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DIFFRACTION RADIATION DEFOCUSING OF AN ELECTRON RING*

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ABSTRACT

The influence upon axial stability in an electron ring of the diffraction radiation reaction force, generated by a ring moving in an acceleration column, is calculated theoretically. A stability criterion is obtained, and numerical examples show that the criterion is not an important constraint upon the choice of parameters or the operation of an electron ring accelerator.

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I. INTRODUCTION

It is well known that the diffraction radiation by an electron ring in the acceleration column of an electron ring accelerator (ERA) is an important effect insofar as it can cause significant loss of energy of the ring[1,2]. The effect of the diffraction radiation upon the internal dynamics of the ring has not so far been studied, although it is clear that the large energy radiation could easily have a significant effect upon ring stability in the axial direction, where the focusing - coming only from ions, images[3], and possibly from the accelerating field -- is weak.

In this note we study the contribution of diffraction radiation to the axial focusing forces of a ring, limiting our analysis, for convenience, to the case of a ring moving at relativistic speeds. We evaluate the defocusing force for two different geometries: In Section II we consider a charged rod and a current carrying rod moving past an infinite array of semi-infinite perfectly conducting plates, which geometry has the advantage that the problem may be analyzed analytically. In Section III we consider a charged ring in an accelerating column consisting of an infinitely long corrugated cylindrical waveguide. The effect of the ring current is not included in this model. In Section IV we evaluate the axial oscillation frequency resulting from defocusing forces, and in Section V we present some numerical examples.

We may obtain a rough estimate of the order-of-magnitude of

the diffraction defocusing from a simple physical model. Consider a charge, Q, moving along the axis of an acceleration column. The complete solution to Maxwell's equation is, in general, difficult to obtain, but roughly speaking there are image charges moving in concert with the charge Q. These images are slightly displaced behind the charge, leading to an axial field, E_z, at the charge and hence a net retarding force. The magnitude of the displacement is difficult to estimate. The gradient of this field, which is what determines the focusing force, is, however, not sensitive to the image charge displacement. Thus, in a column of radius a , the field gradient $\frac{dE_z}{dz}$ in the frame of the moving charge is approximately given by

$$\left(\frac{dE_z}{dz}\right)^* \approx \frac{-Q}{a^3} \qquad (1.1)$$

Thus in the laboratory frame, dE_z/dz is proportional to the relativistic γ-factor of the charge.

The defocusing force of (1.1) will give a shift in the square of the axial oscillation frequency in the ring frame, $(\omega_o{}^* \nu)^2$, of amount

$$\Delta(\omega_o{}^* \nu)^2 = \frac{1}{m_o \gamma_\perp{}^*} \left(\frac{dE_z}{dz}\right)^* , \qquad (1.2)$$

where $\omega_o{}^*$ is the revolution frequency in the ring frame and $\gamma_\perp{}^*$ is the relativistic γ-factor for the circulating electrons. This formula is derived in Section IV, although many readers may consider

it obvious. Thus, from (1.1) and (1.2):

$$\Delta\nu^2 \approx - \frac{Nr_e}{\gamma_\perp^* a} \tag{1.3}$$

where N is the number of electrons in the ring, and r_e is the classical electron radius. Taking $N = 10^{13}$, $a = 10$ cm, and $\gamma_\perp^* = 40$ -- typical parameters of and ERA -- we obtain $\Delta\nu^2 = - 7. \times 10^{-3}$, which is small in comparison with the expected self-focusing.

We have, in this simple-minded discussion, ignored magnetic images which for a smooth accelerating column would greatly reduce the $\Delta\nu^2$. However, the structure of an accelerating column destroys the nearly perfect electric and magnetic cancellation of a smooth pipe and thus our result -- obtained from considering only electric images -- is a fair estimation of the effect.

II SEMI-INFINITE PLATES

In this section we consider as a model of an acceleration column, an infinite set of semi-infinite conducting planes; i.e. a comb. The electron ring is replaced by a charged rod and a current carrying rod moving past the comb. The advantage of this model is that the defocusing force -- just like the radiation loss[4] -- can be calculated analytically.

