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### **Title**

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ABSTRACT

A number of different methods can be used to describe the magnetic properties of oriented Rare Earth Cobalt (REC) material. It will be shown how these different methods of description lead to different ways to think about, and to execute, the design of magnets that are useful for focusing and guiding charged particle beams. It will also be demonstrated that in some of these magnets, the REC material is used in a somewhat unusual way, requiring magnetic properties of the material that are usually not considered to be of great practical importance.

(1) INTRODUCTION

Because of conditions beyond my control, the preparation of a full manuscript was not possible before the deadline. Since this is a somewhat informal workshop, and not a formal conference, I take the option to give here only a summary of the content, and the "punchlines", of my talk, hoping that I will have a more complete paper ready for distribution at the workshop.

Main purpose of talk: tutorial, giving a personal view about the way different descriptions of magnetic properties of oriented Rare Earth Cobalt (REC) lead, alone or in combination, to the design of some permanent magnets that are important for charged particle optics. Only novel designs will be discussed.

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## (2) THREE USEFUL THEOREMS

(2.1) If  $\vec{j} = 0$  everywhere,

$$\int \vec{B} \cdot \vec{H} \, dv = 0 \quad (1)$$

if the integral is taken over whole volume where fields are not zero. (1) is valid no matter what magnetic properties the components of the system have.

(2.2) For all intents and purposes, in REC systems that do not have saturated steel, the fields produced by the REC components superimpose linearly.

(2.3) Easy axis rotation theorem<sup>1</sup>. If in a 2D, soft-steel free, REC system all easy axes are rotated by the angle  $+\beta$ , then all magnetic fields outside the REC are rotated by the angle  $-\beta$  without a change in magnitude.

## (3) DIRECT USE OF $B_{||}(H_{||})$ RELATIONSHIP OF REC

In systems that use unsaturated steel and REC, with the field in the REC homogeneous and parallel to the easy axis, eqn. (1) gives in good approximation (the steel term is omitted):

$$\left( \int B^2 \, dv / \mu_0 \right)_{\text{air, used}} + \left( \int B^2 \, dv / \mu_0 \right)_{\text{air, not used but unavoidable}} \quad (2)$$

$$+ B_{||} \cdot H_{||} \cdot V_{\text{REC}} = 0.$$

Eqn. (2) is useful to get a lower limit on  $V_{\text{REC}}$ . Eqn. (2) means that  $V_{\text{REC}}$  is minimized (for a fixed first term in eqn. (2)) by choosing  $(-B_{||} \cdot H_{||})_{\text{REC}} = \text{Max}$ . Only if the second term in eqn. (2) stays fixed with the first term in eqn. (2) when one changes the geometry. With this caveat, eqn. (2) can be quite useful as a guide in thinking and design.

$-B_{||} \cdot H_{||}$  is commonly called the "energy product". This is unfortunate not only because that term is clearly dimensionally incorrect, but even more so because  $-B_{||} \cdot H_{||}$  is not twice the magnetic energy density in the REC. Fortunately, I cannot think of any application (with the exception of the conversion of magnetic into thermal energy in cryogenic systems) where the real energy density in REC is needed.

(4) REPRESENTATION OF REC BY VACUUM PLUS  
"FROZEN" MAGNETIC DIPOLE DENSITY

If no steel is present, or if the steel configuration is determined from other considerations, this concept leads to easy optimization of orientation of easy axes: If fields are determined for 2 (or 3 for 3D case) orientations of the easy axis of a REC component, the optimum orientation is easily obtained. Idealized, and then practical, magnets obtained by this process: REC multipole magnets<sup>1</sup> for beam transport and focussing (only quadrupoles and sextupoles discussed here), linear array: (materials separators<sup>2</sup>, linear and helical undulators<sup>3</sup>), washer-like arrays for production of periodic solenoidal fields<sup>3</sup>.

Fig. 1 shows the crosssection of segmented pure REC quadrupole with easy axes indicated by arrows. With  $r_1$  and  $r_2$  representing the inside and outside radius of such a structure, a long magnet produces the 2D field given by

$$B_x - iB_y = 2B_r \cdot ((x + iy)/r_1) \cdot (1 - r_1/r_2) \cdot \frac{\sin \pi/M}{\pi/M} \cdot \cos^2 (\pi/M). \quad (3)$$

The number of segments in the magnet is designated by  $M$  and equals 16 for the magnet shown in Fig. 1. More details about these magnets, such as harmonic content, 3D fringe fields at the ends, etc., can be found in Ref. 1 and 3. One important note: if for  $|x + iy|/r_1 = 1$ ,  $r_1/r_2$  and  $M$  are such that  $|B_x| > B_r$ , some of the REC in this static magnet operates in the third quadrant of the  $B_{||}(H_{||})$  curve representation, where some manufacturers do not even supply information!!

