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Author
Heckman, Harry H.

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WHAT'S NEW AND OLD IN THE RANGE-ENERGY RELATION

Harry H. Heckman

Lawrence Radiation Laboratory

University of California

Berkeley, California

A couple of weeks ago when Dr. Lloyd Hyman asked me to step in and give the late Walter Barkas' paper on the range-energy relation for emulsion to this conference, I was quite reluctant to accept. As you all know well, Walt Barkas was the authority on this topic, therefore a very difficult act to follow. Furthermore, I was not acutely aware of the "range-energy problems" of the hypernuclear physicist, having followed the course of hypernuclear physics as an interested, but uninvolved spectator. To partially remedy this difficiency, Drs. Hyman and Sacton undertook valiant efforts to supply me with pre-prints of papers submitted to this conference and a bibliography of pertinent literature on hyperfragments. For this I thank them very much.

Having accepted Dr. Hyman's request, I began to consider on what rational basis I could speak to you on the range-energy relation. From an examination of the apparent inconsistencies in the measured binding energy of the ${}_{\Lambda}H^4$, as determined by the (π^-, He^4) and $(\pi^-, p\,H^3)$ decay modes, and the dependence of the Λ^0 Q-value on the range of the π^-, it seemed to me that the emulsion technique and the range-energy relation were being pushed to their practical limits. The measurements of hyperfragment binding energies are now being done with sufficient statistical accuracy that systematic errors of about ±1% become the dominant source of error.

Such errors certainly arise from inaccuracies in the range-energy relation, and indeed, in the technical procedures and criteria involved in the measurement of, and the corrections to, the "range" of a particle track. Once one commits himself to an experiment that demands that the accumulative error be 1% and less, then nothing in the experiment can be taken for granted. This, I'm sure, would have been Walter Barkas' message to us today.

My own work on the range-energy relation for singly and multiply charged particles in emulsion has been primarily in the experimental aspects of the problem. Certainly the evolution of a range-energy relation necessarily involves stopping power theory, i.e., the Bethe-Bloch formula, and questions relating to the ionization energies, I, shell corrections, relativistic effects, charge neutralization, etc. And as the theory improves, so will the accuracy of the range-energy relation. Still, any improvements in stopping power theory will not be fully appreciated in a practical sense unless the technical details of range measurement are likewise examined, understood, and applied. It is from this latter point of view that I want to discuss the range-energy relation. Admittedly this presentation will be from one who views hyperfragment physics from the outside-in, rather than vice versa. My comments, then, may be quite naive and/or well known to many of you. Hopefully I might inject some points that some of you have not considered. Anyway, I believe it is worthwhile to re-examine this facet of the R-E relation so fundamental to hypernuclear physics.

Let us begin with what is "old" in the range-energy relation.

The range R of a particle of mass M (in units of the proton mass) and charge z at energy $\tau = \frac{T}{M}$ is given by the expression

$$R = \frac{M}{z^2}\left[\lambda(\tau) + z^{8/3}C_z(\beta/z)\right] + M\,C_{-1}\,(\lambda). \qquad (1)$$

The quantity $\lambda(\tau)$ is the range of a particle of positive charge and of unit (proton) mass -- an "ideal" particle in that it does not neutralize or otherwise interact with the stopping medium at low velocity. The term $C_z(\beta/Z)$ is the extension in range owing to the neutralization of the stopping particle, and $C_{-1}(\lambda)$ is a new correction to the range-energy relation that takes into account differences in the stopping power of a negatively charged particle. I shall discuss this latter effect in some detail later.

I want to consider each of these three quantities in order; first, the proton range- energy relation $\lambda(\tau)$. The "standard" R-E relation for emulsion of density $\rho = 3.815$ gm/cm^3, which I believe is valid to its quoted error of about ±1%, is given in Table 10.41, p. 438 of Ref. 1. Let us isolate the low, intermediate, and high energy regions for specific consideration. At low energies $0.1 < \tau < 2.0$ MeV, the range-energy relation is based on the energy loss rates deduced for emulsion from the data of W. Whaling[2], normalized to the proton range observed in emulsion from the thermal neutron reaction $N^{14}(n,p)C^{14}$, $\lambda = 6.47 \pm 0.12\ \mu$ at $\tau = 0.585 \pm 0.001$ MeV.[3] In the intermediate energy region, $2 < \tau < 40$ MeV, the R-E relation given by Barkas[1] is basically that given by him in the original range-energy experiment[4]. A slight correction to the range to account for the finite grain size of the initial and terminal grains of a particle track has been made to the original data to bring the low and intermediate regions of the range-energy relation into agreement at 2.0 MeV.[3] A least-squares fit, valid for $2 < \tau < 14$ MeV, is given by Heckman, et al[3]. It is:

$$\log_{10} \lambda = 1.1343 + 1.5276 \log_{10} \tau + 0.0882 (\log_{10} \tau)^2. \qquad (2)$$

This expression was used in Ref. 3 as the basis for developing the range-energy relation for heavy ions, $6 \leqslant Z \leqslant 18$, for $\tau \lesssim 10$ MeV. At low and intermediate energies, the range-energy relation is empirical. The data were subjected to systematic smoothing, however.[4]

Above $\tau = 40$ MeV, the high energy portion of the range-energy relation, the experimental data[5] were fitted to the Bethe-Bloch stopping power formula. The best fit was obtained for an ionization energy I = 331 eV. The computed R-E relation for emulsion using this value of I is given in Refs. 1 and 4.

