 281, 292, 89]. Both mechanisms require that the axion decay constant f_a is large - $f_a \gtrsim 10^{11}$ GeV. (The estimation of the abundance in the latter mechanism assumes a scaling law of topological defects. See [203, 283, 323] for a possible violation as well as [240] for results that contradict this violation.)

In this chapter, we point out a new production mechanism for axion dark matter under the assumption that the phase transition temperature of the PQ symmetry breaking is far below the symmetry breaking scale. We find that axions are abundantly produced via parametric resonance arising from oscillations of the symmetry breaking field [397, 296, 388, 297] after the phase transition. Since the phase transition temperature is low, the produced axions are not thermalized and remain as dark matter. The axions produced from the late time phase transition can explain the observed dark matter abundance even if the decay constant is much smaller than 10^{11} GeV.

Low phase transition temperatures are natural in supersymmetric theories. This is because the radial direction of the PQ symmetry breaking field, commonly called the saxion, is the scalar partner of the nearly-massless axion. The mass of the saxion is given by a supersymmetry breaking soft mass and is much smaller than the PQ symmetry breaking scale. This small mass in turn yields a relatively low phase transition temperature.

In contrast to the two conventional mechanisms, the produced axions are initially relativistic and red-shift sufficiently to be dark matter. In some of the parameter space, the axions are still warm enough to affect structure formation by an observable amount.

There are intensive ongoing and future experimental efforts to search for the axion with a small decay constant, such as IAXO [406, 42], TASTE [31], Orpheus [376], MADMAX [93], ARIADNE [46, 195], and many others [389, 44, 57, 303]. Astrophysical observations and the above searches for the QCD axion could probe the dynamics of the PQ phase transition.

3.2 Late Time PQ Phase Transition

The Model

We consider a coupling of the PQ symmetry breaking field P to new PQ charged fermions ψ and $\bar{\psi}$,

$$\mathcal{L} = yP\psi\bar{\psi} + \text{h.c.}. \tag{3.1}$$

The new fermions may be identified with the hidden quarks of the KSVZ model [289, 383]. This coupling gives a thermal potential to P,

$$V_T = V(P) + V_{\text{th}}(P, T), \tag{3.2}$$

where V is the vacuum potential of P. In a typical second-order phase transition, the thermal potential can be expanded about $|P|/T \ll 1$ to get the thermal mass and higher corrections:

$$V_{\text{th}}(P, T) = y^2 T^2 |P|^2 + \cdots . \tag{3.3}$$

The critical temperature T_c is then defined as the temperature at which the curvature of the potential about the origin vanishes. If the vacuum potential has the form $V = -m^2|P|^2 + \cdots$,

$$T_c = \frac{m}{y}, \tag{3.4}$$

We define a late time phase transition as a phase transition that satisfies $m \ll f_a$ so that $T_c \ll f_a$.

While a late time phase transition may seem fine-tuned, the above hierarchy is typically encountered in supersymmetric theories where PQ symmetry breaking scales much larger than the mass m are naturally obtained through the stabilization of P by a higher dimensional interaction [339],

$$V = \left(\frac{2^{n-2} m_s^2}{n(n-1) f_a^{2n-2}} \right) |P|^{2n} - \frac{m_s^2}{2n-2} |P|^2 + \frac{m_s^2 f_a^2}{4n} \tag{3.5}$$

with $n > 2$, or through the renormalization group running of the soft mass of P [337],

$$V = \frac{m_s^2}{2}|P|^2 \left(\ln\frac{2|P|^2}{f_a^2} - 1\right) + \frac{1}{4}m_s^2 f_a^2. \tag{3.6}$$

The parameters of these potentials have been chosen such that the saxion mass around the vacuum is m_s ($\sim m$), the vacuum expectation value of $|P|$ is $f_a/\sqrt{2}$, and the vacuum energy at the minimum of the potential vanishes.

The phase transition and thermal inflation

A late time phase transition does not proceed via a first- or second-order phase transition. Since $m_s \ll f_a$, the high temprature expansion is insufficient at T_c and one must follow the evolution of the total thermal potential as the temperature decreases. At high temperatures, $T^4 \gg m_s^2 f_a^2$, the origin is an absolute minimum. For $T^4 < m_s^2 f_a^2$, on the other hand, the minimum at the origin is a local, metastable minimum until T_c. The scalar field P is trapped at the origin by the dip formed by the thermal correction. This is schematically displayed in Fig. 3.1. One might assume that the phase transition is then first order and proceeds via bubble nucleation. However, the numerical results of [242] find that a slightly different process occurs. Before the quantum tunneling rate becomes effective enough for bubble percolation, thermal fluctuations of P are large enough to cause the phase transition. For a weak Yukawa coupling, $y \lesssim 0.1$, this is found to occur for a temperature within a sub percent of T_c [242].

For both of the potentials above, the PQ symmetry breaking field at the origin has a potential energy density $V(0) \propto m_s^2 f_a^2$. This is larger than the radiation energy density at T_c, $\rho_{rad} = \frac{\pi^2}{30} g_* T_c^4$, if

$$y \gtrsim \sqrt{\frac{m_s}{f_a}}, \tag{3.7}$$

which we assume in the following. The case with smaller y can be analyzed in a similar manner. Since the potential energy dominates, a period of so-called thermal inflation [420, 318] occurs with a Hubble scale $H_{\rm PT} \propto \frac{m_s f_a}{M_{\rm Pl}}$ before the phase transition.

Axions from inhomogeneity

Just after the phase transition, the configuration of the PQ symmetry breaking field is inhomogeneous. Under a normal second-order phase transition, the correlation length of the configuration is determined by the Kibble-Zurek mechanism [287, 421]. However, our scenario differs from this mechanism. As discussed above, [242] finds that the phase transition occurs by thermal fluctuations at a temperature $T > T_c$. In order to estimate the correlation length of the inhomogeneous configuration, we assume the weak coupling scenario of [242]

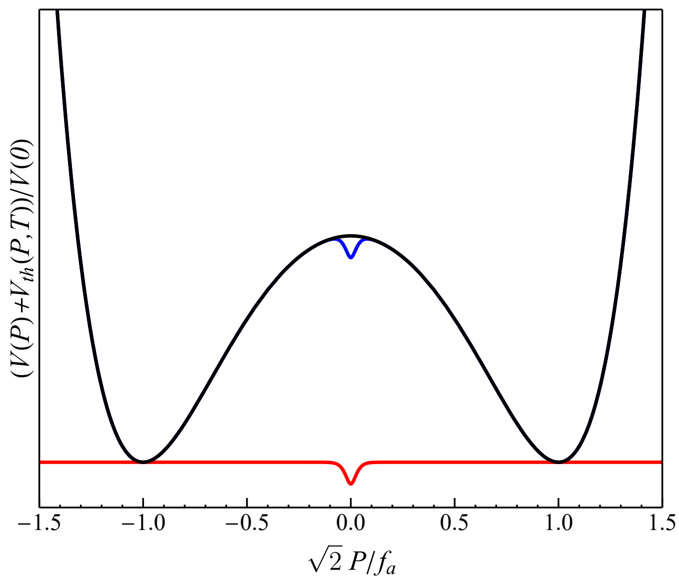

Figure 3.1: The solid black line is the vacuum potential of P as given in Eq. (3.5) with $n = 3$. The solid red line is the thermal correction to the vacuum potential that has been exaggerated to emphasize the structure. The blue line is the sum of the vacuum potential and exaggerated thermal potential.

so that the phase transition occurs at $T_s = \alpha T_c$, where $0 < \alpha - 1 < 10^{-2}$. The correlation length of the scalar field P at T_s is

$$\xi_s = \frac{1}{m_s}\left(\frac{|T_s - T_c|}{T_c}\right)^{-1/2} = m_s^{-1}|1-\alpha|^{-1/2} \tag{3.8}$$

Since the field is correlated on the length scale ξ_s, we can draw an analogy with the Kibble-Zurek mechanism and expect typically one cosmic string per correlation length volume, ξ_s^3, with energy density f_a^2/ξ_s^2. The gradient energy density of the inhomogeneity is f_a^2/ξ_s^2. Typically one cosmic string per correlation length volume, ξ_s^3, exists with energy density f_a^2/ξ_s^2.

The inhomogeneous configuration is quickly homogenized until the correlation length becomes as large as the horizon size. The gradient and string energy should be emitted as axions with typical wavelength ξ_s. The number density of axions produced from the inhomogeneity is then

$$n_a^{\text{inh}} \sim \frac{f_a^2}{\xi_s} = m_s f_a^2 |1-\alpha|^{1/2}. \tag{3.9}$$

The potential energy density $m_s^2 f_a^2$ is converted into the oscillation energy of the saxion, which subsequently red-shifts in proportion to the inverse cube of the scale factor of the universe. We thus normalize the number density of axions by the energy density of the saxion oscillation, n_a/ρ_s, which does not change under red-shifting. For axions coming from the inhomogeneity,

$$\frac{n_a^{\text{inh}}}{\rho_s} = \frac{1}{m_s}|1-\alpha|^{1/2}. \tag{3.10}$$

As we will see in the next section, however, axions produced from the inhomogeneity are subdominant.

3.3 Axions from parametric resonance

Axion Production

The axion population produced by cosmic strings is supplemented and surpassed by a second production mechanism - parametric resonance [397, 296, 388, 297]. After the phase transition, the saxion oscillates with an amplitude on the order of f_a and induces a time-dependent dispersion relation in the equation of motion for axion modes. In certain momentum bands, the axion mode solutions feature instabilties that grow exponentially in time. These modes then yield the axion population produced by the non-perturbative process of parametric resonance.

The production rate of axions via parametric resonance is as large as the frequency of the oscillations $\sim m_s$, since that is the only energy scale appearing in the equation of motion of axions. This is explicitly shown in Appendix A, where we display that the rate of axion production is m_s times an $\mathcal{O}(1)$ constant (Fig. A.1). By comparing this axion production rate with the Hubble rate $\sim m_s(f_a/M_{\text{Pl}}) \ll m_s$, we see that the parametric resonance process is very efficient.

Parametric resonance preferentially creates axions with momenta $k_a \sim m_s/2$ and continues until the newly produced axion energy density is roughly equal to the initial saxion energy density, which is just the potential energy at the origin $V(0) \propto m_s^2 f_a^2$. We label this second contribution to the axion density as n_a^{PR}, so that

$$\frac{n_a^{\text{PR}}}{\rho_s} \simeq \frac{m_s^2 f_a^2}{m_s}\frac{1}{m_s^2 f_a^2} = \frac{1}{m_s}. \tag{3.11}$$

For saxion oscillations with an amplitude of the order of f_a, saxion fluctuations are also produced by parametric resonance and obtain a number density similar to the one in Eq. (3.11).

Comparing Eqs. (3.10) and (3.11), we see that the parametric resonance axions are the dominant contribution to the axion population. In what follows, we only take into account the parametric resonance axions. With this, one finds that the axion number density normalized by the entropy density s is

$$Y_a = \frac{n_a}{s} = \frac{n_a}{\rho_s}\frac{\rho_s}{s} \simeq \frac{T_{RH}}{m_s}, \tag{3.12}$$

where $T_{\rm RH}$ is the reheat temperature by the dissipation of the saxion oscillation and fluctuations after the thermal inflation. To obtain the axion dark matter abundance

$$Y_a^{\rm DM} = \frac{1}{m_a}\frac{\rho_{DM}}{s} = 70\left(\frac{f_a}{10^9\ \mathrm{GeV}}\right), \tag{3.13}$$

the reheat temperature $T_{\rm RH}$ must be above

$$T_{\rm DM} \simeq 0.7\ \mathrm{GeV}\left(\frac{m_s}{10\ \mathrm{MeV}}\right)\left(\frac{f_a}{10^9\ \mathrm{GeV}}\right). \tag{3.14}$$

If $T_{\rm RH}$ is higher than this value, axions are overproduced, but the introduction of extra entropy production from heavy fields can generate the correct abundance. Obtaining this reheat temperature is discussed below.

Validity of Parametric Resonance Scenario

One might be concerned that the inhomogeneity caused by the phase transition could ruin the parametric resonance process, which requires coherent oscillations. We do not anticipate that this is the case since the wavelengths in the resonance band are $\sim 1/m_s$, which is much shorter than the length scale on which the PQ symmetry breaking field is correlated, ξ_s. Hence the oscillations are effectively coherent for the modes in the resonance band.

The above scenario could also be affected if energy from the saxion oscillations is drained into Standard Model fields. This is only a concern if the rate of energy loss to Standard Model fields is comparable to m_s, the rate of axion production from parametric resonance. The saxion coupling to gluons gives a thermalization rate [338, 82]

$$\frac{\Gamma_{\rm gluon}}{m_s} = \left(A\frac{\alpha_3^2 T^3}{f_a^2}\right)\frac{1}{m_s} \lesssim A\alpha_3^2\sqrt{\frac{m_s}{f_a}} \ll 1, \tag{3.15}$$

where A is an $\mathcal{O}(10^{-3})$ constant and we used an inequality $T^4 \lesssim m_s^2 f_a^2$. Thus the energy loss to the Standard Model through gluons is negligible during parametric resonance. Non-perturbative production of gluons is ineffective due to the loop-suppressed coupling between gluons and saxions. The thermal mass of the gluon further reduces the effectiveness of non-perturbative production.

In the sequel, we consider a saxion-Higgs mixing that provides a second avenue for energy loss to Standard Model fields. The ratio of the thermalization rate from the coupling $\lambda S^2 H^\dagger H$ [338] to m_s is

$$\frac{\Gamma_{\rm Higgs}}{m_s} = \left(\frac{\lambda^2 f_a^2}{T}\right)\frac{1}{m_s} \lesssim \frac{10^{-8} m_H^4}{f_a^{1/2} m_s^{3/2} v_{\rm EW}^2}, \tag{3.16}$$

where $m_H = 125$ GeV and $v_{\rm EW} = 246$ GeV. We have used the experimental upper bound $\theta \lesssim 10^{-4}$ [62] on the mixing angle $\theta \sim \frac{\lambda f_a v_{EW}}{m_H^2}$. The last quantity in Eq. (3.16) is much smaller than unity in the viable space discussed below and we conclude that this process is also ineffective. Higgs particles can be produced by parametric resonance, but the produced Higgses are immediately dissipated by decays or scatterings with the thermal bath, and so the Bose-enhancement necessary for efficient parametric resonance is absent. From these results we see that our scenario of parametric resonance is not spoiled by interactions with the Standard Model.

3.4 Model Constraints

Axion Warmness

The axions are produced relativistically and may behave as warm dark matter. To be concrete, we consider a specific model with the PQ potential in Eqs. (3.5) or (3.6). The expressions in the following depend on the potential, but a similar analysis can be performed for more general potentials.

To estimate the warmness, we first note that the ratio between the axion momentum, k_a, and the cube root of the axion number density, $n_a^{1/3}$, is constant throughout the evolution of the universe,

$$\frac{k_a}{n_a^{1/3}} = \left(\frac{n}{2}\right)^{\frac{1}{3}} \left(\frac{m_s}{f_a}\right)^{\frac{2}{3}}, \tag{3.17}$$

where n is an integer larger than 2 in Eq. (3.5) or 1 for the potential in Eq. (3.6). Here it is assumed that half of the potential energy of the saxion is transferred into axions with momenta $k_a = m_s/2$. Using the observed dark matter abundance to fix n_a relative to the entropy density s, we obtain

$$v_a \simeq 6 \times 10^{-4} n^{1/3} \left(\frac{f_a}{10^9 \text{ GeV}}\right)^{\frac{2}{3}} \left(\frac{m_s}{\text{GeV}}\right)^{\frac{2}{3}} \left(\frac{T}{\text{eV}}\right) \tag{3.18}$$

for the velocity of the axions at temperature T. We have assumed $T \ll$ MeV to express the entropy density in terms of T. In Fig. 3.2, we show contours of the axion velocity at $T = 1$ eV for $n = 3$.

The constraint on the warmness of dark matter is frequently estimated for a model where dark matter consists of a massive Weyl fermion with mass $m_{\rm WDM}$ that decouples while relativistic and is later diluted. In such a model, the typical velocity of dark matter, $v_{\rm WDM}$, at temperature T is given by

$$v_{\rm WDM} = \frac{k_{\rm WDM}}{m_{\rm WDM}} \simeq 10^{-4} \left(\frac{T}{1\ {\rm eV}}\right) \left(\frac{3.3\ {\rm keV}}{m_{\rm WDM}}\right)^{\frac{4}{3}}. \tag{3.19}$$

This result, combined with the warm dark matter mass bound, $m_{\rm WDM} > 3.3$ keV [404], yields the generic velocity bound of $v < 10^{-4}$ at $T = 1$ eV. This warmness bound imposes the following constraint on the saxion mass

$$m_s \lesssim 30\ {\rm MeV} \left(\frac{3}{n}\right)^{\frac{1}{2}} \left(\frac{10^9\ {\rm GeV}}{f_a}\right). \tag{3.20}$$

The green shaded region in Fig. 3.2 is disfavored by this constraint. Future observations of 21cm lines can probe $m_{\rm WDM} < 10$-20 keV [391], which corresponds to $v_a \gtrsim 10^{-5}$ at $T = 1$ eV, as indicated by the arrow in Fig. 3.2. For convenience, we provide the correspondence between the mass of this fermionic dark matter and the parameters of our model,

$$m_{\rm WDM} \leftrightarrow \frac{0.8\ {\rm keV}}{n^{1/4}} \left(\frac{10^9\ {\rm GeV}}{f_a}\right)^{\frac{1}{2}} \left(\frac{\rm GeV}{m_s}\right)^{\frac{1}{2}}. \tag{3.21}$$

Stellar Cooling

In addition to the warmness bound, we consider constraints from the cooling of red giant (RG) and horizontal branch (HB) stars [366, 207, 208] by the emission of saxions. We follow the analysis performed in [222, 294]. For RG and HB stars, one must demand that the energy transport by new particles with effective nucleon couplings not exceed 10 erg $g^{-1}s^{-1}$. These constraints are displayed as the blue region in Fig. 3.2 labeled as "RG & HB".

SN1987A and $N_{\rm eff}$

The orange shaded and dashed excluded parameter regions in Fig. 3.2 labeled "SN1987A or $N_{\rm eff}$" arise from the SN1987A constraint of [261], as well as the constraint on the effective number of relativistic degrees of freedom $N_{\rm eff}$ [19]. For SN1987A, the energy loss should not exceed 10^{19} erg $g^{-1}s^{-1}$. This leads to the boundary of the orange shaded region as well as the orange dashed curve. If one takes the SN1987A constraint on energy loss directly, the region below the orange dashed curve would be excluded. However, if one assumes a coupling between the saxion and Standard Model Higgs of the form $\lambda S^2 H^\dagger H$, then the saxion enters the so-called trapping regime and the SN1987A constraint does not apply and the region below the dashed orange curve is permitted. To be in the traping regime, the

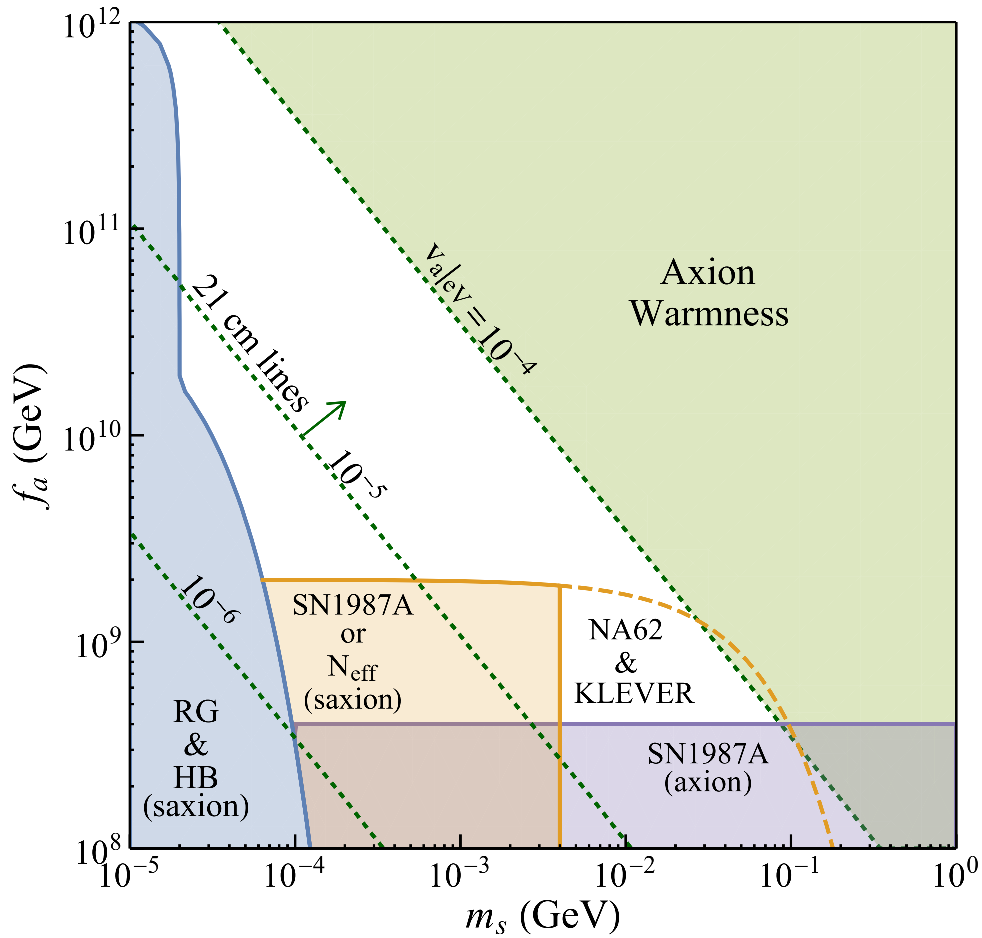

Figure 3.2: Constraints on the saxion mass m_s and the axion decay constant f_a.

saxion-Higgs mixing angle $\theta \sim \frac{\lambda f_a v_{EW}}{m_H^2}$ must be larger than $\sim 10^{-4.5}$ [62]. The orange shaded region remains constrained since the mixing keeps the saxion in thermal equilibrium with electrons even after neutrinos decouple in the early universe. Hence the depletion of the saxion energy heats up photons, resulting in $N_{\rm eff} < 3$. Assuming that neutrinos suddenly decouple at $T \simeq 2$ MeV, we determine a lower bound on the saxion mass of $m_s \gtrsim 4$ MeV. The purple shaded region is the bound related to axions arising from SN1987A [163, 367, 398, 325, 368]. We also note that there is at least an order of magnitude uncertainty in the SN1987A constraints [219, 220, 374, 107, 97, 55]. This could lead to a larger parameter space. The saxion-Higgs mixing results in rare decays of Kaons. As shown in [62], the large mixing in the trapping regime can be probed by NA62 and KLEVER experiments [131, 30].

Saxion Thermalization

The saxion should be thermalized at or above the temperature $T_{\rm DM}$ in Eq. (3.14). We consider the case where the PQ symmetry breaking field P couples to a pair of new fermions f and $\bar{f}$ via the Yukawa coupling

$$\mathcal{L} = \frac{\mu}{f_a} P f \bar{f}, \tag{3.22}$$

where μ is the mass of the fermion. For $T > \mu$, the saxion thermalizes with a rate $\simeq 0.1 T\mu^2/f_a^2$ [58, 338], leading to a reheating temperature

$$T_{\rm RH} \simeq 100 \text{ GeV} \left(\frac{\mu}{100 \text{ GeV}}\right)^2 \left(\frac{10^9 \text{ GeV}}{f_a}\right)^2 . \tag{3.23}$$

If the fermion is charged under the Standard Model gauge group, the mass μ must be above 100 GeV. However, This reheating temperature is larger than the lower bound given in Eq. (3.14) above, and more than enough axion dark matter is produced. $T_{\rm RH} = T_{\rm DM}$ can be obtained through thermalization from coupling the saxion with Standard Model particles or with particles that are neutral under the Standard Model gauge group. Note that too much dark radiation is produced if the new fermions introduced in Eq. (3.22) have masses below $\mathcal{O}(10)$ MeV. If the fermion mass μ required to produce the dark matter abundance is below $\mathcal{O}(10)$ MeV, one must fix the fermion mass to be larger than this scale and introduce additional entropy production to dilute the overproduced axions.

Axion Thermalization

Axions produced in our model are never thermalized. The thermalization rate of an axion is suppressed by the decay constant and the momentum of the axion [335],

$$\Gamma_a = b \frac{k_a^2}{f_a^2} T, \tag{3.24}$$

where b is a constant which depends on the axion coupling. If the axion couples to gluons, b is loop-suppressed and is as small as 10^{-5}. If instead the axion couples to a light fermion in the thermal bath, b may be as large as $O(1)$. During the matter dominated era by the saxion oscillation, $k_a/\rho_s^{1/3}$ remains constant. The momentum of axions is then given by

$$k_a \simeq \left(\frac{m_s \rho_s}{f_a^2}\right)^{\frac{1}{3}} . \tag{3.25}$$

The energy density of the thermal bath never exceeds that of the saxion. Hence the thermalization rate is bounded from above,

$$\Gamma_a < b \frac{m_s^{2/3} \rho_s^{11/12}}{f_a^{10/3}} . \tag{3.26}$$

The ratio between the thermalization rate and the Hubble expansion rate is

$$\frac{\Gamma_a}{H} < b\frac{m_s^{2/3}\rho_s^{5/12}M_{\rm Pl}}{f_a^{10/3}} < b\frac{m_s^{3/2}M_{\rm Pl}}{f_a^{5/2}} \tag{3.27}$$

where the last inequality is saturated right after the phase transition. In the region of parameter space that produces sufficiently cold axion dark matter, the axions are never thermalized. One can see that the late-time phase transition is crucial. If the mass m_s is as large as f_a, the thermalization is effective. After the saxion decays and the radiation dominated era begins, the thermalization rate decreases faster than the Hubble expansion rate and the thermalization of the axions never becomes efficient.

3.5 Conclusion

We have investigated a production mechanism for QCD axion dark matter associated with PQ symmetry breaking at a low temperature. We find that axions are primarily produced by parametric resonance via oscillations of the PQ symmetry breaking field. The low phase transition temperature fits naturally in supersymmetric theories.

The axions produced by this mechanism tend to be warm. The prediction on axion warmness is shown in Fig. 3.2 and constrains the allowed parameter space. Future observations of 21cm lines will probe the parameter space further. Discovery of the QCD axion in laboratories and the determination of dark matter warmness by astrophysical observations will suggest that axion dark matter was produced by parametric resonance.

Fig. 3.2 is one of the primary results of this chapter and contains information beyond the warmness constraint. As outlined above, one also has bounds from energy loss in RG and HB stars and supernovae by saxion emission, as well as axion emission in the supernovae case. We note that our parameter space easily allows for rather low values of the axion decay constant, particularly if strong saxion-Higgs coupling occurs to trap saxions inside the supernova core, or if the traditional SN1987A bound is loosened. The region with large saxion-Higgs mixing can be probed by observations of rare Kaon decays.

There are several uncertainties in our estimation of the warmness. First, we have assumed that half of the energy density of the saxion oscillation is transferred into axions. In reality the transferred fraction will not be exactly half. Second, we have assumed that the momentum as well as the number density of the axions decrease only by the cosmic expansion. However, the momentum/number density can slowly increase/decrease by axion self interactions, see [327] for a related discussion. These two effects will change the prediction on axion velocity by an O(1-10) factor. Whether or not the whole parameter space can be probed depends on these uncertainties, which can be fixed by numerical computation.

We list other known mechanisms to produce axion dark matter for $f_a \ll 10^{11}$ GeV: 1)axion emission from long-lived topological defects which collapse via explicit PQ symmetry breaking [281, 241, 243, 371, 224], 2)parametric resonant production of axions from oscillations of the saxion with a large initial field value [123], 3)a misalignment angle fine-tuned to be close to π [399, 316, 392], 4)dynamical mechanisms that set the misalignment angle close to π [121, 394], 5)the misalignment mechanism with non-standard cosmology [405], and 6)delayed oscillations of the axion field because of a non-zero kinetic energy of the axion field [124, 122]. Among them, 2) can also produce warm axions, but the produced axions are much colder than our mechanism for a given (m_s, f_a). We also note that the large field value assumed in [123] requires that the potential be flat for large field values and is therefore incompatible with the potential in Eq. (3.5).

Our axion production mechanism involves a PQ symmetry breaking field that is initially trapped at the origin. We may consider a generic situation where a PQ symmetry breaking field is trapped at some other point in field space and later begins to oscillate with a large amplitude. One example is a model with the superpotential

$$W = \lambda X(P\bar{P} - V_{\rm PQ}^2), \tag{3.28}$$

where P and $\bar{P}$ are PQ symmetry breaking fields and X is a chiral multiplet that fixes them on the moduli space $P\bar{P} = V_{\rm PQ}^2$. The moduli space is lifted by additional superpotential terms that spontaneously break supersymmetry [99, 100, 225], or by the soft masses of P and $\bar{P}$. Ref. [333] investigates the trapping of the PQ symmetry breaking fields on the moduli space by a thermal potential and finds that oscillations occur in some region of the parameter space. Axions should then be produced via parametric resonance in this setup as well.

Chapter 4

Cosmological Tension of Ultralight Axion Dark Matter and its Solutions

4.1 Introduction

Axions with masses well below the electroweak scale are simple dark matter candidates [363, 11, 152] with novel experimental signatures [357, 286, 248], a potential solution to the apparent incompatibility of cold dark matter with small scale structure [247, 158], and a common prediction of string theory [47, 214]. Axions can arise as pseudo-Goldstone bosons of a spontaneously broken global symmetry or as zero modes of antisymmetric tensor fields after compactification of extra dimensions. In either case, their parametrically suppressed mass can result in a length scale comparable to the size of dwarf galaxies, the so-called *fuzzy* dark matter regime. Dark matter with a macroscopic Compton wavelength allows for novel detection opportunities and many on-going and future experimental efforts search for ultralight axion relics with masses around this limit. Purely gravitational searches look for erasure of structure on small scales as a consequence of the axion's sizable wavelength, limiting the axion mass, $m_a \gtrsim \mathcal{O}(10^{-21}\text{eV})$ [295, 260, 344, 305, 372, 378, 341][1]. If axion dark matter has sizable non-gravitational coupling to the visible sector, there are additional detection strategies dependent on the nature of the coupling.

Axion couplings to the Standard Model fields are intimately related to the shape of the axion potential. For a given axion decay constant, f_a, axion couplings to matter are suppressed by $\sim 1/f_a$ while the ratio of the axion mass to its quartic coupling is typically at least on the order of $\sim f_a$. Such non-quadratic contributions to the axion potential play an important role in cosmology and the interplay between the axion's couplings and potential is the focus of this work.

Measurements of the matter-power spectrum [197, 19, 12] find that the cosmic energy density of dark matter redshifts as $\propto R^{-3}$, where R is the scale factor, until well before

[1]Searches for oscillations in the local stress-energy tensor detectable with pulsar timing arrays [286] set slightly weaker bounds [361, 277].

recombination. Such a scaling is satisfied for an oscillating scalar field if (and only if) the scalar potential is solely composed of a mass term, $\frac{1}{2}m_a^2a^2$. This potential is typically only a good approximation if the field amplitude is sufficiently small and may not hold for ultralight axions in the early universe. Deviation from a simple quadratic term results in a perturbation spectrum that is no longer scale-invariant, constraining the axion potential. This in turn places a powerful bound on conventional ultralight axion dark matter candidates due to the relationship between f_a and the axion-matter couplings.

While this point is implicitly acknowledged in some of the axion literature, its significance is not widely emphasized, and its implications for current and future searches is missing altogether. In this work, we provide the constraints from the matter-power spectrum and thereby motivate a natural region of mass and coupling values wherein the axion can constitute dark matter without any additional model building. Circumventing the cosmological bounds requires breaking the parametric relationships between the axion potential and couplings. There are few techniques that can successfully accomplish this task. We consider the possibility of large charges, kinetic mixing, a clockwork structure, and discrete symmetries in the context of ultralight dark matter. These models typically predict additional light states in the spectrum and we survey their phenomenology.

In section 4.2 we review features of axion models, with particular emphasis on the coupling of axions to visible matter and the axion potential. In section 4.3 we study the impact of ultralight axion dark matter on the matter-power spectrum and derive the associated bound. In section 4.4 we examine the axion detection prospects of various experiments in light of the bounds. In section 4.5 we study the robustness of the constraints by exploring ways to disrupt the relationship between axion-matter couplings and the axion potential. Finally, we conclude in section 4.6.

4.2 Axion Mass and Coupling

The axion decay constant, f_a, relates the terms of the axion potential to its couplings with Standard Model fields. The potential arises from non-perturbative contributions of gauge or string theories and explicitly breaks the continuous shift symmetry of the axion. We use the standard parametrization of the potential, which is a simple cosine of the form

$$V(a) \simeq \mu^4 \cos\frac{a}{f_a}\,, \tag{4.1}$$

where μ is a scale associated with the explicit breaking of the global symmetry. If the potential arises from a composite sector (as in the case of the QCD axion), the explicit breaking scale corresponds to the maximum scale at which states must show up in the spectrum. The full axion potential is expected to be more complicated than the simple cosine above, but we can consider (4.1) to be the first term in a Fourier decomposition of the potential. An important feature of 4.1 is the existence of terms beyond the mass term and that the coefficients of these higher order terms are not arbitrary. The size of the quartic determines the point at which the quadratic approximation breaks down and has significance for axion cosmology.

Axions may also couple to Standard Model fields, with the leading operators obeying the axion shift symmetry. In this work we focus on two types of operators for axion dark matter, the prospective photon and nucleon couplings,[2]

$$\mathcal{L} \supset \frac{C_{a\gamma}\alpha}{8\pi f_a} aF\tilde{F} + \frac{C_{aN}}{f_a}\partial_\mu a\bar{N}\gamma^\mu\gamma_5 N\,, \tag{4.2}$$

where here and throughout we suppress the Lorentz indices on the gauge interactions, $F\tilde{F} \equiv F_{\mu\nu}\tilde{F}^{\mu\nu}$ and $\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$. The parameters $C_{a\gamma}$ and C_{aN} represent combinations of couplings in the UV theory and are $\mathcal{O}(1)$ for generic axions. Demanding that the theory be invariant under axion discrete shift transformations requires the coefficient $C_{a\gamma}$ to be an integer [3] and hence cannot represent a large ratio of scales without additional model building. There may also be contributions to the above couplings from the IR if the axion mixes with dark sector particles, similar to the QCD axion-meson mixing, but such contributions will be unimportant for our considerations. The relationship between the axion potential and its coupling to matter is made manifest in (4.1) and (4.2). To make contact with other studies, we define

$$g_{a\gamma} \equiv \frac{C_{a\gamma}\alpha}{2\pi f_a} \quad , \quad g_{aN} \equiv \frac{C_{aN}}{f_a}\,. \tag{4.3}$$

In principle it is possible that the particle searched for by dark matter experiments is not a true axion, in the sense that it is not shift symmetric, but a light pseudoscalar with a potential,

$$\mathcal{L}_a = \frac{1}{2}m_a^2 a^2\,. \tag{4.4}$$

In this case the corresponding coefficients in-front of the terms in (4.2) do not correspond to any symmetry breaking scale, but are instead completely free parameters associated with the scale of integrating out heavy fields. This would prevent us from using the arguments of section 4.3 to restrict the dark matter parameter space. While such models seem viable, they are highly fine-tuned and do not exhibit the desirable features of axion models. One way to see the tuning is to consider the additional terms in the effective theory that arise when integrating out the heavy fields that lead to the couplings in (4.2). For example, in addition to the $aF\tilde{F}$ term, the low energy theory of a simple pseudoscalar will include terms such as a^2FF, $a^3F\tilde{F}$, etc. These terms will always be generated as they are no longer forbidden by any symmetry, and lead to corrections to the scalar potential which destabilize the light

[2] Other couplings of ultralight axions with matter are the gluon operator, $aG_{\mu\nu}\tilde{G}^{\mu\nu}$, the electron operator, $\partial_\alpha a\bar{e}\gamma^\alpha e$, and the muon operator, $\partial_\alpha a\bar{\mu}\gamma^\alpha\mu$. The gluon operator requires tuning to be sizable around the fuzzy dark matter regime and has other constraints [246, 359, 299], the electron coupling can be probed using torsion pendulums [204], and the muon operator has other strong constraints making it difficult to see experimentally [205, 266].

