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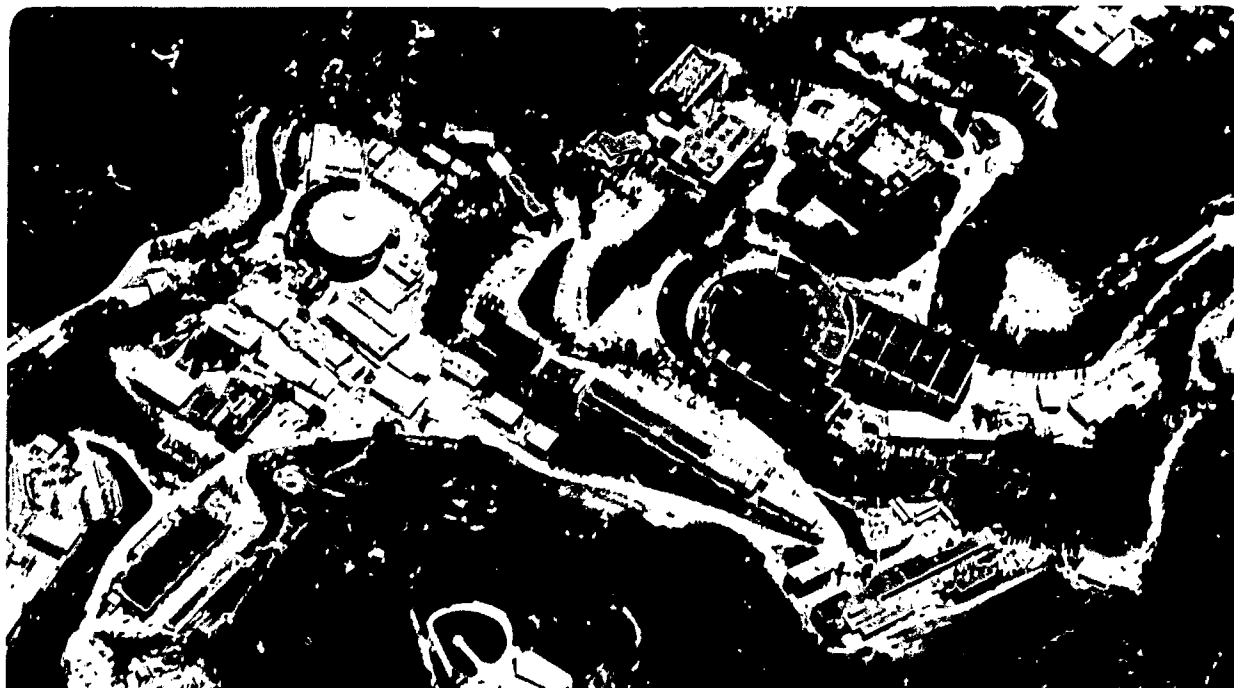
### Recent Results on Jet Physics and Tests of QCD in $e^+e^-$ Annihilation

S. Bethke

November 1989

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## RECENT RESULTS ON JET PHYSICS AND TESTS OF QCD

IN  $e^+e^-$  ANNIHILATION\*

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**Abstract.** Experimental investigations to test specific predictions of Quantum Chromodynamics and to adjust the free parameters of the theory are reviewed. Determinations of the strong coupling constant,  $\alpha_s$ , in  $\Upsilon$ -decays and in the continuum of  $e^+e^-$  annihilations are summarized and discussed. Studies on production rates of multijet hadronic final states in the center of mass energy range of 22 GeV to 93 GeV, including optimizations of both the scale parameter  $\Lambda_{\overline{MS}}$  and the renormalization scale  $\mu^2$  in  $O(\alpha_s^2)$  perturbative QCD, are presented. The status of experimental tests of the nonabelian nature of QCD is summarized.

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## I. INTRODUCTION

One of the basic ingredients of Quantum Chromodynamics (QCD), the gauge theory describing the interactions of quarks and gluons [1], is the principle of “asymptotic freedom”. It determines that the QCD coupling strength,  $\alpha_s$ , decreases with increasing energy. The experimental measurement of  $\alpha_s$  at a certain energy or momentum transfer  $\mu$  fixes the theoretical prediction for the size of  $\alpha_s$  at any other value of  $\mu$ . A meaningful test of QCD and of the applicability of QCD perturbation theory therefore requires precise measurements of  $\alpha_s$  at different energies  $\mu$ .

In this article, recent determinations of  $\alpha_s$  and experimental tests of specific predictions of QCD, performed in  $e^+e^-$  annihilation processes, are reviewed. After a short introduction about the predictions of perturbative QCD in  $e^+e^-$  annihilation, measurements of  $\alpha_s$  from decays of  $\Upsilon$  mesons and from the  $e^+e^-$  continuum are summarized and discussed in chapters III and IV, respectively. Chapter V is devoted to studies of multijet event production rates in the center of mass energy range of 22 GeV to 60 GeV, and to adjustments of the corresponding renormalization scale  $\mu$  and the scale parameter  $\Lambda_{\overline{MS}}$  in second order QCD perturbation theory. The comparison of experimental results on  $\Lambda_{\overline{MS}}$  from different processes and energy ranges is discussed in chapter VI. In chapter VII, the current status of experimental tests of the nonabelian structure of QCD, namely the specific energy dependence of  $\alpha_s$  and the existence of the gluon self coupling, is reviewed. Chapter VIII contains a short update of results which became available after this workshop, including a recent determination of  $\alpha_s$  at  $E_{cm} \approx 60$  GeV and a measurement of jet production rates from hadronic decays of the  $Z^0$  boson at  $E_{cm} \approx 91$  GeV. Chapter IX concludes this review with a final summary and discussion.

## II. PERTURBATIVE QCD IN $e^+e^-$ ANNIHILATION

In perturbative QCD, the expectation value of a typical observable,  $O$ , can be calculated in an expansion like

$$\langle O \rangle = C_1 \cdot \frac{\alpha_s^n(\mu)}{\pi} \cdot \left( 1 + C_2 \cdot \frac{\alpha_s(\mu)}{\pi} + C_3 \cdot \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 + \dots \right), \quad (1)$$

where  $\mu$  is the renormalization scale or energy scale,  $C_i$  are the  $i$ -th order QCD coefficients and  $n$  is the power of  $\alpha_s$  in leading order.  $C_i$  and  $n$  have been computed for many different QCD-related observables. With the exception of the total hadronic cross section, which is calculated up to the third order perturbation theory, the coefficients of most observables are only known up to the second order ( $O(\alpha_s^2)$ ). Numerical values of  $C_i$  and  $n$  are given in Table 1 for the mean value of (1 - Thrust) [2], for the integrated asymmetry of energy-energy correlations [3], for the QCD correction to the total hadronic cross section [4],  $R$ , for the relative production rate of 3-jet events [5;6] and for the leptonic branching ratio of  $\Upsilon$  decays [7].

The coupling constant  $\alpha_s(\mu)$ , in  $2^{nd}$  order perturbation theory, can be written as

O	$n$	$C_1$	$C_2$	$C_3$
$\langle 1 - T \rangle$	1	1.05	9.05	n.c.
$\int_{30^\circ}^{90^\circ} A_{EEC} d\chi$	1	0.77	3.59	n.c.
$\frac{R}{R_{QPM}} - 1$	1	1.0	1.44	64.9
$\frac{\sigma_{3-jet}}{\sigma_{tot}}(y_{cut} = 0.04)$	1	7.13	8.84	n.c.
$\frac{\Gamma(\Upsilon \rightarrow \text{hadrons})}{\Gamma(\Upsilon \rightarrow \mu^+ \mu^-)}$	3	18135.	9.08	n.c.

Table 1. QCD coefficients of some selected observables ( $\mu^2 = E_{cm}^2$ ); n.c. means not calculated.

a function of  $\ln(\mu^2/\Lambda^2)$ :

$$\alpha_s(\mu) = \frac{12\pi}{(33 - 2 \cdot N_f) \cdot \ln(\frac{\mu}{\Lambda})^2} \cdot \left( 1 - 6 \cdot \frac{153 - 19 \cdot N_f}{(33 - 2 \cdot N_f)^2} \cdot \frac{\ln(\ln(\frac{\mu}{\Lambda})^2)}{\ln(\frac{\mu}{\Lambda})^2} \right), \quad (2)$$

where  $N_f$  is the number of quark flavours and  $\Lambda$  is the QCD scale parameter to be determined by experiment. There are alternative expansions which may predict different correlations between  $\Lambda$  and  $\alpha_s$  (see e.g. [8]). In this article, Eq. 2 will be used if numerical values for  $\Lambda$  are quoted.

Up to date, there is no unique and commonly accepted prescription of how to choose the energy scale  $\mu^2$  for a given physical process. While in the continuum of  $e^+e^-$  annihilation  $\mu$  is usually chosen to be the center of mass energy of the hadronic final state ( $\mu^2 = E_{cm}^2$ ), other choices like  $\mu = 0.157 \cdot M_\Upsilon$  or  $\mu = 0.48 \cdot M_\Upsilon$  are commonly used in calculations of partial decay widths of the  $\Upsilon$  resonance. Theoretically, physical observables do not depend on the choice of  $\mu$  if the calculations can be carried out up to infinite order perturbation theory. Finite order calculations, however, depend on the actual definition of  $\mu$  since the next-to-leading QCD coefficients like  $C_2, C_3$  (c.f. Eq. 1) are explicit functions of the energy scale. In  $O(\alpha_s^2)$  and for observables with a leading order power coefficient of  $n = 1$  one gets

$$C_2 \equiv C_2(\mu) = C_2(E_{cm}) + \frac{33 - 2 \cdot N_f}{12} \cdot \ln \frac{\mu^2}{E_{cm}^2}. \quad (3)$$

The renormalization scale has no direct physical interpretation since any physical dependence from the choice of  $\mu$  is only an artifact of the incomplete perturbation series;  $\mu$  is therefore often called an ‘‘unphysical’’ parameter. The impact of different choices of  $\mu^2$  will be further investigated in the course of this article.

