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Essays in International Finance

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Doctor of Philosophy in Economics

by

Chang He

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ABSTRACT OF THE DISSERTATION

Essays in International Finance

by

Chang He
Doctor of Philosophy in Economics
University of California, Los Angeles, 2024
Professor Oleg Itskhoki, Chair

I am an economist working in the areas of International Finance and Macroeconomics. I study the determinants of exchange rates and sovereign risks and how they shape the cross-border movements of financial assets. Understanding these issues is crucial for designing effective monetary and fiscal policies in an open economy. As I show below in the three chapters of my dissertation, much of my work connects macroeconomic theory with rich micro-level data to empirically verify theoretical mechanisms.

In my first chapter of the dissertation (joint with Paula Beltran, IMF), "Inelastic Financial Markets and Foreign Exchange Interventions," we leverage the rebalancings of a local-currency government bond index for emerging countries as a quasi-natural experiment to identify the required size of foreign exchange intervention to stabilize exchange rates. We show that the rebalancings create large and exogenous currency demand shocks that move exchange rates. Our results provide empirical support for models of inelastic financial markets where foreign

exchange intervention serves as an additional policy tool to effectively stabilize exchange rates. Under inelastic financial markets, a managed exchange rate does not have to compromise monetary policy independence even in the presence of free capital mobility, relaxing the classical trilemma constraint. Our results show that, compared with countries with a managed exchange rate regime, countries with a free-floating exchange rate regime are more than twice more effective at stabilizing exchange rates. This is because these countries' volatile exchange rates lead to more inelastic financial markets and generate further departure from the trilemma.

In the second chapter of my dissertation (joint with Xitong Hui, CUHK), "A Theory of International Asset Returns: Country Size and Equity Rebalancing," we provide a theoretical framework to understand the return differences of sovereign bonds issued in different currencies. We develop a continuous-time two-country Lucas tree model with equity constraint and propose that the country-size effect and the equity-rebalancing effect are the key determinants of sovereign bond returns. The country-size effect spills over home production risk to a smaller country through trade and equity rebalancing; equity constraint limits equity rebalancing and creates endogenous uncovered interest parity deviations in both normal and crisis times. In the period of crisis, the larger country's sovereign bond becomes a global safe asset when the country-size effect dominates the equity rebalancing effect, as is the case with the United States.

In the final chapter of my dissertation (joint with Tamon Asonuma, IMF), "Too-little Sovereign Debt Restructurings," we study why sovereign debt restructurings often do not receive sufficient debt relief ("too-little" problem), followed by repeated restructurings. We classify 197 episodes of private external debt restructuring in 1975-2020 and provide novel empirical evidence that (1) restructurings with preemptive strategies are more likely to be "non-cured," requiring a second restructuring within five years; (2) restructuring strategies and outcomes tend to follow the previous restructuring; (3) "cured" post-default restructurings have better GDP growth and debt dynamics over the long horizon than non-cured preemptive restructurings. We propose a simple two-period model with endogenous choices of restructuring strategies to rationalize these stylized facts. The model predicts that the foreign creditor's state-dependent consumption smoothing motive results in small haircuts at preemptive restructurings, leading to new bond issues with high borrowing costs and thus subsequent restructurings.

Apart from the focus on exchange rates and sovereign risks, a common theme across all my work is a passion to work on rich micro-data and finding plausible natural experiments for valid identification. I then use theoretical models in macroeconomics, either new or existing, to rationalize novel empirical findings and address policy-relevant questions. I hope to continue this "micro-to-macro approach" in my future research agenda and continue to pursue my interests in international finance and macroeconomics, with a special focus on issues in exchange rates and sovereign risks.

The dissertation of Chang He is approved.

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CHAPTER 1

Inelastic Financial Markets and Foreign Exchange Interventions

with Paula Beltran[1]

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1.1 Introduction

Are foreign exchange interventions effective at moving exchange rates? And if so, how large should the size of intervention be to stabilize exchange rates? Policymakers frequently resort to large-scale foreign exchange interventions, both in normal and in crisis times.[2] Assessing the effectiveness of the foreign exchange intervention is empirically challenging, because exchange rates, the prevailing macroeconomic conditions, and the intervention itself are jointly endogenous. Several papers have provided empirical evidence on the effects of foreign exchange interventions by resorting to confidential and high-frequency data on intervention episodes (Adler et al., 2019; Fratzscher et al., 2019). Yet, a valid identification calls for a natural experiment that exogenously changes the currency composition of the government bonds in an economy.

In this paper, we overcome this identification challenge by using a quasi-natural experiment to estimate the required size of intervention to stabilize exchange rates. Specifically, we leverage the exogenous currency demand shock from the mechanical rebalancings of J.P. Morgan's Government Bond Index-Emerging Markets (GBI-EM) Global Diversified. Our empirical results provide evidence for models of inelastic financial markets where foreign exchange intervention serves as an effective policy tool to stabilize exchange rates. Through the lens of the

[2] Foreign exchange interventions do not only happen in crisis times. Fratzscher et al. (2019), who used data from central bank interventions from 33 countries, documented that the average daily volume of foreign exchange interventions (either purchase or sale of reserves in the spot market) is 44.3 million USD, with only 0.225 of those intervention days covered in turbulent times.

model, we identify the required size of intervention to stabilize exchange rates for countries with different exchange rate regimes.

The exogenous currency demand shock created by the mechanical rebalancings of the GBI-EM Global Diversified index is crucial for our identification. This is the most widely tracked benchmark index by mutual funds that invest in local-currency government bonds in emerging markets. The monthly rebalancings cap the benchmark weight of each country in the index at 10%, and any excess weight above the cap is redistributed to smaller countries so that all the weights add up to 1. At the rebalancing dates, countries not at the cap experience a positive weight increase not because of an improvement in their economic conditions, but purely as a result of the bigger countries hitting the cap. Thus, the rebalancing feature gives rise to large and exogenous cross-border capital flows for countries not at the weight cap.

We construct our exogenous currency demand shock as the percentage change in the country weights before and after a rebalancing event. Intuitively, the shock captures the change in market value of the local-currency sovereign bonds in the index purely implied by the mechanical rebalancings, independent of the market prices and macroeconomic conditions. For clean identification, we use currency demand shocks only from countries not at the 10% weight cap at the rebalancing dates. A one standard deviation of the shock equals a 4.844% change in market value (or, equivalently, 0.62 billion USD flows on average) of a country's government bonds in the GBI-EM Global Diversified index.

We show that exchange rates respond significantly to the currency demand shock and the effects are persistent for at least three months. On average, a one standard deviation of the currency demand shock appreciates local currencies by 1.2% in the days following a rebalancing event. Despite the significant exchange rate response, central bank monetary policy rates, macroeconomic variables (e.g., GDP, consumption, and net exports), and foreign exchange intervention data do not respond to the currency demand shock. This implies that the macroeconomic conditions are smooth around the index rebalancing events, consistent with the exogeneity assumptions.

The fact that exchange rates respond significantly to the currency demand shock is consistent with models of inelastic financial markets (e.g., Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021). Under these markets, a currency demand shock changes the arbitrageurs' holdings and gives rise to endogenous deviations in the uncovered interest parity (UIP) condition. By comparison, standard macroeconomic models (e.g., Mundell, 1962; Gali and Monacelli, 2005; Farhi and Werning, 2012) assume perfectly elastic financial markets or UIP holds. If financial markets were truly elastic, a currency demand shock would have no impact on either the path of exchange rates or the UIP condition.

Inelastic financial markets have important implications for the effectiveness of foreign exchange interventions at stabilizing exchange rates. Under models of inelastic financial markets, foreign exchange interventions shift the arbitrageurs' risk-bearing capacity in a similar way to the currency demand shock, leading to endogenous deviations in the UIP condition. Therefore, foreign exchange interventions serve as an additional policy tool to effectively stabilize exchange rates, whereas monetary policies can be entirely inward-focused on domestic inflation and output gap. Even under free capital flows, an economy can simultaneously have an independent monetary policy and a managed exchange rate through foreign exchange interventions. We refer to this condition as the "relaxed trilemma" (Basu et al., 2023; Itskhoki and Mukhin, 2023a).

We show that the more inelastic the financial markets, the more effective the foreign exchange interventions. Therefore, the interventions should be more effective for countries with a free-floating currency exchange regime ("free floaters"). Through the lens of our model, the higher exchange rate volatility for free floaters makes the financial markets more inelastic and generates further departure from the trilemma constraint. At the other extreme, in countries where exchange rates are fully pegged (i.e., those with a fixed exchange rate regime, or "peggers"), we are back to the elastic financial market model under the trilemma constraint where foreign exchange interventions are ineffective.

Our findings suggest that the required size of foreign exchange intervention is half as large for free floaters than for managed floaters or peggers (i.e., the intervention works more effectively for free floaters). This can be seen from the larger exchange rate response to the currency demand shock for free floaters. We then convert the exchange rate response to the flows implied by the rebalancings through computing the assets under management of the mutual funds tracking the index. Through the lens of our model, the counterfactual size of intervention required to stabilize exchange rates would have to exactly offset the impact from the currency demand shock. On average, we find that to achieve a 1 percent exchange rate appreciation (resp., depreciation), the average foreign reserves that the central bank needs to sell (resp., buy) in foreign exchange interventions is about 0.1% of GDP.

Related Literature. Our results contribute to various strands of literature in both macroeconomics and finance and are informative to central bank policymakers. First, we contribute to the large empirical literature on the effects of foreign exchange interventions, including Fatum and Hutchison (2003), Blanchard et al. (2015), Fratzscher et al. (2019), and Adler, Lisack and Mano (2019), and the foreign exchange policy framework in Jeanne (2012), Amador, Bianchi, Bocola, and Perri (2019), Cavallino (2019), Fanelli and Straub (2021), Basu et al., (2023), and Itskhoki and Mukhin (2023a). We add to this literature by finding a plausible exogenous currency demand shock through leveraging the rebalancings of a local-currency government bond index as a quasi-natural experiment.

Moreover, our paper connects with the broad finance literature on asset demand estimation and evidence for inelastic financial markets. Empirical studies using index rebalancing (for example, the rebalancings of the S&P 500 index) to estimate asset demand curves date back to Shleifer (1986), followed by a series of studies by Lynch and Mendenhall (1997), Kaul, Mehrotra and Morck (2000), and Chang, Hong and Liskovich (2014) with more refined and cleaner identification strategies. Recent work, such as Pandolfi and Williams (2019), Koijen and Yogo (2019, 2020), Camanho, Hau and Rey (2021), and Moretti et al. (2024), estimates the (global) asset pricing demand system, and Gabaix and Koijen (2022) discuss policy implications for inelastic financial markets. Our paper applies the empirical strategy of index rebalancing traditionally used to estimate asset demand in a new context: foreign exchange interventions.

In addition, our paper speaks to the macro-finance literature on exchange rate dynamics in segmented markets with frictional financial markets. The segmented financial market model we use in this paper builds on Jeanne and Rose (2002), Alvarez, Atkeson and Kehoe (2009), Gabaix and Maggiori (2015), Gourinchas, Ray and Vayanos (2019), Greenwood, Hanson, Stein, and Sunderam (2020), and Itskhoki and Mukhin (2021). Another recent work, by Jiang, Krishnamurthy and Lustig (2022), produces similar exchange dynamics but features incomplete rather than segmented financial markets.

Finally, our work is related to the large literature on exchange rate prediction. The related papers to ours include Fama (1984), Evans and Lyons (2002), Tornell and Gourinchas (2004), Lustig and Verdelhan (2007), Engel (2016), Hassan and Mano (2019), Kremens and Martin (2019), Jiang, Krishnamurthy and Lustig (2022), and Kremens, Martin and Liliana (2023). While these works mostly leverage taste shocks or expectation errors in forecasting exchange rates, our currency demand shocks for predicting exchange rates rely on a quantity shock from the mechanical index rebalancings.

Outline. The rest of the paper is structured as follows. In the first part of the paper, we introduce the exogenous currency demand shock and illustrate its relation to the dynamics of exchange rates and interest rates. To interpret these stylized empirical facts, in the second part of the paper we present an inelastic financial market model where a currency demand shock leads to endogenous deviations in the uncovered interest parity condition. In the third and last part of the paper, we introduce foreign exchange interventions into the inelastic financial market model and estimate the required size of intervention to stabilize exchange rates.

1.2 Introducing the Currency Demand Shock

We leverage the mechanical rebalancing features of a local-currency government bond index for emerging countries to construct an exogenous currency demand shock. We document in detail the rebalancing rules of the index and introduce our measure for the currency demand shock as well as the implied capital flows from the shock.

1.2.1 Mechanical Rebalancings of the GBI-EM Global Diversified

Our empirical strategy relies on the mechanical rebalancings of the Government Bond Index-Emerging Markets (GBI-EM) Global Diversified published by J.P. Morgan. This is the largest local-currency government bond index for emerging countries. An estimated assets under management of more than 200 billion USD of (both active and passive) mutual funds track the index in 2019.[3] At the time of writing, there are 19 emerging countries in the index; each country's weight equals the share of its market value of the local-currency sovereign bonds in the index. A larger country, such as Brazil, has a larger weight in the index than a smaller country, such as Peru or Chile.

The mechanical rebalancings by the GBI-EM Global Diversified index on the country weight cap are crucial for the identification in this paper. The country weight fluctuates daily as the market price of the sovereign bonds moves up or down. However, at the rebalancing date (which is the end of the last business

[3] The 200 billion USD is a large number for the emerging-market sovereign bonds market because the total new issuance of the emerging-market sovereign bonds is merely 160 billion USD in 2019 (Refinitiv data).

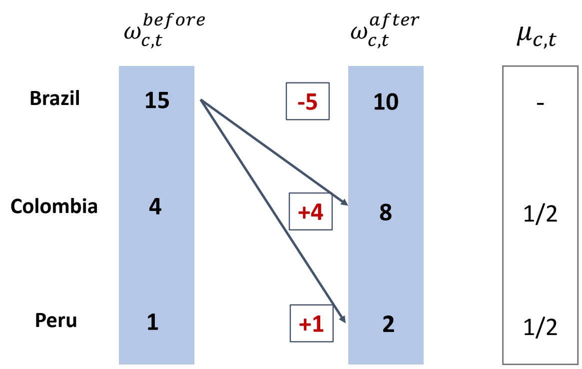
day of each month), the index mechanically caps the country weight at 10% for all countries to limit concentration risk. Any excess weight above the 10% cap is redistributed to smaller countries that are below the cap, proportionally to their allocation so that all country weights add up to 100%. In addition, countries at the cap will follow the "10/10" rule: the country that meets the 10% cap or larger in the GBI-EM index will be staggered over for a 10-month period.[4]

We argue that for countries not at the 10% country-weight cap, their change in weights in the GBI-EM Global Diversified index creates currency demand shocks that are uninformative to the macroeconomic fundamentals of the sovereign. For example, if Brazil's country weight is rebalanced from 15% to 10% and leads to an increase in Peru's country weight, those benchmarked mutual funds have to sell local-currency sovereign bonds of Brazil and buy Peruvian sol in order to purchase local-currency sovereign bonds of Peru. In this rebalancing example, a smaller country experienced a positive currency demand shock on its local-currency bonds *independent* of its own macroeconomic conditions and purely as a result of a larger country hitting the 10% cap.[5]

[4] This means that once Brazil's allocation in the index exceeds 10%, for example, it will be fixed at the cap of 10% at every rebalancing date for a 10-month period, regardless of whether Brazil's allocation exceeds the cap before rebalancing.

[5] The rebalancings can continue recursively for multiple rounds until all the country weights are either at or below the 10% cap. The rebalancings are also done in three layers in order, and the country-weight rebalancing is the last layer following the face-amount inclusion and bond maturity threshold. Appendix A discusses the first two layers of rebalancings and how the countries are chosen to enter or exit the index.

Table 1.2.1: A Simplified Rebalancing Example at the 10% Weight Cap



Note: This table presents a simplified rebalancing example that caps the country weight at 10%. For simplicity, assume there are 11 countries in the index and 8 of them are already at 10%. The rebalancings therefore apply only to Brazil (with weight 15%) above the cap, and Peru (with weight 1%) and Columbia (with weight 4%) below the cap. Each round of rebalancing takes the excess weight of the country and redistributes it to the smaller countries below the cap proportionally to the weight of each country. The rebalancings continue recursively until all the country weights are either at or below the 10% cap. In this example, the currency demand shock $\mu_{c,t}$ for both Colombia and Peru are 1/2 (computed as 4/8 and 1/2, respectively).

1.2.2 Measuring the Currency Demand Shock

We introduce $\mu_{c,t}$ to capture the currency demand shock from the mechanical rebalancings of the GBI-EM Global Diversified index, for country c at the rebalancing date t. As shown in equation (1.1), we define $\omega_{c,t}^{\text{before}}$ and $\omega_{c,t}^{\text{after}}$ as the country weight before and after the rebalancing event, respectively, at the rebalancing date. Taking market price $P_{c,t}$ as given, J.P. Morgan adjusts the country weights (from $\omega_{c,t}^{\text{before}}$ to $\omega_{c,t}^{\text{after}}$) through changing the par value ($\hat{Q}_{c,t}$) of the local-currency sovereign bonds of the countries included in the index:

$$\mu_{c,t} = \frac{\omega_{c,t}^{\text{after}} - \omega_{c,t}^{\text{before}}}{\omega_{c,t}^{\text{after}}}, \tag{1.1}$$

where $\omega_{c,t}^{\text{before}} = \frac{P_{c,t}\hat{Q}_{c,t-1}}{\sum_c P_{c,t-1}\hat{Q}_{c,t-1}} \times \frac{1}{(1+r_t)}$ and $\omega_{c,t}^{\text{after}} = \frac{P_{c,t}\hat{Q}_{c,t}}{\sum_c P_{c,t}\hat{Q}_{c,t}}$; $P_{c,t}$ is the aggregate market price of the local-currency sovereign bonds for country c at the rebalancing date; $\hat{Q}_{c,t-1}$ and $\hat{Q}_{c,t}$ are the aggregate par value of the local-currency sovereign bonds included in the index from the last rebalancing and the current rebalancing, respectively; [6] r_t is the monthly return of the GBI-EM Global Diversified index from the past rebalancing date $t-1$ to the current rebalancing t. Intuitively, $\omega_{c,t}^{\text{before}}$ is the buy-and-hold weight of the country, as the term $\sum_c P_{c,t-1}\hat{Q}_{c,t-1} \times (1+r_t)$ is the market value of the index at the rebalancing date t if rebalancing does not take place. We normalize $\mu_{c,t}$ by its own weight after rebalancing because countries have different market sizes for their sovereign bonds markets, and a 1 billion USD flow would be very different for Brazil compared with a smaller country such as Peru. Table 1.2.1 gives a simplified rebalancing example.

Our main empirical analysis focuses on currency demand shocks from countries that do *not* meet the 10% cap at the rebalancing dates. These countries have to change their weights as a result of the bigger countries meeting the cap (either by exceeding the cap or staggered due to the 10/10 rule). Therefore, their change in weight is independent of their macro-fundamentals, which are smooth around the rebalancing date. In the example in Table 1.2.1, we would use only the change

[6] It is important to distinguish the face amount of sovereign bonds included in the index ($\hat{Q}_{c,t}$) from the face amount of the actual issuance ($Q_{c,t}$) by the sovereign. Appendix A.1 explains the linear extrapolation rule where a portion of the country's actual sovereign bonds outstanding is included in the GBI-EM Global Diversified index.

Table 1.2.2: DISTRIBUTION OF THE CURRENCY DEMAND SHOCK ($\mu_{c,t}$ IN %)

$\mu_{c,t}$, *including* observations at the 10% cap

Obs.	Mean	Std.	Min.	Max.	Median	90%	10%
2,197	-0.152	4.480	-15.627	21.192	-0.031	4.387	-5.371

$\mu_{c,t}$, *excluding* observations at the 10% cap

Obs.	Mean	Std.	Min.	Max.	Median	90%	10%
1,565	-0.162	4.844	-15.627	21.192	-0.156	4.686	-5.643

Note: This table reports the summary statistics on the currency demand shock ($\mu_{c,t}$), in percentage points, implied by the monthly rebalancings of the GBI-EM Global Diversified index. In the top panel we report the distribution statistics including those at the 10% cap, and in the bottom panel we drop the observations at the 10% cap. A negative $\mu_{c,t}$ (< 0) implies that the country is rebalanced downwards, and vice versa for a positive $\mu_{c,t}$. In the empirical analysis, we drop the countries at the 10% cap, for cleaner identification.

in weights from Peru and Columbia for our identification. The currency demand shock for Brazil is therefore not reported.

Table 1.2.2 shows the summary statistics of the currency demand shocks for the countries in our sample. Table 1.B.27 and 1.B.46 in the appendix report the time series of the currency demand shock and weight after the rebalancing for each country. There is significant heterogeneity across countries in meeting the 10% weight cap: Specifically, while Brazil, Mexico, and Poland each had the majority of its time series with weights capped at 10%, Indonesia, Malaysia, Russia, South Africa, Thailand, and Turkey also occasionally met the 10% cap and were staggered for a 10-month period because of the 10/10 rule explained above. Smaller countries (Argentina, Chile, Colombia, Czech Republic, Hungary, Peru, the Philippines, Romania, and Uruguay) never met the 10% cap throughout the sample.

1.2.3 Flows Implied by the Currency Demand Shock

The mechanical rebalancings of the GBI-EM Global Diversified index create large demand shocks on the local-currency government bonds. We show that the mutual funds tracking the index passively and with large asset positions comply with the rebalancing rules, as illustrated by their high-performance R-squared against the returns of the GBI-EM Global Diversified index. We select from the Emerging Portfolio Fund Research (EPFR) dataset all emerging market bond funds whose benchmark indices are the GBI-EM Global Diversified index[7] and regress the monthly returns of each fund on the returns on the index[8]. This gives us a large median R-squared, of 0.92 (Table 1.B.7 in the appendix). We also construct the weighted average return (by asset under management) of these mutual funds and regress the weighted return on the index returns, which results in an even higher R-squared, of 0.97 (Table 1.B.8).

To convert the currency demand shocks to USD flows, we estimate the total assets under management of the mutual funds tracking the GBI-EM Global Diversified index globally. Figure 1.B.3 panel (a) plots the assets under the management of funds tracking the GBI-EM Global Diversified index in the EPFR data from 2016 to 2022. Figure 1.B.3 panel (b) shows the representation of EPFR data in the total mutual fund population as estimated by the Investment Company Institute (ICI)

[7] Details on how we selected mutual funds into the data are reported in Appendix A.

[8] We follow Amihud and Goyenko (2013) and Pandolfi and Williams (2019) and use the return regression to test the performance of mutual funds. The method regresses the fund-level monthly returns on the monthly returns of the GBI-EM Global Diversified: $r_{i,t} = \alpha + \beta r_{B,t}$, where $r_{i,t}$ is the monthly returns from fund i at time t, and $r_{B,t}$ is the monthly returns from the benchmark (i.e.,the J.P. Morgan GBI-EM Global Diversified index in our case). We then collect the fitted R-squared from each return regression. A higher fitted R-squared indicates the fund tracks the benchmark index more closely.

Global. The figures show that EPFR data represent about 60% of the worldwide mutual fund population in 2019.

1.2.4 Data Sources

The main data source we use is the Index Composition and Statistics reports from J.P. Morgan. These reports include monthly information on benchmark weights and rebalancing for their sovereign bonds benchmarks, including the GBI-EM Global Diversified index. Our sample comprises a panel of 18 countries from 2010 to 2021: Argentina, Brazil, Chile, Colombia, Czech Republic (Czechia), Hungary, Indonesia, Malaysia, Mexico, Peru, the Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, and Uruguay.[9] These reports allow us to construct our currency demand shock as introduced above.

The second main data source we use is the EPFR data on the asset positions of the emerging market bond funds. We show that the currency demand shock is correlated with the changes in the asset positions of the mutual funds tracking the GBI-EM Global Diversified index in the EPFR data. Moreover, we use the EPFR data to compute the flows in US dollars implied by the rebalancings by our currency demand shock.

Finally, we combine J.P. Morgan reports and EPFR fund flows data with daily data of exchange rates and data on central bank policy rates from the Bank for International Settlements. We complement these data with sovereign bond yields

[9] We exclude China from the current analysis because there are limited time series on this country in the dataset, as China entered the GBI-EM Global Diversified index only in 2020; we exclude Dominican Republic and Nigeria from the analysis because there are limited data on exchange rates for these countries from the Bank for International Settlements (BIS) statistics.

for each country from Du and Schruger (2016), with the dataset updated until 2021.

1.3 Currency Demand Shocks and Exchange Rates

In this section, we present four novel empirical facts on how the currency demand shock affects exchange rates and interest rates.

Empirical Fact 1: *The currency demand shock moves exchange rates in the short run. A one standard deviation increase in the shock appreciates exchange rates by an average of 1.2% for the cross-country sample.*

Figure 1.1 reports the estimated coefficients of cumulative exchange rate changes on our currency demand shock as measured by $\mu_{c,t}$ in equation (1.1). The regression takes the following form:

$$\Delta e_{c,t+d} = \beta_0 + \beta_\mu \, \mu_{c,t} + \phi \, X_{c,t} + \epsilon_{c,t}, \tag{1.2}$$

where $\mu_{c,t}$ is the currency demand shock defined in equation (1.1); β_0 is the constant; and $X_{c,t}$ is a set of dummies that control for country and date fixed effects, respectively. Standard errors are clustered at the date level. We include time fixed effects at the date level to account for the cyclicality of the global financial cycle (Rey, 2013) and the documented tighter balance sheet constraints for banks toward the quarter ends (Du, Tepper and Verhelhan, 2018).

Exchange rates are measured in local currencies per US dollar, and the exchange rate change $\Delta e_{c,t+d}$ is the cumulative change from the time interval from 28 days before the rebalancing date 0 until d days after this date ($d < 0$ for days before the rebalancing date 0; if $d > 0$, vice versa). Regression estimates for each d days after rebalancing are represented by the red dots in Figure 1.1, with the 95% confidence intervals represented as black bars. We standardize the currency demand shock by its mean and standard deviation according to the distribution in Table 1.2.2. As discussed, in our main empirical analysis we exclude all country-month observations that exceed the 10% threshold from the regression to ensure the currency demand shock is information-free and independent of the macro-fundamentals.[10]

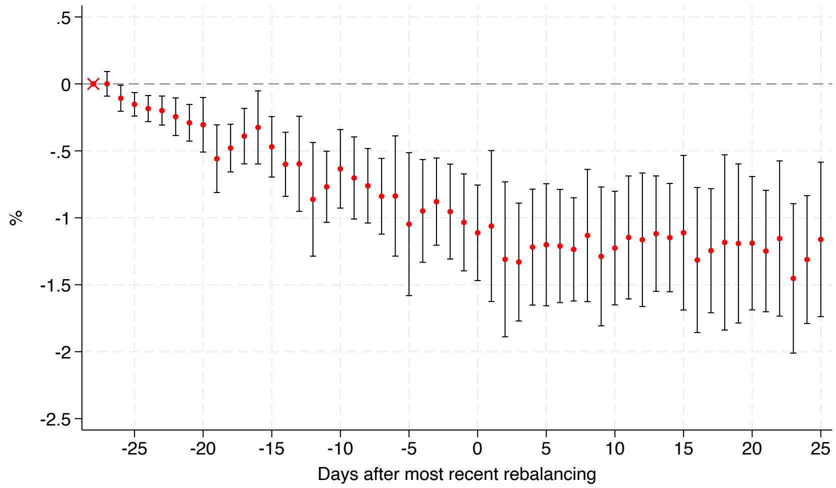
The pooled OLS regression shows that a one standard deviation increase in $\mu_{c,t}$ appreciates local-currency exchange rates significantly, by 1.2%, after one rebalancing event for the cross-country sample. The estimate of 1.2% is the average regression coefficient 0 to 5 days after a rebalancing event, when the exchange rate level does not appreciate further. A one standard deviation increase in $\mu_{c,t}$ is equivalent to a 4.844% (by Table 1.2.2) change in the country weight in the index, which is on average about 0.62 billion USD flows for the emerging countries in our sample.[11]

Remark 1: *Why do exchange rates start to move before rebalancing date 0?*

[10]Nevertheless, we report the regression results that include countries at the 10% threshold in Table 1.B.68 in the appendix, which shows that the estimates are largely identical to those in Fact 1.

[11]Details of the computation are in section 6.1. The basic logic to compute the average USD flows corresponding to a one standard deviation of $\mu_{c,t}$ is the following: 4.84% (std. dev of the shock) $\times$ 6.36% (average weight for a country in the index) $\times$ 120 billion USD (assets under management for the EPFR mutual funds that closely track the GBI-EM Global Diversified index in 2019), divided by 0.6 (share of EPFR mutual fund population in the global funds) = 0.62 billion USD.

Figure 1.1:
Fact 1: Currency Demand Shock Moves Exchange Rates in the Short Run



Note: This figure presents the estimated regression coefficient of the change in exchange rates on the currency demand shock measured by $\mu_{c,t}$ in equation (1.1); $\mu_{c,t}$ is standardized by its mean and standard deviation in the regression. The change in exchange rates (local currencies per US dollar) is measured as the cumulative change starting from 28 days before the recent rebalancing at day 0. The regression is performed in a pooled OLS using time and country fixed effects, with standard errors clustered at the date level. The results are shown as point estimates (red dots) with 95% confidence intervals (black bars) for each regression.

As shown in Figures 1.1 and 1.2, exchange rates respond significantly to the currency demand shock $\mu_{c,t}$ before the rebalancing date 0. We state that these dynamics are expected and strongly support the "efficient market hypothesis" (Fama, 1970). Change in country weights is predicted before the rebalancing date as J.P. Morgan Markets announces its mid-month projections.[12] The mutual funds tracking the index would buy or sell government bonds almost immediately as new information about the next rebalancing feeds in, and exchange rates would move before the rebalancing date as the efficient market hypothesis pre-

[12]Nevertheless, those predictions are imprecise, especially for smaller countries not at the 10% cap.

dicts. The fact that exchange rates start to move before the rebalancing date is also consistent with the movements of stock prices reported in other works on index rebalancings (Kaul, Mehrotra and Morck, 2000; Duffie, 2010;).

Remark 2: *Can other local-currency emerging market sovereign bond indices also contribute to the observed exchange rate movements?*
One concern on identification is that other local-currency emerging market sovereign bond indices may also contribute to the variation in exchange rates. We examine carefully the rebalancing mechanisms of all leading local-currency government bond indices for emerging countries. We find that most of them have different rebalancing schemes and timing compared with the GBI-EM Global Diversified index, with the exception of the Russell FTSE Emerging Markets Government Bond Index (EMGBI-Capped).[13] However, a simple aggregation exercise shows that the total asset positions of the funds tracking the EMGBI-Capped are not even 10% of the positions of the GBI-EM Global Diversified index in our EPFR dataset. Therefore, we consider the variation in exchange rates created by indices other than GBI-EM Global Diversified negligible.

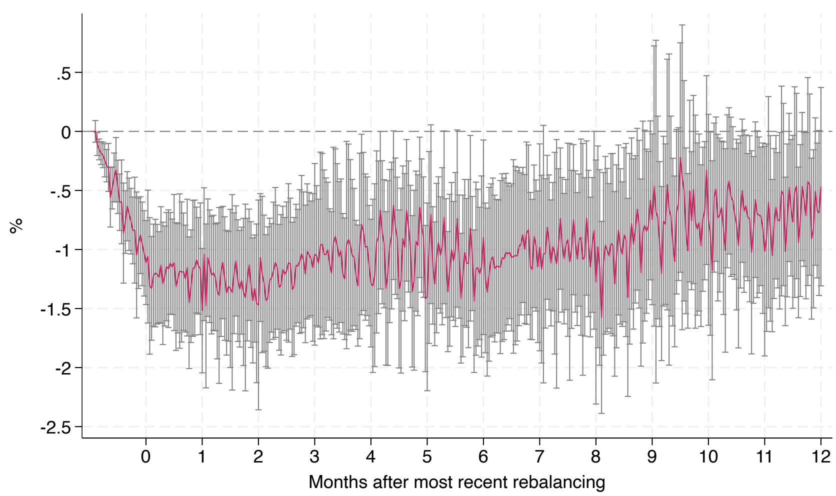
Remark 3: *Could the co-movements of macro-fundamentals across countries contaminate the results on identification?*
Another concern on identification is whether the macro-fundamentals and sovereign bond prices co-move systematically across countries. For example, one might suspect that the inflation-targeting Latin American countries in our sample (namely, Argentina, Brazil, Colombia, Mexico, and Peru) would have strong and positive

[13] FTSE fixed income EMGBI by Russell was introduced in 2018 as a rebranding of an older Citi Group WGBI index. It is an emerging market local-currency government bond index and has an end-of-month country weight cap at 10%.

co-movements in sovereign bond prices throughout our sample. We show in Table 1.B.47 that there is significant heterogeneity across countries in their correlations of aggregate local-currency sovereign bond prices at the rebalancing date, even within the group of Latin American countries. In addition, one should note that the index rebalancings happen at monthly frequency over our decade-long sample. Thus, it is unlikely that the sovereign bond prices of any two countries move in the same direction at every rebalancing date.

Empirical Fact 2: *The currency demand shock has a persistent effect on exchange rates, lasting about three months after a rebalancing event.*

Figure 1.2:
Fact 2: Currency Demand Shock Has Persistent Effects on Exchange Rates



Note: This figure plots the estimated coefficients of the change in the cumulative exchange rate on the currency demand shock measured by $\mu_{c,t}$ in the four-month horizon after a rebalancing event; $\mu_{c,t}$ is standardized by its mean and standard deviation in the regression. The dependent variable is the change in cumulative exchange rates starting from 28 days prior to the first rebalancing event. All regressions are performed in a pooled OLS using time and country fixed effects, with standard errors clustered at the date level. The results are shown as point estimates (red dots) with 95% confidence interval (blacks bars) for each regression.

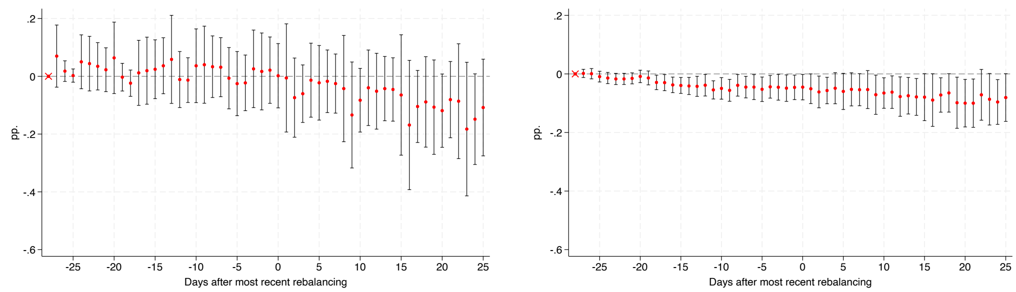
Figure 1.2 shows that the effects of rebalancings on exchange rates do not disappear immediately; instead, they remain significant for at least three months after a rebalancing event. In addition, the same as the response in the short run, the cumulative exchange rates on average appreciate about 1.2% in response to a one standard deviation increase in $\mu_{c,t}$ and do not appreciate further after the first rebalancing event. In addition, the effects on exchange rates gradually lose significance after four months. The regression results include time and country fixed effects, with standard errors clustered at the date level.[14]

Remark 4: *Why does the currency demand shock have persistent effects on exchange rates?* As shown in Figure 1.2, exchange rates have a significant and persistent response to the currency demand shock for at least three months. The fact that exchange rate reversal takes time is consistent with the "slow-moving capital" argument (Duffie, 2010) that the price reversal happens gradually over time as additional capital becomes available following the initial currency shock. In addition, our regression captures a level shift in exchange rates (starting 28 days before a rebalancing event) and there are thus no gains of excess returns for the arbitrageurs in the financial market even if the effects persist for about four months.

Empirical Fact 3: *Policy rates and short yields do not respond to the currency demand shock.*

[14]We do not control for macro-fundamentals because our variables (such as GDP and net foreign asset positions) are much more slow-moving compared with exchange rates, and including them does not alter the baseline results. We also show in Table 1.4.1 that the macro-fundamentals (GDP and net foreign asset positions) are immune to the currency demand shock.

Figure 1.3:
Fact 3: Policy Rates and Yields Do Not Respond to the Currency Demand Shock



Note: Pooled regression coefficients of the change in monetary policy rates (in percentage points, left panel) and change in annualized three-month local-currency government bond yields relative to synthetic USD yields ($i_{c,t} - i^*_{c,t}$) in percentage points, right panel) with 95% confidence intervals. Monetary policy rates and three-month government bond yields are provided at the daily frequency and are defined as the cumulative change from 28 days before the rebalancing date.

Another concern for identification is that central bank policy rates might respond to the rebalancings of the GBI-EM Global Diversified index. If the policy rates were to move, the macro-fundamentals and exchange rates would also respond, violating the exogenous nature of the currency demand. We show that this is not the case.

Central bank policy rates and yields are not responsive to the exogenous currency demand shock.[15] The OLS regression using cumulative changes in central bank policy rates (starting from 28 days before the rebalancing event) on the currency demand shock gives insignificant coefficients for the cross-country regression, as

[15]Pandolfi and Williams (2019) find that a one standard deviation in the flows implied by the rebalancings of the GBI-EM Global Diversified index leads to a small increase in sovereign debt prices of 8 basis points in the window spanning from 5 days before to 5 days after the rebalancing date, for long-term government bonds. Different from their regression, our regressor is the short-term yields, rather than long-term ones, and we use the change in local-currency yields relative to synthetic USD yields.

shown in Figure 1.3 (a). The results show that the central banks are not using monetary policy rates to offset the exchange rate appreciation due to the rebalancings of the index. In addition, Figure 1.3 (b) shows that changes in short-term local-currency government bond yields relative to synthetic USD yields ($i_{c,t} - i^*_{c,t}$) have an insignificant response to the currency demand shock. The regressions in both figures include time and country fixed effects, with standard errors clustered at the date level.

Empirical Fact 4: *The country-specific exchange rate response to the currency demand shocks differs by exchange rate regime, with free floaters being much more responsive than peggers.*

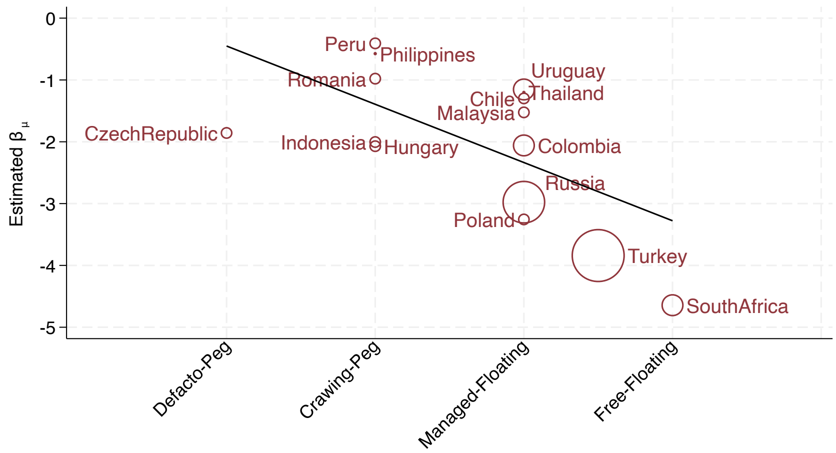
We find heterogenous responses of exchange rates to the currency demand shock across countries. We repeat the exercise in Figure 1.1 for each country and collect the estimated coefficients for the rolling-window regression between day 0 and day 5 after rebalancing, in the empirical specification[16]

$$\Delta e_{c,t+d} = \beta_{0,c} + \beta_{\mu,c}\, \mu_{c,t} + \phi_c\, X_{c,t} + \epsilon_{c,t}, \tag{1.3}$$

where we now estimate the country-specific exchange rate response $\beta_{\mu,c}$. All regressions include constants and a set of dummies $X_{c,t}$ that contain month and

[16]We use the rolling window of 5 days for country-specific regressions because exchange rate data from the BIS are published only on weekdays (excluding weekends and holidays). Using the rolling window ensures we do not leave gaps in our regressions of daily frequency. We do not use a rolling window in the cross-country regressions (Facts 1–3) because it distorts the standard errors.

Figure 1.4:
Fact 4: Free Floaters Respond More to the Currency Demand Shock than Peggers



Note: This figure presents the relation between country-specific exchange rate response to the currency demand shock (measured by $\mu_{c,t}$) and the exchange rate regimes classified by Ilzetzki, Reinhart and Rogoff (2021). The y-axis is the estimated exchange rate response to $\mu_{c,t}$ from the rolling-window regression using the window spanning from 0 to 5 days after rebalancing, with both month and year fixed effects, and standard errors are clustered at the year level. We do not plot the estimates for Brazil and Mexico because they cannot include month fixed effects owing to limited observations (these countries are dropped from the sample because they are often at the 10% cap). We also drop Argentina, whose exchange rate regime is classified beyond the standard free-floating regime. The x-axis is the exchange rate regimes ranging from de facto peg (left) to free floating (right). All regression estimates are significant at the 1% level. The circle size is proportional to the exchange rate volatility of the currency in our sample.

year fixed effects,[17] and standard errors are clustered at the year level. All countries respond to $\mu_{c,t}$ with less than 1% significance and predict the right sign. Specifically, a positive local-currency demand shock (an increase in $\mu_{c,t}$) appreciates local-currency exchange rates and decreases the price of US dollars in units

[17]When month fixed effects are included, we need to drop both Brazil and Mexico from the regression because they have limited observations. Thus, we include only year fixed effects for these countries, and we report their estimates in Tables 1.B.66 and 1.B.67 in the appendix.

of local currency. Tables 1.B.66 and 1.B.67 in the appendix give the country-specific exchange rate response.

There is a clear relation between the country-specific exchange rate response and the exchange rate regimes, as illustrated by the downward trend in Figure 1.4. The y-axis is the country-specific estimated exchange rate response to the currency demand shock ($\mu_{c,t}$); the x-axis is the coarse exchange rate regimes ranging from de facto peg to free floating as classified by Ilzetzki, Reinhart and Rogoff (2021). The figure makes clear that free floaters (e.g., South Africa and Turkey) are much more responsive to $\mu_{c,t}$ compared with either managed floaters (e.g., Colombia, Malaysia, Poland, and Thailand) or peggers (e.g., Czech Republic, Romania, and Peru). In addition, floaters have much larger exchange rate volatility, as indicated by their larger circle size. Taken together, country-specific exchange rate response to the currency demand shock increases with the volatility of the exchange rates and increases as the exchange rate regimes move toward free floating.

1.4 Currency Demand Shocks in Inelastic Financial Markets

In this section, we review the major classes of models in international finance where the uncovered interest parity (UIP) condition does not hold. We show that models with endogenous deviations in the UIP condition in inelastic financial markets can explain the observed empirical facts on our currency demand shocks and exchange rate dynamics.

1.4.1 Inelastic Markets and Uncovered Interest Parity

Our empirical facts that currency demand shocks move exchange rates significantly provide direct evidence that foreign exchange markets are not perfectly elastic. Similar to the argument for the "inelastic markets hypothesis" (Gabaix and Koijen, 2022) for the aggregate equity markets, foreign exchange markets are inelastic in that flows and demand shocks affect asset prices and expected returns in a quantitatively important way. If the markets were perfectly elastic, the currency demand shock should have no traction on exchange rates.

The simplest model where foreign exchange markets are perfectly elastic is the case when the UIP condition holds. We define this condition as follows. Let $i_{c,t}$ and $i^*_{c,t}$ be the returns of home- and foreign-currency bonds, respectively; $e_{c,t}$ is the exchange rate measured in the number of home currencies per US dollar (foreign); $\mathbb{E}_t \Delta e_{c,t+1}$ is the expected change in exchange rates from t to $t+1$. The UIP condition implies zero excess return in the currency carry trade on home- and foreign- currency bonds. In other words, the expected exchange rate change is fully offset by return differentials, and thus no arbitrageur profits.

Definition 1: *UIP holds if the following equation holds:*

$$(i_{c,t} - i^*_{c,t}) - \mathbb{E}_t \Delta e_{c,t+1} = 0. \tag{1.4}$$

Classical macroeconomic models (e.g., Mundell, 1962; Obstfeld and Rogoff, 1995; Gali and Monacelli, 2005) typically assume the UIP condition in equation (1.4) holds. In these models, a currency demand shock plays no role in determining neither the path of exchange rates nor short-term interest rate differentials, because the financial markets are assumed to be perfectly elastic.

However, the assumption of perfectly elastic financial markets does not require the UIP condition to hold. Another class of macroeconomic models with capital control taxes and exogenous risk-premium shocks (Devereux and Engel 2002; Farhi and Werning, 2012) or convenience yields (Jiang, Krishnamurthy, and Lustig, 2018) violate the UIP condition in equation (1.4) but not the assumption of perfectly elastic financial markets. In this class of models, exogenous shocks deviate from the UIP condition (i.e., *exogenous UIP shocks*) and move exchange rates but do not change the equilibrium allocation of assets. Similar to the classical macroeconomic models where UIP holds, currency demand shocks play no role in models with exogenous UIP shocks.

Only in models with inelastic foreign exchange markets would a currency demand shock have traction on exchange rates. In this class of models (e.g., Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021), a currency demand shock changes the risk-bearing capacity of the arbitrageurs who conduct currency carry trade in a segmented financial market that is not perfectly elastic. As the risk-bearing capacity of the arbitrageurs is limited, a currency demand shock translates into movements in exchange rates, changes in the equilibrium allocation of assets, and endogenous deviations from the UIP condition. We therefore also refer to the currency demand shocks in inelastic financial markets as *endogenous UIP shocks*.

1.4.2 Empirical Evidence for Inelastic Financial Markets

We argue that a model of perfectly elastic markets cannot square with the observed empirical facts on our currency demand shocks and exchange rate dynamics. Markets are perfectly elastic both in models where UIP holds and in models with exogenous UIP shocks. We provide empirical evidence that our exogenous currency demand shock would have no bearing on exchange rate movements in these markets.

We start with the following modified UIP condition that includes both the exogenous and endogenous UIP shocks.

Definition 2: The modified UIP condition is given by

$$i_{c,t} - i^*_{c,t} - \mathbb{E}_t \Delta e_{c,t+1} = \underbrace{\tau_{c,t} + \psi_{c,t}}_{\text{exogenous}} + \underbrace{\Lambda_{c,t}}_{\text{endogenous}}, \tag{1.5}$$

where we denote capital control taxes by $\tau_{c,t}$, the exogenous risk-premium shock by $\psi_{c,t}$, and the endogenous risk-premium shock by $\Lambda_{c,t}$. Both $\tau_{c,t}$ and $\psi_{c,t}$ are exogenous UIP shocks, and $\Lambda_{c,t}$ is the endogenous UIP shock.

Strictly speaking, there should be separate capital taxes for home and foreign capital. Without loss of generality, we use net capital tax defined as the difference between home and foreign capital tax. An example of risk-premium shock ($\psi_{c,t} > 0$) is an increase in the world interest rate that makes investors deem home assets more risky than foreign assets, but without changing the equilibrium allocation of assets and exchange rates.

Table 1.4.1: Capital Controls and Macro-fundamentals Are Not Responsive To $\mu_{c,t}$

	(1) Capital controls	(2) GDP	(3) Consumption	(4) NFA	(5) Net exports	(6) Inflation
$\mu_{c,t}$	-0.0375	1.27	0.430	-1.040	0.285	-1.462
	(0.0289)	(0.357)	(6.540)	(0.880)	(2.984)	(0.9602)
Constant	0.523***	1.439***	2.214***	3.947***	0.466***	4.075***
	(0.00002)	(0.112)	(0.0533)	(0.00178)	(0.00543)	(0.0022)
Observations	1793	1247	1368	1992	1835	1956
R^2	0.9781	0.9199	0.9264	0.8637	0.1875	0.6451
Adjusted R^2	0.978	0.919	0.925	0.862	0.175	0.6401

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.02$, *** $p < 0.01$

Note: This table shows the OLS regression results of the following independent variables on the currency demand shock ($\mu_{c,t}$): capital control measures (Fernandez et. al., 2016), nominal GDP, consumption, net foreign asset positions (NFA), net exports, and inflation. Capital controls, GDP (billions of local currency), and inflation are in annual frequency. NFA, consumption and net exports are in billions of local currency and of quarterly frequency. Inflation level is computed from the consumer price index that treats year 2010 as the base year and is of quarterly frequency. All regressions include country and year fixed effects, with standard errors clustered at the year level.

We show that a model with only exogenous UIP shocks cannot square with our stylized empirical facts. Intuitively, both capital control taxes and risk premium for macroeconomic conditions are slow-moving variables compared with the exogenous currency demand shocks, which arrive at monthly frequency. We provide formal econometrics to attest to this idea by using capital control index data from Fernandez et al., (2016) (with the dataset updated to 2021) to proxy $\tau_{c,t}$, and variables of macroeconomic fundamentals (e.g., inflation, consumption, output, net exports) to proxy $\psi_{c,t}$. As shown in Table 1.4.1 in the appendix, both measures for capital taxes τ_t and risk-premium shock $\psi_{c,t}$ are immune to our exogenous currency demand shock $\mu_{c,t}$. Taken together with the results on interest rates (Fact 3), the evidence suggests that models with exogenous UIP shocks cannot explain the observed dynamics in exchange rates (Facts 1 and 2).

1.4.3 A Model of Endogenous UIP Shocks in Inelastic Financial Markets

Our empirical facts point to a model with endogenous UIP shocks. In this section, we present a simple model featuring the financial sector only where a currency demand shock shifts the arbitrageurs' holdings and gives rise to endogenous deviations in UIP. There are two types of agents in the model. Arbitrageurs demand home- and foreign-currency bonds and derive profits from the excess returns in currency carry trades; noise traders have a constant supply schedule of home- and foreign-currency bonds, with their positions $n_{c,t}$ subject to the currency demand shocks $\mu_{c,t}$. Importantly, shocks to noise traders' positions are orthogonal to macroeconomic fundamentals.

The arbitrageurs' holdings and market clearing condition with the noise traders' positions are as follows:

$$i_{c,t} - i^*_{c,t} - \mathbb{E}_t \Delta e_{c,t+1} - (\tau_{c,t} + \psi_{c,t}) = \lambda_{c,t}\, d_{c,t} \tag{1.6}$$

$$n_{c,t} + d_{c,t} = 0, \tag{1.7}$$

where in (1.6), we follow Gabaix and Maggiori (2015) and rewrite the endogenous UIP component Λ_t in equation (1.5) as the arbitrageurs' holdings in local-currency bonds ($d_{c,t}$) times the arbitrageurs' risk-bearing capacity ($\lambda_{c,t}$). The larger the $\lambda_{c,t}$, the lower the arbitrageurs' risk-bearing capacity and the steeper their demand curve. In the limit that $\lambda_t \to \infty$, the international bonds market is completely segmented, with financial autarky. On the other extreme, when $\lambda_{c,t} = 0$, the arbitrageurs are able to take infinite positions and absorb any nonzero excess returns in the currency carry trade. In the case when $\lambda_{c,t} \in (0, \infty)$, the model endogenously generates UIP deviations given by the arbitrageurs' risk-taking capacity.

An exogenous local-currency demand shock[18] (an increase in $\mu_{c,t}$) shifts noise traders' positions $n_{c,t}$ and affects arbitrageurs' holdings through the market clearing condition (1.7). In other words, the exogenous currency demand shock traces out the slope of the demand curve and arbitrageurs' risk-bearing capacity λ_{ct} in

[18] As shown below in the model, a currency demand shock shifts the noise traders' positions and would be seen as shifts in *supply* from the perspective of arbitrageurs. That is why we say the currency demand shock traces out the *demand* curve for the arbitrageurs.

equation (1.6). The steeper the demand curve, the more inelastic the financial market and the lower the arbitrageurs' risk-bearing capacity.

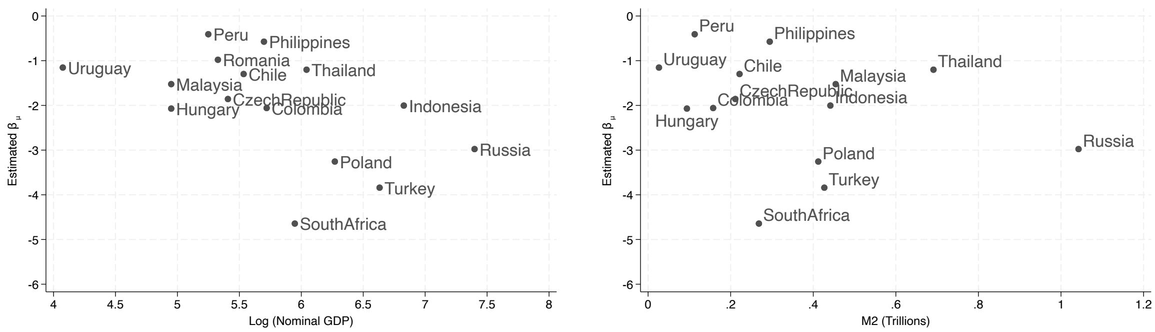
1.4.3.1 Drivers of Risk-Bearing Capacity in Endogenous UIP Models

Given that our currency demand shock $\mu_{c,t}$ does not move interest rates nor the exogenous UIP shocks, the magnitude of the exchange rate responses can identify the arbitrageurs' risk-bearing capacity $\lambda_{c,t}$. There are two caveats here. One is that our empirical specification in equation (1.3) uses the level of exchange rates while the model equation (1.6) is in expected changes. Nevertheless, the magnitude of exchange rate response is a linear function of $\lambda_{c,t}$, and in section 1.5.1.1 we give two examples on solving the exchange rate response. The other caveat is that our measure of $\mu_{c,t}$ is the share of market value while the noise traders' positions are in flows. We show in 1.A.3 in the appendix how to convert $\mu_{c,t}$ into flows, and the relation between the estimated $\beta_{\mu_{c,t}}$ and the arbitrageurs' risk-bearing capacity $\lambda_{c,t}$.