[3] If there is additional axion coupling in the phase of the mass matrix of some new fermions, $C_{a\gamma}$ only needs to sum to an integer with the coefficient of the coupling, see e.g., [173]

scalar. Thus any simple pseudoscalar becomes unnatural and the motivation to consider such a particle as dark matter is rendered null. Therefore, we take the position that the target particles of experimental searches are indeed ultralight axions with a full trigonometric potential, and we now examine the cosmological limitations of such dark matter candidates.

4.3 Axion Matter-Power Spectrum

A scalar field evolving in a purely quadratic potential has a scale-invariant matter-power spectrum, matching that of ΛCDM. However, if the potential contains higher order terms, the scalar equation of motion will possess non-linear terms which impact the growth of perturbations, with positive (negative) contributions wiping out (enhancing) small scale structure. For an axion with field amplitude $a_0(z)$ at redshift z the condition for the axion fluid to behave like cold dark matter is $a_0(z)/f_a \ll 1$. The cosmic microwave background is the most sensitive probe of the matter-power spectrum, measuring deviations at a part per thousand, and sets a bound around recombination on any additional energy density fluctuations, $\delta\rho/\rho \lesssim 10^{-3}$, corresponding to, $a_0(z_{\rm rec})^2/f_a^2 \lesssim 10^{-3}$. It is important to note that this bound does not rely on the specific production mechanism and must be satisfied for any light axion making up the entirety of dark matter.

This constraint was studied quantitatively for misaligned axions in a trigonometric potential in [362] (see also [146] for related discussions). The authors considered an axion with the potential in (4.1) and a field value frozen by Hubble friction until z_c, the redshift at which the axion mass is comparable to the Hubble rate and oscillations begin. The matter-power spectrum then constrains the fraction of dark matter made up by axions as a function of z_c. The authors of [362] find that in order for the axion to constitute all of dark matter, z_c must be $\gtrsim 9 \times 10^4$. This can be translated onto a constraint on f_a by noting that the axion field amplitude is fixed today by the measured dark matter energy density with, $\rho_{\rm DM}(z) = \frac{1}{2}m_a^2 a_0(z)^2$. Since the amplitude redshifts as $a_0(z) \propto (1+z)^{3/2}$, requiring the axion to oscillate before it exceeds its field range, $a_0(z_c) \lesssim f_a$, requires,

$$f_a \gtrsim \sqrt{\frac{2\rho_{\rm DM}(z_0)}{m_a^2}}(1+z_c)^{3/2}\,, \tag{4.5}$$

$$\text{or} \qquad f_a \gtrsim 1.2 \times 10^{13}\ \text{GeV}\left(\frac{10^{-20}\text{eV}}{m_a}\right)\,. \tag{4.6}$$

The rough expressions motivated above, $a_0(z_{\rm rec})^2/f_a^2 \lesssim 10^{-3}$, gives a similar result. Note that while [362] assumed a misalignment mechanism, it is more general, and will apply (approximately) to any axion dark matter production mechanism as suggested by the rough estimate.

The constraint proposed in this work utilizes the matter-power spectrum and is distinct from the work of [39], which presented a bound assuming the misalignment mechanism. The limit in [39] is derived by noting that the maximum energy stored in the axion potential is $\sim \mu^4$ and, assuming a simple cosmology from the start of oscillations to recombination, demanding that this be less than the dark matter energy density at z_c: $\rho_{\rm DM}(z_c) \lesssim \mu^4$. This restricts, $\mu^4 \lesssim {\rm eV}^4 (z_c/z_{\rm eq})^3$, or equivalently,

$$f_a \gtrsim 10^{17}\ {\rm GeV} \left(\frac{10^{-21}\ {\rm eV}}{m_a}\right)^{1/4} \ ({\rm misalignment})\,. \tag{4.7}$$

The authors of [39] also consider temperature-dependent axion masses which relax the misalignment constraint. While both types of bounds in [39] are more stringent than the matter-power spectrum bound, they are also less robust since they rely on misalignment and on having a simple cosmology from z_c to recombination.

The bound on f_a can be translated into a bound on the coupling to photons and nucleons using the relations in (4.2) for given values of $C_{a\gamma}$ and C_{aN}. The results are displayed in Figs. 4.1 and 4.2 in black for several values of the coefficients. Since generic axions models predict $C_{a\gamma}, C_{aN}$ that are at most $\mathcal{O}(1)$, this is a powerful bound on the ultralight axion parameter space. Additional model building beyond the minimal scenario is required to access regions with larger coupling. For comparison, we include the regions constrained assuming misalignment as wavy gray lines for different values of $C_{a\gamma}$ and C_{aN}.

The constraints we derive here assumed the axion field makes up dark matter until prior to recombination. An alternative scenario is the case where an axion is only produced at late times, such as through the decay of a heavier state. While an intriguing possibility, decay of heavy states will produce relativistic axions which will in turn modify the equation of state of the universe. Thus evading the matter-power spectrum bound by tweaking cosmology at late times is a formidable task.

4.4 Comparison with experiments

We now consider the prospects of ultralight axion dark matter searches in light of the matter-power spectrum restriction derived above, starting with a summary of current experimental constraints. Firstly, the Lyman-α-flux power spectra sets a bound on the axion mass, independent of the size of non-linear terms in the potential. These measurements are sensitive to sharp features in the matter-power spectrum on small scales, which would be present if the axion has a mass comparable to the size of dwarf galaxies, and set a bound on the axion mass of $m_a \gtrsim 10^{-21}$ eV [295, 260, 344, 305]. A mass bound of similar magnitude can be determined by utilizing constraints on the subhalo mass function from gravitational lensing and stellar streams [378, 341]. Recently the Lyman-α bound was re-analyzed and strengthened to $m_a \gtrsim 2 \times 10^{-20}$ eV[372]. Since the precise restriction on the axion mass varies between these studies, and so we include both the weakest and strongest bounds in the plots below. In addition, there are astrophysical bounds on axions that are independent of their energy density. Axions released during supernova (SN) 1987A would have produced a flux of axions that could convert as they passed through the galactic magnetic fields [87, 209, 354] and the non-observation of this conversion sets the strongest bounds on low mass axions coupled to photons. For axion-nucleon coupling, the strongest dark matter-independent bounds arise from forbidding excess cooling of SN-1987A [108] and neutron stars [380, 217, 73]. There are also the bounds arising from black hole superradiance [45, 43], but these are relevant for larger masses or smaller couplings than we consider.

There are a large number of searches looking for axion dark matter that rely on its relic abundance. Efforts to discover a photon coupling include looking for deviations in the polarization spectrum of the cosmic microwave background [221] (with updated bounds in [167]), searching for the axion's influence on the polarization of light from astrophysical sources [262, 176, 308, 95], and terrestrial experiments [345, 395, 72].[4] Searches for an axion-nucleon coupling focused on the ultralight regime include axion-wind spin precision [14], using nuclear magnetic resonance [402, 88, 419, 190, 79], and using proton storage rings [205]. Several spin precession experimental setups are considered in [204]. [5]

The photon bounds are compiled in Fig. 4.1 and nucleon bounds in Fig. 4.2. The matter-power spectrum bound derived in section 4.3 is displayed in both figures. We use solid (dashed) lines to denote current (prospective) bounds. We conclude that many experimental proposals in this ultralight regime are inconsistent with a generic axion dark matter and require $C_{a\gamma} \gg 1$ or $C_{aN} \gg 1$. Reaching the large couplings considered in various experiments is an issue of additional model building, and is the focus of the next section.

4.5 Enhanced Axion Couplings

We have presented stringent bounds on axions arising from the relationship between their field range, f_a, and their coupling to photons or nucleons. However, there exist model-building techniques that can relax this relationship, which have often been discussed in the context of axion inflation. These methods may also be applied to ultralight axion dark matter and have distinct low energy phenomenology as a consequence of the lightness of the axion and the requirement of matching the observed matter-power spectrum. In this section we review these mechanisms, provide explicit realizations of such models, and study their phenomenology. We focus on the photon coupling, but similar models can be built for the nucleon coupling.

Large Charges

One way to enhance the axion coupling to visible matter is to introduce fermions with large charges or a large number of fermions (see e.g., [23, 20] for a discussion in the context of inflation). This strategy is limited by the requirement of perturbativity of electromagnetism and the presence of light fermions charged under electromagnetism.

[4] We used the "realistic" projections of [345].

[5] We selected the most stringent bounds from [204, 205, 72] and continued these bounds to lower ULA mass values than what the original works consider.

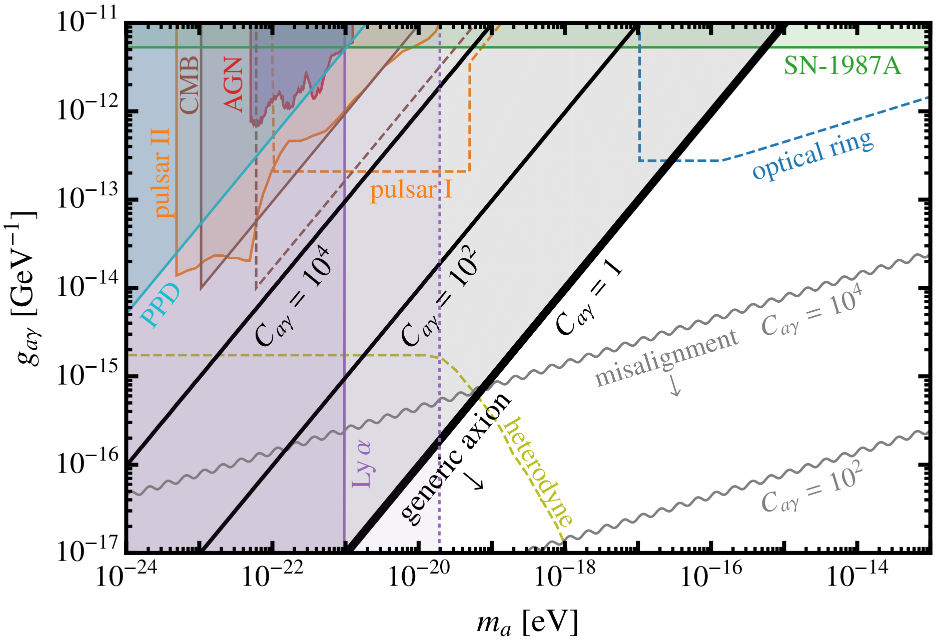

Figure 4.1: Ultralight axion dark matter mass vs. photon-coupling parameter space. Requiring the ultralight axion to exhibit a matter-power spectrum consistent with ΛCDM sets the bound shown in black for serveral values of $C_{a\gamma}$. The region below the $C_{a\gamma} = 1$ line permits natural axions without any additional model building (see text). The solid purple and dotted purple lines display the weakest and strongest bounds, respectively, arising from purely gravitational considerations [380, 217, 108, 73].The lack of axion-to-photon conversion of axions produced during supernova-1987A [354] gives the bound in green. Additional bounds from current (solid) and proposed searches (dashed) are from active galactic nuclei [262](red), protoplanetary disk polarimetry [176] (light blue), CMB birefringence [167] (brown), pulsars [308, 95] (orange), optical rings [345] (dark blue), and heterodyne superconductors [72] (olive). The misalignment bounds for $C_{a\gamma} = 10^2, 10^4$ are displayed by the wavy contours (grey).

To be explicit, consider a KSVZ-like model [289, 383] where a complex scalar Φ (whose phase will be identified with the axion), has Yukawa couplings with a set of Weyl fermions with an electromagnetic charge Q_f. Integrating out the fermions leads to an axion-photon coupling,

$$\mathcal{L} \supset \frac{\alpha Q_f^2 N_f}{8\pi f_a} a F\tilde{F}. \tag{4.8}$$

The presence of charged fermions renormalizes the electric charge as computed through corrections to the photon gauge kinetic term. Perturbativity requires that $N_f Q_f^2 \alpha/4\pi \lesssim 1$.

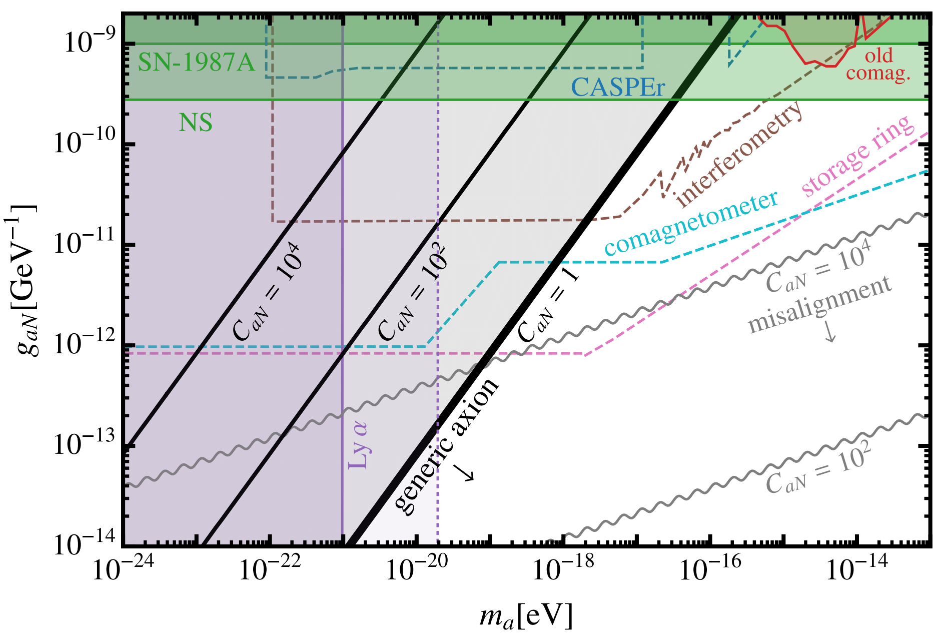

Figure 4.2: Ultralight axion dark matter mass vs. nucleon-coupling parameter space. Requiring the ultralight axion to exhibit a matter-power spectrum consistent with ΛCDM sets the bound shown in black for serveral values of C_{aN}. The region below the $C_{aN} = 1$ line permits natural axions without any additional model building (see text). The solid purple and dotted purple lines display the weakest and strongest bounds, respectively, arising from purely gravitational considerations [380, 217, 108, 73]. Supernova-1987A & neutron star cooling [380, 217, 108, 73] bounds are shown in green and the bound from old comagnetometer data is shown in red [79]. Additional bounds for nucleon couplings are shown from projections of the CASPEr-Zulf experiment [419, 190] (dark blue), atom interferometry (brown) [204], atomic magnetometers (light blue) [204], and storage rings [205] (pink). The misalignment bounds for $C_{aN} = 10^2, 10^4$ are displayed by the wavy contours **(grey).**

Since $C_{a\gamma} = N_f Q_f^2$, the perturbativity constraint sets a bound $C_{a\gamma} \lesssim 4\pi/\alpha$. We conclude that large charges can at most enhance the axion-photon coupling by $\mathcal{O}(10^3)$.[6]

[6] Here we have taken the Peccei-Quinn charges of the fermions to be $\mathcal{O}(1)$. If one chooses larger Peccei-Quinn charges such that the fermion mass only arises through higher dimensional operators, then the photon coupling can be slightly amplified. However, requiring a hierarchy between f_a and the cutoff strongly constrains this possibility [23].

Kinetic Mixing

Kinetic mixing of multiple axion fields can raise the axion coupling to visible matter by (potentially) allowing an axion with a large field range to inherent couplings of an axion with a smaller field range (see [51, 52, 386, 238, 115, 385, 21, 23, 20] for discussions in other contexts). As a simple example consider two axions a_1 and a_2, where a_1 obtains a potential while the lighter axion, a_2 (which is massless here), couples to photons:

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu a_1 \partial^\mu a_1 + \frac{1}{2}\partial_\mu a_2 \partial^\mu a_2 + \varepsilon\, \partial_\mu a_1 \partial a_2 \\ + \mu^4 \cos\frac{a_1}{F_1} + \frac{\alpha}{8\pi F_2} a_2 F_{\mu\nu}\tilde{F}^{\mu\nu} \,. \tag{4.9}$$

The kinetic term can be diagonalized by the shift $a_2 \to a_2 - \varepsilon a_1$, which induces an a_1-photon coupling,

$$\mathcal{L} \supset -\frac{\varepsilon F_1}{F_2}\frac{\alpha}{8\pi F_1} a_1 F_{\mu\nu}\tilde{F}^{\mu\nu} \,. \tag{4.10}$$

Taking a_1 to be the axion dark matter candidate, we conclude that kinetic mixing gives $C_{a\gamma} = \varepsilon F_1/F_2$. If ε is held fixed and the decay constants have a large hierarchy ($F_1 \gg F_2$), then a_1 will have $C_{a\gamma} \gg 1$.

While this appears to be a simple solution, it is not possible to have $C_{a\gamma} \gtrsim 1$ within most field theories. This is a consequence of axions arising as Goldstone bosons of a extended scalar sector and hence the axion kinetic mixing is not a free parameter but must be generated. There are two possible sources for ε: renormalization group flow ("IR") and higher dimensional operators ("UV") contributions. To see the suppression from IR contributions, consider a theory of two axions with a fermion, χ,

$$\mathcal{L} \supset \frac{\partial^\mu a_1}{F_1}\bar{\chi}\gamma_\mu\gamma_5\chi + \frac{\partial^\mu a_2}{F_2}\bar{\chi}\gamma_\mu\gamma_5\chi \,. \tag{4.11}$$

The induced kinetic mixing of the axion is quadratically divergent and goes as ,

$$\varepsilon \sim \frac{\Lambda^2}{(4\pi)^2 F_1 F_2} \,, \tag{4.12}$$

where Λ represents the cutoff scale. Since $\Lambda \lesssim F_{1,2}$ (otherwise the effective theory is inconsistent), the kinetic mixing is bounded by $\varepsilon \lesssim F_2/4\pi F_1$ and will result in $C_{a\gamma} \lesssim 1$.

Alternatively, it is possible to induce an axion kinetic mixing through higher dimensional operators (see e.g., [51, 238]). Taking a_1 and a_2 to be the phases of complex scalar fields Φ_1 and Φ_2, there can be an operator,

$$\mathcal{L} \supset \frac{1}{2M^2}\, \Phi_1^\dagger \overleftrightarrow{\partial} \Phi_1\, \Phi_2^\dagger \overleftrightarrow{\partial} \Phi_2 \,, \tag{4.13}$$

where $\Phi^\dagger \overleftrightarrow{\partial} \Phi \equiv \Phi^\dagger \partial \Phi - (\partial \Phi^\dagger)\Phi$. Once the scalar fields take on their vacuum values, the axions get a mixing term with $\varepsilon = F_1 F_2/M^2$. This is again suppressed since consistency of the effective theory requires $M \gtrsim F_{1,2}$ and cannot result in $C_{a\gamma} \gtrsim 1$.

While these examples show that kinetic mixing is not typically sizable for field theory axions, it is has been suggested that certain string constructions allow for sizable mixing coefficients [23]. While we are not aware of a concrete string construction where this is true, this may be a way to achieve $C_{a\gamma} \gtrsim 1$.

Interestingly, for ultralight axion dark matter, kinetic mixing has additional phenomenological implications. In order for the Lagrangian in (4.9) to result in a photon coupling for a_1 that is not suppressed by a ratio of axion masses, a_2 must be lighter than a_1. Since the a_2 photon coupling is not suppressed by factors of ε, it may be more detectable than a_1 and drastically influence direct constraints, such as from supernova axion cooling or conversion. This would need to be studied with care for a particular realization of a value of ε.

In addition to axion-mixing, kinetic mixing of abelian gauge fields can boost the axion-photon coupling, as considered in [135]. In this case the coupling may be enhanced if the axion-photon coupling inherits the dark photon gauge coupling. To see this explicitly, we consider an axion coupled to a dark U(1) gauge field, A', which kinetically mixes with the electromagnetism,

$$\mathcal{L} \supset \frac{\alpha'}{8\pi F_a} aF'\tilde{F}' - \frac{1}{4}FF - \frac{1}{4}F'F' - \frac{\epsilon}{4}FF', \tag{4.14}$$

where α' is the dark gauge coupling. If A' has a mass below the photon plasma mass, then a basis rotation can be performed to diagonalize the kinetic terms through $A \to A - \epsilon A'$. This transformation leaves the dark photon approximately massless and gives the axion a coupling to photons as

$$\mathcal{L} \supset \frac{\epsilon^2\alpha'}{8\pi F_a} aF\tilde{F}\,, \tag{4.15}$$

such that $C_{a\gamma} = \epsilon^2\alpha'/\alpha$. Direct constraints on dark photons permit $\epsilon \sim 1$ (see [328, 96, 189] for the bounds on ultralight dark photons) while α' can be ~ 1. Taken together, gauge kinetic mixing permits an amplification factor $C_{a\gamma} \sim \mathcal{O}(10^2)$.

So far we have considered the cases of axion-axion and vector-vector mixing. It is also possible for axions to mix with a vector if the axions transform under the gauge symmetry, as is the case for Stückelberg axions (see, e.g., [385, 386] for discussions in the context of inflationary model building, as well as [173, 114]). As a simple model, we consider the case of two Stückelberg axions that have gauge interactions with a dark U(1) gauge field, A', and nearly identical interactions with electromagnetism and a dark confining gauge sector:

$$\begin{aligned}\mathcal{L} \supset\ & \frac{1}{2}\Big(\partial_\mu a_1 - q_1 F_1 A'_\mu\Big)^2 + \frac{1}{2}\Big(\partial_\mu a_2 - q_2 F_2 A'_\mu\Big)^2 \\ & + \frac{\beta\alpha_s}{8\pi}\left(\frac{a_1}{F_1} + \frac{a_2}{F_2}\right)G\tilde{G} + \frac{\alpha}{8\pi}\left(\frac{a_1}{F_1} + \frac{a_2}{F_2}\right)F\tilde{F}.\end{aligned} \tag{4.16}$$

Gauge invariance requires $q_2 = -q_1 \equiv -q$.[7] We perform a field redefinition,

[7]This Lagrangian is invariant under the U(1) gauge transformation $a_1 \to a_1 + q_1 F_1 \alpha$, $a_2 \to a_2 + q_2 F_2 \alpha$, and $A_\mu \to A_\mu + \partial_\mu \alpha$ if $q_1 + q_2 = 0$. The Lagrangian we consider is a simplified version of the setups in [385, 386, 173], but the conclusions are unchanged in the more general scenarios.

$$\begin{aligned} a &= -\bar{F}\left(\frac{a_1}{F_1} + \frac{a_2}{F_2}\right) \\ b &= \bar{F}\left(\frac{a_1}{F_2} - \frac{a_2}{F1}\right), \end{aligned} \tag{4.17}$$

so that the physical axion interactions are,

$$\mathcal{L} \supset -\frac{\beta}{8\pi\bar{F}} aG\tilde{G} - \frac{1}{8\pi\bar{F}} aF\tilde{F}, \tag{4.18}$$

and $\bar{F} = F_1F_2/(F_1^2 + F_2^2)^{1/2}$. The axion b remains charged and provides a mass for the dark gauge boson. The surviving axion, a, is neutral under the dark U(1) and is the dark matter candidate. Since $\bar{F}$ is smaller than F_1 and F_2, a is more strongly coupled to photons than either of the original axions [173]. Nevertheless, this does not result in $C_{a\gamma} \gtrsim 1$. This is because the decay constant of the surviving axion, $\bar{F}$, appears in the anomalous coupling to both the non-abelian and electromagnetic gauge sectors and so the canonical relationship between axion potential and matter coupling is maintained. We conclude that axion-vector mixing cannot be used to evade the cosmological bounds on ultralight dark matter axions.

Clockwork

Clockwork models provide a means to disturb the canonical relationship between the axion potential and photon coupling by introducing a large number of axions, each interacting with both its own confining gauge sector and its "neighbor". After a rotation to the axion mass basis, the lightest axion's potential can be exponentially suppressed without introducing an exponential number of fields. This light axion can be understood as the Goldstone boson of a global symmetry between scalar fields in a UV completion (see, e.g., [113, 166, 133, 23, 20, 321] for discussions in different contexts).

As an explicit model, we consider a set of N axions, a_i, with couplings to N SU(n_i) gauge sectors with field strengths G_i, and a photon coupling only for a_N:

$$\begin{aligned} \mathcal{L} \supset \sum_{i=1}^{N-1} \frac{\alpha_{s,i+1}}{8\pi}\left(\frac{\beta_i a_i}{F_i} + \frac{a_{i+1}}{F_{i+1}}\right) G_{(i+1)}\tilde{G}_{(i+1)} \\ + \frac{\alpha_{s,1}}{8\pi F_1} a_1 G_1 \tilde{G}_1 + \frac{\alpha}{8\pi F_N} a_N F\tilde{F}\,. \end{aligned} \tag{4.19}$$

The β_i factors are integers greater than or equal to unity and we have omitted a bare θ term. Upon confinement, the gauge sectors give rise to the potential for the axions,

$$V(a_i) \simeq \sum_{i=1}^{N-1} \mu_{i+1}^4 \cos\left(\frac{\beta_i a_i}{F_i} + \frac{a_{i+1}}{F_{i+1}}\right) + \mu_1^4 \cos\frac{a_1}{F_1}\,, \tag{4.20}$$

where the μ_i are the confinement scales and represent the maximum possible masses for the dark composite states. To get an enhanced photon coupling, we require

$$\mu_1 \ll \mu_2, \mu_3, ... \tag{4.21}$$

and we take the F_i's to be comparable to each other. In this case, up to $\mathcal{O}(\mu_1/\mu_i)$ corrections, integrating out the heavy axions corresponds to iteratively introducing the substitution:

$$\frac{\beta_i a_i}{F_i} + \frac{a_{i+1}}{F_{i+1}} \simeq 0 \quad \forall i = 1, 2, ... \, . \tag{4.22}$$

This transformation produces the effective Lagrangian

$$\mathcal{L} \simeq \mu_1^4 \cos \frac{a_N}{F_N \prod_i \beta_i} + \frac{\alpha}{8\pi F_N} a_N F\tilde{F} \, . \tag{4.23}$$

The a_N-potential is exponentially suppressed by $\prod_i \beta_i$ while the photon coupling remains unchanged, resulting in an axion potential exponentially flatter than the naive estimate. Redefining the axion decay constant as in (4.1) gives,

$$C_{a\gamma} = \prod_i \beta_i \, , \tag{4.24}$$

thereby boosting the photon coupling relative to a generic axion.

We now consider the phenomenological implications of clockworked axions as dark matter. Firstly, in addition to the light axion, there exist $N-1$ axions with masses proportional to $\mu_{i\geq 2}$ (the only bound on these is from unitarity, requiring $\mu_i \lesssim F_i$[20]). These would be populated in the early universe if the new non-abelian gauge groups confine after reheating (from their own misalignment mechanisms) or if they are thermalized. Assuming the confinement scales $\mu_{i\geq 2}$ are comparable, the energy density of the heaviest axion would dominate. However, the photon couplings of the $N-1$ heavy axions are suppressed by products of β_i relative to the coupling of the lightest axion, and so they cannot be the target particles of the above experimental searches. Furthermore, if the lightest clockwork axion is to be dark matter and the experimental target, the heavier axions must decay into into Standard Model particles (if the axions decayed into lighter axions, they would produce excess dark radiation in conflict with measurements of ΔN_{eff}). This mandates the need for substantial couplings of the heavy axions to the Standard Model and may lead to observable effects in terrestrial experiments.

In addition to the heavy axions, the clockwork model predicts the existence of a non-abelian gauge sector with composite states well below the electroweak scale and masses below μ_1. Demanding that $F_N < M_{\text{pl}}$ results in a confinement scale of ~ 10 keV $\prod_i \beta_i$ for ultralight axion dark matter with a mass $\mathcal{O}(10^{-20})$ eV. Depending on the type of interactions this light gauge sector has with the Standard Model, it may be possible to observe these in terrestrial experiments.

From the low energy perspective, the clockwork model we have described appears to permit arbitrarily large $C_{a\gamma}$ values. However, there may be limitations on this enhancement factor if one attempts to embed the model into a string construction. In heterotic string models, the 4 dimensional gauge groups descend from the rank 16 gauge groups $E_8 \times E_8$ or $SO(32)$. Demanding that the Standard Model's rank 4 gauge group be present in the low energy theory restricts the rank of the dark sector to be ≤ 12 and so N could be severely limited [249][8]. We leave an extensive study of string compactification restrictions on clockwork models to future work.

Discrete Symmetry

Finally, axion couplings to visible matter can be augmented by introducing multiple non-abelian gauge sectors related by a discrete symmetry [245]. When the axion potential contributions from the confinement of each gauge sector are summed together, one finds the potential may be exponentially suppressed compared to the naive expectation.

As an example, we consider a theory with a single axion, a, that couples to N confining gauge sectors with field strengths, $G_{(n)}$, and impose a discrete symmetry under which,

$$\begin{aligned} a &\to a + 2\pi F_a/N \\ G_{(n)} &\to G_{(n+1)} \qquad . \end{aligned} \tag{4.25}$$

The symmetry forces all the non-abelian gauge sectors to share a common gauge coupling and fermion content. Including an axion-photon coupling, the Lagrangian consistent with the symmetry has,

$$\mathcal{L} \supset \frac{\beta\alpha_s}{8\pi}\sum_{n=1}^{N}\left(\frac{a}{F_a}+\frac{2\pi n}{N}\right)G_{(n)}\tilde{G}_{(n)} + \frac{\alpha}{8\pi F_a} aF\tilde{F}\,. \tag{4.26}$$

In contrast to clockwork, the integer β serves no essential purpose here and can be taken to be unity.

Each of the N gauge sectors contribute to the axion potential after they confine. If we were to use the leading contribution to the axion potential from (4.1) for each sector, the total axion potential would vanish. Therefore we must include corrections associated with higher modes in the Fourier expansion of the potential, which depend on the light fermion content of the theory. For a sector with two fermions with masses m_1 and m_2 below the composite scale, chiral perturbation theory yields the leading order potential (see, e.g., [210]),

$$V(a) = -\mu^4 \sum_{n=0}^{N-1}\sqrt{1 - z\sin^2\left(\frac{a}{2F_a}+\frac{\pi n}{N}\right)} \tag{4.27}$$

[8]We assume a generic Calabi-Yau compactification manifold.

with $z = 4m_1m_2/(m_1+m_2)^2$. After the sum in (4.27) is carried out, one finds the axion mass is exponentially suppressed if there is a small hierarchy between the light quark masses. Taking $m_2 > m_1$ the axion mass dependence on N is, approximately,

$$m_a \sim \left(\frac{m_1}{m_2}\right)^{N/2} \frac{\mu^2}{F_a}\,. \tag{4.28}$$

Canonically normalizing the decay constant, we get $C_{a\gamma} \sim (m_2/m_1)^{N/2}$, breaking the relation between the axion mass and photon coupling for $N \gg 1$.

While discrete symmetries produce axions with $C_{a\gamma} \gg 1$, they do not evade the bounds from the matter-power spectrum. This is a consequence of the axion potential from (4.27) giving unusually large higher order axion terms. Unlike clockwork, which keeps the axion potential of the form in (4.1) and simply extends the field range, discrete symmetries break this relationship entirely. To see this behavior, we expand (4.27) about one of its minima, giving the potential,

$$\begin{aligned} V(a) &= \frac{C_2}{2}\frac{\mu^4}{F_a^2}a^2 - \frac{C_4}{4!}\frac{\mu^4}{F_a^4}a^4 + \cdots\,, \\ &= \frac{1}{2}m_a^2a^2 - \frac{1}{4!}\lambda a^4 + \cdots\,, \end{aligned} \tag{4.29}$$

where the C_i's are constants that arise from the sum in (4.27). The coefficient C_2 determines the exponential suppression of the axion mass and C_4 fulfills a similar role for the quartic. It is convenient to recast the mass suppression factor into an axion-photon coupling enhancement factor via $f_a \equiv F_a/\sqrt{C_2}$ such that $m_a = \mu^2/f_a$, $\lambda = C_4\mu^4/C_2f_a^4$, and $C_{a\gamma} = \sqrt{C_2}$.

The key observation is that the dependence on N is different for the two constants C_2 and C_4, as displayed in Fig. 4.3. For large N, C_4 decreases more slowly than C_2 with increasing N. The approximate condition presented above for the axion to behave sufficiently like cold dark matter is,

$$\left.\frac{\lambda a_0^4}{m_a^2a^2}\right|_{\rm eq} \sim \frac{\lambda {\rm eV}^4}{m_a^4} = \frac{C_4}{C_2}\frac{{\rm eV}^4}{m_a^2}\frac{C_{a\gamma}^2}{f_a^2} \lesssim 10^{-3}\,. \tag{4.30}$$

The factor ${\rm eV}^4C_{a\gamma}^2/m_a^2f_a^2$ is restricted to be greater than unity to get a large enhancement in the photon coupling. From Fig. 4.3, we see that C_4/C_2 will also be greater than unity and the bound cannot be satisfied. We conclude that this variety of model cannot be used to boost the axion-photon coupling for ultralight axion dark matter.

4.6 Conclusion

In this work, we considered the experimental prospects of detecting ultralight axion dark matter through its couplings to the visible sector, focusing on photon and nucleon interactions. We presented a stringent bound on axions by requiring that their matter-power

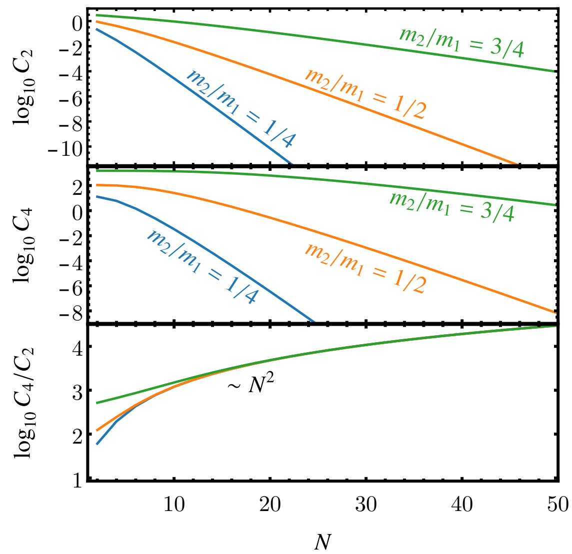

Figure 4.3: The coefficients of the axion potential arising from a discrete symmetry normalized to their expected values. We see the exponential drop in C_4 and C_2, however their ratio grows with N. Thus discrete symmetries strengthen the matter-spectrum bounds instead of weakening them (see text).

spectrum match that of ΛCDM and concluded that generic axions are constrained to have couplings significantly smaller than is often assumed. This bound makes use of the relationship between axion-matter couplings and the axion potential and is independent of the dark matter production mechanism. This discussion displays the tension between experimental projections and cosmological bounds and has not been widely emphasized in previous literature.