### III. DETERMINATIONS OF $\alpha_s$ IN $\Upsilon$ DECAYS

In lowest order QCD, hadronic decays of the  $\Upsilon$  are described as decays into three gluons. Since the partial decay width  $\Gamma_{ggg}$  is proportional to the third power of  $\alpha_s$ ,

measurements of the leptonic branching ratio of the  $\Upsilon$ ,  $B_{\mu\mu} = \Gamma_{\mu\mu}/\Gamma_{ggg}$  and of the ratio  $\Gamma_{\gamma gg}/\Gamma_{ggg}$  provide sensitive determinations of  $\alpha_s$ . A summary of the experimental results of  $\alpha_s$  and of the corresponding QCD parameter  $\Lambda_{\overline{MS}}$ , calculated from Eq. 2 for  $N_f = 4$ , is given in Table 2.

Experiment	Obs.	$\mu$	$\alpha_s(\mu)$	$\Lambda_{\overline{MS}}^{(4)}$ [MeV]	Ref.
Christal Ball ( $\Upsilon$ )	$\frac{\Gamma_{\gamma gg}}{\Gamma_{ggg}}$	1.5 GeV	$0.25 \pm 0.02 \pm 0.04$	$154 \pm 31_{-60}^{+64}$	[9]
ARGUS ( $\Upsilon$ )	$\frac{\Gamma_{\gamma gg}}{\Gamma_{ggg}}$	$0.157 \cdot M_\Upsilon$	$0.225 \pm 0.011 \pm 0.019$	$115 \pm 17 \pm 28$	[10]
CUSB ( $\Upsilon$ )	$\frac{\Gamma_{\gamma gg}}{\Gamma_{ggg}}$	$0.157 \cdot M_\Upsilon$	$0.226_{-0.042}^{+0.067}$	$116_{-57}^{+105}$	[11]
CLEO ( $\Upsilon$ )	$\frac{\Gamma_{\gamma gg}}{\Gamma_{ggg}}$	$0.157 \cdot M_\Upsilon$	$0.27_{-0.02-0.02}^{+0.03+0.03}$	$190_{-30-30}^{+40+40}$	[12]
<i>Average</i>	$\frac{\Gamma_{\gamma gg}}{\Gamma_{ggg}}$			$139 \pm 25$	
CUSB ( $\Upsilon, \Upsilon', \Upsilon''$ )	$\frac{\Gamma_{ggg}}{\Gamma_{\mu\mu}}$	$0.48 \cdot M_\Upsilon$	$0.174 \pm 0.033$	$157 \pm 12$	[13]
<i>Average</i>				$154 \pm 11$	

Table 2. Results of  $\alpha_s$  and of  $\Lambda_{\overline{MS}}$  for 4 quark flavours from  $\Upsilon$  partial decay widths.

The different results from  $\frac{\Gamma_{\gamma gg}}{\Gamma_{ggg}}$  agree well with each other and with the average value of  $\Lambda_{\overline{MS}} = (139 \pm 25)$  MeV, where the error is determined adding the statistical and systematic errors in quadrature. The energy scale chosen for this process is  $\mu = 0.157 \cdot M_\Upsilon$ , as suggested by Brodsky, Lepage and McKenzie (BLM) [14]. For the calculations of  $\frac{\Gamma_{ggg}}{\Gamma_{\mu\mu}}$ , the energy scale according to the suggestion of Grunberg [15] is usually applied. This leads to  $\mu = 0.48 \cdot M_\Upsilon$  and to different experimental values of  $\alpha_s$ . The corresponding value of  $\Lambda_{\overline{MS}}$ , however, is in agreement with the results from  $\frac{\Gamma_{\gamma gg}}{\Gamma_{ggg}}$ , as it should be the case for independent measurements carried out at different energy scales. Therefore the result of  $\Lambda_{\overline{MS}}$ , combined for all measurements of  $\Upsilon$  decays, is given at the end of Table 2. The overall experimental uncertainty of  $\pm 11$  MeV (7% relative) is rather small and is mainly determined, like the average value of  $\Lambda_{\overline{MS}}$ , by the study of  $\frac{\Gamma_{ggg}}{\Gamma_{\mu\mu}}$ , where the experimental observable depends on  $\alpha_s^3$  in leading order QCD (c.f. Table 1).

#### IV. DETERMINATIONS OF $\alpha_s$ IN THE $e^+e^-$ CONTINUUM

Comprehensive summaries of measurements of  $\alpha_s$  in the  $e^+e^-$  continuum above the  $\Upsilon$  resonance have been previously presented [16;8]. The experimental results clustered around  $\alpha_s(34 \text{ GeV}) \approx 0.12 - 0.16$ , which corresponds to  $\Lambda_{\overline{MS}}^{(5)} \approx (220_{-120}^{+270})$  MeV

or, if calculated for 4 quark flavours [17;18], to  $\Lambda_{\overline{MS}}^{(4)} \approx (320_{-160}^{+330})$  MeV. The relatively large uncertainty is mainly determined by the unknown details of the hadronization process and by theoretical uncertainties rather than by the experimental or statistical errors. This situation has not much changed in the past two years, as will be demonstrated by the following summary of the most recent experimental results.

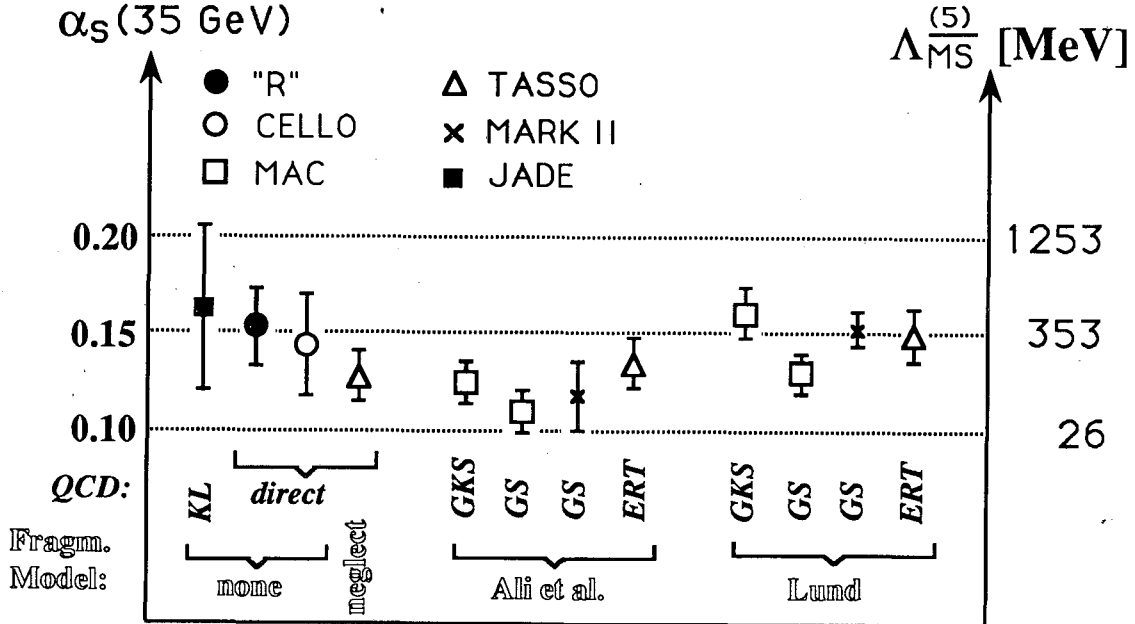


Fig.1. A Summary of recent measurements (1987 - 1989) of  $\alpha_s$  in the  $e^+e^-$  continuum. If taken at different c.m. energies, the results are converted to  $E_{cm} = 35$  GeV. All results are for  $\mu = E_{cm}$ .