Subsequent experiments have indicated that an I value of 331 eV may be slightly high. Barkas and von Friesen[6], by interpolating their measurements of the I values for Al, Pb, and U measured for proton energies up to 750 MeV, deduce I = 304 eV for emulsion. However, a direct measure of this quantity in this experiment gave I = 328 eV. Also, a critical examination of the range-energy relation by Barkas, Dyer, and Heckman[7] was made concurrent with their work on the Σ-hyperon masses. They concluded that, whereas the R-E relation below 40 MeV appeared to be well established, an ionization energy I = 319 ± 9 eV was necessary to obtain equal mass estimates of the Σ^+ from the protonic and mesonic decay modes. Also, to gain momentum balance in $K^- + p \rightarrow \Sigma^+ + \pi^-$, Barkas et al found that an I = 329 ± 20 eV was necessary. Their final estimate was I = 321 ± 8 eV. The Σ and K mass values quoted in this latter work was therefore based on an emulsion range-energy relation, I = 320 eV, tabulated by Barkas and Berger[8].

I would like to cite Reference 8 as a publication that reviews in detail the "state-of-the-art" of the subject on the penetration of charged particles in matter. The article by Barkas and Berger describes well their procedures for improving the computations of the energy losses and ranges of heavy particles in elemental absorbers, given the energy τ of the particle and I of the stopping medium. Because the ranges are given in moles of electrons/cm^2, one must divide the ranges computed for I = 320 eV by the factor 1.734 (= N_e in units of moles/ml) to obtain emulsion ranges in centimeters. We note that the range and energy loss at τ = 40 MeV given by Barkas and Berger (I = 320 eV) are equal to those given in the original range-energy paper.[1,4] Above τ = 100 MeV, however, the Barkas and Berger ranges are about 0.6% less than the 1958 values.

Barkas and Berger have attempted to summarize the entire body of range data by formulas of the form

$$\log \lambda = \log \frac{A}{Z} + \sum_{n=0}^{i} \sum_{m=0}^{i} a_{mn} \left(\log I\right)^m \left(\log \tau\right)^n \tag{3}$$

where the ionization energies I are given by the expressions

$$I/Z = 12 + 7/Z \text{ eV} \quad , \quad I < 163 \text{ eV} \tag{4a}$$

$$I/Z = 9.76 + 58.8\, Z^{-1.19} \quad , \quad I \geq 163 \text{ eV}. \tag{4b}$$

The coefficients a_{mn} for $1 \leq \tau \leq 9$ MeV (i = 2) and $7 \leq \tau \leq 1200$ MeV (i = 3) are given in Table I.

This formula fits the range data to an rms error of ±2% for $1 \leq \tau \leq 9$ MeV, and ± 0.6% for $7 \leq \tau \leq 1200$ MeV. In the latter case, the differential of Eq. 3 gives rates of energy loss that deviates from the input data by only ± 1.3% rms. This matrix formulation of the range-energy data has proven to be particularly useful in computor programs requiring range-energy computations for any element (A, Z).

The statistical accuracy of the range-energy relation in emulsion is usually taken to be ±0.5 to 1.0%. Such an error seems realistic, since checks on the λ vs τ relation over the past decade have not revealed errors in excess of 1%. Although the emulsion data on Λ-binding energies reported by Dr. Sacton this afternoon indicate there may be a discrepancy in the range of energetic pions at about 50 MeV (τ = 350 MeV), it does not appear to be more than 1%, hence within the error limits of the range relation.

Clearly, when any experiment demands range measurements to 1% and better, the techniques and criteria used to actually carry out the range measurement become important. Corrections to and systematic errors in, the range measurement may well introduce errors to order of 1%. Let me mention several points that I believe should be explicitly considered when accurate range measurements are undertaken. For example, how does one obtain the "true range" R from the measured range, R_{meas}? Here, we shall define R_{meas} as equal to the distance between the tangents to the extremities of the first and last grains, and/or to the mid-points of a well-defined (star) origin and termination. Several specific corrections made to R_{meas} to obtain R, assuming no loss of sensitivity, loss of emulsion, etc. at the emulsion surface, are as follows:

(a) For a star center or other well defined origin

$R = R_{meas} - \frac{\alpha}{2}$, where α is the mean grain diameter of the processed emulsion defined by $\alpha = -\frac{L \ln L}{B}$. L and B are the lacunarity and blob density of the track segment.[1]

(b) For a track entering a surface and stopping,

$$R = R_{meas} + L\langle G\rangle - \alpha$$

$$= R_{meas} + \frac{L}{B}[L + \ln L].$$

The term $L\langle G\rangle$ is the mean distance an entering track penetrates the emulsion before the first grain is produced. B and L apply at the emulsion surface under consideration.