We employ exactly the notation of Ref. 4, which reference will have to be consulted to make the present calculation understandable.

The plates are taken in the x-y plane and extend from $-\infty < y < \infty$, $x > o$. They are separated by the distance $2\pi L$, while the rod, located at $x = -x_o$, is parallel to the y-axis and moves in the z-direction with speed v.

1. Charged Rod

We first consider a rod having charge q per unit length. We want to compute the electric field in the z-direction due to the charges and currents on the plates, but we only need this field E_{sz} evaluated at $x = -x_o$, $z = vt + \sigma$, and averaged over one period of the structure. From Eqns. 8 and 23 and the argument leading from Eqn. 23 to Eqn. 36 in Ref. 4, it is easy to see that

$$\langle E(\sigma)\rangle \equiv \langle E_{sz}(-x_o,\ z=vt+\sigma, t)\rangle_t = \mathrm{Im}\left\{\left(\frac{4\pi}{\gamma}\right)\left(\frac{q}{2\pi L}\right)\left(\frac{1-i/\beta\gamma}{1+i/\beta\gamma}\right)\int_o^\infty d\lambda \cdot P^2(\lambda,\gamma)\ \exp\left[-\frac{2x_o\lambda}{L\gamma}+\frac{i\sigma\lambda}{L}\right]\right\}. \tag{2.1}$$

where $\beta = v/c$, and $\gamma = (1-\beta^2)^{-1/2}$, and $P(\lambda,\gamma)$ is given by Eqn. 34 of Ref. 4.

The evaluation of (2.1), in the limit of $\gamma \gg 1$, follows the procedure employed in Section 3 of Ref. 4. In particular, Eqn. 51 is modified to

$$\langle E(\sigma)\rangle \approx \frac{q}{2\pi L}\left(\frac{4\pi}{\gamma}\right)\mathrm{Im}\left\{\left(1-\frac{2i}{\beta\gamma}\right)\int_o^\infty d\lambda\, e^{-B\lambda}\,(1+z)\right\}, \tag{2.2}$$

with

$$B = \left(\frac{2x_o}{L} - \frac{i\sigma_o}{L}\right) \frac{1}{\gamma} \quad , \tag{2.3}$$

$$z = \frac{i}{\gamma} + \sqrt{2}\,(1+i)\,\zeta(1/2)\,\frac{\lambda^{\frac{1}{2}}}{\gamma} \quad , \tag{2.4}$$

and we have written $\sigma = \sigma_o/\gamma$. The expression (2.2) is correct, in the limit of large γ, through the first two terms. Evaluation of the integral yields:

$$\left\langle E(\sigma)\right\rangle = 2q\left\{\frac{\sigma_o}{4x_o^2} + \frac{1}{L\gamma^{\frac{1}{2}}}\,\frac{\zeta(1/2)}{2^{7/2}\pi}\left(\frac{2\pi L}{x_o}\right)^{3/2}\left(1+\frac{3}{4}\,\frac{\sigma_o}{x_o}\right)\right\} \tag{2.5}$$

from which follows:

$$\left\langle E(o)\right\rangle = \frac{q}{L\gamma^{1/2}}\,\frac{\zeta(1/2)}{2^{5/2}\pi}\left(\frac{2\pi L}{x_o}\right)^{3/2} , \tag{2.6}$$

$$\left\langle \frac{dE(\sigma)}{d\sigma}\right\rangle_{\sigma=0} = \frac{q\gamma}{2x_o^2}\left[1 + \frac{3\zeta(1/2)}{2^{5/2}\gamma^{1/2}}\left(\frac{2\pi L}{x_o}\right)^{1/2}\right] \tag{2.7}$$

The formula for $\left\langle E(o)\right\rangle$ shows that the average energy-loss decreases as $\gamma^{-1/2}$ - - which was the major result of Ref. 4. On the other hand, the leading term in the defocusing field varies linearly with γ. It is easy to see that this leading term corresponds in magnitude[5] to what one would expect from an image rod located at $x = + x_o$.