The  $\alpha_s$  measurements of the past two years are summarized in Table 3 and are also shown, in a similar way as previous results were presented in [8;16], in Fig. 1. For the graphical presentation, the experimental results are given as values for  $\alpha_s$  for  $\mu = E_{cm} = 35$  GeV. They are grouped according to the theoretical calculations and according to the hadronization (or fragmentation) model that were used in the analyses:

- The TASSO and the Mark-II collaborations [19;20] both analysed the asymmetry of energy-energy correlations (AEEC) [3] and compared the data to different model calculations based on the  $O(\alpha_s^2)$  QCD calculations of Ellis, Ross and Terrano (ERT) [21;22] and of Gottschalk and Shatz [23]. The fragmentation models used in order to simulate the hadronization of quarks and gluons were the Lund string fragmentation model and an independent fragmentation model [24]. TASSO also adjusted  $\alpha_s$  in a direct fit to analytic  $O(\alpha_s^2)$  calculations of the AEEC, thereby neglecting possible hadronization effects.
- The JADE collaboration provided a measurement of  $\alpha_s$  from the ratio of 3-jet event rates observed at two different center of mass energies, 34 GeV and 44 GeV [25]. Within this method,  $\Lambda_{\overline{MS}}$  is determined by the degree of energy dependence observed in the data and not by absolute cross sections. The result is largely independent from fragmentation models; the experimental error, however, is relatively



Exp.	Obs.	$\mu$ [GeV]	Hadron.	QCD	$\alpha_s(\mu)$	$\Lambda_{\overline{MS}}^{(5)}$ [MeV]
TASSO	AEEC	43.5	<i>none</i>	ERT/AB	$0.125 \pm 0.004 \pm 0.011$	$142_{-70}^{+105}$
"	"	"	Ali	"	$0.129 \pm 0.004 \pm 0.011$	$190_{-90}^{+130}$
"	"	"	Lund	"	$0.143 \pm 0.005 \pm 0.012$	$340_{-140}^{+200}$
Mark-II	AEEC	29.0	Lund	GS	$0.158 \pm 0.003 \pm 0.008$	$380_{-100}^{+110}$
"	"	"	IF	"	0.10 - 0.14	25 - 200
JADE	$\frac{R_3(44\text{GeV})}{R_3(34.6\text{GeV})}$	34 - 44	<i>none</i>	KL	—	$520_{-420}^{+830}$
CELLO	1-T etc.	14-47	<i>none</i>	analytic	—	79 - 628
MAC	$E_{\perp>}^{in}$	29.0	Lund	GS	$0.133 \pm 0.005 \pm 0.009$	$150_{-55}^{+80}$
"	"	"	"	GKS	$0.167 \pm 0.006 \pm 0.011$	$480_{-155}^{+190}$
"	"	"	ALI	GS	$0.112 \pm 0.008 \pm 0.007$	$50_{-25}^{+45}$
"	"	"	"	GKS	$0.128 \pm 0.007 \pm 0.008$	$120_{-50}^{+75}$
<i>all</i>	<i>R</i>	7 - 61	<i>none</i>	$O(\alpha_s^2)$	—	$440_{-220}^{+300}$
"	"	"	"	$O(\alpha_s^3)$	—	$240_{-120}^{+150}$

**Table 3.** Summary of recent measurements (1987 - 1989) of  $\alpha_s$  in the  $e^+e^-$  continuum. The observables, the energy scale  $\mu = E_{cm}$ , the hadronization model and the  $O(\alpha_s^2)$  QCD calculations for which the results are obtained, are given. The values of  $\Lambda_{\overline{MS}}$  are calculated according to Eq. 2 and for 5 quark flavours.

large due to the small size of energy dependent effects in the available range of energy. The theoretical  $O(\alpha_s^2)$  calculations on which this measurement is based are from Kramer and Lampe (KL) [5;6], which are shown to provide similar results as those from Gottschalk and Shatz.

- The CELLO collaboration derived limits for  $\Lambda_{\overline{MS}}$  from measurements of mean values of event shape variables like (1 - Thrust), integrated asymmetries of energy-energy correlations and jet masses at different center of mass energies [26]. Limits of  $\Lambda_{\overline{MS}}$  are determined using analytic  $O(\alpha_s^2)$  QCD calculations and assuming that the sign of hadronization contributions for these observables are known. The result therefore does not depend on the choice of a specific hadronization model.
- The MAC collaboration determined  $\alpha_s$  at  $E_{cm} = 29$  GeV in an analysis of  $E_{\perp>}^{in}$ ,

which is the energy flow of particles in the fat jet of an event, measured in the event plane and transverse to the jet axis [27].  $\alpha_s$  is adjusted in a comparison to different hadronization models based on the  $O(\alpha_s^2)$  calculations of Gottschalk and Shatz and of Gutbrod, Kramer and Schierholz (GKS) [28].

- A fit to the measurements of the normalized total hadronic cross section  $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$  at center of mass energies between 7.0 and 60.8 GeV [29] results in a value of  $\alpha_s$  which does not depend on hadronization effects and which is, regarding the small error of the result, now very competitive to other methods of  $\alpha_s$  determination. Furthermore,  $R$  is the only observable for which the complete third order QCD corrections are calculated [4]. However, the third order corrections turn out to be rather large, reducing the value of  $\alpha_s(35 \text{ GeV})$  by 10% from 0.158 to 0.143. In Fig. 1 the second order results from  $R$  are shown since all the other results are available in  $O(\alpha_s^2)$  only.

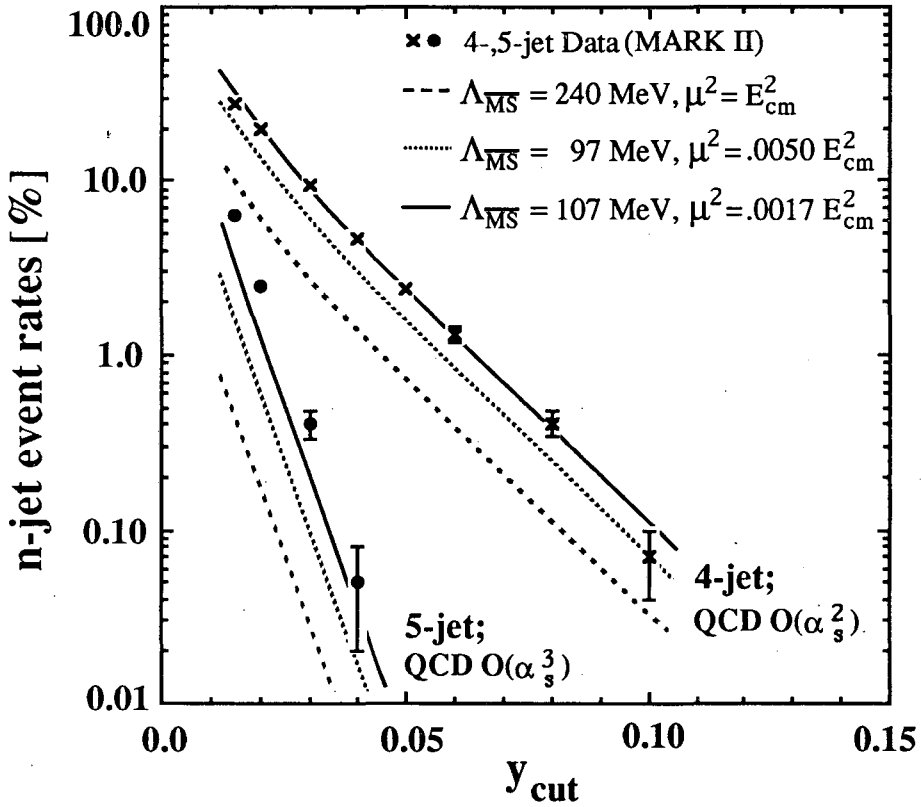
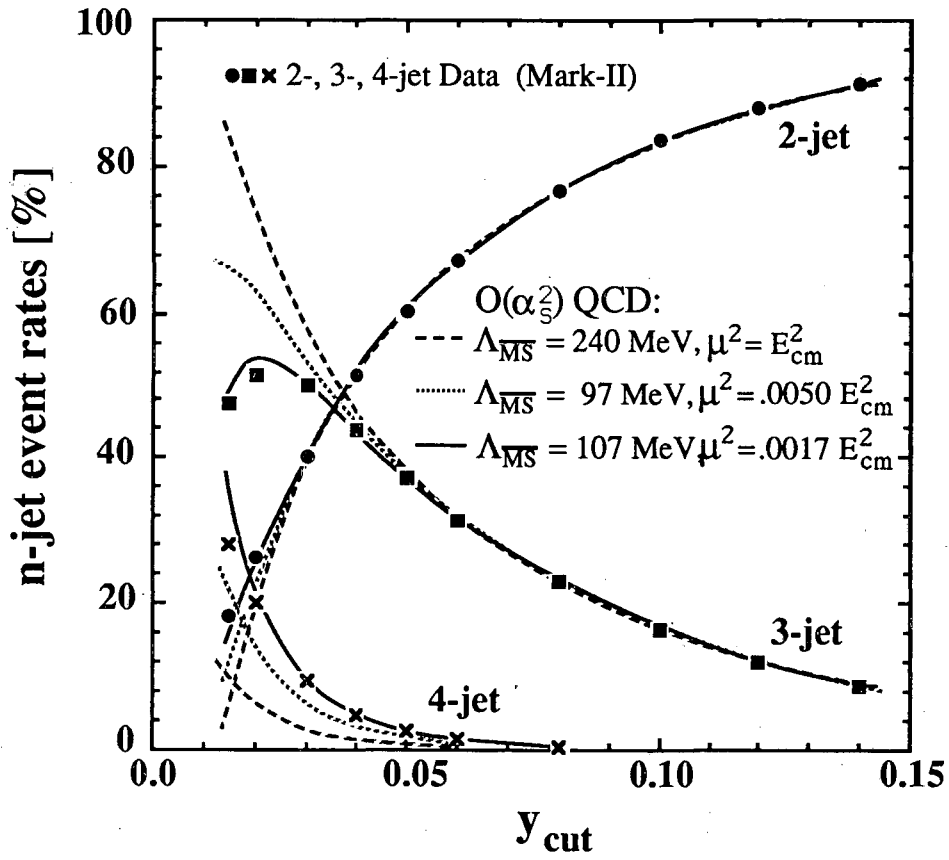
In summary, the latest results as presented in Fig. 1 and in Table 3 are consistent with previous measurements and demonstrate again that the experimental determination of  $\alpha_s$  in general depends on the hadronization model. Analyses which do not explicitly rely on hadronization models usually result in larger errors of  $\alpha_s$ . The difference between the GKS and the GS calculations can be explained by the fact that some of the  $O(\alpha_s^2)$  QCD corrections have been neglected by GKS. Therefore one can conclude that the overall results on  $\alpha_s$  and  $\Lambda_{\overline{MS}}$  in  $O(\alpha_s^2)$  are:

$$\begin{aligned}\alpha_s(35 \text{ GeV}) &= 0.14 \pm 0.02, \\ \Lambda_{\overline{MS}}^{(5)} &= 220_{-120}^{+270} \text{ MeV} \\ \text{or } \Lambda_{\overline{MS}}^{(4)} &= 320_{-160}^{+330} \text{ MeV}.\end{aligned}$$

Note that results given in this chapter are all determined for the renormalization scale  $\mu = E_{cm}$ . The implication of choosing other definitions of  $\mu$  and a comparison between the results obtained in  $\Upsilon$  decays and in the  $e^+e^-$  continuum will be discussed in the following two chapters.