Fig. 2a shows an 18 piece sextupole of the same design family, as originally designed by a user. Used to contain a plasma, electrons can reach the walls where the pieces are radially magnetized, leading to cooling problems there. Guided by the easy axis rotation theorem, the design in Fig. 2b puts the electron loss location between REC blocks where cooling is easy.

Fig. 3 shows a segmented REC undulator. One half of this type of structure has been used to separate magnetic from nonmagnetic materials<sup>2</sup>, and the full structure can be used to bend an electron beam (going generally from left to right) back and forth, thereby emitting synchrotron radiation. Again, description of the 2D and 3D fields produced by such a device can be found in Ref. 3.

#### (5) REPRESENTATION OF REC BY VACUUM PLUS CURRENT SHEETS

This is an extremely good method of description for persons that have experience with the design of electromagnets, because many known and well understood rules and concepts can be directly used.

#### (6) REPRESENTATION OF REC BY VACUUM PLUS MAGNETIC CHARGE SHEETS

This method of description is similar to the one discussed in section 5, but many people feel more comfortable with charges than with currents. In addition, it is generally easier to calculate the scalar potential from charges than it is to calculate the vector potential from currents, making the charge sheet the preferred method of description when 3D fields need to be calculated. This method is quite practical for the calculation of 3D fields produced by a few REC blocks. However, if one needs to calculate 3D fields produced at the ends of pure REC structures like those shown in Figs. 1, 2, and 3, I found the use of the dipole density, coupled with Fourier expansion methods, much more satisfactory. All 3D fields in Ref. 3 were obtained that way.

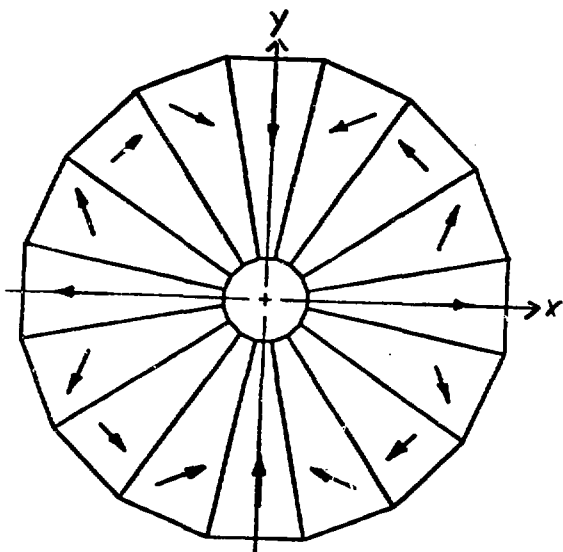


Fig. 1. Cross-section of segmented 16 piece REC quadrupole.

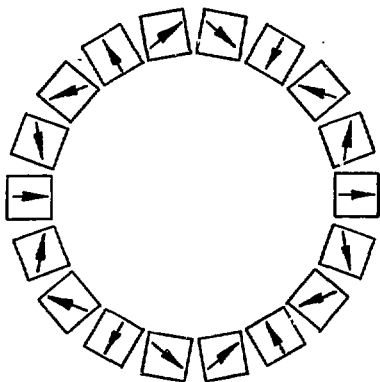


Fig. 2a.

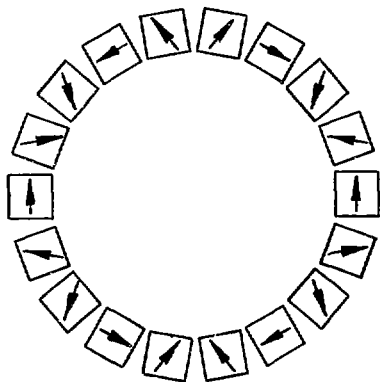


Fig. 2b.

Fig. 2a., Fig. 2b. Two versions of segmented 18 piece REC sextupole.



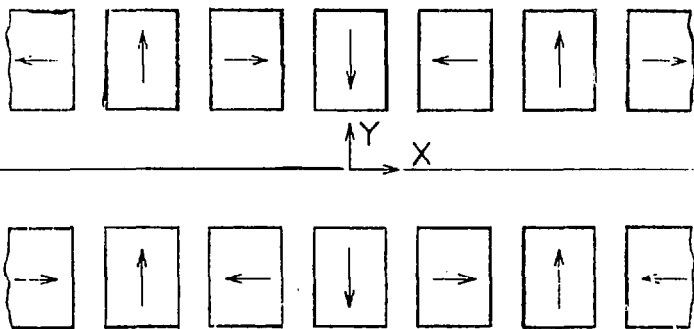


Fig. 3. Crosssection of linear REC undulator.

## REFERENCES

1. K. Halbach, Nucl. Instr. Meth. 169 (1980), 1.
2. P. G. Marston, Paper presented at the World Electrotechnical Congress, Moscow, USSR, 1977.
3. K. Halbach, Paper presented at the International Conf. on Charged Particle Optics, Giessen, W. Germany, 1980 Proceedings to be published in Nucl. Inst. Meth.