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Abstract

The single particle potential in infinite nuclear matter is computed as a function of density and energy in a variety of relativistic mean field models of nuclear matter. A comparison of this potential is made with that computed by Friedman and Pandharipande using the variational method. We also show that the self-consistent mean field Hartree approximation satisfies the Hugenholtz-van Hove theorem. High density behavior of the single particle potential is considered.

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A relativistic quantum field approach to the study of infinite nuclear matter and finite nuclei is a natural idea to pursue. On the basis of such an approach Teller, Duerr and Johnson[1)] were able to make reasonable predictions for spin-orbit splitting in finite nuclei, the well depth of the real part of the optical potential and also its energy dependence. These seemingly unrelated quantities are in fact simple consequences of the Dirac equation for the nucleons, solved in the presence of nuclear interactions of colossal strength. The underlying picture of the model is naively simple. It classifies nuclear interactions according to the irreducible representations of the Lorentz group. Nuclear attraction is generated by a Lorentz scalar meson σ through the interaction $g_s\bar{\psi}\psi\sigma$, while the repulsion is given by a Lorentz vector meson ω_μ through an interaction $g_v\bar{\psi}\gamma_\mu\psi\omega_\mu$.[2)] This distinction plays a fundamental role in the theory. It was shown by Schiff[3)] that this distinction between interactions is sufficient to obtain nuclear matter saturation without introducing hard core repulsion. In the relativistic field approach, nuclear matter saturation is a consequence of the apparent Lorentz covariance of the theory. Nuclear attraction is determined by a Lorentz scalar source term ρ_s and repulsion by a Lorentz vector source function $(\vec{J}, i\rho_v)$.

Recently it was shown that a relativistic mean field model reproduces reasonably well the real part of the optical potential.[4)] The density dependence of this potential has not been investigated. The purpose of this work is to make such an investigation in a variety of relativistic mean field models. We investigate the Walecka model,[2)] the Boguta and Bodmer model,[5)] and a recent model proposed by the author.[6)] We show that all these models satisfy the Hugenholtz-van Hove theorem for an interacting Fermi gas.[7)] A comparison of the predicted single particle potentials at various nuclear densities is made with those of Friedman and Pandharipande, who use Fermi-hypernetted and single-operator-chain summation techniques with a realistic nuclear hamiltonian.[8)]

We assume that symmetric nuclear matter (N = Z) is described by the following Lagrangian

$$\mathcal{L} = -\bar{\psi}(\gamma_\mu \frac{\partial}{\partial x_\mu} + m_N)\psi - \frac{1}{2}\left(\frac{\partial\sigma}{\partial x_\mu}\right)^2 - U(\sigma) - \frac{1}{4}F_{\mu\nu}F_{\mu\nu}$$
$$- \frac{1}{2}m_v^2\omega_\mu\omega_\mu + ig_v\bar{\psi}\gamma_\lambda\psi\omega_\lambda - g_s\bar{\psi}\psi\sigma \quad (1)$$

where

$$F_{\mu\nu} = \frac{\partial}{\partial x_\mu}\omega_\nu - \frac{\partial}{\partial x_\nu}\omega_\mu \quad (2a)$$

$$\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} \quad (2b)$$

and

$$U(\sigma) = \frac{m_s^2}{2}\sigma^2 + \frac{b}{3}\sigma^3 + \frac{c}{4}\sigma^4 \quad (2c)$$

$$\omega_\mu = \begin{pmatrix} \vec{\omega} \\ i\omega_o \end{pmatrix} \quad (2d)$$

For rotationally and translationally invariant nuclear matter, the field equations in the mean field approximation are

$$m_s^2\sigma + b\sigma^2 + c\sigma^3 = -g_s\rho_s \tag{3a}$$

$$\vec{\omega} = 0 \tag{3b}$$

$$m_v^2\omega_o = g_v\rho_v \tag{3c}$$

$$(-i\vec{\alpha}\cdot\vec{\nabla} + \beta m^*)\psi_{\vec{k}} = (E - g_v\omega_o)\psi_{\vec{k}} \tag{3d}$$

The positive energy solutions of the Dirac equation are the usual plane wave solutions with a reduced mass $m^* = m_N + g_s\sigma$. They are

$$\psi_{\vec{k}} \sim \begin{pmatrix} \chi \\ \dfrac{\vec{\sigma}\cdot\vec{k}\ \chi}{\sqrt{k^2+m^{*2}} + m^*} \end{pmatrix} e^{i\vec{k}\cdot\vec{x}} \tag{3e}$$