## V. STUDIES OF JET PRODUCTION RATES

Energetic jets of hadrons are believed to reflect the kinematics of the underlying quarks and gluons. Therefore experimental studies of jet production rates are well suited to test the basic ideas of QCD and to determine the free parameters of the theory. In the past, several detailed studies of relative production rates of 2-, 3-, 4- and 5-jet events have been presented [25;30-33]. It was shown that QCD models which are based on  $O(\alpha_s^2)$  QCD calculations (with the renormalization scale  $\mu = E_{cm}$ ) underestimate the production rates of 4-jet events [30;31]. QCD shower models, which contain higher than  $O(\alpha_s^2)$  QCD contributions and which use typical energy scales much smaller than  $E_{cm}$ , were found to provide a better description of the data. While this was previously interpreted as an apparent need for higher than second order QCD contributions, more recent studies showed that the description of the data by  $O(\alpha_s^2)$  calculations can be significantly improved if the renormalization scale  $\mu$  is chosen to be much smaller than  $E_{cm}$  [34;32;35;36].



**Fig.2.** Two-, three-, four- and five-jet production rates observed at  $E_{cm} = 29 \text{ GeV}$ , compared with the  $O(\alpha_s^2)$  QCD calculations of Kramer and Lampe for different renormalization scales  $\mu$ . The calculations for 5-jet production in  $O(\alpha_s^3)$  are from [37].

This can be seen in Fig. 2, where the jet production rates observed by the Mark-II collaboration at  $E_{cm} = 29$  GeV [33] are compared to the  $O(\alpha_s^2)$  QCD calculations of Kramer and Lampe [5;6] for different choices of  $\mu$  [34]. The relative n-jet production rates  $R_n$  are calculated using the jet algorithm of the JADE collaboration [25;30], where resolvable jets are defined by the requirement that

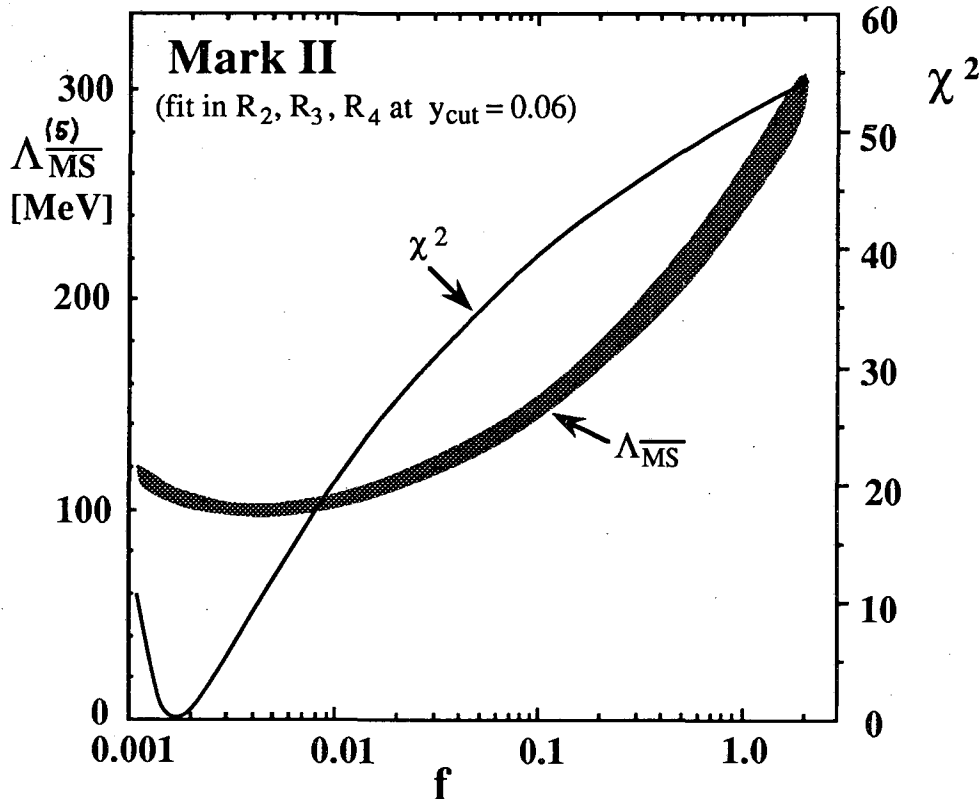
$$y_{ij} \equiv \frac{M_{ij}^2}{E_{vis}^2} > y_{cut} \quad (4)$$

for all pairs of jets  $i$  and  $j$  within a hadronic event.  $M_{ij}$  is the invariant jet pair mass and  $E_{vis}$  is the measured total visible energy of the event. Since the same definition of resolvable jets is used in many theoretical calculations and model studies have shown that the hadronization process affects the association of jets with partons only very little [25;30-33], the data are directly compared to the QCD expectations without using fragmentation models. The values of  $\Lambda_{\overline{MS}}$  for the curves shown in Fig. 2 are optimized such that the 2- and 3-jet rates for  $y_{cut} > 0.06$  are well described.

It can be seen in Fig. 2 that the choice of largely different definitions of  $\mu$  does not alter the good description of the data for  $y_{cut} > 0.08$ . The experimental value of  $\Lambda_{\overline{MS}}$ , however, pretty much depends on the choice of  $\mu$ . This is of course expected since, at the first place,  $\Lambda_{\overline{MS}}$  and  $\mu$  appear as a ratio in the expression of  $\alpha_s$  (c.f. Eq. 2). The additional occurrence of  $\mu$  in the second order QCD coefficient  $C_2$  (see Eqs. 1 and 3) partly absorbs the proportionality of  $\mu$  and  $\Lambda_{\overline{MS}}$  in the  $\alpha_s$  expression, such that  $\Lambda_{\overline{MS}}$  varies less with changes of  $\mu$ . There is a good and a bad message in this observation: the bad one is that experimental values of  $\Lambda_{\overline{MS}}$  indeed depend on the choice of renormalization scale in the  $O(\alpha_s^2)$  calculation; the good one is that the dependence could be worse.

For values of  $y_{cut} \leq 0.06$ , the calculations using small renormalization scales describe the  $y_{cut}$ -dependence of both the 2- and the 3-jet rates much better than the choice  $\mu^2 = E_{cm}^2$ . Furthermore, it can be seen in Fig. 2 that the 4-jet rates and also the 5-jet rates, if the corresponding  $O(\alpha_s^2)$  and  $O(\alpha_s^3)$  [37] calculations are applied using the same values of  $\Lambda_{\overline{MS}}$  and  $\mu$  as for 2- and 3-jets, are much better described with smaller renormalization scales. Theoretically, the latter observation may not be of significance, since the separate treatment of  $\Lambda_{\overline{MS}}$  and  $\mu$  in observables like  $R_4$  and  $R_5$ , which are calculated only in their leading order, is not meaningful and different observables may require different choices of  $\mu$ . Experimentally, however, the consistent description of all experimental jet rates by one set of QCD parameters is very satisfactory.

The functional dependence of  $\Lambda_{\overline{MS}}$  from the choice of  $\mu$  can be evaluated for each observable by simply fixing the value of  $\langle O \rangle$  in a theoretical expression like Eq. 1 and calculating  $\Lambda_{\overline{MS}}$  as a function of  $\mu$ . For the jet production rates in  $O(\alpha_s^2)$ , this dependence is further demonstrated in Fig. 3. Instead of fixing the observable to an arbitrary value, the 2-, 3- and 4-jet rates observed by Mark-II at  $y_{cut} = 0.06$  are used to fit  $\Lambda_{\overline{MS}}$  as a function of  $f = \mu^2/E_{cm}^2$ . It can be seen that the experimental value of  $\Lambda_{\overline{MS}}$  is sensitive to small changes of  $\mu$  around  $\mu = E_{cm}$  ( $f = 1$ ), while it is rather stable around  $f = 0.004$ . The best agreement with the data is achieved around  $f = 0.0017$ , where the corresponding  $\chi^2$  reaches zero (which is expected in a fit of two parameters for two degrees of freedom).