Given the up and coming experimental program, the need to understand the landscape of ultralight axion dark matter models with detectable couplings is clear. As such, we studied various strategies to boost axion couplings that were introduced previously in the literature and applied them to ultralight axion dark matter. In particular, we considered models with large charges, diverse forms of kinetic mixing, a clockwork mechanism, and a discrete symmetry. We examined the extent to which axion couplings can be boosted in each mechanism, if at all, and explored their distinct predictions and phenomenology. In brief, $\mathcal{O}(10^2 - 10^3)$ coupling enhancements are possible by introducing large charges or vector kinetic mixing. Significantly larger enhancements are possible with clockwork models if one takes an agnostic view towards UV completions, but arbitrarily large amplifications may be stymied in string embeddings. Inversely, axion-axion kinetic mixing can only be effective if some string construction allows one to bypass the field theory arguments presented above. Finally, discrete symmetries and axion-photon kinetic mixing are ineffective in raising the axion coupling to visible matter. If a discovery of ultralight axion dark matter is made by a search in the near future, it would be a clear sign of new dynamics with possible implications for other low energy terrestrial experiments.

There are several phenomena not discussed above that may place further restrictions on ultralight axion models. First of all, if a symmetry is restored in the early universe, topological defects such as domain walls and cosmic strings could form. Axion emission from cosmic strings would contribute to the energy density of axions present today and any stable domain walls may dominate the energy density and thereby drastically alter the cosmology. This may further constrain variants of axions models, such as clockwork axions, whose UV completions could have multiple restored symmetries. Additionally, if an axion symmetry is restored, axions may form miniclusters [298] which would contribute to dark matter small scale structure. These may be observed using probes such as microlensing [165], pulsar timing [157, 369], and 21cm cosmology [272]. We leave the consideration of these issues for future work.

We considered ultralight axion dark matter, but the mechanisms discussed here may be applied in other contexts where large axion couplings to the visible sector are desirable. Some examples include inflation (where most of these mechanisms first arose, see text for references), looking for parametric resonance during axion minicluster mergers [235, 234], monodromy axions [265, 68], vector dark matter production [25, 125], and addressing the H_0 tension [202, 413]. Lastly, while we focused primarily on the axion-photon and axion-nucleon couplings, similar bounds can be constructed for axion-electron couplings and, potentially, ultralight neutrino-philic scalars [71] (whose potential likely also needs to arise from breaking of a shift symmetry to be protected against quantum corrections from gravity). We leave a study of such scalars to future work.

Part II

Anomalies in Supergravity Models From String Theory

Overview of Part II

Part II will be a departure from the concrete considerations of Part I. Rather than viable extensions of the Standard Model, here we will consider toy models to develop theoretical understanding necessary for more realistic constructions in the future.

In particular, we will describe features of quantum field theories that serve as low energy descriptions of string compactifications. An important consistency requirement on a quantum theory is the absence or cancellation of quantum anomalies. In the Standard Model, there are potential anomalies that arise from triangle diagrams with the various gauge interactions appearing on each vertex. These anomalies turn out to be zero due to the particle content of the Standard Model and so the theory is safe from inconsistency. However, in the case where anomalies are not identically zero, it is possible to cancel them via the Green-Schwarz mechanism [311, 38, 154, 145, 310, 188, 37, 36, 198, 312]. One can understand this procedure as cancelling the anomalous variation of the quantum corrected Lagrangian under a classical symmetry by imposing a transformation on a particular field in the model. In Chapters 5 and 6, we describe how modular anomalies of supergravity models are canceled in a consistent manner using this mechanism. These modular anomalies differ from the usual gauge and gravity anomalies and include the Kahler moduli that are generic in string compactifications. We describe these anomalies in more detail below.

Chapter 5

Anomaly cancellation in effective supergravity theories from the Heterotic String: two simple examples

5.1 Introduction

On-shell Pauli-Villars regularization of the one-loop divergences of supergravity theories was used to determine the anomaly structure of supergravity in [92, 91]. Pauli-Villars regulator fields allow for the cancellation of all quadratic and logarithmic divergences [182, 181, 179, 180], as well as most linear divergences [92, 91]. If all linear divergences were canceled, the theory would be anomaly free, with noninvariance of the action arising only from Pauli-Villars masses. However there are linear divergences associated with nonrenormalizable gravitino/gaugino interactions that cannot be canceled by PV fields. The resulting chiral anomaly forms a supermultiplet with the corresponding conformal anomaly, provided the ultraviolet cut-off has the appropriate field dependence, in which case uncanceled total derivative terms, such as Gauss-Bonnet, do not drop out from the effective action. The resulting anomaly term that is quadratic in the field strength associated with the space-time curvature, as well as the term quadratic in the Yang-Mills field strength, was shown in [92, 91] to be canceled by the four-dimensional version of the Green-Schwarz mechanism in Z_3 and Z_7 compactifications, in agreement with earlier results [311, 38, 154, 145, 310, 188, 37, 36, 198, 312]. However, the terms in the anomaly that are quadratic and cubic in the parameters of the anomalous transformation are prescription dependent [379, 92, 91]. The choice of PV fields with noninvariant masses used in [92, 91] did not achieve full anomaly cancellation.

Every contribution to the chiral anomaly has a conformal anomaly counterpart, with which it combines to form an "F-term" anomaly. In addition there are "D-term" anomalies associated with logarithmic divergences that have no chiral partner. In a generic supergravity theory, these include terms [92, 91] that are nonlinear in the holomorphic functions

$F^i(T^i)$ of the three diagonal Kähler moduli T^i that characterize modular (or T-duality) transformations:

$$\begin{aligned} T'^i &= \frac{a_i - ib_iT^i}{ic_iT^i + d_i}, \qquad a_ib_i - c_id_i = 1, \qquad a_i, b_i, c_i, d_i \in \mathbf{Z}, \\ \Phi'^a &= e^{-\sum_i q_i^a F^i(T^i)}\Phi^a, \qquad F^i(T^i) = \ln(ic_iT^i + d_i), \end{aligned} \tag{5.1}$$

where Φ^a is any chiral supermultiplet other than a diagonal Kähler modulus, and q_i^a are its modular weights. Only terms in the anomaly that are linear in $F = \sum_i F^i$ can be canceled by the Green-Schwarz term.

In addition, in generic supergravity there are anomalous terms that involve the dilaton superfield S in the chiral supermultiplet formulation–or L in the linear multiplet formulation [74, 16] for the dilaton. Specifically, one expects [174, 311] a term quadratic in the Kähler field strength

$$X_{\mu\nu} = \left(\mathcal{D}_\mu z^i \mathcal{D}_\nu \bar{z}^{\bar{m}} - \mathcal{D}_\nu z^i \mathcal{D}_\mu \bar{z}^{\bar{m}}\right) K_{i\bar{m}} - iF^a_{\mu\nu}(T_a z^i)K_i, \tag{5.2}$$

where $z^i = Z^i|$ is the scalar component of a generic chiral superfield Z^i, $F^a_{\mu\nu}$ is the gauge field strength, T_a is a gauge group generator, and $K(Z,\bar{Z})$ is the Kähler potential. The term quadratic in $X_{\mu\nu}$ was actually found to vanish in [92, 91], but there remained terms linear in $X_{\mu\nu}$ as well as terms involving the Kähler potential in the nonlinear F^i terms mentioned above. Anomaly cancellation by a Green-Schwarz mechanism, to be outlined in the next section, requires that the operators appearing in the anomaly also appear in the real superfield Ω of the (modified) linearity condition for the superfield L:

$$\left(\bar{\mathcal{D}}^2 - 8R\right)(L + \Omega) = \left(\mathcal{D}^2 - 8\bar{R}\right)(L + \Omega) = 0, \qquad \mathcal{D}^2 = \mathcal{D}^\alpha \mathcal{D}_\alpha, \qquad \bar{\mathcal{D}}^2 = \mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} = (\mathcal{D}^2)^\dagger, \tag{5.3}$$

where $\mathcal{D}_\alpha$ is a spinorial derivative and $R = \bar{R}^\dagger$ is the auxiliary field of the supergravity multiplet whose *vev* determines the gravitino mass: $\langle R| \rangle = \frac{1}{2}m_{3/2}$. The action written in terms of L is related to the action written in terms of S by a superfield duality transformation; the standard derivation of the duality transformation requires that Ω be independent of S. It was shown in appendix E of the first reference in [92, 91] that the the duality transformation still goes through with a slight modification if this is not the case. On the other hand it might perhaps be reasonable to impose

$$\frac{\partial \Omega}{\partial S} = 0, \tag{5.4}$$

which is in fact the case for the chiral anomaly found in the string calculation of [379]. We show that it is possible to eliminate all terms that depend on the full Kähler potential K, as well as all terms nonlinear in F, and to reproduce the result given in [379]. However, as discussed in Appendix B, there may be a residual S-dependent contribution of the part of the "D-term" anomaly that arises from uncancelled logarithmic divergences.

In the following section we outline the four-dimensional Green-Schwarz mechanism. In Section 3 we briefly recall the results of [92, 91] and the differences obtained with the present approach. In Sections 4 and 5 we introduce the relevant set of PV fields, outline the conditions for cancellation of ultraviolet divergences and present our results for Z_3 and Z_7 orbifolds. We summarize our results in Section 6. Some details are relegated to Appendices.

5.2 The 4-d Green-Schwarz mechanism

The four dimensional version of the Green-Schwarz (GS) mechanism was originally formulated [311, 38, 154, 145, 310, 188, 37, 36, 198, 312] as a means of canceling the anomaly term quadratic in Yang-Mills fields, using the chiral formulation for the dilaton. The classical Lagrangian for the Yang-Mills field strength reads

$$\mathcal{L}_{\mathrm{YM}} = -\sqrt{g}\frac{s}{8}\sum_a \left(F^a_{\mu\nu} - i\tilde{F}^a_{\mu\nu}\right) F^{\mu\nu}_a + \mathrm{h.c.}, \qquad s = S| \,. \tag{5.1}$$

Under the anomalous modular transformation (5.1) the quantum corrected Lagrangian varies according to

$$\Delta\mathcal{L}_{\mathrm{YM}} = -\frac{\sqrt{g}}{64\pi^2}\sum_{a,i} F^i \left[C_a + \sum_b \left(2q^b_i - 1\right) C^b_a\right] \left(F^a_{\mu\nu} - i\tilde{F}^a_{\mu\nu}\right) F^{\mu\nu}_a + \mathrm{h.c.}, \tag{5.2}$$

where C_a is the quadratic Casimir in the adjoint representation of the gauge group factor $\mathcal{G}_a$ and C^b_a is the Casimir for the representation of the chiral supermultiplet Φ^b. In Z_3 and Z_7 orbifolds one has the universality condition:

$$C_a + \sum_b \left(2q^b_i - 1\right) C^b_a = 8\pi^2 b \quad \forall \quad i, a, \tag{5.3}$$

with $b = 30/8\pi^2$ in the absence of Wilson lines. The dilaton is classically invariant under the modular transformation (5.1). However if we impose the transformation property:

$$\Delta s = -bF = -b\sum_i F^i(t^i), \qquad t^i = T^i\big| \,, \tag{5.4}$$

the variation of the classical Lagrangian (5.1) cancels (5.2).

Now consider the superspace Lagrangian[1]

$$\mathcal{L} = \int d^4\theta E\left(S + \bar{S}\right)\Omega = -\frac{1}{8}\int d^4\theta\frac{E}{R}\left(\bar{\mathcal{D}}^2 - 8R\right)(S\Omega) + \mathrm{h.c.} = -\frac{1}{8}\int d^4\theta\frac{E}{R}S\Phi + \mathrm{h.c.}, \tag{5.5}$$

where E is the superdeterminant of the supervielbein, Ω is the real superfield appearing in (5.3), Φ is its chiral projection:

$$\left(\bar{\mathcal{D}}^2 - 8R\right)\Omega = \Phi, \tag{5.6}$$

[1] We use the Kähler superspace formulation of supergravity [74, 16].

and we used superspace integration by parts [74, 16]. When Φ is replaced by the Yang-Mills superfield strength bilinear $W_a^\alpha W_\alpha^a$, (5.5) is just the Yang-Mills Lagrangian that includes the term in (5.1). If, under the modular transformation (5.1) the quantum Lagrangian varies according to

$$\Delta\mathcal{L}_{\rm anom} = b\int d^4\theta \left[F(T) + \bar{F}(\bar{T})\right]\Omega = -\frac{b}{8}\int d^4\theta \frac{E}{R}F(T)\Phi + \text{h.c.}, \tag{5.7}$$

the full Lagrangian is invariant provided

$$\Delta S = -bF(T). \tag{5.8}$$

However the classical Kähler potential for the dilaton is no longer invariant and must be modified:

$$k_{\rm class}(S,\bar{S}) = -\ln(S+\bar{S}) \to k(S,\bar{S}) = -\ln(S+\bar{S}+V_{GS}), \tag{5.9}$$

where V_{GS} is a real function of the chiral supermultiplets that transforms under (5.1) as

$$\Delta V_{GS} = b\left(F+\bar{F}\right). \tag{5.10}$$

A simple solution consistent with string calculation results [311, 38, 154, 145, 310, 188, 37, 36, 198, 312] is

$$V_{GS} = bg(T,\bar{T}), \tag{5.11}$$

where

$$g(T,\bar{T}) = \sum_i g^i(T^i,\bar{T}^i), \qquad g^i = -\ln(T^i+\bar{T}^i) \tag{5.12}$$

is the Kähler potential for the moduli. The modification (5.9) is the 4d GS term in the chiral formulation.

The 4d GS mechanism is in fact more simply formulated in the linear multiplet formalism for the dilaton. The linear superfield L remains invariant, its Kähler potential is unchanged, and one simply adds a term to the Lagrangian. Using (5.3) and (5.6):

$$\begin{aligned}
\mathcal{L}_{GS} &= -\int d^4\theta E L V_{GS}, \\
\Delta\mathcal{L}_{GS} &= -b\int d^4\theta ELF + \text{h.c.} = \frac{b}{8}\int d^4\theta\frac{E}{R}F\left(\bar{\mathcal{D}}^2 - 8R\right)L + \text{h.c.} \\
&= \frac{b}{8}\int d^4\theta \frac{E}{R}F\Phi + \text{h.c.} = -\Delta\mathcal{L}_{\rm anom}
\end{aligned} \tag{5.13}$$

5.3 The anomaly in Supergravity

As mentioned in the introduction, the quadratic and logarthmic divergences of supergravity can be cancelled [182, 181, 179, 180] by a suitable set of Pauli-Villars (PV) supermultiplets.

It is straightforward to see [92, 91], by an examination of the quadratic divergences, that not all of these fields can have large PV masses that are invariant under nonlinear transformations on the fields that effect a Kähler transformation, such as the modular transformations (5.1), as we will illustrate with an example below.

It was shown in [92, 91] that modular noninvariant masses can be restricted to a subset of PV chiral supermultiplets Φ^C with diagonal Kähler metric:

$$K(\Phi^C, \bar{\Phi}^C) = f^C(Z, \bar{Z})|\Phi^C|^2. \tag{5.1}$$

In particular, those PV fields that have superpotential couplings to light fields and contribute to the renormalization of the Kähler potential can be chosen to have invariant PV masses. The fields in (5.1) acquire masses through superpotential terms:

$$W(\Phi^C, \Phi'^C) = \mu_C \Phi^C \Phi'^C, \tag{5.2}$$

with μ_C constant (in the absence of threshold corrections, as for the cases considered here). We can define a superfield[2]

$$\mathcal{M}_C^2 = \exp(K - f^C - f'^C) = \exp(K - 2\bar{f}^C), \qquad \bar{f}^C = \frac{1}{2}(f^C + f'^C), \tag{5.3}$$

whose lowest component $m_C^2 = \mathcal{M}_C^2|$ is the Φ^C, Φ'^C squared mass. Then the anomalous part of the one-loop corrected supergravity Lagrangian takes the form [92, 91]

$$\mathcal{L}_{\rm anom} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_r = \int d^4\theta E \left(L_0 + L_1 + L_r\right), \tag{5.4}$$

$$L_0 = \frac{1}{8\pi^2}\left[\mathrm{Tr}\eta \ln \mathcal{M}^2 \Omega_0 + K\left(\Omega_{GB} + \Omega_D\right)\right], \qquad L_r = -\frac{1}{192\pi^2}\mathrm{Tr}\eta \int d\ln\mathcal{M}\Omega_r, \tag{5.5}$$

where $\eta = \pm 1$ is the PV signature,

$$\Omega_0 = -\Omega_{\rm GB} + \Omega_{\rm YM} - \frac{1}{12}G_{\dot{\beta}\alpha}G^{\alpha\dot{\beta}} + \frac{1}{3}R\bar{R} - \frac{1}{48}\left(\mathcal{D}^2 R + \bar{\mathcal{D}}^2\bar{R}\right), \tag{5.6}$$

$$\begin{aligned}\Omega_r = & -\frac{\partial}{\partial \ln \mathcal{M}}\left[\frac{1}{4}\left(\mathcal{D}^2 \ln\mathcal{M}\mathcal{D}_{\dot{\beta}}\ln\mathcal{M}\mathcal{D}^{\dot{\beta}}\ln\mathcal{M} + \text{h.c.}\right) - 2G_{\alpha\dot{\beta}}\mathcal{D}^\alpha\ln\mathcal{M}\mathcal{D}^{\dot{\beta}}\ln\mathcal{M}\right.\\ & + \left(\ln\mathcal{M}\left\{\frac{1}{8}\bar{\mathcal{D}}^2\mathcal{D}^2\ln\mathcal{M} + \mathcal{D}^\alpha(R\mathcal{D}_\alpha\ln\mathcal{M})\right\} + \text{h.c.}\right)\\ & + \frac{1}{2}\mathcal{D}^\alpha\ln\mathcal{M}\mathcal{D}_\alpha\ln\mathcal{M}\mathcal{D}_{\dot{\beta}}\ln\mathcal{M}\mathcal{D}^{\dot{\beta}}\ln\mathcal{M}\\ & \left. -(\ln\mathcal{M})^2\left(\frac{1}{4}\mathcal{D}^\alpha L_\alpha + \ln\mathcal{M}\mathcal{D}^\alpha X_\alpha\right)\right],\end{aligned} \tag{5.7}$$

[2]The constants μ_C in (5.2) drop out of the variation $\Delta\mathcal{L}_{\rm anom}$ of the effective action (5.4), and we ignore them throughout.

with

$$X_\alpha = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha K, \qquad L_\alpha = (\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha \ln \mathcal{M}, \tag{5.8}$$

$G_{\dot\beta\alpha}$ is an auxiliary superfield of the gravity supermultiplet, and Ω_D represents the "D-term" anomaly (see Appendix B) that, together with a contribution to the Gauss-Bonnet term $\Omega_{\rm GB}$:

$$\Omega_{\rm GB} = -8\Omega_W - \frac{4}{3}\Omega_X - G_{\dot\beta\alpha}G^{\alpha\dot\beta} + 4R\bar{R}, \tag{5.9}$$

arises from uncanceled total derivatives with logarithmically divergent coefficients as discussed in the introduction. Supersymmetry of these terms requires a field-dependent cut-off:

$$\Lambda = \mu_0 e^{K/4}. \tag{5.10}$$

The constant μ_0 drops out of the effective action (5.4).

The Chern-Simons superfields Ω_W, Ω_X and $\Omega_{\rm YM}$ are defined by their chiral projections:

$$(\bar{\mathcal{D}}^2 - 8R)\Omega_W = W^{\alpha\beta\gamma}W_{\alpha\beta\gamma}, \qquad (\bar{\mathcal{D}}^2 - 8R)\Omega_X = X^\alpha X_\alpha, \qquad (\bar{\mathcal{D}}^2 - 8R)\Omega_{\rm YM} = W^\alpha_a W^a_\alpha. \tag{5.11}$$

where $W_{\alpha\beta\gamma}$ is the superfield strength for space-time curvature.

$\mathcal{L}_1$ is defined by its variation:

$$\Delta L_1 = \frac{1}{8\pi^2}\frac{1}{192}{\rm Tr}\eta\Delta\ln\mathcal{M}^2\Omega'_L = \frac{1}{8\pi^2}\frac{1}{192}{\rm Tr}\eta H\Omega'_L + {\rm h.c.}, \tag{5.12}$$

where under (5.1) $\ln\mathcal{M}^2$ transforms as

$$\Delta\ln\mathcal{M}^2 = H + \bar{H}, \tag{5.13}$$

with H holomorphic. Defining

$$\begin{aligned} (\bar{\mathcal{D}}^2 - 8R)\Omega_f &= f^\alpha f_\alpha, \qquad (\bar{\mathcal{D}}^2 - 8R)\Omega_{\bar f} = \bar{f}^\alpha\bar{f}_\alpha, \qquad (\bar{\mathcal{D}}^2 - 8R)\Omega_{\bar{f}X} = \bar{f}^\alpha X_\alpha, \\ f_\alpha &= -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha f, \qquad \bar{f}_\alpha = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha\bar{f}, \end{aligned} \tag{5.14}$$

we have

$$\begin{aligned} \Omega'_L &= 192\Omega_f - 128\Omega_{\bar f} - 64\Omega_{\bar{f}X}, \\ \Delta L_1 &= \frac{1}{8\pi^2}{\rm Tr}\eta H\left(\Omega_f - \frac{2}{3}\Omega_{\bar f} - \frac{1}{3}\Omega_{\bar{f}X}\right) + {\rm h.c.} \end{aligned} \tag{5.15}$$

The general form of f^C is taken to be

$$\begin{aligned} \ln f^C &= \alpha^C K(Z,\bar{Z}) + \beta^C g(T,\bar{T}) + \delta^C k(S,\bar{S}) + \sum_n q^C_n g^n(T^n,\bar{T}^n), \\ \ln\bar{f}^C &= \bar\alpha^C K + \bar\beta^C g + \bar\delta^C k + \sum_n \bar{q}^C_n g^n, \\ H^C &= \left(1 - 2\bar\gamma^C\right)F(T) - 2\sum\bar{q}^C_n F^n(T^n), \qquad \bar\gamma^C = \bar\alpha^C + \bar\beta^C. \end{aligned} \tag{5.16}$$

The traces in $\Delta\mathcal{L}_{\rm anom}$ can be evaluated using only PV fields with noninvariant masses or using the full set of PV fields, since those with invariant masses, $H^C = 0$, drop out. The contribution ΔL_0 to the anomaly is linear in the parameters α^C, β^C, q_n^C; as a consequence the traces are completely determined by the sum rules [182, 181, 179, 180]

$$\begin{aligned} N' &= \sum_C \eta^C = -N - 29, \qquad N'_G = \sum_\gamma \eta^V_\gamma = -12 - N_G, \\ \sum_C \eta^C \ln f^C &= -10K - \sum_q q^p_n g^n, \end{aligned} \tag{5.17}$$

that are required to assure the cancellation of all quadratic and logarithmic divergences. In (5.17) the index C denotes any chiral PV field, the index γ runs over the Abelian gauge PV superfields that are needed to cancel some gravitational and dilaton-gauge couplings, and the sum over p includes all the light chiral multiplet modular weights with $q_n^S = 0$, $q_n^{T^i} = 2\delta_n^i$. N and N_G are the total number of chiral and gauge supermultiplets, respectively, in the light sector. All PV fields with noninvariant masses have $\delta = 0$, and most[3] with $\delta \neq 0$ have $\alpha = \beta = q_n = 0$. For the purposes of the present analysis we can ignore the latter.

To see that not all the PV chiral multiplets can have invariant masses, there is a quadratically divergent contribution from the light sector given by

$$\mathcal{L}_Q \ni -\sqrt{g}\frac{\Lambda^2}{64\pi^2}\left(3 + N_G - N\right)\mathcal{D}^\alpha X_\alpha\Big|, \tag{5.18}$$

where X_α is defined in (5.8). The Pauli-Villars contribution to the operator in (5.18) is

$$\mathcal{L}_Q^{\rm PV} \ni -\sqrt{g}\frac{\Lambda^2}{64\pi^2}\left(N'_G - N' - 2\alpha\right)\mathcal{D}^\alpha X_\alpha\Big|, \tag{5.19}$$

where $\alpha = \sum \eta^C \alpha^C$. The PV chiral multiplets include a subset θ^a with $N'_\theta = N'_G$ which form massive vector supermultiplets with the PV Abelian gauge supermultiplets; these cancel in (5.19). The remainder get superpotential masses as in (5.2). The pair Φ^C, Φ'^C will have an invariant mass if $\ln \bar{f}^C = \bar{\alpha}^C = \frac{1}{2}$, in which case the total contribution of the pair to (5.19) vanishes identically. Therefore chiral fields with noninvariant masses are needed to cancel (5.18).

In contrast to $\mathcal{L}_0$, the contributions to the anomaly from $\mathcal{L}_1$ and $\mathcal{L}_r$ are nonlinear in the parameters α, β, q, and depend on the details of the PV sector. In [92, 91] the PV sector was constructed in such a way that

$$f^C = f'^C = \bar{f}^C \tag{5.20}$$

[3]There is a set of chiral multiplets in the adjoint representation of the gauge group that has $\ln f = K - k$; these get modular invariant masses though their coupling in the superpotential to a second set with $\ln f = k$. These cancel renormalizable gauge interactions and gauge-gravity interactions, respectively. Together with a third set, that has $f = 1$ and contributes to the anomaly, they cancel the Yang-Mills contribution to the beta-function.

for the PV fields with noninvariant masses. In this case (5.15) reduces to

$$(\Omega'_L)_{[1]} = 64\left(\Omega_{\bar{f}} - \Omega_{\bar{f}X}\right) = \Omega_L - 16\Omega_X, \qquad (\bar{\mathcal{D}}^2 - 8R)\Omega_L = L^\alpha L_\alpha, \tag{5.21}$$

and, for example,

$$\begin{aligned} \mathrm{Tr}\eta H\Omega_{\bar{f}} &= \sum_C \eta_C \left[\left(1 - 2\bar{\gamma}^C\right) F - 2\sum_n \bar{q}^C_n F^n\right] \times \\ & \left(\bar{\alpha}^C X^\alpha + \bar{\beta}^C g^\alpha + \sum_m \bar{q}^C_m g^\alpha_m\right)\left(\bar{\alpha}^C X_\alpha + \bar{\beta}^C g_\alpha + \sum_l \bar{q}^C_l g^l_\alpha\right). \end{aligned} \tag{5.22}$$

The Pauli-Villars modular weights q^C_n are related to the modular weights q^p_n of the light fields by the conditions for the cancellation of UV divergences. In the Z_3 and Z_7 orbifolds considered below, the latter satisfy sum rules of the form:

$$\sum_p q^p_n = A_1, \qquad \sum_p q^p_n q^p_m = A_2 + B_2\delta_{mn}. \tag{5.23}$$

The first sum rule in (5.23) assures the university of the anomaly proportional to $\Omega_0 - \Omega_{\mathrm{YM}}$. However, in the PV sector used in [92, 91] the second equality led to a nonuniversal term:

$$\begin{aligned} \mathrm{Tr}\eta H\Omega_{\bar{f}} &\ni -4\sum_{p,m,n} q^p_n q^p_m F^n g^\alpha_m \left(\bar{\alpha}^C X_\alpha + \bar{\beta}^C g_\alpha\right) \\ &= -4\left(A_2 F g^\alpha + B_2 F^n g^\alpha_n\right)\left(\bar{\alpha}^C X_\alpha + \bar{\beta}^C g_\alpha\right). \end{aligned} \tag{5.24}$$

The sum rule cubic in the modular weights is more complicated, but in general leads to additional nonuniversal terms. These can be avoided by imposing $\bar{q}^C_n = 0$ for fields with noninvariant masses, but if (5.20) is imposed we get

$$\mathrm{Tr}\eta H\Omega'_L = F\left(aX^\alpha X_\alpha + bX^\alpha g_\alpha + cg^\alpha g_\alpha\right), \tag{5.25}$$

which does not include the term proportional to

$$F\sum_n g^\alpha_n g^n_\alpha \tag{5.26}$$

found in the string calculation[4] of [379].

In the following we relax the assumption (5.20), impose $\bar{q}^C_n = 0$, but with $q^C = -q'^C \neq 0$. This still assures a universal anomaly, but allows more freedom in determining its coefficient; in particular, we are able to reproduce the term (5.26).

[4] In fact the four-form $\epsilon^{\mu\nu\rho\sigma} g^n_{\mu\nu} g^n_{\rho\sigma}$ with $g^n_{\mu\nu} = (\partial_\mu t^n \partial_\nu \bar{t}^{\bar{n}}) g^n_{t^n \bar{t}^{\bar{n}}} - (\mu \leftrightarrow \nu)$, that appears in the chiral part of (5.26), vanishes identically. We find it curious that the authors of [379] neglected to comment on this fact. However the associated conformal anomaly is nontrivial.

5.4 Cancellation of UV divergences

The full set of PV fields needed to regulate light field couplings is described in Section 3 of [92, 91]. Among those, here we are primarily concerned with the set $\dot{Z}^P = \dot{Z}^I, \dot{Z}^A$, with negative signature, $\eta^{\dot{Z}} = -1$, that regulates most of the couplings, including all renormalizable couplings, of the light chiral supermultiplets $Z^p = T^i, \Phi^a$. Covariance of the $\dot{Z}^P$ Kähler metric requires that these fields transform under (5.1) like dZ^p:

$$\dot{Z}'^I = e^{-2F^i}\dot{Z}^I, \qquad \dot{Z}'^A = e^{-F^a}\left(\dot{Z}^A - \sum_j F^a_j \Phi^a \dot{Z}^J\right), \qquad F^a = \sum_i F^i(T^i) \tag{5.1}$$

Invariance of the full PV Kähler potential for the $\dot{Z}^P$ and covariance of their superpotential under (5.1):

$$K(\dot{Z}') = K(\dot{Z}), \qquad W(\dot{Z}') = e^{-F(T)}W(\dot{Z}), \tag{5.2}$$

can be made manifest if we supplement [92, 91] these fields with three additional PV fields $\dot{Z}^N,\ N = 1, 2, 3$, with Kähler potential

$$K(\dot{Z}^N) = \sum_{i=n}\left|\dot{Z}^N + \dot{a}\chi^n(T^i)\dot{Z}^i\right|^2, \qquad \chi^n(T'^i) = e^{2F^n}\left(\chi^n(T^i) + F^n_i\right), \tag{5.3}$$

and that transform under (5.1) according to

$$\dot{Z}'^N = \dot{Z}^N - \dot{a}F^n_i(T^i)\dot{Z}^I, \tag{5.4}$$

where $\dot{a}$ is a nonzero constant.[5]

We wish to give these PV fields modular invariant masses. The simplest way to do this is to introduce fields $\dot{Y}_P, \dot{Y}_N$ with the same signature, opposite gauge charges and the inverse Kähler metric. However this would have the effect of canceling the $\dot{Z}$ contributions that are linear in the generalized field strength

$$G_{\mu\nu} = [D_\mu, D_\nu], \tag{5.5}$$

and doubling the quadratic $\dot{Z}$ contributions. Instead we introduce fields $\dot{Y}_P, \dot{Y}_N$ with with gauge charges

$$(T_a)_{\dot{Y}} = -(T^T_a)_{\dot{Z}} = -(T^T_a)_Z, \tag{5.6}$$

and Kähler potential

$$\begin{aligned} K(\dot{Y}) &= e^{\dot{G}}\left(\sum_A e^{-g^a}|\dot{Y}_A|^2 + \sum_I e^{-2g^i}|\dot{Y}_I - \dot{a}\chi^n(T^i)\dot{Y}_N|^2 + |\dot{Y}_N|^2\right), \\ g^a &= \sum_n q^a_n g^n, \qquad \dot{G} = \dot{\alpha}K + \dot{\beta}g, \qquad \dot{\alpha} + \dot{\beta} = 1. \end{aligned} \tag{5.7}$$

[5] Depending on the choice of the functions $\chi^n(T^i)$, one might need to introduce [92, 91] several copies of the sets $\dot{Z}^{P,N}_\lambda$, with constraints on the parameters $\dot{a}_\lambda$ in such a way that no new divergences are introduced by the fields $\dot{Z}^N$.

(5.7) is modular invariant, and the PV mass superpotential

$$W(\dot{Z},\dot{Y}) = \dot{\mu}\left(\dot{Z}^A - \dot{a}^{-1}q_n^a\Phi^a\dot{Z}^N\right)\dot{Y}_A + \sum_{i=n}\dot{\mu}_n\left(\dot{Z}^I\dot{Y}_I + \dot{Z}^N\dot{Y}_N\right), \tag{5.8}$$

is covariant, provided under (5.1)

$$\dot{Y}'_A = e^{-F+F^a}\dot{Y}_A, \qquad \dot{Y}'_I = e^{-F+2F^i}\left(\dot{Y}_I + \dot{a}F_i^n\dot{Y}_N\right), \qquad \dot{Y}'_N = e^{-F}\dot{Y}_N. \tag{5.9}$$

It remains to cancel the divergences introduced by the fields $\dot{Y}$. This was achieved in [92, 91] by an additional set of chiral PV fields, collectively called Ψ, with diagonal metric (5.1), superpotential (5.2), with prefactors (5.16) satisfying (5.20) and $\alpha^\Psi = \delta^\Psi = 0$. In addition $\dot{\alpha} = 0$ in (5.7) was assumed. Here we use a different set of fields, for which we assume only $\delta^C = 0$, as well as allowing $\dot{\alpha} \neq 0$. For this reason we also include in the present analysis the set of fields ϕ^C with prefactors

$$\ln f^{\phi^C} = \alpha^C K \tag{5.10}$$

that regulate certain gravity supermultiplet loops. These must be included together with the PV fields introduced below in implementing the sum rules (5.17). We take the following set:

$$\begin{aligned}
\Phi^P: &\quad \ln f^{\Phi^P} = \sum_n q_n^P g^n = \alpha^\Phi K + \beta^\Phi g - \ln f^{\Phi'^P}, \qquad \alpha^\Phi + \beta^\Phi = 1,\\
\psi^{Pn}: &\quad \ln f^{Pn} = \alpha_\psi^P K + \beta_\psi^P g + q_\psi^P g^n, \qquad \alpha_\psi^P + \beta_\psi^P = \gamma_\psi^P, \qquad \bar{q}_\psi^P = 0,\\
T^P: &\quad \ln f_T^P = \alpha_T^P K + \beta_T^P g, \qquad \alpha_T^P + \beta_T^P = \gamma_T^P.
\end{aligned} \tag{5.11}$$

The pairs Φ^P, Φ'^P have modular invariant masses and do not contribute to the anomaly, but they play an important role in canceling certain divergences. In the case of Z_7 orbifolds we take them to be charged under the two $U(1)$'s of that theory. They have no other gauge charges, the ψ^{Pn} are taken to be gauge neutral, and the T^P have a priori arbitrary gauge charges. For those in real representations of the gauge group one can take $T^P = T'^P$. In Appendix B we display a simple solution to the constraints with some T's in the fundamental and antifundamental representation of the non-Abelian gauge group factors, some with $U(1)$ charges in the Z_7 case, and some gauge singlets.