To understand the drivers of the risk-bearing capacity across countries, we collect the estimated exchange rate responses to the currency demand shock ($\beta_{c,t}$) and plot them against different metrics of macroeconomic fundamentals. We find no correlation between $\beta_{c,t}$ and macroeconomic or financial metrics such as outputs and M2 money supply (Table 1.4.2), but only a strong correlation with the exchange rate regime (and exchange rate volatility).

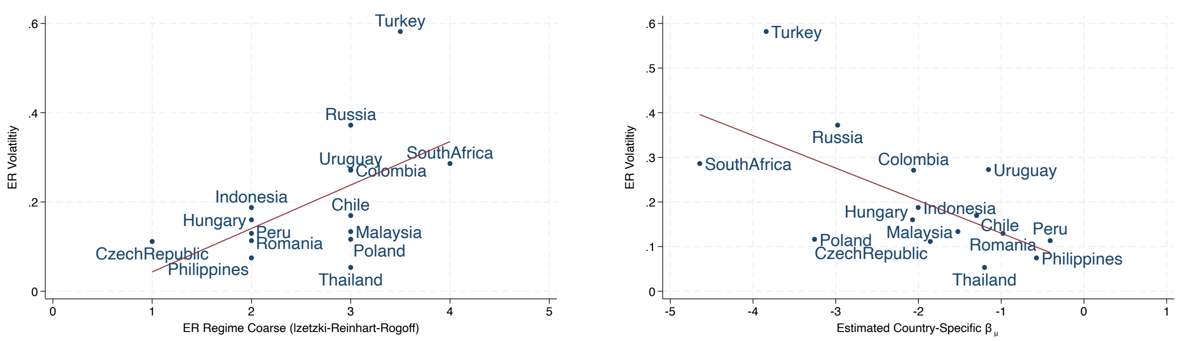
As shown in Empirical Fact 4, floaters have a much larger exchange rate response and much larger exchange rate volatility compared with peggers, as illustrated by the clear downward trend in Figure 1.4 and the relation with exchange rate

Table 1.4.2: Exchange Rate Response Do Not Correlate With Macro-Fundamentals



Note: This figure presents the relation between the country-specific response to the currency demand shock to nominal GDP (left panel) and the country's average M2 supply (right pabel). Nominal GDP is in billions of USD, and M2 in trillions of USD, and we take the average of the sample years from 2009 to 2021.

Table 1.4.3: Exchange Rate Response Correlates With Exchange Rate Volatility



Note: This figure presents the relation between the country-specific exchange rate response to the currency demand shock (measured by $\mu_{c,t}$) and the exchange rate volatility (left panel) and the relation between the exchange rate regime and exchange rate volatility (right panel). The red line is the fitted regression for the x- and y-axis variables.

volatility in Table 1.4.3. The more floating the exchange rates, the larger the exchange rate volatility, the lower the arbitrageurs' risk-bearing capacity (higher $\lambda_{c,t}$) and the more inelastic the financial market. In the next section we formally build a model where the arbitrageurs' risk-bearing capacity endogenously depends on exchange rate volatility.

1.5 Interventions in Inelastic Foreign Exchange Markets

In this section, we introduce foreign exchange interventions in our model of inelastic financial markets with endogenous UIP deviations. We show that under inelastic financial markets, foreign exchange interventions serve as an additional policy tool to stabilize exchange rates without compromising monetary policy independence, regardless of the capital controls.

1.5.1 Endogenous UIP Model with Foreign Exchange Interventions

Consider a small open economy, denoted by c. There are four types of agents in the partially segmented financial market where both home and foreign households can hold only government bonds of their own currency. Households demand home-currency bonds $b_{c,t}$, which are shaped by the macroeconomic fundamentals in the economy. There are also three types of agents who can trade both home- and foreign-currency bonds in the international financial market, namely, noise traders, arbitrageurs, and the government, and we assume without loss of generality that they all reside in the home country. We describe the problem of each of these agents below.

Risk-averse arbitrageurs hold a zero-capital portfolio for home- and foreign-currency bonds $(d_{c,t}, d^*_{c,t})$, with the returns on one local-currency unit holding of such portfolio given by $\tilde{i}_{c,t+1} = i_{c,t} - i^*_{c,t} - \mathbb{E}_t \Delta e_{c,t+1}$. Arbitrageurs choose $(d_{c,t}, d^*_{c,t})$ to maximize the mean-variance preferences over profits in the currency carry trade,

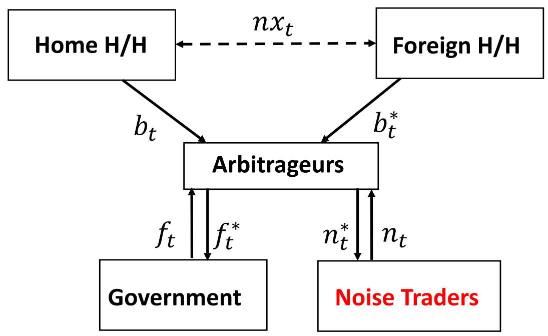
$$d_{c,t} = \frac{1}{\lambda_{c,t}} \big(i_{c,t} - i^*_{c,t} - \mathbb{E}_t \Delta e_{c,t+1} - (\tau_{c,t} + \psi_{c,t}) \big), \tag{1.8}$$

where $\lambda_{c,t} = \omega \; \sigma^2_{e_{c,t}}$ governs the arbitrageurs' risk-bearing ability; parameter ω is the arbitrageurs' risk-aversion coefficient, and $\sigma^2_{e_{c,t}}$ is the equilibrium volatility of exchange rates. The larger the $\lambda_{c,t}$ (or ω and $\sigma^2_{e_{c,t}}$), the lower the arbitrageurs' risk-bearing capacity. We model the risk-bearing capacity to be endogenously dependent on the equilibrium volatility of exchange rates, because our empirical evidence on risk-bearing capacity strongly correlates with exchange rate volatility (Fact 4).

Noise traders hold a zero-capital portfolio $(n_{c,t}, n^*_{c,t})$ and are subject to liquidity demand for local-currency bonds $\mu_{c,t}$. Importantly, $\mu_{c,t}$ is a random variable uncorrelated with the macroeconomic fundamentals. A positive $\mu_{c,t}$ means that the noise traders sell foreign-currency (US dollar) bonds and buy local-currency bonds.

The government holds a portfolio of $(f_{c,t}, f^*_{c,t})$ units of home- and foreign-currency bonds, where $f_{c,t}$, and $f^*_{c,t}$ are policy instruments corresponding to open market operations in foreign exchange interventions for home- and foreign-currency bonds, respectively. A positive (resp., negative) $f_{c,t}$ means buying (resp., selling) local-currency bonds in the foreign exchange interventions.

Figure 1.5: Segmented International Bonds Market



Note: This figure presents the four types of agents in a segmented international bonds market, where home and foreign households (home H/H and foreign H/H, respectively) can hold only government bonds in their own currency. Noise traders' positions are subject to exogenous currency demand shocks that are uncorrelated with the macroeconomic fundamentals.

We also define $b^*_{c,t}$ as the net foreign asset (NFA) position of the home households and government. In our model with only home and foreign countries, $b^*_{c,t}$ is the foreign households' holdings of foreign-currency bonds, as foreign households cannot hold home currency bonds, owing to the segmented financial market. In Figure 1.5 we use a simple diagram to present the four types of agents and their positions in a segmented market.

The market clearing condition for home-currency bond states

$$b_{c,t} + n_{c,t} + d_{c,t} + f_{c,t} = 0. \tag{1.9}$$

Using the zero-capital position of the noise traders and arbitrageurs, one can arrive at the following expression for net foreign assets: $b^*_{c,t} = f^*_{c,t} + n^*_{c,t} + d^*_{c,t}$.

Combining equation (1.9) with equation (1.8) and putting exchange rates on the left-hand-side of the equation, we have

$$\mathbb{E}_t \Delta e_{c,t+1} = i_{c,t} - i^*_{c,t} - (\tau_{c,t} + \psi_{c,t}) \; + \; \lambda_{c,t} \left(b_{c,t} + n_{c,t} + f_{c,t} \right), \tag{1.10}$$

where $\lambda_{c,t} = \omega \; \sigma^2_{e_{c,t}}$, and we substitute the arbitrageurs' holdings using the market clearing condition. A currency demand shock $\mu_{c,t}$ on the local-currency bonds moves the noise traders' holdings $n_{c,t}$ and in turn the arbitrageurs' position, which then leads to movements in exchange rates and endogenous deviations in UIP. Specifically, a positive local-currency demand shock (an increase in $\mu_{c,t}$) appreciates exchange rate levels tomorrow (a decrease in $e_{c,t+1}$), with the size of the appreciation governed by the arbitrageurs' risk-bearing capacity $\lambda_{c,t} = \omega \; \sigma^2_{e_{c,t}}$.

1.5.1.1 Policy Function of Foreign Exchange Interventions

Holding all else constant in equation (1.10), the foreign exchange interventions $f_{c,t}$ stabilize exchange rates by exactly offsetting the noise trader shocks, at the same magnitude and persistence; that is, $f_{c,t} = -n_{c,t}$ to ensure $\partial e_{c,t} / \partial f_{c,t} = -\partial e_{c,t} / \partial n_{c,t}$. This condition requires all variables on the right-hand side of equation (1.10) to be immune to the currency demand shock that moves noise traders' positions $n_{c,t}$. We have already shown that interest rate differentials $(i_{c,t} - i^*_{c,t})$ and exogenous UIP shocks $(\tau_{c,t}, \; \psi_{c,t})$ do not respond to $\mu_{c,t}$. In addition, variables indicating macroeconomic fundamentals $b_{c,t}$ are slow-moving compared with the currency demand shock and would not contaminate the identification. We summarize this statement in Proposition 1.

Proposition 1 *Foreign exchange interventions that use open market operations to stabilize exchange rates need to offset the noise trader shocks with the same magnitude and persistence; that is, $f_{c,t} = -n_{c,t}$. This requires that interest rate differentials and macroeconomic fundamentals (as well as capital control taxes, etc.) to be slow-moving.*

Proof: See Appendix C.

We show empirically that foreign exchange interventions do not respond to the currency demand shocks. Using monthly foreign exchange intervention data from Adler el al. (2021), we find no correlation between spot foreign exchange intervention data (as a share of GDP) and our exogenous currency demand shock $\mu_{c,t}$, as shown in Table 1.5.1. [19] This lack of correlation suggests the central banks are not actively using foreign exchange interventions to offset the noise trader shocks from the exogenous currency demand in equation (1.10). Thus, in the empirical analysis of this paper it is valid to assume $f_{c,t}$ to be independent of the noise traders' positions $n_{c,t}$.

To arrive at the closed-form expression of the policy function of foreign exchange intervention, we provide solutions from two model examples — a partial equilibrium model under the Taylor rule (Engel and West, 2005) and a general equilibrium model with a fully specified goods market and the country's intertemporal budget constraint (Itskhoki and Mukhin, 2021). Through solving these models, one can then match the estimated regression coefficient in the empirical specification in equation (1.3) with the impulse response function of exchange rates in response to the noise trader shocks.

[19] Thailand is the only country in our sample whose proxied foreign exchange intervention data (from Adler et. al (2021)) have a significant response to the currency demand shock.

Table 1.5.1: Foreign Exchange Intervention are Not Response to $\mu_{c,t}$

	(1) Chile	(2) Colombia	(3) Czech Republic	(4) Hungary	(5) Indonesia	(6) Malaysia	(7) Peru
$\mu_{c,t}$	0.561	-0.418	13.58	3.262	-1.456	2.999	-2.455
	(0.324)	(0.253)	(9.501)	(2.192)	(1.280)	(1.970)	(1.508)
Const.	0.044***	0.047***	0.123	0.0384	0.035***	0.006	0.096***
	(0.00008)	(0.0003)	(0.0574)	(0.0191)	(0.002)	(0.00942)	(0.0016)
Obs.	90	127	42	132	81	64	130
R^2	0.7617	0.1857	0.4715	0.1942	0.4775	0.2178	0.3816
Adj. R^2	0.693	0.013	0.167	0.032	0.292	-0.071	0.254

	(1) Philippines	(2) Poland	(3) Romania	(4) Russia	(5) South Africa	(6) Thailand	(7) Turkey	(8) Uruguay
$\mu_{c,t}$	-0.194	6.642	2.071	0.272	0.241	4.067**	0.294	3.086
	(0.462)	(8.425)	(1.479)	(0.955)	(0.394)	(1.393)	(0.651)	(1.983)
Const.	0.145***	0.100*	0.107***	0.153***	-0.060***	0.103***	-0.033**	0.126
	(0.003)	(0.0281)	(0.011)	(0.010)	(0.0003)	(0.004)	(0.010)	(0.045)
Obs.	120	45	89	111	62	118	72	39
R^2	0.495	0.303	0.203	0.283	0.494	0.360	0.382	0.466
Adj. R^2	0.381	-0.058	-0.016	0.114	0.299	0.220	0.156	0.118

Standard errors in parentheses
$^{*}\ p < 0.05$, $^{**}\ p < 0.02$, $^{***}\ p < 0.01$

Note: This table shows the OLS regression results of the foreign exchange intervention data on the currency demand shock ($\mu_{c,t}$). Intervention data are the estimated spot foreign exchange interventions data over GDP at monthly frequency from Adler et al. (2021), which use changes in the balance sheet of central bank reserves rather than publicly available official data. We use the estimated intervention data from Adler et al. (2021) because they have a larger coverage of countries and are meant be capture the covert interventions from the central banks that are not reported otherwise. The regression includes year and month fixed effects, with standard errors clustered at the year level.

Example 1 *In the Taylor-rule model (Engel and West, 2005) with exchange rate target* $\bar{e}_c$*, the home- and foreign monetary policy rates follow the form*

$$
\begin{aligned}
i_{c,t} =& \beta_0 \, (e_{c,t} - \bar{e}_c) + \beta_1 \, y_{c,t} + \beta_2 \, \pi_{c,t} + \nu_{c,t} \quad , \beta_0 \in (0,1) \\
i^*_{c,t} =& \beta_1 \, y^*_{c,t} + \beta_2 \, y^*_{c,t} + \nu^*_{c,t},
\end{aligned}
$$

where $\bar{e}_c$ *is the exchange rate target,* $\pi_{c,t} = p_{c,t} - p_{c,t-1}$ *is the inflation rate, and* $y_{c,t}$ *is the output gap of home country c. The policy function of the foreign exchange intervention is given by*

$$\frac{\partial e_{c,t}}{\partial f_{c,t}} = \frac{\partial e_{c,t}}{\partial n_{c,t}} = \frac{1}{(1 + \beta_0 - \rho)} \lambda_{c,t},$$

where $\lambda_{c,t} = \omega \; \sigma^2_{e_{c,t}}$*, under the assumptions that* $n_{c,t} \sim$ *AR(1) with persistence* ρ*,* $n_{c,t} \perp f_{c,t}$*, and macro-fundamentals are slow-moving compared with the noise trader shocks.*

Proof: See Appendix C.

Example 2 *In the general equilibrium model of Itskhoki and Mukhin (2021) that specifies the budget constraint of a country c,* $\beta \; b^*_{c,t} - b^*_{c,t-1} = nx_{c,t} = \gamma \; e_{c,t} + \xi_{c,t}$*, where* $nx_{c,t}$ *is the net exports and* $b^*_{c,t}$ *the net foreign assets of the home country. The policy function of the foreign exchange intervention is given by*

$$\frac{\partial e_{c,t}}{\partial f_{c,t}} = \frac{\partial e_{c,t}}{\partial n_{c,t}} = \frac{\beta}{(1 - \rho\beta)} \lambda_{c,t},$$

where $\lambda_{c,t} = \omega \; \sigma^2_{e_{c,t}}$*, under the assumptions that* $n_{c,t} \sim$ *AR(1) with persistence* ρ*,* $n_{c,t} \perp f_{c,t}$*, and macro-fundamentals are slow-moving compared with noise trader shocks.*

Proof: See Appendix C.

1.5.2 Implications of Foreign Exchange Interventions and the Relaxed Trilemma

In this section, we discuss the implications of foreign exchange interventions under inelastic financial markets. We define the relaxed trilemma condition following Itskhoki and Mukhin (2023a) for endogenous UIP models with inelastic financial markets. Under inelastic financial markets, foreign exchange intervention serves as an effective policy tool to stabilize exchange rates without compromising monetary policy independence, regardless of the capital controls.

Definition 3: *The relaxed trilemma constraint states that it is possible to have all three of the following conditions simultaneously: an independent monetary policy (inward focused on domestic inflation and output gap), free capital mobility (absence of capital control taxes), and a managed exchange rate. By contrast, under the classical trilemma constraint it is possible to have only two of the three conditions simultaneously.*

Definition 4: *Trilemma-type models are UIP models that bind under the classical trilemma constraint; non-trilemma–type models are UIP models that hold under the relaxed trilemma constraint.*

Definition 1.5.2 contradicts the classical trilemma constraint (Mundell, 1962), which states that it is *not* possible to have all three conditions in definition 1.5.2. The models where the UIP condition holds and the models with exogenous UIP shocks are subject to the classical trilemma constraint; thus, we refer to these models as *trilemma-type* models. If UIP holds, there is free capital mobility by construction and the economy faces the direct trilemma trade-off between an independent monetary policy and a fixed exchange rate, as seen in equation (1.10). If the UIP deviations came from exogenous shocks, monetary policy rates would have to move one on one with exchange rates unless capital control taxes ($\tau_{c,t}$)

and exogenous risk premium ($\psi_{c,t}$) can both be used as policy instruments to offset exchange rates; however, this is clearly not feasible. Thus, under the trilemma constraint, exchange rate stabilization comes at the cost of compromising monetary policy independence.

By contrast, models with endogenous UIP shocks can have all three conditions in the trilemma met, because these models have an additional policy instrument to stabilize exchange rates: foreign exchange interventions. As shown in equation (1.10), foreign exchange interventions conduct open market operations that shift the arbitrageurs' positions, which then lead to endogenous deviations in UIP and move exchange rates. Therefore, the central bank[20] can now stabilize exchange rates through foreign exchange interventions while the monetary policy is entirely domestically focused to close the output gap. In other words, even under perfectly mobile capital flows, the economy no longer has to compromise monetary policy independence to stabilize exchange rates, relaxing the classical trilemma constraint. We thus refer to endogenous UIP models as *non-trilemma–type* models.

1.5.2.1 Empirical Evidence for the Relaxed Trilemma

Empirical Facts 3 and 4 show that there is a significant exchange rate response to the exogenous currency demand shock of almost all currencies but no response of the policy rates. Under trilemma-type models, the movements in exchange rates must be offset one on one by monetary policy rates for exchange rates to be fixed,

[20]The central bank's objective is to minimize the international risk-sharing wedge and domestic output gap.

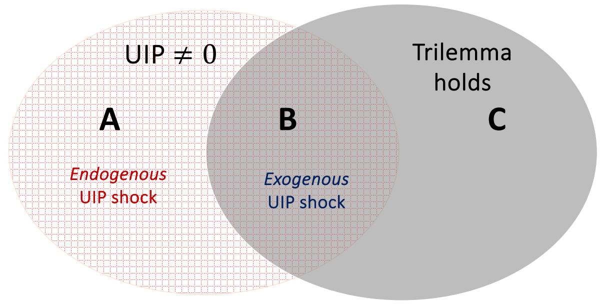
for any given capital control taxes ($\tau_{c,t} \geq 0$ in equation (1.10)). Our evidence provides empirical support for non-trilemma–type models and implies that countries under managed exchange rate regimes (namely, de facto peg, crawling peg, and managed floaters) have used instruments other than monetary policies to manage their exchange rates. We view this finding as the most direct piece of evidence supporting the relaxed trilemma constraint discussed above.

1.5.2.2 Discussion on Non-Trilemma–Type Models and UIP

In this section, we discuss the implications of the major classes of literature in international finance on the trilemma constraint, the UIP condition, and inelastic financial markets. We start with the trilemma-type models where UIP either holds or is subject to exogenous shocks only, and then compare them with non-trilemma–type models with endogenous UIP shocks where foreign exchange (FX) interventions are effective.

The models where UIP holds or the models with exogenous UIP shocks are subject to the classical trilemma constraint. This is because either the models where UIP holds (e.g., Mundell, 1962; Obstfeld and Rogoff, 1995) or the models with exogenous UIP shocks (e.g., Devereux and Engel, 2002; Farhi and Werning, 2012) assume financial markets are perfectly elastic and thus a quantity shock would have no bearing on exchange rates. Even if FX interventions were implemented, they would be ineffective in these models, because they lack the channel where a demand shock endogenously shifts the arbitrageurs' holdings, which in turn leads to deviations in UIP. Therefore, these models are subject to the trilemma

Table 1.5.2: Trilemma Constraint And UIP



Model	Financial market	Papers
endogenous UIP shock	(imperfectly) inelastic	Gabaix and Maggiori (2015), Cavallino (2019), Itskhoki and Mukhin (2021), Fanelli and Straub (2021), Basu et al. (2023)
exogenous UIP shock	perfectly elastic	Devereux and Engel (2002), Farhi and Werning (2012), Jiang, Krishnamurthy and Lustig (2018)
classic trilemma (UIP = 0)	perfectly elastic	Mundell (1962), Dornbusch (1976), Obstfeld and Rogoff (1995), Gali and Monacelli (2005)

Note: This diagram presents the relation between models where UIP fails (left circle) and models where the trilemma constraint holds (right circle). Region A refers to models under the relaxed trilemma and UIP fails (endogenous UIP shock); region B refers to models where UIP fails but the trilemma holds (exogenous UIP shock); region C represents the classic trilemma models where UIP holds. The references for each type of models are listed.

trade-off between monetary policy rates and exchange rates, as discussed in the previous section.

Only in non-trilemma–type models with endogenous UIP deviations (e.g., Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021) can FX interventions effectively stabilize exchange rates. In these models, financial markets are inelastic. FX intervention serves as an additional policy tool to stabilize exchange rates, because demand shocks can have traction on exchange rates under inelastic markets. Thus, FX interventions can now work together with independent monetary policy with no capital controls. Table 1.5.2 presents the relation between classical trilemma models, models with exogenous UIP shocks, and models with endogenous UIP shocks.

1.6 Identifying the Size of Foreign Exchange Interventions

In this section, we identify the required size of FX intervention to stabilize exchange rates and discuss its effectiveness across different exchange rate regimes. We find that free floaters are twice more effective at stabilizing exchange rates on average compared with managed floaters or peggers, with free floaters requiring lower amount of reserves to stabilize exchange rates.

1.6.1 Converting the Estimates to the Size of the Intervention

We convert the estimates from the currency demand shock into implied capital flows in US dollars. We first use the cross-country estimates (Fact 1) and show how to compute the implied flows of the mutual funds tracking the GBI-EM Global Diversified index. We report the average counterfactual required size of FX interventions to stabilize exchange rates and compare it with estimates from the literature.

The caveat in this exercise is that our currency demand shock is measured in change in country weights, whereas the required size of intervention is in capital flows. Regression results in Fact 1 show that a one standard deviation of $\mu_{c,t}$ (4.844% change in country weight in the index, by Table 1.2.2) moves exchange rates by 1.2% after one rebalancing event for the pooled OLS regression with country and time fixed effects. The average country weight in the index is 6.36%. In addition, we estimate that the total positions of mutual funds in the EPFR dataset tracking the index are 120 billion USD in 2019, whereas the EPFR dataset represents about 60% of the global mutual fund population in the Investment Company Institute (ICI) Fact Book (reported in Table 1.B.3 in the appendix).

We can therefore write the following equation to convert our estimate into the USD flows required to stabilize exchange rates by 1%:

$$\text{Required flows to move exchange rates by 1\%} =$$
$$\frac{1}{\beta_{\mu_c}} \times \text{std.}(\mu_{c,t}) \times \text{average country weight } \times \frac{\text{EPFR mutual fund positions}}{\text{Share of EPFR funds in ICI}}.$$

The results of our pooled OLS regression imply that the average required flows to move exchange rates by 1% is $\frac{1}{1.2} \times 4.844\% \times 6.36\% \times \frac{120}{0.6} = 0.51$ billion USD, or about 0.1% of the average annual GDP in 2019 (the average annual nominal GDP in 2019 is 500 billion USD, reported in Table 1.B.1). In other words, the estimated currency elasticity for the cross-country sample is about 0.5. The required size of intervention as a share of broad money supply (M2) is reported in 1.B.2.

Our estimates of currency elasticity are largely consistent with the literature on FX intervention and event studies using index rebalancings, but more on the lower (inelastic) side. Adler et al (2019) focus on FX intervention episodes and estimate the effects of the intervention by relying on an instrumental-variable panel approach. They find that spot intervention with a magnitude of 1% of GDP results in exchange rate depreciation of 1.7% to 2%, meaning that the required size of intervention to move exchange rates by 1% is about 0.5% of GDP. Their results are larger than ours likely because their sample consists of both advanced and emerging countries, with the former having much smaller exchange rate volatility, while our sample consists of only emerging market economics. For the same reason, most papers on currency demand estimation that mainly focus on advanced economies (or a mix of advanced and emerging economies) have estimates larger than ours.[21]

Remark 5: *How do our estimates of currency demand elasticities advance our understanding on foreign exchange interventions compared with the previous literature?*

We believe our estimates on currency demand elasticities from the rebalancings of the GBI-EM Global Diversified index are more suitable for drawing inferences on FX interventions compared with earlier work for the following reasons: First, as documented by Fact 4 and shown in section 1.6.2, we uncover the heterogenous responses across currency regimes between free floaters and managed floaters or peggers. A cross-sectional OLS that includes peggers would create upward bias on the elasticity estimates. Second, the long time series gives us ample variation in our estimation, and the currency demand shock we study matches well with the actual intervention episodes, which typically take place repeatedly over a longer intervention period.[22] By comparison, Hau, Massa, and Peress (2009) use a one-time index reweighting shock to recover currency demand elasticities.

[21]For example, Hau, Massa, and Peress (2009) use the reweighting event of 33 countries in the MSCI Index to estimate currency supply elasticity, and find that an average of 2.6 billion USD is needed for a 1% change in exchange rates. Evans and Lyons (2002) use order flow data for deutsche mark and Japanese yen against the US dollar and estimate that a 1 billion USD daily FX order flows moves exchange rates by 0.5%. Camanho, Hau and Rey (2021) use quarterly rebalancings from the equity funds and find that an average capital flow of 5.5 billion USD amounts to moving exchange rates by 1%, in a quarterly window. Their larger estimates are most likely due to their larger event window of a quarter.

[22]See Fratzscher et al. (2019) for detailed characteristics of foreign exchange interventions.

1.6.2 Size of Foreign Exchange Interventions for Different Currency Regimes

We use the country-specific estimates (Fact 4) to compute the required size of FX intervention to stabilize exchange rates. To do so, we repeat the exercise in the previous section with the country-specific response to the currency demand shock, as well as the country-specific average weight in the GBI-EM Global Diversified index. The counterfactual required size of intervention as a share of GDP to stabilize exchange rates for each country is reported in Table 1.6.1.[23]

We find that to stabilize exchange rates, free floaters require an intervention that is half as large (as a share of GDP) as that required by managed floaters or peggers; thus, free floaters are more effective at using FX interventions. For example, the required FX intervention to move exchange rates by 1% is about 0.038% of GDP for free floaters (0.028% for Turkey and 0.048% for South Africa), while the group average required intervention is 0.069% of GDP for managed floaters and 0.128% of GDP for peggers.

Why are floaters more effective than peggers at stabilizing exchange rates? These empirical results are consistent with the model mechanism in section 1.5. The risk-bearing capacity $\lambda_{c,t} \equiv \omega\sigma^2_{e_{c,t}}$ governs the elasticity of the exchange rate response to the currency demand shock. A more stable or managed exchange rate would therefore imply smaller exchange rate volatility ($\sigma^2_{e_{c,t}}$) and thus a more elastic market. In the limit of exchange rates being fully pegged, we are back to the elastic financial market model under the trilemma constraint where exchange rates are immune to currency demand shocks. In other words, FX interventions

[23] Table 1.B.1 in the appendix also reports 2019 broad money measures (M2) for each country, so one can back out the required size of intervention as a share of M2.

Table 1.6.1: Foreign Exchange Intervention Required To Induce A 1% Exchange Rate Change

Country	ER regime (code)	ER Vol.	FXI	FXI / GDP
Czech Republic	de facto peg (1)	0.112	0.214	0.084%
Peru	crawling peg (2)	0.113	0.548	0.238%
Hungary	crawling peg (2)	0.160	0.464	0.287%
Romania	crawling peg (2)	0.131	0.260	0.104%
Indonesia	crawling peg (2)	0.187	0.453	0.040%
Philippines	crawling peg (2)	0.074	0.065	0.017%
Thailand	managed floating (3)	0.053	0.669	0.119%
Malaysia	managed floating (3)	0.134	0.545	0.148%
Colombia	managed floating (3)	0.271	0.268	0.083%
Chile	managed floating (3)	0.169	0.107	0.041%
Poland	managed floating (3)	0.116	0.285	0.047%
Uruguay	managed floating (3)	0.273	0.016	0.028%
Russia	managed floating (3)	0.372	0.236	0.013%
Turkey	managed floating/free falling (3.5)	0.582	0.203	0.028%
South Africa	free floating (4)	0.286	0.194	0.048%
Group average				
	peg / crawling peg	0.130	0.334	0.128%
	managed floating	0.198	0.304	0.069%
	free floating / falling	0.434	0.198	0.038%
Sample average			0.302	0.088%

Note: This table reports the country-specific required size of foreign exchange intervention (FXI) to stabilize exchange rates by 1%, in billions of US dollars (column 4) and as a share (%) of each country's 2019 nominal GDP (column 5). The exchange rate volatility (ER Vol.), measured as the standard deviation of the log exchange rate level by country, is reported in column 3. In column 2, we sort countries by their coarse exchange rate (ER) regimes (as classified by Iltzetki, Rogoff and Reinhart 2021) from de facto peg to free floating. For countries having multiple exchange rate regime codes during our sample period (2010–2021), as for Mexico and Turkey, we take the average regime code across time.

The required size of intervention is computed using the country-specific exchange rate response to the currency demand shock in the 0–5 day horizon after the rebalancing date. All estimates are significant at the 1% level. A table with the country's GDP, market value in the GBI-EM Global Diversified index, and broad money supply (M2) can be found in the appendix (Table 1.B.1).

are more effective for floaters precisely because they have larger exchange rate volatility (Empirical Fact 4) and a more inelastic financial market, and are thus further away from the trilemma constraint.

Our results on FX interventions being more effective for free floaters are consistent with the event studies of FX interventions (e.g., Fratzscher et al., 2019). Using confidential intervention data from 33 countries, Fratzscher et al. (2019) determine the *success* of interventions (defined as the consistency in the movement of exchange rates during the intervention and its intended direction) across different regimes and find that interventions are most effective for free floaters, with a success rate of 0.53 through pure purchase or sale of foreign exchange reserves. By comparison, the success rate for broad band, narrow band, and other exchange rates regimes are significantly lower.[24]

Remark 6: *What types of foreign exchange interventions can our quasi-natural experiment best speak to?*

The exogenous currency demand shock from our quasi-natural experiment would be most analogous to a sterilized FX intervention in the spot exchange market. Similar to the open market operations in the spot exchange market, the index rebalancings create currency demand shocks that move exchange rates as the mutual fund investors buy or sell their positions of local-currency government bonds. The fact that we find the monetary policy rates are not moving with respect to the currency demand shock makes the experiment most suitable for understanding the effects of sterilized intervention. Nevertheless, our estimates

[24]See Table 5 in Fratzscher et al., (2019).

allow one to separately identify the effects from open market operations in an unsterilized intervention that also employs monetary policy as an instrument.

1.7 Conclusion

In this paper, we use a well-identified currency demand shock on noise traders that gives rise to endogenous uncovered interest parity deviations under an inelastic financial market. Our results show that the exogenous currency demand shock moves exchange rates significantly both in the short- and long-run but does not move monetary policy rates. This finding provides direct support for models with inelastic financial markets and the relaxed trilemma constraint. We assess the effectiveness of foreign exchange interventions for an emerging-market central bank for stabilizing exchange rates under the inelastic financial market hypothesis. When markets are inelastic, foreign exchange intervention works as an additional policy tool to move exchange rates without compromising monetary policy independence, providing evidence relaxing the classical trilemma constraint. Our results contribute to various strands of literature including those on foreign exchange intervention and asset demand estimation, and are informative for policymakers at emerging market central banks.

Online Appendix

1.A Data Description and Background

1.A.1 More on the GBI-EM Global Diversified Index

The GBI-EM Index Family

Published by J.P. Morgan in 2005, the GBI-EM Global Diversified index is the largest local-currency government bond index for emerging countries. It is also the most popular index among the GBI-EM family of six local-currency emerging market government bond indices: three basic versions (i.e., GBI-EM Broad, GBI-EM Global, and GBI-EM Narrow) and a diversified version for each. Each diversified version is created from the corresponding basic version by maintaining the same set of countries but with different country weights, to reduce market concentration risks. Among all basic versions, GBI-EM broad has the broadest coverage of countries, followed by GBI-EM Global, and then GBI-EM Narrow. The three basic versions are compared in Table 1.A.1.

Apart from having different restrictions on capital controls and tax regulations for different versions of the GBI-EM index, all versions have the same control on income capita and credit ratings. A country is chosen to enter (and remain in) the GBI-EM Global diversified index if the country's gross national income

Table 1.A.1: Three Versions of the J.P. Morgan GBI-EM Indices

	GBI-EM Broad	GBI-EM Global	GBI-EM Narrow
Explicit capital control	✓		
Tax/Regulatory constraints	✓	✓	
Direct access by foreigners	✓	✓	✓
No. countries as of 2021	21	19	16
Country criteria	GNI per capita below the IIC for 3 consecutive years		
Instrument criteria	Fixed/Zero coupon; Maturity > 13 months Minimum face amount > US $1 bn.		

Source: J.P. Morgan Market Reports

(GNI) per capita is *below* the J.P. Morgan-defined index income ceiling (IIC) for three consecutive years. A country is chosen to exit the index if the country's GNI per capita is *above* the IIC for three consecutive years and if the country's long-term local-currency sovereign credit rating (the available ratings from S&P, Moody's, and Fitch) is A-/A3/A- (inclusive) or above for three consecutive years. In addition, the government bonds included in the index have to be in local currency and have month-to-maturity of over 13 months as the threshold.

More Details on the Rebalancing Methodology

The monthly rebalancings of the GBI-EM Global Diversified index have three layers, which are done in order on the last weekday of the month. The first layer uses a diversification methodology that includes in the index only a portion of a country's current face amount outstanding. This value — called the adjusted face amount — is based on the respective country's relative size in the index and the average size of all countries in the index. The adjusted face amount is then used to compute the market value of each country in the index. The second layer focuses on the bond maturity threshold that drops from the index the bonds with fewer than 13 months to maturity. As the third and last layer of control, the index rebalancing caps at 10% the weight of each country, computed using the adjusted face amount.

Here we provide more details on the first layer of rebalancing on the country's face amount. The face amount in the *diversified* version is created from its corresponding basic version with different weighting strategies for countries, and it aims to reduce concentration risks. Specifically, the following formula is used to construct the diversified country face amount (FA_c^D) for country c:

$$FA_c^D = \begin{cases} ICA \times 2 & \text{if} \quad FA_{\max} \\ ICA + \frac{ICA}{FA_{\max} - ICA}\left(FA_c - ICA\right) & \text{if} \quad FA_c > ICA \\ FA_c & \text{if} \quad FA_c \leq ICA, \end{cases} \tag{A.1}$$

where FA_c is the face amount of country c. Additionally, ICA is the average face amount of the countries (or currencies) in the index:

$$ICA = \frac{\sum_c \text{Country face amount}}{\text{No. countries in the index}} \tag{A.2}$$

The diversified face amounts (FA^D) are used to compute the country-level weights in the index as follows. The FA^D are multiplied by the dirty price (price plus accrued interest) to compute the market value for each country, which is then divided by the total market value of the entire index to compute the weights. If we were to compare the diversified and non-diversified versions of the GBI-EM, the diversified version would have a much smaller total market value of the entire index. Small countries $(FA_c \leq ICA)$ have the same market value in both indices, although their weights are bigger in the diversified version. For the other countries, their market value is smaller in the diversified version, but their reduction comes from two possible layers of control: the control on country-level face amount and the country weight cap of 10%, as these countries are more likely to hit the cap.

How Often Are the Weights Adjusted?

The weights are updated daily, for both the diversified and non-diversified versions. This is because the market value of the bonds (which uses dirty price) changes every day and all versions use dirty price to compute country weights. However, the rebalancing of the country's diversified face amount, as well as the additional layer of rebalancing of the weight cap of 10%, is done only at the end of each month. This rebalancing at the end of the month creates additional change on the weight to the daily adjustments due to price change. The diversified face amount is then held fixed until the next rebalancing. Therefore, the change in weights in the diversified version before the end of the month reflects the change in market return (or dirty price) only.

How are bonds deleted from the index? Can bonds be deleted both from the maturity threshold (13 months) and from rebalancing of their face amount at the country level? Only rebalancing on the maturity threshold can lead to bonds being dropped from the index. The rebalancing on the face amount keeps the same bonds in the diversified and non-diversified versions of the index while reducing the face amount of the bonds from countries above the ICA in the diversified version.

Therefore, the bonds (number and name) should be the same in both the diversified and non-diversified versions of the index for each country. For example, if we compare a Chinese bond in the GBI-EM Broad with its diversified version in January 2022, the bond should have the same yield and returns in both versions. However, the market value outstanding is smaller in the diversified version for this bond particularly owing to the reduction in the bond's face amount rather than its price (because the bond's face amount is greater than the ICA). Or if we compare a Philippines bond in the two indices, they should have the same metrics on everything including market value because the bond's face amount is below ICA, so it is intact from rebalancing on the country level.

1.A.2 Estimating the Aggregate Flows from EPFR

In panel (a) of Table 1.B.3 we report the estimated assets under management (AUM) of EPFR mutual funds passively tracking the GBI-EM Global Diversified index. In panel (b) of Table 1.B.3 we report the population share of EPFR data in the Investment Company Institute (ICI) database. Scaling up the total AUM of mutual funds in the EPFR tracking the index (panel a) using its population share in ICI (panel b), we arrive at the total mutual funds in the industry tracking the index. Next we explain how to estimate the mutual funds tracking the index in the EPFR dataset.

First, we use EPFR fund flow data to select the mutual funds that closely track the GBI-EM Global Diversified index. To do so, we first filter out all the emerging market bond funds from the EPFR dataset whose benchmark indices are J.P. Morgan GBI-EM Global Diversified: GBI-EM Global Diversified, GBI-EM Global Diversified composite, GBI-EM Global Diversified ESG, or GBI-EM Global Diversified Europe (or LATUM, Asia). We do not include funds whose benchmark names are other indices in the GBI-EM family only (i.e., "GBI-EM Broad") and the investment grade version of the GBI-EM Global Diversified.

Second, we regress the monthly returns of each bond fund in the EPFR dataset on the returns of the GBI-EM Global Diversified and select those funds whose performance R-squared (Amihud and Goyenko, 2013) is at least 0.9. Our final dataset comprises 2113 unique funds, and it merges the funds whose benchmark indices are GBI-EM Global Diversified with those funds whose performance R-squared is at least 0.9.

We use the mutual fund performance R-squared method developed by Amihud and Goyenko (2013) to determine the passivism of the mutual funds in our dataset. The method regresses the fund-level monthly returns on the monthly returns of the GBI-EM Global Diversified. To test the passivism of the mutual funds we selected, we perform the regression in equation (2) on a 12-month rolling window from January 2016 to January 2022, to obtain the R-squared of each regression. The histogram of the estimated R-squared is presented in Table 1.B.7. We use the rolling window rather than the entire time series to gauge the mutual fund performance because the passivism of the mutual funds in our sample could be time-varying. Our regression results show that the mutual funds in our dataset have a medium R-squared performance of 0.9.

As an additional test for the passivism of mutual funds, we construct a hypothetical fund whose return is the weighted average (by assets under management) of all mutual funds in our sample identified as closely tracking the GBI-EM Global Diversified index. We find that the monthly returns of the constructed fund closely track the returns of the GBI-EM Global Diversified, as illustrated in Table 1.B.8. A simple OLS regression using the returns of the constructed fund and the index returns gives an R-squared of 0.97. Taken together, the results in Table 1.B.7 and 1.B.8 show that at the rebalancing date, these funds have to buy or sell their asset positions to match the returns of the GBI-EM Global Diversified that uses the rebalancing scheme discussed above. Table 1.B.3 panel (a) plots the assets under management of the funds tracking the GBI-EM Global Diversified index in the EPFR data from 2016 to 2022.

The final step in computing the global flows of the mutual funds tracking the index is to estimate the population share of the EPFR data in the ICI dataset. The Investment Company Fact Book reports the global mutual fund data population. We aggregate equity, bonds, and money market end-of-month assets for both industrialized and emerging markets from the EPFR data and divide the number we obtain by the global mutual fund data population from the Investment Company Fact Book. The result is the population presentation of the EPFR data in the global mutual fund industry, as reported in panel (b) of Table 1.B.3.

1.A.3 Converting the Currency Demand Shocks into Noise Trader Shocks

We now show how to connect the flows implied by rebalancings (FIR) with the noise traders' positions. Given that we do not observe the entire variation in the noise trader shocks, we decompose noise traders' positions $n_{c,t}$ into two components: The first is the buy-and-hold portfolio of benchmark investments that are subject to mechanical rebalancings ($\tilde{n}_{c,t}$). The second is the part of the noise traders' positions unexplained by rebalancings ($\tilde{e}_{c,t}$). The two components are *additive* and *orthogonal* to each other:

$$n_{c,t} = \tilde{n}_{c,t} + \tilde{e}_{c,t}, \qquad \text{where} \quad \tilde{n}_{c,t} \perp \tilde{e}_{c,t}.$$

The holdings of benchmark investments ($\tilde{n}_t$) are subject to noise trader shocks ($\tilde{\psi}_t$) when rebalancing happens. These shocks are orthogonal to macroeconomic fundamentals, as illustrated in the model. The position $\tilde{n}_t$ at time t is

$$\tilde{n}_{c,t} = \begin{cases} \left(\frac{\tilde{n}_{c,t-1}}{R_{c,t-1}}\right) R_{c,t} & \text{o.w} \\ \tilde{\psi}_t R_{c,t} & \text{if} \quad t \text{ is the rebalancing date.} \end{cases} \tag{A.3}$$

At the rebalancing date,

$$\tilde{n}_{c,t} = \tilde{\psi}_{c,t} R_{c,t} = \underbrace{\tilde{\psi}_{c,t} R_{c,t} - \left(\frac{\tilde{n}_{c,t-1}}{R_{c,t-1}}\right) R_{c,t}}_{\text{flows implied by rebalancings}} + \underbrace{\left(\frac{\tilde{n}_{c,t-1}}{R_{c,t-1}}\right) R_{c,t}}_{\text{market value buy-and-hold}}$$
$$= \text{FIR}_{c,t} + \text{market value}^{BH}_{c,t},$$

where market $\text{value}^{BH}_{c,t}$ is the buy-and-hold market value that equates the face amount of previous rebalancing $t-1$ times the market price at time t. FIR can then be connected with our currency demand shock as in the main text. We can therefore rewrite the noise trader shocks $n_{c,t}$ as

$$n_{c,t} = \text{FIR}_{c,t} + \quad \text{market value}^{BH}_{c,t} + \quad \tilde{e}^{n}_{c,t}, \tag{A.4}$$

where $\tilde{e}^{n}_{c,t} \perp \text{FIR}_{c,t}$; that is, the components of the noise trader shocks unexplained by rebalancings are orthogonal to the flows implied by the rebalancings of the GBI-EM Global Diversified.

1.B Additional Figures and Tables

Table 1.B.1: Country Statistics for Computing the Required Size of Intervention

Country	2019 mkt. value	2019 GDP	2019 broad money (M2)
Argentina	3.83	360.57	
Brazil	92.81	1833.49	1761.21
Chile	29.72	262.98	221.51
Colombia	62.86	321.81	157.54
Czech Republic	38.09	256.02	211.28
Hungary	40.20	161.72	94.05
Indonesia	92.06	1138.96	441.46
Malaysia	55.98	369.14	454.31
Mexico	92.81	1297.19	490.13
Peru	33.07	229.93	112.80
Philippines	2.63	384.63	294.62
Poland	82.88	602.6	412. 15
Romania	24.33	249.67	
Russia	74.20	1764.64	1042.48
South Africa	80.65	400.25	268.35
Thailand	82.92	560.20	691.15
Turkey	36.91	725.20	426.82
Uruguay	1.82	57.82	26.33
Sample average*	49.22	499.04	436.78

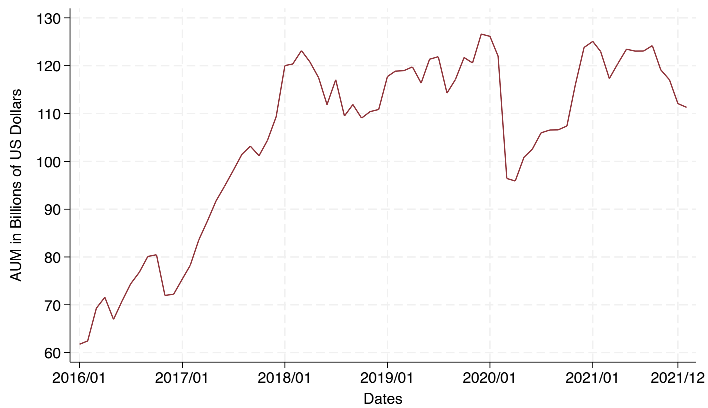
Note: The sample average (last row) excludes Argentina, Brazil, and Mexico during aggregation, because their estimates are not used to infer the required size of intervention in the main text. Column 2 gives the average market value of the local-currency government bonds of each country in the GBI-EM Global Diversified in 2019. Column 3 gives the annual nominal GDP of 2019. Column 4 gives the annual broad money supply (M2) in 2019 from the International Financial Statistics (IFS) of the International Monetary Fund, with the data for Argentina and Romania missing. All values are in billions of US dollars.

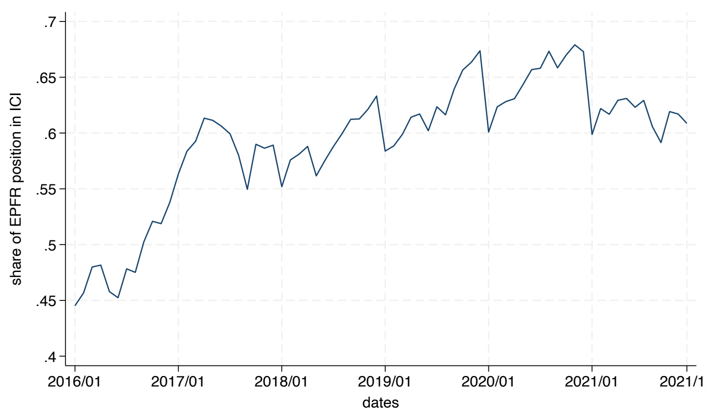
Table 1.B.2:
Intervention Required to Induce 1% Exchange Rate Change (as a Share of M2)

Country	ER regime (code)	ER Vol.	FXI	FXI / M2
Czech Republic	de facto peg (1)	0.112	0.214	0.101%
Peru	crawling peg (2)	0.113	0.548	0.486%
Hungary	crawling peg (2)	0.160	0.464	0.493%
Indonesia	crawling peg (2)	0.187	0.453	0.103%
Philippines	crawling peg (2)	0.074	0.065	0.022%
Thailand	managed floating (3)	0.053	0.669	0.097%
Malaysia	managed floating (3)	0.134	0.545	0.120%
Colombia	managed floating (3)	0.271	0.268	0.170%
Chile	managed floating (3)	0.169	0.107	0.048%
Poland	managed floating (3)	0.116	0.285	0.069%
Uruguay	managed floating (3)	0.273	0.016	0.061%
Russia	managed floating (3)	0.372	0.236	0.023%
Turkey	managed floating / free falling (3.5)	0.582	0.203	0.048%
South Africa	free floating (4)	0.286	0.194	0.072%
Group average				
	peg / crawling peg	0.130	0.334	0.241%
	managed floating	0.198	0.304	0.084%
	free floating / falling	0.434	0.198	0.060%
Sample average			0.302	0.137%

Note: This table reports the country-specific required size of foreign exchange intervention (FXI) to stabilize exchange rates by 1%, in billions of US dollars (column 4) and as a share (%) of each country's 2019 broad money (M2) supply (column 5). The exchange rate volatility, measured as the standard deviation of the log exchange rate level by country, is reported in column 3. We sort countries by their coarse exchange rate regimes (column 2, as classified by Iltzetki, Rogoff and Reinhart 2021) from de facto peg to free floating. For countries having multiple exchange rate regime codes during our sample period (2010–2021), as for Mexico and Turkey, we take the average regime code across time. The required size of intervention is computed using the country-specific exchange rate response to the currency demand shock at the horizon of 0–5 days after the rebalancing date. All estimates are significant at the 1% level.

Table 1.B.5: EPFR Mutual Funds Population Share in ICI

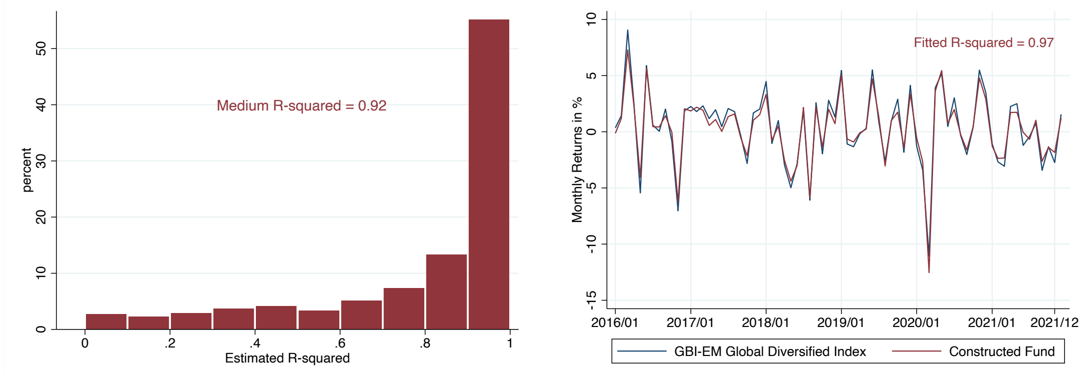




Note: This figure reports the total asset under management (AUM) of bond funds that track the GBI-EM Global Diversified index in the EPFR dataset (panel a) and the share of total EPFR data representation for the entire mutual fund industry (panel b). The bond funds aggregated in panel (a) are in billions of USD and are selected from mutual funds whose benchmark indices track the J.P. Morgan GBI-EM Global Diversified or their performance R-squared is at least 0.8. Observations are in monthly frequency from January 2016 to December 2021.

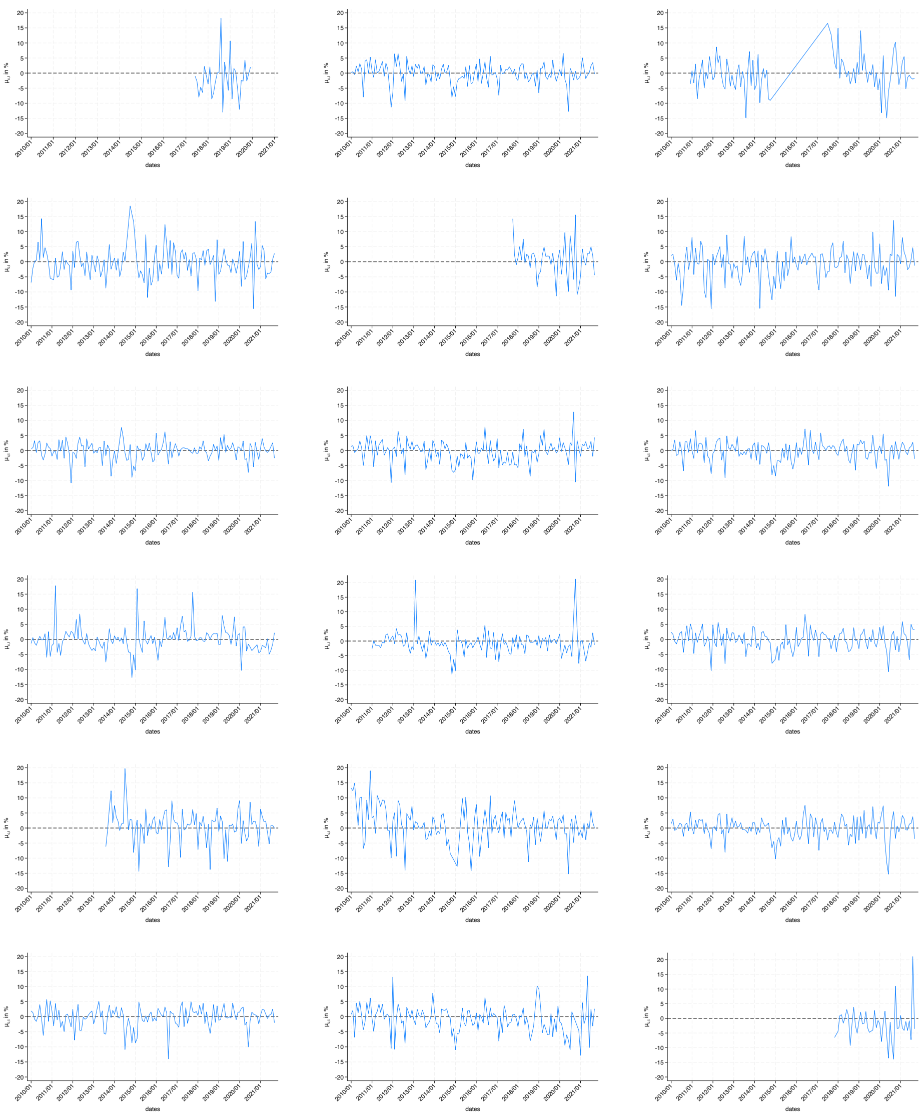
For the share of mutual fund representation in panel (b), we aggregate equity, bonds, and money market end-of month assets for both industrialized and emerging markets from the EPFR data and divide that by the global mutual fund data population from the Investment Company Fact Book. The result is the population presentation of the EPFR data in the global mutual fund industry.