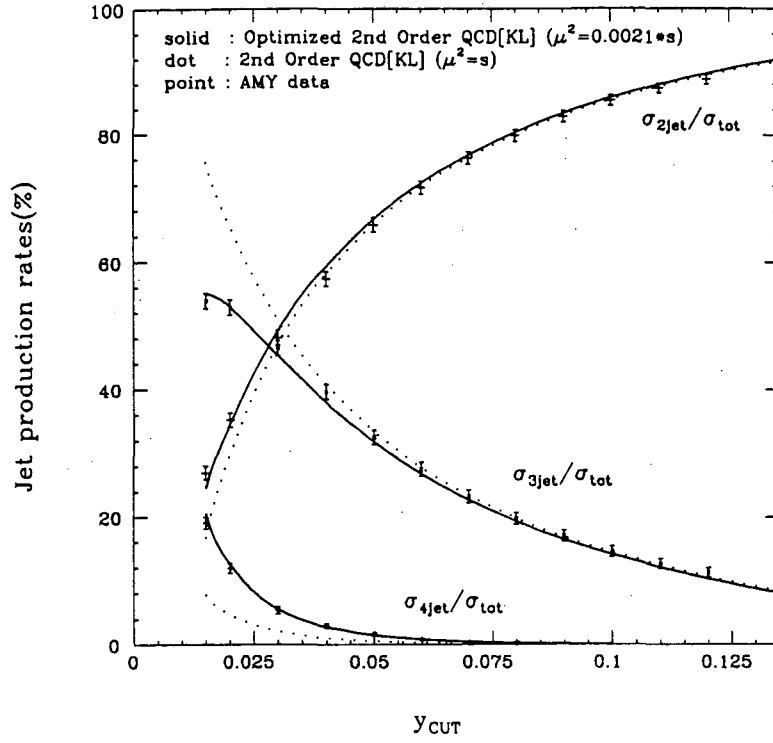
**Fig.3.**  $\Lambda_{\overline{MS}}^{(5)}$  and the corresponding  $\chi^2$ , determined from jet production rates observed at  $E_{cm} = 29$  GeV for  $y_{cut} = 0.06$ , as a function of the renormalization scale factor  $f$  in  $O(\alpha_s^2)$ . The width of the  $\Lambda_{\overline{MS}}^{(5)}$  curve corresponds to the statistical fit error at fixed values of  $f$ .

Similar results are obtained by the AMY collaboration [32] in a study of jet production rates observed at  $E_{cm} = 58$  GeV, as is demonstrated in Fig. 4. The final results of AMY are compatible to those obtained from the Mark-II data at just half the center of mass energy:  $\mu^2/E_{cm}^2 = 0.0027^{+0.0054}_{-0.0012}$  and  $\Lambda_{\overline{MS}}^{(5)} = (110 \pm 37)$  MeV<sup>1</sup>, while  $\mu^2/E_{cm}^2 = 0.002^{+0.0025}_{-0.0009}$  and  $\Lambda_{\overline{MS}}^{(5)} = (107 \pm 19)$  MeV from the Mark-II data [33;34].

## VI. HOW TO COMPARE $\Lambda_{\overline{MS}}$ RESULTS FROM DIFFERENT PROCESSES?

QCD does not predict the value of  $\Lambda_{\overline{MS}}$  or of  $\alpha_s$  at a certain energy  $\mu$ . However, if  $\alpha_s$  is measured at one point of the energy scale, QCD definitely predicts the evolution of  $\alpha_s$  as a function of energy (see Eq. 2). Therefore a meaningful experimental test of QCD is to measure  $\alpha_s$  at different energies and to verify that the measured energy dependence of  $\alpha_s$  is compatible with the theoretical expectation. Equivalently, one expects that the different experimental values of  $\Lambda_{\overline{MS}}$ , calculated in the same perturbative order and for an identical number of quark generations (e.g.  $N_f$

<sup>1</sup>Note that AMY, in their publication and in Fig. 4, quote  $\Lambda_{\overline{MS}}^{(4)} = (171 \pm 58)$  MeV, which here is recalculated for the case of 5 quark flavours.



$\chi^2 / 2 \text{ d.o.f.}$

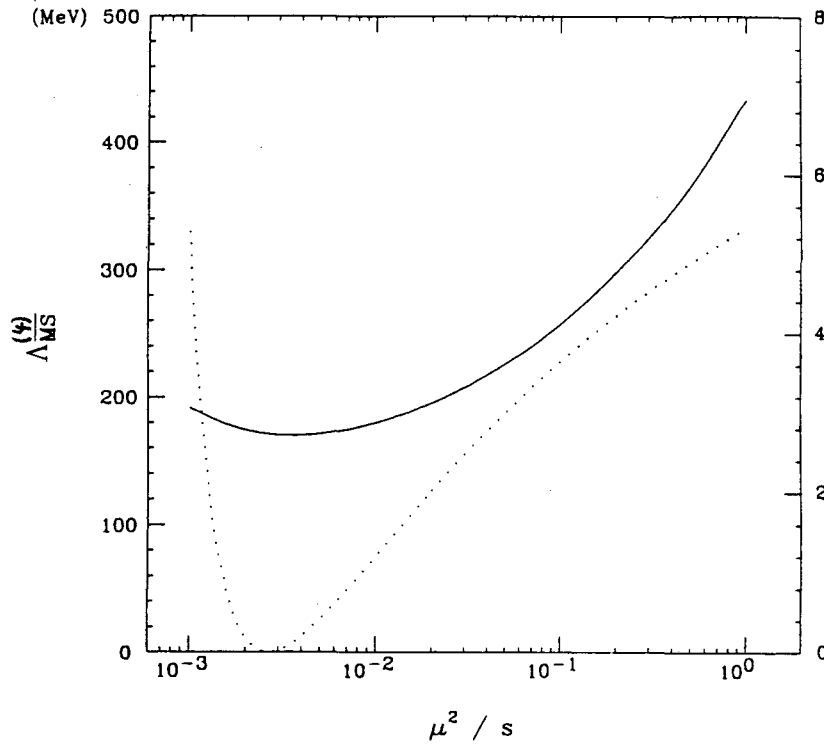


Fig.4. Jet production rates observed by AMY at  $E_{cm} = 58 \text{ GeV}$ , together with the theoretical expectations of  $O(\alpha_s^2)$  QCD calculations for different renormalization scales  $\mu$ , and the functional dependence of  $\Lambda_{\overline{MS}}^{(4)}$  from  $\mu$ , determined in a fit to the experimental jet rates.

$= 4$ ), are compatible with each other. However, the combined results from  $\Upsilon$  decays ( $\Lambda_{\overline{MS}}^{(4)} = (154 \pm 11)$  GeV) and from the continuum measurements ( $\Lambda_{\overline{MS}}^{(4)} = 320_{-160}^{+330}$  MeV), as summarized in chapters II and III, do not agree too well. One should therefore ask the question whether this might be due to a possible failure of QCD in general or whether additional systematic uncertainties must be considered in such comparisons.

One basic difference between the results from  $\Upsilon$  decays and from the  $e^+e^-$  continuum is that the latter are derived by using  $\mu = E_{cm}$  as the renormalization scale in  $O(\alpha_s^2)$  while in the first case the scales suggested by Brodsky, Lepage and McKenzie (BLM) [14] and by Grunberg [15] are used. As was shown above,  $\Lambda_{\overline{MS}}$  depends on the actual choice of  $\mu$ ; therefore comparing different experimental results derived with different choices of  $\mu$  might not be meaningful. It would be logical to demand that a consistent definition of  $\mu$ , e.g. the one by BLM, is used if results from different processes are to be compared with each other.

At this point it is interesting to note that the theoretical suggestions of BLM and of Stevenson [38] to fix the renormalization scale for different physical processes, if applied to the jet production rates in  $O(\alpha_s^2)$ , result in similar scales as the experimental “optimizations” described in the previous chapter [34]. The values of  $\Lambda_{\overline{MS}}$  which correspond to these scales are  $\Lambda_{\overline{MS}}^{(4)} = (167 \pm 30)$  MeV from the jet rates at  $E_{cm} = 29$  GeV [33;34] and  $\Lambda_{\overline{MS}}^{(4)} = (171 \pm 58)$  MeV from the AMY data [32], which both are compatible with the results obtained from  $\Upsilon$  decays.

It is therefore concluded that the values of  $\Lambda_{\overline{MS}}$ , determined in  $\Upsilon$  decays and in the  $e^+e^-$  continuum, are compatible with each other if the renormalization scales for the different processes are chosen in a consistent way, e.g. according to the BLM method.

## VII. TESTS OF THE NONABELIAN NATURE OF QCD

The nonabelian nature of QCD manifests itself in the process of gluon self-coupling and in the specific energy dependence of  $\alpha_s$ , which is predicted to logarithmically decrease with increasing energy scale  $\mu$ . An alternative, abelian gauge theory of the strong interactions would be similar to Quantum Electrodynamics (QED), within which the gauge bosons cannot couple to each other and the coupling constant *rises* with increasing energy. In order to “prove” that QCD is the correct theory of the strong interactions, the experimental verification of at least one or possibly all of the following items is necessary:

- (a) Global data distributions which are sensitive to the differences between the two alternative theories are well described by QCD but disfavour the expectations of “QED”.
- (b) The existence of the gluon self-coupling can be proven, e.g. by observing the typical spin structure of this process in 4-jet final states in  $e^+e^-$  annihilation.
- (c) The coupling constant decreases with increasing energy.