The quadratic and logarithmic divergences we are concerned with here involve the superfield strengths $-i(T_a)W_\alpha^a$, X_α and

$$\Gamma_{D\alpha}^C = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha Z^i\Gamma_{Di}^C, \tag{5.12}$$

associated with the Yang-Mills, Kähler and reparameterization connections, respectively. Since the theories considered here have no gauge anomalies, cancellation of quadratic divergences requires

$$\mathrm{Tr}\eta\Gamma_\alpha = 0, \tag{5.13}$$

and cancellation of logarithmic divergences requires

$$\mathrm{Tr}\eta\Gamma_\alpha\Gamma_\beta = \mathrm{Tr}\eta\Gamma_\alpha T^a = \mathrm{Tr}\eta(T^a)^2 = 0, \tag{5.14}$$

where $\eta = +1$ for light fields. Cancellation of all contributions linear and quadratic in X_α is assured by the conditions in (5.17) together with (B.5) of Appendix B. The Yang-Mills contribution to the term quadratic in W_α is canceled by chiral fields in the adjoint (see footnote on page 57) that we need not consider here. Finally, cancellation of linear divergences requires cancellation of the imaginary part of

$$\mathrm{Tr}\eta X_\chi = \mathrm{Tr}\eta\phi G \cdot \widetilde{G}, \qquad \widetilde{G}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}, \tag{5.15}$$

where $G_{\mu\nu}$ is the field strength associated with the fermion connection;[6] for left-handed fermions:

$$G_{\mu\nu} = -\Gamma^C_{D\mu\nu} + iF^a_{\mu\nu}(T_a)^C_D + \frac{1}{2}X_{\mu\nu}\delta^C_D, \tag{5.16}$$

and for a generic PV superfield Φ^C with diagonal metric, its fermion component χ^C transforms under (5.1) as

$$\chi'^C = e^{\phi^C}\chi^C, \qquad \phi^C = \left(\frac{1}{2} - \alpha^C - \beta^C\right)F - \sum_i F^i(T^i)q^C_i. \tag{5.17}$$

The full set of conditions is extensive, and we evaluate them in Appendix B. In this section we simply outline how to obtain a universal anomaly using PV regularization. For this purpose we focus on terms contributing to UV divergences that could potentially spoil universality. An important feature in our results is the fact that the expression

$$\epsilon^{\mu\nu\rho\sigma}g^i_{\mu\nu}g^i_{\rho\sigma} = 0, \tag{5.18}$$

vanishes identically, and the expressions

$$\begin{aligned} X^{ij} &= \epsilon^{\mu\nu\rho\sigma}\mathrm{Im}F^i g^i_{\mu\nu}g^{j\neq i}_{\rho\sigma} = 4\epsilon^{\mu\nu\rho\sigma}\mathrm{Im}F^i\partial_\mu g^i_\nu\partial_\rho g^j_\sigma = 4\partial_\rho\left(\epsilon^{\mu\nu\rho\sigma}\mathrm{Im}F^i\partial_\mu g^i_\nu g^j_\sigma\right), \\ X^i &= \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Im}F^i g^i_{\mu\nu}X_{\rho\sigma} = 4i\partial_\rho\left(\epsilon^{\mu\nu\rho\sigma}\mathrm{Im}F^i\partial_\mu g^i_\nu\Gamma_\sigma\right), \\ X^{ia} &= \epsilon^{\mu\nu\rho\sigma}\mathrm{Im}F^i g^i_{\mu\nu}F^a_{\rho\sigma} = 4\partial_\rho\left(\epsilon^{\mu\nu\rho\sigma}\mathrm{Im}F^i\partial_\mu g^i_\nu A^a_\sigma\right), \end{aligned} \tag{5.19}$$

are total derivatives, where A^a_μ is an Abelian gauge field,

$$g^i_\mu = -\frac{\partial_\mu t^i - \partial_\mu\bar{t}^{\bar{i}}}{t^i + \bar{t}^{\bar{i}}}, \qquad g^i_{\mu\nu} = \partial_\mu g^i_\nu - \partial_\nu g^i_\mu, \qquad \Gamma_\mu = \frac{i}{4}\left(\mathcal{D}_\mu z^i K_i - \mathcal{D}_\mu\bar{z}^{\bar{m}}K_{\bar{m}}\right), \tag{5.20}$$

and $X_{\mu\nu} = 2i\left(\partial_\mu\Gamma_\nu - \partial_\nu\Gamma_\mu\right)$ is defined in (5.2).

[6]Here we neglect the spin connection which is considered in Appendix B.

If, for example, we replaced $g^n(T^n, \bar{T}^{\bar{n}})$ everywhere by the Kähler potential for the nth untwisted sector, a possibility considered in [92, 91], the above would not hold, and we would be unable to obtain a universal anomaly coefficient. Specifically, we would be not be able to cancel the terms cubic in q_n^p that appear in $X_\chi^{\dot{Y}}$, suggesting that the present construction is the only viable possibility. This agrees with the results of [379], where it was found that the untwisted Kähler moduli are the only chiral supermultiplets that appear in the chiral anomaly (see however footnote page 58).

Reparameterization curvature terms

The functions $\chi^n(T^i)$ in (5.3) and (5.7) do not contribute to the quantities in (5.12) and (5.16) (see footnote page 59), and, using (5.23), one obtains

$$\begin{aligned}
\mathrm{Tr}\dot{\eta}\Gamma_\alpha^{\dot{Y}} &= -\left[(N+2)\dot{\beta} - A_1\right] g_\alpha, \\
\mathrm{Tr}\dot{\eta}\Gamma_\alpha^{\dot{Y}}\Gamma_\beta^{\dot{Y}} &= -2\dot{\alpha}\left[\dot{\beta}(N+2) - A_1\right] X_\alpha g_\beta - \left[\dot{\beta}^2(N+2) - \dot{\beta}A_1 + A_2\right] g_\beta g_\alpha \\
&\quad -B_2 \sum_n g_\alpha^n g_\beta^n. \qquad (5.21)
\end{aligned}$$

In addition we have

$$\begin{aligned}
X_\chi^{\dot{Y}} &= \frac{1}{2}(N+2)F\dot{G}\cdot\tilde{\dot{G}} - F\sum_{p,n} q_n^p \dot{G}\cdot\tilde{g}^n + \frac{1}{2}F\sum_{p,m,n} q_m^p q_n^p g^m\cdot\tilde{g}^n \\
&\quad -\sum_{p,n} q_n^p F^n \dot{G}\cdot\tilde{\dot{G}} + 2\sum_{p,n,m} q_m^p q_n^p F^m \dot{G}\cdot\tilde{g}^n - \sum_{p,l,m,n} q_l^a q_m^a q_n^a F^l g^m\cdot\tilde{g}^n, \qquad (5.22)
\end{aligned}$$

In addition to the sum rules listed in (5.23), we have

$$\begin{aligned}
\sum_p q_i^p q_j^p q_k^p &= A_3 + B_3\delta_{ij}\delta_{ik} + C_3\left[\delta_{ij}\left(\delta_i^1\delta_k^2 + \delta_i^2\delta_k^3 + \delta_i^3\delta_k^1\right) + \mathrm{cyclic}(ijk)\right] \\
&\quad + D_3\left[\delta_{ij}\left(\delta_i^2\delta_k^1 + \delta_i^3\delta_k^2 + \delta_i^1\delta_k^3\right) + \mathrm{cyclic}(ijk)\right], \qquad (5.23)
\end{aligned}$$

with $C_3 = D_3$ for Z_3, $C_3 \neq D_3$ for Z_7. Then using (5.18) and (5.19), (5.22) reduces to

$$\begin{aligned}
X_\chi^{\dot{Y}} &= \frac{1}{2}(N+2)F\dot{G}\cdot\widetilde{\dot{G}} - A_1 F\dot{G}\cdot\tilde{g} + \frac{1}{2}A_2 F g\cdot\tilde{g} - A_1 F\dot{G}\cdot\widetilde{\dot{G}} \\
&\quad + 2A_2 F\dot{G}\cdot\tilde{g} - A_3 F g\cdot\tilde{g} + \text{total derivative}. \qquad (5.24)
\end{aligned}$$

Since $\bar{q}^\Phi = \bar{q}^\psi = 0$, terms cubic in these modular weights do not contribute to X_χ^Φ, X_χ^ψ. Further, since

$$q_m^{Pn} q_l^{Pn} = (q_\psi^P)^2 \delta_m^n \delta_l^n, \qquad (5.25)$$

there are no contributions to X^ψ_χ quadratic in ψ modular weights, and since q^P_ψ is independent of n, X^ψ_χ depends only on $F, g_{\mu\nu}$ and $X_{\mu\nu}$. Then imposing

$$\sum_P \eta^P q^P_n = a_1, \qquad \sum_P \eta^P q^P_n q^P_m = a_2 + b_2 \delta_{mn}, \tag{5.26}$$

X^Φ_χ can also be made to depend only on $F, g_{\mu\nu}$ and $X_{\mu\nu}$. The terms linear in the Φ and ψ modular weights drop out of $(\mathrm{Tr}\eta\Gamma_\alpha)_{\Phi,\psi}$, and one obtains[7]

$$\begin{aligned}
(\mathrm{Tr}\eta\Gamma_\alpha\Gamma_\beta)_\Phi &= \sum_P \eta^P_\Phi G^\Phi_\alpha G^\Phi_\beta - a_1\left(G^\Phi_\alpha g_\beta + g_\alpha G^\Phi_\beta\right) + 2a_2 g_\alpha g_\beta + 2b_2 \sum_n g^n_\alpha g^n_\beta, \\
G^\Phi &= \alpha^\Phi K + \beta^\Phi g, \\
(\mathrm{Tr}\eta\Gamma_\alpha\Gamma_\beta)_\psi &= \sum_P \eta^P_\psi \left[3G^P_\alpha G^P_\beta + q^P_\psi\left(G^P_\alpha g_\beta + g_\alpha G^P_\beta\right)\right] + B_\psi \sum_n g^n_\alpha g^n_\beta, \\
G^P &= \alpha^P K + \beta^P g, \qquad B_\psi = \sum_P \eta^P_\psi (q^P_\psi)^2.
\end{aligned} \tag{5.27}$$

To cancel the last term in (5.21) we require

$$2b_2 + B_\psi = B_2. \tag{5.28}$$

The $\dot{Y}$ and Φ fields do not contribute to the anomaly, and the coefficient of the term (5.26) is determined by B_ψ. The remaining terms in (5.21) and (5.22) can be cancelled by a combination of the full set of PV fields in (5.10) and (5.11), as shown in Appendix B.

Yang-Mills field strengths

The gauge charges[8] and modular weights in Z_3 and Z_7 orbifold compactifications without Wilson lines are given in [379] and Appendix D.5 of [92, 91]. The universality of the anomaly term quadratic in Yang-Mills fields strengths is guaranteed by the universality condition (5.3), as illustrated in Appendix B. Since gauge transformations commute with modular transformations, a set of chiral multiplets Φ^b that transform according to a nontrivial irreducible representation R of a non-Abelian gauge group factor $\mathcal{G}_a$ have the same modular weights q^R_n such that

$$\sum_{b\in R} q^b_n (T_a)^b_b = q^R_n (\mathrm{Tr}T_a)_R = 0. \tag{5.29}$$

Therefore terms linear in Yang-Mills field strengths occur only for Abelian gauge group factors. There are none in Z_3, but two in Z_7, which we refer to as $U(1)_a$, $a = 1, 2$, with

[7]In (5.26) and for Φ in (5.27), the sum is over P only, while for ψ, $\sum_P \equiv \sum_P + \sum_{P'}$, since P and P' are interchangeable in the latter, but not in the former.

[8]We use the standard charge normalization such that (5.3) is satisfied with $C^b_a = (\mathrm{Tr}T^2_a)_{R(b)}$, where $R(b)$ is the gauge group representation of the chiral supermultiplet Φ^b; this differs by a factor $\sqrt{2}$ from the normalization used in [379].

charges Q_a. These are anomaly free; their traces vanish when taken over the full spectrum of chiral multiplets. Defining

$$Q_{an} = \sum_b q_n^b Q_a^b, \qquad Q_{anm} = \sum_b q_n^b q_m^b Q_a^b, \tag{5.30}$$

we have for Z_7:

$$\begin{aligned} Q_{1n} &= \frac{1}{2}(8,2,-10), \qquad Q_{2n} = \frac{1}{2\sqrt{3}}(12,-18,6), \qquad n=(1,2,3), \\ Q_{1nm} &= \frac{1}{2}(5,-4,-1), \qquad Q_{2nm} = \frac{1}{2\sqrt{3}}(-3,-6,9), \\ & nm = (12,23,31). \end{aligned} \tag{5.31}$$

These satisfy

$$\sum_n Q_{an} = 0, \qquad Q_{anm} = -\frac{1}{2}|\epsilon_{nml}| Q_{al}. \tag{5.32}$$

We wish to cancel the $\dot{Y}$-loop contribution to logarithmic divergences

$$(\mathrm{Tr}\eta g_\alpha^n T_a)_{\dot{Y}} = -\sum_b q_n^b Q_a^b g_\alpha^n = -Q_{an} g_\alpha^n, \tag{5.33}$$

and, dropping terms proportional to the last expression in (5.19), $\dot{Y}$ contributions to linear divergences:

$$\begin{aligned} X_\chi^{\dot{Y}} &\ni \sum_b \tilde{F}_{\mu\nu}^a Q_a^b \left[g^{n\mu\nu} q_n^b \left(F - 2q_m^b F^m\right) + 2q_n^b F^n \left\{ \left(\dot{\alpha} - \frac{1}{2}\right) X^{\mu\nu} + \dot{\beta} g^{\mu\nu} \right\} \right] \\ &= \tilde{F}_{\mu\nu}^a F^1 \left\{ \left(Q_{a2} g^{2\mu\nu} + Q_{a3} g^{3\mu\nu}\right) + 2Q_{a1} \left[\left(\dot{\alpha} - \frac{1}{2}\right) X^{\mu\nu} + \dot{\beta}\left(g^{2\mu\nu} + g^{3\mu\nu}\right) \right] \right. \\ &\quad \left. -2\left(Q_{a12} g^{2\mu\nu} + Q_{a13} g^{3\mu\nu}\right) \right\} + \text{cyclic (1,2,3)} + \text{total derivative}. \end{aligned} \tag{5.34}$$

Using (5.32), (5.34) becomes

$$\begin{aligned} X_\chi^{\dot{Y}} &\ni \tilde{F}_{\mu\nu}^a F^1 \left\{ \left(Q_{a2} g^{2\mu\nu} + Q_{a3} g^{3\mu\nu}\right) + 2Q_{a1} \left[\left(\dot{\alpha} - \frac{1}{2}\right) X^{\mu\nu} + \dot{\beta}\left(g^{2\mu\nu} + g^{3\mu\nu}\right) \right] \right. \\ &\quad \left. + \left(Q_{a3} g^{2\mu\nu} + Q_{a2} g^{3\mu\nu}\right) \right\} + \text{cyclic (1,2,3)} + \text{total derivative} \\ &= \tilde{F}_{\mu\nu}^a F^1 \left[\left(Q_{a2} + Q_{a3} + 2\dot{\beta} Q_{a1}\right)\left(g^{2\mu\nu} + g^{3\mu\nu}\right) + Q_{a1}\left(2\dot{\alpha} - 1\right) X^{\mu\nu} \right] \\ &\quad + \text{cyclic (1,2,3)} + \text{total derivative}. \end{aligned} \tag{5.35}$$

Now we assign $U(1)_a$ charges Q_a^P and $-Q_a^P$ to Φ^P and Φ'^P, respectively. This gives a contribution to logarithmic divergences

$$2\sum_P \eta^P q_n^P Q_a^P g_\alpha^n \equiv Q_{an}^\Phi g_\alpha^n. \tag{5.36}$$

Cancellation of (5.33) requires

$$Q^{\Phi}_{an} = Q_{an}, \tag{5.37}$$

The Φ contribution to linear divergences is

$$\begin{aligned} X^{\Phi}_{\chi} &\ni -2\sum_{P}\eta^{P}Q^{P}_{a}q^{P}_{n}\tilde{F}^{a}_{\mu\nu}F^{n}\left[(\alpha^{\Phi}-1)X^{\mu\nu}+\beta^{\Phi}g^{\mu\nu}\right] \\ &= -Q^{\Phi}_{a1}\tilde{F}^{a}_{\mu\nu}F^{1}\left[(\alpha^{\Phi}-1)X^{\mu\nu}+\beta^{\Phi}\left(g^{2\mu\nu}+g^{3\mu\nu}\right)\right] \\ &\qquad +\text{cyclic (1,2,3)} + \text{total derivative.} \end{aligned} \tag{5.38}$$

To cancel the $X^{\mu\nu}$ term we require

$$\dot{\alpha} = \frac{1}{2}\alpha^{\Phi}, \qquad \dot{\beta} = 1 - \frac{1}{2}\alpha^{\Phi} = \frac{1}{2}\beta^{\Phi} + \frac{1}{2}. \tag{5.39}$$

Then

$$\begin{aligned} X^{\dot{Y}}_{\chi} &\ni \tilde{F}^{a}_{\mu\nu}F^{1}\left\{\left[Q_{a2}+Q_{a3}+\left(\beta^{\Phi}+1\right)Q_{a1}\right]\left(g^{2\mu\nu}+g^{3\mu\nu}\right)+Q_{a1}\left(\alpha^{\Phi}-1\right)X^{\mu\nu}\right\} \\ &\qquad +\text{cyclic (1,2,3)} + \text{total derivative} \\ &= \tilde{F}^{a}_{\mu\nu}F^{1}Q_{a1}\left[\beta^{\Phi}\left(g^{2\mu\nu}+g^{3\mu\nu}\right)+\left(\alpha^{\Phi}-1\right)X^{\mu\nu}\right] \\ &\qquad +\text{cyclic (1,2,3)} + \text{total derivative} = -X^{\Phi}_{\chi}, \end{aligned} \tag{5.40}$$

up to a total derivative.

Note that this is a highly nontrivial result. In addition to the importance of the properties in (5.19), the relations (5.32), that are specific to the Z_7 orbifold we are considering, are crucial to the cancellations in this section. Since the Φ have modular invariant masses, the ψ's have no gauge charges, and the T's have n-independent prefactors f^T, no terms linear in the gauge field strengths appear in the anomaly.

Finally we remark that a pair of PV fields Φ^C, Φ'^C with superpotential coupling (5.2) contributes an amount

$$\left(\phi^{C}+\phi'^{C}\right)C^{C}_{a} = \Delta\mathcal{M}^{2}C^{C}_{a} \tag{5.41}$$

to the coefficient of $F^a \cdot \tilde{F}_a$ in (5.15). This vanishes for pairs with invariant masses, and its form assures that the anomaly arising from PV masses in the regulated theory matches the anomaly due to linear divergences in the unregulated theory. In particular it makes no difference whether or not we assign non-Abelian gauge charges to the Φ^P, and their $U(1)_a$ charges have no affect on the term in the anomaly quadratic in the $U(1)_a$ field strengths.

5.5 The anomaly in $\mathbb{Z}_3$ and $\mathbb{Z}_7$ orbifolds

In Appendix B we show that is possible to cancel all the ultraviolet divergences from the $\dot{Y}$ fields with a simple choice of the set (5.11) such that the fields with noninvariant masses have the properties

$$\mathrm{Tr}\eta(\ln\mathcal{M})^{n>1} = \Delta\mathrm{Tr}\eta(\ln\mathcal{M})^{n>1} = \mathrm{Tr}\eta(\Delta\ln\mathcal{M})(\bar{f}_{\alpha})^{n>0} = 0, \tag{5.1}$$

and the anomaly due to the variation of (5.4) reduces to

$$\begin{aligned}\delta\mathcal{L}_{\text{anom}} &= b\int d^4\theta E F\Omega,\\ \Omega &= \Omega_{\text{YM}} - \frac{1}{24}\Omega_{\text{GB}} - \frac{b_{\text{spin}}}{48b}\left(4G_{\dot\beta\alpha}G^{\alpha\dot\beta} - 16R\bar R + \mathcal{D}^2 R + \bar{\mathcal{D}}^2\bar R\right) + \frac{1}{30}\left(\Omega_f + \Omega_D\right)\end{aligned} \tag{5.2}$$

where (see Appendix B)

$$8\pi^2 b_{\text{spin}} = 8\pi^2 b + 1 = 31, \qquad \widetilde\Omega_f = \mathrm{Tr}\eta\Delta\ln\mathcal{M}^2\Omega_f. \tag{5.3}$$

The results for the Gauss-Bonnet and Yang-Mills terms are well-established [311, 38, 154, 145, 310, 188, 37, 36, 198, 312] and result from the universality conditions (5.3) and (B.7), as illustrated in the appendices. The only other term in (5.2) that contains a chiral anomaly is Ω_f, which, using the set (5.11) of PV fields, is a priori a product of the chiral superfields X_α, g_α and g^n_α. We show in Appendix B we may choose the PV parameters such that

$$(\bar{\mathcal{D}}^2 - 8R)\widetilde\Omega_f = 30\sum_n g^\alpha_n g^n_\alpha, \tag{5.4}$$

in agreement with the string callculation of [379].

The anomaly is canceled provided the Lagrangian for the dilaton $S, \bar S$ is specified by the coupling (5.5) and the Kähler potential (5.9), or, equivalently, the linear supfield L satisfies (5.3) and the GS term (5.13) is added to the Lagrangian.

5.6 Conclusion

We have shown that a suitable choice of Pauli-Villars regulator fields allows for a full cancellation of the chiral and conformal anomalies associated, respecively, with the linear and logarithmic divergences in the effective supergravity theories from Z_3 and Z_7 compactification of the weakly coupled heterotic a string without Wilson lines. In particlar we were able to reproduce the form of the chiral anomaly found in a string theory calculation [379] for these two models.

In a future study we will extend our analysis to an example of Z_3 orbifold compactification with Wilson lines and an anomalous $U(1)$.

Chapter 6

Anomaly cancellation in effective supergravity from the heterotic string with an anomalous U(1)

6.1 Introduction

Starting with the determination of the full anomaly structure of Pauli-Villars (PV) regularized supergravity [92], we recently showed [185] that an appropriate choice of PV regulator fields allows for cancellation of all the T-duality (hereafter referred to as "modular") anomalies by the four-dimensional version of the Green-Schwarz term in $\mathbb{Z}_3$ and $\mathbb{Z}_7$ compactifications of the heterotic string without Wilson lines. We further matched our results to a string calculation [379] of the chiral anomaly in those theories. Here we extend our results to a specific $\mathbb{Z}_3$ compactification [252, 171] (hereafter referred to as FIQS) with two Wilson lines and therefore an anomalous $U(1)$, hereafter referred to as $U(1)_X$. In the following section we briefly describe the orbifold model we are studying. In Section 3 we outline the four-dimensional Green-Schwarz mechanism and the structure of the anomaly when an anomalous $U(1)$ is present. In Section 4 we discuss some aspects of the cancellation of ultra-violet (UV) divergences and anomaly matching that are specific to the case with an anomalous $U(1)$, as well as some simplifications with respect to the $\mathbb{Z}_7$ case studied in [185]. We summarize our results in Section 5. The full set of conditions for cancellation of UV divergences and anomaly matching are given in Appendix C, a sample solution to these constraints is presented in Appendix C, and the full spectrum for the FIQS model is displayed in Appendix C. The determination of the correct Pauli-Villars (PV) masses can have implications for soft supersymmetry breaking terms [187, 75, 320].

6.2 The FIQS Model

Here we will give a brief review of the orbifold model we will consider for the rest of the chapter. The FIQS model [252, 171] is a $\mathbb{Z}_3$ orbifold compactification of the 10d $E_8 \otimes E_8$ heterotic string compactified to T^6 with two Wilson lines and a nonstandard embedding for the shift vector. The embeddings of the shift vector and Wilson lines are given by

$$V = \frac{1}{3}(1,1,1,1,2,0,0,0)(2,0,0,0,0,0,0,0)' \tag{6.1}$$

$$a_1 = \frac{1}{3}(0,0,0,0,0,0,0,2)(0,1,1,0,0,0,0,0)' \tag{6.2}$$

$$a_3 = \frac{1}{3}(1,1,1,2,1,0,0,1,1)(1,1,0,0,0,0,0,0)' \tag{6.3}$$

Where the prime indicates that the last 8 elements of the above vectors correspond to the second factor of E_8. With these specifications, the massless spectrum of the FIQS model can be worked out following the standard recipes [250, 251]. The 4D gauge group is $SU(3) \otimes SU(2) \otimes SO(10) \otimes U(1)^8$. The generators of the eight $U(1)$ factors can be written as linear combinations of the $E_8 \otimes E_8$ Cartan subalgebra generators H^I as

$$Q_a = \sum_{I=1}^{16} q_a^I H^I \tag{6.4}$$

The constants q_a^I are determined by requiring that $q_a \cdot q_b = 0$ and $q_a \cdot \alpha_{bj} = 0$, where the α_{bj} are the sixteen dimensional simple root vectors of the nonabelian gauge group factors. Thus the index b corresponds to $SU(3)$, $SU(2)$, or $SO(10)$ and j runs over the rank of each group. One choice of q_a's is [101]:

$$\vec{q}_1 = 6(1,1,1,0,0,0,0,0)(0,0,0,0,0,0,0,0)' \tag{6.5}$$

$$\vec{q}_2 = 6(0,0,0,1,-1,0,0,0)(0,0,0,0,0,0,0,0)' \tag{6.6}$$

$$\vec{q}_3 = 6(0,0,0,0,0,1,0,0)(0,0,0,0,0,0,0,0)' \tag{6.7}$$

$$\vec{q}_4 = 6(0,0,0,0,0,0,1,0)(0,0,0,0,0,0,0,0)' \tag{6.8}$$

$$\vec{q}_5 = 6(0,0,0,0,0,0,0,1)(0,0,0,0,0,0,0,0)' \tag{6.9}$$

$$\vec{q}_6 = 6(0,0,0,0,0,0,0,0)(1,0,0,0,0,0,0,0)' \tag{6.10}$$

$$\vec{q}_7 = 6(0,0,0,0,0,0,0,0)(0,1,0,0,0,0,0,0)' \tag{6.11}$$

$$\vec{q}_8 = 6(0,0,0,0,0,0,0,0)(0,0,1,0,0,0,0,0)' \tag{6.12}$$

To get the charges of the matter fields, one normalizes the $U(1)_a$ generators as

$$Q_a \to \frac{1}{\sqrt{2}\,|q_a|} Q_a, \tag{6.13}$$

where the $\sqrt{2}$ is inserted to adhere to the standard phenomenological normalization. For this choice, one finds that the traces Q_6, Q_7, and Q_8 are all nonzero. One can perform a re-definition of the generators so that only one factor of U(1) has a nonzero trace. In [252, 171], the following re-definition was made:

$$q_6^{(FIQS)} = q_6 + q_7 \tag{6.14}$$
$$q_7^{(FIQS)} = q_7 + q_8 \tag{6.15}$$
$$q_X = q_6 - q_7 + q_8 \tag{6.16}$$

While $\mathrm{Tr}\left[Q_6^{(FIQS)}\right] = \mathrm{Tr}\left[Q_7^{(FIQS)}\right] = 0$ in this basis, one also has $\mathrm{Tr}\left[Q_6^{(FIQS)}Q_7^{(FIQS)}Q_X\right] \neq 0$ which is rather undesirable. Therefore, we will use a different choice such that the above mixed anomaly does not appear. In particular, we define

$$q_6^{(N)} = q_6 - q_8 = q_6^{(FIQS)} - q_7^{(FIQS)} \tag{6.17}$$
$$q_7^{(N)} = q_6 + 2q_7 + q_8 = q_6^{(FIQS)} + q_7^{(FIQS)} \tag{6.18}$$

In what follows, we will simply drop the superscript N and use these as the definition of the $U(1)_6$ and $U(1)_7$ generators. As a final note, the charges defined above are generally not orthogonal to one another, i.e. $\mathrm{Tr}\left[Q_aQ_b\right] \neq 0$ for some $a \neq b$. It is possible to define a new set of charges that are mostly orthogonal to one another, but we will not need to do so for our purposes.

We close this section with some relations among the gauge charges q_a^p and modular weights q_n^p of the chiral superfields Φ^p of the model. These will be useful in the analysis that follows. These include the universality conditions

$$\begin{aligned} 8\pi^2 b &= C_a + \sum_p \left(2q_i^p - 1\right) C_a^p = \frac{1}{24}\left(2\sum_p q_n^p - N + N_G - 21\right) \quad \forall \quad i, a, \\ -2\pi^2\delta_X &= \frac{1}{24}\mathrm{Tr}T_X = \frac{1}{3}\mathrm{Tr}T_X^3 = \mathrm{Tr}(T_a^2 T_X) \quad \forall \quad a \neq X. \end{aligned} \tag{6.19}$$

Here C_a is the quadratic Casimir in the adjoint representation of the gauge group factor $\mathcal{G}_a$ and C_a^p is the Casimir for the representation of the chiral supermultiplet Φ^p, T_a is a generator of $\mathcal{G}_a$, and N, N_G are the number of chiral and gauge supermultiplets respectively, with, in the FIQS model,

$$N = 415, \qquad N_G = 64, \qquad 8\pi^2 b = 6, \qquad -4\pi^2\delta_X = 3\sqrt{6}. \tag{6.20}$$

In addition we will use the sum rules

$$\begin{aligned} \sum_p q_n^p &= A_1, \qquad \sum_p q_m^p q_n^p = A_2 + B_2\delta_{mn}, \\ \sum_p q^l q_m^p q_n^p &= A_3 + B_3\left(\delta_{lm} + \delta_{mn} + \delta_{nl}\right) + C_3\delta_{lm}\delta_{mn}, \\ \sum_b q_a^b q_n^b &= Q_{1a}, \qquad \sum_b q_a^b q_m^b q_n^b = Q_{2a} + P_{2a}\delta_{mn}, \end{aligned} \tag{6.21}$$

with, in particular,

$$B_2 = 42, \qquad P_{2X} = 5\sqrt{6}. \tag{6.22}$$

6.3 Anomalies and anomaly cancellation with an anomalous U(1)

The effective supergravity theory from generic orbifold compactifications with Wilson lines is anomalous under both $U(1)_X$ and T-duality:

$$\begin{aligned} T'^i &= \frac{a_i - ib_iT^i}{ic_iT^i + d_i}, \qquad a_ib_i - c_id_i = 1, \qquad a_i, b_i, c_i, d_i \in \mathbf{Z}, \qquad i = 1,2,3, \\ \Phi'^a &= e^{-\sum_i q_i^a F^i(T^i)}\Phi^a, \qquad F^i(T^i) = \ln(ic_iT^i + d_i), \end{aligned} \tag{6.1}$$

where Φ^a is any chiral supermultiplet other than a diagonal Kähler modulus T^i, and q_i^a are its modular weights.

We are working in the covariant superspace formalism of ref. [74, 16] in which the chiral multiplets $Z^p = T^i, S, \Phi^a$, with S the dilaton superfield, are *covariantly* chiral:

$$\mathcal{D}^{\dot\beta} Z^p = 0, \tag{6.2}$$

with $\mathcal{D}_A,\ A = a, \alpha$ a fully covariant superspace derivative. In particular, under a $U(1)$ gauge transformation

$$Z'^p = g^{q_a^p} Z^p, \qquad \bar{Z}'^p = g^{-q_a^p}\bar{Z}^p, \qquad A'^a_A = A^a_A - g^{-1}\mathcal{D}_A g, \tag{6.3}$$

where g is a hermetian superfield, and A_A is the gauge potential in superspace. Gauge invariance assures that holomorphy of the superfield is maintained under (6.3). If gauge invariance is unbroken, the gauge potential A_A does not appear explicitly in the superspace Lagrangian. Instead the usual Yang-Mills superfield strength W_α is obtained as a component of the two-form superfield strength F_{AB}. One can still introduce [74, 16] a superfield superpotential V_a such that

$$W_\alpha = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha V_a, \qquad V_a' = V_a + \Lambda_a + \bar\Lambda_a, \tag{6.4}$$

but V_a never appears in the Lagrangian and the chiral superfield Λ_a is independent of g in (6.3).

However in the presence of an anomalous $U(1)$, gauge invariance is broken. It is easy to see that the UV divergences cannot be regulated by PV fields that all have $U(1)_X$ invariant masses. There is a quadratically divergent term proportional to $D_X \mathrm{Tr} T_X$, where D_X is the auxiliary field of the $U(1)_X$ supermultiplet, which must be cancelled by the analogous term from the PV sector. Invariant masses require the coupling of PV fields with equal and opposite charges that do not contribute to $(\mathrm{Tr}T_X)_{PV}$. Noninvariant masses arise from the superpotential for PV fields Φ^C:

$$W(\Phi^C, \Phi'^C) = \mu_C \Phi^C \Phi'^C, \tag{6.5}$$

with μ_C constant (in the absence of threshold corrections, as for the cases considered here). If $Q_X^C + Q_X'^C \neq 0$, holomorphy of (6.5) is not respected under (6.3) for $a = X$. For this reason we do not include the $U(1)_X$ connection in the covariant derivative (6.2). Instead of (6.3) we require

$$\Phi'^C = e^{-Q_X^C \Lambda} \Phi^C, \qquad \bar{\Phi}'^C = e^{-Q_X^C \bar{\Lambda}} \bar{\Phi}^C \tag{6.6}$$

under a $U(1)_X$ transformation, and the Kähler potential depends on $U(1)_X$-charged fields through the invariant operators $\bar{\Phi} e^{Q_X V_X} \Phi$.

It was shown in [92] that modular noninvariant masses can be restricted to a subset of PV chiral supermultiplets Φ^C with diagonal Kähler metric:

$$K(\Phi^C, \bar{\Phi}^C) = \exp[f^C(Z, \bar{Z})] |\Phi^C|^2. \tag{6.7}$$

and superpotential (6.5).