(c) For a track that traverses the emulsion, with little or no change in B and L ,

$$R = R_{meas} + \frac{L}{B}\left[2L + \ln L\right] .$$

Range measurements are subject to a number of systematic errors, most of which can easily introduce 1% errors into the measurements. Let me enumerate possible sources of systematic errors that should be taken into account.

A. For short ranges: (1) Density fluctuations in the emulsion of about 1% can be expected, and can result in an increase of the range straggle. (2) Errors in the calibration of the eyepiece reticle can occur owing to non-uniform magnification of the objective lens. (This effect is particularly evident for the highly corrected, long-working distance lens.)

B. In passing from sheet to sheet, errors may arise from (1) loss of sensitivity of surface, e.g. by corrosion, wiping, etc; (2) a surface deposit, making exit points uncertain, and (3) air gaps not correctly being accounted for, if present.

Recommendations: (1) No wiping. (2) The "unseen" portion of the track between emulsion sheets can be estimated through the use of a pair of highly ionizing tracks to determine the apparent "gap" between the sheets. The average "air gap" can be estimated by comparing the average density of the stack with the actual density of the emulsion.

C. Gross distortions of an emulsion stack can be introduced by high-pressure clamping of the stack. The stack tends to contract when the pressure is released. Range corrections up to 0.5% have been made to correct for this effect.[5]

Recommendation: Mill stack with bakelite and plates to check initial and final dimensions of emulsion sheets.

D. Errors that arise from uncertainties in the shrinkage factor and density of the emulsion.

Recommendation: In addition to routine density measurements of the emulsion, internal checks of the shrinkage factor and density can be made via $\pi \rightarrow \mu$ and $\Sigma \rightarrow p$ decays; also, α-stars (the ThX, Rn and RdTh decay series) are useful here if the emulsions have been stored and used under uniform conditions.

E. Errors can be introduced in the R-E relation when it is assumed that I = 320 eV is valid ionization energy for a given emulsion batch, particularly when the emulsion density is significantly different from 3.815 gm/cm^2. Applying a correction to the measured ranges under the assumption that differences in emulsion density arise from differences in water content only may be erroroneous. A quantitative analysis of the chemical composition of the emulsion may be required to eliminate this systematic error in the range-energy relation.

The ultimate procedure is to establish the range energy for the particular emulsion stack one is using. This is a time consuming effort, but this is in fact being done in the experiments reported by Dr. Sacton. Particle decays and reactions that can be used for this calibration are

(a) $\pi \rightarrow \mu + \nu$
(b) $\Sigma^+ \rightarrow p + \pi^0$
(c) $\Sigma^+ \rightarrow \pi^+ + n$
(d) $\Lambda^0 \rightarrow \pi^- + p$
(e) $\tau^+ \rightarrow 3\pi$
(f) $K^+ \rightarrow \pi^+ + \pi^0$

and (g) $K^- + p \rightarrow \Sigma^+ + \pi^-$.

Now let's turn to what is "new" in the range-energy relation. Although not strictly new in the true sense of the word, there have been several published results on the range-momentum relation for slow Ag and Br ion in emulsion.[9,10,11] In the experiment of Henke and Benton,[11] Ag and Br recoils from incident 400-MeV argon ions were used to develope a range vs momentum relation. Because the low velocity Ag and Br ions were only partially ionized, their ranges were particularly useful in extending the $C_z(\beta/z)$ term in Eq. 1 to very low velocities, Fig. 1. Given in this figure are the results of Henke and Benton, compared with the data of Heckman, et al.[3] The two experiments appear to be in good agreement, and support the contention that the quantity C_z is a universal function of (β/z), and is independent of mass. The conclusion, then is that the range-energy for heavy ions at low velocities, e.g. crypto fragments, are calculatable from Eq. 1.