We have numerically evaluated $< E(\sigma)>$ for a number of values

of $\rho = 2\pi L/x_o$ and for γ ranging from 2 to 50. Taking $\rho = 0.5$, for $\gamma = 5$ the asymptotic formula is only in error by 3%, while for $\gamma \geq 10$ the error is less than 1%. For $\rho = 3.5$, the error is about 7% at $\gamma = 5$, but less than 1% for $\gamma \geq 20$.

The numerical calculations are important for evaluating how well $\langle E(\sigma) \rangle$ is approximated by its value and first derivative at $\sigma = 0$. The calculations showed that the diffraction fields (in contrast to the self-field of a rod) were well-approximated by the first two terms of a Taylor series over distances $\sigma \ll x_o/\gamma$; i.e. $\sigma_o \ll x_o$. In applications of this model to an ERA we shall always satisfy this condition; i.e. the ring minor dimensions (in the ring frame) should be smaller than the distance from the ring to the accelerating column wall. Thus, in a ring with non-zero minor dimensions, which in the present model would be approximated by a compact bundle of thin rods, the field due to charges and currents on the plates is adequately described by (2.6) and 2.7) with q corresponding to the total line charge of the ring. The self-fields decrease as γ^{-2} and we can safely neglect them at large γ.

2. Current Carrying Rod

A rod having current in the y-direction of magnitude $q\beta'c$ is treated in Appendix A of Ref. 4. Employing Maxwell's equations to relate H_x to E_y one obtains

$$\langle H(\sigma)\rangle \equiv \left\langle H_{sx}(-x_o,\ z = vt + \sigma,\ t)\right\rangle_t = -\frac{2q\beta'\gamma}{L}\ I_m\left\{\int_0^\infty d\lambda P^2(\lambda,\gamma)\right.$$
$$\left.\exp\left[\frac{-2x_o\lambda}{L\gamma} + \frac{i\sigma\lambda}{L}\right]\right\} \qquad (2.8)$$

The leading terms in the focusing force are easily seen to be

$$\beta'\left\langle\frac{dH(\sigma)}{d\sigma}\right\rangle_{\sigma=0} = -\frac{q(\beta'\gamma)^2}{2x_o^2}\ \gamma\left[1 + \frac{3\,\zeta(\frac{1}{2})}{2^{5/2}\gamma^{\frac{1}{2}}}\ \left(\frac{2\pi L}{x_o}\right)^{\frac{1}{2}}\right] \qquad (2.9)$$

The energy loss, which is evaluated in Ref. 4, varies as $(\beta'\gamma)^2\gamma^{-\frac{1}{2}}$. Since the transverse velocity of electrons, in the ring frame, is approximately constant as the ring is accelerated, the quantity $\beta'\gamma$ is essentially γ-independent (and equal to unity, if the electrons have relativistic transverse velocities before being accelerated axially). Thus the energy loss of a charged rod and a current carrying rod both vary as $\gamma^{-\frac{1}{2}}$ (at large γ) and in fact are equal in magnitude in this limit. In like manner, the focusing force contributions [Eqs. (2.7) and (2.9)] become equal in the limit of large γ. We believe this equality to be a general (geometry-independent) result.

3. Focusing Force

The focusing force on an electron, in the axial direction, is given by

$$F(t) = e\sigma\left[\frac{\partial E_z(\sigma,t)}{\partial\sigma} - \beta'\frac{\partial H_r(\sigma,t)}{\partial\sigma}\right]_{\sigma=0} \qquad (2.10)$$

which we write in the form

$$F(t) = K(t)\sigma \tag{2.11}$$

For the semi-infinite plate model, then, taking $q = Ne/2\pi R$, with R the ring radius

$$K \equiv \langle K(t) \rangle_t = \frac{Ne^2\gamma}{4\pi R x_o^2} \left[1 + \frac{3\zeta(\frac{1}{2})}{2^{5/2}\gamma^{\frac{1}{2}}} \left(\frac{2\pi L}{x_o} \right)^{\frac{1}{2}} \right] \left[1 + (\beta'\gamma)^2 \right] . \tag{2.12}$$

III CORRUGATED CYLINDRICAL WAVEGUIDE

In this section we represent the accelerating column by an infinitely long, periodically corrugated cylindrical waveguide with geometrical parameters as shown in Figure 1. We employ the notation of Ref. 1 which is necessary for the understanding of what follows.