As in [185], we define a superfield

$$\mathcal{M}_C^2 = \mathcal{M}_{C'}^2 = \exp(K - f^C - f'^C) = \exp(K - 2\bar{f}^C), \qquad \bar{f}^C = \frac{1}{2}(f^C + f'^C), \tag{6.8}$$

whose lowest component $m_C^2 = \mathcal{M}_C^2|$ is the Φ^C, Φ'^C squared mass. Then the anomalous part of the one-loop corrected supergravity Lagrangian takes the form [92]

$$\mathcal{L}_{\rm anom} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_r = \int d^4\theta E \left(L_0 + L_1 + L_r\right) \equiv \int d^4\theta E \Omega, \tag{6.9}$$

where E is the superdeterminant of the supervielbein, and

$$L_0 = \frac{1}{8\pi^2} \left[\mathrm{Tr}\eta \ln \mathcal{M}^2 \Omega_0 + K \left(\Omega_{GB} + \Omega_D\right) \right], \tag{6.10}$$

with $\eta = \pm 1$ the PV signature. The operators in (6.10) are given explicitly in [92, 185], except that now

$$\Omega_0 = \Omega_{\rm YM}^0 + \Omega_0', \tag{6.11}$$

where Ω_0' contains the Gauss-Bonnet Chern-Simons superfield and operators composed of auxiliary superfields of the gravity supermultiplet, and

$$\Omega_{\rm YM}^0 = \sum_{a \neq X} \Omega_{\rm YM}^a = \Omega_{\rm YM} - \Omega_{\rm YM}^X, \tag{6.12}$$

is the Yang-Mills Chern-Simons superfield without the $U(1)_X$ term, and and $\Omega_{\rm YM}^a$ is defined by its chiral projection:

$$(\bar{\mathcal{D}}^2 - 8R)\Omega_{\rm YM}^a = W_a^\alpha W_\alpha^a. \tag{6.13}$$

Ω_r is composed of terms linear and higher order in $\ln\mathcal{M}$, and Ω_D represents a "D-term" anomaly [92, 185] that, together with a contribution to the Gauss-Bonnet term $\Omega_{\rm GB}$, arises from uncanceled total derivatives with logarithmically divergent coefficients, requiring the introduction of a field-dependent cut-off:

$$\partial_\mu\Lambda = \frac{1}{4}\partial_\mu K. \tag{6.14}$$

L_1 is defined by its variation:

$$\Delta L_1 = \frac{1}{8\pi^2}\frac{1}{192}{\rm Tr}\eta\Delta\ln\mathcal{M}^2\Omega'_L = \frac{1}{8\pi^2}\frac{1}{192}{\rm Tr}\eta H\Omega'_L + {\rm h.c.}, \tag{6.15}$$

where under (6.1) and (6.6) $\ln\mathcal{M}^2$ transforms as

$$\Delta\ln\mathcal{M}^2 = H + \bar{H}, \tag{6.16}$$

with H holomorphic. Defining

$$\begin{aligned}(\bar{\mathcal{D}}^2 - 8R)\Omega_f &= f^\alpha f_\alpha, \qquad (\bar{\mathcal{D}}^2 - 8R)\Omega_{\bar{f}} = \bar{f}^\alpha\bar{f}_\alpha, \qquad (\bar{\mathcal{D}}^2 - 8R)\Omega_{\bar{f}X} = \bar{f}^\alpha X_\alpha,\\ f_\alpha &= -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha f, \qquad \bar{f}_\alpha = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha\bar{f},\end{aligned} \tag{6.17}$$

we have

$$\begin{aligned}\Omega'_L &= 192\Omega_f - 128\Omega_{\bar{f}} - 64\Omega_{\bar{f}X},\\ \Delta L_1 &= \frac{1}{8\pi^2}{\rm Tr}\eta H\left(\Omega_f - \frac{2}{3}\Omega_{\bar{f}} - \frac{1}{3}\Omega_{\bar{f}X}\right) + {\rm h.c.}\end{aligned} \tag{6.18}$$

In the presence of an anomalous $U(1)_X$ the form of f^C is taken to be

$$\begin{aligned}f^C &= \alpha^C K(Z,\bar{Z}) + \beta^C g(T,\bar{T}) + \delta^C k(S,\bar{S}) + \sum_n q_n^C g^n(T^n,\bar{T}^n) + Q_X^C V_X,\\ \bar{f}^C &= \bar{\alpha}^C K + \bar{\beta}^C g + \bar{\delta}^C k + \sum_n \bar{q}_n^C g^n + \bar{Q}_X^C V_X,\\ H^C &= \left(1 - 2\bar{\gamma}^C\right)F(T) - 2\sum\bar{q}_n^C F^n(T^n) - 2\bar{Q}_X^C\Lambda, \qquad \bar{\gamma}^C = \bar{\alpha}^C + \bar{\beta}^C,\end{aligned} \tag{6.19}$$

where k is the dilaton kähler potential, and g is defined in (6.31) below. The traces in $\Delta\mathcal{L}_{\rm anom}$ can be evaluated using only PV fields with noninvariant masses or using the full set of PV fields, since those with invariant masses, $H^C = 0$, drop out. The contribution ΔL_0 to the anomaly is linear in the parameters $\alpha^C, \beta^C, q_n^C, Q_X^C$, and the trace of the coefficient of Ω'_0 is completely determined by the sum rules [182]

$$\begin{aligned}N' &= \sum_C \eta^C = -N - 29, \qquad N'_G = \sum_\gamma \eta_\gamma^V = -12 - N_G,\\ \sum_C \eta^C f^C &= -10K - \sum_p q_n^p g^n - \sum_a q_X^a V_X,\end{aligned} \tag{6.20}$$

that are required to assure the cancellation of quadratic and logarithmic divergences. In (6.20) the index C denotes any chiral PV field, the index γ runs over the Abelian gauge PV superfields that are needed to cancel some gravitational and dilaton-gauge couplings, and the sum over p includes all the light chiral multiplet modular weights with $q_n^S = 0,\ q_n^{T^i} = 2\delta^i_n$. All PV fields with noninvariant masses have $\delta = 0$, and most[1] with $\delta \neq 0$ have $\alpha = \beta = q_n = 0 = Q_X^C$. For the purposes of the present analysis we can largely ignore the latter. Similarly, the cancellation of linear divergences that give rise to the chiral anomaly proportional to

$$\mathrm{Im}\mathrm{Tr}\phi G\cdot\widetilde{G} \ni \mathrm{Im}\frac{1}{2}\sum_{a\neq X}\left\{F(t)C_a - \sum_p\left[F(t) - 2\sum_n q_n^p F^n(t^n) - 2q_X^p\lambda\right](T_a^p)^2\right\}F^a\cdot\tilde{F}_a \tag{6.21}$$

fixes the coefficient of Ω^0_{YM}. Here $G_{\mu\nu} \ni -iT_aF^a_{\mu\nu}$ is the field strength associated with the fermion connection, $t^i = T^i|$, $\lambda = \Lambda|$ are the lowest components of the chiral supermultiplets T^i, Λ, and a left-handed fermion f transforms as

$$f \to e^{\phi} f \tag{6.22}$$

under modular and $U(1)_X$ transformations; $\phi = -\frac{i}{2}\mathrm{Im}F$ for gauginos, and

$$\phi = \frac{i}{2}\mathrm{Im}F - \sum_n q_n^p F^n(t^n) - q_X^p\lambda \tag{6.23}$$

for chiral fermions χ^p. The compensating PV contribution

$$\mathrm{Im}\left(\mathrm{Tr}\eta\phi G\cdot\widetilde{G}\right)_{PV} \ni \mathrm{Im}\sum_C\eta^C\left(\phi^C + \phi'^C\right)(T_a^C)^2F_a\tilde{F}^a = -\mathrm{Im}\mathrm{Tr}\phi G\cdot\widetilde{G} \tag{6.24}$$

that cancels (6.21) determines the anomaly coefficient of Ω^0_{YM}, since for each pair Φ^C, Φ'^C the sum of fermion phases $\phi^C + \phi'^C = H^C$ is just the holomorphic part of the variation (6.16), (6.19) of the PV mass term $\Delta\ln\mathcal{M}_C^2$.

In the chiral formulation for the dilaton, the anomaly is cancelled by the variation of the superspace Lagrangian

$$\mathcal{L} = \int d^4\theta E\left(S + \bar{S}\right)\Omega. \tag{6.25}$$

where Ω is the real superfield introduced in (6.9). The quantum Lagrangian varies according to

$$\Delta\mathcal{L}_{\mathrm{anom}} = \int d^4\theta\left\{b\left[F(T) + \bar{F}(\bar{T})\right] - \frac{\delta_X}{2}\left(\Lambda + \bar{\Lambda}\right)\right\}\Omega, \tag{6.26}$$

[1] There is a set of chiral multiplets in the adjoint representation of the gauge group that has $f = K - k$; these get modular invariant masses though their coupling in the superpotential to a second set with $f = k$. These cancel renormalizable gauge interactions and gauge-gravity interactions, respectively. Together with a third set, that has $f = 0$ and contributes to the anomaly, they cancel the Yang-Mills contribution to the beta-function.

so the full Lagrangian is invariant provided

$$\Delta S = -bF(T) + \frac{\delta_X}{2}\Lambda, \qquad F = \sum_i F^i. \tag{6.27}$$

However the classical Kähler potential for the dilaton is no longer invariant and must be modified:

$$k_{\rm class}(S,\bar{S}) = -\ln(S+\bar{S}) \to k(S,\bar{S}) = -\ln(S+\bar{S}+V_{GS}), \tag{6.28}$$

where V_{GS} is a real function of V_X and of the chiral supermultiplets; it transforms under (6.1) and (6.4), (6.6) as

$$\Delta V_{GS} = b\left(F+\bar{F}\right) - \frac{\delta_X}{2}\left(\Lambda+\bar{\Lambda}\right). \tag{6.29}$$

A simple solution consistent with string calculation results [38, 50, 150] is

$$V_{GS} = bg(T,\bar{T}) - \frac{\delta_X}{2}V_X, \tag{6.30}$$

where

$$g(T,\bar{T}) = \sum_i g^i(T^i,\bar{T}^i), \qquad g^i = -\ln(T^i+\bar{T}^i) \tag{6.31}$$

is the Kähler potential for the moduli. The modification (6.28) is the 4d Green-Schwarz (GS) term in the chiral formulation. As discussed in [185], the 4d GS mechanism is more simply formulated in the linear multiplet formalism [74, 16]for the dilaton. In this case the linear dilaton superfield L remains invariant, its Kähler potential is unchanged, and instead one adds a term to the Lagrangian:

$$\mathcal{L}_{GS} \;=\; -\int d^4\theta E L V_{GS}, \qquad \Delta\mathcal{L}_{GS} = -\Delta\mathcal{L}_{\rm anom} \tag{6.32}$$

Only terms in the anomaly that are linear in the combination $\tilde{H}$, where

$$\tilde{H} = bF(T) - \frac{\delta_X}{2}\Lambda, \tag{6.33}$$

can be canceled by the Green-Schwarz term. The values of b and δ_X are fixed by the conditions (6.20), (6.24) for the cancellation of divergences, together with the universality conditions (6.19), that hold for all $\mathbb{Z}_3$ and $\mathbb{Z}_7$ orbifold compactifications.

In contrast to $\mathcal{L}_0$, the contributions to the anomaly from $\mathcal{L}_1$ and $\mathcal{L}_r$ are nonlinear in the parameters α, β, q_n, Q_X, and depend on the details of the PV sector. In particular $\mathcal{L}_r$ has no terms linear in $\ln\mathcal{M}$ and must vanish. To insure that the anomaly coefficient depends on the T-moduli only through $F(T)$ we impose [185]

$$\bar{q}_n^C = 0 \tag{6.34}$$

for (almost[2]) all PV fields with noninvariant masses.

[2]The exception is for some PV fields, introduced in Appendix C, needed to cancel divergences from light fields with Abelian gauge charges.

6.4 The anomaly and cancellation of UV divergences in the FIQS model

The full set of conditions for cancellation of the divergences and for obtaining an anomaly linear in $\tilde{H}$, Eq. (6.33), that matches the string result [379] is given in the Appendix C. In this section we outline some features of the case of $\mathbb{Z}_3$ with an anomalous $U(1)_X$. We will be primarily concerned with the contribution of ΔL_1, Eq. (6.18), to the anomaly. This expression is nonlinear in the parameters q_n^C, Q_X^C of the PV fields, and therefore model dependent, as noted above. This was illustrated in [185] where it was shown that cancellation of the modular anomaly requires (6.34). However, the contribution cubic in Q_X^C is model independent. It is given by

$$\Delta L_1(Q_X^3) = -\frac{2(\Lambda+\bar{\Lambda})}{24\pi^2}\mathrm{Tr}\eta\bar{Q}_X\left(3Q_X^2 - 2\bar{Q}_X^2\right)\Omega_{YM}^X = -\frac{2(\Lambda+\bar{\Lambda})}{24\pi^2}\mathrm{Tr}\eta Q_X^3\Omega_{YM}^X, \tag{6.1}$$

where the sum is over all PV fields, and we used the definition (3.6), (6.19) of $\bar{Q}^X$ and the fact that

$$\sum_C \eta^C (Q_X^C)^p (Q_X'^C)^{p'} = \sum_C \eta^C (Q_X^C)^{p'} (Q_X'^C)^p, \tag{6.2}$$

for any powers p, p'. Cancellation of the term in $\mathrm{Tr}\phi G \cdot \widetilde{G}$ that is cubic in Q_X^3 requires

$$-\frac{2(\Lambda+\bar{\Lambda})}{24\pi^2}\mathrm{Tr}\left(\eta Q_X^3\right)\Omega_{YM}^X = \frac{2(\Lambda+\bar{\Lambda})}{24\pi^2}\mathrm{Tr}\left(q_X^3\right)\Omega_{YM}^X = -\frac{\delta_X}{2}(\Lambda+\bar{\Lambda})\Omega_{YM}^X, \tag{6.3}$$

from (6.19), so the anomaly (6.1) is consistent with the requirement for anomaly cancellation.

In contrast, anomaly terms quadratic in Q_X^2 are model dependent. For example, in [92] it was assumed that $\bar{f}^C = f^C$ for all PV fields with noninvariant masses, giving a contribution

$$\Delta L_1(FQ_X^2) = \frac{F+\bar{F}}{24\pi^2}\mathrm{Tr}\eta\left(1-2\bar{\gamma}\right)\left(3Q_X^2 - 2\bar{Q}_X^2\right)\Omega_{YM}^X \tag{6.4}$$

$$= \frac{F+\bar{F}}{24\pi^2}\mathrm{Tr}\eta\left(1-2\bar{\gamma}\right)Q_X^2\Omega_{YM}^X = \frac{F+\bar{F}}{24\pi^2}\mathrm{Tr}q_X^2\Omega_{YM}^X = \frac{b}{3}(F+\bar{F})\Omega_{YM}^X, \tag{6.5}$$

from (6.21) and (6.24) with $a = X$, and (6.19). Here we instead assume, in addition to (6.34), that $\bar{Q}_X^C = 0$ if $1-\bar{\gamma} \neq 0$, that is PV masses can be noninvariant under either T-duality or $U(1)_X$, but not both. In this case the last term in (6.4) drops out and we recover a factor three, in agreement with the requirement for anomaly cancellation.

The full set of PV fields sufficient to regulate light field couplings is described in Section 3 of [92]. These include a set $\dot{Z}^P = \dot{Z}^I, \dot{Z}^A$, with negative signature, $\eta^{\dot{Z}} = -1$, that regulates most of the couplings, including all renormalizable couplings, of the light chiral supermultiplets $Z^p = T^i, \Phi^a$. The $\dot{Z}$ get invariant masses through a superpotential coupling to PV fields $\dot{Y}_P$ with the same signature, opposite gauge charges and the inverse Kähler metric:

$$(T_a)_{\dot{Y}} = -(T_a^T)_{\dot{Z}} = -(T_a^T)_Z. \tag{6.6}$$

It remains to cancel the divergences introduced by the fields $\dot{Y}$. To this end we take the following set:

$$\begin{aligned} \psi^{Pn}: &\quad f^{Pn} = \alpha^P_\psi K + \beta^P_\psi g + q^P_\psi g^n + Q^P_\psi V_X, \qquad \alpha^P_\psi + \beta^P_\psi = \gamma^P_\psi, \qquad \bar{q}^P_\psi = 0, \\ T^P: &\quad f^P_T = \alpha^P_T K + \beta^P_T g + Q^P_T V_X, \qquad \alpha^P_T + \beta^P_T = \gamma^P_T, \\ \phi^C: &\quad f^{\phi^C} = \alpha^C K. \end{aligned} \tag{6.7}$$

In the solution to the constraints given in Appendix C, the ψ^C and T^C are further subdivided, together with additional fields, into sets $S_a,\ a = 1, \ldots, 12$, some of which are charged under the nonanomalous gauge group. The ϕ^C regulate certain gravity supermultiplet loops and nonrenormalizable coupling of chiral multiplets. These must be included together with the other PV fields introduced above in implementing the sum rules (6.20). Their contributions will be included in all the finiteness and anomaly conditions that involve only the parameters α in (6.7); otherwise they play no role in the analysis below. In the expressions given in the remainder of this section, we drop terms that contain only X_α or $X_{\mu\nu}$ since their contributions are included in the sums (6.20) and the additional sum rule [182]

$$\sum_C \eta^C \alpha_C^2 = -4. \tag{6.8}$$

In [185] we also introduced pairs Φ^P, Φ'^P with modular invariant masses that did not contribute to the anomaly, but played an important role in canceling certain divergences. However, because the $\mathbb{Z}_3$ sum rules (6.21) are much simpler than the analogous sum rules for the $\mathbb{Z}_7$ case studied in [185], here we need only the set in (6.7).

The quadratic and logarithmic divergences we are concerned with here involve the superfield strengths $-i(T_a)W^a_\alpha$,

$$\Gamma^C_{D\alpha} = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha Z^p \Gamma^C_{Dp}, \tag{6.9}$$

and

$$X_\alpha = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha K, \tag{6.10}$$

associated with the Yang-Mills, reparameterization and Kähler connections, $i(T_a)^C_D A_\mu,\ \partial_\mu Z^p \Gamma^C_{pD}$ and $\delta^C_D \Gamma_\mu$, respectively, where

$$\Gamma_\mu = \frac{i}{4}\left(\mathcal{D}_\mu z^i K_i - \mathcal{D}_\mu \bar{z}^{\bar{m}} K_{\bar{m}}\right). \tag{6.11}$$

Cancellation of quadratic divergences requires

$$\mathrm{Tr}\eta\Gamma_\alpha = \mathrm{Tr}\eta T_X = 0, \tag{6.12}$$

and cancellation of logarithmic divergences requires

$$\mathrm{Tr}\eta\Gamma_\alpha\Gamma_\beta = \mathrm{Tr}\eta\Gamma_\alpha T^a = \mathrm{Tr}\eta(T^a)^2 = 0, \tag{6.13}$$

where $\eta = +1$ for light fields. Cancellation of all contributions linear and quadratic in X_α is assured by the conditions in (6.20) and (6.8). The Yang-Mills contribution to the term quadratic in W_α is canceled by chiral fields in the adjoint (see footnote on page 73) that we need not consider here. Finally, cancellation of linear divergences requires cancellation of the imaginary part of

$$\mathrm{Tr}\eta X_\chi = \mathrm{Tr}\eta\phi G\cdot\widetilde{G}, \qquad \widetilde{G}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}, \tag{6.14}$$

where $G_{\mu\nu}$ is the field strength associated with the fermion connection;[3] for left-handed fermions:

$$G_{\mu\nu} = -\Gamma^C_{D\mu\nu} + iF^a_{\mu\nu}(T_a)^C_D + \frac{1}{2}X_{\mu\nu}\delta^C_D, \tag{6.15}$$

where

$$\begin{aligned} X_{\mu\nu} &= \left(\mathcal{D}_\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}} - \mathcal{D}_\nu z^i\mathcal{D}_\mu\bar{z}^{\bar{m}}\right)K_{i\bar{m}} - iF^a_{\mu\nu}(T_a z^i)K_i \\ &= 2i\left(\partial_\mu\Gamma_\nu - \partial_\nu\Gamma_\mu\right), \qquad i = p, s, \end{aligned} \tag{6.16}$$

is the field strength associated with the Kähler connection (6.9). For a generic PV superfield Φ^C with diagonal metric, its fermion component χ^C transforms under (6.1) and (6.6) as

$$\chi'^C = e^{\phi^C}\chi^C, \qquad \phi^C = \left(\frac{1}{2} - \alpha^C - \beta^C\right)F - \sum_i F^i(t^i)q^C_i - \lambda Q_X. \tag{6.17}$$

In evaluating (6.14) we will use the fact that the expression[4]

$$\epsilon^{\mu\nu\rho\sigma}g^i_{\mu\nu}g^i_{\rho\sigma} = 0, \tag{6.18}$$

vanishes identically, and the expressions

$$\begin{aligned} X^{ij} &= \epsilon^{\mu\nu\rho\sigma}\mathrm{Im}F^i g^i_{\mu\nu}g^{j\neq i}_{\rho\sigma} = 4\epsilon^{\mu\nu\rho\sigma}\mathrm{Im}F^i\partial_\mu g^i_\nu\partial_\rho g^j_\sigma = 4\partial_\rho\left(\epsilon^{\mu\nu\rho\sigma}\mathrm{Im}F^i\partial_\mu g^i_\nu g^j_\sigma\right), \\ X^i &= \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Im}F^i g^i_{\mu\nu}X_{\rho\sigma} = 4i\partial_\rho\left(\epsilon^{\mu\nu\rho\sigma}\mathrm{Im}F^i\partial_\mu g^i_\nu\Gamma_\sigma\right), \\ X^{ia} &= \epsilon^{\mu\nu\rho\sigma}\mathrm{Im}F^i g^i_{\mu\nu}F^a_{\rho\sigma} = 4\partial_\rho\left(\epsilon^{\mu\nu\rho\sigma}\mathrm{Im}F^i\partial_\mu g^i_\nu A^a_\sigma\right), \end{aligned} \tag{6.19}$$

are total derivatives, where A^a_μ is an Abelian gauge field, and

$$g^i = -\ln(t^i + \bar{t}^{\bar{\imath}}), \qquad g^i_\mu = -\frac{\partial_\mu t^i - \partial_\mu\bar{t}^{\bar{\imath}}}{t^i + \bar{t}^{\bar{\imath}}}, \qquad g^i_{\mu\nu} = \partial_\mu g^i_\nu - \partial_\nu g^i_\mu. \tag{6.20}$$

[3]Here we neglect the spin connection whose contribution was discussed in [185].

[4]It was noted in [185] that the expression (6.18), which is in fact the T-dependent part of the chiral anomaly found in [379], vanishes. The authors of [379] attribute [364] this to their approximation that neglects higher order corrections. However if these corrections take the form $g^i(T^i,\bar{T}^i) \to g^i(T^i,\bar{T}^i) + \Delta^i(T^i,\bar{T}^i)$, our results our unchanged. Note that the functional form of Δ^i is severely restricted by the fact that it has to be invariant under T-duality.

The full Kähler potential for $\dot{Y}$, with no anomalous $U(1)_X$, is given in [92, 185]; here it takes the form

$$\begin{aligned} K(\dot{Y}) &= e^{\dot{G}}\left(\sum_A e^{-g^a-q^a V_X}|\dot{Y}_A|^2 + \sum_I e^{-2g^i}|\dot{Y}_I|^2 + \sum_N |\dot{Y}_N|^2\right) + \ldots, \\ g^a &= \sum_n q^a_n g^n, \qquad \dot{G} = \dot{\alpha}K + \dot{\beta}g, \qquad \dot{\alpha} + \dot{\beta} = 1, \end{aligned} \tag{6.21}$$

where $\dot{Y}_{N=1,2,3}$ (and their counterparts $\dot{Z}^N$) are gauge singlet PV fields needed [182] to make the Kähler potential and superpotential terms for $\dot{Z}, \dot{Y}$ fully invariant, and the ellipsis represents terms that make no contribution to the expressions given below. Using the sum rules in (6.21) and (6.20) we obtain:

$$\begin{aligned} \mathrm{Tr}\dot{\eta}\Gamma^{\dot{Y}}_\alpha &= -\left[(N+2)\dot{\beta} - A_1\right] g_\alpha, \qquad \mathrm{Tr}\dot{\eta}T^{\dot{Y}}_X = \mathrm{Tr}T_X, \\ \mathrm{Tr}\dot{\eta}\Gamma^{\dot{Y}}_\alpha\Gamma^{\dot{Y}}_\beta &= -2\dot{\alpha}\left[\dot{\beta}(N+2) - A_1\right] X_\alpha g_\beta - \left[\dot{\beta}^2(N+2) - \dot{\beta}A_1 + A_2\right] g_\beta g_\alpha \\ &\quad -B_2 \sum_n g^n_\alpha g^n_\beta \\ \mathrm{Tr}\dot{\eta}\Gamma^{\dot{Y}}_\alpha T_a &= \delta_{aX}\mathrm{Tr}T^{\dot{Y}}_X \dot{G}_\alpha - Q_{1a}g_\alpha, \qquad \dot{G}_\alpha = \dot{\alpha}X_\alpha + \dot{\beta}g_\alpha. \end{aligned} \tag{6.22}$$

Using (6.19) and (6.21), the part of $X^{\dot{Y}}$ that is independent of gauge charges takes the form:

$$\begin{aligned} X^{\dot{Y}}_\chi \ni \ & \frac{1}{2}\left[(N+2) - 2A_1\right] F\dot{G}\cdot\tilde{\dot{G}} - (A_1 - 2A_2)\, F\dot{G}\cdot\tilde{g} - A_3 F g\cdot\tilde{g} \\ & +\text{total derivative}, \qquad \dot{G}_{\mu\nu} = \dot{\alpha}X_{\mu\nu} + \dot{\beta}g_{\mu\nu}. \end{aligned} \tag{6.23}$$

The modular weights for the ψ satisfy

$$\begin{aligned} \sum_{m,n} g^n q^{P_m}_n &= g q^P_\psi, \qquad \sum_P \eta^P_\psi q^{P_k}_l q^{P_k}_n q^{P_k}_n = 0, \\ \sum_{l,m,n} g^m g^n q^{P_l}_m q^{P_l}_n &= (q^P_\psi)^2 \sum_n g^n g^n. \end{aligned} \tag{6.24}$$

Like $X^{\dot{Y}}_\chi$, X^ψ_χ depends only on $F, g_{\mu\nu}$ and $X_{\mu\nu}$, and (6.22) and (6.23) can be cancelled by some combination of the fields in (6.7), with the condition

$$\sum_P \eta^P_\psi (q^P_\psi)^2 = B_2. \tag{6.25}$$

The pure T-moduli anomaly is given by

$$\Delta L_1(Fg^2) = \frac{F}{8\pi^2}\mathrm{Tr}\eta_\psi\,(1 - 2\bar{\gamma}_\psi)\, q^2_\psi \Omega_g, \qquad (\bar{\mathcal{D}}^2 - 8R)\Omega_g = \sum_n g^\alpha_n g^n_\alpha. \tag{6.26}$$

Consistency with string results [257] requires

$$\mathrm{Tr}\eta_\psi\left(1-2\bar{\gamma}_\psi\right)q_\psi^2=-8\pi^2 b \tag{6.27}$$

Finally, we require

$$\Delta L_1(Q_X g^2)=-\frac{2\Lambda}{8\pi^2}\mathrm{Tr}\eta\bar{Q}_X\Omega_f=\frac{1}{2}\Lambda\delta_X\Omega_g. \tag{6.28}$$

Using (6.24), the condition (6.28) requires

$$\sum_P \eta_\psi^P\bar{Q}_\psi^P(q_\psi^P)^2=-4\pi^2\delta_X. \tag{6.29}$$

All other other contributions to ΔL_1 are required to vanish.

We conclude this section by noting that cancellation of divergences linear in the $U(1)_a$ field strengths is much simpler than for the $\mathbb{Z}_7$ case considered in [185], as outlined below.

The gauge charges for the FIQS ([252, 171]) model are listed[5] in Appendix C. The universality of the anomaly term quadratic in Yang-Mills fields strengths is guaranteed by the universality condition (6.19), as discussed in Section 6.3. Since gauge transformations commute with modular transformations, a set of chiral multiplets Φ^b that transform according to a nontrivial irreducible representation R of a nonabelian gauge group factor $\mathcal{G}_a$ have the same modular weights q_n^R such that

$$\sum_{b\in R} q_n^b(T_a)_b^b=q_n^R(\mathrm{Tr}T_a)_R=0. \tag{6.30}$$

Therefore terms linear in Yang-Mills field strengths occur only for Abelian gauge group factors. We need to cancel the $\dot{Y}$-loop contribution to logarithmic divergences

$$\left(\mathrm{Tr}\eta\sum_n q_n g_\alpha^n T_a\right)_{\dot{Y}}=-\sum_{b,n}q_n^b Q_a^b g_\alpha^n=-Q_{1a}g_\alpha, \tag{6.31}$$

and, dropping terms proportional to the last expression in (6.19), the relevant $\dot{Y}$ contributions to linear divergences:

$$\begin{aligned} X_\chi^{\dot{Y}} \ni\ & \sum_{a,b,n} Q_a^b\tilde{F}^a\cdot\left[g^n q_n^b\left(F-2\sum_m q_m^b F^m\right)+2q_n^b F^n\left(\dot{G}-\frac{1}{2}X\right)\right] \\ =\ & \sum_a \tilde{F}^a\cdot\left\{\left[g\left(1+2\dot{\beta}\right)+X\left(2\dot{\alpha}-1\right)\right]Q_{1a}F-2\sum_n g^n F^n Q_{2a}\right\}, \end{aligned} \tag{6.32}$$

[5]We have made some corrections to the $U(1)_a$, charges given in [252, 171].

where we used (6.21). The last term in (6.32) is cancelled by

$$X_{\chi}^{\psi} \ni -2 \sum_{a,P,l,m,n} \eta_{\psi}^{P} Q_{a}^{P} q_{m}^{P_l} q_{n}^{P_l} F^{m} \tilde{F}^{a} \cdot g^{n} = -2 \sum_{a,P} \eta_{\psi}^{P} Q_{a}^{P} (q^{P})^{2} \tilde{F}^{a} \cdot \sum_{n} g^{n} F^{n}, \quad (6.33)$$

provided

$$\sum_{P} \eta_{\psi}^{P} Q_{a}^{P} (q^{P})^{2} = -Q_{2a}. \quad (6.34)$$

The remaining terms in (6.32), as well as (6.31) can be cancelled by a combination of the fields in (6.7). For $a = X$ there are additional terms proportional to $(\mathrm{Tr}\eta T_X)_{PV} = -\mathrm{Tr}T_X$.

6.5 The final anomaly in the FIQS model

In Appendix C we show that is possible to cancel all the ultraviolet divergences from the $\dot{Y}$ fields with a choice of the set (6.7) such that the fields with noninvariant masses have the properties

$$\mathrm{Tr}\eta(\ln \mathcal{M})^{n>1} = \Delta \mathrm{Tr}\eta(\ln \mathcal{M})^{n>1} = \mathrm{Tr}\eta(\Delta \ln \mathcal{M})(\bar{f}_{\alpha})^{n>0} = 0. \quad (6.1)$$

Then, including the results of [185], the anomaly due to the variation of (6.9) takes the form

$$\delta \mathcal{L}_{\mathrm{anom}} = \int d^4\theta E \left(bF - \frac{1}{2} \delta_X \Lambda \right) \Omega + \int d^4\theta E b F \Omega', \quad (6.2)$$

where

$$\begin{aligned} \Omega &= \Omega_{\mathrm{YM}} - \Omega_{\mathrm{GB}} + \Omega_{g}, \\ \Omega' &= -\frac{b_{\mathrm{spin}}}{48b} \left(4G_{\dot{\beta}\alpha} G^{\alpha\dot{\beta}} - 16R\bar{R} + \mathcal{D}^2 R + \bar{\mathcal{D}}^2 \bar{R} \right) - \frac{1}{8\pi^2 b} \Omega_D, \end{aligned} \quad (6.3)$$

where Ω_g is defined in (6.26), and b_{spin} governs the contributions from PV masses, as opposed to those arising from uncancelled divergences:

$$8\pi^2 b_{\mathrm{spin}} = 8\pi^2 b + 1, \quad (6.4)$$

with $8\pi^2 b = 6$ in the FIQS model. In the absence of an anomamous $U(1)$, $\Lambda = 0$, the anomaly can be cancelled by the four dimensional GS mechanism as described in [185]. However with $\Lambda \neq 0$, the anomaly as written in (6.3) is no longer universal and cannot be cancelled by the GS term alone. However all of the "D-terms", in other words the full expression Ω', can be removed [90] by adding counterterms to the Lagrangian, giving a universal anomaly which can now be cancelled by the GS term.[6]

The results for the Gauss-Bonnet and Yang-Mills terms are well-established [38] and result from the universality conditions (6.19).

[6] The elimination of Ω_D further obviates the need for a modification of the linear-chiral duality transformation, a possibility condsidered in Appendix B of [185] and Appendix E of [92].

6.6 Conclusion

We have shown that a suitable choice of Pauli-Villars regulator fields allows for a full cancellation of the chiral and conformal anomalies associated, respectively, with the linear and logarithmic divergences in the effective supergravity theory from a $\mathbb{Z}_3$ orbifold compactification with Wilson lines and an anomalous $U(1)$.

A future work [257] will compare this result with that obtained directly from string theory.

Part III

Cosmology & The String Swampland

Overview of Part III

In the remainder of this dissertation, we apply concepts from the string swampland to inflation and dark energy. Dark energy is perhaps the most mysterious known aspect of our Universe. Dark matter is also mysterious, but we are fairly certain it consists of unknown particles. However, we have a much more tenuous grasp on the characteristics dark energy. Observations suggest that dark energy can be modeled as a cosmological constant in Einstein's equations. If we invoke a cosmological constant explanation for dark energy, then the mystery is translated into the small scale associated with the phenomena, $\sim \mathcal{O}(\text{meV})$. Another possibility is that dark energy is not constant but instead arises from the dynamics of a scalar field known as quintessence.

As we will describe below, there are some theoretical arguments from string theory that dark energy should be quintessence as opposed to a cosmological constant. In this final chapter, we describe experimental prospects of differentiating between quintessence and a cosmological constant and how the swampland conjectures relate the phenomena of inflation and dark energy.

Chapter 7

What does inflation say about dark energy given the swampland conjectures?

7.1 Introduction

The discovery of the accelerating expansion of the Universe [358, 370] was a huge surprise to the community. Because gravity only *pulls*, it should put a brake on the expansion of the Universe after the Big Bang and hence the expansion should decelerate. Acceleration implies there is a substance in the Universe that *pushes* the expansion. It was dubbed *dark energy*. The most discussed candidate for dark energy is the cosmological constant Λ, a finite energy density of the vacuum, due to the simple way it can be implemented into cosmological models based on general relativity. However, despite being consistent with data [19], the 120 orders of magnitude difference between the observed vacuum energy density ($\rho \approx (\mathrm{meV})^4$) and the naïve theoretical expectation ($\rho \approx M_{\mathrm{Pl}}^4$) still remains the most challenging problem in modern physics [412].

Since dark energy and the cosmological constant problem inevitably involve quantum gravity, string theory, as a theory of quantum gravity, should address these topics. The attempts to construct de Sitter solutions (spacetime solutions to general relativity with a positive Λ) in string theory [83, 196, 270] have lead to the notion of the string landscape. The landscape consists of an enormous number of vacua, each described by different low-energy effective field theories (EFTs) of different fields and parameters. String theory therefore supports the anthropic argument [410], namely that the value of the observed dark energy density is what it is because otherwise human civilization could not exist. If we really live in a (meta-)stable vacuum in the string landscape where a constant vacuum energy explains dark energy, then there is no point in measuring the dark energy equation of state parameter $w = p/\rho$, where p and ρ are the pressure and energy density of the dark energy, respectively.

String theory seems to lead to many possible low-energy EFTs, so conversely one can ask what criteria a given low-energy EFT should satisfy in order to be contained in the string landscape. For the last decade, several criteria of this kind, dubbed *swampland conjectures*, have been proposed [400, 348, 41]. These can have important cosmological implications. For instance, one of the relatively well-established conjectures is the *distance swampland conjecture* [348, 351, 61, 293, 401, 80, 352, 314, 229, 117, 211, 231, 81, 304] which implies that scalar fields in a low-energy EFT of a consistent theory of quantum gravity cannot have field excursions much larger than the Planck scale since otherwise an infinite tower of states becomes exponentially light and the validity of the EFT breaks down. In other words, one has the constraint[1]

$$\Delta\phi \lesssim \alpha M_{\rm Pl}, \qquad \alpha \approx O(1). \tag{7.1}$$

In the context of inflation, field excursions are related to the tensor-to-scalar ratio r by the Lyth bound [130],

$$\frac{\Delta\phi}{M_{\rm Pl}} \simeq \sqrt{\frac{r}{8}}\mathcal{N} \tag{7.2}$$

where $\mathcal{N}$ is the number of e-folds of inflationary expansion. Clearly the distance conjecture, Eq. (7.1), limits the possibility of measuring tensor modes and hence primordial B-modes in the cosmic microwave background (CMB). Naively, with $\mathcal{N} \gtrsim 50$, we find $r \lesssim 0.003$, which is on the edge of observability for future experiments [10, 393].

The attempts to construct de Sitter solutions or inflationary models in string theory [270, 271, 390, 54, 59, 415, 60, 155, 375, 77, 119, 116] have sparked discussions on various issues with such constructions, as well as no-go theorems [319, 396, 236, 132, 326, 105, 104, 98, 418, 387, 206, 194, 65, 78, 64, 136, 300, 365, 140, 268, 267, 35, 330, 382, 33, 137]. Motivated by the obstructions encountered in various attempts, the *de Sitter swampland conjecture* was proposed [346], which states that the scalar potential of a low-energy limit of quantum gravity must satisfy

$$M_{\rm Pl}|\nabla V| \geq c\,V, \qquad c \approx O(1) > 0 \tag{7.3}$$

where ∇ denotes the gradient with respect to the field space, and the norm of the gradient is defined by the metric on field space. Whether the conjecture holds true is still an open debate [275, 169, 69, 70, 228, 273, 360, 274, 9, 118, 141, 373, 32, 34, 129, 27, 276, 269, 329, 66, 193]. Yet, even before the debate is settled, it is interesting and important to investigate both its consequences in cosmology and potential modifications or extensions [112, 233, 232, 322, 340, 290, 309, 63, 324, 128, 144, 147, 284, 191, 15, 138, 407, 85, 218, 84, 148, 160, 307, 215, 282, 336, 49, 139, 408, 177, 230, 347, 192, 353, 76, 377, 306]. The primary implication of this condition is that the observed positive energy density of our Universe should correspond to the potential of a rolling quintessence field rather than a positive Λ [24]. The fact that one can easily embed any quintessence model into supergravity [86, 111] in a rather simple fashion, despite the difficulty that supersymmetry breaking generically spoils the flatness of

[1]We note that α can be greater than unity. We restrict ourselves to a more conservative approach and keep $\alpha \sim 1$, but values as large as $2\log(M_{pl}/H_{inf})$ could be permitted.

the quintessence potential, is also encouraging. This raises the hope that $w \neq -1$ might be detected.