Table 1.B.8: Weighted (by Positions) Average Returns



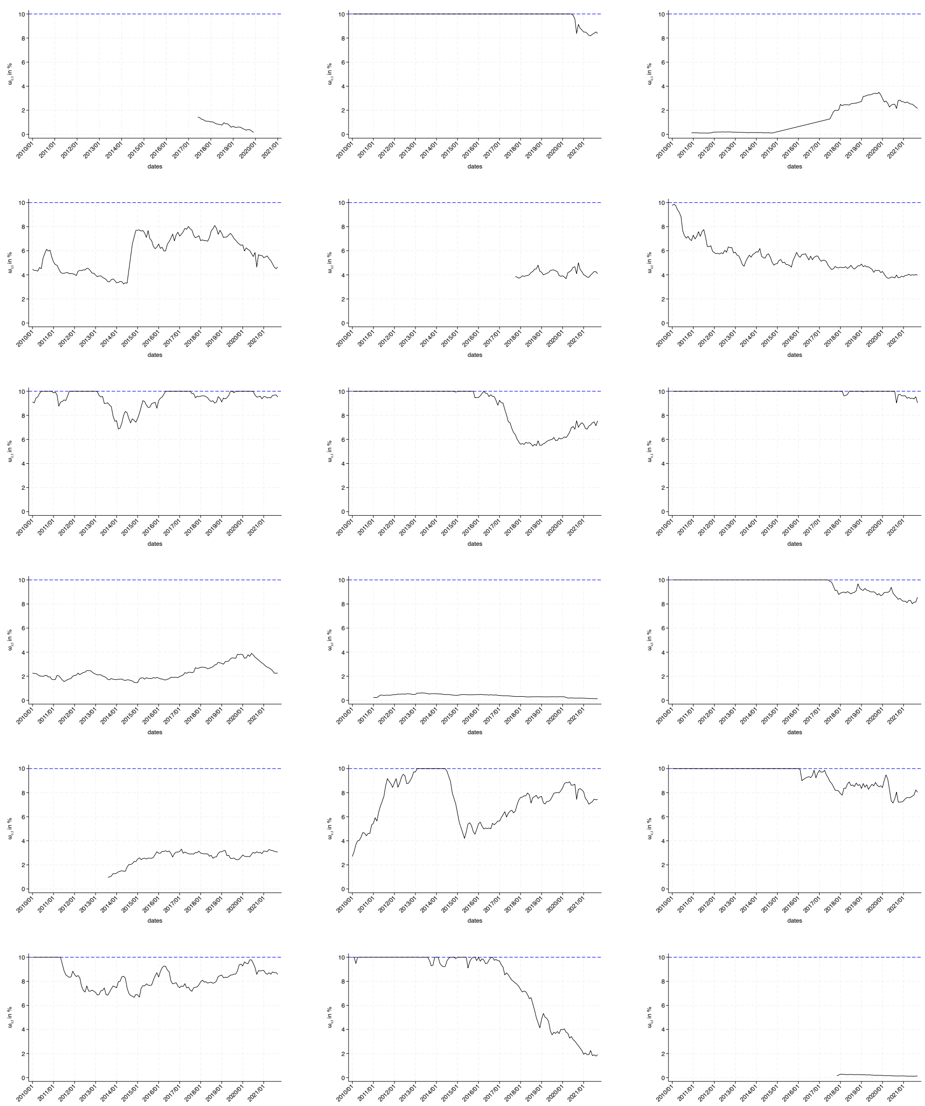
Note: The left panel presents the histogram of estimated R-squared of 12-month rolling window regressions of monthly fund returns on the returns of the GBI-EM Global Diversified index; the median R-squared is 0.92. The right panel plots the returns of the GBI-EM Global Diversified index and the returns of weighted (by asset under management) mutual funds tracking the index; the performance R-squared is 0.97.

Table 1.B.27: Currency Demand Shock ($\mu_{c,t}$ in %) in Time Series



Note: The figure depicts the monthly currency demand shock ($\mu_{c,t}$, measured in percentage points) for each country in the GBI-EM Global Diversified index between 2010 and 2021. Missing values in a given month means the country is not included in the GBI-EM Global Diversified index in that month.

Table 1.B.46: Weights after Rebalancing ($\omega_{c,t}$, in %) in Time Series



Note: This figure shows the monthly weights after rebalancing ($\omega_{c,t}$, measured in percentage points) for each country between 2010 and 2021 (black curves). The dotted blue line indicates the 10% country weight cap that is mechanically imposed at each rebalancing date. Missing values in a given month means the country is not included in the GBI-EM Global Diversified index in that month.

Table 1.B.47: Correlation Matrix of Sovereign Bond Prices across Countries

	Argentina	Brazil	Chile	Colombia	Czech Republic	Hungary	Indonesia	Malaysia	Mexico	Peru	Phillippines	Poland	Romania	Russia	South Africa	Thailand	Turkey	Uruguay
Argentina	1	-0.8375	-0.8129	0.2588	0.5674	0.3118	0.6365	-0.6978	0.2861	-0.1942	0.1479	-0.7477	0.5894	0.1125	0.0567	-0.3082	0.6161	0.6515
Brazil	-0.8375	1	0.8259	0.136	0.0007	-0.16	0.1876	0.6409	-0.241	0.6052	0.3611	-0.0826	-0.6697	0.6915	-0.1826	0.3843	-0.2603	-0.1387
Chile	-0.8129	0.8259	1	-0.0893	-0.0991	0.2982	0.0205	0.7709	0.2172	0.5313	0.5934	0.5881	-0.0891	0.6214	-0.2868	0.6411	0.0394	-0.0867
Colombia	0.2588	0.136	-0.0893	1	0.0854	-0.6275	0.7322	0.3045	0.4828	0.6615	0.1472	-0.1345	-0.2691	0.4563	0.8199	-0.3833	0.5485	0.2267
Czech Republic	0.5674	0.0007	-0.0991	0.0854	1	0.5746	0.5731	-0.0654	0.2933	0.0762	0.1848	-0.0742	0.7641	0.0734	0.0455	0.0894	0.5688	0.3162
Hungary	0.3118	-0.16	0.2982	-0.6275	0.5746	1	-0.6249	-0.306	-0.117	-0.3587	0.2331	0.5308	0.6694	-0.4758	-0.5827	0.3045	-0.1248	-0.1467
Indonesia	0.6365	0.1876	0.0205	0.7322	0.5731	-0.6249	1	0.494	0.4165	0.605	0.1923	-0.2212	-0.058	0.4223	0.6708	-0.1097	0.4343	0.7759
Malaysia	-0.6978	0.6409	0.7709	0.3045	-0.0654	-0.306	0.494	1	0.3731	0.6203	0.5879	0.1779	-0.2167	0.6014	0.075	0.3943	0.1972	0.2707
Mexico	0.2861	-0.241	0.2172	0.4828	0.2933	-0.117	0.4165	0.3731	1	0.3078	0.5318	0.5108	0.5711	-0.09	0.5411	-0.057	0.7291	0.5352
Peru	-0.1942	0.6052	0.5313	0.6615	0.0762	-0.3587	0.605	0.6203	0.3078	1	0.5845	0.1084	-0.5196	0.6308	0.4414	-0.1035	0.3259	0.3149
Phillippines	0.1479	0.3611	0.5934	0.1472	0.1848	0.2331	0.1923	0.5879	0.5318	0.5845	1	0.6392	0.1009	0.3058	0.043	0.3197	0.4332	0.6155
Poland	-0.7477	-0.0826	0.5881	-0.1345	-0.0742	0.5308	-0.2212	0.1779	0.5108	0.1084	0.6392	1	0.4257	-0.3521	-0.1133	0.0005	0.2222	0.2845
Romania	0.5894	-0.6697	-0.0891	-0.2691	0.7641	0.6694	-0.058	-0.2167	0.5711	-0.5196	0.1009	0.4257	1	-0.7123	0.2355	0.1476	0.6135	0.7802
Russia	0.1125	0.6915	0.6214	0.4563	0.0734	-0.4758	0.4223	0.6014	-0.09	0.6308	0.3058	-0.3521	-0.7123	1	0.0026	0.1857	-0.0248	0.288
South Africa	0.0567	-0.1826	-0.2868	0.8199	0.0455	-0.5827	0.6708	0.075	0.5411	0.4414	0.043	-0.1133	0.2355	0.0026	1	-0.423	0.5578	0.0321
Thailand	-0.3082	0.3843	0.6411	-0.3833	0.0894	0.3045	-0.1097	0.3943	-0.057	-0.1035	0.3197	0.0005	0.1476	0.1857	-0.423	1	-0.0057	0.0783
Turkey	0.6161	-0.2603	0.0394	0.5485	0.5688	-0.1248	0.4343	0.1972	0.7291	0.3259	0.4332	0.2222	0.6135	-0.0248	0.5578	-0.423	1	0.2783
Uruguay	0.6515	-0.1387	-0.0867	0.2267	0.3162	-0.1467	0.7759	0.2707	0.5352	0.3149	0.6155	0.2845	0.7802	0.288	0.0321	0.0783	0.2783	1

Note: This table reports the correlation coefficient in aggregate local-currency sovereign bond prices at the rebalancing date ($P_{c,t}$ in equation (1.1) on the currency demand shock) across countries. Each entry in the matrix is the time-series correlation in prices between the two countries over the sample period from 2010 to 2021 at monthly frequency. The cells highlighted in red indicate positive correlations, and those highlighted in blue indicate negative correlations, with the darker shade implying a stronger correlation in magnitude. Diagonal entries are shown in the darkest shade of red because their correlation coefficients equal 1.

Table 1.B.67: COUNTRY-SPECIFIC EXCHANGE RATE RESPONSE TO $\mu_{c,t}$

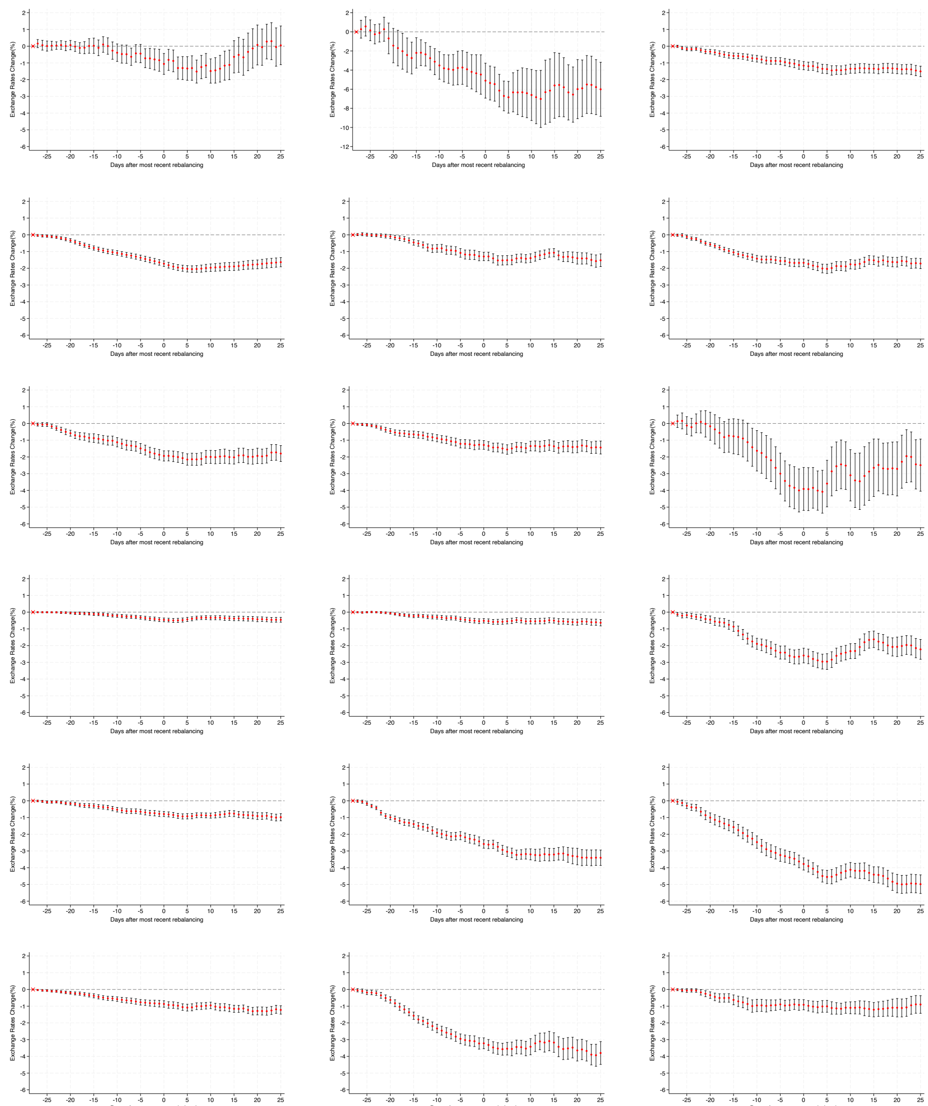
	(1) Argentina	(2) Brazil	(3) Chile	(4) Colombia	(5) Czechia	(6) Hungary	(7) Indonesia	(8) Malaysia	(9) Mexico
$\mu_{c,t}$	-1.371***	-6.842***	-1.298***	-2.059***	-1.857***	-2.071***	-2.004***	-1.524***	-3.591***
	(0.355)	(.)	(0.829)	(0.096)	(0.155)	(0.139)	(0.179)	(0.163)	(0.695)
Const.	2.059***	0.306	0.096	0.344***	0.036	0.086	0.287***	0.105	0.096
	(0.434)	(0.411)	(0.134)	(0.094)	(0.130)	(0.135)	(0.094)	(0.110)	(0.336)
Year FE.	yes	yes	yes	yes	yes	yes	yes	yes	yes
Mon.FE.	yes	no	yes	yes	yes	yes	yes	yes	no
Obs.	109	47	335	816	172	527	331	266	59
R^2	0.387	0.629	0.361	0.479	0.678	0.403	0.558	0.450	0.461
Adj.R^2	0.296	0.612	0.318	0.463	0.644	0.374	0.525	0.408	0.421

	(1) Peru	(2) Philippines	(3) Poland	(4) Romania	(5) Russia	(6) South Africa	(7) Thailand	(8) Turkey	(9) Uruguay
$\mu_{c,t}$	-0.408***	-0.574***	-3.256***	-0.979***	-2.977***	-4.644***	-1.201***	-3.841***	-1.154***
	(0.064)	(0.079)	(0.302)	(0.086)	(0.146)	(0.196)	(0.116)	(0.185)	(0.186)
Const.	0.145***	0.035	-0.130	0.405***	1.007***	0.187	-0.037	0.084	0.803***
	(0.049)	(0.060)	(0.134)	(0.084)	(0.152)	(0.150)	(0.064)	(0.160)	(0.181)
Year FE.	yes	yes	yes	yes	yes	yes	yes	yes	yes
Mon.FE.	yes	yes	yes	yes	yes	yes	yes	yes	yes
Obs.	461	476	183	364	446	249	466	285	143
R^2	0.319	0.244	0.623	0.426	0.603	0.770	0.334	0.653	0.527
Adj.R^2	0.281	0.205	0.588	0.392	0.581	0.752	0.301	0.626	0.467

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

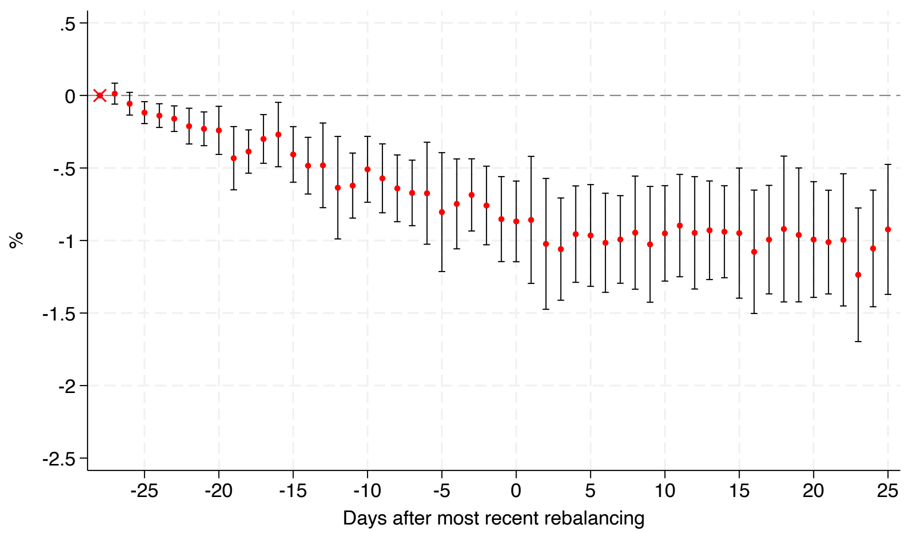
Note: This figure shows the regression coefficient of country-level cumulative exchange rate change (in % or $100 \times \Delta \log(.)$) in response to $\mu_{c,t}$. Exchange rate change is defined as the change from 28 days before the current rebalancing to z days after rebalancing, and we use the rolling window regression with $z \in [0, 5]$ (i.e., 0–5 days after the rebalancing date). We use the rolling window regression because daily exchange rate data on weekends and holidays are missing.

Table 1.B.66: Exchange Rates Change on $\mu_{c,t}$ with Year and Month Fixed Effects



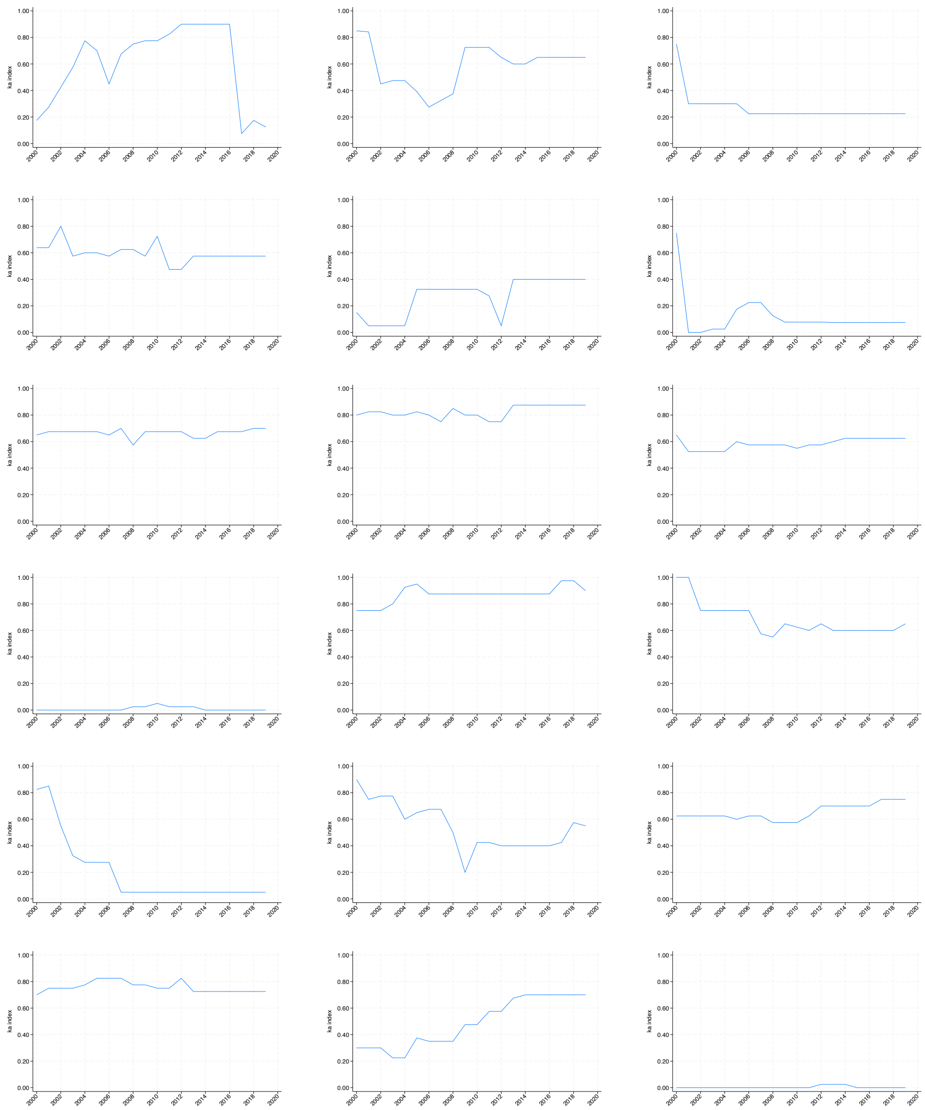
Note: This figure shows the regression coefficient of country-level cumulative exchange rate change (in % or $100 \times \Delta \log(.)$) in response to $\mu_{c,t}$. Exchange rate change is defined as the change since 28 days before the current rebalancing. Black bars indicate a confidence interval of 95%. The regressions for Mexico and Brazil have year fixed effects because there are limited observations for these countries. All graphs have the same y-axis scale except for Brazil due to its large estimates.

Table 1.B.68: CUMULATIVE EXCHANGE RATE CHANGE ON $\mu_{c,t}$ (INCLUDING AT 10% CAP)



Note: This figure presents the estimated regression coefficient of the exchange rate change on the currency demand shock measured by $\mu_{c,t}$, which is standardized by its mean and standard deviation in the regression. Different from Fact 1 in the main text, this regression includes observations at the 10% threshold. Exchange rate change (local currency per USD) is defined as the cumulative change starting from 28 days before the recent rebalancing at day 0. The regression is performed in a pooled OLS using time and country fixed effects, with standard errors clustered at the date level. The results are reported in point estimates (red dots) with 95% confidence intervals (black bars).

Table 1.B.87: Capital Controls Overall Restriction Index



Note: This figure presents the overall capital restriction index (the average of capital inflow and outflow restriction) for each country in our dataset, provided by Fernandez et al. (2016), with data updated to 2021. The measure is in annual frequency.

1.C Derivation and Proofs

1.C.1 Optimal Policies of the Central Bank

The policy objective of the central bank is to maintain the trade-off between the output gap ($x_{c,t}$) stabilization and the international risk sharing wedge ($z_{c,t}$):

$$\min_{x_{c,t}, z_{c,t}, e_{c,t}, b^*_{c,t}, f^*_{c,t}, \sigma^2_{e_{c,t}}} \quad \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma\, z^2_{c,t} + (1-\gamma)x^2_{c,t}\right]$$

$$\text{subject to} \quad \beta b^*_{c,t} = b^*_{c,t-1} - z_{c,t}$$

$$\mathbb{E}\Delta z_{c,t+1} = -\omega\sigma^2_{e_{c,t}}(b^*_{c,t} - n^*_{c,t} - f^*_{c,t}),$$

where the two constraints are the country budget constraint and the international risk-sharing wedge. Here $b^*_{c,t}$ is the net foreign asset position of the home country, and the international risk-sharing wedge is measured as the deviation from the uncovered interest parity (UIP) condition. Parameter γ is the weight on the international risk-sharing wedge and a measure of the degree of openness of the economy. Given initial net foreign assets $b^*_{c,-1}$ and the exogenous path of noise trader shocks $n^*_{c,t}$, monetary policy chooses the direct path of the output gap $x_{c,t}$, while foreign exchange intervention $f^*_{c,t}$ chooses the path of the risk-sharing wedge $z_{c,t}$. The goal of the policymaker is thus to minimize the weighted average of the volatility of the output gap $x_{c,t}$ and the risk-sharing wedge $z_{c,t}$.

If both policy instruments are available and unconstrained, the optimal policy fully stabilizes both wedges, the output gap $x_{c,t} = 0$ and the risk sharing wedge $z_{c,t} = 0$. The demand for currency with foreign exchange interventions through the open market operations is given by $f^*_{c,t} = -n^*_{c,t}$ (or $f_{c,t} = -n_{c,t}$), as stated in Proposition 1.

To see this, note that the constrained optimum allocation (derivation omitted) features $x_t = z_t = 0$ for all z_t. Such allocation can be delivered by a combination of monetary policy and foreign exchange (FX) policies, with monetary policy stabilizing the output gap ($x_{c,t} = 0$), and optimal FX interventions $f^*_{c,t} = -n^*_{c,t}$ to ensure $z_{c,t} = 0$. As a result, the risk-sharing wedge is fully offset, and the optimal international risk sharing is restored independently of the currency demand shocks $n^*_{c,t}$.

Optimal policies above can be implemented using a conventional Taylor interest rule targeting the output gap and a similar policy rule for FX interventions that target UIP deviations. Specifically, FX interventions $f^*_{c,t} = -\mathbb{E}_t \Delta z_{c,t+1}$ and $f^*_{c,t} = -n^*_{c,t}$. That is, the optimal FX interventions should "lean against the wind" intensively until the UIP wedge is fully eliminated. The implementation does not require observing the shocks and distinguishing between macro-fundamental and non-fundamental sources of variation in exchange rates.

1.C.2 Proof of Examples 1 and 2

In this section, we provide two model examples and their solutions of exchange rates in response to the noise trader shocks (or currency demand shocks). These impulse responses of exchange rates can be directly mapped into the estimated coefficient from our empirical results. We start with a model with endogenous deviation from uncovered interest parity (UIP) with inelastic financial markets. We then combine the UIP equation with both a partial equilibrium model (Engel and West, 2005) and a general equilibrium model (Itskhoki and Mukhin, 2021) to solve for exchange rates and their impulse response functions to the noise trader shocks.

The modified UIP equation with endogenous UIP shocks as given in equation (1.5) is

$$i_{c,t} - i^*_{c,t} - \mathbb{E}_t \Delta e_{c,t+1} = \tau_{c,t} + \psi_{c,t} - \omega\, \sigma^2_{e_{c,t}} \left(b_{c,t} + n_{c,t} + f_{c,t}\right), \tag{A.1}$$

where we have substituted the risk-bearing capacity $\lambda_{c,t} = \omega\, \sigma^2_{e_{c,t}}$. Capital control taxes ($\tau_{c,t}$) and risk-premium shock ($\psi_{c,t}$) impose exogenous UIP deviations. We can rearrange equation (A.1) as

$$\mathbb{E}_t \Delta e_{c,t+1} = \underbrace{(i_{c,t} - i^*_{c,t}) - \tau_{c,t} - \psi_{c,t}}_{\equiv -x_{c,t}} \quad + \quad \underbrace{\omega\, \sigma^2_{e_{c,t}} \left(b_{c,t} + n_{c,t} + f_{c,t}\right)}_{\equiv -u_{c,t}}, \tag{A.2}$$

where $x_{c,t}$ is the component of exchange rate $e_{c,t}$ where the classical trilemma constraint holds. The term $u_{c,t}$ is the additional component that generates endogenous UIP deviation when the classical trilemma constraint no longer binds, as FX interventions $f_{c,t}$ can now work as an additional policy tool to stabilize exchange rates under inelastic financial markets. Specifically, under trilemma models where the classical trilemma constraint holds, the risk-bearing capacity of the arbitrageurs $\lambda_{c,t} = 0$, due to either the risk-aversion of the arbitrageurs $\bar{\omega} = 0$ or to exchange rates being fixed (so that $\sigma^2_{e_c} = 0$). The term non-trilemma u_t therefore vanishes under trilemma models, whose UIP deviations can come only from exogenous UIP shocks.

If we continue to iterate equation (A.2) forward, we have

$$e_{c,t} = \mathbb{E}_t\, e_{c,\infty} + \mathbb{E}_t \sum_{j=0}^{\infty} x_{c,t+j} + \mathbb{E}_t \sum_{j=0}^{\infty} u_{c,t+j}, \tag{A.3}$$

and the expectation term vanishes as $e_{c,\infty} = 0$ if exchange rate $e_{c,t}$ follows a stationary process.

Example 1: Engel and West's (2005) Taylor Rule Model

We solve for the impulse exchange rate response to the noise trader shocks under a partial equilibrium model with Taylor rule as specified by Engel and West (2005). Let $\pi_{c,t} = p_{c,t} - p_{c,t-1}$ be the inflation rate, and $y_{c,t}$, the output gap of home country c. Monetary policy in the home country (emerging country) follows a Taylor rule of the form

$$i_{c,t} = \beta_0(e_{c,t} - \bar{e}_{c,t}) + \beta_1\, y_{c,t} + \beta_2\, \pi_{c,t} + v_{c,t},$$

where exchange rate target $\bar{e}_t$ ensures purchasing power parity (PPP) so that $\bar{e}_{c,t} = p_{c,t} - p^*_{c,t}$ and $\beta_0 \in (0,1)$.

Monetary policy in the foreign country (US) follows the Tylor rule of the form

$$i^*_{c,t} = \beta_1 \, y^*_{c,t} + \beta_2 \pi^*_{c,t} + v^*_{c,t}$$

Interest rate difference $i_{c,t} - i^*_{c,t}$ can thus be written as

$$i_{c,t} - i^*_{c,t} = \beta_0 \, (e_{c,t} - \bar{e}_{c,t}) + \beta_1 (y_{c,t} - y^*_{c,t}) + \beta_2 (\pi_{c,t} - \pi^*_{c,t}) + (v_{c,t} - v^*_{c,t}). \quad \text{(A.4)}$$

Now combine the interest rate differential expression in equation (A.4) with the UIP condition in equation (A.2) to substitute out $(i_{c,t} - i^*_{c,t})$

$$\mathbb{E}_t \, e_{c,t+1} = e_{c,t} - \tau_{c,t} - \psi_{c,t} + \beta_0 \, (e_{c,t} - \bar{e}_{c,t}) + \beta_1 (y_{c,t} - y^*_{c,t}) + \beta_2 (\pi_{c,t} - \pi^*_{c,t}) + (v_{c,t} - v^*_{c,t}) - u_{c,t},$$

and it can be further simplified to

$$\begin{aligned}(1+\beta_0) e_{c,t} = {} & \tau_{c,t} + \psi_{c,t} + \mathbb{E}_t e_{c,t+1} + \beta_0 (p_{c,t} - p^*_{c,t}) - \beta_1 (y_{c,t} - y^*_{c,t}) \\ & - \beta_2 (\pi_{c,t} - \pi^*_{c,t}) - (v_{c,t} - v^*_{c,t}) + u_{c,t} \Rightarrow e_{c,t} \\ & = \frac{1}{1+\beta_0} (\tau_{c,t} + \psi_{c,t}) + \frac{\beta_0}{1+\beta_0} (p_{c,t} - p^*_{c,t}) - \frac{\beta_1}{1+\beta_0} (y_{c,t} - y^*_{c,t}) - \frac{\beta_2}{1+\beta_0} (\pi_{c,t} - \pi^*_{c,t}) - \\ & \cdots - \frac{1}{1+\beta_0} (v_{c,t} - v^*_{c,t}) + \frac{1}{1+\beta_0} u_{c,t} + \frac{1}{1+\beta_0} \mathbb{E}_t e_{c,t+1}.\end{aligned}$$

We can write the solution of exchange rate under the Taylor rule as in equation (A.2) by separating its trilemma and non-trilemma components:

$$e_{c,t} = X_{c,t} + U_{c,t} + \frac{1}{1+\beta_0}\mathbb{E}_t\, e_{c,t+1}, \tag{A.5}$$

where $\beta_0 \in (0,1)$, $U_{c,t} \equiv \frac{1}{1+\beta_0}u_{c,t} = -\frac{1}{1+\beta_0}\ \omega\sigma^2_{e_{c,t}}(b_{c,t} + n_{c,t} + f_{c,t})$ is the non-trilemma component and $X_t \equiv \frac{1}{1+\beta_0}(\tau_{c,t} + \psi_{c,t}) + \frac{\beta_0}{1+\beta_0}(p_{c,t} - p^*_{c,t}) - \frac{\beta_1}{1+\beta_0}(y_{c,t} - y^*_{c,t}) - \frac{\beta_2}{1+\beta_0}(\pi_{c,t} - \pi^*_{c,t}) - \frac{1}{1+\beta_0}(v_{c,t} - v^*_{c,t})$ is the trilemma component.

Iterating equation (A.5) forward, we have

$$e_{c,t} = \mathbb{E}_t \sum_{j=0}^{\infty} \frac{1}{(1+\beta_0)^j} X_{c,t+j} \ + \ \mathbb{E}_t \sum_{j=0}^{\infty} \frac{1}{(1+\beta_0)^j} U_{c,t+j} \ + \ \mathbb{E}_t \lim_{j\to\infty} \frac{1}{(1+\beta_0)^j} e_{c,\infty},$$

where $\lim_{j\to\infty} \frac{1}{(1+\beta_0)^j} = 0$ in the limit, together with $e_{c,\infty} = 0$ under the stationary process, the expectation term of exchange rates vanishes.

If we impose the assumption that $n_{c,t}$ inside the non-trilemma component $U_{c,t}$ is an AR(1) process with persistence ρ, foreign exchange interventions are independent of noise trader shocks $f_{c,t} \perp n_{c,t}$, and that macro-fundamentals are slow-moving compared with noise trader shocks $n_{c,t}$, we can solve for the impulse response of exchange rate $e_{c,t}$ in response to $n_{c,t}$ as

$$\frac{\partial e_{c,t}}{\partial n_{c,t}} = \frac{-\omega\sigma^2_{e_{c,t}}}{(1+\beta_0-\rho)} < 0. \tag{A.6}$$

On impact, a positive noise trader shock (or a positive local-currency demand shock from the increase in country weight in the GBI-EM Global Diversified index) appreciates home currency and leads to a decrease in exchange rate $e_{c,t}$, which is defined in the number of local currencies per USD. Therefore, the model prediction gives the right sign as suggested by our empirical evidence.

Example 2: Itskhoki and Mukhin's (2021) General Equilibrium Model

We now solve for the impulse exchange rate response to the noise trader shocks under a general equilibrium model with the country's intertemporal budget constraint as specified by Itskhoki and Mukhin (2021). The log-linearized intertemporal budget constraint in Itskhoki and Mukhin (2021) states

$$\beta b^*_{c,t} - b^*_{c,t-1} = nx_{c,t} = \lambda\, e_{c,t} + \xi_{c,t}, \tag{A.7}$$

where $b^*_{c,t}$ is the net foreign asset position of country c at time t; $nx_{c,t}$ is the net exports; and $e_{c,t}$ is the level of exchange rates. Parameter β is the discount factor; λ (> 0) is a structural parameter pinned down from the price equations in the equilibrium goods market; and $\xi_{c,t}$ is a shock to the net exports $nx_{c,t}$ and is orthogonal to $e_{c,t}$.

We iterate the country budget constraint forward and get

$$b^*_{c,t-1} + \mathbb{E}_t\, \lambda \sum_{j=0}^{\infty} \beta^j e_{c,t+j} = \lim_{T\to\infty} \beta^T b^*_{c,t+T-1} = 0, \tag{A.8}$$

where we impose the no-Ponzi game condition (NPGC) on the country's intertemporal budget constraint.

The country's intertemporal budget constraint uses the net foreign asset position $b^*_{c,t}$ of home households (which equals foreign households' holding of foreign-currency bonds), while the UIP condition in equation (A.2) uses home households' holding of home-currency bonds. We therefore need to rewrite equation (A.2) using $b^*_{c,t}$:

$$\mathbb{E}_t \Delta e_{c,t+1} = \underbrace{(i_{c,t} - i^*_{c,t}) - \tau_{c,t} - \psi_{c,t}}_{\equiv -x_{c,t}} \quad + \quad \underbrace{\omega\, \sigma^2_{e_{c,t}} (n^*_{c,t} + f^*_{c,t})}_{\equiv -u^*_{c,t}}, \tag{A.9}$$

where we use the market clearing condition of home- and foreign-currency bonds to substitute the zero-capital position of the arbitrageurs' holdings. In addition, we normalize $b^*_{c,t} = 0$ without loss of generality, to simplify the derivations below. We use notation $u^*_{c,t}$ (rather than $u_{c,t}$) to represent the non-trilemma component because carry-trade returns are now for the holdings of foreign currency.

We iterate equation (A.9) forward, as we did for equation (A.2), to derive an expression of $\mathbb{E}_t\, e_{t+j}$:

$$\mathbb{E}_t\, e_{t+j} = \mathbb{E}_t\, e_{c,\infty} + \mathbb{E}_t \sum_{k=0}^{\infty} x_{c,t+j+k} + \mathbb{E}_t \sum_{k=0}^{\infty} u^*_{c,t+j+k}. \tag{A.10}$$

We can then combine equation (A.10) with the country budget constraint in equation (A.8):

$$
\begin{aligned}
& b^*_{c,t-1} + \lambda \sum_{j=0}^{\infty} \beta^j \ \mathbb{E}_t e_{c,t+j} = 0 \\
\Rightarrow & b^*_{c,t-1} + \lambda \sum_{k=0}^{\infty} \beta^j \Big(\mathbb{E}_t \ e_{c,\infty} + \mathbb{E}_t \sum_{k=0}^{\infty} x_{c,t+j+k} + \mathbb{E}_t \sum_{j=0}^{\infty} u^*_{c,t+j+k} \Big) = 0 \\
\Rightarrow & b^*_{c,t-1} + \frac{\lambda}{1-\beta} \mathbb{E}_t \ e_{c,\infty} + \lambda \ \mathbb{E}_t \sum_j \sum_k \beta^j x_{c,t+j+k} + \lambda \ \mathbb{E}_t \sum_j \sum_k \beta^j u^*_{c,t+j+k} = 0 \\
\Rightarrow & b^*_{c,t-1} + \frac{\lambda}{1-\beta} \underbrace{\Big(e_{c,t} - \mathbb{E}_t \ \sum_{j=0}^{\infty} x_{c,t+j} - \mathbb{E}_t \ \sum_{j=0}^{\infty} u^*_{c,t+j} \Big)}_{= \mathbb{E}_t \ e_{c,\infty}} + \lambda \ \mathbb{E}_t \sum_j \sum_k \beta^j x_{c,t+j+k} \quad + \cdots \\
& \cdots \qquad \lambda \ \mathbb{E}_t \sum_j \sum_k \beta^j u^*_{c,t+j+k} = 0,
\end{aligned}
$$

where the last line substitutes the expression of $\mathbb{E}_t \ e_{c,\infty}$ from equation (A.10). From above, we have the relation between $e_{c,t}$ and $b^*_{c,t-1}$:

$$
\frac{\lambda}{1-\beta} e_{c,t} + b^*_{c,t-1} + X_{c,t} + U^*_{c,t} = 0, \tag{A.11}
$$

where $X_{c,t} \equiv -\frac{\lambda}{1-\beta} \sum_j \ \mathbb{E}_t x_{c,t+j} + \lambda \sum_j \sum_k \beta^j \ \mathbb{E}_t x_{c,t+j+k}$ is the trilemma component of the UIP equation, and $U^*_{c,t} \equiv -\frac{\lambda}{1-\beta} \sum_j \ \mathbb{E}_t u^*_{c,t+j} + \lambda \sum_j \sum_k \beta^j \ \mathbb{E}_t u^*_{c,t+j+k}$ is the non-trilemma component of the UIP equation and generates endogenous UIP deviations from the noise trader shocks.

To arrive at the closed-form solution of the exchange rate response to the noise trader shock, we impose the following assumptions, as in the model example 1 under Taylor rule: We assume that noise traders' positions $n^*_{c,t}$ inside the non-trilemma component follows an AR(1) process with persistence ρ, that foreign exchange interventions in foreign-currency bonds $f^*_{c,t}$ are independent of noise trader shocks ($f^*_{c,t} \perp n^*_{c,t}$), and that macro-fundamentals are slow-moving compared with noise trader shocks $n^*_{c,t}$.

Under these three assumptions, we can simplify equation (A.11) to

$$\frac{\lambda}{1-\beta} e_{c,t} + b^*_{c,t-1} + X_{c,t} - \frac{\beta\lambda\ \omega\sigma^2_{e_{c,t}}}{(1-\rho\beta)(1-\beta)}\ n^*_t + \tilde{U}^*_{c,t} = 0, \tag{A.12}$$

where $\tilde{U}^*_{c,t} \equiv U^*_{c,t} + \frac{\beta\lambda\ \omega\sigma^2_{e_{c,t}}}{(1-\rho\beta)(1-\beta)}\ n^*_t$ are the residuals of the non-trilemma component of $U^*_{c,t}$, such as foreign exchange interventions $f^*_{c,t}$, that are independent of noise traders' positions $n^*_{c,t}$.

We can therefore compute the impulse response of exchange rate level e_t to the noise trader shock to holdings of foreign-currency holdings $n^*_{c,t}$:

$$\frac{\partial e_{c,t}}{\partial n^*_{c,t}} = \frac{\beta\ \omega\sigma^2_{e_{c,t}}}{(1-\rho\beta)} > 0 \tag{A.13}$$

as $\rho, \beta \in (0,1)$. This is consistent with the empirical evidence because here noise traders' positions $n^*_{c,t}$ are measured in foreign currency rather than local currency. A positive foreign-currency demand shock appreciates foreign currency and depreciates local currency as the relative demand for local currency drops, resulting in an increase in local-currency exchange rate level $e_{c,t}$, which is measured in number of local currencies per dollar. Thus, the inequality in equation

(A.13) is equivalent to saying $\frac{\partial e_{c,t}}{\partial n_{c,t}} < 0$, the same as the prediction from model example 1 under Taylor rule.

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CHAPTER 2

A Theory of International Asset Returns: Country Size and Equity Rebalancing

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2.1 Introduction

Sovereign bonds valued as safe assets by global investors pay lower expected returns that can not be compensated by exchange rates movements. Such persistent difference in sovereign bond return, reflecting variations in sovereign bond safety, is also known as the failure of the *uncovered interest rate parity* (UIP). The failure of UIP to hold in data has been a long-standing puzzle in International Finance since the pioneering work of Fama (1984). Moreover, recent literature has documented that the UIP premium reverses sign[2] and that the reversal seems to be systematically correlated with the period of crisis (Corsetti and Martin, 2020) and the global risk appetite (Kalemli-Ozcan and Varela, 2023).

What determines the relative returns of sovereign bonds issued in different currencies (UIP premium), in both crisis and normal times? Sovereign bond returns is at the core of international macroeconomics and finance, yet there's no unifying theory that jointly explains the sovereign bonds safety both in normal and crisis times, especially through the dynamics in equity markets. Both the international bonds market and equity markets experienced dramatic fall during the period of crisis. However, the literature has either focused on the bond markets to study the exchange rate dynamics while leaving the equity markets unattended (Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021) , or on the portfolio re-

[2]Recent literature documents that high interest rate currencies for advanced economies have higher expected returns over the near future and the UIP reverses sign after about eight quarters (Engel, 2016; Dahlquist et al., 2023)

balancing dynamics in the equity markets solely (Hau and Rey, 2008; Camanho, Hau, and Rey, 2022).

This paper leverages the insights on portfolio rebalancing from the equity markets to generate rich dynamics in the foreign exchange and the sovereign bonds market. Our theory shows that the relative size of the country as well as the equity rebalancing channel jointly determine sovereign bond safety. Using a two-country Lucas tree model with equity constraints, we characterize the model mechanism in closed-form and reconcile the observed UIP patterns both in normal and in crisis times. We propose that the interaction between *country-size effect* and the *equity-rebalancing effect* due to equity constraint is the key driver of UIP patterns. Our model mechanisms can qualitatively explain the pattern of UIP reversals combined with the country-size and equity-rebalancing effect in the period of financial crisis of 2008 for both the G10 and emerging market economy (EME) currencies.

In normal times, the two countries in our model – a smaller home country and a larger foreign country – can perfectly share consumption risks through freely adjusting their equity and bond holdings. The *country-size effect* makes the larger country's bond a global safe asset in normal times as the larger country constitutes most of total world consumption risks through the international trade and financial market. Therefore, investors are willing to pay a safety premium for the larger country bond by receiving lower expected returns. This rationalizes the observed UIP premium during normal times. Throughout the paper, we treat the

U.S. as the larger foreign country, and the G10- or EME-currency countries as the smaller home country.

In the period of crisis, both the home and foreign countries are constrained in their equity holdings and have to take on more home risks in consumption than they would ideally prefer. However, the two model mechanisms – namely the country-size and equity-rebalancing effect – work differently for the smaller home and the larger foreign country in crisis. For investors in the smaller home country, the equity-rebalancing effect *competes* with the country-size effect. If the country-size effect dominates, home bond becomes safer for home investors in crisis and UIP reversal occurs; if the equity-rebalancing effect dominates, the larger foreign country's bond (U.S.) remains safer for home investors and the flight-to-safety occurs. For investors in the foreign larger country, the equity-rebalancing effect *collaborates* with the country-size effect and the safety of the larger country's bond is strengthened for home investors in the period of crisis.

In our model, the equity constraint is the key and only departure from an otherwise standard two-country Lucas tree model as in in Cochrane, Longstaff, and Santa-Clara (2007). Each country has to hold at least a fraction of their domestic equity - they can not issue as much domestic equity share as they would like to. The equity constraints deliver the equity home bias well-documented in the literature (Hau and Rey, 2008; Coeurdacier and Rey, 2013). In this paper, we argue that shocks that tightens the equity constraint facing home country drives the system into crisis. That is, the maximum holding of foreign equity by home country decreases during crisis. The equity constraints in our model fall into the balance sheet constraints class and is supported by empirical evidence (Du, Tepper, and Verdelhan, 2018b; Du, Hebert, and Huber, 2019).[3]

Both the country-size spillover and equity-rebalancing effect during the financial crisis are well-founded by empirical evidence. On the country-size effect, we found that the relative size of G10 currencies relative to the U.S. peaks at the 2008 financial crisis, consistent with the strong and negative correlation on the UIP premium and country size on the advanced countries as documented by Hassan (2013). By comparison, we didn't find such correlation for the EME-currencies, suggesting that the country-size effect might not the driver for UIP reversals for the EMEs. On the equity-rebalancing effect, we found that the effect is present for both the G10 and EMEs with the larger foreign country (U.S.) holdings of foreign assets increased during the period of financial crisis, while at the same time the home country (G10 or EME) holdings of home assets have no sizable change. Our empirical evidence on equity rebalancing is consistent with the increase in equity home bias during the financial crisis (Wynter, 2019; Atkeson, Heathcote, and Perri, 2022).

Our model predictions also reconcile with the empirical facts on deviations from the *covered interest rate parity* (CIP) and convenient yields. The failure of CIP implies a breakdown of the no-arbitrage condition, contrasts the friction-less market assumption, and points to models with financial frictions. There is market seg-

[3]For example, Du, Tepper, and Verdelhan (2018b) uses banking regulation to test the balance sheet constraints and shows that the balance sheet constraints have impact on asset prices. Du, Hebert, and Huber (2019) provides direct evidence that the risk of balance sheet constraints becoming tighter is priced.

mentation during crisis time in our model with equity constraints. Such market segmentation limits arbitrage across markets and law of one price is violated in crisis time, which generates the deviations from CIP.

2.1.1 Literature review

This paper builds on and contributes to the study of currency risk premia, safe asset determination, and portfolio rebalancing.

First, our study on currency risk premia is related to the discussion of safe asset, UIP puzzles, and exchange rate risk hedging. Gopinath and Stein (2020) shows that a currency that hedges exchange rate risk endogenously has lower return and becomes the dominant currency due to the complementarity in trade invoicing and banking, taken exchange rates as exogenous in the model. Gabaix and Maggiori (2015) studies exchange rate determination in a segmented international financial market with global financiers facing credit constraints and explains the UIP violation, taken bond demand as given. Itskhoki and Mukhin (2021) also looks at a segmented international financial market and introduce exogenous noise trader shocks into a standard international real business cycle model to account for the UIP violation puzzle. In our model, exchange rates, the demand for bonds as well as equities are all endogenously determined.

Second, our study on sovereign bond safety contributes to the existing literature by jointly explaining bond safety in both normal and crisis times. The literature proposed various fundamental determinants of bond safety but cannot The documented determinants include coordination of investors (He, Krishnamurthy, and Milbradt, 2019), financial depth (Maggiori, 2017), heterogeneous risk aversion coefficients (Gourinchas and Rey, 2007), rare disaster and heterogeneous disaster resilience (Farhi and Gabaix, 2016; Corsetti and Marin, 2020), and country size effect solely (Martin, 2011; Hassan, 2013). While each of the existing theory can explain only one of the empirical facts mentioned above, our paper jointly explain UIP violation in both normal and crisis times, as well as the facts on the flight to safety, CIP deviations and convenience yields in the period of crisis.

In addition, our two-country Lucas-tree model builds on classic continuous-time asset pricing framework. Starting from fiction-less models: Cochrane, Longstaff, and Santa-Clara (2007) solves a two-tree model with perfect substitutable goods and leaves no space for exchange rates. Pavlova and Rigobon (2007) solves a two-tree model with log-linear preference, where the country size spillover effect does not show up as a result of the knife-edge case of CES consumption. Martin (2011) solves the price levels in a general two-trees model with and shocks following any Levy process whereas we instead focus on optimal portfolio trade-off and solve for intertemporal risk pricing and Euler equations. Continuing to models with financial frictions: Pavlova and Rigobon (2008) builds a center-periphery three-country model with exogenous country size parameters and general portfolio constraints to study contagion and exchange rate movements in crisis. Garleanu and Pedersen (2011) shows that deviations from law of one price emerges in a heterogeneous risk-averse agents model with linear margin constraints.

Lastly, the portfolio rebalancing literature has either focused on the bond markets to study the exchange rate dynamics while leaving the equity markets unattended (Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021), or on the portfolio rebalancing dynamics in the equity markets solely (Hau and Rey, 2004; Camanho et al., 2022). This paper bridges this gap.

2.1.2 Outline

The rest of the paper is organized as follows. Section 2.2 presents the empirical facts on UIP reversal, country-size effect and the equity-rebalancing effect during the financial crisis. Section 2.3 introduces and set up the model. Section 2.4 discusses the model mechanisms and predictions in the complete market setting with no equity constraints. Section 2.5 introduces the equity constraints and addresses the model solutions in both normal and crisis times. Section 2.6 explains how the model predictions reconcile with the empirical facts on UIP reversal, flight-to-safety and CIP. Section 2.7 considers model solutions when there are changes in the equity constraint and trade elasticity due to financial development. The last section concludes.

2.2 Motivating Empirical Facts

2.2.1 Data Description

We combine a few public databases to provide empirical evidence on the pattern of UIP premium and its relation with country size and the equity portfolio rebalancings at the financial crisis. The data on exchange rates and government bond yields used to construct UIP premium are from the Bank for International Settlements (BIS). The data on nominal GDP are retrieved from the International Financial Statistics (IFS) provided by the International Monetary Fund (IMF).

We use both the Coordinated Portfolio Investment Survey (CPIS) provided by the IMF and the market capitalization database from the World Bank to compute the equity portfolio holdings measure. The CPIS dataset has a wide coverage of countries but it only reports cross-border demand and not demand for domestic equities held by domestic investors. We therefore follow Koijen and Yogo (2020) and use the total market capitalization of all domestic listed firms reported by the World Bank as the total supply of domestic equities. We then use the total foreign demand aggregated from the CPIS data to subtract from the total supply of domestic equities to back out the holdings of domestic equities.

Our sample includes both the group of G10 currencies and the group for emerging market currencies. The G10 currencies besdies the U.S. Dollar (USD) are Euro (EUR), Pound Sterling (GBP), Japanese Yen (JPY), Australian Dollar (AUD), New Zealand Dollar (NZD), Canadian Dollar (CAD), Swiss Franc (CHF), Norwegian

Krone (NOK) and the Swedish Krona (SEK). We have 12 emerging market currencies in our sample chosen for their availability in the exchange rates, government bond yields and equity holdings data. These 12 currencies are Brazillian Real (BRL), Chilean Peso (CLP), Colombian Peso (COP), Hungarian Forint (HUF), Indonesian Rupiah (IDR), Indian Rupee (INR), Mexican Peso (MXN), Malaysian Ringgit (MYR), Philippine Peso (PHP), Russian Ruble (RUB), Thai Baht (THB) and South African Rand (ZAR).