The source terms ρ_s and ρ_v are then given as a summation of nucleons up to the top of the Fermi sphere k_F.

$$\rho_s = \frac{4}{(2\pi)^3}\int^{k_F} d^3k\bar{\psi}_{\vec{k}}\psi_{\vec{k}} \tag{4a}$$

$$= \frac{4}{(2\pi)^3}\int^{k_F} d^3k \frac{m^*}{\sqrt{k^2+m^{*2}}} \tag{4b}$$

$$\rho_v = \frac{4}{(2\pi)^3}\int^{k_F} d^3k\ \psi^\dagger_{\vec{k}}\psi_{\vec{k}} \tag{4c}$$

$$= \frac{2}{3\pi^2} k_F^3 \tag{4d}$$

The single particle energy, as function of momentum $\vec{k}$ is given by

$$E = g_v\omega_o + \sqrt{k^2 + m^{*2}} \tag{5}$$

and the Fermi energy is given by

$$E_F = g_v\omega_o + \sqrt{k_F^2 + m^{*2}} \tag{6}$$

The energy density ε is given by

$$\varepsilon = \frac{1}{2}\left(\frac{g_v}{m_v}\right)^2 \rho_v^2 + U(\sigma) + \frac{4}{(2\pi)^3}\int^{k_F} d^3k\, \sqrt{k^2 + m^{*2}} \tag{7}$$

and the pressure for absolute zero temperature is

$$P = \rho_v^2 \frac{d}{d\rho_v}\left(\frac{\varepsilon}{\rho_v}\right) \tag{8}$$

Weisskopf showed, in an independent particle model, the equality of the Fermi energy and the energy per particle at saturation.[9)] Subsequently, Hugenholtz and van Hove proved that for an interacting Fermi gas at zero absolute temperature the exact solution of the many body problem relates the Fermi energy, energy per particle, and the pressure through the relation[7)]

$$E_F = \frac{\varepsilon + P}{\rho_v} \tag{9a}$$

$$= \frac{d}{d\rho_v}(\varepsilon) \tag{9b}$$

At saturation, P = 0, and we recover the original Weisskopf relation $E_F = \varepsilon/\rho_v$. Hugenholtz and van Hove used their results to check the validity of the approximations made in the Brueckner theory of nuclear matter. It was also used as criteria to check the validity of approximations made in the mass operatory theory.[10)] We shall now show that the Hugenholtz-van Hove theorem holds in the self-consistent Hartree mean field approximation. The validity of the theorem in the Hartree approximation is surprising, since the original proof is for the exact solution of the many body problem. To establish the relationship of Eq. (9a-9b), we directly calculate the derivative of the energy density ε with respect to the particle density ρ_v. One should note that the effective mass m* is density dependent. We have

$$\frac{d\varepsilon}{d\rho_v} = \left(\frac{g_v}{m_v}\right)^2 \rho_v + \sqrt{k_F^2 + m^{*2}} + \left\{ g_s \rho_s + \frac{\partial U}{\partial \sigma} \right\} \frac{\partial \sigma}{\partial \rho_v} \tag{10}$$

$$= E_F$$

The bracketed term in Eq. (10) vanishes identically on account of the field equation Eq. (3a). The validity of the Hugenholtz-van Hove theorem in a self-consistent mean field approximation suggests that the formal diagrammatic structure of the nonrelativistic many body theory of nuclear matter and that of the relativistic field approach are inherently quite different. The field equations Eq. (3a-3d) were derived in the Hartree approximation. Nonetheless, Eq. (6a) for the single particle energy in nuclear matter shows that in this Hartree approximation a quasi particle approximation is valid. The interactions have been absorbed in the effective mass m* and the field ω_0. This is not the case in the Hartree approximation in nonrelativistic theory, where the Fock term is mandatory.