The complete vector potential $\underset{\sim}{A}(\underset{\sim}{r},t)$ is given as a sum over the eigenfunctions of the empty waveguide $\underset{\sim}{A}_\lambda(\underset{\sim}{r})$:

$$\underset{\sim}{A}(\underset{\sim}{r},t) = \sum_\lambda q_\lambda(t)\, \underset{\sim}{A}_\lambda(\underset{\sim}{r}), \tag{3.1}$$

where the functions $q_\lambda(t)$ obey the equation

$$\ddot{q}_\lambda + \omega_\lambda^2 q_\lambda = N_c^{-1} \int_{V_N} \underset{\sim}{j}\cdot\underset{\sim}{A}_\lambda^* \, dV = f_\lambda . \tag{3.2}$$

N_c is the number of cells and V_N is their volume.

If the azimuthal motion of the electrons is neglected, an electron ring with charge Q and geometrical parameters as shown in Figure 1,

travelling with speed v, has the current density

$$j_z = \frac{Qv}{\pi h(R_2^2 - R_1^2)} H(\tfrac{1}{2}h - |z - vt|)\, H(\rho - R_1)\, H(r_2 - \rho) \qquad (3.3)$$

where H(x) is the Heaviside step function. Performing the integration in (3.2) with A_λ^* from Ref. 1 yields :

$$f_\lambda = -\frac{Qvi}{N_c \omega_\lambda} \sum_m A_m S(\tfrac{1}{2}\beta_m h)\, J(\chi_m) \quad \exp(i\beta_m vt) \qquad (3.4)$$

The factors $S(x) = x^{-1} \sin x$ and J take into account the finite dimensions of the electron ring ; J is given by

$$J(\chi_m) = \frac{2\left[R_2 J_1(\chi_m R_2) - R_1 J_1(\chi_m R_1)\right]}{\chi_m (R_2^2 - R_1^2)\, J_o(\chi_m a)} \qquad (3.5)$$

The propagation constants β_m and χ_m are defined in Ref. 1 by $\beta_o = \omega_\lambda/v - 2\pi\ell/d$ with ℓ chosen such that $|\beta_o| \leq \pi/d$, $\beta_m = \beta_o + 2\pi m/d$, $\chi_m^2 = \omega_\lambda^2/c^2 - \beta_m^2$.

With the N_c cavities centred at z = 0, and with j from (3.3), $f_\lambda(t) = 0$ for $|t| \geq T = \frac{1}{2}N_c d/v$, and hence for $t \geq T$ $q_\lambda(t)$ is given by

$$q_\lambda(t) = \omega_\lambda^{-1} \int_{-T}^{+T} f_\lambda(t') \sin \omega_\lambda(t - t')\, dt' \qquad (3.6)$$

which becomes

$$q_\lambda(t) = -\frac{Qdi}{2\omega_\lambda^2} \sum_m A_m S(\tfrac{1}{2}\beta_m h)\, J(\chi_m) \left[(S(\phi^+) + S(\phi^-)) \sin \omega_\lambda t + (S(\phi^+) - S(\phi^-))\, i \cos \omega_\lambda t\right] \qquad (3.7)$$

where $\phi^{\pm} = \frac{1}{2} N_c d\, (\omega_\lambda/v \pm \beta_m)$, and $S(\phi) = \phi^{-1} \sin \phi$ as above.

For $N_c \to \infty$, the contributions to q_λ come from two resonances with $\omega_\lambda \pm \beta_\ell v = 0$ in the notation of Ref. 1. In that limit we find :

$$\lim_{N_c \to \infty} q_\lambda(t) = - Qdi\omega_\lambda^{-2} A_\ell S(\tfrac{1}{2}\omega_\lambda h/v) J(\omega_\lambda/\gamma v) \sin \omega_\lambda t \qquad (3.8)$$

This result is multiplied by a factor of two because two resonance conditions are fulfilled at the same frequency by waves travelling in opposite directions which are counted as one mode in Ref. 1.