2.2.2 Definition on UIP premium

To fix ideas, let us define UIP premium as the excess return of home currency asset against the U.S. Dollar (foreign currency). The home currency can be any G10 currencies other than the USD or an EME currency. The UIP premium in log points is therefore:

$$\mathbb{E}_t[\lambda_{t+h}] \equiv (i_t - i_t^{\text{US}}) \; - \; (\mathbb{E}_t \, s_{t+h} - s_t) \tag{2.1}$$

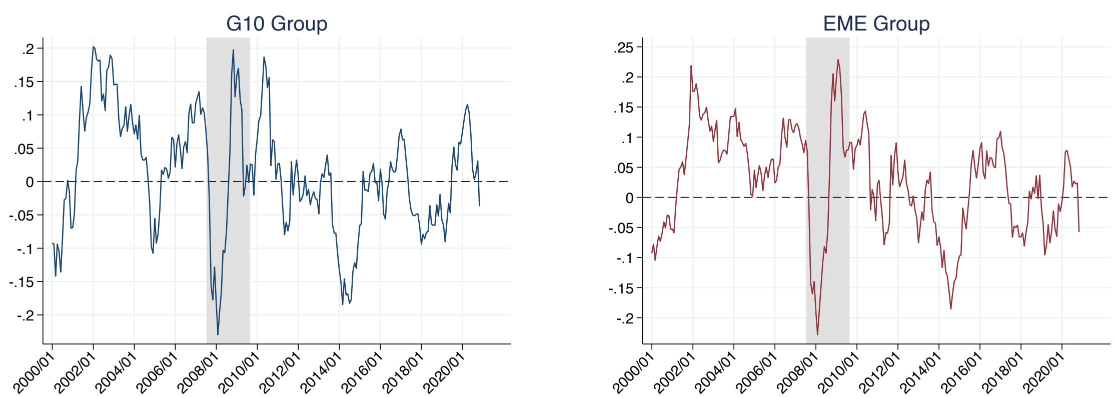
where i_t and i_t^{US} are local and U.S. annualized one-year government bond yields; h is the 12-month horizon. Exchange rate s is in units of local currency per USD; an increase in s would imply local currency depreciation against the USD. When $\mathbb{E}_t[\lambda_{t+h}] = 0$, the UIP condition holds and there's no excess return from the currency carry trade. If $\mathbb{E}_t \;\; \lambda_{t+h} > 0$, there's positive excess returns for the currency trade that longs home currencies and shorts the USD; vice versa for $\mathbb{E}_t[\lambda_{t+h}] < 0$. In the data, we use realized exchange rates to measure UIP premium due to the lack of survey data on exchange rates expectation.

2.2.3 Three Stylized Empirical Facts

We highlight three stylized facts linking reversals in UIP premiums, the country-size effects and the equity rebalancing channel at the financial crisis for both the G10 and EME currency group.

Stylized Fact I: *UIP premium – defined as the excess returns in local currencies against the USD – reverses sign during the financial crisis for both the G10 and EME currencies.*

Figure 2.1: Annualized UIP Premium for G10 and EME Group



Note: This panel presents the annualized average UIP premium as excess return in local currencies against the USD for countries in the G10 currency group (left) and in the emerging market economies group (right). The UIP premiums are in log points and calculated for one-year government bond yields. The grey area corresponds to months of the Great Financial Crisis in 2008.

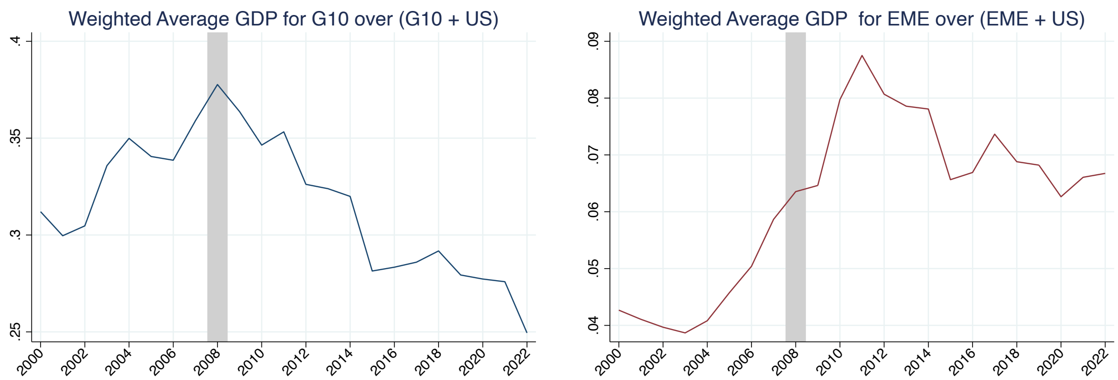
Our work is motivated by the empirical facts on UIP reversals in the period of crisis, as shown in Figure 2.1. The pattern of UIP reversals during the the period of crisis is robust for both the G10 and the emerging market economies. Average realized local-currency premium falls dramatically and turn negative in the start of the great financial crisis of 2008/09 before reverting back to positive. A negative local-currency excess return implies that investors are losing profits investing in local currencies against the USD. We also plot UIP premium at the currency level and found that most currencies share the same feature of UIP reversal during the crisis, as reported in Table 2.B.1 and 2.B.2 in the Appendix.

While we use the 2008 great financial crisis for illustration in Figure 2.1, the empirical facts on UIP reversals in crisis times are *not* unique to the great financial crisis, as verified in several recent papers.[4] For example, Kalemli-Ozcan and Varela (2023) shows that the realized UIP premium in both the advanced and the emerging market economies correlate strongly with the VIX index, a proxy for global risk perception that has been widely used in the international finance literature (see for example, Rey, 2015). Specifically, the correlation between UIP premium and VIX is positive and highly statistically significant, suggesting that higher global risk associates with higher UIP premia in local currencies against the US Dollar. In addition, Corsetti and Marin (2020) documented systematic UIP reversals for the carry trade in the Pound Sterling (GBP) against the US. Dollar that co-move with the episodes of crises periods in the US.

[4]Note that while several recent papers, including Kalemli-Ozcan and Varela (2023) and Dahlquist et al. (2023), look at the reversal of local-currency UIP premium from the negative to the positive sign, we focus on the reversal of UIP premium that turns negative in the period of crisis.

Stylized Fact II: *The relative country size for the G10 currency group peaks during the financial crisis while it remains flat for the emerging market currency group.*

Figure 2.2: Relative GDP Ratio for G10-currency and EME Group



Note: This panel of figures present the weighted average nominal GDP for G-10 currency group (left) and emerging market economy currency group (right) over its sum with the US nominal GDP. The weights of its currency are its share of GDP within the G10 currency (or EME) group. The shaded area represents the 2008 financial crisis.

We construct the relative country size of the G10-currency (or EME) group to the world as the weighted average nominal GDP of G10 over its sum with the US of the same year. We treat G10-currency (or EME) as the home country, and the U.S. as the foreign and the larger country. Therefore, the share of country size constructed is a proxy for the relative country size in the world. Figure 2.2 presents the time-series on this relative country size and shows that the size for G10-currency group peaks at the great financial crisis of 2008, with an increase of more than 3% in the 2008 financial crisis. In comparison, the EME group relative size was rather flat with no visiable change during the 2008 financial crisis and only peaks three years after. Table 2.B.7 in the appendix gives the time series for country-specific relative size.

The finding that the relative country-size for G10-currencies, but not EMEs, peaks at the financial crisis of 2008 suggests that the country-size effect might not be the driver for the UIP reversal in the EMEs. To confirm this, we preform OLS regressions for annualized UIP premium on the country-specific size relative to the world, as reported in Table 2.B.3 and 2.B.4. The relation between the UIP premium and relative country size is negative and strongly significant for both the full sample and the sub-sample using only years before and after the 2008 financial crisis. This suggests that the increase in the relative country-size of G10 in the 2008 financial crisis can potentially explain the fact on UIP reversal addressed above. In comparison, the OLS results for EMEs are largely inconclusive, with the full sample OLS coefficient producing the wrong sign on the relation between UIP premium and GDP and the sub-sample results largely insignificant.

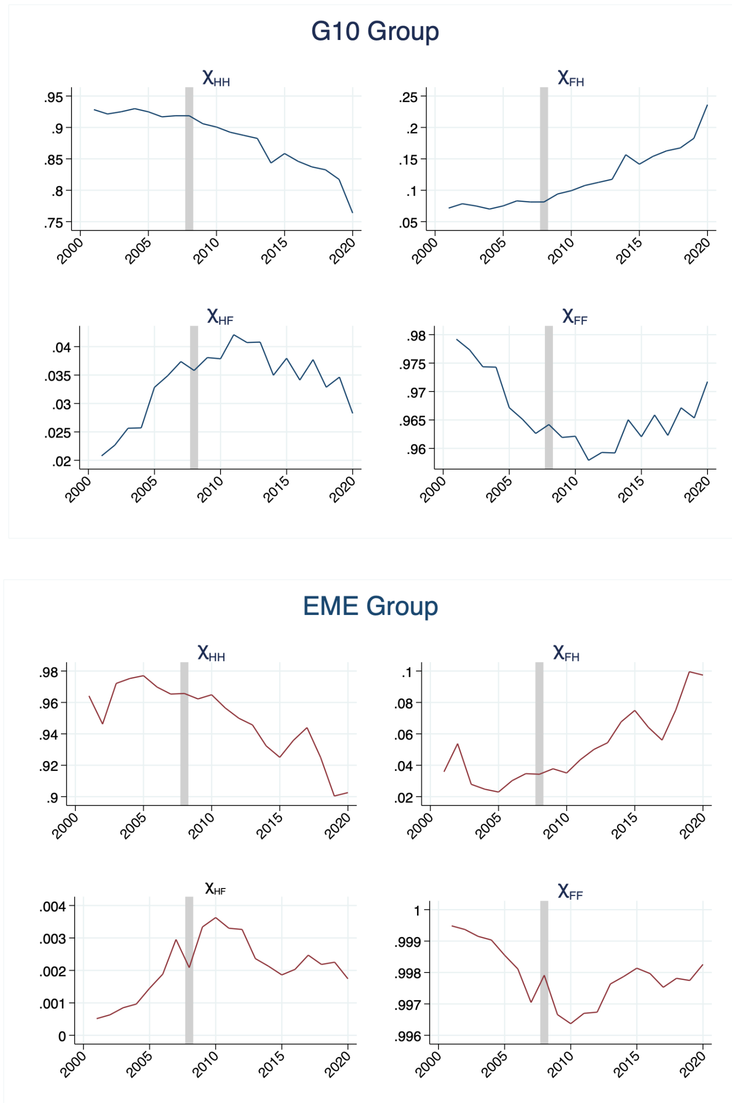
We also verify the relation between the UIP premium and the *within-group* country size for the G10 and EME group separately. Consistent with Hassan (2013), we found negative and statistically significant correlation between the UIP premium and the within-G10 country size share for the advanced economies (our G10 currency group). However, the relation between UIP and country size breaks down for the group of emerging market economies. Scatter plots in Table 2.B.8 attest to this claim. The OLS regression with year and country fixed effects also confirms the statistically significant relation for the G10-currency group, as reported in Table 2.B.5, and the insignificant relation for the EME group, as reported by 2.B.6.

Stylized Fact III: *Equity re-balancing effects are present for both the G10- and EME-currency group in the 2008 financial crisis, with an increase in the foreign (US) investor holdings of foreign equities ($\chi_{F,F}$) and no change in the home (G10 or EME) investor holdings of home equities ($\chi_{H,H}$).*

We provide four measures on cross-border equities holdings[5] to gauge the effects on equity rebalancing channel during the financial crisis. All measures treat G10(or EME) currencies other than the USD as home and the USD as foreign. We define $\chi_{H,H}$ as the share of home investors' holdings of home equities over the total supply of home equities; $\chi_{F,H}$ is foreign investors' holdings of home equities over the total supply of home equities; $\chi_{H,F}$ is home investors' holdings of foreign equities over the total supply of foreign equities; Finally, $\chi_{F,F}$ is foreign investors' holdings of foreign equities over the total supply of foreign equities. By construction, $\chi_{H,H}$ and $\chi_{F,H}$ add up to 1 as they share the total supply of home equities as the same denominator; $\chi_{H,F}$ and $\chi_{F,F}$ also add up to 1 and both use the total supply of foreign (USD) equities as the denominator.

[5]Note that the share of equity holdings needs to be distinguished from the equity *portfolio shares* (denoted by θ) that will be introduced later in the model section of the paper. For example, we use $\theta_{H,H}$ to define the share of home equities held by home investors over the *total wealth* of home investors.

Figure 2.3: Equity Shares for G10- and EME-currency group



Note: This panel of figures present the average share of equity holdings or the G10 (top, blue) and the EME group (bottom, red). For all figures, home (H) is G10 or EME country, and foreign (F) is the U.S. We use the weighted average across all country holdings in the G10-currency group for computing the equity share for G10 (top); we use the simple average across EME-currency group for computing the equity share in the EME group (bottom) as we don't have a fully balanced panel. The equity share $\chi_{H,H}$ is defined as home investors' holdings of home equities over the total supply of home equities; $\chi_{F,H}$ is foreign investors' holdings of home equities over the total supply of home equities; $\chi_{H,F}$ is home investors' holdings of foreign equities over the total supply of foreign equities; and $\chi_{F,F}$ is foreign investors' holdings of foreign equities over the total supply of foreign equities. The shaded line indicates the 2008 financial crisis.

Figure 2.3 presents the time series of these four measures on equity holdings. Consistent with the literature that documents the phenomenon on equity home bias for the U.S as the global safe assets (Atkeson, Heathcote, and Perri, 2022), we find that the share of holdings of US equities of by US investors (denoted by $\chi_{F,F}$, as USD is foreign) increases during the 2008 financial crisis. The finding is robust for using both the G10-currency group and EME-currency as home. By construction, the home investor holdings of US equities ($\chi_{H,F}$) dropped at the financial crisis. At the same time, there's no sizable change in the share of holdings on home (G10 or EME) equities during the 2008 financial crisis, as shown by the graphs on $\chi_{H,H}$ and $\chi_{F,H}$.

2.3 Model Set-up

Time is continuous and infinite horizon, $t \in [0, +\infty)$. There are two countries in the world, home country (denoted by H) and foreign country (denoted by F). For ease of illustration, we will call home country the UK (one of G10-currency countries) and foreign country the US.

Technology Each country is endowed with a tree producing domestic good. The two trees evolve as follows,

$$\frac{dY_{H,t}}{Y_{H,t}} = \mu_{H,t}\,dt + \sigma_{H,t}\,dZ_{H,t}$$
$$\frac{dY_{F,t}}{Y_{F,t}} = \mu_{F,t}\,dt + \sigma_{F,t}\,dZ_{F,t}$$

where $\{\mu_{H,t}, \mu_{F,t}, \sigma_{H,t}, \sigma_{F,t}\}$ are exogenous parameters (or processes). For simplicity, we assume throughout the paper that $\mu_{H,t} = \mu_{F,t} = \mu$ and $\sigma_{H,t} = \sigma_{F,t} = \sigma$.

Preferences In order to highlight our mechanism, we assume homogeneous preference, logarithmic utility, and no consumption home bias for the representative agents of the two countries. The final consumption is a CES aggregate of the two goods produced by the two countries. The expected utility of the representative agent in country i, takes the form

$$\mathbb{E} \int_0^\infty e^{-\rho t} \log C_{H,t}\, dt$$

where

$$C_{H,t} = \left[a^{\frac{1}{\eta}} C_{HH,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} C_{HF,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

a is the share parameter [6]. η is the elasticity of substitution between the two goods, assumed to be greater than 1 and smaller than infinity.

Numeraire Define 1 unit of the CES basket of total output $\overline{Y}_t$ as numeraire throughout the paper,

$$\overline{Y}_t \equiv \left[a^{\frac{1}{\eta}} Y_{H,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} Y_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

Denote the process of total output $\overline{Y}_t$ as

$$\frac{d\overline{Y}_t}{\overline{Y}_t} = \overline{\mu}_t dt + \overline{\sigma}_t dZ_t$$

[6] Unlike Pavlova and Riggobon (2008) where a represents the country size, here in our model a is not a key parameter of interest.

where $dZ_t = [dZ_{H,t} \;\; dZ_{F,t}]^T$.

International trade market and exchange rate The international trade market (of home and foreign goods) is frictionless. Denote p_t^H as the price of home good and p_t^F as the price of foreign good. The real exchange rate is given by the relative price of home and foreign good[7],

$$e_t \equiv \frac{p_t^H}{p_t^F} \tag{2.2}$$

and e_t is also the terms of trade in this model. And denote the endogenous process of real exchange rate, e_t, as

$$\frac{de_t}{e_t} = \mu_t^e dt + (\sigma_t^e)^T dZ_t$$

Equity Each country can issue domestic equity shares in unit supply. The equities are risky claims to domestic trees. Denote S_t^H and S_t^F as the total value of domestic equity and foreign equity respectively. Define $\chi_t^{H,H}$ as the the share of home stock market (apple tree) held by home investor, $\chi_t^{H,F}$ as the share of foreign stock market (orange tree) held by home investor. And similarly define $\chi_t^{F,H}$ as the share of home stock market (apple tree) held by foreign investor and $\chi_t^{F,F}$ as the share of foreign stock market (orange tree) held by foreign investor.

[7] An increase of e_t corresponds to an appreciation of home currency relative to foreign currency.

Equity constraint Importantly, equity constraint for home country:

$$0 \leq \chi_t^{H,F} \leq \overline{\chi}^{H,F} \tag{2.3}$$

This equation is saying that home investor can not hold more than $\overline{\chi}^{H,F}$ share of foreign equity, nor short-sell foreign equity[8].

And similarly we have equity constraint for foreign country:

$$0 \leq \chi_t^{F,H} \leq \overline{\chi}^{F,H} \tag{2.4}$$

That is, foreign investor can not hold more than $\overline{\chi}^{F,H}$ share of home equity, nor short-sell home equity.

Sovereign Bond Each country can issue (sovereign) bond in zero net supply. The bonds, denoted as B_t^H and B_t^F, are instantaneously *risk-free in domestic goods* but *not* risk-free in terms of numeraire[9]. The price of home (foreign) bond B_t^H (B_t^F) in terms of numeraire is the same as the price of home (foreign) good p_t^H (p_t^F). Denote $B_t^{H,H}$ as the home bond held by home investors[10] and $B_t^{F,H}$ as the home bond held by foreign investors. And denote $B_t^{H,F}$ as the foreign bond held by home investors and $B_t^{F,F}$ the foreign bond held by foreign investors.

[8]Here one can replace the lower bound 0 to a negative number, say $\underline{\chi}^F$. The key thing is that $\chi_t^{H,F}$ (the share of foreign equity held by home investor) is lower bounded.

[9]Their returns are subject to exchange rate risks through price changes

[10]$B_t^{H,H} > 0$ means lending and $B_t^{H,H} < 0$ means borrowing

Asset returns We introduce notations for asset returns which are *endogenous* processes. Recall that B_t^H is instantaneously risk-less bond in home good and denote the return process of home bond (in terms of numeraire) as:

$$dr_t^{B^H} = \frac{d(p_t^H B_t^H)}{p_{H,t} B_t^H} = (\mu_{p^H,t} + r_t^H)\,dt + \sigma_{p^H,t}\,dZ_t$$

where $\mu_{p^H,t}$ and $\sigma_{p^H,t}$ are given by the endogenous process

$$\frac{dp_t^H}{p_t^H} = \mu_{p^H,t} t\,dt + \sigma_{p^H,t}\,dZ_t$$

Similarly denote the return process of foreign bond (in terms of numeraire) as:

$$dr_t^{B^F} = \frac{d(p_t^F B_t^F)}{p_{F,t} B_t^F} = (\mu_{p^F,t} + r_t^F)\,dt + \sigma_{p^F,t}\,dZ_t$$

Recall that S_t^H is the total value of home equity and define q_t^H as the per unit price of home equity in terms of numeraire $\overline{Y}_t$, that is, $S_t^H = q_t^H \overline{Y}_t$. And postulate the endogenous process of q_t^H as follows

$$\frac{dq_t^H}{q_t^H} = \mu_{q^H,t} dt + \sigma_{q^H,t} dZ \tag{2.5}$$

The return of home equity in terms of numeraire is given by

$$dr_t^{S^H} = \underbrace{\frac{p_t^H Y_{H,t}}{q_t^H \overline{Y}_t} dt}_{\text{dividend yield}} + \underbrace{\frac{d(q_t^H \overline{Y}_t)}{q_t^H \overline{Y}_t}}_{\text{capital gain}}$$

and similarly

$$dr_t^{S^F} = \underbrace{\frac{p_t^F Y_{F,t}}{q_t^F \overline{Y}_t} dt}_{\text{dividend yield}} + \underbrace{\frac{d(q_t^F \overline{Y}_t)}{q_t^F \overline{Y}_t}}_{\text{capital gain}}$$

Forward market Since there is no friction on the bond markets, there naturally exists a FX forward market for home bond and foreign bond. Home investor can enter a FX forward contract (long in home currency and short in foreign currency) with zero cost today which will deliver an instantaneous return $dr_t^{B^H} - dr_t^{B^F}$.[11]

Wealth and portfolio shares we introduce notations for wealth and portfolio shares, which will be determined in equilibrium. Denote the aggregate wealth of home country as W_t^H and the aggregate wealth of foreign country as W_t^F.

Denote $\theta_t^{H,S^H} = \frac{\chi_t^{H,H} S_t^H}{W_t^H}$ as the portfolio share of home equity for home country, $\theta_t^{H,S^F} = \frac{\chi_t^{H,F} S_t^F}{W_t^H}$ as the portfolio share of foreign equity for home country. And similarly, denote $\theta_t^{F,S^H} = \frac{\chi_t^{F,H} S_t^H}{W_t^F}$ as the portfolio share of home equity for foreign country and $\theta_t^{F,S^F} = \frac{\chi_t^{F,F} S_t^F}{W_t^F}$ as the portfolio share of foreign equity for foreign country.

We can similarly define portfolio shares of bonds in home and foreign country. Denote $\theta_t^{H,B^H} = \frac{p_t^H B_t^{H,H}}{W_t^H}$ as the portfolio share of home bond for home country, $\theta_t^{H,B^F} = \frac{p_t^H B_t^{H,F}}{W_t^H}$ as the portfolio share of foreign bond for home country, And similarly, denote $\theta_t^{F,B^H} = \frac{p_t^H B_t^{F,H}}{W_t^F}$ as the portfolio share of home bond for foreign country and $\theta_t^{F,B^F} = \frac{p_t^F B_t^{F,F}}{W_t^F}$ as the portfolio share of foreign bond for foreign country.

[11] In equilibrium, investors will exactly do so as discussed in appendix

Country Size Define relative size of home country as follows.

$$s_t = \frac{a^{\frac{1}{\eta}} Y_{H,t}^{\frac{\eta-1}{\eta}}}{a^{\frac{1}{\eta}} Y_{H,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} Y_{F,t}^{\frac{\eta-1}{\eta}}} = a^{\frac{1}{\eta}} \left(\frac{Y_{H,t}}{\overline{Y_t}} \right)^{\frac{\eta-1}{\eta}} \tag{2.6}$$

Optimization problems The optimization problem for home country is as follows:

$$\begin{aligned} \max_{\{C_{HH,t}, C_{HF,t}, \chi_t^{H,H}, \chi_t^{H,f}, \theta_t^{H,B^H}, \theta_t^{H,B^F}\}_{t=0}^{\infty}} \quad & \mathbb{E}\left[\int_0^{\infty} e^{-\rho t} \log\left(\left[a^{\frac{1}{\eta}} C_{HH,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} C_{HF,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right) dt \right] \\ \text{s.t.} \quad \frac{dW_t^H}{W_t^H} =& \frac{\chi_t^{H,H} S_t^H}{W_t^H} dr_t^{S^H} + \frac{\chi_t^{H,F} S_t^F}{W_t^H} dr_t^{S^F} + \theta_t^{H,B^H} dr_t^{B^H} + \theta_t^{H,B^F} dr_t^{B^F} \\ & - \frac{p_t^H C_{HH,t} + p_t^F C_{FF,t}}{W_t^H} dt \\ 1 =& \frac{\chi_t^{H,H} S_t^H}{W_t^H} + \frac{\chi_t^{H,F} S_t^F}{W_t^H} + \theta_t^{H,B^H} + \theta_t^{H,B^F} \\ 0 \leq & \chi_t^{H,F} \leq \overline{\chi}^{H,F} \end{aligned} \tag{2.7}$$

The optimization problem for foreign country is similar and discussed in appendix.

Market clearing conditions Home equity market clears,

$$\chi_t^{H,H} + \chi_t^{F,H} = 1 \tag{2.8}$$

Foreign equity market clears,

$$\chi_t^{H,F} + \chi_t^{F,F} = 1 \tag{2.9}$$

Home bond market clears,

$$B_t^{H,H} + B_t^{F,H} = 0 \tag{2.10}$$

And foreign bond market clears,

$$B_t^{H,F} + B_t^{F,F} = 0 \tag{2.11}$$

Total consumption of home (foreign) good equals total production of home (foreign) good,

$$C_{HH,t} + C_{FH,t} = Y_{H,t} \tag{2.12}$$

$$C_{HF,t} + C_{FF,t} = Y_{F,t} \tag{2.13}$$

2.4 Complete market model

Before solving the model with equity constraints, it is useful to solve for the complete market case which works as a clear illustration of the country size spillover effect.

Postulate two stochastic discount factor processes for the two countries, $\xi_{H,t} = e^{-\rho t}\frac{1}{C_{H,t}}$ and $\xi_{F,t} = e^{-\rho t}\frac{1}{C_{F,t}}$, as

$$\frac{d\xi_{H,t}}{\xi_{H,t}} = -r^f_{H,t}\,dt - m^T_{H,t}\,dZ_t$$

$$\frac{d\xi_{F,t}}{\xi_{F,t}} = -r^f_{F,t}\,dt - m^T_{F,t}\,dZ_t$$

respectively. $m_{H,t}$ is the vector of risk prices in home country and also the consumption risk[12] in the logarithmic utility case.

In the complete market case, there exists a unique stochastic discount factor ξ_t such that $\frac{d\xi_{H,t}}{\xi_{H,t}} = \frac{d\xi_{F,t}}{\xi_{F,t}} = \frac{d\xi_t}{\xi_t}$.

2.4.1 Sovereign bond safety and country size spillover effect

Proposition 1: (Sovereign bond safety) *Expected return difference between home bond and foreign bond is given by*

$$\frac{\mathbb{E}_t\left[dr_t^{B^H} - dr_t^{B^F}\right]}{dt} = m_{H,t}^T \sigma_t^e = m_{F,t}^T \sigma_t^e \tag{2.14}$$

where $dr_t^{B^H}$ *is the return process for home bond,* $dr_t^{B^F}$ *is the return process for foreign bond,* σ_t^e *is the exchange rate risk.*

Proof: See appendix. ■

If home country's consumption risk is positively correlated with exchange rate risk (domestic consumption is low when domestic currency depreciates), then home bond earns a positive risk premium. If home country's consumption risk is negatively correlated with its exchange rate (domestic consumption is high when domestic currency depreciates), then home bond earns a negative safety premium.

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[12]Consumption risk of a country is defined as the volatility vector of consumption process of that country, $\frac{dC_{H,t}}{C_{H,t}}$.

The intuition is as follows: A bond is considered safe if it has high value when consumption is low, because the bond insures investors against bad times. In our example, US treasury pays lower expected return than UK government bond if GBP depreciates against USD when consumption is low. Because in this case, US treasury is a good hedge for consumption risk and is viewed as safe, while UK government bond does not hedge consumption risk and is viewed as risky. Lustig and Verdelhan (2007) provides empirical evidence for proposition 1.

In our model, uncertainty comes from production fluctuations of the trees. When UK production declines due to a negative shock, the supply of UK good declines, and the relative price of UK good should go up, implying a higher expected return of UK bond. However, this is not the whole story for bond safety. Because the final consumption is an aggregate of both countries' goods, another competing force emerges: the demand for US good increases because of consumption smoothing motive. The final consumption shifts more towards US good than before due to a supply drop of UK good. This positive demand shock for US good will put upward pressure on the expected return of US bond. The next proposition shows that the relative magnitude of the supply force and demand force is determined by the relative country size.

Proposition 2: (Country size spillover effect) *Solving for* (2.14), *we have*

$$\frac{\mathbb{E}_t[dr_t^{B^H} - dr_t^{B^F}]}{dt} = \overline{\sigma}_t^T \sigma_t^e = \begin{bmatrix} s_t\sigma_H & (1-s_t)\sigma_F \end{bmatrix} \begin{bmatrix} -\frac{1}{\eta}\sigma_H \\ \frac{1}{\eta}\sigma_F \end{bmatrix} \tag{2.15}$$

$$= \frac{1}{\eta}(-s_t\sigma_H^2 + (1-s_t)\sigma_F^2) \tag{2.16}$$

and the safety threshold

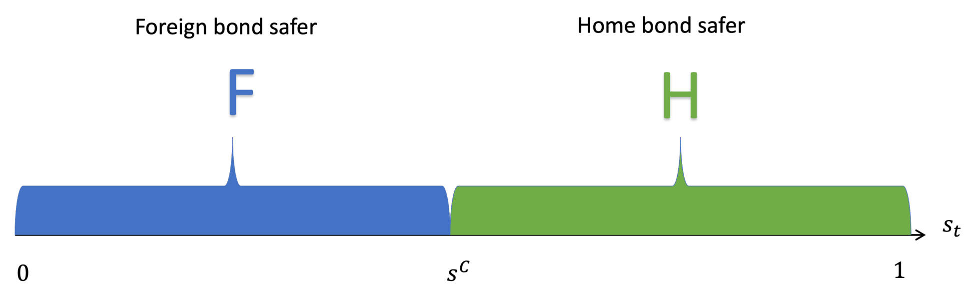
$$s^C = \frac{\sigma_F^2}{\sigma_H^2 + \sigma_F^2} \tag{2.17}$$

If $s_t < s^C$, home country is a relatively small country and home bond is riskier than foreign bond. If $s_t > s^C$, home country is a relatively large country, country bond is safer than foreign bond.

Proof: See appendix. ■

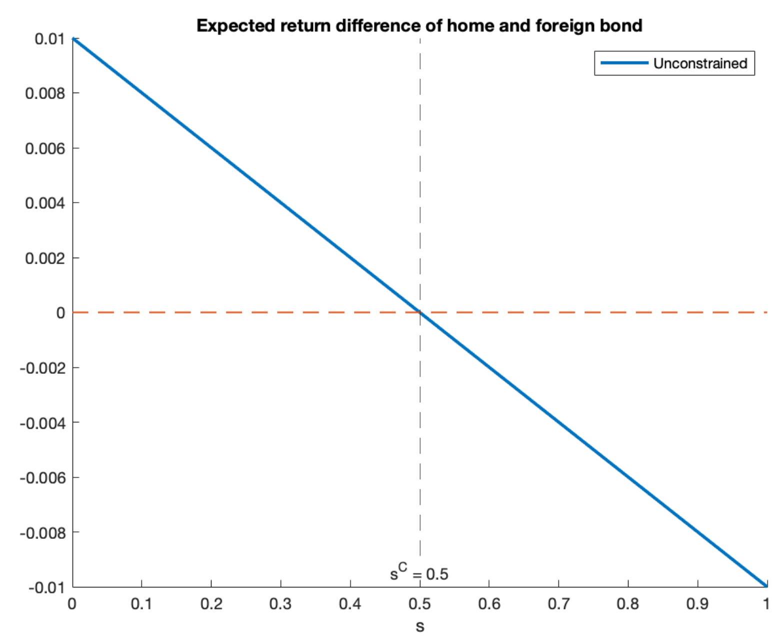
Country size spillover effect states that larger country's bond is safer. US treasury is safer than UK government bond because the size of US economy is a larger than the size of UK economy. Since US contributes a larger share to world consumption, the world consumption risk also consists largely US risk. US treasury becomes a safe asset and pays lower expected return because it is a better hedge for world consumption risk. This is often referred to as the exorbitant privilege of the US Dollar. Going back to the supply force and demand force discussed earlier: the larger the country's share in the world consumption, the larger the magnitude of supply or demand change of its good. When the small country becomes smaller, the large country's production becomes more dominant in world consumption, strengthening the hedging benefit of the large country's bond. On the other hand, when the small country grows larger, world consumption depends less on the large country's production, reducing the hedging benefit of the large country's bond. Country size spillover effect is stronger when there is more asymmetry in s_t and $1 - s_t$. Expected return differences of sovereign bonds are sizable and persistent across countries as discussed in Hassan (2013), which also provides empirical evidence for country size spillover effect.

Figure 2.4: safety region of home and foreign bond



Note: This figure shows the safety region of home and foreign bond as the relative size of home country s_t changes. s^C is the safety threshold where UIP changes sign.

Figure 2.5: Expected return difference of home and foreign bond, complete market



Note: The y-axis of this figure shows the expected return difference of home and foreign bond. The x-axis is the relative size of home country s. The dotted vertical line s^C represents the endogenous threshold for UIP changing sign. Parameter values used: growth volatility $\sigma_H^2 = \sigma_F^2 = \sigma^2 = 0.04$, and trade elasticity $\eta = 4$.

2.4.2 Persistence and asymmetry

In the simplest complete market case, on top of country size spillover effect, there are also persistence effect and asymmetry effect on prices of risks through changes in country size s_t.

The volatility [13] of country size s_t and the prices of risks during normal times are given by

$$\sigma_{s_t} = \frac{\eta - 1}{\eta}(1 - s_t)\begin{bmatrix} \sigma_H \\ -\sigma_F \end{bmatrix} \tag{2.18}$$

and

$$m_{H,t} = m_{F,t} = \overline{\sigma}_t = \begin{bmatrix} \underbrace{s_t \sigma_H}_{\text{price of home country risk}} \\ \underbrace{(1 - s_t)\sigma_F}_{\text{price of foreign country risk}} \end{bmatrix} \tag{2.19}$$

Persistence A temporary negative shock to home country's production immediately reduces home country's relative country size, decreases the price of home country risk. In addition, a smaller s_t also affects the magnitude of future shocks on country size s_t as well as prices of risks $\overline{\sigma}_t$. Unlike classic works in macro-finance literatrue ((Bernanke et al., 1999), where the persistence of a temporary shock is due to changes in current and future investment, the persistence here in our benchmark model without investment is purely from changes in country size s_t.

[13] Throughout this paper, we denote the volatility of the process $\frac{dX_t}{X_t}$ as volatility of X_t.

Asymmetry A negative shock to home country production affects both price of home country risk and price of foreign country risk through country size spillover effect. This shock affect prices of risks asymmetrically through changes in country size s_t. As in equation (2.19), the decline of home risk price is mitigated by the smaller size of home country, s_t, while increase of foreign country risk price is amplified by the larger size of foreign country, $1 - s_t$. The same shock thus affects prices of the two countries' bonds and equities asymmetrically.

2.5 Model with equity constraints

Adding another key ingredient to the model, the equity constraints, we proceed in two steps.

First step, we explore what happens with only one equity constraint for foreign country's holding of home equity, $0 \leq \chi_t^{F,H} \leq \overline{\chi}^{F,H}$. There is a maximum limit on home equity share held by foreign investors and no short-selling of home country's equity is allowed. As shown in the following proposition **??**, the two countries can still perfectly share consumption risk and have the same prices of risks as in the complete market case.

Second step, we explore the full model with equity constraints for both countries. There exists an endogenous crisis regime in the model which results in asymmetry and instability of the system.

To highlight the mechanism and simplify some algebra for illustration purpose, we assume symmetric parameters for the two trees $\mu_1 = \mu_2 = \mu$ and $\sigma_H = \sigma_F =$

σ hereafter. And taking advantage of symmetry, we focus on analysing home country (Home). The symmetric assumption makes sense in the example of UK and US, as the two countries have similar growth rates and volatilities. And we will focus on empirically relevant case where home country (UK) is a small country relative to foreign country (US).

2.5.1 Normal regime

Proposition 3: *With only one equity constraint for foreign country's holding of home equity,* $0 \leq \chi_t^{F,H} \leq \overline{\chi}^{F,H}$*, the two countries can perfectly share consumption risk and replicate the complete market case result in the sense that proposition 1 and proposition 2 still hold true.*

Proof: See appendix. ■

An intuitive way to look at proposition 3 is to count the risks and assets. There are two sources of risks, from the two trees. Even with one equity holding constraint, there are still another three assets that can be freely traded which can span all the possible states of the world. So investors in the two countries can still replicate first best risk-sharing through portfolio re-balancing. Similar to the complete market case, there is indeterminacy in the model with respect to portfolio holdings but not asset returns.

Proposition 4: *With the only equity constraint for foreign country's holding of home equity,* $0 \leq \chi_t^{F,H} \leq \overline{\chi}^{F,H}$*, a special specification for discount rate* $\rho = (\frac{\eta-1}{\eta}\sigma)^2$*, initial* *condition s_0, home country's equity shares and bond holdings are given by*

$$\chi_t^{H,H} \in [1 - \overline{\chi}^{F,H}, 1]$$
$$\chi_t^{H,F} = \frac{n_0 - \chi_t^{H,H}\rho q_t^H}{1 - \rho q_t^H}$$
$$\theta_t^{H,B^H} = \frac{\rho (q_t^H)'(s_t) s_t (1 - s_t)(\chi_t^{H,H} - \chi_t^{H,F})(\eta - 1)}{n_0}$$
$$\theta_t^{H,B^F} = -\theta_t^{H,B^H}$$

where

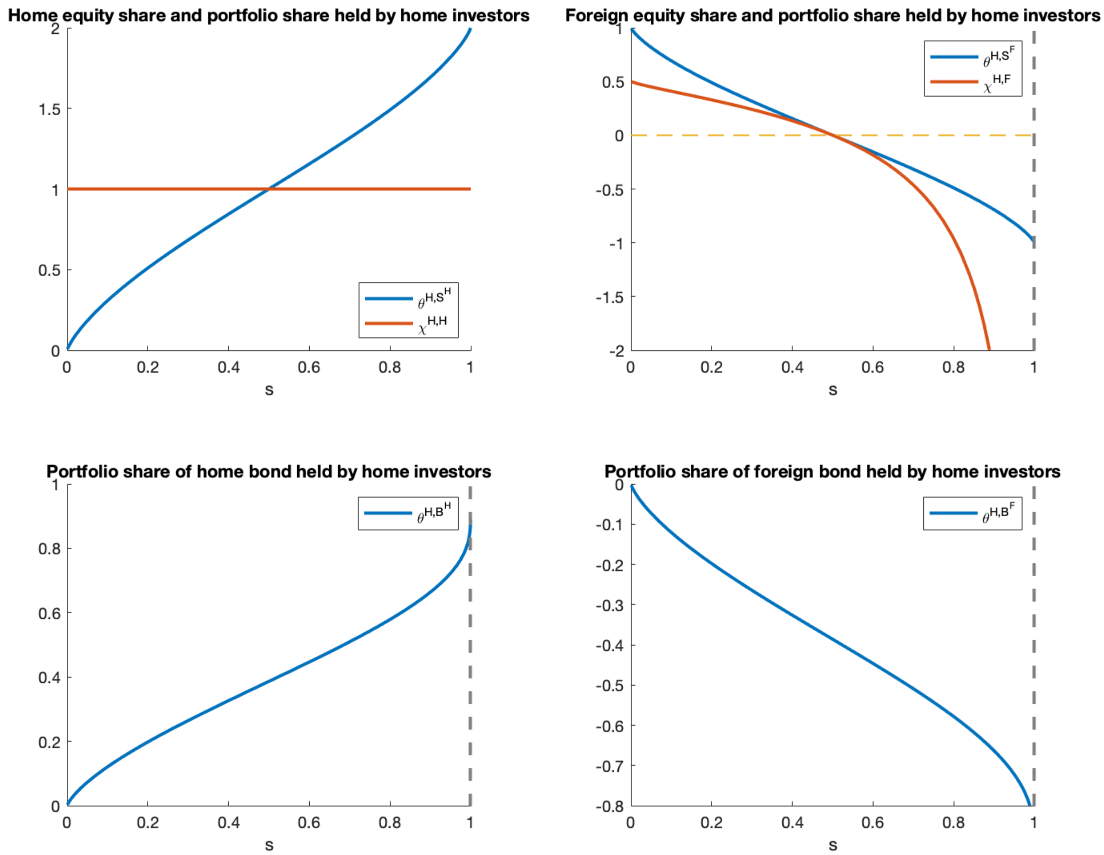
$$q_t^H(s) = \frac{1}{2\rho}\Big(1 + \frac{1-s}{s}\ln(1-s) - \frac{s}{1-s}\ln(s)\Big)$$

is the per unit price of home equity and taking derivatives with respect to s_t, we have

$$(q_t^H)'(s_t) = -\frac{1}{2\rho}\frac{1}{s(1-s)}\Big(1 + \frac{1-s}{s}\ln(1-s) + \frac{s}{1-s}\ln(s)\Big)$$

And $n_0 = q^H(s_0)$ is the initial wealth share of home country. **Proof:** See appendix. ■

Figure 2.6: Equity shares and portfolio shares of home investors



Note: This panel of figures present the equity shares and portfolio shares of home investors solved from the model. For all panels, the x-axis is the relative size of home country s. For the top left panel, the equity share $\chi^{H,H}$ is defined as home investors' holdings of home equities over the total supply of home equities; and the portfolio share of home equity θ^{H,S^H} is defined as home investors' holdings of home equities over their total wealth. For top right panel, $\chi^{H,F}$ is home investors' holdings of foreign equities over the total supply of foreign equities; and the portfolio share of foreign equity θ^{H,S^F} is defined as home investors' holdings of foreign equities over their total wealth. For the bottom left panel, θ^{H,B^H} is defined as home investors' holdings of home bond over their total wealth And for the bottom right panel, θ^{H,B^F} is defined as home investors' holdings of foreign bond over their total wealth. Parameter vlues used: volatility of GDP growth $\sigma^2 = 0.04$, trade elasticity $\eta = 2$, and equity share $\chi^{H,H} = 1$.

From proposition 4 and figure 2.6, we see that the net borrowing in bonds between the two countries is zero. The two countries smooth their consumption by holding equities and use bonds to help achieve perfect risk-sharing. Both countries go short in foreign bond and long in domestic bond. Because domestic bond is a better hedge for domestic risk, the two countries can offload the extra domestic risk from domestic equity holding requirement by lending in domestic bond and borrowing in foreign bond.

Comparing figure 2.5 and the right-bottom panel of figure 2.6, we see that countries borrow more in foreign bond when their country size grows larger and domestic bond becomes safer, fixing home country's holding of domestic equity share. Because when a country grows larger, its domestic equity price increases, leading to a heavier portfolio weight on domestic equity and thus more domestic risk exposure, which requires more hedging. This is consistent with the empirical fact documented by Du, Pflueger, and Schreger (2020). Until now, the model with only country size spillover effect can explain the UIP violation in normal times and find empirical support from earlier work. However, the model does not have space for crisis yet and is thus silent about what happens in crisis.

2.5.2 Crisis regime

Moving on to second step, with equity holding constraints for both countries, an endogenous crisis regime emerges and the system moves into the crisis regime when one country falls too small.

Proposition 5: (Crisis regime) *The system moves in to crisis regime if* $s_t \in [0, s^U] \cup [1 - s^U, 1]$*, where* s^U *is the crisis boundary and solves*

$$q_1(s^U) = \frac{n_0 - \overline{\chi}^{H,F}}{\rho(1 - \overline{\chi}^{H,F})} \tag{2.20}$$

If $s_t \in [0, s^U]$*, we have*

$$\chi_t^{H,H} = 1, \quad \chi_t^{H,F} = \overline{\chi}^{H,F}$$

If $s_t \in [1 - s^U, 1]$*, we have*

$$\chi_t^{H,H} = 1 - \overline{\chi}^{F,H}, \quad \chi_t^{H,F} = 0$$

Proof: See appendix. ■

The crisis boundary s^U is the left margin where both countries' equity holding constraints bind and $1 - s^U$ is the right margin where the equity holding constraints bind in the opposite direction. When $s_t < s^U$, the two countries can perfectly share exchange rate risk through freely adjusting their equity holdings and trading on the FX market. The gains and losses from FX market will be delivered by capital flows induced by equity trading, until both equity constraints bind. we refer to the constrained region as *crisis regime*. In crisis regime, risk-sharing is limited, and asset returns vary discontinuously from in normal regime due to the constraint on equity rebalancing.

2.5.2.1 Safety spectrum

In crisis regime, equity reblancing is constrained and risk-sharing is limited. This market segmentation drives a wedge between normal time SDF and crisis time SDF, thus a wedge of risk prices between normal times and crisis time. We refer to the effect of the constraints on equity reblancing as the *equity rebalancing effect*[14].

Proposition 6: (equity rebalancing effect) *In crisis region, there exists a wedge between the normal time SDF and crisis time SDF, due to lack of equity rebalancing. For home country investors, denote this wedge as* σ_{n_t}.

If $0 < s_t < s^U$,

$$\sigma_{n_t} = \frac{(1-\overline{\chi}^{H,F})s_t}{(1-\overline{\chi}^{H,F})s_t + \overline{\chi}^{H,F}} \frac{\eta - 1}{\eta}(1-s_t)\begin{bmatrix} \sigma_H \\ -\sigma_F \end{bmatrix} \tag{2.21}$$

If $1 - s^U < s_t < 1$,

$$\sigma_{n_t} = \frac{\eta - 1}{\eta}(1-s_t)\begin{bmatrix} \sigma_H \\ -\sigma_F \end{bmatrix} \tag{2.22}$$

symmetrically for foreign investors.

Proof: See appendix. ■

With two equity constraints and the crisis region, the model exhibits a safety spectrum for each country with four regions identified by three key safety thresholds.

[14] To be precise, this is "the lack of equity rebalacing" effect

Proposition 7: (Safety spectrum) *With equity holding constraints for both countries,* $0 \leq \chi_t^{F,H} \leq \overline{\chi}^{F,H}$ *and* $0 \leq \chi_t^{H,F} \leq \overline{\chi}^{H,F}$*, and reasonable parameter restrictions on* $(\overline{\chi}^{F,H}, \overline{\chi}^{H,F}, \eta)$*, there are three key thresholds, normal time safety threshold* s^C*, crisis boundary* s^U*, and crisis time safety threshold* s^A*,*

$$s^C = \frac{1}{2} \tag{2.23}$$

$$q^H(s^U) = \frac{n_0 - \overline{\chi}^{H,F}}{\rho(1 - \overline{\chi}^{H,F})} \tag{2.24}$$

$$\frac{2(1-\overline{\chi}^{H,F})(s^A)^2 + (2\overline{\chi}^{H,F}\eta + (1-\overline{\chi}^{H,F})(\eta-2))s^A - \eta\overline{\chi}^{H,F}}{(1-\overline{\chi}^{H,F})s^A + \overline{\chi}^{H,F}} = 0 \tag{2.25}$$

such that

$$0 < s^A < s^U < s^C \tag{2.26}$$

When $s^U < s_t < 1 - s^U$*, the system stays in normal regime, country size determines bond safety as in proposition 2:*

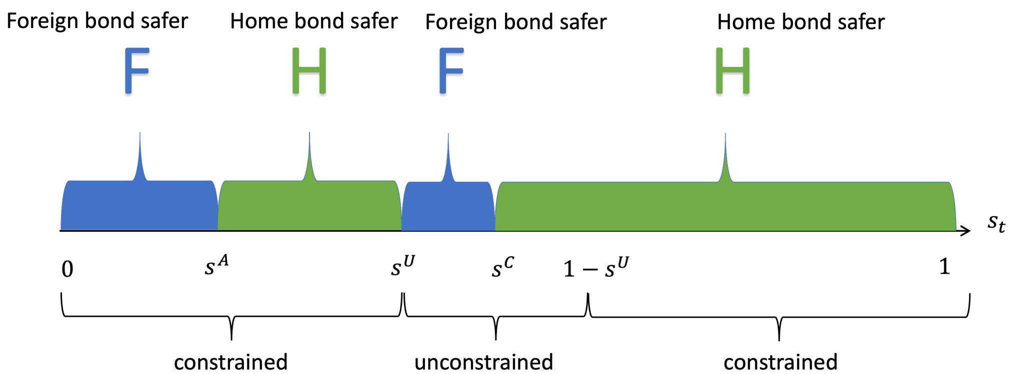
If $s^U < s_t < s^C$*, home country is a relatively small country, home country's bond is risky.*

If $s^C < s_t < 1 - s^U$*, home country is a relatively large country, home country's bond is safe.*

When $0 < s_t < s^U$ *or* $1 - s^U < s_t < 1$*, the system moves into crisis regime. Country size spillover effect and the equity rebalancing effect will jointly determine sovereign bond safety.*

When $0 < s_t < s^U$*, the system moves into crisis regime where country size spillover effect competes with equity rebalancing effect:*

Figure 2.7: safety region of home and foreign bond for home investors



Note: This figure shows the safety region of home and foreign bond as the relative size of home country s_t changes, from the perspective of home investors. s^U is the boundary of crisis regime (constrained) as home country falls in relative country size, symmetrically for $1 - s^U$. s^C and s^A are the safety thresholds where UIP changes sign in normal regime (unconstrained) and crisis regime (constrained).

If $s^A < s_t < s^U$*, equity rebalancing effect dominates, home country's bond is safe for domestic investors.*

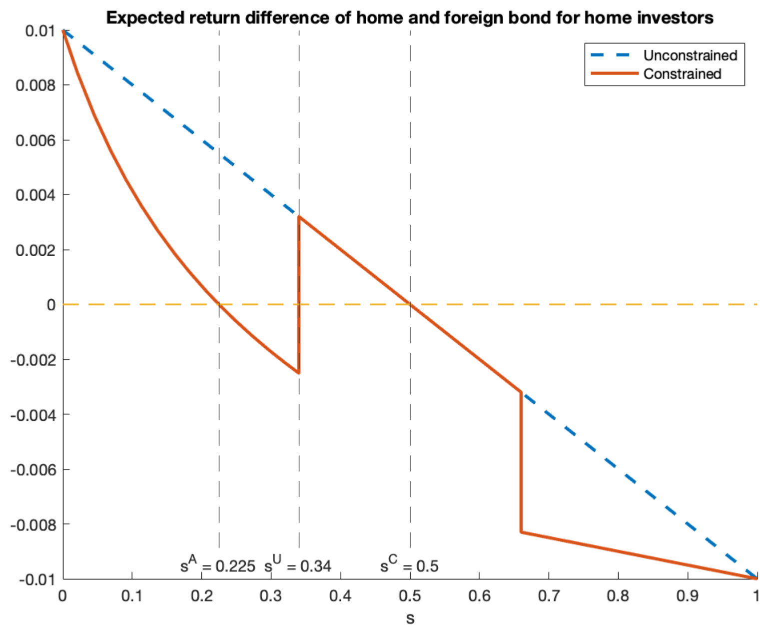
If $0 < s_t < s^A$*, country size spillover effect dominates, home country's bond is risky for domestic investors.*

When $1 - s^U < s_t < 1$*, the system moves into crisis regime where country size spillover effect joins forces with equity rebalancing effect: home country's bond is safe for domestic investors.*

Proof: See appendix. ∎

As shown in figure 2.7, cut by the three thresholds, there are four regions along the safety spectrum. Blue regions represent where foreign bond is safer for domestic investors, and green regions represent where domestic bond is safer. Fig-

Figure 2.8: Expected return difference of home and foreign bond for home investors



Note: The y-axis of this figure shows the expected return difference of home and foreign bond from the perspective of home investors. The x-axis is the relative size of home country s. The dotted blue line (unconstrained) corresponds to the complete market model. The solid orange line (constrained) corresponds to the full model with two equity constraints. The three dotted vertical lines (from left to right) s^A, s^U, s^C represents the endogenous threshold for UIP changing sign in crisis regime, the boundary of crisis regime, and the threshold for UIP changing sign in normal regime, respectively. Parameter values used: $\eta = 4$, $\overline{\chi}^{F,H} = \overline{\chi}^{H,F} = 0.2$, $\sigma^2 = 0.04$.

ure 2.8 shows the expected return difference between domestic bond and foreign bond for home country investors.

In normal regime where $s^U < s_t < 1 - s^U$, there is only the familiar country size spillover effect: the larger country's bond is safer. If home country's size continues falling below s^U, the risk-sharing is limited by the equity holding constraints.

In crisis regime, investors in both countries are forced to hold more domestic risk and less foreign risk compared to the perfect risk-sharing scenario in normal regime. Because of limited risk-sharing, domestic bond becomes safer for domestic investors in crisis regime than in normal regime, as it is a better hedge for domestic risk.

equity rebalancing effect improves safety of the domestic bond for domestic investors in crisis regime while country size spillover effect improves safety of the larger country's bond. So in crisis regime, country size spillover effect competes with equity rebalancing effect for the smaller country's investors but collaborates for the larger country's investors.

If the smaller country's size falls in between s^A and s^U, equity rebalancing effect dominates country size spillover effect. The smaller country's domestic bond is safe for domestic investors. If the smaller country falls below s^A, country size spillover effect dominates equity rebalancing effect. The smaller country's domestic bond is risky for domestic investors.

Whereas for the larger country when $1 - s_t > s^C$, its domestic bond is always safe for domestic investors. The safety of the larger country's domestic bond is discontinuously strengthened when $1 - s_t > s^U$ due to equity rebalancing effect.

2.5.2.2 Domestic amplification

equity rebalancing amplifies the effect of domestic shock. This domestic amplification exists both in the "time series" (compared to normal regime) and in the "cross secrion" (compared to foreign country).