To test the content of the relativistic mean field theory, it is important to study the single particle energy as a function of energy and density. This directly reveals whether the shell potential in the theory is a reasonable one. There have been a number of calculations for the real part of the optical potential in various many body theories, the most recent one being that of Friedman and Pandharipande using the variational method with Fermi-hypernetted and single operator-chain summation techniques.[8,10] Thus, we have a good idea how a realistic nucleon-nucleon interaction projects itself onto the single particle properties in nuclear matter not only as a function of energy but also of density. To see how compatible the relativistic mean field approach is with refined many body calculations in the single particle sector, we study the single particle potential in three distinct relativistic mean field models. All of them saturate nuclear matter at a density of $0.1625/fm^3$ with a binding energy per particle of -15.75 MeV. Let $C_s = g_s/m_s\ m_N$, $C_v = g_v/m_v\ m_N$, $\overline{b} = b/(m_N g_s^3)$ and $\overline{c} = c/g_s^4$. The first model, considered by Boguta and Bodmer,[5] (called BB) is defined by taking $C_s = 8$, $C_v = 2$, $\overline{b} = 0.471$, $\overline{c} = 9.1$. This model has the feature that the nuclear surface energy is reasonable and the nuclear compressibility is K = 175 MeV. A very different model is obtained by taking $C_s = 17.65$ and $C_v = 15.60$ with b = c = 0. This model was considered by Walecka[2] and we call it the W-model. It has many appealing phenomenological features,[11] but the nuclear surface energy is incorrect and the nuclear compressibility is K = 550 MeV. A third model was recently considered by the author and corresponds to $C_s = 15.6$, $C_v = 12.5$, $\overline{b} = 2.08 \times 10^{-3}$, $\overline{c} = 3.6 \times 10^{-4}$. We call it the B-model. It has a reasonable surface energy and K = 290 MeV. The three models differ in the predicted effective mass m* at saturation. BB-model has $m^*/m_N = 0.935$, W-model has $m^*/m_N = 0.56$, and B-model has $m^*/m_N = 0.69$. The B-model is interesting in that it has a reasonable compressibility and the effective mass corresponds most closely to the one required by the phenomenological analysis of Nobel[12)] and theoretical calculations of Jeukenne et al.[10)]

The real part of the nonlocal optical potential is defined to be

$$E = \sqrt{k^2 + m^2} + U_{eff} \tag{11}$$

where

$$U_{eff} = E - \sqrt{(E - g_v \omega_o)^2 + m_N^2 - m^{*2}} \tag{12}$$

In Fig. 1a, we show the comparison of the Friedman-Pandharipande (F-P) results at $\rho_v = \rho_o = 0.1625/fm^3$ with those of the relativistic mean field models. A compendium of Wood-Saxon well depths needed to fit the data is also shown. They are taken from Friedman and Pandharipande.[8)] The W-model shows a significant deviation from the F-P results and from the Wood-Saxon potential fits. This was already noted by Jaminon, Mahaux and Rochus.[13)] The BB-model is even worse here. The B-model agrees with F-P calculation for $0 \leqslant e \leqslant 100$ MeV. For $e < 0$, the W-model agrees very well with F-P results, while the BB-model is very bad even here. The B-model shows about a 10% deviation. In Fig. 1b we show U_{eff} for ρ_v = 3/4 ρ_o, 1/2 ρ_o and 1/4 ρ_o, respectively, as a function of $e = E - m_N$ for all three models.

The density dependence of U_{eff} for the W-model and the B-model is in reasonable agreement with the variational calculations even for low densities. This is surprising, since one would have expected that correlations would play a very important role here.

In Fig. 2a we show $U_{eff}(e)$ for normal and compressed nuclear matter for the B-model and in Fig. 2b, the same is shown for the W-model. Both are significantly different from each other and from F-P results. In the W-model, $U_{eff}(e)$, for $\rho_V = \rho_0$, vanishes at 130 MeV. For the B-model this occurs at 210 MeV. In the F-P calculation it vanishes at 300 MeV.

In this work, we have studied the single particle properties in infinite nuclear matter as a function of both the density and energy in a variety of relativistic mean field models. Our calculation reveals that a phenomenologically reasonable model, such as the B-model, predicts the real part of the single particle potential in reasonable agreement with those calculated in a refined many body calculation of Friedman and Pandharipande using the variational method with Fermi-hypernetted and single-operator-chain summation techniques. This agreement is in density and energy. There does not appear to be any essential contradiction between the conventional many body approach and the relativistic mean field calculations. What is different is the interpretation. We have stressed the phenomenological aspects of the models. It was shown by Serr and Walecka[11)] that quantum corrections are indeed very important and change the properties of nuclear matter. But if one readjusts the coupling constants C_s and C_v, after higher order corrections were computed, to saturate nuclear matter at correct density and binding energy, the equation of state is only slightly modified. Thus a diagrammatic term-by-term comparison, though highly desirable for a fundamental theory of nuclear matter, can be misleading in a phenomenological approach. A phenomenological model of nuclear matter is justified only if it has considerable predictive power. We have shown in this work and previous works that this is indeed the case. It means, as suggested by Boguta and Bodmer, that the complicated many body effects, though important and seemingly neglected in the relativistic mean field approximation, can indeed be absorbed into a few phenomenological constants.