The current status of measurements of  $\alpha_s$  does not provide a convincing verification of

item (c) due to the systematic uncertainties discussed above. An alternative solution, however, is to show that

- (d) Observables whose energy dependence is only determined by  $\alpha_s$ , exhibit the characteristic energy dependence proposed by QCD,

without determining  $\alpha_s$  itself. In the following paragraphs, the status of the experimental studies of items (a), (b) and (d) will be summarized.

#### (a) A Comparison of Jet Production Rates with QCD and “QED”

A “QED” like theory for the strong interactions can be obtained from the corresponding QCD calculations by replacing the group constants of SU(3) (QCD) by the ones of U(1) (“QED”):  $C_F = 1$ ,  $N_C = 0$  and  $T_R = 6 \cdot T_R^{QCD}$ . It was recently shown [34] that the jet production rates predicted by a second order perturbative “QED” calculation are noticeably different from QCD. This is demonstrated in Fig. 5, where the jet production rates as a function of  $y_{cut}$ , observed by Mark-II at  $E_{cm} = 29$  GeV [33], are compared with the expectations of “QED” and QCD, both of which are calculated in complete second order perturbation theory. Also shown are the results of a phase space model (PS), which is derived from an  $O(\alpha_s^2)$  parton generator [23] by simply setting the QCD matrix elements to unity. The free parameters of the theoretical models, namely  $\Lambda_{\overline{MS}}$  and  $\mu$  for QCD, the abelian coupling strength  $\alpha_A$  and  $\mu$  for QED and the relative normalization of 3- and 4-parton events for the phase space model, are adjusted to describe the 2-, 3- and 4-jet rates observed at  $y_{cut} = 0.04^1$ .

As can be seen in Fig. 5, all three theoretical models can be adjusted to describe the data at the optimization point, but only QCD reproduces the  $y_{cut}$  dependence observed in the experiment. “QED” and PS are not compatible with the data over *ranges* of  $y_{cut}$ . The difference between QCD and QED indicates that the 2- and 3-jet event rates are indeed sensitive to the specific structure of QCD. These differences apparently are just a matter of the next-to-leading order corrections to  $R_2$  and  $R_3$ , since the  $y_{cut}$  dependence of  $R_4$ , which is only determined in leading order, is the same for both QCD and “QED”.

From these studies it is concluded that the abelian vector theory in second order perturbation theory and a simple phase space model are not adequate to describe the experimental jet production rates. This conclusion, however, is strictly valid only under the assumption that higher than second order corrections or an abelian hadronization model, about both of which nothing is known so far, do not drastically alter the “QED” predictions shown in Fig. 5.

#### (b) A Test of the Triple Gluon Coupling

Several experimental observables have been proposed which should be sensitive to the specific spin structure of the triple gluon vertex within 4-jet events. The AMY

<sup>1</sup>The resulting parameters are  $\Lambda_{\overline{MS}} = 107$  MeV and  $\mu^2 = 0.0016 \cdot E_{cm}^2$  for QCD and  $\alpha_A = 0.315$  and  $\mu^2 = 0.0025 \cdot E_{cm}^2$  for “QED”.



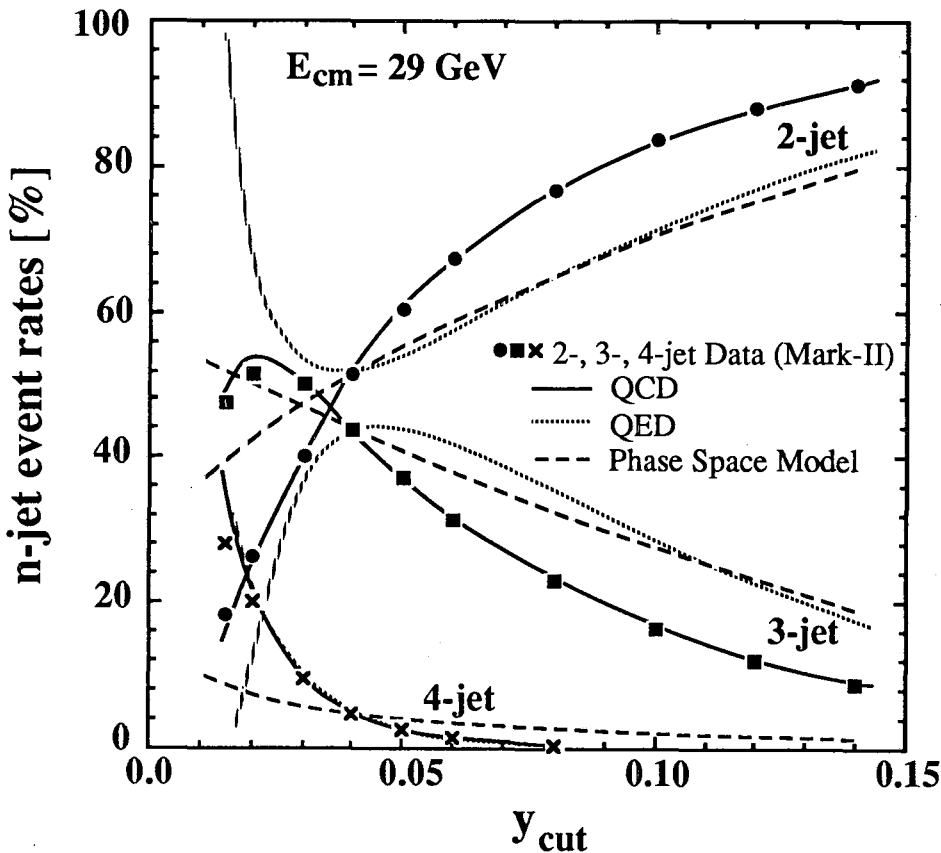


Fig. 5. Two-, three- and four-jet event rates observed at  $E_{cm} = 29$  GeV as a function of the jet resolution parameter  $y_{cut}$ , compared with  $O(\alpha_s^2)$  QCD, an  $O(\alpha_A^2)$  abelian vector theory and a simple phase space model, adjusted to describe the jet rates at  $y_{cut} = 0.04$ .

collaboration [32] selected a sample of 139 4-jet events ( $E_{cm} = 50 - 57$  GeV) and analysed them in terms of two of such observables, the angles  $\theta_{BZ}$  proposed by Bengtsson and Zerwas [39] and  $\theta_{NR^*}$ , proposed by Nachtmann and Reiter [40] and modified by Bengtsson [41]. Both these angles are calculated from the reconstructed jet axes of each event. The measured distributions of these angles are shown in Fig. 6. The data are compared with the corresponding results from an  $O(\alpha_s^2)$  QCD plus fragmentation model (full line) and to a second model in which the 4-parton QCD generator was replaced by an abelian version (dashed line).

The data are well described by the QCD model, while the abelian model is disfavoured with a significance of about 2 to 2.5 standard deviations. While this result is certainly very encouraging, a larger statistical significance is clearly needed in order to rule out the abelian model on the basis of this analysis. It should also be noted that the abelian model used in this study is not complete in  $O(\alpha_A^2)$ , since only the 4-jet generator has been replaced by the abelian version and the relative normalization of the different jet classes has been retained from the QCD model. Furthermore, as already mentioned in the previous paragraph, it cannot be excluded that the influence of higher order corrections and of an hypothetical abelian fragmentation model could explain the differences between the data and the “QED” model observed in this analysis.

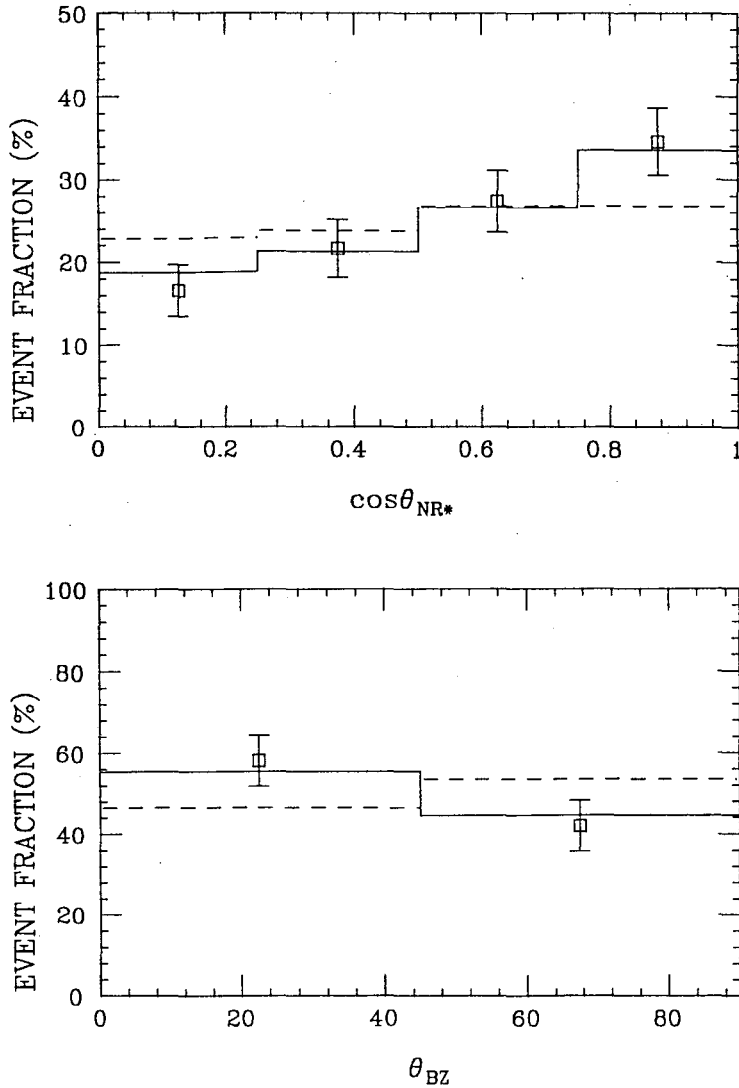


Fig.6.  $|\cos\theta_{NR^*}|$  and  $\theta_{BZ}$  distributions of 4-jet events, compared to an  $O(\alpha_s^2)$  QCD model (full line) and to an abelian model (dashed line); from AMY.