During the past few years there has been mounting evidence that the range of a particle in matter depends on the sign of the charge as well as mass and velocity.[7,12,13] That the range of the Σ^- produced in the $K^- + p \rightarrow \Sigma^- + \pi^+$ reaction at rest is some 25μ greater than expected from the range energy relation demonstrates this effect most clearly. However, range difference measurements tell us only that at some velocity, the difference in the ranges of a positive and negative particle is some quantity ΔR. Lacking is information on the how and where this range difference comes about. An obvious next step in the pursuit of this problem is to observe directly the difference in the energy-loss rates of stopping positive and negative particles. It was with this objective in mind that Peter Lindstrom and I undertook the experiment that I now want to describe. It was our feeling that, because range differences of several percent had been measured for positive and negative pions over ranges of the order of 100 μ, the rates of ionization may well differ by some 10% at low velocities. If so, this could be observed as differences in the grain densities of stopping pions in nuclear emulsion.

The data I shall discuss were obtained from a stack of Ilford G.5 emulsions that was exposed to beams of stopping π^+ and π^- mesons. As some of you may well know, the tracks of stopping pions in G.5 emulsions are highly saturated. It was therefore necessary to limit the development of the emulsion so that ionization measurements were possible, yet would permit the pions to be unambiguously identified as to charge by the emulsion scanner. Figure 2 illustrates how we divided the last 200 μ of the stopping π^- and π^+ tracks into cells, nominally 5 to 50 μ in length. The first five of these are super-imposed on the tracks in this illustration. The data we recorded for each cell of the charge-identified pions were (1) the number of blobs, B, (2) the linear fraction of the cell that consisted of gaps, L, and (3) the start and end coordinates of the cell. There was one operational procedure we had to establish however before any measurements could be made -- namely, where to start the measurements. We have tried to demonstrate this problem in this slide. In case of the π^-, the last grain could easily be the first grain of a heavily ionizing star prong. On the other hand, if we use the μ-meson decay track as a guide, there is no blob at all at the ending of the π^+. Therefore, it was decided that the grain density measurements had to begin at the first well defined blob of the stopping pion track that was seperated from the end by a measurable gap. We eliminated, therefore, the ambiguous terminal blob of the track. The actual starting points of the measurements were distributed about an average 1.1 μ from the pion endings.

Table II gives the results of our grain density measurements. Listed for each cell are the average range intervals over which the B and L measurements were made, the mean π^+ velocity and two estimates of the grain density ratio g_+/g_-. In Column (a), the grain density ratios are given by the ratios $(B/L)_+/(B/L)_-$ and in (b) by $\ln L_+/\ln L_-$. These data are based upon a total of 1.85×10^5 blob and gap length measurements and are the compilation of the results of five scanners from eleven different emulsion plates. The two values of g_+/g_- are not independent measurements, but do serve to check on the overall accuracy of the grain density measurements.

The data demonstrate quite clearly that there are grain density differences between the positive and negative pions. At the lowest velocity we were able to measure ($\beta = 0.051$), the grain density of the π^+ is greater than that of the π^- by some 7 to 8 per cent. Our results indicate that the grain density ratios decrease monotonically with increasing velocity, becoming consistent with unity at the higher velocities. How these grain density measurements translate into energy loss differences between positive and negative pions, plotted vs the pion range is shown in Fig. 3. The two data points for each cell (indicated by the hatched areas over the range scale) were obtained from the two estimates of the grain density ratio. At $\langle\beta\rangle = 0.051$, the stopping power of the π^+ meson is about 60 MeV/cm greater than that of the π^-. This corresponds to about 14% difference in the stopping powers. For velocities $\beta \gtrsim 0.14$, the energy-loss rates for the positive and negative pions are equal within the 1% statistical errors.

These data on energy loss differences can be also presented in terms of the differences in range. In this form we can compare directly the results of this experiment with existing data on range differences. Fig. 4, then, gives the differences in range between the π^- and π^+, ΔR, as a function of the range of the π^+. The data points are values of the range differences for pion ranges between 1.1 and 200 μ and correspond to velocity interval $\beta = 0.035$ to 0.183. The data are fitted quite well by an exponential function, asymptotic to $\Delta R = 6$ μ, having a characteristic length of 45 ± 10μ. This curve is drawn through the data points. If we express this curve in terms of λ, then we obtain the quantity $C_{-1}(\lambda)$, which is to be added to the expression for R (Eq. 1). It is $C_{-1}(\lambda) = (40 \pm 15)\left[1 - \exp\left(\frac{-\lambda}{300 \pm 70}\right)\right]$.