The electric field gradient for the λ-th mode is

$$\left.\frac{\partial E_{z\lambda}}{\partial z}\right|_{z = vt} = - \dot{q}_\lambda(t) \left.\frac{\partial A_{z\lambda}}{\partial z}\right|_{z = vt} \qquad (3.9)$$

The z-derivative of the vector potential, averaged over the minor ring dimensions, follows from Ref.1 :

$$\left.\frac{\partial A_z}{\partial z}\right|_{z = vt} = -\omega_\lambda^{-1} \sum_m A_m \beta_m S(\tfrac{1}{2}\beta_m h) J(\chi_m) \exp(-i\beta_m vt) \qquad (3.10)$$

Using (3.8), (3.9) and (3.10) we find the electric field gradient in the limit $N_c \to \infty$:

$$\lim_{N_c \to \infty} \left.\frac{\partial E_{z\lambda}}{\partial z}\right|_{z = vt} = Qdi\omega_\lambda^{-2} A_1 S(\tfrac{1}{2}\omega_\lambda h/v) J(\omega_\lambda/\gamma v) \sum_m A_m \beta_m S(\tfrac{1}{2}\beta_m h) J(\chi_m) \cos \omega_\lambda t \exp(-i\beta_m vt) \qquad (3.11)$$

When this expression is averaged over the time necessary to traverse one period of the structure the sum reduces to a single term and yields :

$$\left\langle \lim_{N_c \to \infty} \left.\frac{\partial E_{z\lambda}}{\partial z}\right|_{z = vt} \right\rangle = \frac{Qdi}{2\omega_\lambda v} \left[A_\ell S(\tfrac{1}{2}\omega_\lambda h/v) J(\omega_\lambda/\gamma v) \right]^2 \qquad (3.12)$$

Since A_ℓ is not available in closed form, it is advantageous to compare (3.12) to the energy U_λ radiated in the λ-th mode in one period of the structure, calculated in Ref.1. We find

$$\left\langle \lim_{N_c \to \infty} \left. \frac{\partial E_{z\lambda}}{\partial z} \right|_{z = vt} \right\rangle = \frac{\omega_\lambda U_\lambda}{vdQ} \tag{3.13}$$

The total electric field gradient

$$\left\langle \left. \frac{\partial E_z}{\partial z} \right|_{z = vt} \right\rangle$$

is obtained by summing (3.13) over all modes. Because of the factor ω_λ it converges less rapidly as a function of ω_λ than the energy loss U_λ.

Finally, we wish to remind the reader that in this section we have neglected ring current effects.

IV EVALUATION OF THE AXIAL FREQUENCY

In order to evaluate internal ring dynamics it is convenient to work in the frame of reference in which the ring is at rest. In this frame, axial motion of electrons is described by

$$\frac{d^2 z^*}{dt^{*2}} + \omega_o^{*2} \nu_o^2 z^* = \frac{F^*(t^*)}{m_o \gamma_\perp^*} , \tag{4.1}$$

where ω_o^* is the revolution frequency, ν_o describes the focusing due to ions, images, and the accelerating wave, $\gamma_\perp^*$ is the relativistic γ-factor for the circulating electron, and F^* is the axial force on an electron due to the diffraction radiation. The absence of a star on ν_o follows from its invariance under Lorentz transformation. We

have neglected the change in energy of an electron in the interval of an axial oscillation.