#### (d) Jet Production rates and the Running $\alpha_s$

Investigations of jet production rates make it possible to study the energy dependence of  $\alpha_s$  without determining explicit values of  $\alpha_s$  [25;31-33]. In  $O(\alpha_s^2)$ , the relative production rates  $R_n$  of 2-, 3- and 4-jet events are functions of  $\alpha_s$  and of  $\alpha_s^2$  as given in Eq. 1. The coefficients  $C_1$  and  $C_2$  depend on the actual value of the resolution parameter  $y_{cut}$  as well as on the prescription to recombine unresolvable partons, but do not exhibit an explicit dependence on the energy. Therefore, for a given value of  $y_{cut}$ , the energy dependence of the jet production rates  $R_n$  is only determined by the energy dependence of  $\alpha_s$ .

When reconstructing jets with the jet algorithm of JADE [25;30], hadronization corrections to the n-parton event rates are small and energy independent for values of  $y_{cut}$  that correspond to jet pair masses greater than 7 GeV [25]. The energy dependence of the experimental jet rates can therefore directly be compared to the  $O(\alpha_s^2)$

QCD expectations, with the restriction that hadronization effects may cause an overall uncertainty of about  $\pm 6\%$  in  $\alpha_s$ , [25;32;33].

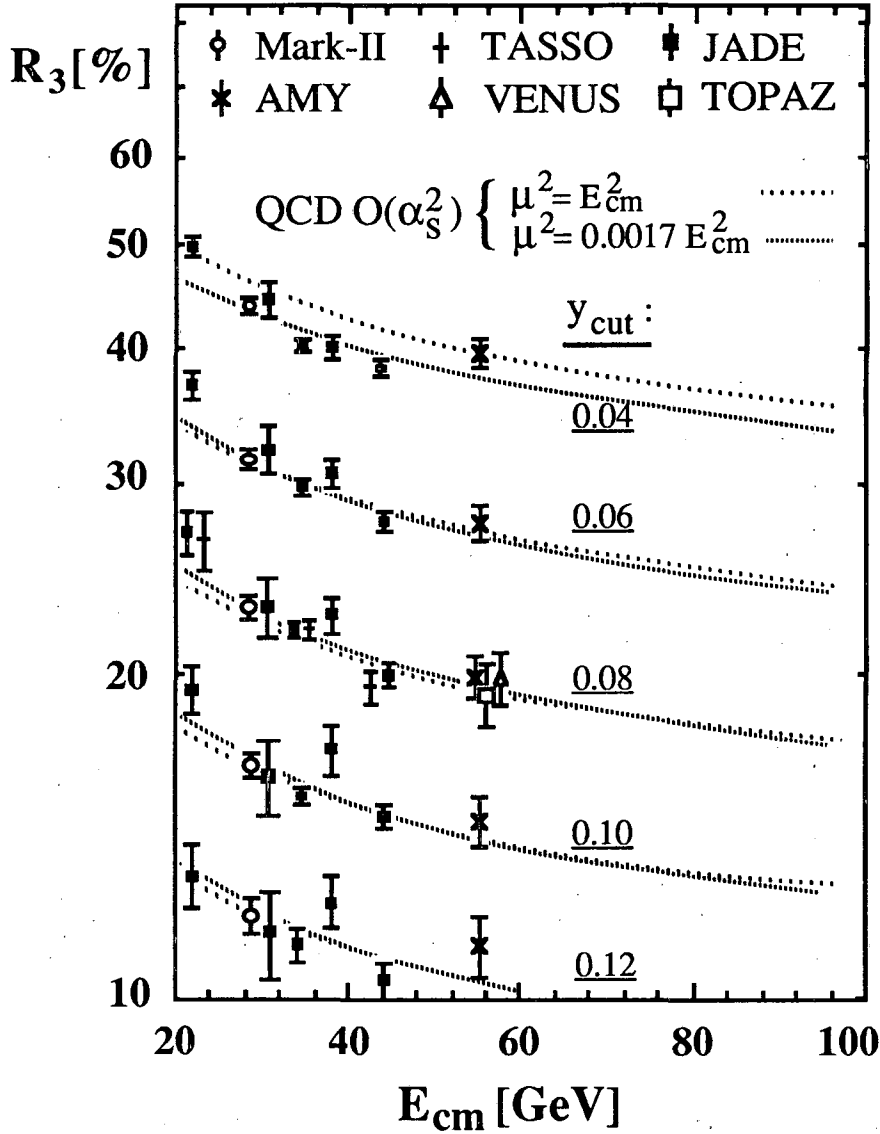


Fig.7. Three-jet event production rates observed at different center of mass energies, compared to  $O(\alpha_s^2)$  calculations with two different renormalization scales  $\mu^2$ .

The 3-jet event production rates observed by JADE [25], TASSO [31], Mark-II [33], AMY [32], VENUS and TOPAZ [42] in the center of mass energy range from 22 GeV to 60 GeV for various values of  $y_{cut}$ , are shown in Fig. 7. The data are compared to the calculations of Kramer and Lampe for two renormalization scales  $\mu^2 = E_{cm}^2$  ( $\Lambda_{\overline{MS}} = 230$  MeV) and  $\mu^2 = 0.0017 \cdot E_{cm}^2$  ( $\Lambda_{\overline{MS}} = 105$  MeV). The data are compatible with each other and with the theoretical expectations. While both QCD calculations describe the energy dependence as well as the absolute normalization of the data for all  $y_{cut} \geq 0.06$ , only the calculation with the smaller renormalization scale

also provides a simultaneous description at  $y_{cut} = 0.04$ . The assumption of an energy independent coupling strength, however, is not compatible with the data: in this case,  $R_3$  is expected not to depend on the center of mass energy.

The following fit results, obtained at  $y_{cut} = 0.08$  where the most data are available, express the significance of these observations (the combined value of the AMY, VENUS and TOPAZ results is used in the fits; the data at  $E_{cm} = 22$  GeV are not included [25]): The hypothesis of an energy independent coupling constant results in  $\chi^2 = 33.6$  for 7 degrees of freedom, while  $\chi^2 = 6.0$  and 5.3 for QCD with  $\mu^2 = E_{cm}^2$  and  $\mu^2 = 0.0017 \cdot E_{cm}^2$ , respectively. The possibility of  $\alpha_s = \text{constant}$  can therefore be excluded with a significance of 4.6 standard deviations.

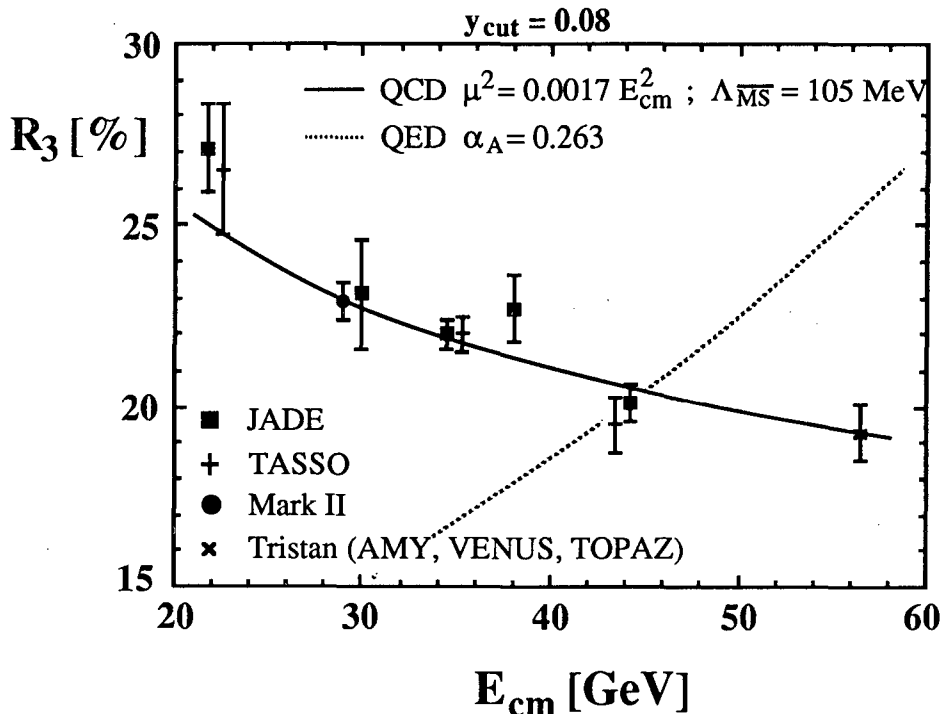


Fig.8. Three-jet event production rates at  $y_{cut} = 0.08$ , compared to  $O(\alpha_s^2)$  QCD and to an  $O(\alpha_A^2)$  QED calculation.