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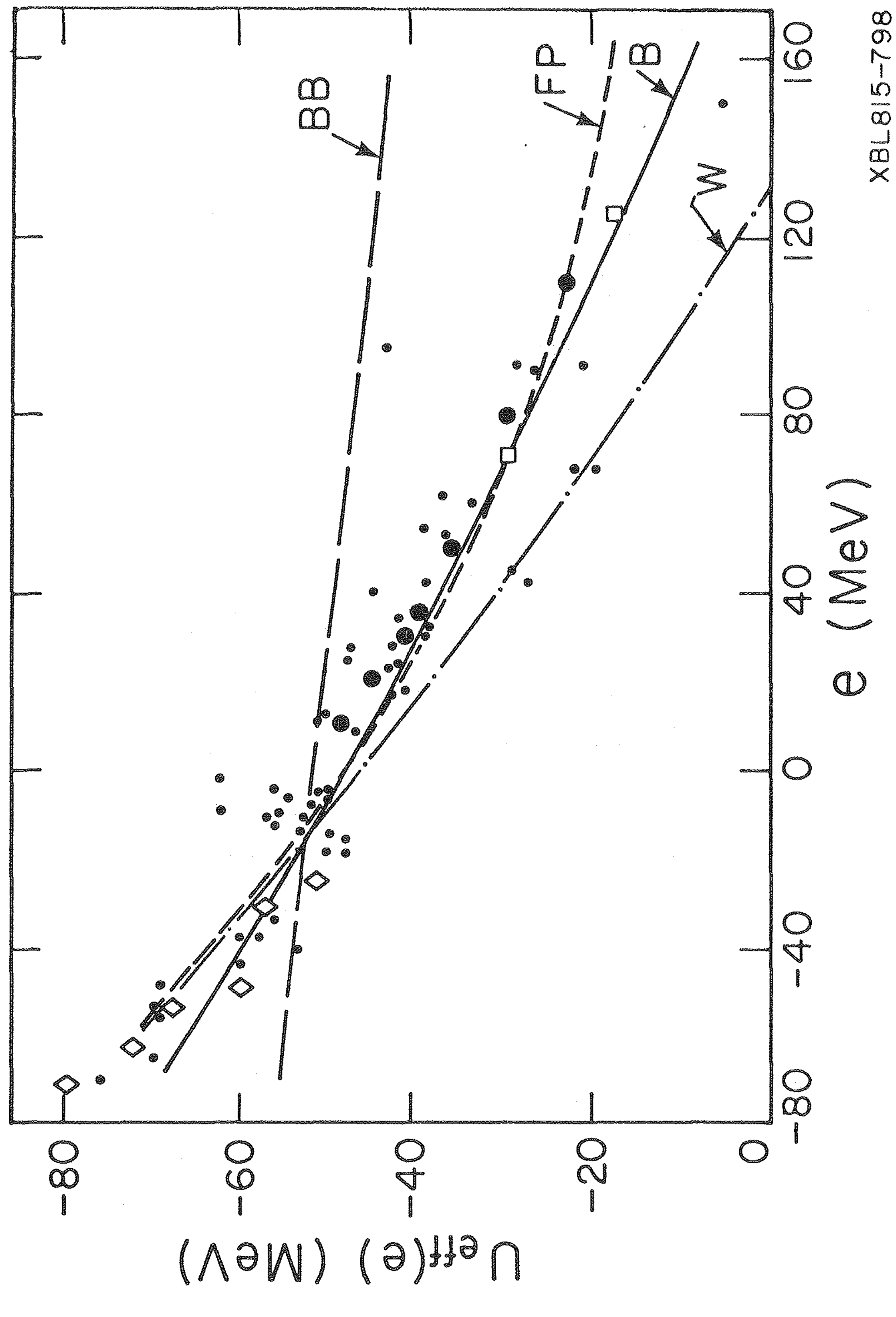
Figure Captions

Fig. 1a. Real part of the optical potential U_{eff} for the W-model, the B-model, the BB model, compared with the results of Friedman-Pandharipande (F-P) variational calculation. The compendium of Wood-Saxon well depths is taken from F-P.

Fig. 1b. Density and energy dependence of the single particle potential in various mean field models (BB, B and W) compared with F-P variational calculations.