In the crisis regime $[0, s^U]$, the prices of risks for home country investors is given by

$$m_{H,t} = \frac{(1-\overline{\chi}^{H,F})s_t}{(1-\overline{\chi}^{H,F})s_t + \overline{\chi}^{H,F}}\sigma_{s_t} + \overline{\sigma}_t = \begin{bmatrix} \underbrace{\left[\frac{(1-\overline{\chi}^{H,F})}{(1-\overline{\chi}^{H,F})s_t + \overline{\chi}^{H,F}}\frac{\eta-1}{\eta}(1-s_t)+1\right] s_t\sigma_H}_{\geq s_t\sigma_H,\text{ normal time price of home country risk}} \\ \underbrace{\left[1 - \frac{(1-\overline{\chi}^{H,F})s_t}{(1-\overline{\chi}^{H,F})s_t + \overline{\chi}^{H,F}}\frac{\eta-1}{\eta}\right](1-s_t)\sigma_F}_{\leq (1-s_t)\sigma_F,\text{ normal time price of foreign country risk}} \end{bmatrix} \tag{2.27}$$

and the prices of risks for foreign country investors is given by

$$m_{F,t} = \sigma_{1-s_t} + \overline{\sigma}_t = \begin{bmatrix} \underbrace{\frac{1}{\eta}s_t\sigma_H}_{\leq s_t\sigma_H,\text{ normal time price of home country risk}} \\ \underbrace{\left[\frac{\eta-1}{\eta}s_t + (1-s_t)\right]\sigma_F}_{\geq (1-s_t)\sigma_F,\text{ normal time price of foreign country risk}} \end{bmatrix} \tag{2.28}$$

For home country investors, compared to in normal times, the effect of a shock on domestic risk price is amplified by the factor

$$\frac{(1-\overline{\chi}^{H,F})}{(1-\overline{\chi}^{H,F})s_t+\overline{\chi}^{H,F}}\frac{\eta-1}{\eta}(1-s_t)+1>1$$

and the effect of a shock on foreign risk price is mitigated by the factor

$$1-\frac{(1-\overline{\chi}^{H,F})s_t}{(1-\overline{\chi}^{H,F})s_t+\overline{\chi}^{H,F}}\frac{\eta-1}{\eta}<1$$

Similarly for foreign country investors, the effect of a shock on domestic risk price is amplified by the factor

$$\frac{\eta-1}{\eta}\frac{s_t}{1-s_t}+1>1$$

and the effect of a shock on foreign risk price is mitigated by the factor

$$\frac{1}{\eta}<1$$

In crisis regime, domestic risk price response to a shock is amplified and foreign risk price response to a shock is mitigated compared to in normal times due to lack of equity rebalancing. Domestic amplification improves the hedging benefit of domestic bond in crisis compared to in normal times.

In another dimension, comparing the prices of risks between home country investors and foreign country investors, we have

$$\left[\frac{(1-\overline{\chi}^{H,F})}{(1-\overline{\chi}^{H,F})s_t+\overline{\chi}^{H,F}}\frac{\eta-1}{\eta}(1-s_t)+1\right]s_t\sigma_H>s_t\sigma_H>\frac{1}{\eta}s_t\sigma_H$$

where the first term is domestic risk price for home country investors in crisis regime $\left[0, s^U\right]$, the second term is home country risk price in normal regime $\left[s^U, s^C\right]$, and the third term is foreign risk price for foreign country investors in crisis regime $\left[0, s^U\right]$. And similarly

$$\left[1 - \frac{(1-\overline{\chi}^{H,F})s_t}{(1-\overline{\chi}^{H,F})s_t + \overline{\chi}^{H,F}} \frac{\eta - 1}{\eta}\right](1-s_t)\sigma_F < (1-s_t)\sigma_F < \left[\frac{\eta-1}{\eta}s_t + (1-s_t)\right]\sigma_F$$

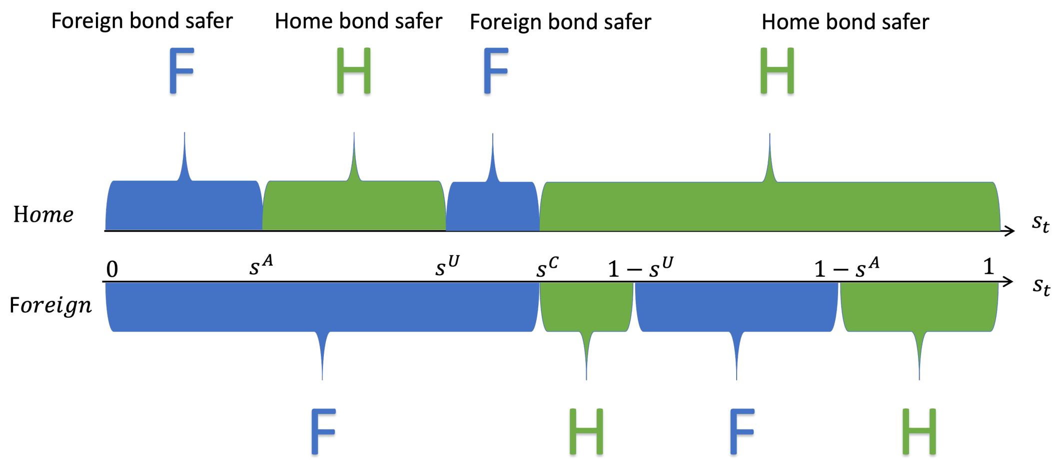
where the first term is foreign risk price for home country investors in crisis regime $\left[0, s^U\right]$, the second term is foreign country risk price in normal regime $\left[s^U, s^C\right]$, and the third term is domestic risk price for foreign country investors in crisis regime $\left[0, s^U\right]$. In crisis regime, domestic investors hold more domestic risk than foreign investors and thus require a higher risk premium than foreign investors. Domestic amplification drives up domestic risk price and pushes down foreign risk price in crisis for investors in both countries.

2.5.2.3 Domestic and global safety

In crisis regime, because of domestic amplification, domestic safety status of bonds may or may not coincide with global safety status of bonds. As shown in the following proposition 8, the smaller country's bond is domestically safe in mild crisis when country sizes are mildly asymmetric and the larger country's bond is globally safe in deep crisis when country sizes are sufficiently asymmetric.

Proposition 8: (Domestic and global safety) *Assume that home country is the smaller country, $s_t < s^C$. The smaller country's bond is domestically safe if $s_t \in [s^A, s^U]$.*

Figure 2.9: Global safety region of home and foreign bond

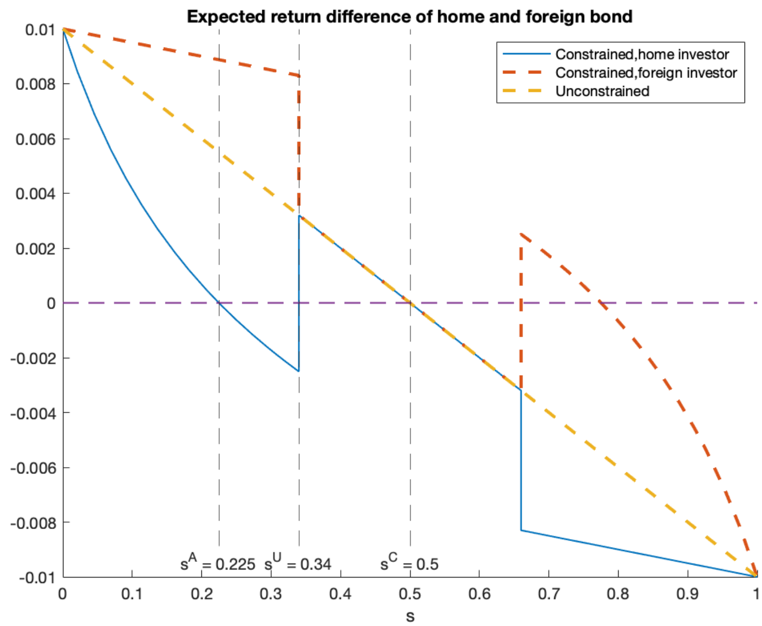


Note: This figure shows the safety region of home and foreign bond as the relative size of home country s_t changes, from the perspective of both home (the upper axis) and foreign investors (the lower axis). s^U is the boundary of crisis regime (constrained) as home country falls in relative country size, symmetrically for $1 - s^U$. s^C and s^A (symmetrically for $1 - s^A$) are the safety thresholds where UIP changes sign in normal regime (unconstrained) and crisis regime (constrained).

The larger country's bond is globally safe if and only if $s_t \in [0, s^A] \cup [s^U, s^C]$.

Proof: See appendix. ■

Figure 2.10: Expected return difference of home and foreign bond for home and foreign investors



Note: The y-axis of this figure shows the expected return difference of home and foreign bond. The x-axis is the relative size of home country s. The solid blue line (constrained, home investor) and the dotted orange line (constrained, foreign investors) shows the expected return difference of home and foreign bond in the full model (with two equity constraints) from the perspective of home investors and foreign investors, respectively. The dotted yellow line (unconstrained) shows the expected return difference of home and foreign bond in complete market model. And the three dotted vertical lines (from left to right) s^A, s^U, s^C represents the endogenous threshold for UIP changing sign in crisis regime, the boundary of crisis regime, and the threshold for UIP changing sign in normal regime, respectively. Parameter values used: $\eta = 4$, $\overline{\chi}^{F,H} = \overline{\chi}^{H,F} = 0.2$, $\sigma^2 = 0.04$.

In crisis regime, binding equity constraints drives a wedge between SDFs of home and foreign investors. Different pricing kernels result in different returns for the same asset. Investors in the two countries disagree on expected return difference between bonds in crisis, as shown in figure 2.10. Technically, the heterogeneity in SDFs comes from heterogeneity in constraints. The two countries face asymmetric complementary margin requirements on their equity holdings which results in asymmetric Lagrangian multipliers associated with the binding constraints.

For the smaller country, when falling into crisis, country size spillover effect competes with equity rebalancing effect. In mild crisis, equity rebalancing effect dominates country size spillover effect. The smaller country's bond becomes domestically safe. In deep crisis, country size spillover effect dominates equity rebalancing effect. The larger country's bond becomes safe for investors in the smaller country. For investors in the larger country, domestic bond is always safe and domestic bond safety status gets strengthened upon entering crisis regime, see the jumps in figure 2.10. So the larger country's bond is globally safe in normal times and in deep crisis, as shown in figure 2.9.

2.5.2.4 Market segmentation

Proposition 9: (Market segmentation) *In crisis regime, bond holdings of the two countries are given by*

$$\theta_t^{H,B^H} = \theta_t^{H,B^F} = 0, \quad \theta_t^{F,B^H} = \theta_t^{F,B^F} = 0$$

Proof: See appendix. ■

Forced by the constraints, investors in both countries hold more domestic risk than desired and would like to offload domestic risk to foreign investors. However, no such security is available because domestic amplification is resulted from dispersion in consumption prices (in terms of numeraire) of the two countries created by binding equity constraints and applies to any real asset. Real bond returns for investors bear the extra domestic risk coming from consumption price and no bond is held or traded between the two countries in crisis even though there is no friction in bond markets. Liquidity drains between the two countries in every asset market[15] and financial dichotoour emerges in crisis regime.

2.5.2.5 Non-linearity and systemic risk

Non-linearity The non-linearity in asset returns shows up for investors in the smaller country who hold both domestic equity and foreign equity. The non-linearity factor for home country is given by

$$\frac{(1-\overline{\chi}^{H,F})s_t}{(1-\overline{\chi}^{H,F})s_t+\overline{\chi}^{H,F}} \tag{2.29}$$

Taking derivative with respect to s_t, we have

$$\frac{\overline{\chi}^{H,F}(1-\overline{\chi}^{H,F})}{\left[(1-\overline{\chi}^{H,F})s_t+\overline{\chi}^{H,F}\right]^2}$$

where we see the non-linearity effect gets stronger when s_t is smaller, consistent with figure 2.8.

Systemic risk At crisis boundaries s^U and $1-s^U$, there are endogenous jumps between normal regime and crisis regime for both countries, which is the systemic risk in the model. The discontinuous change in expected return difference between sovereign bonds for home country investors at s^U (in absolute value),

[15] There is still trade happening between the two countries and potential trade in assets within domestic investors.

denoted as Δ_{11} is given by

$$\Delta_{11} = \frac{(1-\overline{\chi}^{H,F})s^U}{(1-\overline{\chi}^{H,F})s^U + \overline{\chi}^{H,F}} \frac{\eta - 1}{\eta^2}(1 - s^U)(\sigma_H^2 + \sigma_F^2) \tag{2.30}$$

and similarly denote Δ_{21} as the absolute change in expected return difference between sovereign bonds for foreign country investors at s^U, where

$$\Delta_{21} = \frac{\eta - 1}{\eta^2} s^U (\sigma_H^2 + \sigma_F^2) \tag{2.31}$$

Since $s^U < \frac{1-\overline{\chi}^{H,F}}{2(1-\overline{\chi}^{H,F})}$, we have

$$\Delta_{11} > \Delta_{21}$$

that is, the smaller country suffers greater systemic risk instability when it falls into crisis.

2.6 UIP reversal, flight-to-safety, CIP deviations and convenience yields

The model with crisis regime can explain several puzzles strongly associated with crisis periods: UIP reversal, flight to safety, CIP deviations and convenience yields.

First, in crisis regime, both countries price risks differently from normal regime. Binding equity constraints distort asset returns and asset safety in crisis, which explains the UIP reversal and flight to safety in crisis. Second, in crisis regime, the two countries disagree on prices of risks with each other, which explains the CIP deviations and convenience yields: in crisis, UK investors and US investors perceive different returns for exactly the same bond, the US Treasury. The gap between the actual return of US Treasury and the perceived return of US Treasury by foreign investors is also known as the CIP deviations or the convenience yield of the US Treasury, which is a signature of the 2008 Great Financial Crisis (GFC).

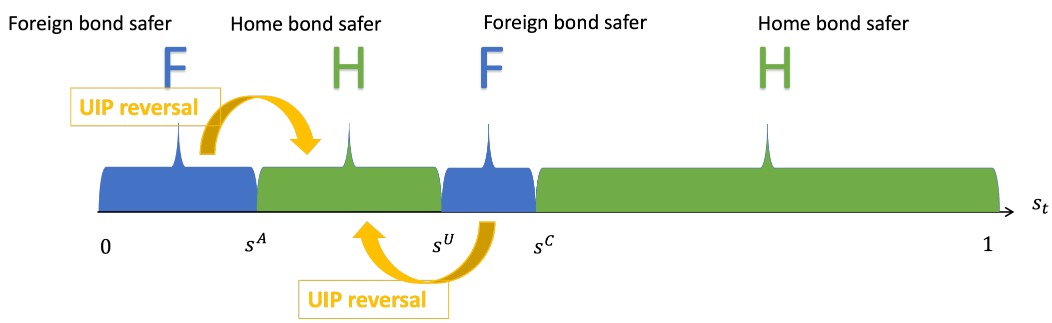
2.6.1 UIP reversal

UIP reversal is the opposite direction of UIP violation in normal times. As documented in Corsetti and Marin (2020), in normal times, US Treasury is a safer asset than UK government bond for UK investors and pays lower expected return as its safety premium. While when crisis hit, UK government bond pays lower expected return and becomes safer than US Treasury for UK investors.

Mapping into the model, in normal times, UK country size s_t is above the crisis boundary s^U but below the normal time safety threshold s^C. US Treasury is safer than UK government bond because of country size spillover effect. If UK economy or US economy suffers a rare loss, s_t falls into $[s^A, s^U]$ or rises to above s^C, UK government bond reverses to become a domestic safe bond and pays lower expected return for domestic investors compared to the US Treasury. In addition, if s_t rises to $[s^C, 1 - s^U]$ or $[1 - s^A, 1]$, UK government bond becomes a global safe bond. As shown in figure 2.11, UIP reversal between UK and US government bond happens when UK country size s_t falls into the left blue area $[0, s^A]$ from the green area $[s^A, s^U]$.

The model can also speak to emerging economies. For emerging economies whose initial country size is too small and is below s^A, if s_t rises into $[s^A, s^U]$ because the emerging economy grows larger due to its higher growth rate or the larger country, say US, falls into crisis due to rare losses, the emerging economy's domestic bond reverses from being the riskier bond to become the safer bond than US Treasury for domestic investors. As shown in figure 2.11, UIP reversal happens for emerging countries when their domestic country size s_t rises into the green area $[s^A, s^U]$ from the left blue area $[0, s^A]$.

Figure 2.11: UIP reversal



Note: This figure shows how UIP reversal happens as the relative size of home country s_t changes. s^A, s^U, s^C represents the endogenous threshold for UIP changing sign in crisis regime, the boundary of crisis regime, and the threshold for UIP changing sign in normal regime, respectively.

2.6.2 Flight to safety

In a flight-to-safety phenomena, investors demand safe assets and push down the return of safe assets in crisis. US Treasury and German bond are good examples for being the global safe asset in crisis and pay lower expected returns than in normal times.

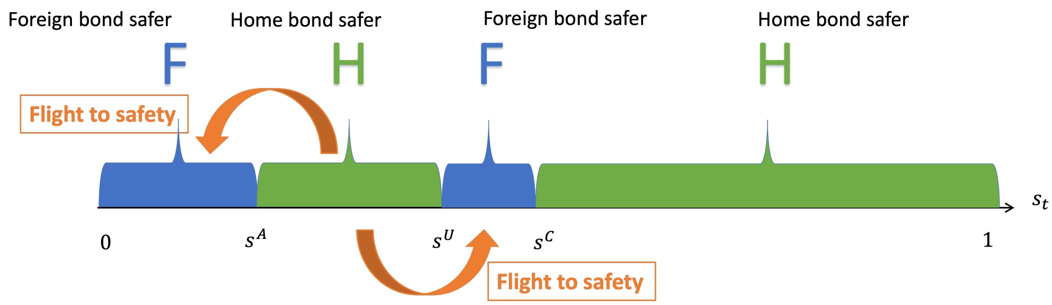
Mapping into the model, if UK country size s_t falls below s^U, the safety of US Treasury for US domestic investors is strengthened by equity rebalancing effect and US Treasury yield drops. If UK country size s_t falls below s^A, US Treasury is the safe asset for both UK investors and US investors, the global safe asset. The smaller UK country size s_t falls, the safer the US Treasury. Because country size spillover effect, which improves US Treasury safety, gets stronger with falling s_t. When UK country size s_t falls into crisis regime $[0, s^U]$, US investors find US Treasury even safer than in normal times and UK investors find US Treasury the safer bond than UK government bond if s_t falls below s^A.

In the case of the European debt crisis, German bond is the safe asset and pays historically low return. In the model, when periphery countries falls deep in crisis and their country size s_t drops below s^A, German bond is the global safe asset and investors are willing to accept an extra low return as the safety premium.

In a deep crisis, country size spillover effect dominates equity rebalancing effect and the larger country's bond is the global safe asset. From the point of view of investors in the smaller country, a flight-to-safety to the larger country's bond happens when s_t falls into the left blue area as shown in figure 2.12.

For emerging countries whose initial country size s_t is in the green area $[s^A, s^U]$, if US economy suffers from rare losses and s_t rises into blue area $[s^U, s^C]$, US Treasury becomes a global safe asset. As shown in figure 2.12, a flight-to-safety to the larger country's bond also happens when s_t rises into the blue area $[s^U, s^C]$ from the green area $[s^A, s^U]$.

Figure 2.12: Flight to safety



Note: This figure shows how flight-to-safety happens as the relative size of home country s_t changes. s^A, s^U, s^C represents the endogenous threshold for UIP changing sign in crisis regime, the boundary of crisis regime, and the threshold for UIP changing sign in normal regime, respectively.

2.6.3 CIP deviations and convenience yields

CIP deviations is a failure of the Law of One Price: assets with the same underlying dividend flow pay different returns. Government bond convenience yield is the return difference between risk-free rate and government bond yield. The relative convenience yield between sovereign bonds are often related to CIP deviations. deviations. Du, Im, and Schreger (2018a) studies the US Treasury premium which is defined as the relative convenience yield between US Treasury and other countries' government bonds by measuring CIP deviations between government bond yields. In our model, the larger country's bond enjoys a positive convenience yield relative to the smaller country's bond in crisis regime.

In crisis region, if home (G-10) investors want to borrow US dollar, they can not directly borrow from US dollar cash market with rate dr_t^{F,B^F} because of market segmentation. However, they can borrow domestic currency at rate dr^{H,B^H} and simultaneously enter a forward contract $-dr^{H,B^H} + dr^{H,B^F}$ to sell domestic currency for US dollar in the future. The implied US dollar rate from FX swap market (or the synthetic dollar rate) is thus dr^{H,B^F}.

Proposition 10: *The CIP condition is violated in crisis regime. The direct US dollar rate from cash market is lower than the synthetic dollar rate implied from FX swap market, that is*

$$\frac{\mathbb{E}_t\left[dr_t^{F,B^F} - dr^{H,B_t^F}\right]}{dt} = r_{F,t}^f - r_{H,t}^f - \frac{1}{1-n_t}\sigma_{n_t}\sigma_{e_t} < 0 \tag{2.32}$$

Proof: See appendix. ■

2.7 Financial development and trade elasticity

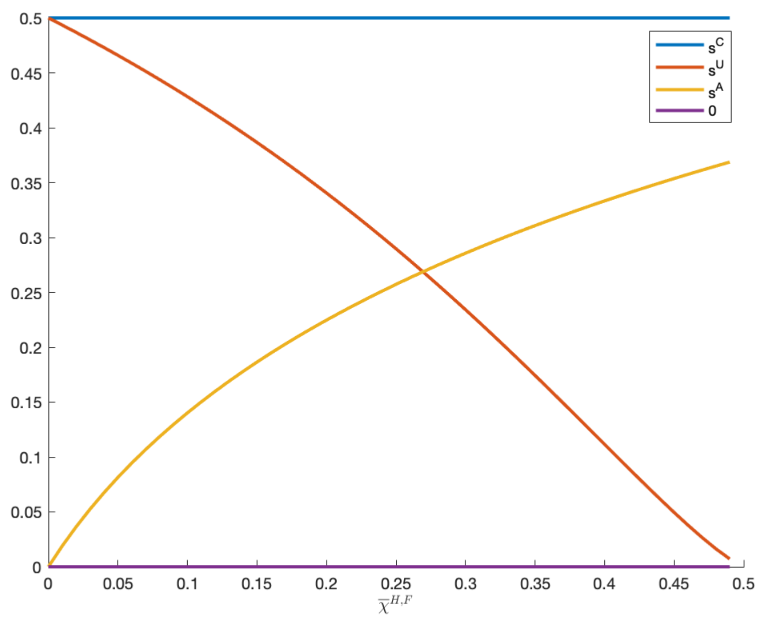
With a stable country size s_t, bond safety can also change with a shift of the safety spectrum due to changes in financial fiction parameters $1-\overline{\chi}^{F,H}$, $\overline{\chi}^{H,F}$, and trade elasticity η.

2.7.1 Financial development

As discussed in section 2.5.2, it is the foreign country's financial development that matters for bond safety when domestic country size shrinks and falls into crisis regime. In crisis regime $[0, s^U]$, a tightening of the larger country's equity holding constraint (i.e., a smaller $\overline{\chi}^{H,F}$), has impact on two safety thresholds, the crisis boundary s^U and crisis time safety threshold s^A, the non-linear domestic amplification for home country investors, and the systemic risk instability. Tighter equity constraint makes it harder for consumption smoothing and risk-sharing when there is asymmetry in country sizes, which shifts s^U to the right and expands the crisis regime. Chances of entering and the time spent in the crisis regime are increased. Meanwhile, a tighter constraint strengthens equity rebalancing effect and shifts crisis time safety threshold s^A to the left. The safety region of domestic bond in crisis regime is expanded due to increased hedging benefit of domestic bond.

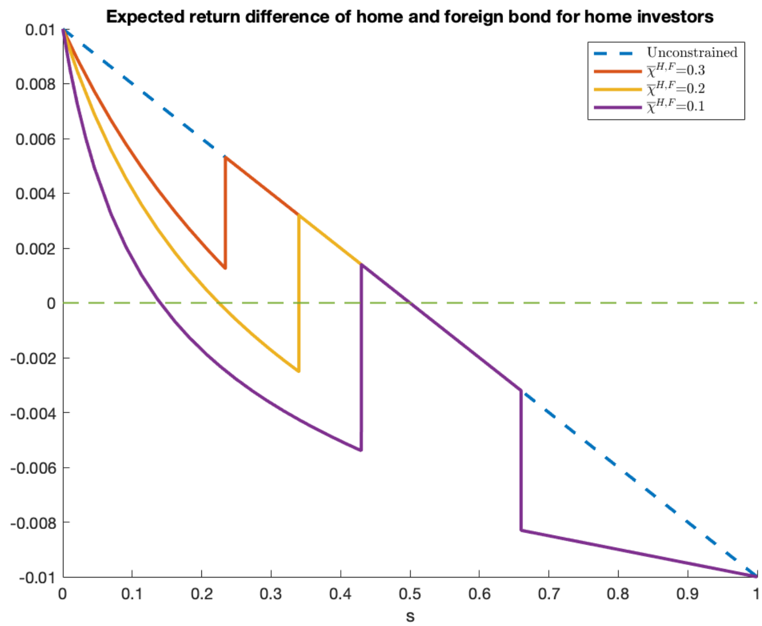
On the other hand, financial development (i.e., a larger $\overline{\chi}^{H,F}$) reduces the crisis regime coverage, as well as the safety region of domestic bond in crisis regime until the left threshold s^A exceeds the right threshold s^U. With a sufficient loose constraint, equity rebalancing effect is too weak to reverse country size spillover effect and foreign bond is still the safer bond in crisis regime for domestic investors. See equation (2.20), (2.25), (2.21) and figure 2.13. Financial development also matters for the non-linear domestic amplification effect and systemic risk instability, see figure 2.14. The larger country's financial development reduces systemic risk instability for both countries when the smaller country falls too small, see equation (2.30) and (2.31). And the domestic non-linear effect weakens with foreign financial development, see equation (2.29).

Figure 2.13: Changes of safety thresholds with respect to equity limit $\overline{\chi}^{H,F}$



Note: This figure shows how the safety thresholds s^C (UIP changing sign in normal regime, the blue line), s^U (boundary of crisis region, the orange line), and s^A (UIP changing sign in crisis regime, the yelow line) change with respect to equity limit $\overline{\chi}^{H,F}$. The y-axis of this figure is the relative size of home country s. The x-axis is the tightness of equity constraint $\overline{\chi}^{H,F}$ (the upper limit of foreign equity share held by home investors). The equity constraint is tighter with smaller $\overline{\chi}^{H,F}$. Parameter values used: $\eta = 4$, $\overline{\chi}^{F,H} = 0.2$, $\sigma^2 = 0.04$.

Figure 2.14: Changes of expected return difference of home and foreign bond with respect to equity limit $\overline{\chi}^{H,F}$



Note: The y-axis of this figure shows the expected return difference of home and foreign bond from the perspective of home investors. The x-axis is the relative size of home country s. The dotted blue line (unconstrained) corresponds to the complete market model. The solid lines corresponds to the full model with different tightness of equity constraint $\overline{\chi}^{H,F}$ (the upper limit of foreign equity share held by home investors). The equity constraint is tighter with smaller $\overline{\chi}^{H,F}$. Parameter values used: $\eta = 4$, $\overline{\chi}^{F,H} = 0.2$, $\sigma^2 = 0.04$.

2.8 Conclusion

This paper provides a theory of sovereign bond safety which is jointly determined by country size and equity rebalancing. Country size spillover effect improves the safety of the larger country's bond, which explains normal time UIP violation. Equity rebalancing, the equity holding constraints in the model, creates endogenous systemic risk instability between normal regime and crisis regime where domestic risk is amplified. The interaction between country size and equity rebalancing in crisis regime explains UIP reversal, flight to safety, sovereign bond CIP deviations and convenience yields at the same time.

Online Appendix

2.A Additional Proofs and Derivations

Trade market: Notice that international trade is a static problem for both countries. Since there is no friction in the international trade market and homogeneous preferences, we have that

$$\frac{C_{HH,t}}{C_{HF,t}} = \frac{C_{FH,t}}{C_{FF,t}} = \frac{Y_{H,t}}{Y_{F,t}} \tag{A.1}$$

Using market clearing condition for home good and foreign good,

$$C_{HH,t} + C_{FH,t} = Y_{H,t}$$
$$C_{HF,t} + C_{FF,t} = Y_{F,t}$$

we have

$$C_{H,t} + C_{F,t} = \overline{Y}_t$$

where

$$C_{H,t} = \left[a^{\frac{1}{\eta}} C_{HH,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} C_{HF,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

$$C_{F,t} = \left[a^{\frac{1}{\eta}} C_{FH,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} C_{FF,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

$$\overline{Y}_t = \left[a^{\frac{1}{\eta}} Y_{H,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} Y_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

As a result, the prices of the two goods produced by the two trees are given by

$$p_t^H = \left(\frac{Y_{H,t}}{a\overline{Y}_t}\right)^{-\frac{1}{\eta}} \quad \text{and} \quad p_t^F = \left(\frac{Y_{F,t}}{(1-a)\overline{Y}_t}\right)^{-\frac{1}{\eta}} \tag{A.2}$$

We have that

$$C_{H,t} = p_t^H C_{HH,t} + p_t^F C_{HF,t}$$

$$C_{F,t} = p_t^H C_{FH,t} + p_t^F C_{FF,t}$$

Recall that country size if defined as follows

$$s_t = \frac{a^{\frac{1}{\eta}} Y_{H,t}^{\frac{\eta-1}{\eta}}}{a^{\frac{1}{\eta}} Y_{H,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} Y_{F,t}^{\frac{\eta-1}{\eta}}} = a^{\frac{1}{\eta}} \left(\frac{Y_{H,t}}{\overline{Y_t}} \right)^{\frac{\eta-1}{\eta}}$$

as home country's share of the world total output, i.e. the *country size* of home country, which will turn out to be an important state variable. We have that

$$p_t^H Y_{H,t} = s_t \overline{Y_t} \quad \text{and} \quad p_t^F Y_{F,t} = (1-s_t)\overline{Y_t}$$

The aggregate wealth of home country is

$$W_t^H = \chi_t^{H,H} S_t^H + \chi_t^{H,F} S_t^F + p_t^H B_t^{H,H} + p_t^F B_t^{H,F}$$

The aggregate wealth of foreign country is

$$W_t^F = \chi_t^{F,H} S_t^H + \chi_t^{F,F} S_t^F + p_t^H B_t^{F,H} + p_t^F B_t^{F,F}$$

The optimization problem for home country is as follows:

$$\begin{aligned}
\max_{\{C_{HH,t}, C_{HF,t}, \chi_t^{H,H}, \chi_t^{H,f}, \theta_t^{H,B^H}, \theta_t^{H,B^F}\}_{t=0}^{\infty}} \quad & \mathbb{E}\left[\int_0^\infty e^{-\rho t} \log\left(\left[a^{\frac{1}{\eta}} C_{HH,t}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} C_{HF,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right) dt \right] \\
\text{s.t.} \quad \frac{dW_t^H}{W_t^H} &= \frac{\chi_t^{H,H} S_t^H}{W_t^H} dr_t^{S^H} + \frac{\chi_t^{H,F} S_t^F}{W_t^H} dr_t^{S^F} + \theta_t^{H,B^H} dr_t^{B^H} + \theta_t^{H,B^F} dr_t^{B^F} \\
&\quad - \frac{p_t^H C_{HH,t} + p_t^F C_{FF,t}}{W_t^H} dt \\
1 &= \frac{\chi_t^{H,H} S_t^H}{W_t^H} + \frac{\chi_t^{H,F} S_t^F}{W_t^H} + \theta_t^{H,B^H} + \theta_t^{H,B^F} \\
0 &\le \chi_t^{H,F} \le \overline{\chi}^{H,F}
\end{aligned} \tag{A.3}$$

Define home country's wealth share as

$$n_t = \frac{W_t^H}{W_t^H + W_t^F} \tag{A.4}$$

Lemma: Home country's wealth share $n_t = \frac{W_t^H}{W_t^H + W_t^F}$ is a function of country size s_t. And we always have that

$$p_t^H B_t^{H,H} + p_t^F B_t^{H,F} = 0 \tag{A.5}$$

Proof: From optimization problems, we have that

$$C_{H,t} = \rho W_t^H \quad \text{and} \quad C_{F,t} = \rho W_t^H$$

In the aggregate, we have that total consumption equals total output

$$\begin{aligned} C_{HH,t} + C_{FH,t} &= Y_{H,t} \\ C_{HF,t} + C_{FF,t} &= Y_{F,t} \end{aligned}$$

And using the result from trade market optimization, we have

$$C_{H,t} + C_{F,t} = \overline{Y_t}$$

That is,

$$\rho W_t^H + \rho W_t^F = \overline{Y_t}$$

The total wealth in the world is given by

$$W_t^H + W_t^F = \frac{\overline{Y_t}}{\rho}$$

There are two state variables, wealth share n_t and country size s_t. In equilibrium, all the prices and quantities must be functions of state variables n_t and s_t. We can rewrite home country's wealth as

$$p_t^H B_t^{H,H} + p_t^F B_t^{H,F} = [n_t - (\chi_t^{H,H} q_t^H + \chi_t^{H,F} q_t^F)]\overline{Y_t}$$

where $q_t^H = \frac{S_t^H}{\overline{Y}_t}$ and $q_t^F = \frac{S_t^F}{\overline{Y}_t}$ are per unit price of home and foreign equity, respectively. Recall that

$$p_t^H Y_{H,t} = s_t \overline{Y_t} \quad \text{and} \quad p_t^F Y_{F,t} = (1-s_t)\overline{Y_t}$$

we have

$$p_t^H = a^{\frac{1}{\eta-1}} s_t^{-\frac{1}{\eta-1}} \quad \text{and} \quad p_t^F = (1-a)^{\frac{1}{\eta-1}} (1-s_t)^{-\frac{1}{\eta-1}}$$

And in equilibrium $\theta_t^{H,B^H} = \frac{p_t^H B_t^{H,H}}{W_t^H}$, $\theta_t^{H,B^F} = \frac{p_t^F B_t^{H,F}}{W_t^H}$, q_t^H, q_t^F, $\chi_t^{H,H}$ and $\chi_t^{H,F}$ must be functions of state variables n_t and s_t. We have that

$$n_t = \frac{\chi_t^{H,H} q_t^H + \chi_t^{H,F} q_t^F}{1 - (\theta_t^{H,B^H} + \theta_t^{H,B^F})} \equiv f(s_t, n_t) \tag{A.6}$$

Equation (A.6) is an implicit function and we can solve for n_t as a function of s_t. Now we have that in equilibrium $\theta_t^{H,B^H} = \frac{p_t^H B_t^{H,H}}{W_t^H}$, $\theta_t^{H,B^F} = \frac{p_t^F B_t^{H,F}}{W_t^H}$, q_t^H, q_t^F, $\chi_t^{H,H}$ and $\chi_t^{H,F}$ must be functions of the *only* state variable s_t.
Recall the dynamic budget constraint of home country,

$$\begin{aligned}\frac{dW_t^H}{W_t^H} = \frac{\chi_t^{H,H} S_t^H}{W_t^H} dr_t^{S^H} + \frac{\chi_t^{H,F} S_t^F}{W_t^H} dr_t^{S^F} + \theta_t^{H,B^H} dr_t^{B^H} + \theta_t^{H,B^F} dr_t^{B^F} \\ - \frac{p_t^H C_{HH,t} + p_t^F C_{FF,t}}{W_t^H} dt\end{aligned}$$

And the asset return processes,

$$dr_t^{S^H} = \underbrace{\frac{p_t^H Y_{H,t}}{q_t^H \overline{Y}_t} dt}_{\text{dividend yield}} + \underbrace{\frac{d(q_t^H \overline{Y}_t)}{q_t^H \overline{Y}_t}}_{\text{capital gain}} = \mu_t^{S^H} dt + \left(\frac{(q^H)'(s_t)s_t}{q_t^H}\sigma_{s_t} + \overline{\sigma}_t\right) dZ_t$$

and similarly

$$dr_t^{S^F} = \underbrace{\frac{p_t^F Y_{F,t}}{q_t^F \overline{Y}_t} dt}_{\text{dividend yield}} + \underbrace{\frac{d(q_t^F \overline{Y}_t)}{q_t^F \overline{Y}_t}}_{\text{capital gain}} = \mu_t^{S^F} dt + \left(\frac{(q^F)'(s_t)s_t}{q_t^F}\sigma_{s_t} + \overline{\sigma}_t\right) dZ_t$$

$$dr_t^{B^H} = \frac{d(p_t^H B_t^H)}{p_{H,t} B_t^H} = (\mu_{p^H,t} + r_t^H)\, dt + \sigma_{p^H,t}\, dZ_t$$

$$dr_t^{B^F} = \frac{d(p_t^F B_t^F)}{p_{F,t} B_t^F} = (\mu_{p^F,t} + r_t^F)\, dt + \sigma_{p^F,t}\, dZ_t$$

And we have

$$\sigma_{p_t^H} = -\frac{1}{\eta - 1}\sigma_{s_t}$$

$$\sigma_{p_t^F} = \frac{s_t}{(\eta - 1)(1 - s_t)}\sigma_{s_t}$$

Since $\rho W_t^H = n_t \overline{Y_t}$, we have

$$\frac{dW_t^H}{W_t^H} = \mu_t^{W^H} dt + (\frac{n'(s_t)s_t}{n_t}\sigma_{s_t} + \overline{\sigma}_t)\, dZ_t$$

Note that

$$\overline{\sigma}_t = [s_t \sigma^H, (1 - s_t)\sigma^F]$$

$$\sigma_{s_t} = \frac{\eta - 1}{\eta}(1 - s_t)[\sigma^H, -\sigma^F]$$

are *linearly independent* for non-degenerate s_t. Matching terms for $\overline{\sigma}_t$, we must have that

$$\frac{\chi_t^{H,H} S_t^H}{W_t^H} + \frac{\chi_t^{H,F} S_t^F}{W_t^H} = 1 \tag{A.7}$$

That is

$$p_t^H B_t^{H,H} + p_t^F B_t^{H,F} = 0 \tag{A.8}$$

■

Corollary The total capital flow of home country induced by equity trade is given by

$$\begin{aligned} dQ_t^H = {} & S_t^H d\chi_t^{H,H} - \chi_t^{H,H}(p_t^H Y_{H,t})dt + S_t^F d\chi_t^{H,F} - \chi_t^{H,F}(p_t^F Y_{F,t})dt \\ & + d\chi_t^{H,H} dS^H + d\chi_t^{H,F} dS_t^F \end{aligned} \tag{A.9}$$

And such capital flow must be financed and absorbed by trading in bonds and consumption goods

$$dQ_t^H = (\theta_t^{B^{H,H}} dr_t^{B^H} + \theta_t^{B^{H,F}} dr_t^{B^F}) W_t^H - (p_t^H C_{HH,t} + p_t^F C_{HF,t}) dt \tag{A.10}$$

Similarly the total capital flow of foreign country induced by equity trade is given by

$$\begin{aligned} dQ_t^F &= S_t^H d\chi_t^{F,H} - \chi_t^{F,H}(p_t^H Y_{H,t})dt + S_t^F d\chi_t^{F,F} - \chi_t^{F,F}(p_t^F Y_{F,t})dt \\ &\quad + d\chi_t^{F,H} dS^H + d\chi_t^{F,F} dS_t^F \end{aligned} \tag{A.11}$$

and such capital flow must be financed and absorbed by trading in bonds and consumption goods

$$dQ_t^F = (\theta_t^{B^{F,H}} dr_t^{B^H} + \theta_t^{B^{F,F}} dr_t^{B^F}) W_t^F - (p_t^H C_{FH,t} + p_t^F C_{FF,t}) dt \tag{A.12}$$

Proof: This follows from Lemma 1. Since we have

$$W_t^H = \chi_t^{H,H} S_t^H + \chi_t^{H,F} S_t^F$$

Taking total differentiation on both sides, we have that

$$dW_t^H = \chi_t^{H,H} S_t^H \, dr_t^{S^H} + \chi_t^{H,F} S_t^F \, dr_t^{S^F} + dQ_t^H \tag{A.13}$$

Combining with dynamic budget constraint

$$\begin{aligned} \frac{dW_t^H}{W_t^H} &= \frac{\chi_t^{H,H} S_t^H}{W_t^H} dr_t^{S^H} + \frac{\chi_t^{H,F} S_t^F}{W_t^H} dr_t^{S^F} + \theta_t^{H,B^H} dr_t^{B^H} + \theta_t^{H,B^F} dr_t^{B^F} \\ &\quad - \frac{p_t^H C_{HH,t} + p_t^F C_{FF,t}}{W_t^H} dt \end{aligned}$$

We have that

$$dQ_t^H = (\theta_t^{B^{H,H}} dr_t^{B^H} + \theta_t^{B^{H,F}} dr_t^{B^F}) W_t^H - (p_t^H C_{HH,t} + p_t^F C_{HF,t}) dt$$

Similar proof for foreign country. ■

Lemma: In crisis region $s_t \in [0, s^u]$, we have that

$$\chi_t^{H,H} = 1, \quad \chi_t^{H,F} = \overline{\chi}^{H,F}$$

$$p_t^H B_t^{H,H} = p_t^F B_t^{H,F} = 0$$

Proof: At the crisis region boundary $s_t = s^u$, we have $\chi_t^{H,H} = 1$ and $\chi_t^{H,F} = \overline{\chi}^{H,F}$,

$$dQ_t^H = S_t^H d\chi_t^{H,H} - (p_t^H Y_{H,t})dt + S_t^F d\chi_t^{H,F} - \overline{\chi}^{H,F}(p_t^F Y_{F,t})dt + d\chi_t^{H,H} dS^H + d\chi_t^{H,F} dS_t^F$$

For *any* realization of $ds_t < 0$ at $s_t = s^u$, it must be that $d\chi_t^{H,H} \leq 0$ and $d\chi_t^{H,F} \leq 0$. To satisfy this, we must have $d\chi_t^{H,H}$ and $d\chi_t^{H,F}$ are deterministic for any realization of $ds_t < 0$ (thus, any $s < s^u$) at $s_t = s^u$.
Collecting terms for σ_{s_t} and $\overline{\sigma}_t$, we have

$$\begin{aligned} dQ_t^H - \mathbb{E}[dQ_t^H] = [(S_t^H d\chi_t^{H,H} + S_t^F d\chi_t^{H,F})\overline{\sigma}_t + \\ \cdots (S_t^H d\chi_t^{H,H} \frac{(q^H)'(s_t)s_t}{q_t^H} + S_t^F d\chi_t^{H,F} \frac{(q^F)'(s_t)s_t}{q_t^F})\sigma_{s_t})]\, dZ_t \end{aligned} \quad \text{(A.14)}$$

On the other side, we have

$$dQ_t^H = (\theta_t^{B^{H,H}} dr_t^{B^H} + \theta_t^{B^{H,F}} dr_t^{B^F})W_t^H - (p_t^H C_{HH,t} + p_t^F C_{HF,t})dt$$

which only consists of σ_{s_t} risk. As a result of matching terms for $\overline{\sigma}_t$, we have

$$S_t^H d\chi_t^{H,H} + S_t^F d\chi_t^{H,F} = 0$$

That is, $d\chi_t^{H,H} = d\chi_t^{H,F} = 0$. It follows that matching terms for σ_{s_t} on both sides should also be 0, and we have

$$-\theta_t^{B^{H,H}} + \frac{s_t}{1-s_t}\theta_t^{B^{H,F}} = 0$$

Combining with

$$\theta_t^{B^{H,H}} + \theta_t^{B^{H,F}} = 0$$

We have that in crisis region,

$$\theta_t^{B^{H,H}} = \theta_t^{B^{H,F}} = 0$$

■

While in crisis when $s_t \in [0, s^U]$, $P_t^H = (1 - \overline{\chi}^{H,F})s_t + \overline{\chi}^{H,F}$ and $P_t^F = (1 - \overline{\chi}^{H,F})(1 - s_t)$.

Because the two countries have different consumption prices which are non-degenerate stochastic processes, the same (real) bond corresponds to different return processes in the two countries. That is, real bond returns bear consumption price risks and real bonds can not help overcome consumption price deviations. As a result, any real bond will not be traded in the constrained equilibrium [16]. The equity holding constraints creates financial friction that can not be overcome by sovereign bonds.

With one equity constraint Recall the wealth of home country and its evolution

$$W_t^H = \chi_t^{H,H} S_t^H + \chi_t^{H,F} S_t^F + p_t^H B_t^{H,H} + p_t^F B_t^{F,F} \tag{A.15}$$

$$\frac{dW_t^H}{W_t^H} = \frac{\rho \chi_t^{H,H} q_t^H}{n_t} dr_t^{S^H} + \frac{\rho \chi_t^{H,F} q_t^F}{n_t} dr_t^{S^F} + \frac{p_t^H B_{1t}}{W_t^H} dr_t^{B_1} + \frac{p_t^F B_{2t}}{W_t^H} dr_t^{B_2} - \rho dt \tag{A.16}$$

Denote

$$\frac{dW_t^H}{W_t^H} = \mu_{W_t^H} dt + \sigma_{W_t^H} dZ_t \tag{A.17}$$

$$\frac{dW_t^F}{W_t^F} = \mu_{W_t^F} dt + \sigma_{W_t^F} dZ_t \tag{A.18}$$

$$\frac{dn_t}{n_t} = \mu_{n_t} dt + \sigma_{n_t} dZ_t \tag{A.19}$$

There are two risks in this world: the aggregate consumption risk, $\overline{\sigma}_t$, and the distribution risk, σ_{s_t}. Since there are four financial assets, there is some redundancy. With only one equity constraint, the two countries can still perfectly share consumption risk. So we have

$$\sigma_{n_t} = \sigma_{1-n_t} = 0 \tag{A.20}$$

and

$$\sigma_{W_t^H} = \sigma_{W_t^F} = \overline{\sigma}_t \tag{A.21}$$

To find the portfolio weights on each asset, we have

$$\overline{\sigma}_t = \frac{\rho \chi_t^{H,H} q_t^H}{n_t} (\sigma_{q_t^H} + \overline{\sigma}_t) + \frac{\rho \chi_t^{H,F} q_t^F}{n_t} (\sigma_{q_t^F} + \overline{\sigma}_t) + \frac{p_t^H B_t^{H,H}}{W_t^H} \sigma_{p_t^H} + \frac{p_t^F B_t^{H,F}}{W_t^H} \sigma_{p_t^F} \tag{A.22}$$

[16] The symmetric setting in discount rate and preferences matters. (conjecture)

Now we need to find $\sigma_{q_t^H}$. In the complete market case, we have

$$q_t^H = \mathbb{E}_t\Big[\int_0^\infty e^{-\rho\tau} s_\tau d\tau\Big] \tag{A.23}$$

Using Ito's lemma we have

$$\sigma_{q_t^H} = \frac{(q^H)'(s_t)s_t}{q_t^H}\sigma_{s_t} \tag{A.24}$$

and since

$$q_t^F = \frac{1}{\rho} - q_t^H \tag{A.25}$$

we have

$$\sigma_{q_t^F} = -\frac{q_t^H}{q_t^F}\sigma_{q_t^H} = -\frac{(q^H)'(s_t)s_t}{q_t^F}\sigma_{s_t} \tag{A.26}$$

And we have

$$\sigma_{p_t^H} = -\frac{1}{\eta - 1}\sigma_{s_t} \tag{A.27}$$

$$\sigma_{p_t^F} = \frac{s_t}{(\eta - 1)(1 - s_t)}\sigma_{s_t} \tag{A.28}$$

Note that

$$\overline{\sigma}_t^T = [s_t\sigma_1, (1 - s_t)\sigma_2] \tag{A.29}$$

$$\sigma_{s_t}^T = \frac{\eta - 1}{\eta}(1 - s_t)[\sigma_1, -\sigma_2] \tag{A.30}$$

are *linearly independent*. Now coming back to the risk of home country's wealth (A.22) and matching $\overline{\sigma}_t$ term, we have

$$\frac{\rho\chi_t^{H,H}q_t^H}{n_t} + \frac{\rho\chi_t^{H,F}q_t^F}{n_t} = 1 \tag{A.31}$$

That is

$$W_t^H = \chi_t^{H,F}S_t^H + \chi_t^{H,F}S_t^F \tag{A.32}$$

and thus

$$p_t^H B_t^{H,H} + p_t^F B_t^{H,F} = 0 \tag{A.33}$$

To determine portfolio weights on bonds, we match σ_{s_t} terms in home country's wealth

$$\frac{\rho\chi_t^{H,H}q_t^H}{n_t}\frac{(q^H)'(\frac{s_t}{n_t})s_t}{q_t^H}+\frac{\rho\chi_t^{H,F}q_t^F}{n_t}(-\frac{(q^H)'(s_t)s_t}{q_t^F})$$
$$\cdots+\frac{p_t^HB_t^{H,H}}{W_t^H}(-\frac{1}{\eta-1}-\frac{s_t}{(\eta-1)(1-s_t)})=0 \quad \text{(A.34)}$$

Simplified to

$$\frac{\rho(q^H)'(s_t)s_t(\chi_t^{H,H}-\chi_t^{H,F})}{n_t}-\frac{p_t^HB_t^{H,H}}{W_t^H}\frac{1}{(1-s_t)(\eta-1)}=0 \quad \text{(A.35)}$$

So we have

$$\frac{p_t^HB_t^{H,H}}{W_t^H}=\frac{\rho\chi_t^{H,H}(q^H)'(s_t)s_t(1-s_t)(\chi_t^{H,H}-\chi_t^{H,F})(\eta-1)}{n_t}>0 \quad \text{(A.36)}$$

and

$$\frac{p_t^FB_t^{H,F}}{W_t^H}=-\frac{p_t^HB_t^{H,H}}{W_t^H}<0 \quad \text{(A.37)}$$

We can also find the drift of home country wealth share n_t by looking at the drift term of the wealth. Using market clearing condition $B_t^{H,H}=-B_t^{F,H}$ and $B_t^{H,F}=-B_t^{F,F}$,

$$\mu_{W_t^F}=-\frac{n_t}{1-n_t}\mu_{n_t}+\overline{\mu}_t \quad \text{(A.38)}$$
$$=\frac{\mathbb{E}_t[dr_t^{S^F}]}{dt}-\frac{n_t}{1-n_t}\frac{p_t^HB_t^{H,H}}{W_t^H}m_t\sigma_{e_t}^T-\rho \quad \text{(A.39)}$$

and we have that

$$\mu_{q_t^H}=\rho-\frac{s_t}{q_t^H}+\mu_{n_t}-\sigma_{n_t}^2+\sigma_{q_t^H}\sigma_{n_t} \quad \text{(A.40)}$$

and $\sigma_{n_t}=0$,

$$\mu_{q_t^H}=\rho-\frac{s_t}{q_t^H}+\mu_{n_t} \quad \text{(A.41)}$$

and thus

$$\frac{\mathbb{E}_t[dr_t^{S^F}]}{dt} = \frac{1-s_t}{q_t^F} + \left(-\frac{q_t^H}{q_t^F}\mu_{q_t^H} + \overline{\mu}_t + \sigma_{q_t^F}\overline{\sigma}_t^T\right) \tag{A.42}$$

Substituting into (A.38), and solving for μ_{n_t}, we have

$$\mu_{n_t} = \frac{\frac{q_t^H}{q_t^F}\sigma_{q_t^H}\overline{\sigma}_t^T + \frac{n_t\theta_t^{H,B^H}}{1-n_t}\sigma_{e_t}\overline{\sigma}_t^T}{\frac{n_t}{1-n_t} - \frac{q_t^H}{q_t^F}} = 0 \tag{A.43}$$

as

$$\frac{q_t^H}{q_t^F}\sigma_{q_t^H}\overline{\sigma}_t^T = -\frac{(q^H)'(\frac{s_t}{n_t})s_t(1-s_t)(\eta-1)}{q_t^F}\sigma_{e_t}\overline{\sigma}_t^T = \frac{n_t\theta_t^{H,B^H}}{1-n_t}\sigma_{e_t}\overline{\sigma}_t^T \tag{A.44}$$

With two equity holding constraints The second step is to explore what will happen with two equity holding constraints. Now the two countries can not always perfectly share consumption risk and the wealth shares are not always constant.