The energy dependence of 3-jet event rates, predicted by the abelian vector model (“QED”) as calculated in complete  $O(\alpha_A^2)$ , are shown at  $y_{cut} = 0.08$  in Fig. 8, together with the data and the  $O(\alpha_s^2)$  QCD result as shown in Fig. 6. For the “QED”-like theory, the abelian coupling constant  $\alpha_A$  and the renormalization scale  $\mu^2$  are adjusted to describe the experimental jet rates at  $E_{cm} = 44$  GeV, resulting in  $\alpha_A = 0.263^{+0.043}_{-0.051}$  and  $\mu^2/E_{cm}^2 = 0.0059^{+0.0019}_{-0.0029}$ . Note that the  $O(\alpha_A^2)$  calculation provides physical results, i.e. positive jet rates, only for small renormalization scales; fixing  $\mu^2$  to  $E_{cm}^2$  results in negative 3-jet rates [34]. The energy dependence of  $\alpha_A$  is calculated by an analytic solution of the renormalization group equation in  $O(\alpha_A^2)$  [43]. As expected, in “QED” the production rates of 3-jet events *rise* with increasing centre of mass energy; a

prediction which is clearly ruled out by the data with large significance. Even the fact that nothing is known about the possible influence of an abelian hadronization model can hardly affect this conclusion: the differences between the data and the "QED" predictions are too large to be explained by any reasonable assumption about hadronization. Also higher order contributions are unlikely to change these predictions by a large amount, since the energy dependence of  $\alpha_A$  is almost identical in  $O(\alpha_A)$  and  $O(\alpha_A^2)$ .

### VIII. ADDENDUM: THE LATEST RESULTS SINCE THE WORKSHOP

Between the end of this workshop (1 July 1989) and the time where this article is written (November 1989), several new experimental results which contribute to the topics discussed in this review became available. In this context, the measurement of  $\alpha_s$  at  $E_{cm} = 53.3$  GeV and 59.5 GeV by the TOPAZ collaboration [44], the determination of  $\Lambda_{\overline{MS}}$  from the total hadronic cross section [45], updated to include data obtained at the  $Z^0$  resonance [46], and the analysis of jet production rates in hadronic decays of the  $Z^0$  boson around  $E_{cm} = 91$  GeV by the OPAL collaboration [47] are of particular interest. They will therefore shortly be reviewed.

The TOPAZ collaboration determined  $\alpha_s$  in a measurement of the asymmetry of energy-energy correlations at  $E_{cm} = 53.3$  GeV and 59.5 GeV [44]. The data are compared to QCD model calculations [24] which are based on  $O(\alpha_s^2)$  calculations of Gottschalk and Shatz [23] for the renormalization scale  $\mu = E_{cm}$ . The results,  $\alpha_s(53.3 \text{ GeV}) = 0.129 \pm 0.007 \pm 0.010$  ( $\Lambda_{\overline{MS}}^{(5)} = 230_{-105}^{+160}$  MeV) and  $\alpha_s(59.5 \text{ GeV}) = 0.122 \pm 0.008 \pm 0.010$  ( $\Lambda_{\overline{MS}}^{(5)} = 185_{-100}^{+155}$  MeV), are compatible with the results and expectations from measurements at lower center of mass energies; see chapter IV. The new measurement extends the energy range in which similar determinations of  $\alpha_s$  are available by almost a factor of two. The systematic uncertainties of these and the previous results, however, still do not allow a definite proof of the energy dependence of  $\alpha_s$ .

The result on  $\Lambda_{\overline{MS}}$  from a combined analysis of the total hadronic cross section,  $R$ , has been updated [45] by including the data from Mark-II obtained on the  $Z^0$  resonance around  $E_{cm} = 91$  GeV [46]. The result of  $\Lambda_{\overline{MS}}^{(5)} = 260_{-130}^{+160}$  MeV in  $O(\alpha_s^3)$  has changed only very little by the inclusion of the  $Z^0$  data.

Jet production rates of hadronic decays of the  $Z^0$  boson were studied by Mark-II [48] and by OPAL [47]. Both collaborations observed that the new data, at the highest  $e^+e^-$  center of mass energies available, are well described by QCD model calculations with parameters optimized at  $E_{cm} = 29$  and 35 GeV. While the data of Mark-II do not yet comprise sufficient statistics in order to significantly contribute to the studies summarized in chapter VI-d, the analysis of OPAL is based on more than 10000 hadronic  $Z^0$  decays and thus adds important information to the energy dependence of jet production rates.

An update of Fig. 7, now also containing the results of OPAL at  $E_{cm} = 91$  GeV, is given in Fig. 9. The OPAL data agree with the theoretical expectations

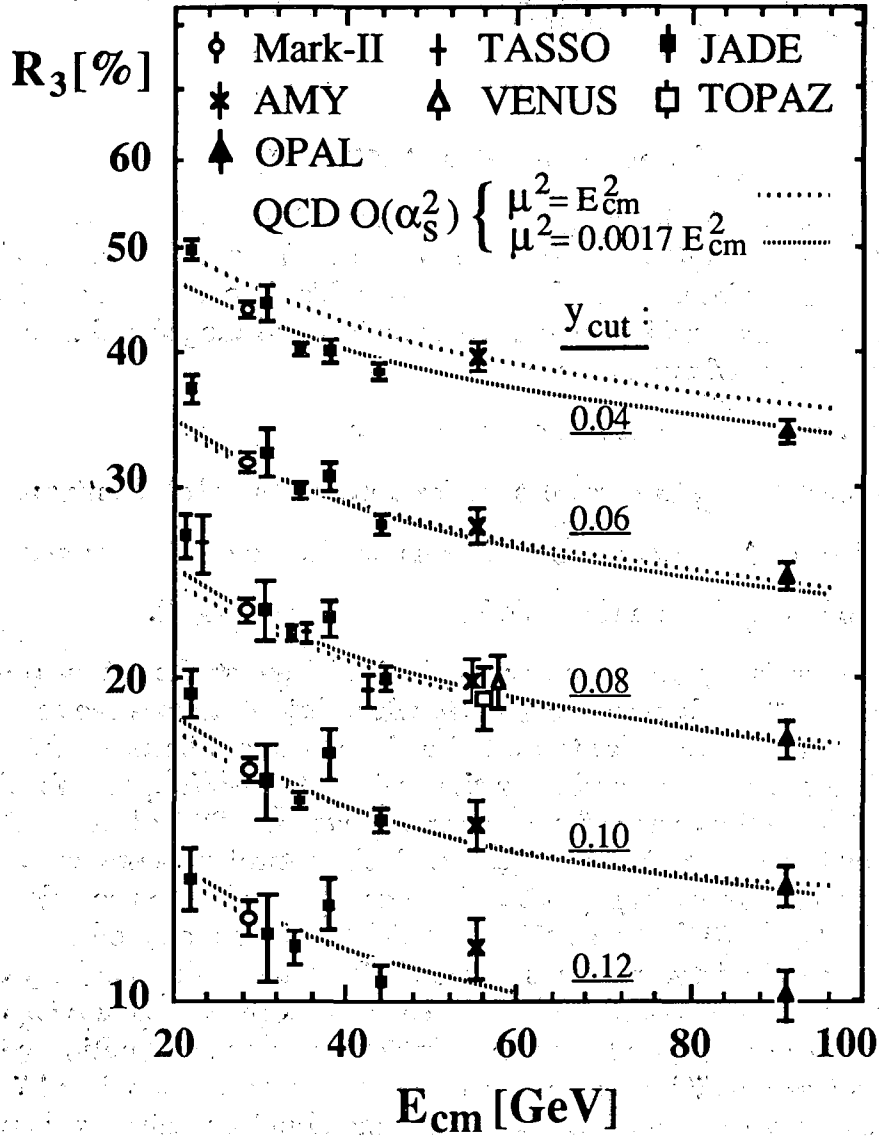


Fig.9. Three-jet event production rates as a function of the center of mass energy, compared to the same  $O(\alpha_s^2)$  calculations as shown in Fig. 7.

which are adjusted to describe the lower energy data. At  $y_{cut} = 0.08$  and for the data from 29 GeV to 91 GeV center of mass energy, the  $\chi^2$  of the hypothesis of an energy independent coupling constant is now 45.8 for 8 degrees of freedom. The QCD fits provide a significantly better description of the data with  $\chi^2 = 5.3$  and 6.2 for 8 degrees of freedom for  $\mu^2 = 0.0017 \cdot E_{cm}^2$  ( $\Lambda_{\overline{MS}} = 107 \pm 4$  MeV) and  $\mu^2 = E_{cm}^2$  ( $\Lambda_{\overline{MS}} = 250 \pm 11$  MeV), respectively. The differences in  $\chi^2$  rule out the possibility of an energy independent  $\alpha_s$  with a significance of 5.7 standard deviations.

OPAL also adjusts  $\Lambda_{\overline{MS}}$  and the renormalization scale  $\mu$  in  $O(\alpha_s^2)$  in the differential distribution  $D_2(y) = \frac{R_2(y) - R_2(y - \Delta y)}{\Delta y}$  ( $y \equiv y_{cut}$ ). The results are  $\mu^2/E_{cm}^2 = 0.001 - 0.003$  and  $\Lambda_{\overline{MS}}^{(5)} = 80$  MeV - 180 MeV, where the errors are largely dominated by the

Within the present accuracy of the experimental measurements and the theoretical calculations, a consistent description of the data in the entire energy range available so far is only provided by QCD.

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