The de Sitter conjecture forbids (meta-)stable vacua with positive energy density, so it is not surprising that the inflationary paradigm has apparent conflicts with the conjecture and one may call for a paradigm shift. Nonetheless, one can also adopt a conservative approach and regard the conjecture as a parametric constraint where the inequality holds but the number c may not be strictly $\mathcal{O}(1)$ [147]. From this perspective, constraints on inflation can then be used to constrain c.

However, if we follow this route, the optimism that one can observe $w \neq -1$ is greatly diminished. To see this, recall that in single-field slow-roll inflation, the slow-roll parameters of the potential are defined as

$$\epsilon_V \equiv \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta_V \equiv M_{\rm Pl}^2 \frac{V''}{V}, \tag{7.4}$$

where the primes denote derivatives with respect to the inflaton. The distance conjecture limits the inflaton field excursion $\Delta\phi \approx \sqrt{2\epsilon_V}\,\mathcal{N} \lesssim \mathcal{O}(1)$ and therefore the necessary number of e-folds $\mathcal{N} \approx 50$ forces $c \lesssim \sqrt{2\epsilon_V} \lesssim \mathcal{N}^{-1} \sim 0.02$. On the other hand, the number c in Eq. (7.3) is meant to be *universal* in a given EFT. Therefore, the current accelerating expansion must involve a quintessence field Q whose potential V_Q must satisfy

$$1 + w = \frac{2(V_Q')^2}{(V_Q')^2 + 6V_Q^2} > \frac{2c^2}{6+c^2} \equiv \Delta \gtrsim 1.33 \times 10^{-4}. \tag{7.5}$$

Although this does not exclude observable quintessence, given the fact that so far almost all observations are consistent with a cosmological constant, such a small lower bound on possible deviation of w from -1 makes it questionable if it is worthwhile to push the sensitivity of the observations further. We may never know whether the Universe is de Sitter or quintessence.

However, the original de Sitter conjecture, Eq. (7.3), was so strong that even the Higgs potential was in tension with it [144]. The conjecture was also in tension with the well-understood supersymmetric AdS solutions [129]. Recently the *refined de Sitter swampland conjecture* was proposed [191, 349], which states that the scalar potential of a low-energy theory that can be consistently coupled to quantum gravity should satisfy *either*

$$M_{\rm Pl}|\nabla V| \geq c\,V, \qquad c \approx O(1) > 0, \tag{7.6}$$

or

$$M_{\rm Pl}^2 \min(\nabla_i \nabla_j V) \leq -c' V, \qquad c' \approx O(1) > 0, \tag{7.7}$$

where min(...) denotes the minimum eigenvalue of the Hessian $\nabla_i\nabla_j V$ in an orthonormal frame of the scalar field space. With this refinement, the aforementioned conflicts with the Higgs potential and the SUSY AdS solutions are resolved. The refined conjecture also raises new possibilities for inflation. In particular, one can evade the strict bound on c arising from the distance conjecture by having the scalar potential satisfy the second condition Eq. (7.7) of the new conjecture during part (or all) of inflation. As such, one may regain the hope that observable time-varying dark energy with $w \neq -1$ can be obtained. See also [22] for a recent discussion on w in consideration of the refined dS conjecture.

7.2 Single-Field Slow-Roll Inflation Models

Due to the above tension between the de Sitter conjecture and the requirements of inflation, we assume that the inflaton potential switches from one de Sitter condition to another as the inflaton rolls, an idea also utilized in [177]. To be specific, we take the following step-function approach to keep the discussion general and simple: we apply the first condition, Eq. (7.6), for the initial $\mathcal{N}_1$ e-folds and apply the second condition, Eq. (7.7), for the remaining $\mathcal{N}_2 = \mathcal{N}_{tot} - \mathcal{N}_1$ e-folds. In our analysis we set $\mathcal{N}_{tot} = 50$. We assume ϵ_V and η_V are approximately constant for each interval so that we have

$$\sqrt{2\epsilon_V^{(1)}} \geq c \text{ and } \eta_V^{(2)} \leq -c'. \tag{7.8}$$

Additionally, Eq. (7.1) requires that

$$\sqrt{2\epsilon_V^{(1)}}\mathcal{N}_1 + \sqrt{2\epsilon_V^{(2)}}\mathcal{N}_2 \leq \alpha \sim O(1). \tag{7.9}$$

To maximize c, we assume $\epsilon_V^{(2)} < 10^{-4}$ so that the contribution of the second era to Eq. (7.1) is negligible. Combining Eq. (7.8) and Eq. (7.9), we have

$$c < \frac{\alpha - \sqrt{2\epsilon_V^{(2)}}\mathcal{N}_2}{\mathcal{N}_1} . \tag{7.10}$$

We can also obtain a bound for c' from the spectral tilt $n_s = 1 - 2\epsilon - \eta$, where the Hubble slow-roll parameters are

$$\epsilon = -\frac{\dot{H}}{H^2}, \qquad \eta = \frac{\dot{\epsilon}}{H\epsilon} . \tag{7.11}$$

For single-field inflation models, these are related to the slow-roll parameters of the potential as $\epsilon_V = \epsilon$ and $\eta_V = 2\epsilon - \frac{1}{2}\eta$. Therefore, we can constrain η_V and hence the second parameter of the refined de Sitter conjecture as

$$c' < \frac{1}{2}\left(1 - n_s(k) - 6\epsilon_V^{(2)}\right) , \tag{7.12}$$

where we are allowing for a k-dependent spectral tilt. Since we assume $\epsilon_V^{(2)}$ is small, our bounds simplify to

$$(c', c) < \left(\frac{1 - n_s(k)}{2}, \frac{\alpha}{\mathcal{N}_1}\right) . \tag{7.13}$$

Eq. (7.13) is valid until $\mathcal{N}_1 = \mathcal{N}_{tot}$, at which point the derivation on the bound of c' above no longer applies, and the only constraint one finds is that $c < \alpha/\mathcal{N}_{tot}$. To proceed, we utilize the Planck analysis based on TT, TE, EE, lowE, lensing and BAO [19], which gives

$$dn_s/d\ln k = -0.0041 \pm 0.0067, \tag{7.14}$$
$$n_s = 0.9659 \pm 0.0040, \tag{7.15}$$

at $k_* = 0.05\text{Mpc}^{-1}$. We add errors in quadrature, ignoring correlations, and use

$$n_s(k) = 0.9659 - 0.0041 \ln \frac{k}{k_*} \pm \sqrt{(0.0040)^2 + \left(0.0067 \ln \frac{k}{k_*}\right)^2} \,. \tag{7.16}$$

A smaller n_s allows for larger c' in Eq. (7.13), so we take the 1σ allowed lower end in order to place our bounds. The weak correlation between n_s and $dn_s/d\ln k$ we see in Fig. 26 of [19] actually works in our favor and ignoring correlation is therefore the more conservative approach (*i.e.*, gives a smaller allowed range) [2]. Using the simple relationship $\mathcal{N}_1 = \ln(k/a_0 H_0)$, where a_0 is the present scale factor and H_0 is the present Hubble scale, we can constrain the swampland parameters in single-field inflation as shown in Fig. 7.1. The current CMB constraints on the spectral index and its running are limited to $\mathcal{N}_1 \lesssim 10$. This range is denoted by the solid lines in Fig. 7.1. Beyond this there are no strong observational constraints and we extend our analysis by extrapolating Eq. (7.16) to $\mathcal{N}_1 \geq 10$ shown by the dashed lines in Fig. 7.1. The unshaded regions indicate values of (c', c) that satisfy the above inequalities. The vertical asymptotes correspond to satisfying Eq. (7.7) for the entirety of the inflationary epoch, $\mathcal{N}_1 = 0$, so that c is left completely arbitrary but c' has a strict upper bound that is much less than the $\mathcal{O}(1)$ expectation. The horizontal dotted lines correspond to satisfying the first constraint Eq. (7.6) for all of inflation, $\mathcal{N}_2 = 0$, which leaves c' arbitrary but severely limits c. The horizontal black dashed lines indicate the lowest values of c that yield the given Δ defined in Eq. (7.5) as the lower bound on $1 + w$ from the constraint Eq. (7.6). Finally, the grey region excludes values of c that may satisfy Eq.(7.13), depending on the value of α, but conflicts with the constraint $r_{0.002} < 0.064$ [26], as $r = 16\epsilon \geq 8c^2$. The grey excluded region has a left vertical boundary since the constraint applies only to $k > 0.002\ \text{Mpc}^{-1}$.

We also comment on the observability of the tensor mode r. The swampland distance conjecture, Eq. (7.1), combined with the Lyth bound, Eq. (7.2), is normally believed to

[2]The Planck 2018 paper [19] also shows the analysis where they allow for the running of running $d^2 n_s/d\ln k^2$. Unfortunately they do not show the correlation and we cannot use it for our purposes. In fact, the extrapolation of $n_s(k)$ to small scales from the Planck data is most likely too restrictive, as the allowed range for the primordial power $P_\zeta(k)$ blows up for $k \gtrsim 0.2\ \text{Mpc}^{-1}$ (see Fig. 20 in [26]).

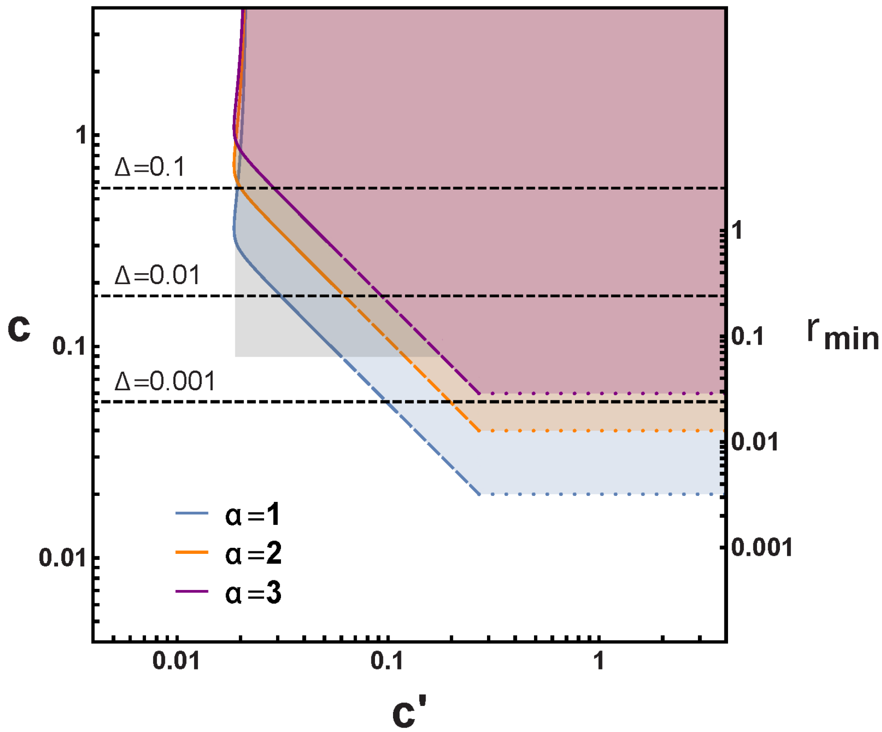

Figure 7.1: Bounds on swampland parameters for generic single-field inflation models at the 1σ level assuming the running of n_s can be extended to $\mathcal{N}_{tot} = 50$ e-folds. The unshaded region is the allowed parameter space. The solid lines are for $\mathcal{N}_1 \leq 10$; the dashed lines are for $10 < \mathcal{N}_1 < 50$, and the horizontal dotted lines correspond to $\mathcal{N}_1 = 50$, i.e. the first constraint Eq. (7.6) applies to the whole inflationary period. The values of c excluded by [26] are shaded in grey. We required the distance conjecture with $\Delta\phi \leq \alpha M_{Pl}$, and display the minimum values for $1 + w \geq \Delta$ with black dashed lines. With the original de Sitter conjecture, c had to be below the dotted horizontal lines but there were no constraints on c'.

disfavor observably large r, assuming $\alpha \approx 1$. The best sensitivity anticipated in the future is $r \sim 10^{-3}$ [10, 393]. There is a parameter region in Figure 7.1 where $r \geq r_{\min} \equiv 8c^2$ is close to the current observational bound. Physically this is because, in our spirit of a step function approximation, we can allow for a brief initial period, say $\mathcal{N}_0 \sim 4$, where the upper bound on ϵ from the distance conjecture,$\epsilon \lesssim \mathcal{N}_0^{-2}/2 \sim 0.03$, is relaxed. Thus it is possible to have r large enough to saturate the observational bound at low ℓ. This is encouraging, especially for space-born CMB B-mode experiments such as LiteBIRD [393].

7.3 Multi-Field Slow-Roll Inflation Models

The constraints discussed above are due to the tight relations between n_s, ϵ_V, η_V, and r in single-field slow-roll inflation models. It is natural to ask whether the constraints can be relaxed in multi-field models. In our analysis below, we take the conservative assumption that the swampland distance conjecture applies to the proper length of the trajectory, instead of the geodesic distance between the starting and ending points in the field space.

We discuss here a class of multi-field models where directions orthogonal to the slow-roll direction are massive, $M \gtrsim H$. The inflaton therefore rolls near the bottom of the valley, which has "bends" in the multi-dimensional field space. The main difference here is that the local angular velocities of the inflaton around the bends can modify the effective sound speed c_s of fluctuations. As a result, we have the modified relation [237]

$$\begin{aligned} 12\eta_V = &(c_s^{-2} - 1)\frac{M^2}{H^2} + 2\frac{M^2}{H^2} + 3(4\epsilon - \eta) \\ &- 2\sqrt{\left(\frac{M^2}{H^2} - \frac{3}{2}(4\epsilon - \eta)\right)^2 + 9(c_s^{-2} - 1)\frac{M^2}{H^2}}\ . \end{aligned} \tag{7.17}$$

Here, η_V is the minimum eigenvalue of the Hessian and M is the effective mass of the field orthogonal to the slow-roll direction, and c_s is given by

$$c_s^{-2} = 1 + \frac{4\Omega^2}{M^2}\ , \tag{7.18}$$

where Ω is the local angular velocity describing the bend of the inflaton trajectory in the potential. Note that in the limit $\Omega \to 0$, the sound speed reduces to unity and η_V to the expression of the single-field models. Allowing for a significant deviation of c_s from unity relaxes the constraints on (c, c'), as shown in Fig. 7.2, where we set $M = H$. This allows for larger values of c and c' compared to the single-field case, which are preferred by the swampland conjecture. Note that lowering the sound speed further will not achieve $\mathcal{O}(1)$ values for c' because our scenario relies on having negative η_V. As c_s is reduced from unity, η_V initially becomes more negative and widens the allowed parameter space. Beyond some critical value $c_s \approx 0.3$, further reduction of c_s makes η_V less negative, thereby narrowing the allowed parameter space. For $c_s \lesssim 0.2$, η_V becomes positive and our analysis no longer holds. Empirically, we find that $c_s \sim 0.24$ maximizes the allowed parameter region in the (c', c)-plane. The grey shaded regions again correspond to experimental constraints on $r = 16\epsilon c_s$, but their area is greatly reduced as c_s decreases.

It is also interesting to note that we expect primordial equilateral and orthogonal non-Gaussianities once $c_s \neq 1$ in this class of models [237],

$$f_{\rm NL}^{\rm equil} = -(c_s^{-2} - 1)(0.275 + 0.078c_s^2), \tag{7.19}$$
$$f_{\rm NL}^{\rm ortho} = (c_s^{-2} - 1)(0.0159 - 0.0167c_s^2). \tag{7.20}$$

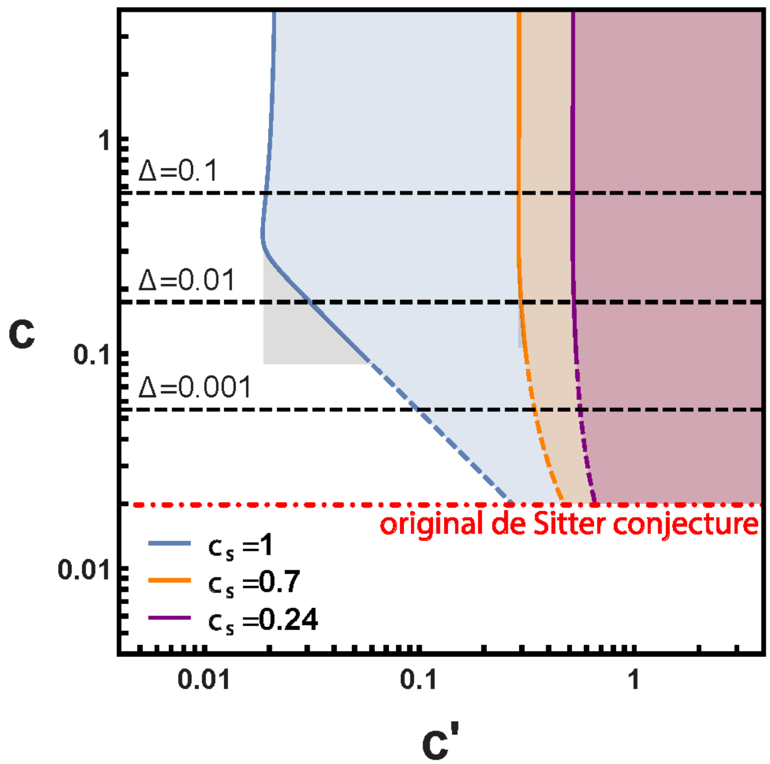

Figure 7.2: Bounds on swampland parameters for generic multi-field inflation models. We took $\alpha = 1$ and $M = H$. c_s is the sound speed for fluctuations, and the rest is the same as in Fig. 7.1. With the original de Sitter conjecture, Eq. (7.3), and single-field slow-roll models, c had to be below the red dot-dashed horizontal line.

Here we have ignored the third order parameter. The current observational constraint on the sound speed is $c_s \geq 0.024$ (see Eq. (89) of [17]), which is an order of magnitude below the limit we can reach in our setup, as shown in Fig. 7.2. Future observations combining CMB lensing, galaxy and 21cm surveys, Lyman α forest, *etc.* have the potential to improve the constraint on $f_{\rm NL}$ by an order of magnitude or more [302].

7.4 Implications for Dark Energy

The de Sitter conjecture states that constants c and c' are *universal* and should apply to all sectors in a given EFT. Therefore, we can use inflationary physics to get a handle on the values of c and c' and apply this knowledge to the quintessence potential V_Q. When this argument is applied to single-field inflation models with conjectures Eq. (7.3) and Eq. (7.1), one deduces that there may be little hope in finding $w \neq -1$ due to the small lower bound seen in Eq. (7.5). This depressing outlook is drastically changed in light of Eqs. (7.6) and (7.7), as Fig. 7.1 illustrates. We see that the refined de Sitter conjecture has allowed for the possibility of having Δ bounded from below such that it must be larger than a few per cent and should be observable to experiments. Current and future experiments, such as DES [1], HSC [2], DESI [3], PFS [4], LSST [5], Euclid [6], and WFIRST [7], are aiming for an accuracy of about a percent in w. The cost for this is that c' must be much lower than the $\mathcal{O}(1)$ expectation of [191, 349] in the single-field case. This seems to indicate that single-field inflation falls more in line with the modified de Sitter conjecture discussed in [340], where the smallest Hessian eigenvalue needs only be negative when $|\nabla V| < cV$.

This state of affairs is altered by considering multi-field inflation models. Not only could Δ be forced to be as large as several per cent, it is also possible to have both c and c' approximately $\mathcal{O}(1)$ as long as the sound speed is low enough, as seen in Fig. 7.2. In either the single-field or multi-field scenario, a better theoretical understanding of the magnitude of c' is essential to understand the consistency of the swampland conjectures and inflation.

7.5 Conclusion

In this chapter, we studied the consequences of the latest swampland conjecture on inflation and dark energy. The original de Sitter conjecture raised the hope that measuring the dark energy equation of state w would be promising while simultaneously dashing that hope since consistency with single-field inflation suggests that the deviation from $w = -1$ would likely be unobservable. As we have shown, this situation is much more encouraging with the refined de Sitter conjecture. Not only could $w \neq -1$ be observable even with a single-field inflationary scenario, but tensor modes could be as well. If one considers multi-field inflationary scenarios, then the prospect for observing $w \neq -1$ is better and one gains improved agreement with the swampland conjectures.

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Appendix A

Parametric Resonance

In this appendix, we numerically study the process of parametric resonance for our model. To be concrete, let us consider the potential in Eq. (3.5) with $n = 3$. We decompose the PQ symmetry breaking field as

$$P = \frac{1}{\sqrt{2}} \left(f_a + s\left(t\right) + \sigma\left(t, x\right) + ia\left(t, x\right) \right), \tag{A.1}$$

where s is a zero mode and σ and a are saxion and axion fluctuations, respectively. The linearized equation of motion of the axion in a mode k is

$$\ddot{a}_k + \left(k^2 + \frac{m_s^2}{f_a} s + \frac{3m_s^2}{2f_a^2} s^2 + \frac{m_s^2}{f_a^3} s^3 + \frac{m_s^2}{4f_a^4} s^4 \right) a_k = 0. \tag{A.2}$$

For a small saxion oscillation amplitude, a quadratic approximation can be used to give the saxion profile, $s = S_0 \sin(m_s t)$, and Eq. (A.2) can be transformed into the Mathieu equation for the axion with a wave number k,

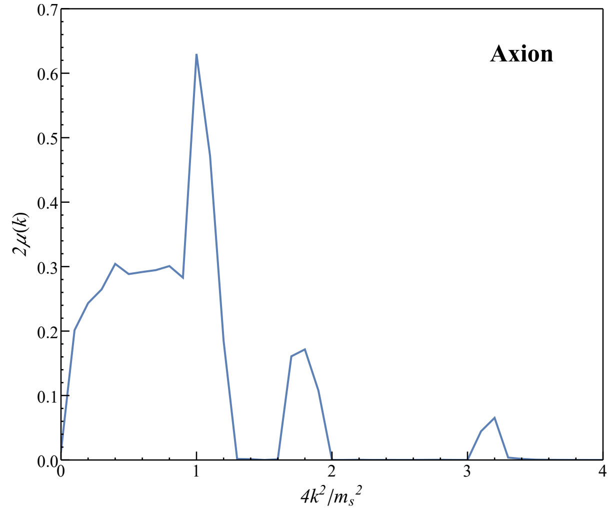
$$a_k''(z) + \left(A_k - 2q \cos\left(2z\right) \right) a_k(z) = 0 \tag{A.3}$$

where $m_s t = 2z - \pi/2$, $A_k = 4k^2/m_s^2$, and $q = 2S_0/f_a$. For certain values of (A_k, q), the mode solutions exhibit exponential growth of the form $a_k \sim \exp\left(\mu\left(k\right) m_s t\right)$ [297], where $\mu(k)$ is the so-called Floquet index.

Since the amplitude of the saxion oscillations is as large as f_a, the quadratic approximation may not be valid and so we numerically solve the saxion zero mode's nonlinear equation of motion, which is given in this case by

$$\ddot{s} + \left(\frac{m_s^2}{4f_a^4} s^4 + \frac{5m_s^2}{4f_a^3} s^3 + \frac{5m_s^2}{2f_a^2} s^2 + \frac{5m_s^2}{2f_a} s + m_s^2 \right) s = 0. \tag{A.4}$$

The profile determined from this equation of motion is then used to examine the growth of axion modes. This process can also be done for saxion perturbation modes σ_k, which have



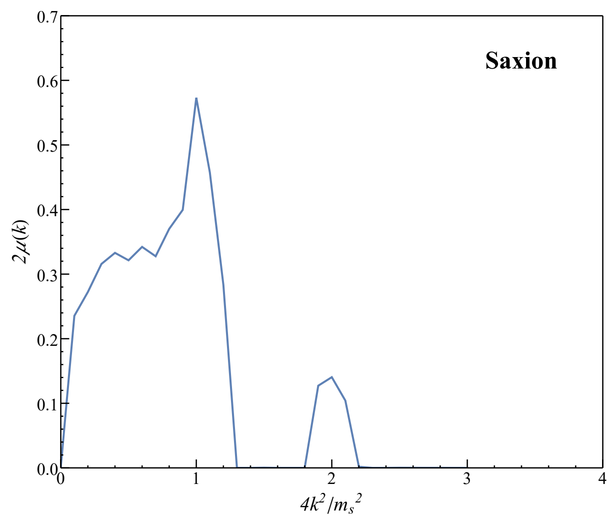


Figure A.1: Floquet indices for axion (top) and saxion (bottom) modes with a background saxion amplitude of $0.8f_a$.

the equation of motion

$$\ddot{\sigma}_k + \left(k^2 + m_s^2 + \frac{5m_s^2}{f_a}s + \frac{15m_s^2}{2f_a^2}s^2 + \frac{5m_s^2}{f_a^3}s^3 + \frac{5m_s^2}{4f_a^4}s^4\right)\sigma_k = 0. \tag{A.5}$$

The numerical results for the indices $\mu(k)$ for both the axion and saxion are displayed in Fig. A.1. The axion and the saxion have similar index profiles and both plots feature a sharp peak at the mode $k \simeq m_s/2$.

Appendix B

Anomaly Cancellation Details for the $\mathbb{Z}_3$ and $\mathbb{Z}_7$ Models

Cancellation of ultraviolet divergences and evaluation of the anomaly

In this appendix, we will specify a choice of PV fields that cancel leftover divergences from the invariant mass PV sector of [92] (referred to in what follows as BG) and reproduce the universal chiral anomaly of [379]. Aside from the residual divergences discussed in Appendix B, the PV fields introduced in BG eliminate all the divergences from the light sector of the two string models we are considering, but have some leftover divergences arising from the $\dot{Y}$ fields if one excludes the noninvariant mass PV sector of BG. Since we must alter the noninvariant mass sector of BG to produce a universal anomaly, our strategy here will be to introduce fields with parameters that decouple as much as possible from the BG fields but still cancel the divergences of the $\dot{Y}$. These new fields replace the BG set that was collectively denoted by Ψ. We expand the sum rules of BG to accommodate the more general Kähler potential of PV fields we consider in (5.16), and find that the sum rules that the PV fields must satisfy to cancel divergences are

$$\sum_C \eta^C = N' = -N - 29 \tag{B.1}$$

$$\sum_\gamma \eta_\gamma^V = N'_G = -12 - N_G \tag{B.2}$$

$$\sum_C \eta^C \alpha^C = -10 \tag{B.3}$$

$$\sum_C \eta^C \left(\beta^C g + \sum_n q_n^C g^n\right) = -A_1 g \tag{B.4}$$

$$-4 = \sum_C \eta^C \alpha^C \alpha^C \tag{B.5}$$

$$2 = \sum_C \eta^C \delta^C \tag{B.6}$$

$$0 = \sum_C \eta^C \left(\alpha^C \beta^C (X_\alpha g_\beta + g_\alpha X_\beta) + \sum_n \alpha^C q_n^C (g_\alpha^n X_\beta + X_\alpha g_\beta^n)\right) \tag{B.7}$$

$$\sum_C \eta^C \left(\beta^C \beta^C g_\alpha g_\beta + \sum_n \beta^C q_n^C (g_\alpha^n g_\beta + g_\alpha g_\beta^n) + \sum_{n,m} q_n^C q_m^C g_\alpha^n g_\beta^m\right) = -A_2 g_\alpha g_\beta - B_2 \sum_n g_\alpha^n g_\beta^n \tag{B.8}$$

$$\sum_C \eta^C C_{(G)}^C = C_{(G)}^M \tag{B.9}$$

$$\sum_C \eta^C (T_a)_C^C \alpha^C = 0 \tag{B.10}$$

$$\sum_C \eta^C (T_a)_C^C \left(\beta^C + q_n^C\right) = -\sum_p q_n^p (T_a)_p^p \tag{B.11}$$

$$\sum_C \eta^C \left(\frac{F}{2} - \gamma^C F - \sum_n q_n^C F^n\right)\left(\alpha^C - \frac{1}{2}\right)^2 X \cdot \widetilde{X} = -\frac{1}{4}\left(\frac{N}{2} - A_1\right) F X \cdot \widetilde{X} \tag{B.12}$$

$$F \sum_p \left(\frac{A_1}{2} - A_2\right) X \cdot \tilde{g} = \sum_C \eta^C \left(\frac{F}{2} - \gamma^C F - \sum_n q_n^C F^n\right) \times \left(\alpha^C - \frac{1}{2}\right)\left(2\beta^C X \cdot \tilde{g} + \sum_n 2 q_n^C X \cdot \tilde{g}^n\right) \tag{B.13}$$

$$-F\left(\frac{A_2}{2} - A_3\right) g \cdot \tilde{g} = \sum_C \eta^C \left(\frac{F}{2} - \gamma^C F - \sum_n q_n^C F^n\right) \times \left(\beta^C \beta^C g \cdot \tilde{g} + \sum_n 2\beta^C q_n^C g \cdot \tilde{g}^n + \sum_{n,m} q_n^C q_m^C g^n \cdot \tilde{g}^m\right) \tag{B.14}$$

$$\frac{1}{2}\sum_p \left(\frac{F}{2} - \sum_n q_n^p F^n\right)(T_{(G)})_p^p = \sum_C \eta^C \left(\frac{F}{2} - \gamma^C F - \sum_n q_n^C F^n\right)\left(\alpha^C - \frac{1}{2}\right)(T_a)_C^C \tag{B.15}$$

$$\sum_C \eta^C \left(\frac{F}{2} - \gamma^C F - \sum_n q_n^C F^n\right)(T_{(G)})_C^C \beta^C = 0 \tag{B.16}$$

$$\sum_C \eta^C \left(\frac{F}{2} - \gamma^C F - \sum_n q_n^C F^n\right)(T_{(G)})_C^C q_n^C = -\sum_p \left(\frac{F}{2} - \sum_n q_n^p F^n\right)(T_{(G)})_p^p q_n^p \quad \text{(B.17)}$$

$$\sum_{C,J} \eta^C \left(\frac{F}{2} - \gamma^C F - \sum_n q_n^C F^n\right)(T_a)_J^C (T_b)_C^J = -\sum_{p,q} \left(\frac{F}{2} - \sum_n q_n^p F^n\right)(T_a)_q^p (T_b)_p^q \quad \text{(B.18)}$$

where we have used the sum rules for the light field modular weights (5.23), (5.23) and Eq. (5.18) and Eq. (5.19). F is defined by Eq. (5.4). On the left hand side of each condition we are summing over all PV fields, while the right hand sides correspond to summing over the parameters of the light fields. Since a subset of the the BG PV fields already eliminate divergences from the light sectors of the string models, we can recast the above conditions by setting the right hand side of all conditions to zero and summing over only the $\dot{Y}$, Φ, ϕ, T, and ψ fields. To match the anomaly calculated in [379], we also require

$$0 = \sum_C \eta^C (1-2\bar{\gamma}^C)\left(\alpha^C\alpha^C - \frac{2}{3}\bar{\alpha}^C\bar{\alpha}^C - \frac{1}{3}\bar{\alpha}^C\right) \quad \text{(B.19)}$$

$$0 = \sum_C \eta^C (1-2\bar{\gamma}^C)\left(2\beta^C\alpha^C - \frac{4}{3}\bar{\beta}^C\bar{\alpha}^C - \frac{1}{3}\bar{\beta}^C\right) \quad \text{(B.20)}$$

$$0 = \sum_C \eta^C (1-2\bar{\gamma}^C)\left(\beta^C\beta^C - \frac{2}{3}\bar{\beta}^C\bar{\beta}^C\right) \quad \text{(B.21)}$$

$$0 = \sum_C \eta^C \left(1-2\bar{\gamma}^C\right)^2 \quad \text{(B.22)}$$

$$0 = \sum_C \eta^C q_n^C \alpha^C (1-2\bar{\gamma}^C) \quad \text{(B.23)}$$

$$0 = \sum_C \eta^C q_n^C \beta^C (1-2\bar{\gamma}^C) \quad \text{(B.24)}$$

$$0 = \sum_C \eta^C \bar{\alpha}^C (1-2\bar{\gamma}^C)^2 \quad \text{(B.25)}$$

$$0 = \sum_C \eta^C \bar{\beta}^C (1-2\bar{\gamma}^C)^2 \quad \text{(B.26)}$$

$$0 = \sum_C \eta^C \bar{\alpha}^C (1-2\bar{\gamma}^C)(1-2\bar{\alpha}^C) \quad \text{(B.27)}$$

$$0 = \sum_C \eta^C \bar{\beta}^C (1-2\bar{\alpha}^C)(1-2\bar{\gamma}^C) \quad \text{(B.28)}$$

$$0 = \sum_C \eta^C \bar{\alpha}^C \bar{\beta}^C (1-2\bar{\gamma}^C) \quad \text{(B.29)}$$

$$0 = \sum_C \eta^C \bar{\beta}^C \bar{\beta}^C (1 - 2\bar{\gamma}^C) \quad \text{(B.30)}$$

$$0 = \sum_C \eta^C \bar{\alpha}^C \bar{\alpha}^C (1 - 2\bar{\gamma}^C)^2 \quad \text{(B.31)}$$

$$0 = \sum_C \eta^C \bar{\beta}^C \bar{\beta}^C (1 - 2\bar{\gamma}^C)^2 \quad \text{(B.32)}$$

$$0 = \sum_C \eta^C \bar{\alpha}^C \bar{\beta}^C (1 - 2\bar{\gamma}^C)^2 \quad \text{(B.33)}$$

$$0 = \sum_C \eta^C \bar{\beta}^C (1 - 2\bar{\gamma}^C) \quad \text{(B.34)}$$

$$0 = \sum_C \eta^C \bar{\beta}^C \bar{\beta}^C \bar{\beta}^C (1 - 2\bar{\gamma}^C) \quad \text{(B.35)}$$

$$30\delta_{nm} = \sum_C \eta^C q_n^C q_m^C (1 - 2\bar{\gamma}^C) \quad \text{(B.36)}$$

$$2\sum_p q_n^p + 3 - N + N_G = \sum_C \eta^C (1 - 2\alpha^C + 2q_n^C) \quad \text{(B.37)}$$

In this second set of conditions, only fields with noninvariant masses contribute to the sums since $\bar{\gamma} = \frac{1}{2}$ for fields with invariant masses. We now describe a particular choice of $\{\Phi, \mathrm{T}, \psi\}$ fields that lead to an easily solvable system for their parameters. We must also supplement these fields with the ϕ^C fields, since some of these have noninvariant masses. Starting with divergences related to gauge interactions, we introduce a pair of T fields for each non-Abelian simple factor of the string model gauge group: $(\mathrm{T}^P_{(\mathcal{G})1}, \mathrm{T}'^P_{(\mathcal{G})1})$, $(\mathrm{T}^P_{(\mathcal{G})2}, \mathrm{T}'^P_{(\mathcal{G})2})$, where $\mathcal{G}$ specifies the simple group factor. We take the $\mathrm{T}_{(\mathcal{G})1}$ ($\mathrm{T}'_{(\mathcal{G})1}$) to be in the fundamental (antifundamental) representation of $\mathcal{G}$ while the $\mathrm{T}_{(\mathcal{G})2}$ are gauge singlets. Then Eq. (B.9) gives

$$C^M_{\mathcal{G}} = 2C^f_{(\mathcal{G})} \sum_P \eta^P_{\mathrm{T}_{(\mathcal{G})1}} \quad \text{(B.38)}$$

$$C^f_{(\mathcal{G})} N_{(\mathcal{G})} = 2N_{\mathrm{T}_{(\mathcal{G})1}} C^f_{(\mathcal{G})}, \qquad \sum_P \eta^P_{\mathrm{T}_{(\mathcal{G})1}} = \frac{N_{(\mathcal{G})}}{2}, \quad \text{(B.39)}$$

for non-Abelian gauge groups and

$$\sum_p Q^p_a Q^p_a = 2\sum_P (\eta_\Phi)^P (Q^\Phi)^P_a (Q^\Phi)^P_a + 2\sum_P (\eta_{\mathrm{T}_{a1}})^P (Q^{\mathrm{T}})^P_a (Q^{\mathrm{T}})^P_a \quad \text{(B.40)}$$

for Abelian groups. These are just the conditions needed to cancel the $\dot{Y}$ factor of C^M_G. The Φ's enter in Eq. B.40 since they are given $U(1)_a$ charges as per the prescription in Section 4.2. Note also that Eq B.39 works for the two models considered here since the number $N_{(\mathcal{G})}$ of fundamentals in $\mathcal{G}$ is even for all the gauge groups. We will constrain these T fields so that they do not contribute to any divergences other than those arising from gauge interactions.