Because of our inability to measure grain densities for ranges between 0 and 1.1 μ, we have no information on the differences in the energy losses of the pions in this range interval. It is, in fact, the unknown behavior of the energy losses for the pions for ranges less than 1.1 μ that introduces the largest uncertainty in the estimate of the total range difference. This is illustrated here by the dashed curves above and below the data points. The top curve is the range difference expected if the energy lost by the π^+ meson in the first micron exceeds that of the π^- by an amount $\Delta E = 15$ keV, an arbitrary, but perhaps not unrealistic, value. The lower curve applies if $\Delta E = -15$ keV. We also have included in this figure the three measurements of range differences in emulsion. These are: (a) the Σ^- data,[7] where R and ΔR are normalized by a factor equal to the $\pi:\Sigma^-$ mass ratio, (b) The range difference of 1.6 MeV $\pi^{\pm}$ mesons;[12] and, (c) at 725 μ range, the 5.5 ± 3.2 μ range difference between the negative and positive pion observed in the mass-ratio experiment.[13]

This figure presents the current status of emulsion data on the stopping power differences between positive and negative particles. I believe it is demonstrated here that the reported range differences can be accounted for by the results of this latest experiment. The sign and magnitude of the differences in the energy-loss rates we have observed are sufficient to reproduce satisfactorily the range difference data. The pion data are within their respective experimental errors. The Σ^- data appear to be low, but there may still be unknowns in the behavior of stopping Σ^- hyperons. Possible evidence for this is the excessive range straggle of the sample of Σ^- hyperons used by Barkas, et al to reveal the Σ^--range anomaly.[7]

The experimental evidence thus shows that the rates of energy loss for positive particles exceed those for negative particles at equal velocities, when these velocities are comparable to those of the atomic electrons of the stopping medium. Experiments to determine the energy-loss differences at very low velocities are clearly needed. How the stopping powers of positive and negative particles depend on the atomic number of the stopping material is another problem that should be examined. Theoretical guide lines are conspicuously absent and are urgently needed.

A promising direction for theory is to examine how the Bethe-Bloch formula can be extended by using higher Born approximations. Barkas, who carried out some preliminary investigations toward this end, found that the second-order Born approximation introduces a term in the energy loss expression that is proportional to z^3 of the incident particle. Such a term is of the correct nature to account for the observations, but no estimate was made as to its magnitude.

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Figure Captions

Figure 1 Range extension correction, C_z vs 137 β/z. Solid line and error bars are the data of Henke and Benton.[11] The dashed curve is from Heckman, et al,[3] drawn through the O^{16}, Ne^{20}, and Ar^{40} points. The fission data are from Vigneron, Compt. Rend. 231, 1473 (1950).

Figure 2 Illustration of stopping π^- and π^+ meson in underdeveloped G.5 emulsion. The first five cells (5 to 25 μ in length) in which grain density measurements were made are superimposed on the tracks in this figure. Grain-density measurements were made between 1.1 and 200 μ from the pion endings.

Figure 3 The difference between the total rates of energy loss for positive and negative pions, $\iota_+ - \iota_-$, vs range. The energy-loss differences evaluated from the grain density ratios given in columns (a) and (b), Table II, are denoted by the symbols x and ⊛, respectively. The hatched areas above the range scale indicate the interval of range over which the ionization measurements were made.

Figure 4 The differences between the π^- and π^+ ranges, $\Delta R = R(\pi^-)-R(\pi^+)$ vs the pion range, as derived from the energy loss differences, Fig. 3. For ranges greater than 1.1 μ, ΔR can be represented by the function

$$\Delta R = 6\left[1 - \exp\left(-\frac{R-1.1}{45}\right)\right].$$

This curve is drawn through the data points. The dashed curves above and below the data illustrate how ΔR depends on the (unknown) difference in energy loss between 0 and 1.1 μ. The top curve applies if the total energy lost by the π^+ in the first micron exceeds that of the π^- by ΔE = 15 keV.

The lower curve applies if ΔE = -15 keV. The range differences reported in References 7, 12, and 13 are also shown.

Table II

Grain density ratios g_+/g_- as evaluated from (a) $(B/L)_+ \, (B/L)_-^{-1}$ and (b) $\ln L_+/\ln L_-$.

Cell	Range (μ)	⟨β⟩	g_+/g_- (a) $\frac{(B/L)_+}{(B/L)_-}$	g_+/g_- (b) $\frac{\ln L_+}{\ln L_-}$
1	1.1 - 5.1	0.051	1.078 ± 0.022	1.070 ± 0.019
2	-10.1	0.071	1.030 ± 0.016	1.035 ± 0.013
3	-15.2	0.084	1.018 ± 0.015	1.018 ± 0.016
4	-25.2	0.097	1.050 ± 0.013	1.050 ± 0.012
5	-50.3	0.117	1.002 ± 0.009	1.020 ± 0.008
6	-100.2	0.142	0.992 ± 0.011	0.988 ± 0.011
7	-149.9	0.163	1.002 ± 0.014	0.996 ± 0.012
8	-199.9	0.178	1.006 ± 0.014	1.001 ± 0.013