From (2.11), and the invariance of longitudinal force under Lorentz transformation, we have

$$F^*(t^*) = K(t)\,\sigma, \tag{4.2}$$

which, since $z^* = \sigma\,\gamma_{\|}$, becomes

$$F^*(t^*) = \frac{K(t)z^*}{\gamma_{\|}}\,. \tag{4.3}$$

In the case of relativistic axial ring velocity $(\gamma_{\|} \gg 1)$ and for closely spaced accelerating cavities, the time variation of $K(t)$ is rapid compared with an axial oscillation period, and we may average $K(t)$ over time. Thus letting

$$K \equiv \langle K(t) \rangle\,, \tag{4.4}$$

and combining (4.1), (4.3), and (4.4), we have

$$\frac{d^2z^*}{dt^{*2}} + \omega_o^{*2}\,\nu^2\,z^* = 0, \tag{4.5}$$

where the total axial betatron oscillation frequency ν is given by

$$\nu^2 = \nu_o^2 - \frac{K}{m_o\,\gamma_{\perp}^*\,\omega_o^{*2}\,\gamma_{\|}}\,. \tag{4.6}$$

We introduce the quantity B, by writing

$$K = \frac{Ne^2\gamma_{||}}{2\pi R} B, \tag{4.7}$$

where N is the number of electrons in the electron ring and R is the ring major radius. Clearly B has the dimensions of inverse length squared. The factor $\gamma_{||}$ has been inserted merely for convenience. From (4.6) and (4.7)

$$\nu^2 = \nu_o^2 - \frac{N r_e R B}{2\pi \gamma_\perp^*} \tag{4.8}$$

In (4.8), r_e is the classical electron radius and we have employed $\omega_o^* \approx c/R$ in deriving the equation.

The quantity B, upon which ν^2 depends, is a function of the geometry of the accelerating structure and of ring speed. For $B > 0$, the diffraction radiation reaction is a defocusing effect. Axial stability follows if ν^2 is positive, and hence to obtain stability when $B > 0$ requires that a non-zero amount of focusing be supplied by ions, images, or the accelerating wave.

We have not concerned ourselves with radial motion in this note as the focusing - - from ions, images, and the external field - - is strong in this direction, and there is no near danger of loss of radial stability. Crossing of a resonance by a relativistic ring would - - presumably - - not be serious.

V NUMERICAL EXAMPLES

In this section we evaluate (4.8) for the structures discussed in Sections II and III.

The semi-infinite plate model, after replacing the charge per unit length q by $Ne/2\pi R$ and setting $\beta'\gamma = 1$, yields, from (2.12),

$$B = \frac{1}{x_o^2} \left[1 - 0.778 \left(\frac{2\pi L}{\gamma_{||} x_o} \right)^{\frac{1}{2}} \right] . \tag{5.1}$$

The corrugated cylindrical waveguide model for a charged ring (ring currents ignored) yields

$$B = \frac{2\pi R \eta}{a^3} , \tag{5.2}$$

where the coefficient $\eta(b/a, d/a, g/a, \gamma)$ is a weak function of all of its arguments. Computations for a large number of cases indicate that $\eta \lesssim 0.5$. As remarked at the end of Section II.2, we expect that in the relativistic limit ring current effects should introduce an additional factor of 2 in η.

Taking as typical values, $N = 10^{13}$, $R = 3.0$ cm, $\gamma_\perp^* = 40$ one finds, from (4.9)

$$\nu^2 = \nu_o^2 - 3.2 \times 10^{-2} B. \tag{5.3}$$

Thus (5.1) with $x_o \approx 10.0$ cm and $\gamma_{||} \gg 1$ yields $\nu^2 = \nu_o^2 - 1.6 \times 10^{-4}$; while (5.2) with $R = 3.0$ cm, $a = 10.0$ cm and $\eta = 1.0$ (to

include magnetic effects) yields $\nu^2 = \nu_o^2 - 6.0 \times 10^{-4}$.

These defocusing effects are small, and presumably can be easily overcome in practice by means of ion focusing or image focusing.

VI ACKNOWLEDGMENTS

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FOOTNOTES AND REFERENCES

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5. The sign is just wrong, however. The reason for this is that in the plate structure, since the plates are perpendicular to the direction of motion of the rod, the boundary conditions are satisfied - - to fair approximation - - by an image rod of the same sign as the rod (thus minimizing H_x along the plates); hence the reversed sign in $(dE/d\sigma)$.