To do this, we enforce the following

$$(\alpha_{\mathrm{T}_{(\mathcal{G})2}}) = (\alpha_{\mathrm{T}_{(\mathcal{G})1}}) \tag{B.41}$$
$$(\beta_{\mathrm{T}_{(\mathcal{G})2}}) = (\beta_{\mathrm{T}_{(\mathcal{G})1}}) \tag{B.42}$$
$$(\alpha'_{\mathrm{T}_{(\mathcal{G})2}}) = (\alpha'_{\mathrm{T}_{(\mathcal{G})1}}) \tag{B.43}$$
$$(\beta'_{\mathrm{T}_{(\mathcal{G})2}}) = (\beta'_{\mathrm{T}_{(\mathcal{G})1}}) \tag{B.44}$$
$$(\eta_{\mathrm{T}_{(\mathcal{G})2}})^P = -(\eta_{\mathrm{T}_{(\mathcal{G})1}})^P \tag{B.45}$$

For the T fields charged under non-Abelian group factors, we impose that $\bar{\gamma}^P_{\mathrm{T}_{(\mathcal{G})1}}$ is independent of P so that we can cancel all remaining divergences from non-Abelian interactions by demanding

$$\frac{C_{\mathrm{GS}} - C_{(\mathcal{G})}}{2} = \frac{C^f_{(\mathcal{G})} N_{(\mathcal{G})}}{2} \left(1 - 2\bar{\gamma}_{\mathrm{T}_{(\mathcal{G})1}}\right), \tag{B.46}$$

for every non-Abelian group factor $\mathcal{G}$, where

$$C_{\mathrm{GS}} = 8\pi^2 b = 30 \tag{B.47}$$

is the adjoint Casimir for E_8, which is the gauge group of the pure Yang-Mills hidden sector of the models considered here. For the Abelian divergences, we will not force $\bar{\gamma}^P_{\mathrm{T}_{(\mathcal{G})1}}$ to be independent of P, but we will require Eq. (B.43) and that the primed and unprimed parameters are identical:

$$\begin{aligned}(\alpha_{\mathrm{T}_{(\mathcal{G})1}})^P &= (\alpha'_{\mathrm{T}_{(\mathcal{G})1}})^P \\ (\beta_{\mathrm{T}_{(\mathcal{G})1}})^P &= (\beta'_{\mathrm{T}_{(\mathcal{G})1}})^P.\end{aligned} \tag{B.48}$$

With these conditions, the remaining divergences from Abelian interactions are canelled by imposing

$$\frac{C_{GS}}{2} = \sum_M (\eta_{\mathrm{T}_{(a)1}})^P \left(1 - 2\gamma^P_{\mathrm{T}_{(a)1}}\right) (Q^{\mathrm{T}})^P_a (Q^{\mathrm{T}})^P_a. \tag{B.49}$$

With the above restrictions, the charged T fields will eliminate only gauge-related divergences and not contribute to any of ther other sum rules listed above. Turning to the ψ and ϕ fields, we choose parameters such that

$$\bar{\gamma}^P_\psi = \bar{\alpha}^P_\psi = \bar{\beta}^P_\psi = 0. \tag{B.50}$$

Then the second set of conditions reduces to

$$0 = \sum_P (\eta_\psi)^P + \sum_C \hat{\eta}^C \left(1 - 2\bar{\hat{\alpha}}^C\right)^2 \tag{B.51}$$

$$\begin{aligned} 0 &= \sum_P (\eta_\psi)^P (\alpha_\psi)^P (\alpha_\psi)^P \\ &\quad + \sum_C \hat{\eta}^C (1 - 2\bar{\hat{\alpha}}^C) \left(\hat{\alpha}^C \hat{\alpha}^C - \frac{2}{3} \bar{\hat{\alpha}}^C \bar{\hat{\alpha}}^C - \frac{1}{3} \bar{\hat{\alpha}}^C \right) \end{aligned} \tag{B.52}$$

$$0 = \sum_C \hat{\eta}^C \bar{\hat{\alpha}}^C \left(1 - 2\bar{\hat{\alpha}}^C\right)^2 \tag{B.53}$$

$$0 = \sum_C \hat{\eta}^C \bar{\hat{\alpha}}^C \bar{\hat{\alpha}}^C \left(1 - 2\bar{\hat{\alpha}}^C\right)^2 \tag{B.54}$$

$$2 \sum_p q_n^p + 3 - N + N_G = \sum_C \hat{\eta}^C (1 - 2\hat{\alpha}^C) \tag{B.55}$$

$$0 = \sum_P (\eta_\psi)^P \tag{B.56}$$

$$0 = \sum_P (\eta_\psi)^P (\alpha_\psi)^P (\alpha_\psi)^P \tag{B.57}$$

$$0 = \sum_P (\eta_\psi)^P (\alpha_\psi)^P (\beta_\psi)^P \tag{B.58}$$

$$0 = \sum_P (\eta_\psi)^P (\beta_\psi)^P (\beta_\psi)^P \tag{B.59}$$

$$0 = \sum_P (\eta_\psi)^P q_\psi^P \alpha_\psi^P \tag{B.60}$$

$$0 = \sum_P (\eta_\psi)^P q_\psi^P \beta_\psi^P \tag{B.61}$$

$$30 = 2 \sum_P (\eta_\psi)^P q_\psi^P q_\psi^P \tag{B.62}$$

Finally, to cancel all the divergences as required by the sum rules in (B.1)–(B.14), we introduce gauge singlet T fields ($\mathrm{T}_3^P, \mathrm{T}_3'^P$) with invariant masses. These fields, along with the Φ and ψ fields, are enough to regulate those divergences of the $\dot{Y}$ fields that do not involve gauge couplings. While we have solved this system to obtain numerical solutions, the results are not particularly enlightening and we will not reproduce them here.

Residual linear and logarithmic divergences

There are two sources of the chiral anomaly involving space-time curvature. The first arises from the spin connection in the fermion covariant derivatives. The three sum rules in (5.17) assure that the linear divergent terms from the PV fermion spin connection cancel those from the light fields, and the residual anomaly arises from the PV masses, giving a supersymmetric contribution

$$\Delta\mathcal{L}_{\rm spin} = -b_{\rm sp}\int EF\Omega_{\rm GB} + {\rm h.c.}, \tag{B.1}$$

which is the variation of the first term in $\mathcal{L}_0$ in (5.5), with

$$8\pi^2 b_{\rm sp} = \frac{1}{24}\left(N' - N'_G - 2\alpha + 2\sum_p q_p^n\right) = \frac{1}{24}\left(2\sum_p q_n^p + 3 - N + N_G\right) = 31 \;\; \forall \;\; n \tag{B.2}$$

for Z_3 and Z_7 orbifolds without Wilson lines. The second contribution arises from the affine connection in the gravitino covariant derivative; it has no PV counterpart and is not canceled. However there is a residual conformal anomaly associated with the linear divergence arising from the Gauss-Bonnet term which is a total derivative, and which is uniquely determined [37, 36] by the spins of the particles in the loop. For PV regulated supergravity we have

$$\mathcal{L}_{\rm GB} = \frac{\sqrt{g}b_{\rm BG}}{2}\left(r_{\mu\nu\rho\sigma}r^{\mu\nu\rho\sigma} - 4r_{\mu\nu}r^{\mu\nu} + r^2\right)\ln\Lambda, \tag{B.3}$$

with

$$8\pi^2 b_{\rm GB} = \frac{1}{48}(N + N' - 3N_G - 3N'_G + 41) = 1. \tag{B.4}$$

The variation of (B.3) forms a supersymmetric operator with the chiral anomaly from the gravitino affine connection provided the cut-off takes the value in (5.10), giving a contribution

$$\Delta\mathcal{L}_{\rm aff} = b_{\rm GB}\int EF\Omega_{\rm GB} + {\rm h.c.}, \tag{B.5}$$

which is the variation of the $K\Omega_{\rm GB}$ term in (5.5), and combines with (B.2) to give

$$\Delta\mathcal{L}_{\rm anom} \ni -b\int EF\Omega_{\rm GB} + {\rm h.c.}, \tag{B.6}$$

where

$$8\pi^2 b = 8\pi^2(b_{\rm sp} - b_{\rm GB}) = \frac{1}{24}\left(2\sum_p q_n^p - N + N_G - 21\right) = 30. \qquad \forall \;\; n \tag{B.7}$$

There is also a linear divergence arising from an off-diagonal gravitino-gaugino connection in the fermion covariant derivative. This also combines with an uncanceled logarithmically divergent total derivative to form an anomaly supermultiplet if the cut-off satisfies (5.10). It was shown in Appendix B.3 of [92] that this anomaly can be canceled for a particular choice of masses for certain PV fields that regulate gauge and gravity sector loops.

Finally, there are "D-term" anomalies that arise from uncanceled logarithmically divergent terms with no chiral anomaly counterpart. These were simply dropped in the evaluation of on-shell ultraviolet divergences in [183] and [184]. Since they have no chiral anomaly partner they are more difficult to identify than the above terms. With the cut-off (5.10), the conformal anomaly includes a contribution

$$\Delta\mathcal{L}_{\rm conf} \ni \frac{\sqrt{g}}{8\pi^2}{\rm Re}FK_{i\bar{m}}D^\mu\left[\mathcal{D}_\mu\bar{z}^{\bar{m}}\left(2F^i\big|\,R\big| - D_\nu\mathcal{D}^\nu z^i\right) + D_\nu\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\nu z^i + {\rm h.c.}\right], \tag{B.8}$$

where

$$F^i = -\frac{1}{4}\mathcal{D}^2 Z^i \tag{B.9}$$

is the auxiliary field of the chiral supermultiplet Z^i, that was identified in [92] as arising from a total derivative dropped in the evaluation [183] of UV divergences for gravity coupled to chiral matter. When Yang-Mills couplings are included [184], there are many more terms, and digging out total derivatives is much more difficult. We find the additional light field contribution:

$$\Delta\mathcal{L}_{\rm conf} \ni -\frac{\sqrt{g}}{8\pi^2}{\rm Re}FD^\mu\left[\frac{3}{2}K_{i\bar{m}}\mathcal{D}_\mu\bar{z}^{\bar{m}}D^a(T_a z)^i + \frac{\partial_\mu\bar{s}}{s+\bar{s}}D^a D_a\right]\Bigg| + {\rm h.c.}, \tag{B.10}$$

where

$$D^a = -\frac{1}{2}\mathcal{D}_\alpha W^a_\alpha \tag{B.11}$$

is the auxiliary field for the superfield strength W^a_α, and we evaluated the result of [184] using the classical Kähler potential in (5.9) for the dilaton. We also find a contribution [182] from the PV sector

$$\Delta\mathcal{L}_{\rm conf} \ni \frac{\sqrt{g}}{16\pi^2}{\rm Re}FD^\mu\left[K_{i\bar{m}}\mathcal{D}_\mu\bar{z}^{\bar{m}}D^a(T_a z)^i - \frac{\partial_\mu\bar{s}}{s+\bar{s}}\left(F^i\bar{F}_i - 8R\bar{R}\right)\right]\Bigg| + {\rm h.c.} \tag{B.12}$$

With the classical Kähler potential in (5.9) the equations of motion give

$$F^s\big| = 2(s+\bar{s})\bar{R}\big|, \qquad \bar{F}_s\big| = \frac{2}{s+\bar{s}}R\Bigg|, \tag{B.13}$$

and the dilaton-dependent contribution can be written

$$\begin{aligned}\Delta\mathcal{L}_{\rm conf}(s,\bar{s}) \;\;=\;\; & -\frac{\sqrt{g}}{8\pi^2}{\rm Re}FD^\mu\left[(\partial_\mu\bar{s}\Box s - \partial_\nu\partial_\mu\bar{s}\partial^\nu s + {\rm h.c.})\right.\\ & \left.+\frac{1}{2}\frac{\partial_\mu\bar{s}}{s+\bar{s}}\left(F^p\bar{F}_p - 12R\bar{R} + 2D^a D_a\right)\big|\right], \end{aligned}\tag{B.14}$$

However we cannot be certain that we have identified all the uncanceled total derivatives. It is also possible that one might be able to modify the PV sector parameter such that the dilaton dependence can be canceled, as was the case for F-term anomaly arising from the off-diagonal gaugino-gravitino connection mentioned above.

Appendix C

Anomaly Cancellation Details for the FIQS Model

Conditions for the cancellation of ultraviolet divergences and the evaluation of the anomaly

Notation

We pair PV fields according to their mass terms. A pair of PV fields $(\Phi^P,\ \Phi'^P)$ has a superpotential coupling

$$W_{PV} = \sum_P \mu_P \Phi'^P \Phi^P \tag{C.1}$$

and a Kähler potential

$$K_{PV} = \sum_P e^{f^P} \Phi^{*P} \Phi^P + \sum_P e^{f'^P} \Phi'^{*P} \Phi'^P, \tag{C.2}$$

where

$$f^P = \alpha^P K + \beta^P g + \sum_n q_n^P g^n \tag{C.3}$$

with an identical definition holding for f'^P but with primes on the constants $\{\alpha^P, \beta^P, q_n^P\}$. While we will not use it often, summing over the index C means summing over PV fields and then their primed partners whereas summing over P means summing over only the unprimed or primed fields, depending on the quantity being summed. For example,

$$\sum_C \eta^C \alpha^C = \sum_P \eta_P \alpha^P + \sum_P \eta_P \alpha'^P. \tag{C.4}$$

However, to reduce clutter, we will abbreviate the above. When summing over primed and unprimed fields, we will use "Tr". When summing over only primed or unprimed ones, we will use "Sum". Thus the above would be written as

$$\mathrm{Tr}[\eta\alpha] = \mathrm{Sum}[\eta\alpha] + \mathrm{Sum}[\eta\alpha']. \tag{C.5}$$

We will also encounter sums over various combinations of U(1) charges, $\mathrm{U}(1)_X$ charges, and modular weights. To abbreviate these, especially when dealing with the quantum numbers of the light fields, we will define

$$\begin{aligned}
Q_{1a} &= \mathrm{Sum}[\eta Q_a q_n] && \text{(C.6)}\\
Q_{2a} + P_{2a}\delta_{nm} &= \mathrm{Sum}[\eta Q_a q_n q_m] && \text{(C.7)}\\
R_a &= \mathrm{Sum}[\eta Q_a Q_X q_n] && \text{(C.8)}\\
R_{ab} &= \mathrm{Sum}[\eta Q_a Q_b q_n] && \text{(C.9)}\\
S_a &= \mathrm{Sum}[\eta Q_a Q_X] && \text{(C.10)}\\
S_{ab} &= \mathrm{Sum}[\eta Q_a Q_b]. && \text{(C.11)}
\end{aligned}$$

Conditions for Regularization

The terms we must cancel come from linear, logarithmic, and quadratic divergences. It is helpful to organize these terms by forming subsets based on whether terms depend on nonabelian gauge interactions, nonanomalous Abelian gauge interactions, anomalous Abelian gauge interactions, or none of the above. We will refer to these groupings as nonabelian divergences, $\mathrm{U}(1)_a$ divergences, $\mathrm{U}(1)_X$ divergences, and modular divergences, respectively. As an overview, the divergences come from the terms

$$\begin{aligned}
&\mathrm{Tr}[\eta\Gamma_\alpha] && \text{(C.12)}\\
&\mathrm{Tr}[\eta\Gamma_\alpha\Gamma_\beta] && \text{(C.13)}\\
&\mathrm{Tr}[\eta\Gamma_\alpha T_a] && \text{(C.14)}\\
&\mathrm{Tr}[\eta T_a T_b] && \text{(C.15)}\\
&\mathrm{Tr}[\eta Q_a], && \text{(C.16)}
\end{aligned}$$

where

$$\begin{aligned}
\Gamma^C_{D\alpha} &= -\frac{1}{8}\left(\bar{\mathcal{D}}^2 - 8R\right)\mathcal{D}_\alpha Z^i \Gamma^C_{Di} && \text{(C.17)}\\
\phi^C &= \left(\frac{1}{2} - \alpha^C - \beta^C\right)F - \sum_i F^i q^C_i - q^C_X\Lambda && \text{(C.18)}\\
G_{\mu\nu} &= \Gamma^C_{C\mu\nu} - \frac{1}{2}X_{\mu\nu}\delta^C_D - iF^a_{\mu\nu}(T_a)^C_D - iF^X_{\mu\nu}(Q_X)^C_D. && \text{(C.19)}
\end{aligned}$$

for our PV fields defined above.

The PV fields involved in this procedure are numerous. We take all of the PV fields described in sections 3 and 4 of [92] and supplement them with further fields. However, to cancel the divergences above, we need only focus on the $\dot{Y}$ and $\hat{\phi}$ fields of [92]. We now group all the terms in the above expressions with our organizational scheme.

Modular Divergences

To cancel all the modular divergences, we require

$$0 = -\mathrm{Tr}\left[\eta\beta\left(\frac{1}{2}-\alpha\right)^2\right] - \mathrm{Tr}\left[\eta q_n\left(\frac{1}{2}-\alpha\right)^2\right] \tag{C.20}$$

$$\begin{aligned} 0 &= -\frac{1}{2}\mathrm{Tr}\left[\eta(1-2\alpha)\beta(1-2\gamma)\right] + \mathrm{Tr}\left[\eta\beta q_n(1-2\alpha)\right] \\ &\quad -\frac{1}{2}\mathrm{Tr}\left[\eta(1-2\alpha)(1-2\gamma)q_n\right] + \mathrm{Tr}\left[\eta q_n q_m(1-2\alpha)\right] \end{aligned} \tag{C.21}$$

$$\begin{aligned} 0 &= \frac{1}{2}\mathrm{Tr}\left[\eta\beta^2(1-2\gamma)\right] - \mathrm{Tr}\left[\eta\beta^2 q_n\right] + \mathrm{Tr}\left[\eta\beta(1-2\gamma)q_n\right] - 2\mathrm{Tr}\left[\eta\beta q_n q_m\right] \\ &\quad +\frac{1}{2}\mathrm{Tr}\left[\eta(1-2\gamma)q_n q_m\right] - \mathrm{Tr}\left[\eta q_n q_m q_k\right]. \end{aligned} \tag{C.22}$$

U(1)$_X$ Divergences

To cancel all the U(1)$_X$ divergences, we need

$$0 = \mathrm{Tr}[\eta Q_X] \tag{C.23}$$

$$0 = \mathrm{Tr}[\eta Q_X\beta] + \mathrm{Tr}[\eta Q_X q_m] \tag{C.24}$$

$$0 = \mathrm{Tr}[\eta Q_X\alpha] \tag{C.25}$$

$$0 = \mathrm{Tr}\left(\eta Q_X\left(\alpha-\frac{1}{2}\right)^2\right) \tag{C.26}$$

$$0 = -\mathrm{Tr}\left(\eta Q_X\beta\left(\alpha-\frac{1}{2}\right)\right) + \mathrm{Tr}\left(\eta Q_X q_n\left(\alpha-\frac{1}{2}\right)\right) \tag{C.27}$$

$$0 = \mathrm{Tr}\left(\eta Q_X\beta^2\right) + 2\mathrm{Tr}\left(\eta Q_X q_n\beta\right) + \mathrm{Tr}\left(\eta Q_X q_n q_m\right) \tag{C.28}$$

$$0 = \mathrm{Tr}\left(\eta Q_X^3\right) \tag{C.29}$$

$$0 = \mathrm{Tr}\left(\eta Q_X^2\left(\frac{1}{2}-\gamma\right)\right) - \mathrm{Tr}\left(\eta Q_X^2 q_n\right) \tag{C.30}$$

$$0 = \mathrm{Tr}\left(\eta Q_X^2\left(\alpha-\frac{1}{2}\right)\right) \tag{C.31}$$

$$0 = 2\mathrm{Tr}\left(\eta Q_X\left(\alpha-\frac{1}{2}\right)\left(\frac{1}{2}-\gamma\right)\right) - 2\mathrm{Tr}\left(\eta Q_X q_n\left(\alpha-\frac{1}{2}\right)\right) \quad \text{(C.32)}$$

$$0 = -2\mathrm{Tr}\left(\eta Q_X^2\beta\right) - 2\mathrm{Tr}\left(\eta Q_X^2 q_n\right) \quad \text{(C.33)}$$

$$0 = 2\mathrm{Tr}\left(\eta Q_X\beta\left(\frac{1}{2}-\gamma\right)\right) - 2\mathrm{Tr}\left(\eta Q_X q_n\beta\right) + 2\mathrm{Tr}\left(\eta Q_X q_n\left(\frac{1}{2}-\gamma\right)\right) -2\mathrm{Tr}\left(\eta Q_X q_n q_m\right). \quad \text{(C.34)}$$

Note that only fields that have $\bar{Q}_X \neq 0$ will contribute to Eq. (C.29).

Non-Abelian Divergences

To cancel the non-Abelian divergences, we need

$$0 = \mathrm{Tr}[\eta T_a T_b] \quad \text{(C.35)}$$

$$0 = \mathrm{Tr}[\eta Q_X T_a T_b] \quad \text{(C.36)}$$

$$0 = \mathrm{Tr}\left[\eta T_a T_b\left(\gamma-\frac{1}{2}\right)\right], \quad \text{(C.37)}$$

where T^a is a generator of a non-Abelian gauge group factor.

U(1)$_a$ Divergences

Finally, the conditions for canceling the abelian divergences are

$$0 = \mathrm{Tr}[\eta Q_a] \quad \text{(C.38)}$$

$$0 = \mathrm{Tr}[\eta Q_a\alpha] \quad \text{(C.39)}$$

$$0 = \mathrm{Tr}[\eta Q_a\beta] + \mathrm{Tr}[\eta q_n Q_a] \quad \text{(C.40)}$$

$$0 = \mathrm{Tr}[\eta Q_X Q_a Q_b] \quad \text{(C.41)}$$

$$0 = \mathrm{Tr}[\eta Q_X Q_a\beta] + \mathrm{Tr}[\eta Q_X q_n Q_a] \quad \text{(C.42)}$$

$$0 = \mathrm{Tr}\left[\eta Q_X Q_a\left(\alpha-\frac{1}{2}\right)\right] \quad \text{(C.43)}$$

$$0 = -\mathrm{Tr}\left[\eta Q_X Q_a\left(\frac{1}{2}-\gamma\right)\right] + \mathrm{Tr}[\eta Q_X Q_a q_n] \quad \text{(C.44)}$$

$$0 = \mathrm{Tr}\left[\eta Q_a\left(\alpha-\frac{1}{2}\right)\left(\left(\frac{1}{2}-\gamma\right)-q_n\right)\right] \quad \text{(C.45)}$$

$$0 = \mathrm{Tr}\left[\eta Q_a\beta\left(\left(\frac{1}{2}-\gamma\right)-q_n\right)\right] + \mathrm{Tr}\left[\eta Q_a q_n\left(\left(\frac{1}{2}-\gamma\right)-q_n\right)\right] \quad \text{(C.46)}$$

$$0 = \mathrm{Tr}\left[\eta Q_a Q_b\left(\left(\gamma-\frac{1}{2}\right)+q_n\right)\right]. \quad \text{(C.47)}$$

In all of the above sets, we have assumed that the modular weights of all PV fields satisfy sum rules reminiscent of those satisfied by the light sector, (5.23). Indeed, this will be baked directly into our choice of PV fields. We have also used the total derivative identities (5.19). In addition to the above conditions, we must enforce the sum rules of [92]:

$$-N-29 = \mathrm{Tr}[\eta] \tag{C.48}$$
$$-10 = \mathrm{Tr}[\eta\alpha] \tag{C.49}$$
$$-4 = \mathrm{Tr}[\eta\alpha^2] \tag{C.50}$$
$$0 = \mathrm{Tr}\Big[\eta\beta\Big] \tag{C.51}$$
$$0 = \mathrm{Tr}\Big[\eta\beta^2\Big] \tag{C.52}$$
$$0 = \mathrm{Tr}\Big[\eta\beta\alpha\Big]. \tag{C.53}$$

Conditions for Anomaly Matching

By drawing an analogy with the calculation of [379], we infer that in four dimensions the anomaly polynomial for the FIQS model has the form [257]

$$I_6 = \Big(-\frac{b}{4\pi}\sum_{i=1}^{3} G_i + \frac{\delta_X}{8\pi}F_X\Big)\Big(\mathrm{tr}(R^2) - \sum_n (F_n^{SU(3)})^2 - \sum_n (F_n^{SU(2)})^2 - \sum_n (F_n^{SO(10)})^2 - \sum_{a=1}^{7}(F_a)^2 - (F_X)^2 + 2\sum_i G_i^2\Big) \tag{C.54}$$

where

$$G_i = dZ_i \tag{C.55}$$
$$Z_i = \frac{1}{2i}\frac{d(T^i-\bar{T}^i)}{T^i+\bar{T}^i} \tag{C.56}$$
$$\tag{C.57}$$

and

$$\mathrm{tr}(R^2) = R^a{}_b R^b{}_a \tag{C.58}$$
$$= \frac{1}{4}R^\tau{}_{\epsilon\mu\nu}R^\epsilon{}_{\tau\rho\sigma}dx^\mu dx^\nu dx^\rho dx^\sigma \tag{C.59}$$
$$(F_A)^2 = \frac{1}{4}F_{A\mu\nu}F_{A\rho\sigma}dx^\mu dx^\nu dx^\rho dx^\sigma \tag{C.60}$$

In the above, we have implicitly assumed wedge products in the multiplication of differential forms. To get the 4D anomaly from the 6-form anomaly polynomial, one goes through the usual descent equations:

$$2\pi I_6 = dI_5 \tag{C.61}$$
$$\delta I_5 = dI_4 \tag{C.62}$$

For example, under a modular transformation, $Z_i \to Z_i + d\mathrm{Im}(F^i)$ so that the modular-gravity-gravity anomaly has the form

$$\int I_4 \supset \int -\frac{3}{32\pi^2}\left(\sum_{i=1}^{3}\mathrm{Im}(F^i)\right)R^{\tau}{}_{\omega\mu\nu}R^{\omega}{}_{\tau\rho\sigma}\epsilon^{\mu\nu\rho\sigma}\sqrt{g}d^4x \tag{C.63}$$

which is precisely what one would expect if one considers the modular-gravity-gravity anomaly to have the same form as a U(1)-gravity-gravity anomaly. To match this anomaly, we look at the anomalous contributions of PV fields with masses that are noninvariant under modular and $\mathrm{U}(1)_X$ transformations. The general form of their contribution is

$$\mathcal{L}_{\mathrm{anom}} = \int d^4\theta E(L_0 + L_1 + L_r) \tag{C.64}$$

with

$$L_0 = \frac{1}{8\pi^2}\left(\mathrm{Tr}[\eta\ln(\mathcal{M}^2)]\Omega_0 + K(\Omega_{GB}+\Omega_D)\right) \tag{C.65}$$

$$L_r = -\frac{1}{192\pi^2}\mathrm{Tr}\left[\eta\int d\ln(\mathcal{M})\Omega_r\right]. \tag{C.66}$$

Focusing on the second term of Eq. (C.64) , we again break up terms based on whether they contribute to the $\mathrm{U}(1)_X$ related anomalies or the pure modular anomaly.

$\mathbf{U(1)}_X$ Anomaly Conditions

To match the anomalies involving $\mathrm{U}(1)_X$, we require

$$0 = \frac{2}{3}\mathrm{Tr}\left[\eta\bar{Q}_X\left(2\bar{\alpha}^2 + \bar{\alpha} - 3\alpha^2\right)\right] \tag{C.67}$$

$$0 = \frac{2}{3}\mathrm{Tr}\left[\eta\bar{Q}_X\left(\bar{\beta} + 4\bar{\alpha}\bar{\beta} - 6\alpha\beta\right)\right] \tag{C.68}$$

$$0 = \frac{2}{3}\mathrm{Tr}\left[\eta\bar{Q}_X\left(2\bar{\beta}^2 - 3\beta^2\right)\right] \tag{C.69}$$

$$0 = -4\mathrm{Tr}\left[\eta\left(\alpha\bar{Q}_X q_n\right)\right] \tag{C.70}$$

$$0 = -4\mathrm{Tr}\left[\eta\left(\beta\bar{Q}_X q_n\right)\right] \tag{C.71}$$

$$8\pi^2\delta_X\delta_{mn} = -2\mathrm{Tr}\left[\eta\left(\bar{Q}_X q_n q_m\right)\right] \tag{C.72}$$

$$0 = \frac{1}{3}\mathrm{Tr}\left[\eta\left(Q_X\left(-4\bar{\alpha} + 6\alpha - 1\right)\left(1 - 2\bar{\gamma}\right)\right)\right] \tag{C.73}$$

$$0 = \frac{2}{3}\mathrm{Tr}\left[\eta Q_X\left(1-2\bar{\gamma}\right)\left(3\beta Q_X - 2\bar{\beta}\bar{Q}_X\right)\right)\right] \quad (C.74)$$

$$0 = 2\mathrm{Tr}\left[\eta\left(Q_X q_n\left(1-2\bar{\gamma}\right)\right)\right] \quad (C.75)$$

$$8\pi^2 b = \frac{1}{3}\mathrm{Tr}\left[\eta\left(1-2\bar{\gamma}\right)\left(3Q_X^2 - 2\bar{Q}_X^2\right)\right] \quad (C.76)$$

$$0 = \frac{2}{3}\mathrm{Tr}\left[\eta\bar{Q}_X\left(4\bar{\alpha}\bar{Q}_X + \bar{Q}_X - 6\alpha Q_X\right)\right] \quad (C.77)$$

$$0 = \frac{1}{3}\mathrm{Tr}\left[\eta\left(8\bar{\beta}\bar{Q}_X^2 - 12\beta Q_X\bar{Q}_X\right)\right] \quad (C.78)$$

$$0 = -4\mathrm{Tr}\left[\eta\left(Q_X\bar{Q}_X q_n\right)\right] \quad (C.79)$$

$$-4\pi^2\delta_X = \mathrm{Tr}\left[\eta\left(\frac{4\bar{Q}_X^3}{3} - 2Q_X^2\bar{Q}_X\right)\right] = -\frac{2}{3}\mathrm{Tr}\left[\eta Q_X^3\right]. \quad (C.80)$$

Note that the last term is fixed by cancellation of the linear divergence term Eq.(C.29).

Pure Modular Anomaly Conditions

To match the pure modular anomaly, we require

$$0 = \frac{1}{3}\mathrm{Tr}\left[\eta\left(1-2\bar{\gamma}\right)\left(-2\bar{\alpha}^2 - \bar{\alpha} + 3\alpha^2\right)\right] \quad (C.81)$$

$$0 = \frac{1}{3}\mathrm{Tr}\left[\eta\left(1-2\bar{\gamma}\right)\left(3\beta^2 - 2\bar{\beta}^2\right)\right] + 2\mathrm{Tr}\left[\eta\beta\left(1-2\bar{\gamma}\right)q_n\right] \quad (C.82)$$

$$0 = \frac{1}{3}\mathrm{Tr}\left[\eta\left(1-2\bar{\gamma}\right)\left(6\alpha\beta - \left(4\bar{\alpha}+1\right)\bar{\beta}\right)\right] + 2\mathrm{Tr}\left[\eta\alpha\left(1-2\bar{\gamma}\right)q_n\right] \quad (C.83)$$

$$-8\pi^2 b\delta_{mn} = \mathrm{Tr}\left[\eta q_m q_n\left(1-2\bar{\gamma}\right)\right]. \quad (C.84)$$

As for the third term of Eq. (C.64), we need it to vanish identically. This can be achieved so long as the following are satisfied

$$0 = \mathrm{Tr}\left[\eta x(1-2\bar{\gamma})^2\right] \quad (C.85)$$

$$0 = \mathrm{Tr}\left[\eta x\bar{q}_X(1-2\bar{\gamma})\right] \quad (C.86)$$

$$0 = \mathrm{Tr}\left[\eta x\bar{q}_X^2\right] \quad (C.87)$$

$$0 = \mathrm{Tr}\left[\eta\bar{\alpha}\bar{\beta}(1-2\bar{\gamma})\right] \quad (C.88)$$

$$0 = \mathrm{Tr}\left[\eta\bar{\alpha}\bar{\beta}\bar{q}_X)\right] \quad (C.89)$$

$$0 = \mathrm{Tr}\left[\eta\bar{\beta}^k(1-2\bar{\gamma})\right] \tag{C.90}$$

$$0 = \mathrm{Tr}\left[\eta\bar{\beta}^k\bar{q}_X\right]\mathrm{Tr}\left[\eta\bar{\beta}^3\bar{q}_X\right], \tag{C.91}$$

where $x = 1, \bar{\alpha}, \bar{\beta}, \bar{q}_X, \bar{\alpha}^2, \bar{\beta}^2, \bar{q}_X^2, \bar{\alpha}\bar{\beta}, \bar{\alpha}\bar{q}_X, \bar{\beta}\bar{q}_X$ and $k = 1, 2, 3$.

Solution to the Pauli-Villars Regularization Conditions

We will now elucidate a solution to the system described above. The solution consists of sets S_a, $a = 1, 2, \ldots$ of PV fields that address each of the divergence and anomaly sets of conditions more or less separately. For example, it is possible to introduce PV fields that cancel only the nonabelian divergences and contribute to no other conditions. We will try to follow the same strategy for all the sets of conditions described above. It is not entirely possible to do so - for example, fields that solve the modular anomaly conditions will generically contribute to modular divergences. Of course, this is far from the only way to tackle the system, but it is straightforward method to illustrate that a solution can be found. To this end, we define the notion of clone fields for PV fields. For a given pair of PV fields $\left(\Phi^P, \Phi'^P\right)$, we define clone fields $\left(\Phi^P_{cl}, \Phi'^P_{cl}\right)$ that have almost the same parameters (α, β, q_n, $\ldots$) and quantum numbers as the original pair but with negative signature. We say almost here because this notion is only useful if the $\left(\Phi^P, \Phi'^P\right)$ have quantum numbers different from the clones so that the two sets cancel each other's contributions to some subset of the conditions, but not all conditions. As a concrete example, which will be described below, one can introduce PV fields with nonabelian gauge interactions to eliminate divergences associated with those same interactions. One can then introduce clone PV fields without gauge interactions that exactly cancel the contributions of the gauge charged PV fields to all other terms. The primary advantage of this technique is tidiness.

PV Fields for $\mathrm{U}(1)_X$ Anomaly Matching

The fields described here will satisfy Eqs. (C.67)–(C.80) and will contribute to some of the $\mathrm{U}(1)_X$ divergence conditions (C.24)–(C.34). In particular, only PV fields with $\bar{Q}_X \neq 0$ contribute to Eq. (C.29), so this condition will be taken care of by this sector only. The sets of PV fields we need are

- S_1: A set of PV fields with modular invariant masses,$\alpha_1 = \alpha'_1 = \bar{\gamma}_1 = 1/2$, and $\bar{q}_n^{(1)} = 0$ and modular weights of the form $(q^{(1)})^C_m = q^P_{(1)}\delta^n_m$ and clone fields with no $\mathrm{U}(1)_X$.