Table I

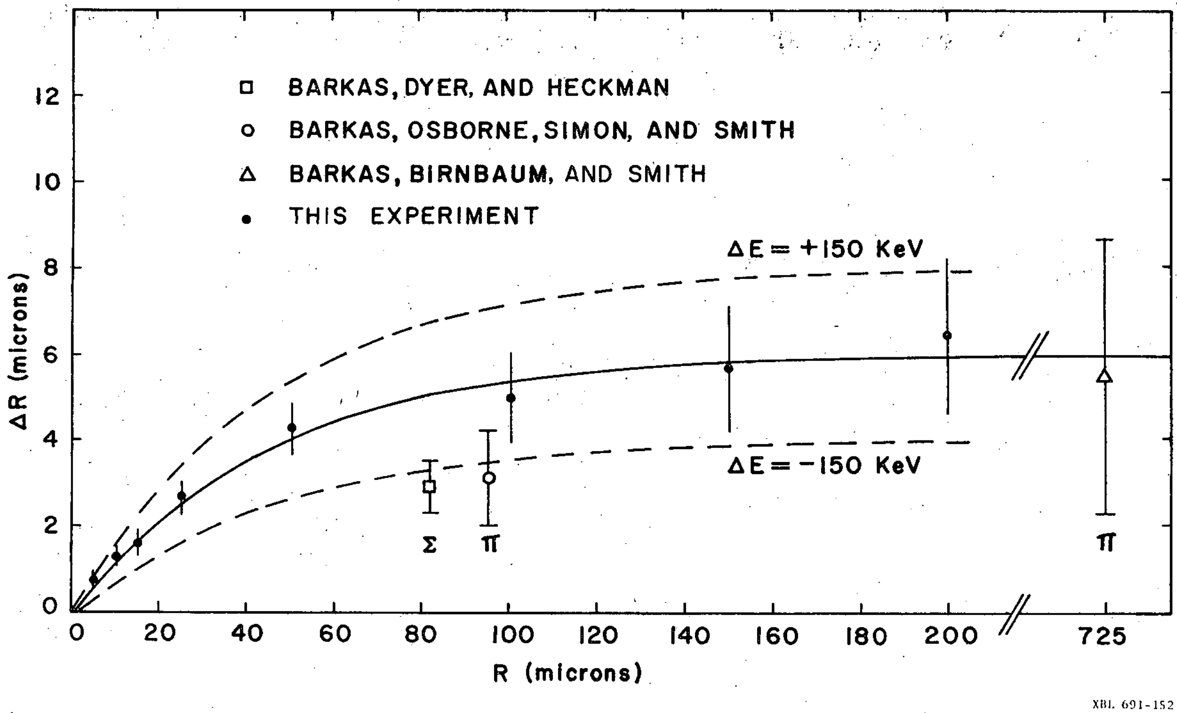
Coefficients a_{mn} in Equation 3.

(a) $1 \leq \tau \leq 9$ MeV

m \ n	0	1	2
0	-7.5265×10^{-1}	2.5398	-2.4598×10^{-1}
1	7.3736×10^{-2}	-3.1200×10^{-1}	1.1548×10^{-1}
2	4.0556×10^{-2}	1.8664×10^{-2}	-9.9661×10^{-3}

(b) $7 \leq \tau \leq 1200$ MeV

m \ n	0	1	2	3
0	-8.0155	1.8371	4.5233×10^{-2}	-5.9898×10^{-3}
1	3.6916×10^{-1}	-1.4520×10^{-2}	-9.5873×10^{-4}	-5.2315×10^{-4}
2	-1.4307×10^{-2}	-3.0142×10^{-2}	7.1303×10^{-3}	-3.3802×10^{-4}
3	3.4718×10^{-3}	2.3603×10^{-3}	-6.8538×10^{-4}	3.9405×10^{-5}



BARKAS, DYER, AND HECKMAN
BARKAS, OSBORNE, SIMON, AND SMITH
BARKAS, BIRNBAUM, AND SMITH
THIS EXPERIMENT
ΔE = +150 KeV
ΔE = −150 KeV
Σ
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R (microns)
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XBL 691-152