FIGURE CAPTIONS

Fig. 1. Geometry of a corrugated cylindrical waveguide with a charged ring.

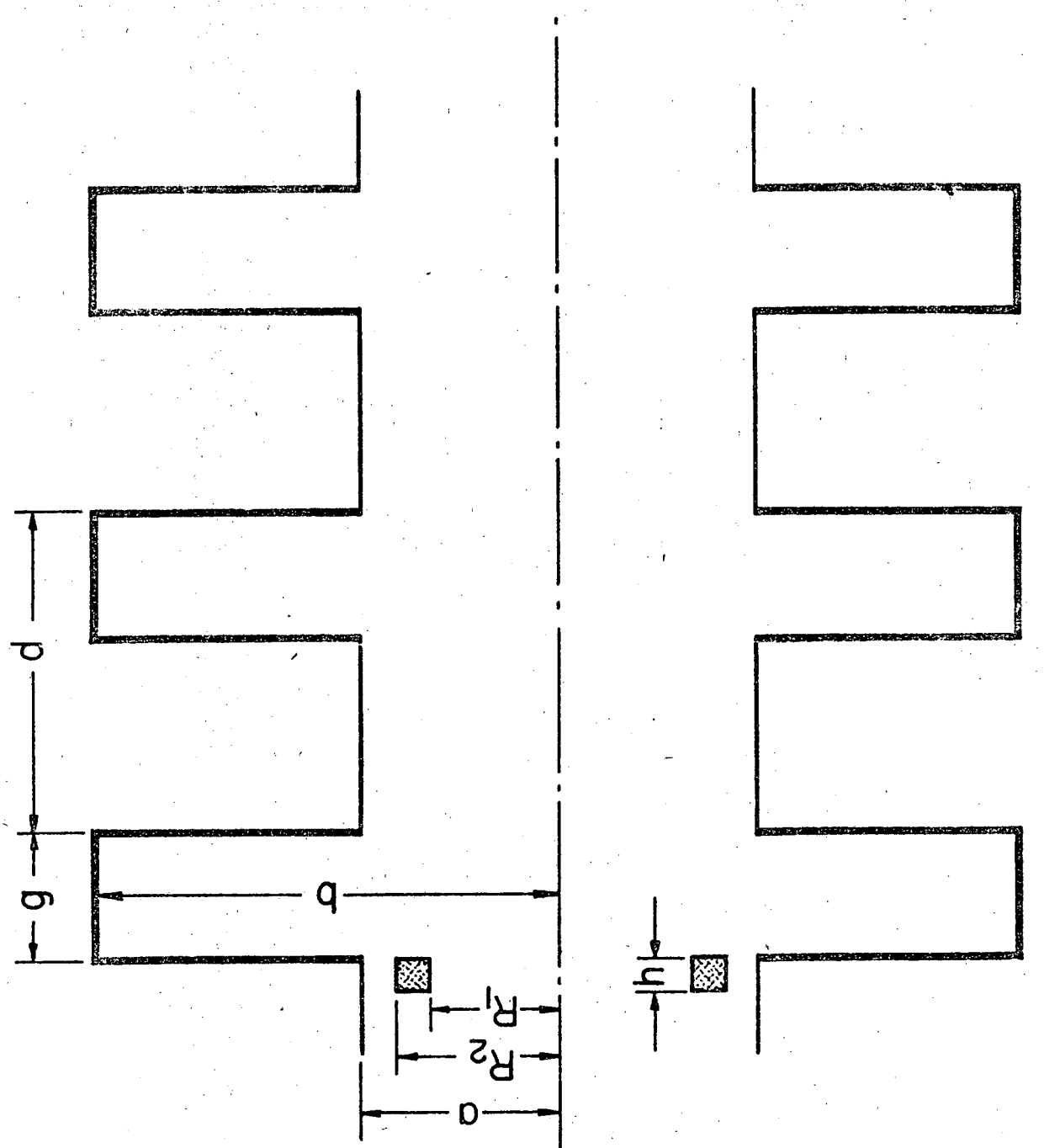

Fig. 1

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