- S_2 : A set of PV fields with $\bar{\alpha}_2 = \bar{\beta}_2 = \bar{\gamma}_2 = \bar{Q}_X^{(2)} = (q^{(2)})^C_n = 0$ and clone fields with no $\mathrm{U}(1)_X$ charge.

We then place the following conditions on the parameters for these fields:

$$\begin{aligned}
\mathrm{Sum}\left[Q_X^{(L)}\right] &= -\mathrm{Sum}\left[\eta Q_X^{(1)}\right] && \text{(C.1)}\\
\mathrm{Sum}\left[(Q_X^{L})^3\right] &= -\mathrm{Tr}\left[\eta_1 (Q_X^{(1)})^3\right] && \text{(C.2)}\\
0 &= \mathrm{Tr}\left[\eta_1 \bar{Q}_X^{(1)}\left(1-3\alpha_1^2\right)\right] && \text{(C.3)}\\
0 &= \left[\eta_1 \bar{Q}_X^{(1)} \alpha_1 q_n^{(1)}\right] && \text{(C.4)}\\
0 &= \mathrm{Tr}\left[\eta_1 \alpha_1 \bar{Q}_X^{(1)} Q_X^{(1)}\right] && \text{(C.5)}\\
0 &= \mathrm{Tr}\left[\eta (\bar{Q}_X^{(1)})^2\right] && \text{(C.6)}\\
0 &= \mathrm{Tr}\left[\eta (\bar{Q}_X^{(1)})^3\right] && \text{(C.7)}\\
0 &= \mathrm{Tr}\left[\eta (\bar{Q}_X^{(1)})^4\right] && \text{(C.8)}\\
0 &= \mathrm{Tr}\left[\eta \bar{Q}_X^{(1)} q_n^{(1)}\right] && \text{(C.9)}\\
0 &= \mathrm{Tr}\left[\eta \bar{Q}_X^{(1)} Q_X^{(1)} q_n^{(1)}\right] && \text{(C.10)}\\
-4\pi^2 \delta_X \delta_{nm} &= \mathrm{Tr}\left[\eta \bar{Q}_X^{(1)} q_n^{(1)} q_m^{(1)}\right] && \text{(C.11)}\\
2\pi^2 \delta_X &= -\frac{1}{3}\mathrm{Sum}\left[(Q_X^{(L)})^3\right] = \mathrm{Tr}\left[\eta \bar{Q}_X^{(1)} (Q_X^{(1)})^2\right]. && \text{(C.12)}
\end{aligned}$$

Once again, the first condition is a linear divergent term that can only be cancelled by fields with masses that are noninvariant under $\mathrm{U}(1)_X$. This in turn forces the correct coefficient for the pure $\mathrm{U}(1)_X$ anomaly in the last condition. While the second set must satisfy

$$\begin{aligned}
0 &= \mathrm{Tr}[\eta_2] && \text{(C.13)}\\
0 &= \mathrm{Tr}\left[\eta_2 \alpha_2 Q_X^{(2)}\right] && \text{(C.14)}\\
0 &= \mathrm{Tr}\left[\eta_2 \beta_2 Q_X^{(2)}\right] && \text{(C.15)}\\
8\pi^2 b &= \mathrm{Tr}\left[\eta_2 (Q_X^{(2)})^2\right]. && \text{(C.16)}
\end{aligned}$$

The first condition here comes from Eq. (C.85) and potentially can be relaxed.

PV Fields for Modular Anomaly Matching

The fields described here will satisfy conditions (C.81)–(C.84) and contribute to the modular divergence conditions (C.20)–(C.22). The sets are

- S_3: A set of pairs of PV fields with $\beta_3 = \beta_3' = 0$, $\bar{q}_n^{(3)} = q_n'^{(3)} = 0$.
- S_4: A set of pairs of PV fields with $\alpha_4 = \alpha_4' = \beta_4 = \beta_4' = \bar{q}_4^n = 0$, $(q^{(4)})_m^C = (q^{(4)})^P \delta_m^n$, and clone fields with no modular weights.

These fields will also contribute to the modular divergence conditions, as outlined below. We also have to consider the $\hat{\phi}$ fields of [92] here since they have noninvariant masses under a modular transformation. These fields have no β or modular weight parameters but do have $f_{\hat{\phi}} = \hat{\alpha}K$. then the conditions the S_3, S_4, and $\hat{\phi}$ fields must satisfy are

$$0 = \mathrm{Tr}\left[\hat{\eta}(1-2\bar{\hat{\alpha}})^2\right] + \mathrm{Tr}\left[\eta_3(1-2\bar{\alpha}_3)^2\right] \tag{C.17}$$

$$0 = \mathrm{Tr}\left[\hat{\eta}\bar{\hat{\alpha}}(1-2\bar{\hat{\alpha}})^2\right] + \mathrm{Tr}\left[\eta_3\bar{\alpha}_3(1-2\bar{\alpha}_3)^2\right] \tag{C.18}$$

$$0 = \mathrm{Tr}\left[\hat{\eta}\bar{\hat{\alpha}}^2(1-2\bar{\hat{\alpha}})^2\right] + \mathrm{Tr}\left[\eta_3\bar{\alpha}_3^2(1-2\bar{\alpha}_3)^2\right] \tag{C.19}$$

$$0 = \mathrm{Tr}\left[\hat{\eta}\left(1-2\bar{\hat{\alpha}}\right)\left(-2\bar{\hat{\alpha}}^2-\bar{\hat{\alpha}}+3\hat{\alpha}^2\right)\right] + \mathrm{Tr}\left[\eta_3\left(1-2\bar{\alpha}_3\right)\left(-2\bar{\alpha}_3^2-\bar{\alpha}_3+3\alpha_3^2\right)\right] \tag{C.20}$$

and

$$-8\pi^2 b = \mathrm{Tr}\left[\eta_4 q_4^P q_4^P\right] = 2\mathrm{Sum}\left[\eta_4 q_4^P q_4^P\right]. \tag{C.21}$$

PV Fields for the Regulation of Modular Divergences

Here we introduce fields that can cancel the contributions to Eqs. (C.20)–(C.22) from the $\dot{Y}$, S_3, and S_4 and contribute to the sum rules in Eqs. (3.37), (3.38) and (A.16) of [92]. The only new set we introduce here is

- S_5 : A set of pairs of PV fields with $\bar{\gamma}_5 = \frac{1}{2}$ and $(\bar{q}^{(5)})_n^C = 0$ with $(q^{(5)})_m^C = (q^{(5)})^P \delta_m^n$.

Then the conditions we must satisfy are

$$\begin{aligned} 0 = {} & (N+2)\dot{\beta}\left(\frac{1}{2}-\dot{\beta}\right)^2 - A_1\left(\frac{1}{2}-\dot{\beta}\right)^2 - \mathrm{Sum}\left[\eta_5\beta_5\left(\frac{1}{2}-\alpha_5\right)^2\right] - \mathrm{Sum}\left[\eta_5\beta_5'\left(\frac{1}{2}-\alpha_5'\right)^2\right] \\ & -\mathrm{Sum}\left[\eta_5 q_5^P\left(\frac{1}{2}-\alpha_5\right)^2\right] + \mathrm{Sum}\left[\eta_5 q_5^P\left(\frac{1}{2}-\alpha_5'\right)^2\right] \end{aligned} \tag{C.22}$$

$$\begin{aligned}
0 &= (N+2)\dot{\beta}\left(\frac{1}{2}-\dot{\beta}\right) - A_1\left(\frac{1}{2}-\dot{\beta}\right) - 2A_2\dot{\beta}\left(\frac{1}{2}-\dot{\beta}\right) + 2A_2\left(\frac{1}{2}-\dot{\beta}\right) \\
&\quad -\frac{1}{2}\left(\mathrm{Sum}\left[\eta_5\beta_5(1-2\alpha_5)(1-2\gamma_5)\right] + \mathrm{Sum}\left[\eta_5\beta_5'(1-2\alpha_5')(1-2\gamma_5')\right]\right) \\
&\quad +\mathrm{Sum}\left[\eta_5 q_5^P\beta_5(1-2\alpha_5)\right] - \mathrm{Sum}\left[\eta_5 q_5^P\beta_5'(1-2\alpha_5')\right] - \frac{1}{2}\mathrm{Sum}\left[\eta_5 q_5^P(1-2\alpha_5)(1-2\gamma_5)\right] \\
&\quad -\frac{1}{2}\mathrm{Sum}\left[\eta_5 q_5^P(1-2\alpha_5')(1-2\gamma_5')\right] + \mathrm{Sum}\left[\eta_5 q_5^P q_5^P(1-2\alpha_5)\right] + \mathrm{Sum}\left[\eta_5 q_5^P q_5^P(1-2\alpha_5')\right] \\
&\quad +2\mathrm{Sum}\left[\eta_4 q_4^P q_4^P\right] \qquad \text{(C.23)} \\
0 &= (N+2)\frac{\dot{\beta}^2}{2} - A_1\dot{\beta} + \frac{A_2}{2} - A_1\dot{\beta}^2 + 2A_2\dot{\beta} - A_3 + \frac{1}{2}\left(\mathrm{Sum}\left[\eta_5\beta_5^2(1-2\gamma_5)\right] + \mathrm{Sum}\left[\eta_5\beta_5'^2(1-2\gamma_5')\right]\right) \\
&\quad -\left(\mathrm{Sum}\left[\eta_5 q_5^P\beta_5^2\right] - \mathrm{Sum}\left[\eta_5 q_5^P\beta_5'^2\right]\right) + \left(\mathrm{Sum}\left[\eta_5\beta_5 q_5^P(1-2\gamma_5)\right] - \mathrm{Sum}\left[\eta_5\beta_5' q_5^P(1-2\gamma_5')\right]\right) \\
&\quad -2\left(\mathrm{Sum}\left[\eta_5\beta_5 q_5^P q_5^P\right] + \mathrm{Sum}\left[\eta_5\beta_5' q_5^P q_5^P\right]\right) + \frac{1}{2}\left(\mathrm{Sum}\left[\eta_5(1-2\gamma_5)q_5^P q_5^P\right] + \mathrm{Sum}\left[\eta_5(1-2\gamma_5')q_5^P q_5^P\right]\right) \\
&\quad +\mathrm{Sum}\left[\eta_4 q_4^P q_4^P\right]. \qquad \text{(C.24)}
\end{aligned}$$

We include an explicit P in the modular weights simply to remind ourselves that we sum over the "P" index and not the "n" index since C = (P,n).

PV Fields for the Regulation of U(1)$_X$ Divergences

Here we introduce fields that cancel the contributions to Eqs. (C.24)–(C.34) from the $\dot{Y}$, S_1, and S_2. Note that we will omit Eq. (C.29) since has been taken care of above. We introduce the following set:

- S_6: A set of pairs of PV fields with $Q_X^{(6)} = -Q_X'^{(6)}$ and $\bar{q}_n^{(6)} = 0$ and clone fields without U(1)$_X$ charge.

Then the conditions we must satisfy are

$$\begin{aligned}
0 &= 12C_{GS}'\dot{\beta} + 2\mathrm{Sum}\left[\eta_2 Q_{(2)}^X\beta_2\right] + \mathrm{Sum}\left[\eta_6 Q_{(6)}^X\beta_6\right] - \mathrm{Sum}\left[\eta_6 Q_{(6)}^X\beta_6'\right] \qquad \text{(C.25)} \\
0 &= 12C_{GS}'(1-\dot{\beta}) + 2\mathrm{Sum}\left[\eta_2 Q_{(2)}^X\alpha_2\right] + \mathrm{Sum}\left[\eta_6 Q_{(6)}^X\alpha_6\right] - \mathrm{Sum}\left[\eta_6 Q_{(6)}^X\alpha_6'\right] \qquad \text{(C.26)} \\
0 &= 12C_{GS}'\left(\frac{1}{2}-\dot{\beta}\right)^2 + \mathrm{Sum}\left[\eta_6 Q_{(6)}^X\left(\alpha_6-\frac{1}{2}\right)\right] - \mathrm{Sum}\left[\eta_6 Q_{(6)}^X\left(\alpha_6'-\frac{1}{2}\right)\right] \qquad \text{(C.27)} \\
0 &= 12C_{GS}'\dot{\beta}\left(\frac{1}{2}-\dot{\beta}\right) + Q_{1X}^{(L)}\left(\frac{1}{2}-\dot{\beta}\right) + \mathrm{Sum}\left[\eta_2 Q_{(2)}^X q_2^P\right] + \mathrm{Sum}\left[\eta_6 Q_{(6)}^X\beta_6\left(\alpha_6-\frac{1}{2}\right)\right] \\
&\quad -\mathrm{Sum}\left[\eta_6 Q_{(6)}^X\beta_6'\left(\alpha_6'-\frac{1}{2}\right)\right] - \mathrm{Sum}\left[\eta_6 Q_{(6)}^X q_6^P\left(\alpha_6-\frac{1}{2}\right)\right] - \mathrm{Sum}\left[\eta_6 Q_{(6)}^X q_6^P\left(\alpha_6'-\frac{1}{2}\right)\right] \qquad \text{(C.28)}
\end{aligned}$$

$$\begin{aligned}
0 &= 12\dot{\beta}^2 C'_{GS} - 2\dot{\beta}Q^{(L)}_{1X} + Q^{(L)}_{2X} + \mathrm{Sum}\left[\eta_1 Q^X_{(1)} q^P_1 q^P_1\right] + \mathrm{Sum}\left[\eta_1 Q'^X_{(1)} q^P_1 q^P_1\right] + \mathrm{Sum}\left[\eta_6 Q^X_{(6)} \beta^2_6\right] \\
&\quad - \mathrm{Sum}\left[\eta_6 Q^X_{(6)} \beta'^2_6\right] + 2\mathrm{Sum}\left[\eta_6 Q^X_{(6)} \beta_6 q^P_6\right] + 2\mathrm{Sum}\left[\eta_6 Q^X_{(6)} \beta'_6 q^P_6\right] \qquad \text{(C.29)} \\
0 &= \frac{1}{2}\mathrm{Tr}\left[(Q^X_{(L)})^2\right] - R^{(L)}_X - \mathrm{Sum}\left[\eta_1 (Q^X_{(1)})^2 q^P_1\right] + \mathrm{Sum}\left[\eta_1 (Q'^X_{(1)})^2 q^P_1\right] + \mathrm{Sum}\left[\eta_2 (Q^X_{(2)})^2\right] \\
&\quad + \mathrm{Sum}\left[\eta_6 (Q^X_{(6)})^2 \left(\frac{1}{2} - \gamma_6\right)\right] + \mathrm{Sum}\left[\eta_6 (Q^X_{(6)})^2 \left(\frac{1}{2} - \gamma'_6\right)\right] \qquad \text{(C.30)} \\
0 &= \mathrm{Tr}\left[(Q^X_{(L)})^2 \left(\frac{1}{2} - \dot{\beta}\right)\right] - \mathrm{Sum}\left[\eta_2 (Q^X_{(2)})^2\right] + \mathrm{Sum}\left[\eta_6 (Q^X_{(6)})^2 \left(\alpha_6 - \frac{1}{2}\right)\right] \\
&\quad + \mathrm{Sum}\left[\eta_6 (Q'^X_{(6)})^2 \left(\alpha_6 - \frac{1}{2}\right)\right] \qquad \text{(C.31)} \\
0 &= -\frac{1}{2} Q^{(L)}_{1X} \left(\frac{1}{2} - \dot{\beta}\right) + R^{(L)}_X \left(\frac{1}{2} - \dot{\beta}\right) + \mathrm{Sum}\left[\eta_2 Q^X_{(2)} (\alpha_2 + \gamma_2)\right] + \mathrm{Sum}\left[\eta_2 Q^X_{(2)} q^P_2\right] \\
&\quad + \mathrm{Sum}\left[\eta_6 Q^X_{(6)} \left(\alpha_6 - \frac{1}{2}\right)\left(\frac{1}{2} - \gamma_6\right)\right] - \mathrm{Sum}\left[\eta_6 Q^X_{(6)} \left(\alpha'_6 - \frac{1}{2}\right)\left(\frac{1}{2} - \gamma'_6\right)\right] - \mathrm{Sum}\left[\eta_6 Q^X_{(6)} q^P_6 \left(\alpha_6 - \frac{1}{2}\right)\right] \\
&\quad - \mathrm{Sum}\left[\eta_6 Q^X_{(6)} q^P_6 \left(\alpha'_6 - \frac{1}{2}\right)\right] \qquad \text{(C.32)} \\
0 &= -\dot{\beta}\mathrm{Tr}\left[(Q^X_{(L)})^2\right] + R^{(L)}_X + \mathrm{Sum}\left[\eta_1 (Q^X_{(1)})^2 q^P_1\right] - \mathrm{Sum}\left[\eta_1 (Q'^X_{(1)})^2 q^P_1\right] + \mathrm{Sum}\left[\eta_6 (Q^X_{(6)})^2 \beta_6\right] \\
&\quad + \mathrm{Sum}\left[\eta_6 (Q^X_{(6)})^2 \beta'_6\right] \qquad \text{(C.33)} \\
0 &= -6\dot{\beta} C'_{GS} + \beta Q^{(L)}_{1X} + \frac{1}{2} Q^{(L)}_{1X} - Q^{(L)}_{2X} - \mathrm{Sum}\left[\eta_1 Q^X_{(1)} q^P_1 q^P_1\right] - \mathrm{Sum}\left[\eta_1 Q'^X_{(1)} q^P_1 q^P_1\right] + \mathrm{Sum}\left[\eta_2 Q^X_{(2)} \beta_2\right] \\
&\quad + \mathrm{Sum}\left[\eta_2 Q^X_{(2)} q^P_2\right] + \mathrm{Sum}\left[\eta_6 Q^X_{(6)} \beta_6 \left(\frac{1}{2} - \gamma_6\right)\right] - \mathrm{Sum}\left[\eta_6 Q^X_{(6)} \beta'_6 \left(\frac{1}{2} - \gamma'_6\right)\right] - \mathrm{Sum}\left[\eta_6 Q^X_{(6)} \beta_6 q^P_6\right] \\
&\quad - \mathrm{Sum}\left[\eta_6 Q^X_{(6)} \beta'_6 q^P_6\right] + \mathrm{Sum}\left[\eta_6 Q^X_{(6)} q^P_6 \left(\frac{1}{2} - \gamma_6\right)\right] + \mathrm{Sum}\left[\eta_6 Q^X_{(6)} q^P_6 \left(\frac{1}{2} - \gamma'_6\right)\right]. \qquad \text{(C.34)}
\end{aligned}$$

PV Fields for the Regulation of Nonabelian Divergences

Here we introduce fields to cancel Eqs. (C.35)-(C.37). We introduce a separate PV set for each of the nonabelian factors of the FIQS gauge group as follows

- S_7: A set of pairs of PV fields in the fundamental of SU(3) (anti-fundamental for the primed fields) with no modular weights, uniform constants, and clone fields with no gauge charges. By uniform coefficients, we mean that α^C and β^C are independent of index within the set: $\alpha^C = \alpha$ and $\beta^C = \beta$.

- S_8: A set of pairs of PV fields in the fundamental of SU(2) with no modular weights, uniform constants, and clone fields with no gauge charges.

- S_9: A set of pairs of PV fields in the **16** (and $\overline{\mathbf{16}}$ for primed fields) of SO(10) and a set of pairs of PV fields in the **10** of SO(10), all with no modular weights, uniform constants, and clone fields with no gauge charges.
- S_{10}: A set of PV fields with $\gamma = \gamma' = 1/2$, zero modular weights, a nonzero trace $\mathrm{U}(1)_X$ charge matrix, and charged under the nonabelian gauge groups in the same reps as the light fields and clone fields without nonabelian gauge charges.

Let us discuss this choice briefly. First we need to check the number of fields in a given representation. This is because we care about the quantity

$$C^M_{(\mathcal{G})} = C^m_{(\mathcal{G})} N_{(\mathcal{G})}, \tag{C.35}$$

which comes from the first term in the list above. The technique in [185] relies on having an even number of light fields in a given representation for all the gauge factors. Let us check if this is the case for the FIQS model. See Appendix C for a detailed breakdown of the FIQS spectrum. For the SU(3) of FIQS, the total number of triplets charged under this gauge group is

$$\begin{aligned} N^{SU(3)}_{Q_L} + N^{SU(3)}_{u_L} + N^{SU(3)}_{u_2} + \sum_{i=1}^{2} N^{SU(3)}_{d_i} + \sum_{j=1}^{4} N^{SU(3)}_{D_j} + \sum_{j=1}^{2} N^{SU(3)}_{\bar{D}_j} \\ = 6 + 3 + 12 + 15 = 36. \end{aligned} \tag{C.36}$$

For the SU(2) of FIQS, there are

$$\begin{aligned} N^{SU(2)}_{Q_L} + \sum_{i=1}^{4} N^{SU(2)}_{\bar{G}_i} + \sum_{i=1}^{5} N^{SU(2)}_{G_i} + \sum_{i=1}^{4} N^{SU(2)}_{F_i} \\ = 9 + 3 + 33 + 3 = 48 \end{aligned} \tag{C.37}$$

doublets. Note that we have used the fact that each state in the table of Appendix C has a degeneracy of 3, with the exception of the states Y_1, Y_2, and Y_3. The number of states charged under the SU(3) and SU(2) groups are indeed even, but this is not the case for SO(10), since there are only 3 **16**'s charged under this gauge factor. To resolve this, we first list the Casimirs for the first few SO(10) reps.

$$\text{Fundamental} \quad \mathbf{10} : C_{10} = 1 \tag{C.38}$$

$$\text{Spinor} \quad \mathbf{16} : C_{16} = 2 \tag{C.39}$$

$$\text{Adjoint} \quad \mathbf{45} : C_{45} = 8 \tag{C.40}$$

Note that these satisfy the sum rule (5.12) of [92] when considering the fields charged under SO(10):

$$C_{45} - 3C_{16} + 2C_{16} \sum_i \delta^i_n = 8 - 6 + 4 = 6 \tag{C.41}$$

The first divergence we cancel is $\mathrm{Tr}(\eta T_a T_b)$. The $\dot{Y}$ give the negative of the contribution of the light fields, so in the case of SO(10) this trace is simply $-3C_{16} = -6$. Since PV fields come in pairs, we cancel this with at least 2 fields and we have

$$3C_{16} = 2\sum_P \eta^P C^P \tag{C.42}$$

Thus, we have two options. We can have a PV pair in the **16** (and $\overline{\mathbf{16}}$) plus a PV pair in the **10** or we can have 3 pairs of PV fields in the **10**. The other divergence from gauge interactions we have to get rid of is the linear divergence proportional to the Casimir. We note that the $\dot{Y}$'s here give

$$(-1)\left(\frac{F}{2} - F + \sum_n q_n^a F^n\right) C_{(\mathcal{G}_a)} = \left(-\frac{1}{2}\right)(C_{GS} - C_{\mathcal{G}}) \tag{C.43}$$

since $\dot{\alpha} + \dot{\beta} = 1$. The overall sign is the sum of the signatures. Cancellation then requires

$$\begin{aligned} \frac{C_{GS} - C_{\mathcal{G}}}{2} &= \sum_C \eta^C C_{\mathcal{G}_C}\left(\frac{1}{2} - \gamma^C\right) && \text{(C.44)} \\ &= \sum_P \eta^P C_{\mathcal{G}_P}\left(1 - 2\bar{\gamma}^P\right). && \text{(C.45)} \end{aligned}$$

provided that the PV fields have no modular weights. The first sum is over all PV fields whereas the second is over PV pairs. Both of our potential solutions can work here since we have either 1 or 2 free parameters in the γ's. In the list of sets of PV fields above, we opted for the combination of PV fields in the **10** and **16** of SO(10). For the last nonabelian divergence, Eq. (C.36), we explicitly write out the contribution from the $\dot{Y}$ so that is takes the form

$$0 = \mathrm{Tr}(Q_X^L)C_{\mathcal{G}}^m + \mathrm{Tr}\left[\eta Q_X^{PV} T_a T_b\right], \tag{C.46}$$

where $C_{\mathcal{G}}^m$ is the Casimir of the representation of the matter fields. If we consider fields from the set S_{10}, then this becomes

$$\begin{aligned} -\mathrm{Tr}(Q_X^L) &= \mathrm{Tr}(Q_X^{PV}) = 2\mathrm{Sum}\left[\eta\bar{Q}_X^{PV}\right] && \text{(C.47)} \\ & && \text{(C.48)} \end{aligned}$$

The fields in S_{10} contribute to Eq. (C.35) but not to Eq. (C.37) since we have restricted their γ parameters to be $\gamma = \frac{1}{2}$. Their contribution to Eq. (C.35) is not an issue since we can simply include more fields in the other sets described in this section to cancel their contribution. Finally, the clone fields ensure that none of the sets described in this section contribute to other conditions.

PV Fields for the Regulation of Abelian Divergences

Here we satisfy the conditions Eqs. (C.40)–(C.47). The $\dot{Y}$ contribute here, and to cancel them we will need to introduce fields with $\bar{q}_n \neq 0$, which is different from all other fields considered thus far. This would alter some of the expressions we have used above, but we will not consider these alterations since we will employ clone fields that cancel contributions to previously considered terms from the fields introduced here. Specifically, we consider

- S_{11}: A set of pairs of PV fields such that the unprimed fields have the same abelian gauge charges as the light fields (including $\mathrm{U}(1)_X$), $\alpha_{11}^P = \dot{\alpha}$, $\beta_{11}^P = \dot{\beta}$, $q_n^{(11)} = -q_n^{(L)}$, $\alpha'^P = \frac{1}{2}$, $\beta'^P = Q'^{PV}_X = q'^{(11)}_n = 0$, and positive signature and clone fields with no $\mathrm{U}(1)_a$ charges.

- S_{12}: A set of pairs of PV fields with no β parameters or modular weights and with $\alpha_{12}^P = \alpha'^P_{12} = 1/2$, $Q'^X_{(12)} = 0$, $Q^X_{(12)} = 4Q^X_{(L)}$, and $\mathrm{U}(1)_a$ charges $Q^a_{(12)} = Q^a_{(L)}/\sqrt{2}$ and negative signature and clone fields with no $\mathrm{U}(1)_a$ charges.

These satisfy

$$
\begin{aligned}
0 &= -S_{ab}^{(L)} + 2\mathrm{Sum}\left[\eta_{11} Q^a_{(11)} Q^b_{(11)}\right] + 2\mathrm{Sum}\left[\eta_{12} Q^a_{(12)} Q^b_{(12)}\right] && \text{(C.49)}\\
0 &= -2\pi^2\delta_X + \mathrm{Sum}\left[\eta_{11} Q^X_{(11)} Q^a_{(11)} Q^b_{(11)}\right] + \mathrm{Sum}\left[\eta_{12} Q^X_{(12)} Q^a_{(12)} Q^b_{(12)}\right] && \text{(C.50)}\\
0 &= -\dot{\beta} S_a^{(L)} + R_a^{(L)} + \mathrm{Sum}\left[\eta_{11} Q^a_{(11)} Q^X_{(11)} \beta_{11}\right] + \mathrm{Sum}\left[\eta_{11} q_n^{(11)} Q^a_{(11)} Q^X_{(11)}\right] && \text{(C.51)}\\
0 &= -\left(\frac{1}{2} - \dot{\beta}\right) S_a^{(L)} + \mathrm{Sum}\left[\eta_{11} Q^X_{(11)} Q^a_{(11)} \left(\alpha_{11} - \frac{1}{2}\right)\right] && \text{(C.52)}\\
0 &= -\frac{1}{2} S_a^{(L)} + R_a^{(L)} + \mathrm{Sum}\left[\eta_{11} Q^X_{(11)} Q^a_{(11)} \left(\gamma_{11} - \frac{1}{2}\right)\right] + \mathrm{Sum}\left[\eta_{11} Q^X_{(11)} Q^a_{(11)} q_n^{(11)}\right] && \text{(C.53)}\\
0 &= -\frac{1}{2} S_{ab}^{(L)} + R_{ab}^{(L)} + \mathrm{Sum}\left[\eta_{11} Q^a_{(11)} Q^b_{(11)} \left(\left(\gamma_{11} - \frac{1}{2}\right) + q_n^{(11)}\right)\right], && \text{(C.54)}
\end{aligned}
$$

where again a subscript or superscript (L) implies a trace over the corresponding values of the light fields. Note that we have omitted some conditions that are automatically zero. There are also terms in the above that vanish for the choice of U(1) charges defined in this work but do not vanish for other choices. If one substitutes the parameters of S_{11} and S_{12} as per the discussion above, one sees that all the remaining conditions above are satisfied.

The FIQS spectrum

The FIQS model was described in [252, 172, 171, 102, 103]. The modular weights in this model are simple: the fields in the ith untwisted sector have $q_n^i = \delta_n^i$, and the twisted sector fields have $q_n = \frac{2}{3}$, except for the Y^i with $q_n^i = \delta_n^i + \frac{2}{3}$. Here we will focus in particular on the U(1) charges of the low-energy matter spectrum. The U(1) charge generators arising from the Cartan subalgebra of the $E_8 \times E_8$ and the corresponding charges were worked out in [102, 103].Table 2 of [172] lists the charges of the massless spectrum. However, the linear combinations of generators given in [252, 171] have a mixed anomaly:

$$\mathrm{Tr}(Q_6 Q_7 Q_X) = 1296. \tag{C.1}$$

To avoid this, one should re-define Q_6 and Q_7. The fix is very simple:

$$Q_6' = Q_6 - Q_7, \tag{C.2}$$
$$Q_7' = Q_6 + Q_7. \tag{C.3}$$

Below we produce a table of the new charge designations.

Table C.1: U(1) charges of the FIQS massless spectrum

(n_1, n_3)	Field	Rep	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6^N	Q_7^N	X
untwisted	Q_L	(3,2)	-6	-6	0	0	0	0	0	0
	u_L	($\bar{3}$,1)	6	0	0	-6	0	0	0	0
	$\bar{G}_1$	(1,2)	0	6	0	6	0	0	0	0
	$16'$	1	0	0	0	0	0	0	0	9
(0,0)	D_1	(3,1)	0	4	0	0	0	4	4	4
	$\bar{G}_2$	(1,2)	6	-2	0	0	0	4	4	4
	$\bar{A}_1$	1	-3	-2	-3	-3	-3	4	4	4
	$\bar{A}_2$	1	-3	-2	3	-3	3	4	4	4
	A_1	1	-3	-2	-3	3	3	4	4	4
	A_2	1	-3	-2	3	3	-3	4	4	4
(1,0)	S_4	1	6	4	0	0	-2	2	10	4
	S_5	1	6	4	0	0	-2	-4	-8	4
	S_6	1	6	4	0	0	-2	2	-2	-8
	$\bar{A}_3$	1	-3	-2	-3	-3	1	2	10	4
	$\bar{A}_4$	1	-3	-2	-3	-3	1	-4	-8	4
	$\bar{A}_5$	1	-3	-2	-3	-3	1	2	-2	-8
	A_3	1	-3	-2	3	3	1	2	10	4
	A_4	1	-3	-2	3	3	1	-4	-8	4
	A_5	1	-3	-2	3	3	1	2	-2	-8
(-1,0)	S_7	1	6	4	0	0	2	6	-2	4
	S_8	1	6	4	0	0	2	0	4	-8
	S_9	1	6	4	0	0	2	-6	-2	4
	$\bar{A}_6$	1	-3	-2	3	-3	-1	6	-2	4
	$\bar{A}_7$	1	-3	-2	3	-3	-1	0	4	-8
	$\bar{A}_8$	1	-3	-2	3	-3	-1	-6	-2	4
	A_6	1	-3	-2	-3	3	-1	6	-2	4
	A_7	1	-3	-2	-3	3	-1	0	4	-8
	A_8	1	-3	-2	-3	3	-1	-6	-2	4
(0,1)	d_1	($\bar{3}$, 1)	0	0	0	2	2	0	-8	4
	F_1	(1,2)	3	0	-3	-1	-1	0	-8	4
	$\bar{A}_9$	1	3	6	3	-1	-1	0	-8	4
	A_9	1	3	-6	3	-1	-1	0	-8	4
	$\bar{l}_1$	1	-6	0	0	-4	2	0	-8	4
	S_{10}	1	-6	0	0	2	-4	0	-8	4

(n_1, n_3)	Field	Rep	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6^N	Q_7^N	X
(1,1)	D_2	(3,1)	6	0	0	2	0	-2	-2	4
	u_2	$(\bar{3}$,1)	0	0	0	-4	0	-2	-2	4
	F_2	(1,2)	3	0	3	-1	-3	-2	-2	4
	F_3	(1,2)	3	0	-3	-1	3	-2	-2	4
	S_1	1	-6	0	0	2	0	4	4	-8
	Y_1	1	-6	0	0	2	0	-2	-2	4
	$\bar{A}_{10}$	1	3	6	-3	-1	-3	-2	-2	4
	$\bar{A}_{11}$	1	3	6	3	-1	3	-2	-2	4
	A_{10}	1	3	-6	3	-1	3	-2	-2	4
	A_{11}	1	3	-6	-3	-1	-3	-2	-2	4
(-1,1)	d_2	$(\bar{3},1)$	0	0	0	2	-2	-4	4	4
	F_4	(1,2)	3	0	3	-1	1	-4	4	4
	$\bar{A}_{12}$	1	3	6	-3	-1	1	-4	4	4
	A_{12}	1	3	-6	-3	-1	1	-4	4	4
	$\bar{l}_2$	1	-6	0	0	-4	-2	-4	4	4
	S_{11}	1	-6	0	0	2	4	-4	4	4
(0,-1)	$\bar{D}_1$	$(\bar{3},1)$	-3	2	-3	1	1	2	-2	4
	D_3	(3,1)	3	2	3	1	1	2	-2	4
	$\bar{G}_3$	(1,2)	0	2	0	4	-2	2	-2	4
	G_1	(1,2)	0	2	0	-2	4	2	-2	4
	S_2	1	0	-4	0	-2	-2	-4	4	-8
	Y_2	1	0	-4	0	-2	-2	2	-2	4
	l_1	1	0	-4	0	4	4	2	-2	4
	$\bar{l}_3$	1	0	8	0	-2	-2	2	-2	4
	$\bar{A}_{13}$	1	-9	2	3	1	1	2	-2	4
	A_{13}	1	9	2	-3	1	1	2	-2	4
(n_1, n_3)	Field	Rep	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6^N	Q_7^N	X
(1,-1)	$\bar{D}_2$	$(\bar{3},1)$	-3	2	3	1	-1	0	4	4
	D_4	(3,1)	3	2	-3	1	-1	0	4	4
	$\bar{G}_4$	(1,2)	0	2	0	4	2	0	4	4
	G_2	(1,2)	0	2	0	-2	-4	0	4	4
	S_3	1	0	-4	0	-2	2	0	-8	-8
	Y_3	1	0	-4	0	-2	2	0	4	4
	l_2	1	0	-4	0	4	-4	0	4	4
	$\bar{l}_4$	1	0	8	0	-2	2	0	4	4
	$\bar{A}_{14}$	1	-9	2	-3	1	-1	0	4	4
	A_{14}	1	9	2	3	1	-1	0	4	4
(-1,-1)	G_3	(1,2)	0	2	0	-2	0	4	-8	4
	G_4	(1,2)	0	2	0	-2	0	-2	10	4
	G_5	(1,2)	0	2	0	-2	0	-2	-2	-8
	l_3	1	0	-4	0	4	0	-2	10	4
	l_4	1	0	-4	0	4	0	4	-8	4
	l_5	1	0	-4	0	4	0	-2	-2	-8