Fig. 1

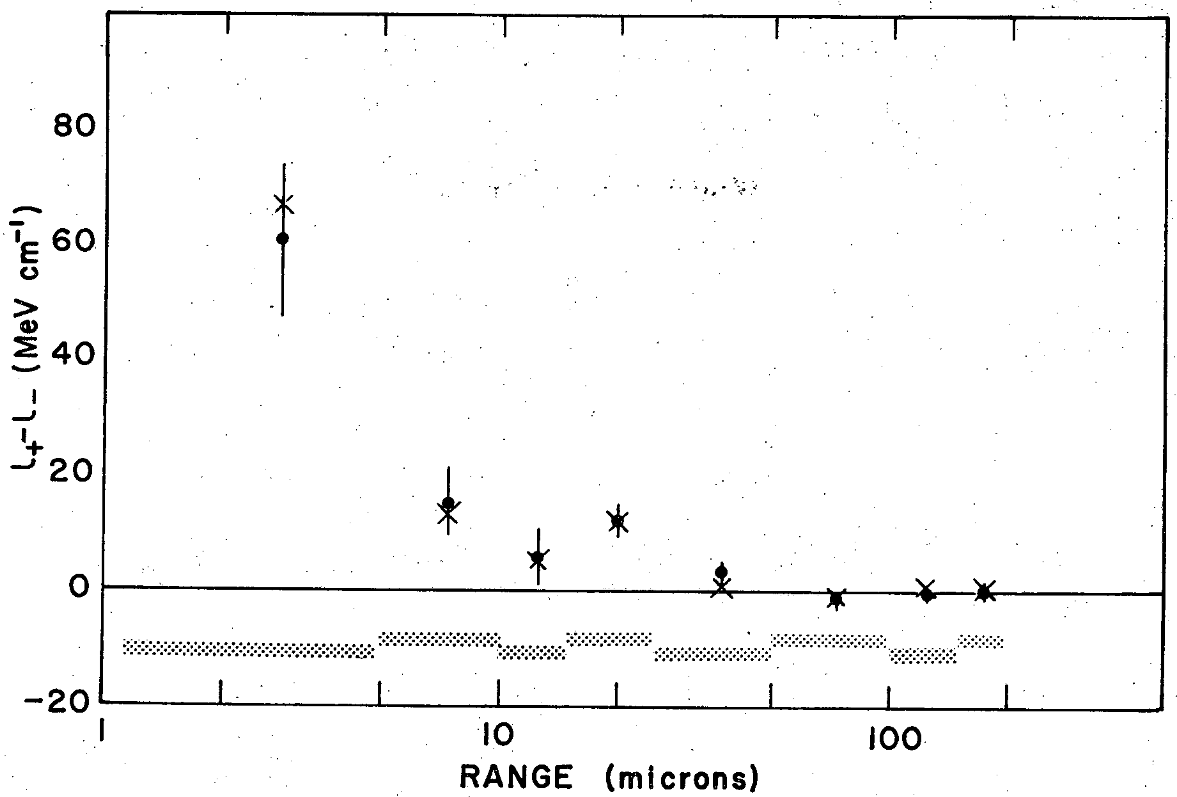

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Fig. 2

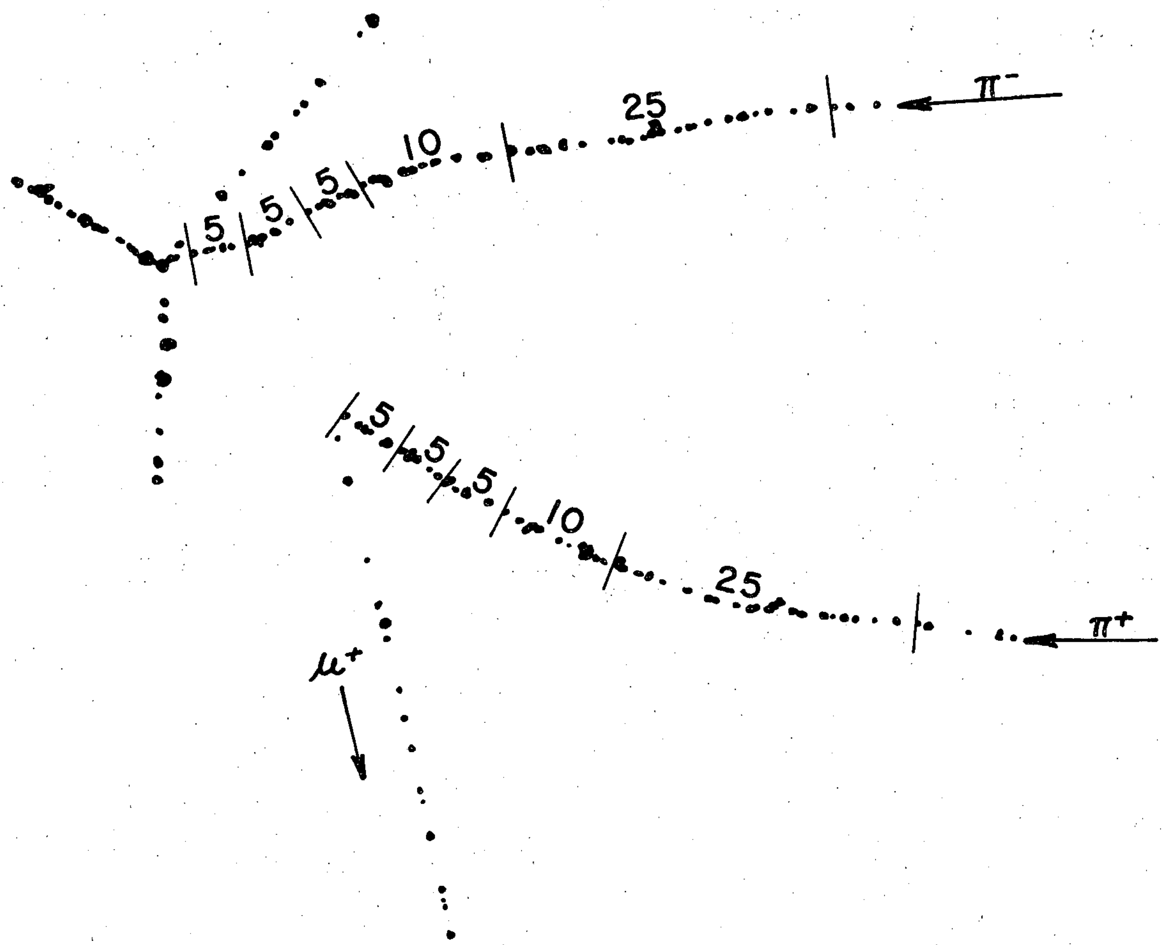