Full model special case: with two equity holding constraints $0 \le \chi_t^{H,F} \le \overline{\chi}^{H,F}$, $0 \le \chi_t^{F,H} \le \overline{\chi}^{F,H}$, and symmetric parameters $\sigma_1 = \sigma_2 = \sigma$ there are three safety thresholds s^c, s^u and s^a, $n_0 = \frac{1}{2}$:

$$s^c = \frac{1}{2} \tag{A.45}$$

$$q_1(s^u) = \frac{1-2\overline{\chi}^{H,F}}{2\rho(1-\overline{\chi}^{H,F})} \tag{A.46}$$

and

$$0 < s^a < s^u \tag{A.47}$$

with parameter restrictions on $(\overline{\chi}^{H,F}, \eta)$. The equity shares are

$$\chi_{1t} = 1 \tag{A.48}$$

and

$$\chi_t^{H,F} = \begin{cases} \overline{\chi}^{H,F} & if \quad s_t < s^U(\overline{\chi}^{H,F}) \\ \frac{1-2\rho q_t^H}{2(1-\rho q_t^H)} & if \quad s^U(\overline{\chi}^{H,F}) < s_t < \frac{1}{2} \\ 0 & if \quad s_t > \frac{1}{2} \end{cases} \tag{A.49}$$

bond holdings are (in equilibrium)

$$\theta_t^{H,B^H} = \theta_t^{H,B^F} = \theta_t^{F,B^H} = \theta_t^{F,B^F} = 0 \tag{A.50}$$

where

$$\frac{q_t^{H,H}}{n_t} = \mathbb{E}_t\Big[\int_0^\infty e^{-\rho\tau}\frac{s_\tau}{n_\tau}d\tau\Big] \tag{A.51}$$

Proof: The equity holding constraint binds when $s_t < s^u$. There are no cross-border bond trading. The implied bond returns (within domestic country) are given as follows

$$\frac{\mathbb{E}_t[dr_t^{B^{H,H}}]}{dt} - r_t^{H,f} = m_{1t}^T\sigma_{p_t^H} = (\sigma_{n_t} + \overline{\sigma}_t)\sigma_{p_t^H} \tag{A.52}$$

$$\frac{\mathbb{E}_t[dr_t^{B^{H,F}}]}{dt} - r_t^{H,f} = m_{1t}^T\sigma_{p_t^F} = (\sigma_{n_t} + \overline{\sigma}_t)\sigma_{p_t^F} \tag{A.53}$$

$$\frac{\mathbb{E}_t[dr_t^{B^{F,H}}]}{dt} - r_t^{F,f} = m_{2t}^T\sigma_{p_t^H} = (-\frac{n_t}{1-n_t}\sigma_{n_t} + \overline{\sigma}_t)\sigma_{p_t^H} \tag{A.54}$$

$$\frac{\mathbb{E}_t[dr_t^{B^{F,F}}]}{dt} - r_t^{F,f} = m_{2t}^T\sigma_{p_t^F} = (-\frac{n_t}{1-n_t}\sigma_{n_t} + \overline{\sigma}_t)\sigma_{p_t^F} \tag{A.55}$$

where $-r_t^{H,f}$ and $-m_{1t}$ are the drift and volatility of home country's SDF and similarly $-r_t^{F,f}$ and $-m_{2t}$ are for foreign country.

$$r_t^{H,f} = \rho + \mu_{n_t} + \overline{\mu}_t + \sigma_{n_t}\overline{\sigma}_t - (\sigma_{n_t} + \overline{\sigma}_t)^2 \tag{A.56}$$

$$r_t^{F,f} = \rho - \frac{n_t}{1-n_t}\mu_{n_t} + \overline{\mu}_t - \frac{n_t}{1-n_t}\sigma_{n_t}\overline{\sigma}_t - (-\frac{n_t}{1-n_t}\sigma_{n_t} + \overline{\sigma}_t)^2 \tag{A.57}$$

$$m_{1t} = \sigma_{n_t} + \overline{\sigma}_t \tag{A.58}$$

$$m_{2t} = \sigma_{1-n_t} + \overline{\sigma}_t \tag{A.59}$$

When $s_t < s(\overline{\chi}^{H,F})$, we can write out Country 1 and Country 2's wealth as

$$W_t^H = S_t^{H,F} + \overline{\chi}^{H,F} S_t^{H,F} \tag{A.60}$$

$$W_t^F = (1 - \overline{\chi}^{H,F}) S_t^{F,F} \tag{A.61}$$

And from the optimization of logarithmic utility, we have

$$\rho W_t^H = C_{H,t} = n_t \overline{Y}_t \tag{A.62}$$

$$\rho W_t^F = C_{F,t} = (1 - n_t)\overline{Y}_t \tag{A.63}$$

Looking at the volatility of W_t^H,

$$\sigma_{W_t^H} = \sigma_{n_t} + \overline{\sigma}_t \tag{A.64}$$

$$= \frac{\rho q_t^{H,H}}{n_t}(\sigma_{q_t^{H,H}} + \overline{\sigma}_t) + \frac{\rho \overline{\chi}^{H,F} q_t^{H,F}}{n_t}(\sigma_{q^{H,F}} + \overline{\sigma}_t) \tag{A.65}$$

and the volatility of W_t^F,

$$\sigma_{W_t^F} = -\frac{n_t}{1 - n_t}\sigma_{n_t} + \overline{\sigma}_t \tag{A.66}$$

$$= \frac{\rho(1 - \overline{\chi}^{H,F}) q_t^{F,F}}{1 - n_t}(\sigma_{q_t^{F,F}} + \overline{\sigma}_t) \tag{A.67}$$

For both countries, portfolio weights add up to 1

$$\frac{\rho q_t^{H,H}}{n_t} + \frac{\rho \overline{\chi}^{H,F} q_t^{H,F}}{n_t} = 1 \tag{A.68}$$

$$\frac{\rho(1 - \overline{\chi}^{H,F}) q_t^{F,F}}{1 - n_t} = 1 \tag{A.69}$$

so we have

$$\frac{\rho q_t^H}{n_t} + \frac{\rho \overline{\chi}^{H,F} q_t^{H,F}}{n_t} + \frac{\rho(1 - \overline{\chi}^{H,F}) q_t^{F,F}}{1 - n_t} = 2 \tag{A.70}$$

where

$$\frac{q_t^{H,H}}{n_t} = \mathbb{E}_t\Big[\int_0^\infty e^{-\rho\tau}\frac{s_\tau}{n_\tau}d\tau\Big] \tag{A.71}$$

$$\frac{q_t^{H,F}}{n_t} = \mathbb{E}_t\Big[\int_0^\infty e^{-\rho\tau}\frac{1-s_\tau}{n_\tau}d\tau\Big] \tag{A.72}$$

$$\frac{q_t^{F,F}}{1-n_t} = \mathbb{E}_t\Big[\int_0^\infty e^{-\rho\tau}\frac{1-s_\tau}{1-n_\tau}d\tau\Big] \tag{A.73}$$

That is,

$$\mathbb{E}_t\Big[\int_0^\infty e^{-\rho\tau}(\frac{s_\tau+\overline{\chi}^{H,F}(1-s_\tau)}{n_\tau}+\frac{(1-\overline{\chi}^{H,F})(1-s_\tau)}{1-n_\tau})d\tau\Big]=\frac{2}{\rho} \tag{A.74}$$

Using Feynman-Kac formula, we have

$$\frac{s_t+\overline{\chi}^{H,F}(1-s_t)}{n_t}+\frac{(1-\overline{\chi}^{H,F})(1-s_t)}{1-n_t}=2 \tag{A.75}$$

So n_t is a function of the only state variable s_t (the other solution $n_t = \frac{1}{2}$ is not achievable with equity constraint binding).

$$n_t = (1-\overline{\chi}^{H,F})s_t + \overline{\chi}^{H,F} \tag{A.76}$$

$$1-n_t = (1-\overline{\chi}^{H,F})(1-s_t) \tag{A.77}$$

$$\theta_t^{H,B^H} = \theta_t^{H,B^F} = 0 \tag{A.78}$$

And also we have

$$\frac{C_{H,t}}{C_{F,t}} = \frac{n_t}{1-n_t} \tag{A.79}$$

Now we can solve for prices of risks,

$$m_{1t} = \sigma_{n_t} + \overline{\sigma}_t = \frac{(1-\overline{\chi}^{H,F})s_t}{(1-\overline{\chi}^{H,F})s_t+\overline{\chi}^{H,F}}\sigma_{s_t} + \overline{\sigma}_t \tag{A.80}$$

and solve for s^a using

$$\frac{\mathbb{E}_t[dr_t^{B^{H,H}} - dr_t^{B^{H,F}}]}{dt} = m_{1t}(\sigma_{p_t^H} - \sigma_{p_t^F}) < 0 \tag{A.81}$$

we have

$$\frac{2(1-\overline{\chi}^{H,F})s^2 + (2\overline{\chi}^{H,F}\eta + (1-\overline{\chi}^{H,F})(\eta-2))s - \eta\overline{\chi}^{H,F}}{(1-\overline{\chi}^{H,F})s_t + \overline{\chi}^{H,F}} > 0 \tag{A.82}$$

For $0 < s^a < s^u$, we need the right range for parameter pair $(\overline{\chi}^{H,F}, \eta)$. For example, if $\overline{\chi}^{H,F} = 0$, we have $s^u = \frac{1}{2}$ and $s^a = \frac{2-\eta}{2}$. And if $\eta = \infty$, we have $s^a = \frac{\overline{\chi}^{H,F}}{1+\overline{\chi}^{H,F}}$.

With a special parameter case, $\rho = (\frac{\eta-1}{\eta}\sigma)^2$, we can solve everything we need analytically. The price of equity 1 in unconstrained case is

$$q_t^H(s) = \frac{1}{2\rho}\Big(1 + \frac{1-s}{s}\ln(1-s) - \frac{s}{1-s}\ln(s)\Big) \tag{A.83}$$

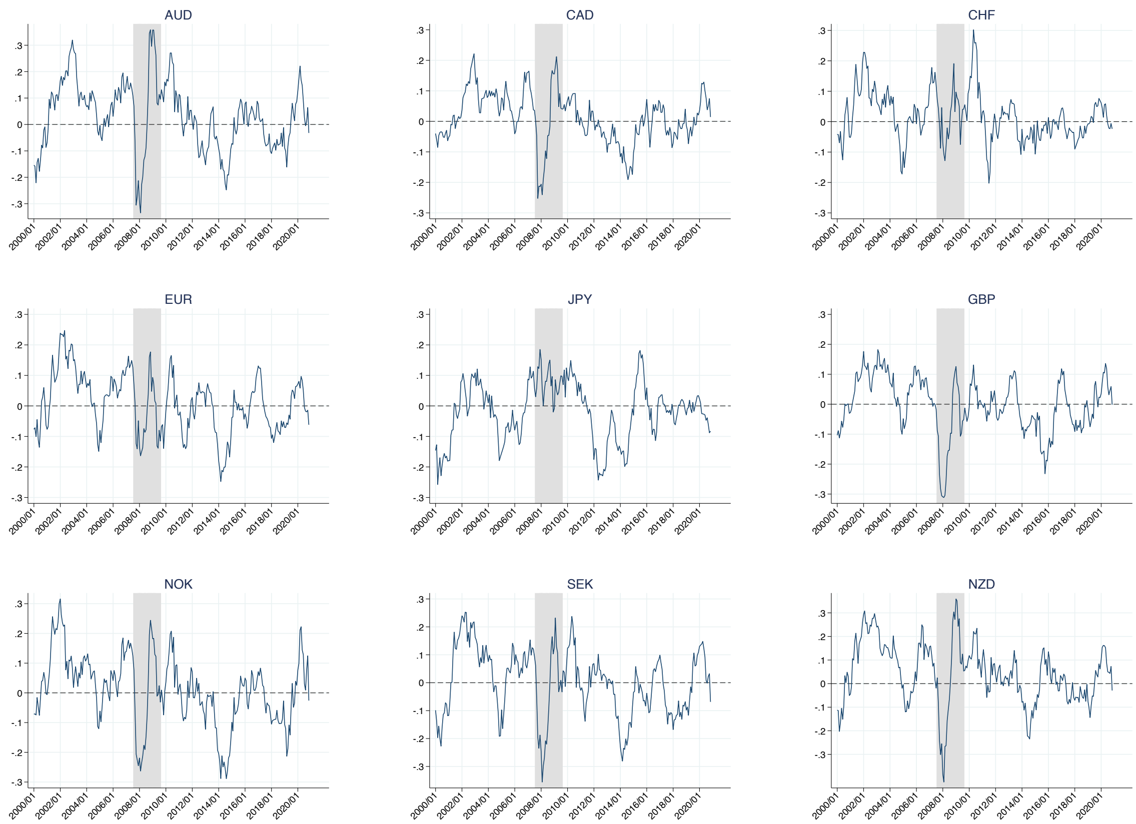
$$q'_{1t}(s) = -\frac{1}{2\rho}\frac{1}{s(1-s)}\Big(1 + \frac{1-s}{s}\ln(1-s) + \frac{s}{1-s}\ln(s)\Big) \tag{A.84}$$

$$q''_{1t}(s) = -\frac{1}{2\rho}\frac{1}{s^2(1-s)^2}\Big((2s-1) - \frac{(1-s)^2}{s}\ln(1-s) + \frac{s^2}{1-s}\ln(s)\Big) \tag{A.85}$$

■

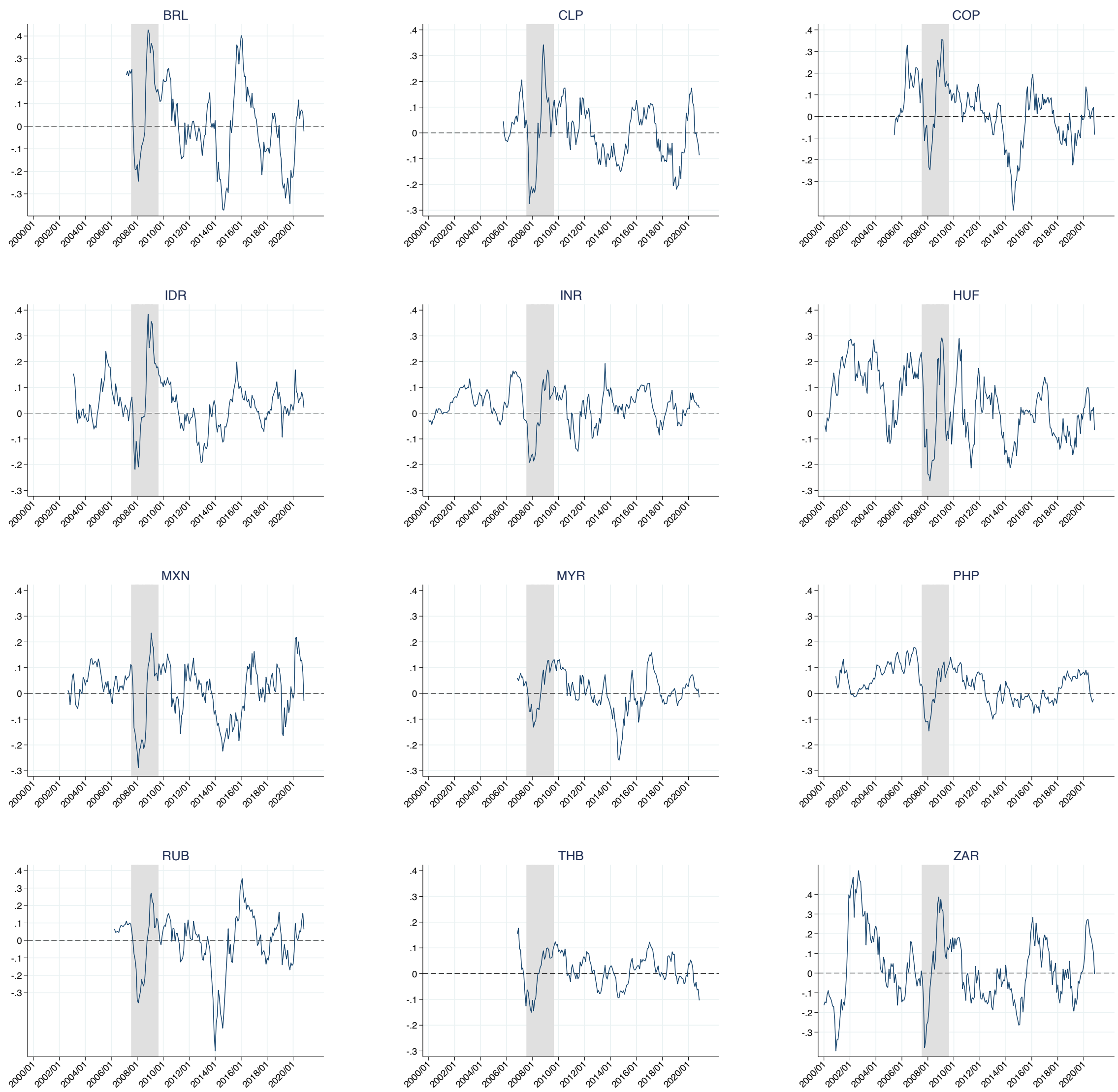
2.B Additional Graphs and Tables

Table 2.B.1: Annualized UIP Premium for G10 Currencies



Note: This panel of figures present the pattern of UIP premiums for G-10 currencies over the 2000 - 2021 sample. The UIP premiums are defined as the annualized excess returns of local-currency one-year government bond yields against the synthetic USD yields and are in log points. The grey area corresponds to the months of the Great Financial Crisis in 2008.

Table 2.B.2: Annualized UIP Premium for EME currencies



Note: This panel of figures present the pattern of UIP premiums for emerging market economies over the 2000 - 2021 sample. The UIP premiums are defined as the annualized excess returns of local-currency one-year government bond yields against the synthetic USD yields and are in log points. The grey area corresponds to the months of the Great Financial Crisis in 2008.

Table 2.B.3: G10-currency annualized UIP on relative country size

	(1) Full Sample	(2) Year $<$ 2008	(3) Year $>$ 2008
Share of GDP over World	-0.901***	-0.655***	-0.999***
	(0.106)	(0.209)	(0.160)
Constant	0.118***	0.123***	0.108***
	(0.0126)	(0.0249)	(0.0190)
Country + Year FE	yes	yes	yes
Observations	2257	864	1393
R^2	0.4331	0.4555	0.3733
Adjusted R^2	0.426	0.445	0.364

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table presents the OLS results for monthly UIP deviations on the relative size of the country to the “world GDP" (defined as sum of the country and the U.S.). The UIP deviations are annualized and defined in excess returns in local currency against the USD and measured in log points. The relative country size of each country is computed as its GDP over the sum with the U.S. GDP. Column (1) covers the full sample at monthly frequency from 2000 to 2021; column (2) uses the sub-sample before the 2008 financial crisis and column (3) uses the post-crisis sub-sample. All regressions include year and country fixed effects.

Table 2.B.4: EME-currency annualized UIP on the relative country size

	(1) Full Sample	(2) Year < 2008	(3) Year > 2008
Share of GDP over World	0.480**	-2.225**	-0.114
	(0.219)	(1.057)	(0.316)
Constant	0.000862	0.132***	0.0102
	(0.00919)	(0.0322)	(0.0142)
Country + Year FE	yes	yes	yes
Observations	1640	416	1224
R^2	0.2922	0.2638	0.2921
Adjusted R^2	0.280	0.236	0.280

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table presents the OLS results for monthly UIP deviations on the relative size of the country to the "world GDP" (defined as sum of the country and the U.S.). The UIP deviations are annualized and defined in excess returns in local currency against the USD and measured in log points. The relative country size of each country is computed as its GDP over the sum with the U.S. GDP. Column (1) covers the full sample at monthly frequency from 2000 to 2021; column (2) uses the sub-sample before the 2008 financial crisis and column (3) uses the post-crisis sub-sample. All regressions include year and country fixed effects.

Table 2.B.5: G10-currency annualized UIP on within-G10-group country size

	(1) Full Sample	(2) Year $<$ 2008	(3) Year $>$ 2008
Within-G10 GDP Share (%)	-1.072***	-0.665***	-2.195***
	(0.108)	(0.156)	(0.267)
Constant	11.88***	11.32***	20.95***
	(1.094)	(1.582)	(2.679)
Country + Year FE	yes	yes	yes
Observations	2508	960	1548
R^2	0.4408	0.4632	0.3796
Adjusted R^2	0.434	0.453	0.371

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table presents the OLS results for monthly UIP deviations on the relative size of the country (measured by nominal GDP) *within* the G-10 currency group. The UIP deviations are annualized and defined in excess returns in local currency against the USD and measured in log points. The relative country size of each country is in percentage points and computed as its share over the total nominal GDP of all G10 countries (other than the US). Column (1) covers the full sample at monthly frequency from 2000 to 2021; column (2) uses the sub-sample before the 2008 financial crisis and column (3) uses the post-crisis sub-sample. All regressions include year and country fixed effects.

Table 2.B.6: EME-currency annualized UIP on within-EME-group country size

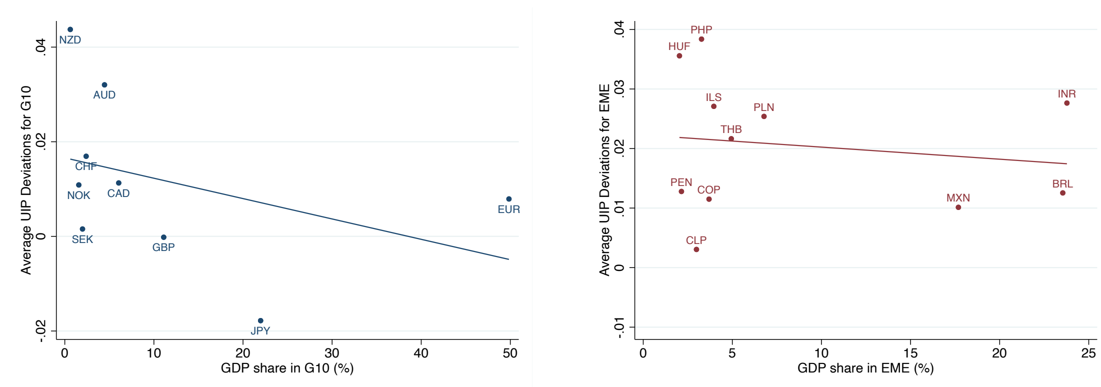
	(1) Full Sample	(2) Year $<$ 2008	(3) Year $>$ 2008
Within-EME GDP Share (%)	0.0732	-0.598	0.0400
	(0.0865)	(0.377)	(0.117)
Constant	1.374*	13.18***	0.0162
	(0.832)	(3.718)	(1.096)
Country + Year FE	yes	yes	yes
Observations	2206	533	1673
R^2	0.2977	0.2334	0.2906
Adjusted R^2	0.288	0.207	0.281

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

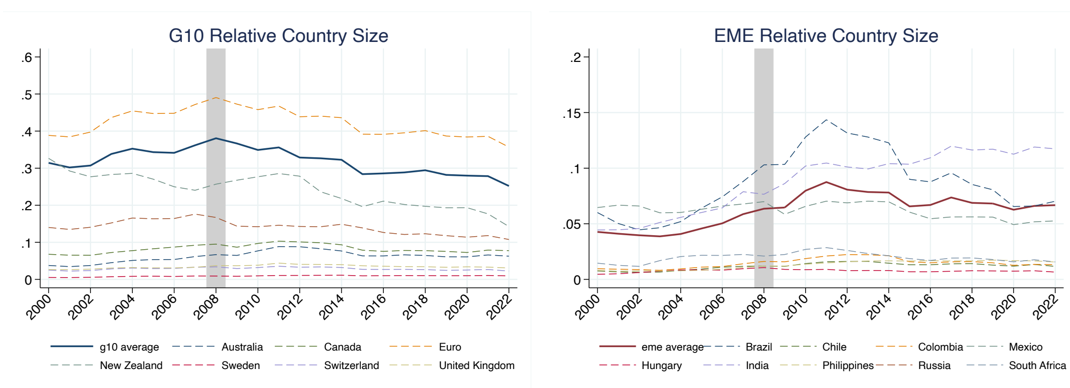
Note: This table presents the OLS results for monthly UIP deviations on the relative size of the country (measured by nominal GDP) in the EME group. The UIP deviations are annualized and defined in excess returns in local currency against the USD and measured in log points. The relative country size of each country is in percentage points and computed as its share over the total nominal GDP of all EME countries. Column (1) covers the full sample at monthly frequency from 2000 to 2021; column (2) uses the sub-sample before the 2008 financial crisis and column (3) uses the post-crisis sub-sample. All columns include year and country fixed effects in the OLS.

Table 2.B.8: Average UIP Premium and Country Size Share within the G10 (or EME) Group



Note: This panel of figures present the scatter plot of average UIP premium in our sample against the average country size share (measured by nominal GDP) for each currency in their respective G10 currency group (left) or EME group (right). UIP deviations are in log points and are annualized and averaged across time for the sample period of 2000 to 2021. Each dot represent a currency labeled by its currency name.

Table 2.B.7: Relative G10 (or EME) Country Size



Note: This panel of figures present the ratio of nominal GDP of a country over the sum of GDP of the country and of the U.S. The figure for G10 currency countries are on the left and emerging market economies are on the right. Each line represents the country-specific GDP ratio. The bolded line is the weighted average ratio for the G10 group (left) and the EME group (right).

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CHAPTER 3

“Too-Little" Sovereign Debt Restructurings

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3.1 Introduction

Sovereign debt restructurings have often been both “too late” and “too little” (IMF, 2013). Unsustainable debt situations often fester years before they are resolved (“too late”); when restructurings do occur, they often do not restore debt sustainability in a durable manner and result in repeated restructurings (“too little”). Several papers have explored the “too late” problem through modeling the frictions that cause delays in debt renegotiations between the sovereign and foreign creditors (Benjamin and Wright, 2009; Bai and Zhang, 2012; Asonuma and Joo, 2020).

However, the literature on sovereign debt has yet to provide a clear mechanism that explains the “too-little” problem in sovereign debt restructurings. On the one hand, inadequately-sized (limited debt relief) restructurings (“too little”) can alleviate the output costs and shorten the market exclusion of the sovereign in the short run compared to a larger-sized restructuring (Cruces and Trebesch, 2013; Trebesch and Zabel, 2017). On the other hand, since the inadequately-sized (limited debt relief) restructurings do not necessarily restore debt sustainability, the sovereign is more likely to experience another restructuring in the years following the current episode suffering further output costs and debt overhang.

This paper contributes to fill this gap. We proxy the “too-little" problem by considering whether or not a second restructuring is required over the medium term (i.e., a particularly stark manifestation of this "too-little" problem)[2] Restructurings are considered "cured" when they are not followed by another restructuring within five years after completion, and "non-cured" when they are followed by a second restructuring during that time frame. We combine that dimension with the classification of restructuring strategies in Asonuma and Trebesch (2016), which considers whether the restructuring takes place *preemptively* (before payments are missed) or *post-default* (after payments are missed). We classify 197 private external debt restructuring episodes in 1975-2020 along these outcome and strategy dimensions to present novel empirical facts and construct a two-period sovereign debt model with two types of debt restructurings and provide a quantitative analysis applied to both Argentina and Uruguay to replicate empirical findings.

We present five new stylized facts on the "too-little" restructuring problem. First, preemptive restructurings are more likely to end up non-cured than post-default restructurings. Second, haircuts—how much the foreign creditors suffer by debt restructuring (i.e., debt relief for sovereign debtors)—higher in the cured than in the non-cured group, within each restructuring strategy. Third, post-default restructurings which end up cured have lower debt over medium term than preemptive restructurings which end up non-cured. Fourth, restructuring strategies and outcomes tend to follow the previous restructuring (are "sticky"). Fifth, hair-

[2]We use five years as the benchmark for our "medium-term" analysis in the empirical section. Our measure is conservative as using the second restructuring to define "too-little" is a particularly stark manifestation of the problem.

cuts negatively correlate with foreign creditors' income, within the preemptive restructuring group.

These stylized facts pose a new question on the theoretical literature on sovereign debt. Why are preemptive restructurings more likely to be non-cured and experience subsequent restructurings despite their swift debt crisis resolution such as short restructuring duration and quick market re-access?

To explore this question, we build a two-period sovereign debt model which includes endogenous choice of preemptive and post-default renegotiations between a risk averse sovereign debtor and a representative risk averse foreign creditor.[3] Our general equilibrium model follows the classical set up of Eaton and Gersovitz (1981) as in recent quantitative work on sovereign debt. There are two types of debt restructurings (preemptive and post-default), which follow a conventional bargaining between the sovereign debtor and foreign creditor over haircuts (e.g., Yue, 2010; Benjamin and Wright, 2009). The key innovation of our model is to embed how different outcomes of the current debt restructurings (over the extent to which it restores solvency) influences the sovereign debtor's borrowing and debt dynamics, and in turn, its choice among repayment, preemptive restructuring and default/post-default restructuring in the subsequent periods.

[3]Our general equilibrium model of debt renegotiation with a representative (risk averse) foreign creditor follows closely Asonuma and Joo (2020).Relaxing the model to include a multiple of risk averse creditors does not provide additional insights but increases technical difficulty to track the model. In a partial equilibrium model of sovereign debt, Pitchford and Wright (2012) explore the role of the multiple foreign creditors in a decentralized bargaining to explain holdout creditors.

Our model provides two theoretical predictions. The first prediction is that within preemptive restructurings, when the foreign creditor has high income, he is less eager to settle without high recovered debt payments and the sovereign has no option but to agree on low haircuts (i.e., limited debt relief). This is because the representative foreign creditor is risk averse facing his income fluctuations, and how valuable the recovered debt payments are for him to smooth consumption varies depending on his income, "state-dependent consumption smoothing". He expects low haircuts in the future round after a high income realization which is an outside option in the current round. In order to reach settlement in the current round, the sovereign has no option but to propose low haircuts at least equivalent to his outside option. Our prediction is similar to that in post-default restructurings in Asonuma and Joo (2020) which shows that when the foreign creditor has high income, post-default renegotiations become more prolonged and result in lower haircuts than when he has high income.

The second prediction is that both small haircuts (i.e., limited debt relief) and quick market re-access in preemptive debt restructurings result in a higher debt level and higher borrowing costs, leading to a second debt restructuring. Asonuma and Trebesch (2016) show that lower haircuts which are more favorable to the foreign creditor, result in quicker re-access to the international capital market for preemptive restructurings than post-default restructurings. When the initial restructuring results in low haircuts, while the sovereign regains access to the market quickly, its debt level remains high and issues new bonds with high borrowing costs (high bond spreads). The sovereign has less scope to accumulate debt and is more likely to restructure debt again (a “non-cured preemptive restructuring”).

We apply quantitative exercise to both debt restructurings in Argentina and Uruguay in 1985-2020. Our quantitative results expect to replicate the stylized facts and support our theoretical predictions.

Related Literature. Our paper speaks to several important strands of literature on sovereign debt. First of all, our paper contributes to empirical strands of literature on sovereign debt restructuring such as Benjamin and Wright (2009), Sturzenegger and Zettelmeyer (2008), Sturzenegger and Zettelmeyer (2006), Reinhart and Rogoff (2009), Cruces and Trebesch (2013), Kaminsky and Vega-Garcia (2016), Reinhart and Trebesch (2016), Asonuma and Trebesch (2016) and Asonuma and Joo (2020). These papers explore in particular restructuring choice (preemptive vs. post-default), duration, haircuts, market re-access, debt dynamics and output costs in sovereign debt restructurings. Our paper provides new findings on whether two types of debt restructurings achieve debt sustainability over the medium term, i.e., cured vs. non-cured and on "too little" debt restructurings.

Our theoretical analysis is related to previous studies that theoretically model a bargaining game between a sovereign debtor and its creditors –Bulow and Rogoff (1989), Kovrijnykh and Szentes (2007), Bi (2012), Bai and Zhang (2012), D’Erasmo et al. (2008), Yue (2010), Pitchford and Wright (2012), Benjamin and Wright (2009), Hatchondo, Martinez, and Padilla (2014), and Asonuma and Trebesch (2016). To our knowledge, we are the first to explore theoretically a difference in two types of debt restructurings (preemptive vs. post-default) on debt sustainability over the medium term.[4]

3.2 Empirical Facts on "Too Little" Restructuring Problems

3.2.1 Data and Definition

Throughout the paper we focus on default and restructuring events between sovereigns and private external creditors such as international banks or bondholders. The original dataset we build on covers 197 private external debt restructuring episodes between 1975 and 2020,as defined in the most dataset for Asonuma and Trebesch (2016).[5] We truncate this dataset by dropping the episodes with overlapping years as well as bond debt restructurings and end up with 175 unique country-year restructuring episodes.[6] The various demensions of restructuring features our dataset combines measures of restructuring haircuts (Cruces and Trebesch, 2013), classification of preemptive vs. post-default restructurings (Asonuma and Trebesch, 2016), restructuring duration (Asonuma and Trebesch, 2016), and the data on the creditor committees and chairs of each restructuring episode (Asonuma and Joo, 2020).

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[4]By comparison, Amador and Phelan (2021) and Asonuma (2016) explain the mechanism of repeated sovereign defaults.

[5]Please check https://sites.google.com/site/christophtrebesch/data for the most updated dataset on Asonuma and Trebesch (2016).

[6]The detailed explanation on the methodology employed constructing the dataset can be seen in the Appendix A.1.

We classify restructuring episodes by their strategies as well as outcomes. In terms of restructuring strategies, we divide restructurings into *preemptive* restructurings and *post-default* restructurings based on whether a debtor exchanges outsanding debt without missing any contractual payments towards the creditors involved. Unlike Asonuma and Trebesch (2016), we do not distinguish between strict and weak preemptive restructurings and treat an episode as preemptive as long as there's no unilateral defaults prior to the negotiations. In terms of restructuring outcomes, we divide restructurings into “cured" and “non-cured" episodes depending on whether a debt exchange reoccurs for the same debtor in the near future. We use a horizon of five years to define re-occurring restructurings and confirm that our baseline results are robust to the horizon of three, five and seven years.[7]

In summary, the definitions by restructuring strategies and outcomes are given as below:

- (By restructuring strategies) ***Preemptive Restructurings*** are debt exchanges implemented with no missing payments at all (no legal default) or those in which some payments are missed, but only temporarily and after the start of negotiations with creditor representatives (no unilateral default).
- (By restructuring strategies) ***Post-default Restructurings*** are debt exchanges in which payments are missed unilaterally and without the agreement of creditor representatives (unilateral default prior to negotiations).

[7] The episodes that are changed under different definitions of cured vs. non-cured are reported in the Appendix A.2.

- (By restructuring outcomes) ***Non-Cured Restructurings*** are debt exchanges that reoccur for the same debtor within five years after the completion of the current episode.

- (By restructuring outcomes) ***Cured Restructurings*** are debt exchanges that have *no* recurring exchanges in five years after the completion of the current episode.

We can therefore divide all restructurings episodes into the 2-strategy $\times$ 2-outcome cases, namely *Preemptive/Cured (PC)*, *Preemptive/Non-Cured (PNC)*, *Post-default/Cured (DC)* and *Post-default/Non-Cured (DNC)*. We will refer to these four groups of restructurings repeatedly throughout the paper.

In this paper, we use the “cured" vs. “non-cured" outcome to define the “too-little" restructurings, that sovereign debt restructurings often do not achieve sufficient debt relief (haircuts), debt overhang remains, leading to recurring restructurings. An additional debt restructuring is a particularly stark manifestation of the “too-little” problem. We use the outcome of repeated restructurings rather than haircuts as the latter are highly endogenous to the macroeconomic fundamentals including the pre-crisis debt level of the sovereign. Moreover, it’s empirically implausible to measure the optimal haircuts the sovereign would need to achieve debt sustainability with no more repeated restructurings. Thus, we define the“too-little" sovereign restructurings problem below:

- The ***"too-little" restructurings*** refer to sovereign debt exchanges that are "non-cured," that is, debt exchanges that reoccur for the same debtor within five years after the completion of the current episode.

3.2.2 Five Stylized Empirical Facts

In this section, we present five stylized empirical facts on sovereign debt restructurings that shed light on the "too-little" restructuring problem.[8]

Stylized Fact I: Preemptive restructurings are more likely to end up non-cured than post-default restructurings.

We divide all restructurings into the four groups classified above and compute the conditional probability of non-cured for each restructuring strategy. As shown in Table 3.1, the conditional probability of non-cured restructurings under preemptive strategy (72%) is much higher than that under post-default (42%). There are a total of 175 unique country-year restructuring episodes reported in Table 3.1. Appendix A.2 lists all the restructuring episodes in each of the four groups defined.

We show the fact that preemptive restructurings are more likely to be non-cured is consistent for the sub-sample of restructurings after the 1990s. To do so, we drop the episodes whose end years are before 1990 and report the statistics as in Table 3.1. The results shown in Table 3.B.1 on the sub-sample make clear that

[8] In line with empirical literature on sovereign debt (e.g., Borensztein and Panizza (2009), our empirical analysis focuses on debt, GDP growth, creditor losses (haircuts) and likelihood of a second restructuring as costs of debt restructurings.

while 42% of the preemptive restructurings end up non-cured, only 8% of the post-default restructuring are non-cured, a ratio more than five times smaller compared with the former.

Table 3.1: Number of Episodes for Strategies × Outcomes (Stylized Fact I)

	Cured	Non-cured	cond. prob. for non-cured
Preemptive	19	50	72%
Post-default	61	45	42%

Note: This table reports the number of episodes for the four restructuring groups, namely Preemptive/Cured (PC), Preemptive/Non-Cured (PNC), Post-default/Cured (DC) and Post-default/Non-Cured (DNC), as well the conditional probability of non-cured under the preemptive and post-default restructuring strategy. There's a total of 175 restructuring episodes in this table.

Stylized Fact II: Haircuts are higher in the cured than the non-cured group, within each restructuring strategy.

Using the definition of net-present-value (NPV) haircuts[9] (Cruces and Trebesch, 2013; Sturzenegger and Zettelmeyer, 2008), we find that the average haircuts are higher in the cured than the non-cured group. The result also holds within the restructuring strategy of preemptive or post-default, as shown in Table 3.2. Under preemptive restructuring strategies, preemptive/cured (PC) have haircuts almost

[9]The NPV haircuts introduced by Sturzenegger and Zettelmeyer (2008) and Cruces and Trebesch (2013)) uses the present value of the old debt and account for the characteristics of both new and old debt:

$$\text{Haircuts}_t = 1 - \frac{\text{Present Value of } \textit{New} \text{ Debt } (r_t)}{\text{Present Value of } \textit{Old} \text{ Debt}(r_t)}$$

twice as high as preemptive/non-cured (PNC); under the post-default restructuring strategies, post-default/cured (DC) also have much higher haircuts than the post-default/non-cured (DNC). In addition, the difference in haircuts within each restructuring strategy is statistically significant at 5% level, as reported in Table 3.3.

Table 3.2: Mean NPV-Haircuts in Different Restructuring Groups (Stylized Fact II)

Group	PC	PNC	DC	DNC
Mean Haircuts (%)	32	18.5	59	43

Note: This table reports the average NPV haircuts (in percentage points) for the four restructuring groups, namely preemptive/cured (PC), preemptive/non-cured (PNC), post-default/cured (DC) and post-default/non-cured (DNC).

Table 3.3: T-Test for Differences in NPV-Haircuts (Stylized Fact II)

H_A : mean (PC) - mean (PNC) $>$ 0 $\quad$ H_0 : mean (PC) - mean (PNC) = 0

mean. diff.	std.err.	t-stats	p-value	df.
13	6.7	1.967	0.03	24

H_A : mean (DC) - mean (DNC) $>$ 0 $\quad$ H_0 : mean (DC) - mean (DNC) = 0

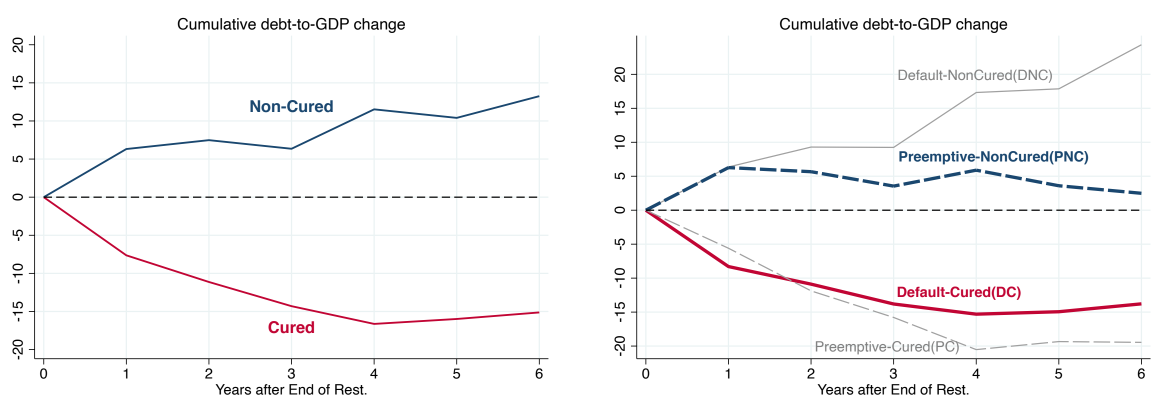
mean. diff.	std.err.	t-stats	p-value	df.
16	9	1.74	0.04	65

Note: This table reports the t-test result of the difference in mean NPV haircuts between the preemptive/cured (PC) and preemptive/non-cured (PNC) group, as reported in the top panel, and the t-test result of the difference in mean NPV haircuts between the post-default/cured (DC) and post-default/non-cured (DNC), as reported in the bottom panel. NPV haircuts are measured in percentage points.

Stylized Fact III: Post-default-cured (DC) restructurings have lower debt over medium term than preemptive-non-cured (PNC).

Although preemptive strategies have appealing short-run benefits (Asonuma and Trebesch, 2016; Asonuma, Chamon, Erce, and Sasahara, 2021) at the start of the restructuring, what drives the debt dynamics over medium term is the cured vs. the non-cured outcome. Specifically, restructurings that are post-default cured (PC) have significantly lower public-debt-to-GDP ratio (thus less debt overhang problem) than preemptive restructurings that are non-cured (PNC) over medium term. This implies that over the medium term, the benefits from cured restructuring outcome outweigh the costs of adopting post-default restructuring strategy in the short run, at least in terms of the debt-to-GDP dynamics.

Table 3.4: Public Debt-to-GDP in the Post-Restructuring Years (Stylized Fact III)



Note: This panel of figures present the cumulative change of public debt-to-GDP ratio (in percentage points) following the end of the restructuring (Year = 0). Public debt-to-GDP ratio at the end of the restructuring year is normalized to zero. Panel (a) reports the cumulative change of average public debt-to-GDP ratio for the cured and the non-cured restructuring group. Panel (a) reports the average change of the same variable for the four restructuring groups, namely preemptive/cured (PC), preemptive/non-cured (PNC), post-default/cured (DC) and post-default/non-cured (DNC).

We show that compared with the non-cured restructurings, the *cured* restructurings have more significant *reduction* in their public debt-to-GDP ratio in the post-restructuring years, even under the post-default restructuring strategies. The left panel of Table 3.4 groups all restructurings by the cured and non-cured outcomes and reports the average cumulative change of the public-debt-to-GDP in each group in the years following the end of the restructuring.[10] On average, the cured restructuring group has significant reduction in their public debt-to-GDP ratio in the years following the end of restructuring, in contrast with the continual rise the ratio for the non-cured restructuring group. The right panel of Table 3.4 reports the public debt-to-GDP ratio for the four groups by strategies and outcomes. Cured restructurings, even with post-default strategies (DC), have significant debt reduction in the years following the end of restructuring compared to the non-cured restructurings with preemptive strategies (PNC).

We also show that the difference in average public debt-to-GDP ratio across groups in Table 3.4 are significant. Table 3.B.2 in the appendix shows that the differences for the group averages are statistically significant for the five years following the completion of the restructuring at 1% level. Figure 3.B.1 in the Appendix reports the same dynamics for the subsample excluding restructuring episodes whose end years are in the 1980s and 1970s. Our results make clear that

[10]We do not distinguish external and domestic debt and use the public debt data provided by historical public debt database (HPDD) constructed by IMF. This dataset has the longest time-series coverage for the sample in our paper.

our stylized fact is not driven by observations in the early years with prolonged post-default restructuring episodes.

Stylized Fact IV: Restructuring strategies and outcomes are sticky.

We show that countries that adopt preemptive (or post-default) strategies are likely to choose preemptive (or post-default) again, if a second restructuring is needed. This can be seen in the left four columns of Table 3.5, where we give the transition matrix of the number of restructurings that transition from preemptive/non-cured (PNC) or post-default/non-cured (DNC) into the all four groups of restructurings by the same debtor. The table also makes clear that a large proportion of non-cured countries still end up non-cured (58% for the preemptive group and 55% for the post-default group), implying the stickiness in restructuring outcomes. In addition, countries are rather consistent in adopting their restructuring strategies. More than 70% of the preemptive/non-cured (PNC) group stills chooses preemptive strategies, and over 80% of the post-default/non-cured (DNC) group sticks post-default strategies when another restructuring is needed.

Appendix A.2 reports all episodes in each entry of the transition matrix in Table 3.5. Note that the observations for the non-cured group is smaller than the total number of non-cured episodes as reported in Table 3.1 since some episodes cannot be classified as cured or not by the same we write this paper.

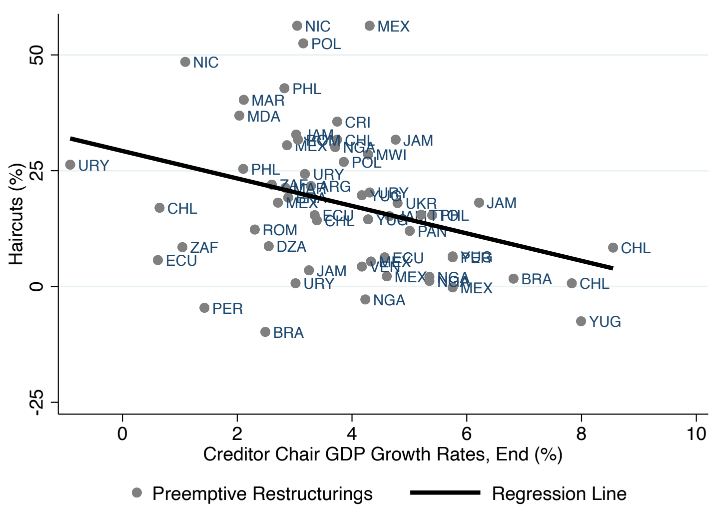
Table 3.5: Transition matrix from non-cured to other states (Stylized Fact IV)

	PC	PNC	DC	DNC	same outc.	same strat.	same strat. & outc.
PNC	9	25	11	3	58%	71%	52%
DNC	2	6	18	18	55%	82%	41%

Note: This table gives the transition matrix of the number of restructurings that transition from preemptive/non-cured (PNC) or post-default/non-cured (DNC) into the all four groups of restructurings by the same debtor. The last three columns reports the conditional probability of preemptive/non-cured (PNC) or post-default/non-cured (DNC) to stay in the same outcome, the same restructuring strategy, and both the same restructuring strategy and outcome.

Stylized Fact V: Haircuts negatively correlate with foreign creditors' income, within the preemptive restructurings group.

Figure 3.1: Haircuts and Creditor GDP Growth for Preemptive Restructurings



Note: The figures shows the unconditional scatter plot of NPV haircuts (%) and the creditor chair GDP growth (%) measured at the end of restructurings. Each Dot is a preemptive sovereign debt restructuring episode and labeled by the name of the debtor country. The black line is the correlation between haircuts (%) and the creditor chair GDP growth (%) with no additional controls except for the constant.

We focus on preemptive restructurings and plot the relation between haircuts and the foreign creditor chair GDP growth measured at the end of restructurings, as shown in Figure 3.1. We find a strong negative correlation between haircuts and the creditor's growth, that is, an improvement of the foreign creditor chair business and financial cycles (eg., measured by GDP growth rates) leads to a significant reduction in haircuts. Our results are largely consistent with Asonuma and Joo (2020) on the sample of post-default restructurings.[11]

Table 3.6: Haircuts and Creditor GDP Growth (Stylized Fact V)

	(1)	(2)	(3)
Creditor Chair GDP growth, End	-2.965***	-2.466**	-2.475**
	(1.095)	(1.202)	(1.212)
Dummy for Post-1989		5.740	7.450
		(5.709)	(5.552)
Length of Restructuring (years)			0.124
			(0.313)
Debtor External-Debt-to-GDP (%)			0.192***
			(0.0559)
Constant	29.26***	26.38***	11.81
	(4.587)	(5.406)	(7.264)
Observations	58	58	54
R^2	0.1157	0.1317	0.2744
Adjusted R^2	0.100	0.100	0.215

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

[11] We report the scatter plot (as well as the regression results) on the relation between haircuts and creditor chair GDP growth for the post-default episodes in the Appendix B in Figure 3.B.4.

Note: Dependent variable is NPV haircuts (%) and all observations include preemptive restructurings only. Creditor chair GDP growth are computed at the end of the restructuring and are in percentage points.

We provide the econometric support for the negative correlation between haircuts and creditor income in Table 3.6. It reports the cross-sectional regression results of the NPV haircuts on creditor chair GDP growth at the end of the restructurings. Column (1) has no controls except for the constant; Column (2) adds the dummy for restructuring start year post-1989 as the control and column (3) adds additional controls such as the length of restructurings and the debtor's external-debt-to-GDP ratio. Our main variable of interest – creditor chair GDP growth at the end of the restructuring – remains negative and significant in all specifications. On average, a one percentage point increase in the creditor chair GDP growth leads to about 2.5 to 3 percentage points reduction in the NPV haircuts, for the preemptive restructurings group.

3.3 Theoretical Model

We build a simple two-period model with two types of debt renegotiations to explore the mechanisms of "too-little" sovereign debt restructurings. Importantly, the two-period model captures how the outcomes of previous preemptive and post-default restructurings influence the likelihood of subsequent preemptive and post-default debt restructurings.

3.3.1 Assumptions in the Model

There are two agents in the model: a sovereign (government) and a foreign creditor. The sovereign is risk averse, facing a fluctuating income stream and taking the world risk-free interest rate (r^*) as given as its economy is too small relative to the rest of the world. However, the sovereign has access to the segmented international capital market in that it can only borrow from a representative risk averse foreign creditor who also faces income fluctuation and whose borrowing and lending decisions also do not influence the world interest rate. The sovereign's and the foreign creditor's current utility (i.e., one-period) functions are defined as follows:

$$u(c_t) \qquad \text{and} \qquad v(c_t^*)$$

These are strictly increasing and strictly concave, and satisfy the Inada conditions. c_t and c_t^* denote consumption of the sovereign and foreign creditor in period t, respectively. The discount factor of the sovereign denoted by β reflects both time preference (impatience) and the probability that the current government will remain in power in the next period. The discount factor of the creditor denoted by $\frac{1}{(1+r^*)}$ and captures only his time preference. The assumption of the risk averse creditor follows Lizarazo (2013), Aguiar et al. (2016) and Asonuma and Joo (2020).

In each period, both the sovereign and the foreign creditor receive stochastic endowment streams y_t^h and y_t^f, respectively. We denote $\mathbf{y}_t$, a vector of two income processes as $\mathbf{y}_t = [y_t^h, y_t^f]$. The income process is stochastic and drawn from a compact set $Y = [y^h_{\min}, y^h_{\max}] \times [y^f_{\min}, y^f_{\max}] \subset R^2_+$. $\mu(\mathbf{y}_{t+1}|\mathbf{y}_t)$ is a probability distribution of a vector of shocks $\mathbf{y}_{t+1}$ conditional on its previous realization $\mathbf{y}_t$. The goods endowed in two countries are identical and tradable. The sovereign is in current account deficit (surplus)—its consumption is larger (less) than endowment—when it repays debt and issues new debt (has new savings). On the contrary, when the sovereign defaults on his debt repayment and loses market access, it has a balanced current account and its consumption is equal to endowment.

Both the sovereign bond and risk-free bond markets are incomplete. On the one hand, the sovereign can borrow and lend only via the one-period, zero-coupon sovereign bonds. We denote b_{t+1} as the amount of bonds to be repaid in the next period whose set is shown by $B = [b_{\min}, b_{\max}] \subset R$, where $b_{\min} \leq 0 \leq b_{\max}$. We set the lower bound for the sovereign's bond holding at $b_{\min} > -y^h_{\max}/r^*$, which is the highest debt that the sovereign can repay. The upper bound $b_{\max}$ is the highest level of assets that the sovereign may accumulate. On the other hand, the foreign creditor can smooth his consumption through borrowing and lending via both one-period, zero-coupon sovereign bonds and risk-free bonds. Denote b^*_{t+1} and b^{*f}_{t+1} as the amounts of sovereign bonds and risk-free bonds to be repaid in the next period whose sets are shown by $B^* = [b^*_{\min}, b^*_{\max}] \subset R$ and $B^{*f} = [b^{*f}_{\min}, b^{*f}_{\max}] \subset R$, where $b^*_{\min} \leq 0 \leq b^*_{\max}$ and $b^{*f}_{\min} \leq 0 \leq b^{*f}_{\max}$, respectively. Information on both income processes and bond issuance of two parties is perfect and symmetric. We assume the price of sovereign bonds to be $q(b_{t+1}, b^{*f}_{t+1}, \mathbf{y}_t)$ with the sovereign's asset position b_{t+1}, the foreign creditor's holding of risk-free bonds b_{t+1}^{*f}, and a vector of income shocks $\mathbf{y}_t$. The price of sovereign bonds is determined in equilibrium. We also assume $q^f(= 1/(1 + r^*))$ to be the price of risk-free bonds that the foreign creditor takes as given.

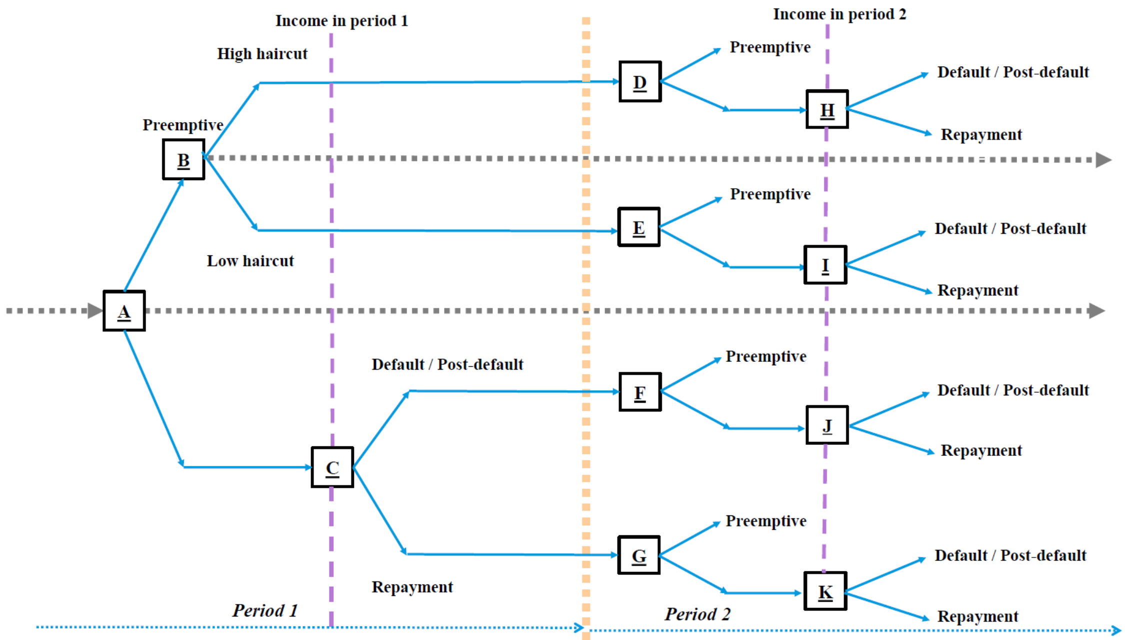
We assume that the foreign creditor always commits to repay its debt. On the contrary, the sovereign is not committed to repay debt and makes its decisions before and after the income realization. Before the income realization, the sovereign chooses whether to restructure debt preemptively or pass the option. If it chooses to restructure preemptively, it is then subject to smaller output costs. When a preemptive restructuring is chosen, the sovereign and the creditor negotiate over debt via one-round bargaining. At the renegotiations, the sovereign chooses whether to propose an offer with haircuts (recovery rates) or to pass. The foreign creditor decides whether to accept or to reject the proposal. If the offer with haircuts is proposed and accepted, then the sovereign regains access in the current period and the foreign creditor receives his recovered debt payments (with debt relief). Otherwise, both parties result in no settlement and move on to non-preemptive choice.

After the income realization, the sovereign chooses whether to repay or default. If the sovereign chooses to repay its debt, it will preserve its access to the international capital market in the next period. On the contrary, if the sovereign chooses to default, it is then subject to exclusion from the international capital market, arrear accumulation and direct output costs. When a default occurs, the sovereign and the foreign creditor negotiate over unpaid debt via one-round bargaining. At the renegotiation, the sovereign chooses whether to propose an offer with haircuts (recovery rates) or to pass. The foreign creditor decides whether to accept or to reject the proposal. If the offer with haircuts is proposed and accepted, then the sovereign regains access in the next period after temporal exclusion in the current period and the foreign creditor receives his recovered debt payments. Otherwise, both parties result in no settlement and the sovereign remains in permanent autarky.

3.3.2 Timing of the Model

Figure 3.2 summarizes the timing of decisions in two periods. The sovereign can choose either a preemptive restructuring or passing the preemptive choice before income realization. After income realization, the sovereign choose either repayment or a post-default restructuring. In the period 1, we also distinguish high-haircut and low-haircut preemptive restructurings, while we do not make such distinction in the period 2 as the period 2 is the last period.

Figure 3.2: Timing of the Model



Note: This figure presents the renegotiation choices of the sovereign in two periods both before and after the income realization. The sovereign's renegotiation choice are preemptive restructurings, post-defaults, and repayment.

1. The sovereign starts the period 1 with initial debt. We are in node A.
2. The sovereign decides whether to take a preemptive option or to pass the option.
   (a) If the sovereign opts a preemptive option, we move to the upper branch of the tree and are in node B. The sovereign suffers smaller output costs. At the preemptive restructuring, haircuts are determined. If "high" haircuts are achieved, the sovereign issues new debt and we move to the upper branch of the tree. Otherwise (if "low" haircuts are

achieved), the sovereign issues new debt and we move to the middle-upper branch of the tree.

(b) If the sovereign passes the preemptive option, we move to the lower branch of the tree and are in node $\underline{C}$.

3. Income in the period 1 realizes. The sovereign decides whether to repay debt in full or to default.

   (a) In node $\underline{F}$ (post-default node), if it defaults, we move to the middle-lower branch of the tree. The sovereign suffers larger output costs (compared with node $\underline{B}$). At the post-default restructuring, haircuts are determined. The sovereign cannot issue new debt due to temporary financial autarky as a result of default.

   (b) In node $\underline{G}$ (repayment node), the sovereign repays debt in full, we move to the lower branch of the tree. The sovereign issues new debt.

4. The sovereign starts the period 2 with initial debt. We are in either node $\underline{D}$, $\underline{E}$, $\underline{F}$ or $\underline{G}$. The sovereign decides whether to take a preemptive option or to pass the option.

   (a) If the sovereign opts a preemptive option, we move to the upper sub-branch of each branch. It suffers smaller output costs. At the preemptive restructuring, haircuts are determined.

   (b) If the sovereign passes the preemptive option, we move to the upper sub-branch of each branch.

5. Income in the period 2 realizes. The sovereign decides whether to repay debt in full or to default.
   (a) If it defaults, we move to the middle sub-branch of each branch. It suffers larger output costs. At the post-default restructuring, haircuts are determined.
   (b) If it repays debt in full, we move to the lower sub-branch of each branch.

3.3.3 Sovereign's Problem

3.3.3.1 Problem in the Period 2

We start from the sovereign's problem in the period 2. The sovereign's objective is to maximize utility at the period 2. Prior to the income realization in the period 2, the sovereign decides whether to take the preemptive restructuring ($\mathcal{P}$) or to pass the preemptive restructuring ($\mathcal{NP}$), at the node D, E, F, or G. The sovereign's ex-ante value function is:

$$V^{\text{EX-ANTE}}(b_2, b_2^{*f}, \mathbf{y}_1|i) = \max\left\{V^{\mathcal{P}}(b_2, b_2^{*f}, \mathbf{y}_1|i),\ V^{\mathcal{NP}}(b_2, b_2^{*f}, \mathbf{y}_1|i)\right\},\ i = \underline{\text{D}}, \underline{\text{E}}, \underline{\text{F}}, \underline{\text{G}} \tag{3.1}$$

where $V^{\mathcal{P}}(b_2, b_2^{*f}, \mathbf{y}_1 \mid i)$ is the ex-ante value of a preemptive restructuring at node i:

$$V^{\mathcal{P}}(b_2, b_2^{*f}, \mathbf{y}_1 \mid i) = \int_{Y} u(c_2)\ d\ \mu(\mathbf{y}_2|\mathbf{y}_1) \qquad i = \underline{D}, \underline{E}, \underline{F}, \underline{G}$$
$$s.t. \quad c_2 = (1-\lambda_{\mathcal{P}}) \int_{\mathbf{Y}} y_2^h\ d\ \mu(\mathbf{y}_2|\mathbf{y}_1) + \delta(b_2, b_2^{*f}, \mathbf{y}_1) b_2 \tag{3.2}$$

with $\delta(b_2, b_2^{*f}, \mathbf{y}_1)$ is the recovery rate on debt determined at a preemptive restructuring.

$V^{\mathcal{NP}}(b_2, b_2^{*f}, \mathbf{y}_1 \mid i)$ is the ex-ante value of passing a preemptive restructuring ($\mathcal{NP}$):

$$V^{\mathcal{NP}}(b_2, b_2^{*f}, \mathbf{y}_1 \mid i) = \int_{\mathbf{Y}} V(b_2, b_2^{*f}, \mathbf{y}_2 \mid j)\ d\ \mu(\mathbf{y}_2|\mathbf{y}_1)\ i = \underline{D}, \underline{E}, \underline{F}, \underline{G} \text{ and } j = \underline{H}, \underline{I}, \underline{J}, \underline{K} \tag{3.3}$$

The sovereign's preemptive restructuring choice can be characterized by a preemptive restructuring set $\mathcal{P}(b_2, b_2^{*f}|i) \subset A$, a set of income vector $\mathbf{y}_1$ at which the sovereign finds preemptive restructuring choice optimal, that is,

$$\mathcal{P}\ (b_2, b_2^{*f} \mid i) = \left\{ \mathbf{y}_1 \subset \mathbf{Y} \,\middle|\, V^{\mathcal{P}}(b_2, b_2^{*f}, \mathbf{y}_1|i)\ \geq\ V^{\mathcal{NP}}(b_2, b_2^{*f}, \mathbf{y}_1|i) \right\} \qquad i = \underline{D}, \underline{E}, \underline{F}, \underline{G} \tag{3.4}$$

Upon realization of income in the period 2, the sovereign decides whether to repay the debt ($\mathcal{R}$) or to default ($\mathcal{D}$):

$$V(b_2, b_2^{*f}, \mathbf{y}_2|i) = \max\left\{V^{\mathcal{R}}(b_2, b_2^{*f}, \mathbf{y}_2|i),\ V^{\mathcal{D}}(b_2, b_2^{*f}, \mathbf{y}_2|i)\right\} \qquad i = \underline{\text{D}}, \underline{\text{E}}, \underline{\text{F}}, \underline{\text{G}} \tag{3.5}$$

where $V^{\mathcal{R}}(b_2, b_2^{*f}, \mathbf{y}_2|i)$ is the value of repayment:

$$\begin{aligned} V^{\mathcal{R}}(b_2, b_2^{*f}, \mathbf{y}_2|i) &= u(c_2) \qquad i = \underline{\text{H}}, \underline{\text{I}}, \underline{\text{J}}, \underline{\text{K}} \\ s.t. \quad c_2 &= y_2^h + b_2 \end{aligned} \tag{3.6}$$

and $V^{\mathcal{D}}(b_2, b_2^{*f}, \mathbf{y}_2|i)$ is the value of default/post-default:

$$\begin{aligned} V^{\mathcal{D}}(b_2, b_2^{*f}, \mathbf{y}_2|i) &= u(c_2) \qquad i = \underline{\text{H}}, \underline{\text{I}}, \underline{\text{J}}, \underline{\text{K}} \\ s.t. \quad c_2 &= (1 - \lambda_{\mathcal{D}})\ y_2^h + \alpha(b_2, b_2^{*f}, \mathbf{y}_2)\ (1 + r^*)\ b_2 \end{aligned} \tag{3.7}$$

where $(1 + r^*)\ b_2$ is the defaulted debt with arrear interests accumulation and $\alpha(b_2, b_2^{*f}, \mathbf{y}_2)$ is the recover rate on on the defaulted debt determined at a post-default restructuring. The sovereign's post-default restructuring choice can be characterized by a post-default restructuring set $\mathcal{D}\ (b_2, b_2^{*f}) \subset Y$, which is a set of income vector $\mathbf{y}_2$ at which the sovereign finds default/post-default restructuring choice optimal:

$$\mathcal{D}\ (b_2, b_2^{*f} \mid i) = \left\{ \mathbf{y}_2 \subset \mathbf{Y} \,\middle|\, V^{\mathcal{D}}\ (b_2, b_2^{*f}, \mathbf{y}_2 \mid i) \geq V^{\mathcal{R}}(b_2, b_2^{*f}, \mathbf{y}_2 \mid i) \right\} \quad i = \underline{\text{H}}, \underline{\text{I}}, \underline{\text{J}}, \underline{\text{K}} \tag{3.8}$$

3.3.3.2 Problem in the Period 1

We move onto the sovereign's problem in the period 1. Prior to the income realization in the period 1, the sovereign decides whether to take a preemptive restructuring ($\mathcal{P}$) or not ($\mathcal{NP}$), at node $\underline{\text{A}}$ and its ex-ante value function is defined as follows:

$$V^{\text{EX-ANTE}}(b_1, b_1^{*f}, \mathbf{y}_0 | \ \underline{\text{A}}) = \max\left\{ V^{\mathcal{P}}(b_1, b_1^{*f}, \mathbf{y}_0 | \ \underline{\text{A}}),\ V^{\mathcal{NP}}(b_1, b_1^{*f}, \mathbf{y}_0 | \ \underline{\text{A}}) \right\} \tag{3.9}$$

and $V^{\mathcal{P}}(b_1, b_1^{*f}, \mathbf{y}_0 | \ \underline{\text{A}})$ is the ex-ante value of a preemptive restructuring:

$$V^{\mathcal{P}}(b_1, b_1^{*f}, \mathbf{y}_0 | \ \underline{\text{A}}) = \mathbf{1}_{\underline{\text{B}}_{\mathcal{P}H}}\ V^{\mathcal{P}}(b_1, b_1^{*f}, \mathbf{y}_1 \mid \underline{\text{B}}_{\mathcal{P}^H}) + \left(1 - \mathbf{1}_{\underline{\text{B}}_{\mathcal{P}H}}\right)\ V^{\mathcal{P}}(b_1, b_1^{*f}, \mathbf{y}_1 \mid \underline{\text{B}}_{\mathcal{P}^L}) \tag{3.10}$$

where $\mathbf{1}_{\underline{\text{B}}_{\mathcal{P}H}}$ is an indicator and equals 1 if a preemptive restructuring at node $\underline{\text{B}}$ results in high haircut ($\mathcal{P}^H$); and 0 if results the preemptive restructuring results in low haircut ($\mathcal{P}^L$). Specifically, $V^{\mathcal{P}}(b_1, b_1^{*f}, \mathbf{y}_1 \mid \underline{\text{B}}_{\mathcal{P}^H})$ and $V^{\mathcal{P}}(b_1, b_2^{*f}, \mathbf{y}_1 \mid \underline{\text{B}}_{\mathcal{P}^L})$

are defined as:

$$V^{\mathcal{P}}(b_1, b_1^{*f}, \mathbf{y}_0 |\ \underline{\mathrm{B}}_{\mathcal{P}^i}) = \max_{b_2} \int_{\mathbf{Y}} \left[u(c_1) + \beta\, V(b_2, b_2^{*f}, \mathbf{y}_2)\; d\; \mu(\mathbf{y}_2|\mathbf{y}_1)\right] d\; \mu(\mathbf{y}_1|\mathbf{y}_0)$$

$$s.t. \quad c_1 + q(b_2^*, b_2^{*f}, \mathbf{y}_0)\; b_2 = \; (1 - \lambda_{\mathcal{P}}) \int_{\mathbf{Y}} y_1^h d\; \mu(\mathbf{y}_1|\mathbf{y}_0) + \delta^{\mathcal{P}^i}(b_1, b_1^{*f}, \mathbf{y}_0) b_1 \tag{3.11}$$

for $\mathcal{P}^i = \mathcal{P}^H, \mathcal{P}^L$, where b_2 is the amount of new bonds to issue and $q(b_1, b_1^{*f}, \mathbf{y}_0)$ is the price of the newly issued bonds. $\delta^{\mathcal{P}^i}(b_1, b_1^{*f}, \mathbf{y}_0)$ is the recovery rates on the debt determined at a preemptive restructuring, with $\delta^{\mathcal{P}^{\mathcal{H}}}$ for high-haircut preemptive and $\delta^{\mathcal{P}^{\mathcal{L}}}$ for low-haircut preemptive. $\delta^{\mathcal{P}^i}(b_1, b_1^{*f}, \mathbf{y}_0)$ for $\mathcal{P}^i = \mathcal{P}^H, \mathcal{P}^L$ are defined in Section 3.5.

$V^{\mathcal{NP}}(b_1, b_1^{*f}, \mathbf{y}_0 |\ \underline{\mathrm{A}})$ is the ex-ante value of a passing preemptive restructuring ($\mathcal{NP}$):

$$V^{\mathcal{NP}}(b_1, b_1^{*f}, \mathbf{y}_0 |\ \underline{\mathrm{A}}) = \int_{\mathbf{Y}} V(b_1, b_1^{*f}, \mathbf{y}_1 |\ \underline{\mathrm{C}})\; d\; \mu(\mathbf{y}_1|\mathbf{y}_0) \tag{3.12}$$

The sovereign's preemptive restructuring choice can be characterize by a preemptive restructuring set $\mathcal{P}(b_1, b_1^{*f}) \subset Y$, which is a set of income vector y_0 at which the sovereign finds preemptive restructuring choice optimal:

$$\mathcal{P}(b_1, b_1^{*f} |\ \underline{\mathrm{A}}) = \left\{ \mathbf{y}_0 \subset \mathbf{Y} \middle|\ V^{\mathcal{P}}(b_1, b_1^{*f}, \mathbf{y}_0 |\ \underline{\mathrm{A}}) \geq V^{\mathcal{NP}}(b_1, b_1^{*f}, \mathbf{y}_0 |\ \underline{\mathrm{A}}) \right\} \tag{3.13}$$

After income realization in period 1, the sovereign decides whether to repay ($\mathcal{R}$) or to default ($\mathcal{D}$) at node $\underline{\mathrm{C}}$:

$$V(b_1, b_1^{*f}, \mathbf{y}_1 |\ \underline{C}) = \ \max \left\{ V^{\mathcal{R}}(b_1, b_1^{*f}, \mathbf{y}_1 |\ \underline{C}),\ V^{\mathcal{D}}(b_1, b_1^{*f}, \mathbf{y}_1 |\ \underline{C}) \right\} \tag{3.14}$$

where $V^{\mathcal{R}}(b_1, b_1^{*f}, \mathbf{y}_1 |\ \underline{C})$ is the value of repayment ($\mathcal{R}$):

$$\begin{aligned} V^{\mathcal{R}}(b_1, b_1^{*f}, \mathbf{y}_1 |\ \underline{C}) = & \max_{b_2}\ u(c_1) + \beta\ V(b_2, b_2^{*f}, \mathbf{y}_2)\ d\ \mu(\mathbf{y}_2 | \mathbf{y}_1) \\ & s.t. \quad c_1 + q(b_2^*, q_2^{*f}, \mathbf{y}_1)\ b_2 = y_1^h + b_1 \end{aligned} \tag{3.15}$$

and $V^{\mathcal{D}}(b_1, b_1^{*f}, \mathbf{y}_1 |\ \underline{C})$ is the value of default/post-default ($\mathcal{D}$):

$$\begin{aligned} V^{\mathcal{D}}(b_1, b_1^{*f}, \mathbf{y}_1 |\ \underline{C}) = & \max_{b_2}\ u(c_1) + \beta\ V(b_2, b_2^{*f}, \mathbf{y}_2)\ d\ \mu(\mathbf{y}_2 | \mathbf{y}_1) \\ & s.t. \quad c_1 = (1 - \lambda_{\mathcal{D}})\ y_1^h + \alpha(b_1, b_1^{*f}, \mathbf{y}_1)\ (1 + r^*)\ b_1 \end{aligned} \tag{3.16}$$

where $(1 + r^*) b_1$ is defaulted debt with arrear interests accumulation and $\alpha(b_1, b_1^{*f}, \mathbf{y}_1)$ is the recovery rate on the defaulted debt determined at a post-default restructuring.

The sovereign's default/post-default restructuring choice can be characterized by post-default restructuring set $\mathcal{D}(b_1, b_1^{*f}) \subset Y$, which is a set of income vector $\mathbf{y}_1$ at which the sovereign finds default/post-default restructuring choice optimal.

$$\mathcal{D}(b_1, b_1^{*f} |\ \underline{C}) = \left\{ \mathbf{y}_1 \subset \mathbf{Y} \middle| V^{\mathcal{D}}(b_1, b_1^{*f}, \mathbf{y}_1 |\ \underline{C}) \geq V^{\mathcal{R}}(b_1, b_1^{*f}, \mathbf{y}_1 |\ \underline{C}) \right\} \tag{3.17}$$

3.3.4 Foreign Creditor's Problem

We describe the foreign creditor's problem in the periods 1 and 2 both before and after the income realization.

3.3.4.1 Problem in the Period 2

We start from the foreign creditor's problem in the period 2. The foreign creditor's problem is to maximize his utility at the period 2. Prior to income realization in the period 2, the creditor's ex-ante value function is defined as:

$$V^{*,\text{EX-ANTE}}(b_2, b_2^{*f}, \mathbf{y}_1|i) = \mathbf{1}^{\mathcal{P}}(b_2, b_2^{*f}, \mathbf{y}_1|i) V^{*,\mathcal{P}}(b_2, b_2^{*f}, \mathbf{y}_1|i) + \left(1 - \mathbf{1}^{\mathcal{P}}\right) V^{*,\mathcal{NP}}(b_2, b_2^{*f}, \mathbf{y}_1|i) \quad (3.18)$$

where $V^{*,\mathcal{P}}(b_2, b_2^{*f}, \mathbf{y}_1|i),\ i = \underline{\text{D}}, \underline{\text{E}}, \underline{\text{F}}, \underline{\text{G}}$ is the ex-ante value of a preemptive restructuring at node i:

$$\begin{aligned} V^{*,\mathcal{P}}(b_2, b_2^{*f}, \mathbf{y}_1|i) &= \int_{\mathbf{Y}} v(c_1) d\, \mu(\mathbf{y}_2|\mathbf{y}_1) \qquad i = \underline{\text{D}}, \underline{\text{E}}, \underline{\text{F}}, \underline{\text{G}} \\ s.t. \quad c_2^* &= \int_{\mathbf{Y}} y_2^f \, d\, \mu(\mathbf{y}_2|\mathbf{y}_1) - \delta(b_2, b_2^{*f}, \mathbf{y}_1)\, b_2^* \end{aligned} \quad (3.19)$$

and $V^{*,\mathcal{NP}}(b_2, b_2^{*f}, \mathbf{y}_1|i)$ is the ex-ante value of passing a preemptive restructuring:

$$V^{*,\mathcal{NP}}(b_2, b_2^{*f}, \mathbf{y}_1|i) = \int_{\mathbf{Y}} V^*(b_2, b_2^{*f}, \mathbf{y}_2|j)\, d\, (\mathbf{y}_2|\mathbf{y}_1),\ i = \underline{\text{D}}, \underline{\text{E}}, \underline{\text{F}}, \underline{\text{G}}, \text{ and } j = \underline{\text{H}}, \underline{\text{I}}, \underline{\text{J}}, \underline{\text{K}} \quad (3.20)$$

After realization of income in the period 2, the foreign creditor's ex-post value function is defined as follows:

$$V^*(b_2, b_2^{*f}, \mathbf{y}_2|j) = \left(1 - \mathbf{1}^{\mathcal{D}}(b_2, b_2^{*f}, \mathbf{y}_2|i)\right) V^{*,\mathcal{R}}(b_2, b_2^{*f}, \mathbf{y}_2|i) + \mathbf{1}^{\mathcal{D}}(b_2, b_2^{*f}, \mathbf{y}_2|i)\, V^{*,\mathcal{D}}(b_2, b_2^{*f}, \mathbf{y}_2|j) \quad (3.21)$$

where $V^{*,\mathcal{R}}(b_2, b_2^{*f}, \mathbf{y}_2|i),\ i = \underline{\text{H}}, \underline{\text{I}}, \underline{\text{J}}, \underline{\text{K}}$ is the value of sovereign's repayment choice:

$$\begin{aligned} V^{*,\mathcal{R}}(b_2, b_2^{*f}, \mathbf{y}_2|i) &= \nu(c_2) \qquad i = \underline{\text{H}}, \underline{\text{I}}, \underline{\text{J}}, \underline{\text{K}} \\ s.t. \quad c_2^* &= y_2^f + b_2^* + b_2^{*f} \end{aligned} \quad (3.22)$$

and $V^{*,\mathcal{D}}(b_2, b_2^{*f}, \mathbf{y}_2|i)$ is the value of sovereign's default/post-default choice:

$$\begin{aligned} V^{*,\mathcal{D}}(b_2, b_2^{*f}, \mathbf{y}_2|i) &= \nu(c_2) \qquad i = \underline{\text{H}}, \underline{\text{I}}, \underline{\text{J}}, \underline{\text{K}} \\ s.t. \quad c_2^* &= y_2^f + \alpha(b_2, b_2^{*f}, \mathbf{y}_2)\, b_2^* + b_2^{*f} \end{aligned} \quad (3.23)$$

3.3.4.2 Problem in the Period 1

We move onto the foreign creditor's problem in the period 1. Prior to the realization of income in the period 1, the foreign creditor's ex-ante value function is defined as follows:

$$V^{*,\text{EX-ANTE}}(b_1, b_1^{*f}, \mathbf{y}_0|\underline{\text{A}}) = \mathbf{1}^{\mathcal{P}}(b_1, b_1^{*f}, \mathbf{y}_0|\underline{\text{A}}) V^{*,\mathcal{P}}(b_1, b_1^{*f}, \mathbf{y}_0|\underline{\text{A}}) + \big(1 - \mathbf{1}^{\mathcal{P}}(b_1, b_1^{*f}, \mathbf{y}_0|\underline{\text{A}})\big) V^{*,\mathcal{NP}}(b_1, b_1^{*f}, \mathbf{y}_0|\underline{\text{A}}) \quad (3.24)$$

$V^{*,\mathcal{P}}(b_1, b_1^{*f}, y_0|\ \underline{\text{A}})$ is the ex-ante value of a preemptive restructuring:

$$V^{\mathcal{P}}(b_1, b_1^{*f}, \mathbf{y}_0 \mid \underline{\text{A}}) = \mathbf{1}_{\underline{\text{B}}_{\mathcal{P}H}}\ V^{*,\mathcal{P}}(b_1, b_1^{*f}, \mathbf{y}_0|\ \underline{\text{B}}_{\mathcal{P}^H}) + \big(1 - \mathbf{1}_{\underline{\text{B}}_{\mathcal{P}H}}\big)\ V^{*,\mathcal{P}}(b_1, b_1^{*f}, \mathbf{y}_0|\ \underline{\text{B}}_{\mathcal{P}^L}) \quad (3.25)$$

$$V^{*,\mathcal{P}}(b_1, b_1^{*f}, \mathbf{y}_0|\ \underline{\text{B}}_i) = \max_{b_2, b_2^{*f}} \int_{\mathbf{Y}} \Big[v(c_1^*) + \frac{1}{(1+r^*)} V(b_2, b_2^{*f}, \mathbf{y}_2)\ d\ \mu(\mathbf{y}_2|\mathbf{y}_1) \Big]\ d\ \mu(\mathbf{y}_1|\mathbf{y}_0)$$
$$s.t. \quad c_1^* + q(b_2, b_2^{*f}, \mathbf{y}_0)\ b_2^* \ = \ \int_{\mathbf{Y}} y_1^f\ d\ \mu(\mathbf{y}_1|\mathbf{y}_0) + \delta_i(b_1, b_1^{*f}, \mathbf{y}_0)\ b_1^*, \quad (3.26)$$

where $i = \mathcal{P}^H, \mathcal{P}^L$, and $V^{*,\mathcal{NP}}(b_1, \mathbf{y}_0|\ \underline{A})$ is the ex-ante value of passing preemptive restructuring:

$$V^{*,\mathcal{P}}(b_1, b_1^{*f}, \mathbf{y}_0|\ \underline{A}) = \int_{\mathbf{Y}} V^*(b_1, b_1^{*f}, \mathbf{y}_0|\underline{\text{C}})\ d\ \mu(\mathbf{y}_1|\mathbf{y}_0) \quad (3.27)$$

The price of bonds is determined by the foreign creditor's problem defined above:

$$q(b_2, b_2^{*f}, \mathbf{y}_0) = \begin{cases} \frac{1}{1+r^*} \quad \text{if} \quad b_2^* \leq 0 \\ \frac{\int_{\mathbf{Y}} \delta(b_2, b_2^{*f}, \mathbf{y}_1)\, d\,\mu(\mathbf{y}_1|\mathbf{y}_0)}{(1+r^*)} \quad \text{if } b_2^* > 0 \text{ and } \mathbf{1}^{\mathcal{P}}(b_2, b_2^{*f}, \mathbf{y}_1|\, \underline{\mathrm{B}}_i) = 1,\ i = \mathcal{P}^H, \mathcal{P}^L \\ \frac{\int_{\mathbf{Y}} \left\{ \left(1 - \mathbf{1}^{\mathcal{D}}(b_2, b_2^{*f}, \mathbf{y}_2|i)\right) + \mathbf{1}^{\mathcal{D}}(b_2, b_2^{*f}, \mathbf{y}_2|i) \int_{\mathbf{Y}} \alpha(b_2, b_2^{*f}, \mathbf{y}_2|i)\, d\,\mu(\mathbf{y}_2, \mathbf{y}_1) \right\} d\,\mu(\mathbf{y}_1, \mathbf{y}_0)}{(1+r^*)} \text{ o.w} \end{cases} \tag{3.28}$$

where $\mathbf{1}^{\mathcal{D}}$ is the indicator function th at equals 1 if default ($\mathcal{D}$) happens, and 0 otherwise.

After income realization in the period 1, the foreign creditor's ex-post value is defined as:

$$V^*(b_1, b_1^{*f}, \mathbf{y}_1|\underline{\mathrm{C}}) = \left(1 - \mathbf{1}^{\mathcal{D}}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{\mathrm{C}})\right) V^{*,\mathcal{R}}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{\mathrm{C}}) + \mathbf{1}^{\mathcal{D}}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{\mathrm{C}})\ V^{*,\mathcal{D}}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{\mathrm{C}}) \tag{3.29}$$

$V^{*,\mathcal{R}}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{\mathrm{C}})$ is the value of repayment:

$$V^{*,\mathcal{R}}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{\mathrm{C}}) = \max_{b_2^*}\ \nu(c_1^*) + \beta\, V^*(b_2, b_2^{*f}, \mathbf{y}_2)\, d\,\mu(\mathbf{y}_2|\mathbf{y}_1)$$
$$s.t. \quad c_1^* + q(b_2, b_2^{*f}, \mathbf{y}_1)\, b_2^* + \frac{1}{(1+r^*)} b_2^{*f} = y_1^f + b_1^* + b_1^{*f} \tag{3.30}$$

and $V^{*,\mathcal{D}}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{\mathrm{C}})$ is the value of default/post-default:

$$V^{*,\mathcal{D}}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{\mathrm{C}}) = \max_{b_2^*}\ \nu(c_1^*) + \beta\, V^*(b_2, b_2^{*f}, \mathbf{y}_2)\, d\,\mu(\mathbf{y}_2|\mathbf{y}_1)$$
$$s.t. \quad c_1^* - \alpha(b_2, b_2^{*f}, \mathbf{y}_1) b_1^* + \frac{1}{(1+r^*)} b_2^f = y_1^f + b_1^{*f} \tag{3.31}$$

The price of bonds is determined by the foreign creditor:

$$q(b_2, b_2^{*f}, \mathbf{y}_1) = \begin{cases} \frac{1}{1+r^*} \quad \text{if} \qquad b_2^* \ \leq \ 0 \\ \frac{\delta(b_2, b_2^{*f}, \mathbf{y}_1)}{(1+r^*)} \qquad \text{if} \qquad b_2^* > 0 \text{ and } \mathbf{1}^{\mathcal{P}}(b_2, b_2^{*f}, \mathbf{y}_1 | \ \underline{\text{B}}_i) = 1 \quad i = \mathcal{P}^H, \mathcal{P}^L \\ \frac{\left(1 - \mathbf{1}^{\mathcal{D}}(b_2, b_2^{*f}, \mathbf{y}_2 | i)\right) + \mathbf{1}^{\mathcal{D}}(b_2, b_2^{*f}, \mathbf{y}_2 | i) \ \int_{\mathbf{Y}} \alpha(b_2, b_2^{*f}, \mathbf{y}_2 | i) \ d \ \mu(\mathbf{y}_2, \mathbf{y}_1))}{(1+r^*)} \qquad \text{o.w} \end{cases} \tag{3.32}$$

where $\mathbf{1}^{\mathcal{D}}$ is the indicator function that equals 1 if default ($\mathcal{D}$) happens, and 0 otherwise.

3.3.5 Debt Renegotiation

Two types of debt renegotiations (preemptive and post-default) are symmetric and follow the form of a two-player one-round bargaining game with complete information.[12] For simplicity, we assume that the sovereign debtor always be a proposer, i.e., no change in the proposer. Appendix C considers the case the foreign creditor always be a proposer. In every round, the sovereign debtor proposes recovery rates or passes. If it proposes, then the foreign creditor chooses to accept or reject the proposal. If the offer with recovery rates (haircuts) is proposed and accepted, both parties agree on debt settlement. The sovereign repays the recovered debt payments to the foreign creditor. Otherwise, i.e., when the sovereign debtor passes or the foreign creditor rejects the offer after the sovereign debtor

[12] An alternative one-round bargaining game is Nash bargaining as in Yue (2010) and Asonuma and Trebesch (2016). However, the bargaining set-up assumes pre-determined (exogenous) sharing of surplus obtained from the bargaining game which does not necessarily reflect the reality of debt renegotiations.

proposes, there is no settlement. The sovereign debtor does not repay any recovered debt payments to the foreign creditor and is excluded from the market permanently.

As in Asonuma and Trebesch (2016), timing and outside options for two parties differ between preemptive and post-default renegotiations. Post-default renegotiation takes place after the current income realization and the sovereign debtor's default choice. Outside options are permanent autarky for the sovereign debtor and no recovered debt payments for the foreign creditor. Preemptive renegotiation takes place before the current income realization Outside options are passing a preemptive option for the sovereign debtor and ex-ante expected return on sovereign bonds for the foreign creditor.

We define some basic concepts of the game. A one-round bargaining game may be denoted by $(C, \beta, \frac{1}{1+r^*})$ where for each vector of income processes $\mathbf{y} \in Y$, $C(\mathbf{y})$ is the set of feasible utility vectors that may be agreed upon in that state. β and $1/(1+r^*)$ are the discount factors for the sovereign debtor (B) and foreign creditor (L), respectively. A payoff function is an element $\Delta(\mathbf{y}) \in C(\mathbf{y})$, where $\Delta_i(\mathbf{y})$ is the utility to player $i = B, L$.

As in Merlo and Wilson (1995), we focus on a game with stationary strategies, that is, the players' actions depend only on the current state $(b_t, b_t^{*f}, \mathbf{y}_t)$ $t = 1, 2$ and the current offer. We denote the sovereign debtor B's and the foreign creditor L's equilibrium strategies as follows:

- For post-default restructurings in $t = 1, 2$:

1. $\theta_B(b_t, b_t^{*f}, \mathbf{y}_t) = 1$ (propose) when the sovereign debtor B proposes and $\theta_L\ (b_t, b_t^{*f}, \mathbf{y}_t) = 1$ (accept) when the foreign creditor L accepts the offer.

2. $\theta_B(b_t, b_t^{*f}, \mathbf{y}_t) = 0$ (pass) when the sovereign debtor B passes and $\theta_L(b_t, b_t^{*f}, \mathbf{y}_t) = 0$ (reject) when the foreign creditor L rejects the offer.

- For preemptive restructurings in $t = 1, 2$,

1. $\tilde{\theta}_B(b_t, b_t^{*f}, \mathbf{y}_{t-1}) = 1$ (propose) when the sovereign debtor B proposes and $\tilde{\theta}_L(b_t, b_t^{*f}, \mathbf{y}_{t-1}) = 1$ (accept) when the foreign creditor L accepts the offer.

2. $\tilde{\theta}_B(b_t, b_t^{*f}, \mathbf{y}_{t-1}) = 0$ (pass) when the sovereign debtor B passes and $\tilde{\theta}_L(b_t, b_t^{*f}, \mathbf{y}_{t-1}) = 0$ (reject) when the foreign creditor L rejects the offer.

A stationary subgame perfect (SP) equilibrium is defined as the player B and L's equilibrium stationary strategies $\theta_B(.)$ and $\theta_L(.)$ for post-default restructurings, and $\tilde{\theta}_B(.)$ and $\tilde{\theta}_L(.)$ for preemptive restructurings; the payoff functions Γ and Γ^* for post-default restructurings, and Ψ and Ψ^* for preemptive restructurings. The expected payoff at preemptive and post-default debt renegotiations for the borrower B and lender L in period t, are shown as:

$$\begin{aligned}
\Gamma(b_t, b_t^{*f}, \mathbf{y}_t) &= \Gamma^B(b_t, b_t^{*f}, \mathbf{y}_t) \\
\Gamma^*(b_t, b_t^{*f}, \mathbf{y}_t) &= \Gamma^{*B}(b_t, b_t^{*f}, \mathbf{y}_t) \\
\Psi(b_t, b_t^{*f}, \mathbf{y}_{t-1}) &= \Psi^B(b_t, b_t^{*f}, \mathbf{y}_{t-1}) \\
\Psi^*(b_t, b_t^{*f}, \mathbf{y}_{t-1}) &= \Psi^{*B}(b_t, b_t^{*f}, \mathbf{y}_{t-1})
\end{aligned} \tag{3.33}$$

for $t = 1, 2$. Here the superscript denotes the identity of the proposer: Γ^B (Γ^{*B}) and Ψ^B (Ψ^{*B}) represents the borrower's (lender's) payoff when the borrower proposes.

3.3.5.1 Renegotiation in the Period 2

First, we start with post-default debt renegotiation. We denote the proposed recovery rates as α_2^B, the borrower's values of proposing and passing as V^{PRO} and V^{PASS}, and the lender's values of accepting and rejecting as $V^{*,ACT}$ and $V^{*,REJ}$, respectively. When the borrower B proposes and the lender L accepts the offer, the sovereign debtor repays the recovered debt payments, which is $-\alpha_2^B(b_2, b_2^{*f}, \mathbf{y}_2)(1 + r^*)b_2$, and remains excluded from the market temporarily and cannot issue new bond in the current period.

$$\begin{aligned}
V^{PRO}(b_2, b_2^{*f}, \mathbf{y}_2 \,|i) &= u(c_2) \qquad i = \underline{H}, \underline{I}, \underline{J}, \underline{K} \\
s.t. \quad c_2 &= (1 - \lambda_{\mathcal{D}}) y_2^h \;+\; \alpha_2^B(b_2, b_2^{*f}, \mathbf{y}_2)(1 + r^*)\; b_2
\end{aligned} \tag{3.34}$$

and,

$$V^{*,ACT}(b_2, b_2^{*f}, \mathbf{y}_2|i) \;=\; \nu(c_2) \qquad i = \underline{H}, \underline{I}, \underline{J}, \underline{K}$$
$$s.t. \quad c_2^* \;= y_2^f \;+\; \alpha_2^B(b_2, b_2^{*f}, \mathbf{y}_2)(1+r^*)\; b_2^* \;+\; b_2^{*f}$$

When the borrower B passes and the lender rejects, the sovereign debtor does not pay any recovered debt payments to the foreign creditor and it remains excluded from the market permanently.

$$\begin{aligned} V^{PASS}(b_2, b_2^{*f}, \mathbf{y}_2|i) \;&= u(c_2) \qquad i = \underline{H}, \underline{I}, \underline{J}, \underline{K} \\ s.t. \quad c_2 \;&= (1-\lambda_{\mathcal{D}})y_2^h \end{aligned} \tag{3.35}$$

and,

$$\begin{aligned} V^{*,REJ}(b_2, b_2^{*f}, \mathbf{y}_2|i) \;&= \nu(c_2^*) \qquad i = \underline{H}, \underline{I}, \underline{J}, \underline{K} \\ s.t. \quad c_2^* \;&= y_2^f + b_2^{*f} \end{aligned} \tag{3.36}$$

In equilibrium, the agreed recovery rates $\alpha_2^B(b_2, b_2^{*f}, \mathbf{y}_2)$ satisfy the following:

$$\begin{aligned} \alpha_2^B(b_2, b_2^{*f}, \mathbf{y}_2|i) \;=\; \text{argmax}\; & V^{PRO}(b_2, b_2^{*f}, \mathbf{y}_2|i) \qquad i = \underline{H}, \underline{I}, \underline{J}, \underline{K} \\ s.t. \quad & V^{PRO}(b_2, b_2^{*f}, \mathbf{y}_2|i) \geq V^{PASS}(b_2, b_2^{*f}, \mathbf{y}_2|i) \\ & V^{*,ACT}(b_2, b_2^{*f}, \mathbf{y}_2|i) \geq V^{*,REJ}(b_2, b_2^{*f}, \mathbf{y}_2|i) \end{aligned} \tag{3.37}$$

If both parties reach an agreement, the two parties' payoffs are as follows:

$$\begin{aligned} \Gamma^B(b_2, b_2^{*f}, \mathbf{y}_2|i) \;&= V^{PRO}(b_2, b_2^{*f}, \mathbf{y}_2|i) \\ \Gamma^{*B}(b_2^*, b_2^{*f}, \mathbf{y}_2|i) \;&= V^{*,ACT}(b_2^*, b_2^{*f}, \mathbf{y}_2|i) \end{aligned} \tag{3.38}$$

Otherwise,

$$\begin{aligned}\Gamma^B(b_2, b_2^{*f}, \mathbf{y}_2|i) &= V^{PASS}(b_2, b_2^{*f}, \mathbf{y}_2|i) \\ \Gamma^{*B}(b_2^*, b_2^{*f}, \mathbf{y}_2|i) &= V^{*,REJ}(b_2^*, b_2^{*f}, \mathbf{y}_2|i) \end{aligned} \tag{3.39}$$

The renegotiation settlement for post-default case at node H,I,J,K can be characterized by settlement set $R(b_2, b_2^{*f}|i) \subset Y$. It is a set of vectors of income processes y_2 at which both parties reach an agreement:

$$\begin{aligned} R(b_2, b_2^{*f}, \mathbf{y}_2|i) = \Big\{ & y_2 \in Y | V^{PRO}(b_2, b_2^{*f}, \mathbf{y}_2|i) \geq V^{PASS}(b_2, b_2^{*f}, \mathbf{y}_2|i) \\ & \& \ V^{*,ACT}(b_2, b_2^{*f}, \mathbf{y}_2|i) \geq V^{*,REJ}(b_2, b_2^{*f}, \mathbf{y}_2|i) \Big\} \end{aligned} \tag{3.40}$$

for $i = \underline{H}, \underline{I}, \underline{J}, \underline{K}$.

Second, we consider preemptive debt renegotiation. We denote the proposed recovery rates as $\delta_2^B(.)$, the borrower's values of proposing and passing as V^{PRO} and V^{PASS}, and the lender's values of accepting and rejecting as $V^{*,ACT}$ and $V^{*,REJ}$, respectively. When the borrower B proposes and the lender accepts the offer, the sovereign debtor repays the recovered debt payments $-\delta_2^B(b_2, b_2^{*f}, \mathbf{y}_1)b_2$ and regains access to the market.

$$\begin{aligned} V^{PRO}(b_2, b_2^{*f}, \mathbf{y}_1|i) &= \int_Y u(c_2)\ d\ \mu(\mathbf{y}_2|\mathbf{y}_1) \qquad i = \underline{D}, \underline{E}, \underline{F}, \underline{G} \\ s.t. \quad c_2 &= (1-\lambda_{\mathcal{D}}) \int_Y y_2^h\ d\ \mu(\mathbf{y}_2|\mathbf{y}_1) \ + \ \delta_2^B(b_2, b_2^{*f}, \mathbf{y}_1)b_2 \end{aligned} \tag{3.41}$$

and,

$$\begin{aligned} V^{*,ACT}(b_2, b_2^{*f}, \mathbf{y}_1|i) &= \int_Y \nu(c_2^*)\; d\; \mu(\mathbf{y}_2|\mathbf{y}_1) \qquad i = \underline{D}, \underline{E}, \underline{F}, \underline{G} \\ s.t. \quad c_2^* &= \int_Y y_2^f\; d\mu(\mathbf{y}_2|\mathbf{y}_1) \; + \delta_2^B(b_2, b_2^{*f}, y_1) b_2^* \; + \; b_2^{*f} \end{aligned} \tag{3.42}$$

When the borrower B passes and the lender L rejects, the sovereign debtor passes its option for preemptive restructurings.

$$\begin{aligned} V^{PASS}(b_2, b_2^{*f}, \mathbf{y}_1|i) &= \int_Y V(b_2, b_2^{*f}, \mathbf{y}_2|j)\; d\mu(\mathbf{y}_2|\mathbf{y}_1),\; i = \underline{D}, \underline{E}, \underline{F}, \underline{G} \text{ and } j = \underline{H}, \underline{I}, \underline{J}, \underline{K} \\ V^{*,REJ}(b_2, b_2^{*f}, \mathbf{y}_1|i) &= \int_Y V^*(b_2, b_2^{*f}, \mathbf{y}_2|j)\; d\mu(\mathbf{y}_2|\mathbf{y}_1),\; i = \underline{D}, \underline{E}, \underline{F}, \underline{G} \text{ and } j = \underline{H}, \underline{I}, \underline{J}, \underline{K} \end{aligned} \tag{3.43}$$

In equilibrium, the agreed recovery rates $\delta_2^B(b_2, b_2^{*f}, \mathbf{y}_1)$ satisfy the following:

$$\begin{aligned} \delta_2^B(b_2, b_2^{*f}, \mathbf{y}_1|i) &= \text{argmax } V^{PRO}(b_2, b_2^{*f}, \mathbf{y}_1|i) \qquad i = \underline{D}, \underline{E}, \underline{F}, \underline{G} \\ & s.t. \quad V^{PRO}(b_2, b_2^{*f}, \mathbf{y}_1|i) \; \geq \; V^{PASS}(b_2, b_2^{*f}, \mathbf{y}_1|i) \\ & V^{*,ACT}(b_2, b_2^{*f}, \mathbf{y}_1|i) \; \geq \; V^{*,REJ}(b_2, b_2^{*f}, \mathbf{y}_1|i) \end{aligned} \tag{3.44}$$

If both parties reach an agreement, the two parties' payoffs are as follows:

$$\begin{aligned} \Psi^B(b_2, b_2^{*f}, \mathbf{y}_1|i) &= V^{PRO}(b_2, b_2^{*f}, \mathbf{y}_1|i) \\ \Psi^{*B}(b_2, b_2^{*f}, \mathbf{y}_1|i) &= V^{*,ACT}(b_2, b_2^{*f}, \mathbf{y}_1|i) \end{aligned} \tag{3.45}$$

Otherwise,

$$\Psi^{B}(b_2, b_2^{*f}, \mathbf{y}_1|i) \ = V^{PASS}(b_2, b_2^{*f}, \mathbf{y}_1|i)$$
$$\Psi^{*B}(b_2, b_2^{*f}, \mathbf{y}_1|i) \ = V^{*,REJ}(b_2, b_2^{*f}, \mathbf{y}_1|i) \quad (3.46)$$

The renegotiation settlement for preemptive case at node D, E, F, G can be characterized by settlement set $R^{\mathcal{P}}(b_2, b_2^{*f}|i) \subset Y$. It is a set of vectors of income processes $\mathbf{y}_1$ at which both parties reach an agreement:

$$R^{\mathcal{P}}(b_2, b_2^{*f}|i) = \Big\{\mathbf{y}_1 \in Y | V^{PRO}(b_2, b_2^{*f}, \mathbf{y}_1|i) \geq V^{PASS}(b_2, b_2^{*f}, \mathbf{y}_1|i)$$
$$\text{and} \quad V^{*,ACT}(b_2, b_2^{*f}, \mathbf{y}_1|i) \geq V^{*,REJ}(b_2, b_2^{*f}, \mathbf{y}_1|i)\Big\} \quad (3.47)$$

with $i = \underline{D}, \underline{E}, \underline{F}, \underline{G}$.

3.3.5.2 Renegotiation in the Period 1

First, we start with post-default debt renegotiation. We denote the proposed recovery rates as α_1^B, the borrower's values of proposing and passing as V^{PRO} and V^{PASS}, and the lender's values of accepting and rejecting as $V^{*,ACT}$ and $V^{*,REJ}$, respectively. When the borrower B proposes and the lender accepts the offer, the sovereign debtor repays the recovered debt payments, which is, $-\alpha_1^B(b_1, b_1^{*f}, \mathbf{y}_1)(1 + r^*)\ b_1$, and remains excluded from the market temporarily and cannot issue new bond in the current period.

$$V^{PRO}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{C}) \;= u(c_1) + \beta V(b_2, b_2^{*f}, \mathbf{y}_2)\; d\;(\mathbf{y}_2|\mathbf{y}_1)$$

$$s.t. \quad c_1 \;= (1-\lambda_{\mathcal{D}}) y_1^h \;+\; \alpha_1^B(b_1, b_1^{*f}, \mathbf{y}_1)(1+r^*)\; b_1 \tag{3.48}$$

and,

$$V^{*,ACT}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{C}) \;= \nu(c_1^*) \;+\; \beta V^*(b_2, b_2^{*f}, \mathbf{y}_2)\; d\; \mu(\mathbf{y}_2|\mathbf{y}_1)$$

$$s.t. \quad c_1^* \;= y_1^f \;+\; \alpha_1^B(b_1, b_1^{*f}, \mathbf{y}_1)(1+r^*)\; b_1^* \tag{3.49}$$

When the borrower B passes and the lender rejects, the sovereign debtor does not pay any recovered debt payments to the foreign creditor and it remains excluded from the market permanently.

$$V^{PASS}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{C}) \;= u(c_1) \;+\; \beta V(b_2, b_2^{*f}, \mathbf{y}_2)\; d\mu(\mathbf{y}_2|\mathbf{y}_1)$$

$$s.t. \qquad c_1 \;= (1-\lambda_{\mathcal{D}})\; y_1^h \tag{3.50}$$

and,

$$V^{*,REJ}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{C}) \;= \nu(c_1^*) \;+\; \beta V^*(b_2, b_2^{*f}, \mathbf{y}_2)\; d\; \mu(\mathbf{y}_2|\mathbf{y}_1)$$

$$s.t. \quad c_1^* \;= y_1^f \;+\; b_1^{*f} \tag{3.51}$$

In equilibrium, the agreed recovery rates $\alpha_1^B(b_2, b_2^{*f}, \mathbf{y}_2)$ satisfy the following:

$$\begin{aligned} \alpha_1^B(b_1, b_1^{*f}, \mathbf{y}_1) \; &= \text{argmax } V^{PRO}(b_1, b_1^{*f}, \mathbf{y}_1 | \underline{C}) \\ s.t. \quad V^{PRO}(b_1, b_1^{*f}, \mathbf{y}_1 | \underline{C}) \; &\geq \; V^{PASS}(b_1, b_1^{*f}, \mathbf{y}_1 | \underline{C}) \\ V^{*,ACT}(b_1, b_1^{*f}, \mathbf{y}_1 | \underline{C}) \; &\geq \; V^{*,REJ}(b_1, b_1^{*f}, \mathbf{y}_1 | \underline{C}) \end{aligned} \tag{3.52}$$

If both parties reach an agreement, the two parties' payoffs are as follows:

$$\begin{aligned} \Gamma^B(b_1, b_1^{*f}, \mathbf{y}_1) \; &= V^{PRO}(b_1, b_1^{*f}, \mathbf{y}_1 | \underline{C}) \\ \Gamma^{*B}(b_1, b_1^{*f}, \mathbf{y}_1) \; &= V^{*,ACT}(b_1, b_1^{*f}, \mathbf{y}_1 | \underline{C}) \end{aligned} \tag{3.53}$$

Otherwise,

$$\begin{aligned} \Gamma^B(b_1, b_1^{*f}, \mathbf{y}_1) \; &= V^{PASS}(b_1, b_1^{*f}, \mathbf{y}_1 | \underline{C}) \\ \Gamma^{*B}(b_1, b_1^{*f}, \mathbf{y}_1) \; &= V^{*,REJ}(b_1, b_1^{*f}, \mathbf{y}_1 | \underline{C}) \end{aligned} \tag{3.54}$$

The renegotiation settlement for post-default case at node $\underline{C}$ can be characterized by settlement set $R(b_1, b_1^{*f}) \subset Y$. It is a set of s of income processes at which both parties reach an agreement:

$$\begin{aligned} R(b_1, b_1^{*f}) = \{ \mathbf{y}_1 \in Y \mid V^{PRO}(b_1, b_1^{*f}, \mathbf{y}_1 | \underline{C}) \geq V^{PASS}(b_1, b_1^{*f}, \mathbf{y}_1 | \underline{C}) \\ \& \; V^{*,ACT}(b_1, b_1^{*f}, \mathbf{y}_1 | C) \geq V^{*,REJ}(b_1, b_1^{*f}, \mathbf{y}_1 | \underline{C}) \} \end{aligned} \tag{3.55}$$

Second, we consider preemptive debt renegotiation. We denote the proposed recovery rates as δ_1^B, the borrower's values of proposing and passing as V^{PRO}

and V^{PASS}, and the lender's values of accepting and rejecting as $V^{*,ACT}$ and $V^{*,REJ}$, respectively. When the borrower B proposes and the lender accepts the offer, the sovereign debtor repays the recovered debt payments $-\delta_1^B(b_1, b_1^{*f}, \mathbf{y}_0)b_1$ and regains access to the market.

$$\begin{aligned} V^{PRO}(b_1, b_1^{*f}, \mathbf{y}_0) \; &= \; \max_{b_2} \int_Y \left[u(c_1) \; + \; \beta V(b_2, b_2^{*f}, \mathbf{y}_2) \; d \; \mu(\mathbf{y}_2|\mathbf{y}_1) \right] \; d\mu(\mathbf{y}_1|\mathbf{y}_0) \\ s.t. \quad c_1 \; + \; q(b_2, b_2^{*f}, \mathbf{y}_0)b_2 \; &= \; (1 - \lambda_{\mathcal{P}}) \int_Y y_1^h \; d \; \mu(\mathbf{y}_1|\mathbf{y}_0) \; + \; \delta_1^B(b_1, b_1^{*f}, \mathbf{y}_0) \; b_1 \end{aligned} \tag{3.56}$$

and,

$$\begin{aligned} V^{*,ACT}(b_1, b_1^{*f}, \mathbf{y}_0) \; &= \; \max_{b_2^*} \int_Y \left[v(c_1^*) \; + \; \beta V^*(b_2, b_2^{*f}, \mathbf{y}_2) \; d \; \mu(\mathbf{y}_2|\mathbf{y}_1) \right] \; d \; \mu(\mathbf{y}_1|\mathbf{y}_0) \\ s.t. \quad c_1^* \; + \; q(b_2, b_2^{*f}, \mathbf{y}_0)b_2^* \; + \frac{1}{1+r^*} b_2^{*f} \; &= \; \int_Y y_1^f \; d \; \mu(\mathbf{y}_1|\mathbf{y}_0) \; + \; \delta_1^B(b_1, b_1^{*f}, \mathbf{y}_0) \; b_1^* \end{aligned} \tag{3.57}$$

When the borrower B passes and the lender rejects, the sovereign debtor passes its preemptive option.

$$\begin{aligned} V^{PASS}(b_1, b_1^{*f}, \mathbf{y}_0) \; &= \; \int_Y V(b_1, b_1^{*f}, \mathbf{y}_1|\underline{C}) \; d\mu(\mathbf{y}_1|\mathbf{y}_0) \\ V^{*,REJ}(b_1, b_1^{*f}, \mathbf{y}_0) \; &= \; \int_Y V^*(b_1, b_1^{*f}, \mathbf{y}_1|\underline{C}) \; d \; \mu(\mathbf{y}_1|\mathbf{y}_0) \end{aligned} \tag{3.58}$$

In equilibrium, the agreed recovery rates $\delta_2^B(b_2, b_2^{*f}, \mathbf{y}_1)$ satisfy the following:

$$
\begin{aligned}
\delta_1^B(b_1, b_1^{*f}, \mathbf{y}_1) \;&= \text{argmax } V^{PRO}(b_1, b_1^{*f}, \mathbf{y}_0|\underline{C}) \\
s.t. \quad & V^{PRO}(b_1, b_1^{*f}, \mathbf{y}_0) \;\geq\; V^{PASS}(b_1, b_1^{*f}, \mathbf{y}_0) \\
& V^{*,ACT}(b_1, b_1^{*f}, \mathbf{y}_0) \geq V^{*,REJ}(b_1, b_1^{*f}, \mathbf{y}_0)
\end{aligned} \tag{3.59}
$$

If both parties reach an agreement, the two parties' payoffs are as follows:

$$
\begin{aligned}
\Psi^B(b_1, b_1^{*f}, \mathbf{y}_0) \;&= V^{PRO}(b_1, b_1^{*f}, \mathbf{y}_0) \\
\Psi^{*B}(b_1, b_1^{*f}, \mathbf{y}_0) \;&= V^{ACT}(b_1, b_1^{*f}, \mathbf{y}_0)
\end{aligned} \tag{3.60}
$$

Otherwise,

$$
\begin{aligned}
\Psi^B(b_1, b_1^{*f}, \mathbf{y}_0) \;&= V^{PASS}(b_1, b_1^{*f}, \mathbf{y}_0) \\
\Psi^{*B}(b_1, b_1^{*f}, \mathbf{y}_0) \;&= V^{*,REJ}(b_1, b_1^{*f}, \mathbf{y}_0)
\end{aligned} \tag{3.61}
$$

The renegotiation settlement for preemptive case at node A can be characterized by settlement set $R^{\mathcal{P}}(b_1, b_1^{*f}|\underline{A}) \subset Y$. It is a set of vectors of income processes at which both parties reach an agreement:

$$
\begin{aligned}
R^{\mathcal{P}}(b_1, b_1^{*f}|\underline{A}) = \Big\{ \mathbf{y}_0 \in Y \;\Big|\; & V^{PRO}(b_1, b_1^{*f}, \mathbf{y}_0|\underline{A}) \geq V^{PASS}(b_1, b_1^{*f}, \mathbf{y}_0|\underline{A}) \\
& \& \; V^{*,ACT}(b_1, b_1^{*f}, \mathbf{y}_0|\underline{A}) \geq V^{*,REJ}(b_1, b_1^{*f}, \mathbf{y}_0|\underline{A}) \Big\}
\end{aligned} \tag{3.62}
$$

3.3.6 Market Clearing Conditions

If the sovereign repays its debt, a market clearing condition for goods is as follows:

$$\begin{aligned} c_1 \; + \; c_1^* \; &= y_1^h \; + \; y_1^f \\ c_2 \; + \; c_2^* \; &= y_2^h \; + \; y_2^f \end{aligned} \tag{3.63}$$

On the contrary, if the sovereign defaults, the following market clearing condition for goods holds:

$$\begin{aligned} c_1 \; &= (1 - \lambda_{\mathcal{D}}) y_1^h \, , \quad c_1^* = \; y_1^f \\ c_2 \; &= (1 - \lambda_{\mathcal{D}}) y_2^h \, , \quad c_2^* = \; y_2^f \end{aligned} \tag{3.64}$$

A market clearing condition for sovereign bonds is as follows:

$$\begin{aligned} b_2 + b_2^* &= 0 \\ b_1 + b_1^* &= 0 \end{aligned} \tag{3.65}$$

3.3.7 Equilibrium

A recursive equilibrium is defined as a set of functions for (a). the sovereign's ex-ante and ex post value functions, consumption, assets/debt, sets of preemptive restructuring and default/post-default restructuring, (b). the foreign creditor's consumption and assets/debt, (c). the sovereign's and the foreign creditor's decision functions, payoffs, recovery rates, settlement sets (given that the sovereign debtor proposes), (d). sovereign bond price such that:

1. The sovereign's value function, consumption, assets/debt, sets of preemptive restructuring and default/post-default restructuring satisfy its optimization problem (1)-(17).