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Fig. 3

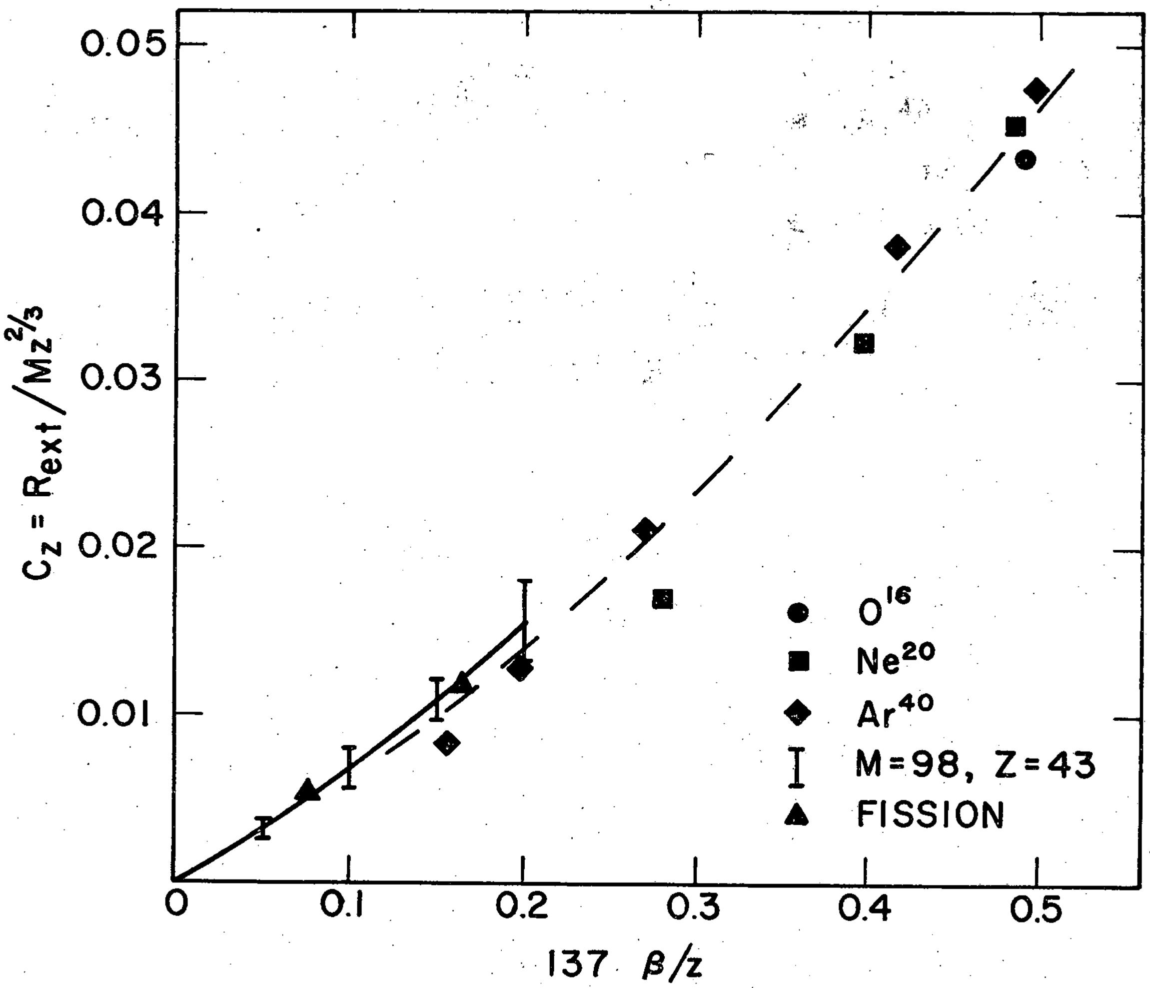

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Fig. 4