2. The foreign creditor's consumption and assets/debt, sovereign bond prices satisfy his optimization problem (18)-(32).

3. Both parties' decisions, payoffs and recovery rates solve the one-round preemptive and post-default debt renegotiation problems (33)-(62)

4. The market clearing conditions for goods and sovereign bonds (63)–(65) are satisfied.

In equilibrium, default/post-default restructuring probability and preemptive restructuring probability are defined by the sovereign's default/post-default restructuring set and preemptive restructuring set:

$$
\begin{aligned}
p^{\mathcal{D}}(b_2, b_2^{*f}, \mathbf{y}_1|i) &= \int_{\mathcal{D}(b_2)} d\,\mu(\mathbf{y}_2|\mathbf{y}_1), \qquad i = \underline{D}, \underline{E}, \underline{F}, \underline{G} \quad \& \quad \text{Non-preemptive} \\
p^{\mathcal{D}}(b_1, b_1^{*f}, \mathbf{y}_0|\underline{C}) &= \int_{\mathcal{D}(b_1)} d\,\mu(y_1|y_0) \qquad \& \quad \text{Non-preemptive} \\
p^{\mathcal{P}}(b_2, b_2^{*f}, \mathbf{y}_1|i) &= \int_{\mathcal{D}(b_2)} d\,\mu(\mathbf{y}_2|\mathbf{y}_1) \qquad i = \underline{D}, \underline{E}, \underline{F}, \underline{G} \\
p^{\mathcal{P}}(b_2, b_2^{*f}, \mathbf{y}_1|\underline{A}) &= \int_{\mathcal{D}(b_2)} d\,\mu(\mathbf{y}_1|\mathbf{y}_0)
\end{aligned}
\tag{3.66}
$$

3.4 Quantitative Analysis

We apply the quantitative analysis of our model to two countries experiencing sequences of debt restructurings: (i) Argentina - post-default restructurings in 1988-93, 2001-05, 2019-20, (ii) Uruguay - preemptive restructurings in 1987-88, 1989-91, 2003. Our quantitative analysis of model is expected to replicate five stylized facts.

3.4.1 Parameters and Functional Forms

For our quantitative analysis, we follow parameter values and functional forms which are commonly used in the literature of sovereign debt. The sovereign's and the foreign creditor's utility is constant relative risk aversion (CRRA):

$$u(c_t) = \frac{(1 - c_t)^{1-\sigma}}{1-\sigma} \quad \text{and} \quad v(c_t^*) = \frac{(1 - c_t^*)^{1-\sigma^*}}{1-\sigma^*} \tag{3.67}$$

where σ and σ^* are the degree of risk aversion of the sovereign and the creditor which are $\sigma = \sigma^* = 2$. This follow the real business cycle (RBC) literature on advanced and emerging market countries—following Aguiar et al. (2016) and Lizarazo (2013). The risk-free interest rate is set to 1 percent corresponding to the average quarterly interest rate on the 3-month US Treasury bills (Aguiar et al. (2016); Yue (2010)).. We follow the convention in the literature to set a parameter value for creditor discount rate matching one-to-one with that for the risk-free interest rate (e.g., Aguiar et al. (2016): 0.98 vs. 1.7). The value of $\beta^* = 0.99$ is in the range of commonly used in the RBC literature on advanced economies. Output cost parameters are set as $\lambda_d = 0.02$ and $\lambda_p = 0.015$ following empirical estimates in Sturzenegger (2004) and Asonuma and Trebesch (2016) and assume symmetric lump-sum output costs of debt restructurings (e.g., Aguiar and Gopinath (2006); Yue (2010)).

The endowment processes are calibrated to match the quarterly seasonally adjusted GDP data from the Ministry of Economy and Production in Argentina (MECON) and the National Institute of Statistics and Censuses (INDEC) for Argentina and the Central Bank of Uruguay (CBU) for the sovereign, and the US Bureau of Economic Analysis (BEA) over 1993Q1–2013Q3. The data are detrended using a Hodrick–Prescott filter with a smoothing parameter of 1,600. As in previous studies (Arellano (2008); D'Erasmo et al. (2008); Benjamin and Wright (2009)), we assume the income processes of the sovereign and the foreign creditor to follow log normal AR(1) processes.

We consider independent processes and set the processes as follows:

$$\log(y_t^i) = \epsilon_{y,t}^i \log(y_{t-1}^i) + \epsilon_{y,t}^i \quad i = h, f \tag{3.68}$$

where an endowment shock $\epsilon_{y,t}^i$ is i.i.d. $N(0, \sigma^{i,2})$. With two separate autoregressive model results, we obtain $\epsilon_{y,t}^h = 0.85$ and $\sigma^{h,2} = 0.0017$ for Argentina, $\epsilon_{y,t}^h = 0.90$ and $\sigma^{h,2} = 0.0015$ for Uruguay, $\epsilon_{y,t}^f = 0.89$ and $\sigma^{f,2} = 0.0001$ for the US. Calibrated stochastic process is approximated as a discrete Markov chain of equally spaced grids by applying Tauchen (1986)'s quadrature approach.

Sturzenegger and Zettelmeyer (2006) and Asonuma and Trebesch (2016) report that Argentina and Uruguay experienced 7 and 6 debt restructurings, respectively, in 1820–2020. [13] Moreover, Sturzenegger and Zettelmeyer (2008) shows that the recovery rate (haircut) in Argentina 2001–05 post-default restructuring and Uruguayan 2003 preemptive restructuring were 25.0% (75.0%) and 87.1% (12.9%), respectively. We specify the sovereign's discount factor and bargaining power $\beta = 0.80$, $\phi = 0.93$ (Argentina) and $\beta = 0.80$, $\phi = 0.70$ (Uruguay) to replicate the average default frequency and recovery rate of 3.26% and 25.0% for Argentina and of 3.26% and 75.0% for Uruguay. [14]

3.4.2 Intuition: Underlying Mechanism

Before explaining calibration results, we provide intuition on underlying mechanism of "too little" sovereign debt restructuring. The key mechanism lies in the foreign creditor's consumption smoothing motive at the preemptive debt restructurings and the sovereign's new borrowing after preemptive restructurings. We explain in two steps: first, we explore determinants of haircuts at both preemptive and post-default restructurings; second, we explore how outcomes of both preemptive and post-default restructurings (i.e., haircuts) influence the sovereign's borrowing and choice of a subsequent restructuring.

[13] Asonuma and Trebesch (2016) provide the updated dataset covering 197 private external debt restructurings in 1975–2020 including Argentina 2019–20 episode.

[14] The sovereign's discount factor parameters, $\beta = 0.80$ for Argentina and $\beta = 0.80$ for Uruguay follow closely $\beta = 0.80$ for Argentina and $\beta = 0.80$ for Uruguay in Asonuma and Trebesch (2016) .

First, as shown in Asonuma and Trebesch (2016), preemptive debt restructurings have lower haircuts (higher recovery rates) because the foreign creditor has a high outside option, defined as his utility value without a preemptive option. He is more willing to reject an exchange offer and anticipate the sovereign's full repayment after good income realization. To be accepted by the foreign creditor ex ante, the sovereign has to propose an exchange offer which provides at least the foreign creditor's utility value in the absence of settlement on a preemptive restructuring (corresponding to his value of passing).

Most importantly, the risk averse foreign creditor's outside option is state-dependent and varies with his consumption-smoothing motive, in contrast with the risk-neutral creditor whose outside option remains constant. *When the foreign creditor has bad income*, he is more eager to recoup losses on debt subject to a preemptive restructuring for his consumption-smoothing purpose (i.e., high marginal utility of consumption from recovered debt payments) in the current round of preemptive renegotiations. Furthermore, his outside option remains relatively low because his income is expected to persist. The foreign creditor can accept moderately high haircuts (moderately low recovery rates) in the current round as long as these are comparable to the moderately high expected haircuts in future rounds. As a result, the sovereign can achieve settlement on preemptive restructuring with moderately high haircuts (i.e., a"preemptive restructuring with high haircut").

When the foreign creditor has high income, he is less eager to recoup losses on debt for his consumption-smoothing motive (i.e., low marginal utility of consumption from recovered debt payments) in the current round of preemptive renegotiations. His outside option remains relatively high because his income is expected to persist. The foreign creditor can accept moderately low haircuts (moderately high recovery rates) in the current round expecting that they receive the low expected haircuts in future rounds. As a result, the sovereign reach settlement on preemptive restructuring with low haircuts (i.e., a "preemptive restructuring with low haircut"). The underlying mechanism of the foreign creditor state-dependent consumption-smoothing motive in preemptive restructurings is the same with Asonuma and Joo (2020) in post-default restructurings.

Post-default restructurings have higher haircuts (smaller recovery rates). Once the sovereign defaults, it offers worse terms (i.e., high haircuts). The foreign creditor is willing to accept the offer because he has a low outside option such as his utility value with no recovered debt payments. As a result, the sovereign reach settlement on post-default restructuring with high haircuts (i.e., a "post-default restructuring with high haircut").

Second, as shown in Asonuma and Trebesch (2016), preemptive debt restructurings have quick re-access to the international capital market because the sovereign achieves quick settlement with short duration and low haircuts. The foreign creditor is more willing to provide new financing to the sovereign due to short restructuring duration and favorable outcome (low creditor losses). *When the sovereign achieves moderately high haircuts*, its debt level turns to moderate and issues new bonds with low borrowing costs (low bond spreads). The sovereign has more time to accumulate debt to high level and is less likely to restructure its debt (a “cured preemptive restructuring"). *When the sovereign results in low haircuts*, its debt level remains high and issues new bonds with high borrowing costs (high bond spreads). The sovereign has less time to accumulate debt to high level and is more likely to restructure its debt (a “non-cured preemptive restructuring").

Post-default restructurings have long re-access to the international capital market. This is because the sovereign has long restructuring duration and results in high haircuts. The foreign creditor is less reluctant to provide new financing to the sovereign due to long restructuring duration and high creditor losses (high haircuts). With high haircuts, the sovereign debt level turns to low and needs to spend more time to normalize its access to the international market and issue new bonds. It takes more time for the sovereign to accumulate debt to high level and is less likely to default or restructure its debt (a “cured post-default restructuring").

3.5 Conclusion

This paper studies the“too-little" (repeated) problem of sovereign debt restructurings both empirically and theoretically. Our empirical facts show that preemptive restructurings are more likely experience the “too-little" problem despite their swift debt crisis resolution. We build a two-period sovereign debt model with endogenous choice of restructuring strategies and renegotiations between the sovereign debtor and foreign creditor to rationalize the empirical facts. We also apply a quantitative exercise to both debt restructurings in Argentina and Uruguay in 1985-2020 to replicate the stylized facts and theoretical predictions.

Online Appendix

3.A Data Description and Definitions

3.A.1 Methodology

We truncate the original sovereign debt restructuring dataset of 197 observations from Asonuma and Trebesch (2016) down to the 175 observations that we use in this paper. We first drop the episodes of bond debt restructurings. These three bond restructuring episodes indicated by their country name and the start of the restructuring year are Pakistan 1999, Pomona 1987, and Dominican Republic 2004. We also drop the six episodes that started in or after 2017 regardless of whether the restructurings have concluded or not as we cannot tell if these episodes are "cured" restructurings at the time this paper is written. These six episodes with start year in or after 2017 are Argentina 2019, Belize 2020, Bardados 2018, Ecuador 2020, Mongolia 2017, and Chad 2017. This reduce the sample size to 188 observations.

In addition, We need to clean data for countries with overlapping and recurring restructurings to avoid the double counting in the empirical analysis of the paper. We drop restructuring episodes whose start and end year are both contained by an equal or longer episode by the same country. For example, Mexico has two restructurings that both started in 1984 and ended in 1985 and we keep one of these two episodes. As another example, Russia has two restructurings that both ended in 2000 while one started in 1999 and the other one started in 1998. We drop the first episode that started in 1999 for Russia the it is contained by the other longer episode that ended in the same year.

The 13 episodes that we drop due to overlapping issues are: Brazil 1989, Chile 1983, Mexico 1984, Nigeria 1982, Poland 1982, Poland 1986, Russia 1998, Russia 1999, Turkey 1976, Turkey 1981, and Ukraine 1998, Ukraine 2015, Yugoslavia 1983. This gives our final observation of 175 episodes.

3.A.2 Summary of restructuring episodes

Observations in the four restructuring groups

We list the restructuring episodes in the four groups we defined in this paper, namely Post-Default-Cured (DC), Default-Noncured (DNC), Preemptive-Cured (PC) and Preemptive-Noncured (PNC). Our baseline results of cured/non-cured uses the 5 year horizon. The detailed definition for each restructuring group is discussed in section 2.1 of the paper.

The 61 observations that are in the **Post-Default-Cured (DC**) group in chronicle order by the start the restructuring year are Albania 1991, Algeria 1993, Argentina 1988, Argentina 2001, Bolivia 1988, Bosnia and Herzegovina 1992, Brazil 1989, Bulgaria 1990, Cameroon 1985, Costa Rica 1986, Cote d'Ivoire 2011, Croatia 1991, Cuba 1985, Dominican Republic 1987, Dominican Republic 2004, Ecuador 1999, Ecuador 2008, Ethiopia 1990, Gabon 1989, Gambia 1984, Grenada 2013, Guinea

1991, Guyana 1993, Honduras 1990, Iraq 1986, Jordan 1989, Kenya 1992, Liberia 1980, Macedonia 1992, Madagascar 1987, Malawi 1987, Mauritania 1992, Moldova 2001, Mozambique 1983, Mozambique 2016, Nicaragua 1985, Niger 1986, North Macedonia 1992, Pakistan 1998, Panama 1987, Paraguay 1986, Poland 1989, Peru 1984, Russian 1998, Sao Tome and Principe 1984, Senegal 1992, Serbia 1992, Seychelles 2008, Sierra Leone 1980, Slovenia 1992, Sudan 1975, Tanzania 1981, Togo 1991, Uganda 1979, Venezuela 1989, Vietnam 1982, Yemen 1983, and Zambia 1983.

The 45 observations in the **Post-Default-Non-Cured (DNC)** group are Congo (Zaire) 1975, Turkey 1976, Nicaragua 1978, Bolivia 1980, Costa Rica 1981, Honduras 1981, Madagascar 1981, Poland 1981, Romania 1981, Senegal 1981, Argentina 1982, Congo (Zaire) 1982, Dominican Republic 1982, Guyana 1982, Madagascar 1982, Nigeria 1982, Poland 1982, Congo (Zaire) 1983, Cote d'Ivoire 1983, Cuba 1983, Morocco 1983, Nicaragua 1983, Nigeria 1983, Philippines 1983, Poland 1983, Venezuela 1983, Brazil 1984, Congo (Zaire) 1984, Costa Rica 1984, Cuba 1984, Madagascar 1985, Congo (Zaire) 1985, Guinea 1985, Congo (Zaire) 1986, Ecuador 1986, Gabon 1986, Nigeria 1986, Poland 1986, Poland 1986, Nigeria 1987, Togo 1987, Nigeria 1988, Poland 1988, Russia 1991, and Cote d'Ivoire 2000.

The 19 observations in the **Preemptive-Cured (PC)** group in chronicle order by the start the restructuring year are: Turkey 1981, Senegal 1985, Romania 1986, Yugoslavia 1987, Mexico 1988, Trinidad and Tobago 1988, Morocco 1989, Uruguay 1989, Chile 1990, Philippines 1990, South Africa 1992, Ukraine 2000, Dominica

2003, Uruguay 2003, Grenada 2004, Belize 2006, Greece 2011, St. Kitts and Nevis 2011, Ukraine 2015.

Finally, the 50 observations in the **Preemptive-Non-Cured (PNC)** group in chronicle order by the start the restructuring year are: Peru 1976, Jamaica 1977, Jamaica 1978, Peru 1979, Jamaica 1980, Nicaragua 1981, Nicaragua 1982, Brazil 1982, Ecuador 1982, Malawi 1982, Mexico 1982, Nicaragua 1982, Brazil 1983, Chile 1983, Ecuador 1983, Jamaica 1983, Niger 1983, Peru 1983, Romania 1983, Uruguay 1983, Yugoslavia 1983, Brazil 1984, Chile 1984, Ecuador 1984, Jamaica 1984, Mexico 1984, Niger 1984, Panama 1984, Yugoslavia 1984, South Africa 1985, Uruguay 1985, Chile 1986, Jamaica 1986, Mexico 1986, Philippines 1986, Venezuela 1986, Mexico 1987, Uruguay 1987, Philippines 1988, South Africa 1989, Algeria 1990, Senegal 1990, Ukraine 1998, Ukraine 1999, Moldova 2002, Belize 2012, Chad 2014, Mozambique 2015, Belize 2016.

Observations in the transition matrix

There're 9 observations that transit from **PNC to PC**. These restructuring episdoes in chronicle order are Romania 1983, Yugoslavia 1984, Morocco 1985, Chile 1986, Mexico 1987, Uruguay 1987, Philippines 1988, South Africa 1989, and Ukraine 1999.

There're 25 observations from **PNC to PNC**: Peru 1976, Jamaica 1977, Jamaica 1978, Peru 1979, Jamaica 1980, Nicaragua 1981, Brazil 1982, Mexico 1982, Ecuador

1982, Brazil 1983, Uruguay 1983, Chile 1983, Ecuador 1983, Niger 1983, Jamaica 1983, Uruguay 1983, Yugoslavia 1983, Chile 1984, Jamaica 1984, Mexico 1984, South Africa 1985, Uruguay 1985, Philippines 1986, Mexico 1986, Ukraine 1998, and Belize 2012.

There're 11 observations from **PNC to DC**: Malawi 1982, Peru 1983, Niger 1984, Panama 1984, Argentina 1985, Jamaica 1986, Venezuela 1986, Algeria 1990, Senegal 1990, Moldova 2002, and Mozambique 2015. There're 3 observations from **PNC to DNC**: Brazil 1982, Nicaragua 1984, and Ecuador 1984.

The observations that transition from the PNC group to the other groups are smaller than the total number of observations in the PNC of 50 reported above, since episodes cannot be known whether it's cured or not at the time we write this paper. For example, Belize experienced in 2012, 2016 and 2020 and adopted preemptive strategies for all. We can classify the transition from 2016 to 2016 as PNC to PNC but cannot decide on the transition from 2016 to 2020 as it's yet to be determined if the Belize 2020 restructuring will be cured or not.

There're 2 observations from **DNC to PC**: Turkey 1976 and Senegal 1981. There're 6 observations transition from **DNC to PNC**: Nicaragua 1978, Romania 1981, Argentina 1982, Venezuela 1983, Philippines 1983, and Morocco 1983.

There're 18 observations from **DNC to DC**: Bolivia 1980, Honduras 1981, Guyana 1982, Dominican Republic 1982, Nicaragua 1983, Congo 1983, Costa Rica 1984, Cuba 1984, Brazil 1984, Guinea 1985, Brazil 1986, Gabon 1986, Congo (Zaire)

1986, Ecuador 1986, Nigeria 1986, Togo 1987, Poland 1988, Russia 1991, and Cote d'Ivoire 2000.

There're 18 observations from **DNC to DNC**: Congo (Zaire) 1975, Poland 1981, Costa Rica 1981, Madagascar 1981, Poland 1981, Nigeria 1982, Poland 1982, Congo (Zaire) 1982, Cuba 1983, Poland 1983, Nigeria 1983, Congo (Zaire) 1983, Cote d'Ivoire 1983, Congo (Zaire) 1984, Congo (Zaire) 1985, Poland 1986, and Poland 1986, Nigeria 1987.

Different definitions on "Cured" vs. "Non-Cured"

If we use the definition of three-year horizon – that is, define a restructuring episode as cured if there's no recurring restructuring in three years following the conclusion of the current episode – seven episodes that were classified as non-cured under the five-year definition now become cured. These episodes with preemptive strategies are: Belize 2012, Belize 2016, Malawi 1982 and Peru 1979. These episodes that adopted post-default strategies are: Guinea 1985, Ecuador 1986, and Togo 1987.

If we use the definition of seven-year horizon, two episodes that were classified as cured under the five-year definition now become non-cured. They both adopted preemptive strategies and are Belize 2006 and Senegal 1985.

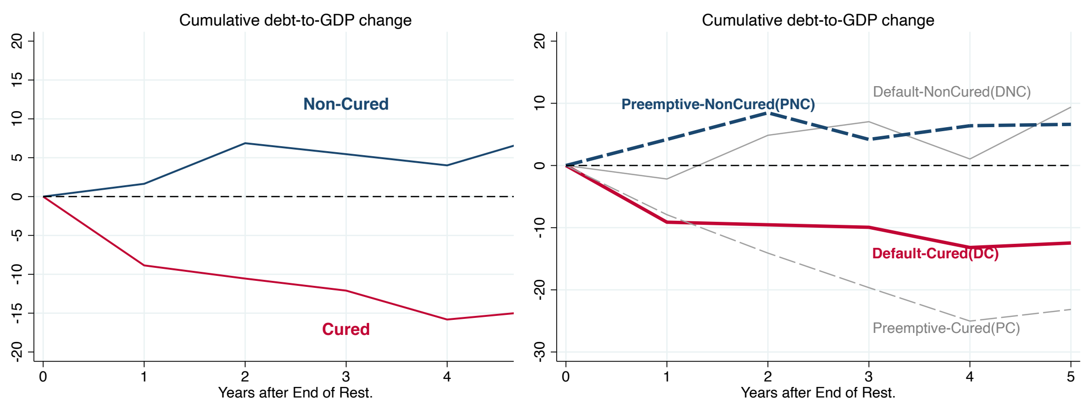
3.B Additional Graphs and Tables

Table 3.B.1: Number of Episodes for Strategies × Outcomes (Years ≥ 1990)

	Cured	Non-cured	cond. prob. for non-cured
Preemptive	14	10	42%
Post-default	55	5	8%

Note: This table reports the number of episodes for the four restructuring groups, namely Preemptive/Cured (PC), Preemptive/Non-Cured (PNC), Post-default/Cured (DC) and Post-default/Non-Cured (DNC), as well the conditional probability of non-cured under the preemptive and post-default restructuring strategy for the sub-sample *without* those episodes whose end years are before 1990. There's a total of 84 restructuring episodes in this table.

Figure 3.B.3: Average across four groups



Note: This panel of figures present the cumulative change of public debt-to-GDP ratio (in percentage points) following the end of the restructuring (Year = 0) for the sub-sample *without* those episodes whose end years are before 1990. Public debt-to-GDP ratio at the end of the restructuring year is normalized to zero. Panel (a) reports the cumulative change of average public debt-to-GDP ratio for the cured and the non-cured restructuring group. Panel (a) reports the average change of the same variable for the four restructuring groups, namely preemptive/cured (PC), preemptive/non-cured (PNC), post-default/cured (DC) and post-default/non-cured (DNC).

Table 3.B.2: T-test on debt-to-GDP ratio across different groups

H_A : mean (PNC) - mean (DC) >0 H_0 : mean (PNC) - mean (DC) = 0

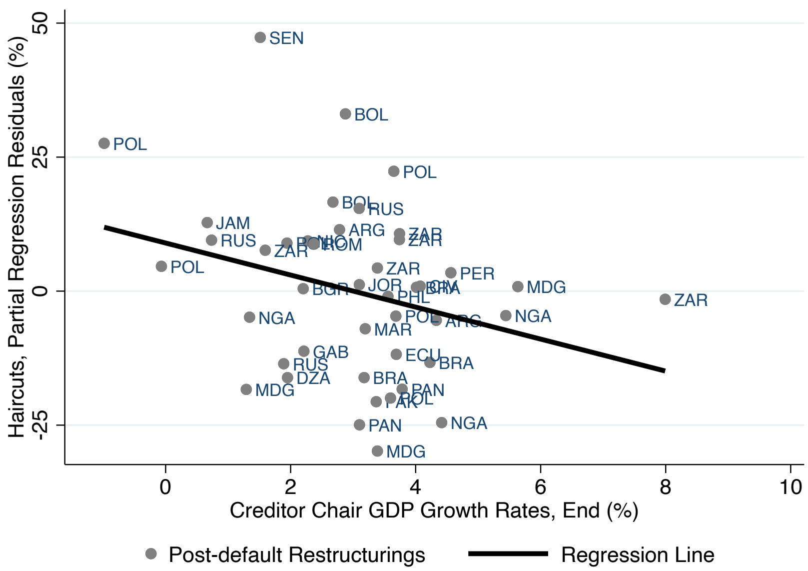
Year	1	2	3	4	5	6
diff.	18.25	24.15	21.58	28.63	26.54	24.54
std. err.	4.75	8.52	8.97	10.2	11.0	12.1
t-stats	3.83	2.83	2.41	2.81	2.413	2.03
p-value	0.0001	0.003	0.009	0.003	0.009	0.02
df.	82	80	78	78	76	75

H_A : mean (non-cured) - mean (cured) >0 H_0 : mean (non-cured) - mean (cured) = 0

Year	1	2	3	4	5	6
diff.	16.78	25.6	-25.1	35.1	40.3	38.8
std. err.	3.40	5.76	6.22	7.2	10.4	9.64
t-stats	4.93	4.46	4.04	4.88	3.88	4.03
p-value	0.00	0.00	0.00	0.00	0.0001	0.00
df.	131	130	127	127	122	121

Note: This table reports the t-test result of the difference in mean cumulative changes in public-debt-to-GDP ratio between the preemptive/cured (PC) and preemptive/non-cured (PNC) group, as reported in the top panel, and the t-test result of the difference in public-debt-to-GDP ratio between the non-cured and cured group, as reported in the bottom panel. Public-debt-to-GDP ratios are normalized to zero at year 0.

Figure 3.B.4: Haircuts and Creditor Chair GDP Growth Rates for Post-Default Restructurings



Note: The figures shows the unconditional scatter plot of NPV haircuts (%) and the creditor chair GDP growth (%) measured at the end of restructurings. Each Dot is a poest-default sovereign debt restructuring episode and labeled by the name of the debtor country. The black line is the correlation between haircuts (%) and the creditor chair GDP growth (%) with no additional controls except for the constant.

Table 3.B.3: Number of Episodes for Preemptive Restructurings

	NPV below mean	NPV above mean
Num. Episodes	58	11
Cond. prob. for Non-cured	0.76	0.55

Note: This table reports the number of episodes for the preemptive restructuring groups, divided by haircuts above mean and below the entire sample mean (of both preemptive and post-default restructurings). The mean NPV haircuts for the entire sample is 37.4%. The cured/non-cured definition follows the 5-year horizon (with the results robust under the 7-year horizon).

Table 3.B.4: Number of Episodes for Post-Default Restructurings

	NPV below mean	NPV above mean
Num. Episodes	41	65
Cond. prob. for Non-cured	0.63	0.29

Note: This table reports the number of episodes for the post-default restructuring groups, divided by haircuts above mean and below the entire sample mean (of both preemptive and post-default restructurings). The mean NPV haircuts for the entire sample is 37.4%. The cured/non-cured definition follows the 5-year horizon (with the results robust under the 7-year horizon).

3.C Additional Theoretical Derivations

This appendix explores the case where the lender L proposes. The expected payoff at preemptive and post-default debt renegotiations for the borrower B and lender L in period t, are shown as:

$$\begin{aligned} \Gamma(b_t, b_t^{*f}, \mathbf{y}_t) &= \Gamma^L(b_t, b_t^{*f}, \mathbf{y}_t) \\ \Gamma^*(b_t, b_t^{*f}, \mathbf{y}_t) &= \Gamma^{*L}(b_t, b_t^{*f}, \mathbf{y}_t) \\ \Psi(b_t, b_t^{*f}, \mathbf{y}_{t-1}) &= \Psi^L(b_t, b_t^{*f}, \mathbf{y}_{t-1}) \\ \Psi^*(b_t, b_t^{*f}, \mathbf{y}_{t-1}) &= \Psi^{*L}(b_t, b_t^{*f}, \mathbf{y}_{t-1}) \end{aligned} \tag{3.C.1}$$

for $t = 1, 2$. Here the superscript denotes the identity of the proposer: $\Gamma^L(\Gamma^{*L})$ and $\Psi^L(\Psi^{*L})$ represents the borrower's (lender's) payoff when the lender proposes.

3.C.1 Renegotiation in the period 2

First, we start with post-default debt renegotiation. We denote the proposed recovery rates as $\alpha_2^L(.)$, the lender (L)'s values of proposing and passing as $V^{*,PRO}$ and $V^{*,PASS}$, and the borrower (B)'s values of accepting and rejecting as V^{ACT} and V^{REJ}, respectively. When the lender L proposes and the borrower B accepts the offer, the sovereign debtor repays the recovered debt payments, which is

$-\alpha_2^L(b_2, b_2^{*f}, \mathbf{y}_2)(1+r^*)b_2$, and remains excluded from the market temporarily and cannot issue new bond in the current period.

$$\begin{aligned} V^{ACT}(b_2, b_2^{*f}, \mathbf{y}_2\,|i) &= u(c_2) \qquad i = \underline{H}, \underline{I}, \underline{J}, \underline{K} \\ s.t. \quad c_2 &= (1-\lambda_{\mathcal{D}})y_2^h \;+\; \alpha_2^L(b_2^*, b_2^{*f}, \mathbf{y}_2)(1+r^*)\; b_2 \end{aligned} \tag{3.C.2}$$

and,

$$\begin{aligned} V^{*,PRO}(b_2^*, b_2^{*f}, \mathbf{y}_2|i) &= \nu(c_2) \qquad i = \underline{H}, \underline{I}, \underline{J}, \underline{K} \\ s.t. \quad c_2^* &= y_2^f \;+\; \alpha_2^L(b_2^*, b_2^{*f}, \mathbf{y}_2)(1+r^*)\; b_2^* \;+\; b_2^{*f} \end{aligned}$$

When the lender L passes and the borrower B rejects, the sovereign debtor does not pay any recovered debt payments to the foreign creditor and it remains excluded from the market permanently.

$$\begin{aligned} V^{REJ}(b_2, b_2^{*f}, \mathbf{y}_2|i) &= u(c_2) \qquad i = \underline{H}, \underline{I}, \underline{J}, \underline{K} \\ s.t. \quad c_2 &= (1-\lambda_{\mathcal{D}})y_2^h \end{aligned} \tag{3.C.3}$$

and,

$$\begin{aligned} V^{*,PASS}(b_2^*, b_2^{*f}, \mathbf{y}_2|i) &= \nu(c_2^*) \qquad i = \underline{H}, \underline{I}, \underline{J}, \underline{K} \\ s.t. \quad c_2^* &= y_2^f + b_2^{*f} \end{aligned} \tag{3.C.4}$$

In equilibrium, the agreed recovery rates $\alpha_2^L(b_2, b_2^{*f}, \mathbf{y}_2)$ satisfy the following:

$$\begin{aligned} \alpha_2^L(b_2, b_2^{*f}, \mathbf{y}_2|i) \;=\; \text{argmax}\; & V^{*,PRO}(b_2, b_2^{*f}, \mathbf{y}_2|i) \qquad i = \underline{H}, \underline{I}, \underline{J}, \underline{K} \\ s.t. \quad & V^{*,PRO}(b_2, b_2^{*f}, \mathbf{y}_2|i) \geq V^{*,PASS}(b_2, b_2^{*f}, \mathbf{y}_2|i) \\ & V^{ACT}(b_2, b_2^{*f}, \mathbf{y}_2|i) \geq V^{REJ}(b_2, b_2^{*f}, \mathbf{y}_2|i) \end{aligned} \tag{3.C.5}$$

If both parties reach an agreement, the two parties' payoffs are as follows:

$$\begin{aligned} \Gamma^L(b_2, b_2^{*f}, \mathbf{y}_2|i) \;&=\; V^{ACT}(b_2, b_2^{*f}, \mathbf{y}_2|i) \\ \Gamma^{*L}(b_2^*, b_2^{*f}, \mathbf{y}_2|i) \;&=\; V^{*,PRO}(b_2^*, b_2^{*f}, \mathbf{y}_2|i) \end{aligned} \tag{3.C.6}$$

Otherwise,

$$\begin{aligned} \Gamma^L(b_2, b_2^{*f}, \mathbf{y}_2|i) \;&=\; V^{REJ}(b_2, b_2^{*f}, \mathbf{y}_2|i) \\ \Gamma^{*L}(b_2^*, b_2^{*f}, \mathbf{y}_2|i) \;&=\; V^{*,PASS}(b_2^*, b_2^{*f}, \mathbf{y}_2|i) \end{aligned} \tag{3.C.7}$$

The renegotiation settlement for post-default case at node $\underline{H}$,$\underline{I}$,$\underline{J}$,$\underline{K}$ can be characterized by settlement set $R^L(b_2, b_2^{*f}|i) \subset \mathbf{Y}$. It is a set of vectors of income processes $\mathbf{y}_2$ at which both parties reach an agreement:

$$\begin{aligned} R^L(b_2, b_2^{*f}, \mathbf{y}_2|i) = \Big\{ & y_2 \in Y | V^{*,PRO}(b_2, b_2^{*f}, \mathbf{y}_2|i) \geq V^{*,PASS}(b_2, b_2^{*f}, \mathbf{y}_2|i) \\ & \& \; V^{ACT}(b_2^*, b_2^{*f}, \mathbf{y}_2|i) \geq V^{REJ}(b_2^*, b_2^{*f}, \mathbf{y}_2|i) \Big\} \end{aligned} \tag{3.C.8}$$

for $i = \underline{H}, \underline{I}, \underline{J}, \underline{K}$.

Second, we consider preemptive debt renegotiation. We denote the proposed recovery rates as $\delta_2^L(.)$, the lender's values of proposing and passing as $V^{*,PRO}$ and $V^{*,PASS}$, and the borrower's values of accepting and rejecting as V^{ACT} and V^{REJ}, respectively. When the borrower B proposes and the lender accepts the offer, the sovereign debtor repays the recovered debt payments $-\delta_2^L(b_2, b_2^{*f}, \mathbf{y}_1)b_2$ and regains access to the market.

$$\begin{aligned} V^{ACT}(b_2, b_2^{*f}, \mathbf{y}_1|i) &= \int_Y u(c_2)\, d\, \mu(\mathbf{y}_2|\mathbf{y}_1) \qquad i = \underline{D}, \underline{E}, \underline{F}, \underline{G} \\ s.t. \quad c_2 &= (1-\lambda_{\mathcal{D}}) \int_Y y_2^h\, d\, \mu(\mathbf{y}_2|\mathbf{y}_1) \;+\; \delta_2^L(b_2, b_2^{*f}, \mathbf{y}_1)b_2 \end{aligned} \tag{3.C.9}$$

and,

$$\begin{aligned} V^{*,PRO}(b_2^*, b_2^{*f}, \mathbf{y}_1|i) &= \int_Y \nu(c_2^*)\, d\, \mu(\mathbf{y}_2|\mathbf{y}_1) \qquad i = \underline{D}, \underline{E}, \underline{F}, \underline{G} \\ s.t. \quad c_2^* &= \int_Y y_2^f\, d\mu(\mathbf{y}_2|\mathbf{y}_1) \;+ \delta_2^L(b_2, b_2^{*f}, y_1)b_2^* \;+\; b_2^{*f} \end{aligned} \tag{3.C.10}$$

When the lender L passes and the borrower B rejects, the sovereign debtor passes its option for preemptive restructurings.

$$\begin{aligned} V^{REJ}(b_2, b_2^{*f}, \mathbf{y}_1|i) &= \int_Y \; V(b_2, b_2^{*f}, \mathbf{y}_2|j)\, d\mu(\mathbf{y}_2|\mathbf{y}_1)\; i = \underline{D}, \underline{E}, \underline{F}, \underline{G} \;\&\; = \underline{H}, \underline{I}, \underline{J}, \underline{K} \\ V^{*,pass}(b_2^*, b_2^{*f}, \mathbf{y}_1|i) &= \int_Y \; V^*(b_2^*, b_2^{*f}, \mathbf{y}_2|j)\, d\mu(\mathbf{y}_2|\mathbf{y}_1)\; i = \underline{D}, \underline{E}, \underline{F}, \underline{G} \;\&\; j = \underline{H}, \underline{I}, \underline{J}, \underline{K} \end{aligned} \tag{3.C.11}$$

In equilibrium, the agreed recovery rates $\delta_2^L(b_2, b_2^{*f}, \mathbf{y}_1)$ satisfy the following:

$$\begin{aligned}
\delta_2^L(b_2, b_2^{*f}, \mathbf{y}_1|i) \; &= \text{argmax } V^{PRO}(b_2, b_2^{*f}, \mathbf{y}_1|i) \qquad i = \underline{D}, \underline{E}, \underline{F}, \underline{G} \\
s.t. \quad V^{*,PRO}(b_2, b_2^{*f}, \mathbf{y}_1|i) \; &\geq \; V^{*,PASS}(b_2, b_2^{*f}, \mathbf{y}_1|i) \\
V^{ACT}(b_2^{*}, b_2^{*f}, \mathbf{y}_1|i) \; &\geq \; V^{REJ}(b_2^{*}, b_2^{*f}, \mathbf{y}_1|i) \qquad (3.C.12)
\end{aligned}$$

If both parties reach an agreement, the two parties' payoffs are as follows:

$$\begin{aligned}
\Psi^L(b_2, b_2^{*f}, \mathbf{y}_1|i) \; &= V^{ACT}(b_2, b_2^{*f}, \mathbf{y}_1|i) \\
\Psi^{*L}(b_2^{*}, b_2^{*f}, \mathbf{y}_1|i) \; &= V^{*,PRO}(b_2^{*}, b_2^{*f}, \mathbf{y}_1|i) \qquad (3.C.13)
\end{aligned}$$

Otherwise,

$$\begin{aligned}
\Psi^L(b_2, b_2^{*f}, \mathbf{y}_1|i) \; &= V^{REJ}(b_2, b_2^{*f}, \mathbf{y}_1|i) \\
\Psi^{*L}(b_2^{*}, b_2^{*f}, \mathbf{y}_1|i) \; &= V^{*,PASS}(b_2^{*}, b_2^{*f}, \mathbf{y}_1|i) \qquad (3.C.14)
\end{aligned}$$

The renegotiation settlement for preemptive case at node $\underline{D}$, $\underline{E}$, $\underline{F}$, $\underline{G}$ can be characterized by settlement set $R^{L,\mathcal{P}}(b_2, b_2^{*f}|i) \subset Y$. It is a set of vectors of income processes $\mathbf{y}_1$ at which both parties reach an agreement:

$$\begin{aligned}
R^{L,\mathcal{P}}(b_2, b_2^{*f}|i) = \Big\{ y_1 \in Y | V^{*,PRO}(b_2, b_2^{*f}, \mathbf{y}_1|i) \geq V^{*,PASS}(b_2, b_2^{*f}, \mathbf{y}_1|i) \\
\& \; V^{ACT}(b_2^{*}, b_2^{*f}, \mathbf{y}_1|i) \geq V^{REJ}(b_2^{*}, b_2^{*f}, \mathbf{y}_1|i) \Big\} \quad (3.C.15)
\end{aligned}$$

with $i = \underline{D}, \underline{E}, \underline{F}, \underline{G}$.

3.C.1.1 Renegotiation in the Period 1

First, we start with post-default debt renegotiation. We denote the proposed recovery rates as $\alpha_1^L(.)$, the lender's values of proposing and passing as $V^{*,PRO}$ and $V^{*,PASS}$, and the borrower's values of accepting and rejecting as V^{ACT} and V^{REJ}, respectively. When the lender L proposes and the borrower (B) accepts the offer, the sovereign debtor repays the recovered debt payments, which is, $-\alpha_1^L(b_1, b_1^{*f}, \mathbf{y}_1)(1+r^*)\ b_1$, and remains excluded from the market temporarily and cannot issue new bond in the current period.

$$
\begin{aligned}
V^{ACT}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{C}) &= u(c_1) + \beta V(b_2, b_2^{*f}, \mathbf{y}_2)\ d\ (\mathbf{y}_2|\mathbf{y}_1) \\
s.t. \quad c_1 &= (1-\lambda_{\mathcal{D}})y_1^h \ + \ \alpha_1^L(b_1^*, b_1^{*f}, \mathbf{y}_1)(1+r^*)\ b_1
\end{aligned}
\tag{3.C.16}
$$

and,

$$
\begin{aligned}
V^{*,PRO}(b_1^*, b_1^{*f}, \mathbf{y}_1|\underline{C}) &= v(c_1^*) \ + \ \beta V^*(b_2^*, b_2^{*f}, \mathbf{y}_2)\ d\ \mu(\mathbf{y}_2|\mathbf{y}_1) \\
s.t. \quad c_1^* &= y_1^f \ + \ \alpha_1^L(b_1^*, b_1^{*f}, \mathbf{y}_1)(1+r^*)\ b_1^*
\end{aligned}
\tag{3.C.17}
$$

When the lender L passes and the borrower B rejects, the sovereign debtor does not pay any recovered debt payments to the foreign creditor and it remains excluded from the market permanently.

$$
\begin{aligned}
V^{REJ}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{C}) \; &= u(c_1) \; + \; \beta \; V(b_2, b_2^{*f}, \mathbf{y}_2) \; d\mu(\mathbf{y}_2|\mathbf{y}_1) \\
s.t. \qquad c_1 \; &= (1 - \lambda_{\mathcal{D}}) \; y_1^h
\end{aligned} \tag{3.C.18}
$$

and,

$$
\begin{aligned}
V^{*,PASS}(b_1^*, b_1^{*f}, \mathbf{y}_1|\underline{C}) \; &= \nu(c_1^*) \; + \; \beta \; V^*(b_2^*, b_2^{*f}, \mathbf{y}_2) \; d \; \mu(\mathbf{y}_2|\mathbf{y}_1) \\
s.t. \quad c_1^* \; &= y_1^f \; + \; b_1^{*f}
\end{aligned} \tag{3.C.19}
$$

In equilibrium, the agreed recovery rates $\alpha_1^L(b_2, b_2^{*f}, \mathbf{y}_2)$ satisfy the following:

$$
\begin{aligned}
\alpha_1^L(b_1, b_1^{*f}, \mathbf{y}_1) \; &= \text{argmax} \; V^{*,PRO}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{C}) \\
& s.t. \quad V^{ACT}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{C}) \; \geq \; V^{REJ}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{C}) \\
& \qquad V^{*,PRO}(b_1^*, b_1^{*f}, \mathbf{y}_1|\underline{C}) \; \geq \; V^{*,PASS}(b_1^*, b_1^{*f}, \mathbf{y}_1|\underline{C})
\end{aligned} \tag{3.C.20}
$$

If both parties reach an agreement, the two parties' payoffs are as follows:

$$
\begin{aligned}
\Gamma^L(b_1, b_1^{*f}, \mathbf{y}_1) \; &= V^{ACT}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{C}) \\
\Gamma^{*L}(b_1^*, b_1^{*f}, \mathbf{y}_1) \; &= V^{*,PRO}(b_1^*, b_1^{*f}, \mathbf{y}_1|\underline{C})
\end{aligned} \tag{3.C.21}
$$

Otherwise,

$$\Gamma^L(b_1, b_1^{*f}, \mathbf{y}_1) \ = V^{REJ}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{C})$$
$$\Gamma^{*L}(b_1^*, b_1^{*f}, \mathbf{y}_1) \ = V^{*,PASS}(b_1^*, b_1^{*f}, \mathbf{y}_1|\underline{C}) \tag{3.C.22}$$

The renegotiation settlement for post-default case at node $\underline{C}$ can be characterized by settlement set $R^L(b_1, b_1^{*f}) \subset \mathbf{Y}$. It is a set of s of income processes at which both parties reach an agreement:

$$R^L(b_1, b_1^{*f}) = \{y_1 \in Y \mid V^{*,PRO}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{C}) \geq V^{*,PASS}(b_1, b_1^{*f}, \mathbf{y}_1|\underline{C})$$
$$\& \ V^{ACT}(b_1^*, b_1^{*f}, \mathbf{y}_1|C) \geq V^{REJ}(b_1^*, b_1^{*f}, \mathbf{y}_1|\underline{C})\} \tag{3.C.23}$$

Second, we consider preemptive debt renegotiation. We denote the proposed recovery rates as δ_1^B, the lender's values of proposing and passing as $V^{*,PRO}$ and $V^{*,PASS}$, and the borrower's values of accepting and rejecting as V^{ACT} and V^{REJ}, respectively. When the lender L proposes and the borrower B accepts the offer, the sovereign debtor repays the recovered debt payments $-\delta_1^L(b_1, b_1^{*f}, \mathbf{y}_0)b_1$ and regains access to the market.

$$V^{ACT}(b_1, b_1^{*f}, \mathbf{y}_0) \ = \max_{b_2} \int_Y \left[u(c_1) \ + \ \beta V(b_2, b_2^{*f}, \mathbf{y}_2) \ d \ \mu(\mathbf{y}_2|\mathbf{y}_1)\right] \ d\mu(\mathbf{y}_1|\mathbf{y}_0)$$
$$s.t. \quad c_1 \ + \ q(b_2^*, b_2^{*f}, \mathbf{y}_0)b_2 \ = \ (1 - \lambda_{\mathcal{P}}) \int_Y y_1^h \ d \ \mu(\mathbf{y}_1|\mathbf{y}_0) \ + \ \delta_1^L(b_1, b_1^{*f}, \mathbf{y}_0) \ b_1 \tag{3.C.24}$$

and,

$$V^{*,PRO}(b_1^*, b_1^{*f}, \mathbf{y}_0) \;=\; \max_{b_2^*} \int_Y \left[\nu(c_1^*) \;+\; \beta V^*(b_2^*, b_2^{*f}, \mathbf{y}_2)\; d\; \mu(\mathbf{y}_2|\mathbf{y}_1)\right]\; d\; \mu(\mathbf{y}_1|\mathbf{y}_0)$$

$$s.t. \quad c_1^* \;+\; q(b_2^*, b_2^{*f}, \mathbf{y}_0) b_2^* \;+ \frac{1}{1+r^*} b_2^{*f} \;=\; \int_Y y_1^f\; d\; \mu(\mathbf{y}_1|\mathbf{y}_0) \;+\; \delta_1^L(b_1, b_1^{*f}, \mathbf{y}_0)\; b_1^* \tag{3.C.25}$$

When the lender L passes and the borrower B rejects, the sovereign debtor passes its preemptive option.

$$\begin{aligned} V^{REJ}(b_1, b_1^{*f}, \mathbf{y}_0) \;&=\; \int_Y V(b_1, b_1^{*f}, \mathbf{y}_1|\underline{C})\; d\mu(\mathbf{y}_1|\mathbf{y}_0) \\ V^{*,PASS}(b_1^*, b_1^{*f}, \mathbf{y}_0) \;&=\; \int_Y V^*(b_1^*, b_1^{*f}, \mathbf{y}_1|\underline{C})\; d\; \mu(\mathbf{y}_1|\mathbf{y}_0) \end{aligned} \tag{3.C.26}$$

In equilibrium, the agreed recovery rates $\delta_2^L(b_2, b_2^{*f}, \mathbf{y}_1)$ satisfy the following:

$$\begin{aligned} \delta_1^B(b_1, b_1^{*f}, \mathbf{y}_1) \; &= \text{argmax}\; V^{PRO}(b_1, b_1^{*f}, \mathbf{y}_0|\underline{C}) \\ & s.t. \quad V^{PRO}(b_1, b_1^{*f}, \mathbf{y}_0) \;\geq\; V^{PASS}(b_1, b_1^{*f}, \mathbf{y}_0) \\ & \qquad\quad V^{*,ACT}(b_1^*, b_1^{*f}, \mathbf{y}_0) \geq V^{*,REJ}(b_1^*, b_1^{*f}, \mathbf{y}_0) \end{aligned} \tag{3.C.27}$$

If both parties reach an agreement, the two parties' payoffs are as follows:

$$\begin{aligned} \Psi^L(b_1, b_1^{*f}, \mathbf{y}_0) \; &= V^{ACT}(b_1, b_1^{*f}, \mathbf{y}_0) \\ \Psi^{*L}(b_1^*, b_1^{*f}, \mathbf{y}_0) \; &= V^{*,PRO}(b_1^*, b_1^{*f}, \mathbf{y}_0) \end{aligned} \tag{3.C.28}$$

Otherwise,

$$
\begin{aligned}
\Psi^{L}(b_1, b_1^{*f}, \mathbf{y}_0) &= V^{REJ}(b_1, b_1^{*f}, \mathbf{y}_0) \\
\Psi^{*L}(b_1^{*}, b_1^{*f}, \mathbf{y}_0) &= V^{*,PASS}(b_1^{*}, b_1^{*f}, \mathbf{y}_0)
\end{aligned} \tag{3.C.29}
$$

The renegotiation settlement for preemptive case at node A can be characterized by settlement set $R^{L,\mathcal{P}}(b_1, b_1^{*f}|\underline{A}) \subset Y$. It is a set of vectors of income processes at which both parties reach an agreement:

$$
\begin{aligned}
R^{L,\mathcal{P}}(b_1, b_1^{*f}|\underline{A}) = \Big\{ \mathbf{y}_0 \in Y \Big| & V^{*,PRO}(b_1, b_1^{*f}, \mathbf{y}_0|\underline{A}) \geq V^{*,PASS}(b_1, b_1^{*f}, \mathbf{y}_0|\underline{A}) \\
& \& V^{ACT}(b_1, b_1^{*f}, \mathbf{y}_0|\underline{A}) \geq V^{REJ}(b_1^{*}, b_1^{*f}, \mathbf{y}_0|\underline{A}) \Big\}
\end{aligned} \tag{3.C.30}
$$

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