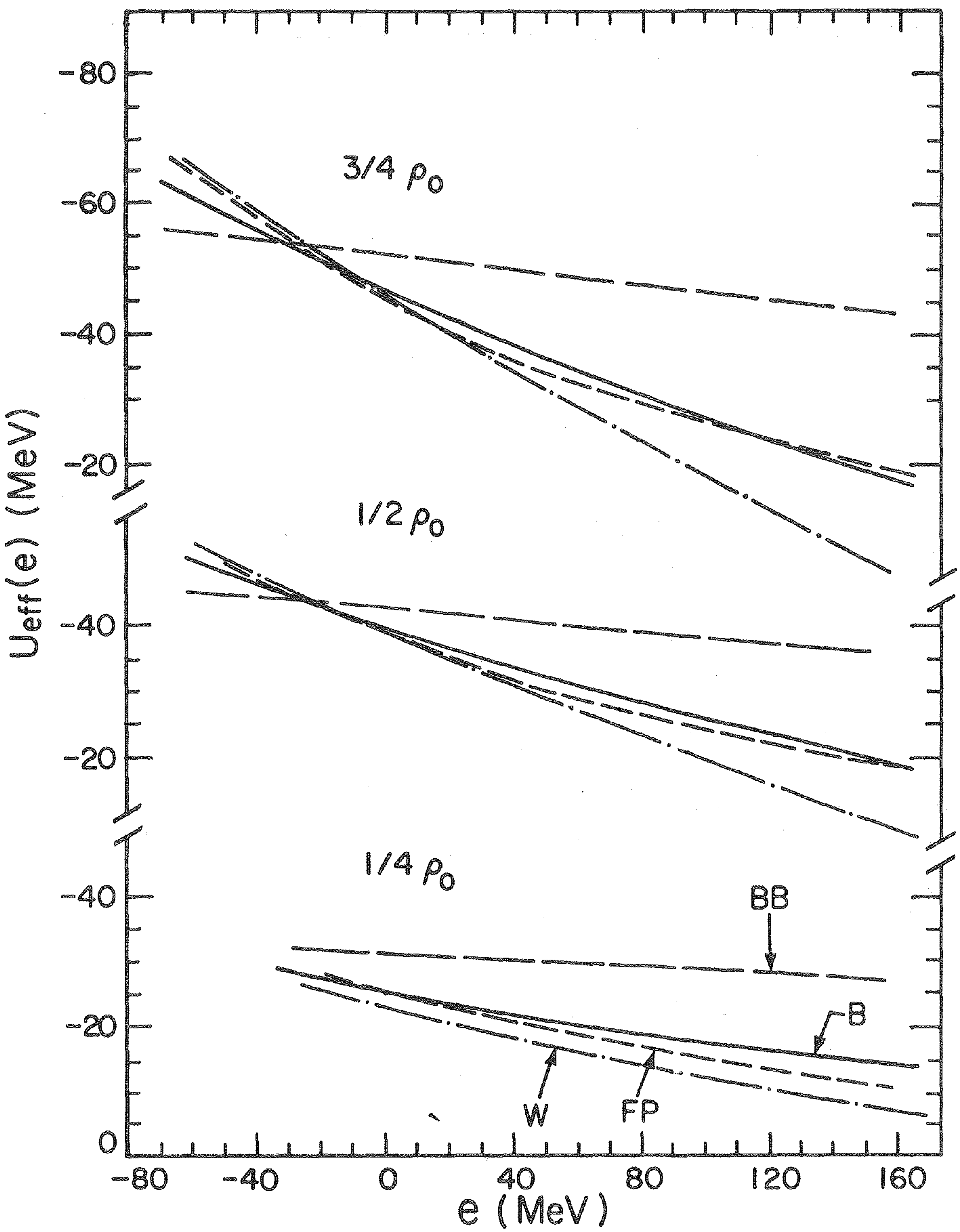
Fig. 2a. B-model calculations of the single particle potential U_{eff} as a function of energy and various densities. The light line divides the kinematically allowed regions.

Fig. 2b. W-model calculations of the single particle potential U_{eff} as a function of energy and various densities. The light line divides the kinematically allowed regions.



BB
FP
B
W
$U_{eff}(e)$ (MeV)
e (MeV)
-80
-60
-40
-20
0
-80
-40
0
40
80
120
160
XBL815-798

Fig. 1a



XBL815-797

Fig. 1b

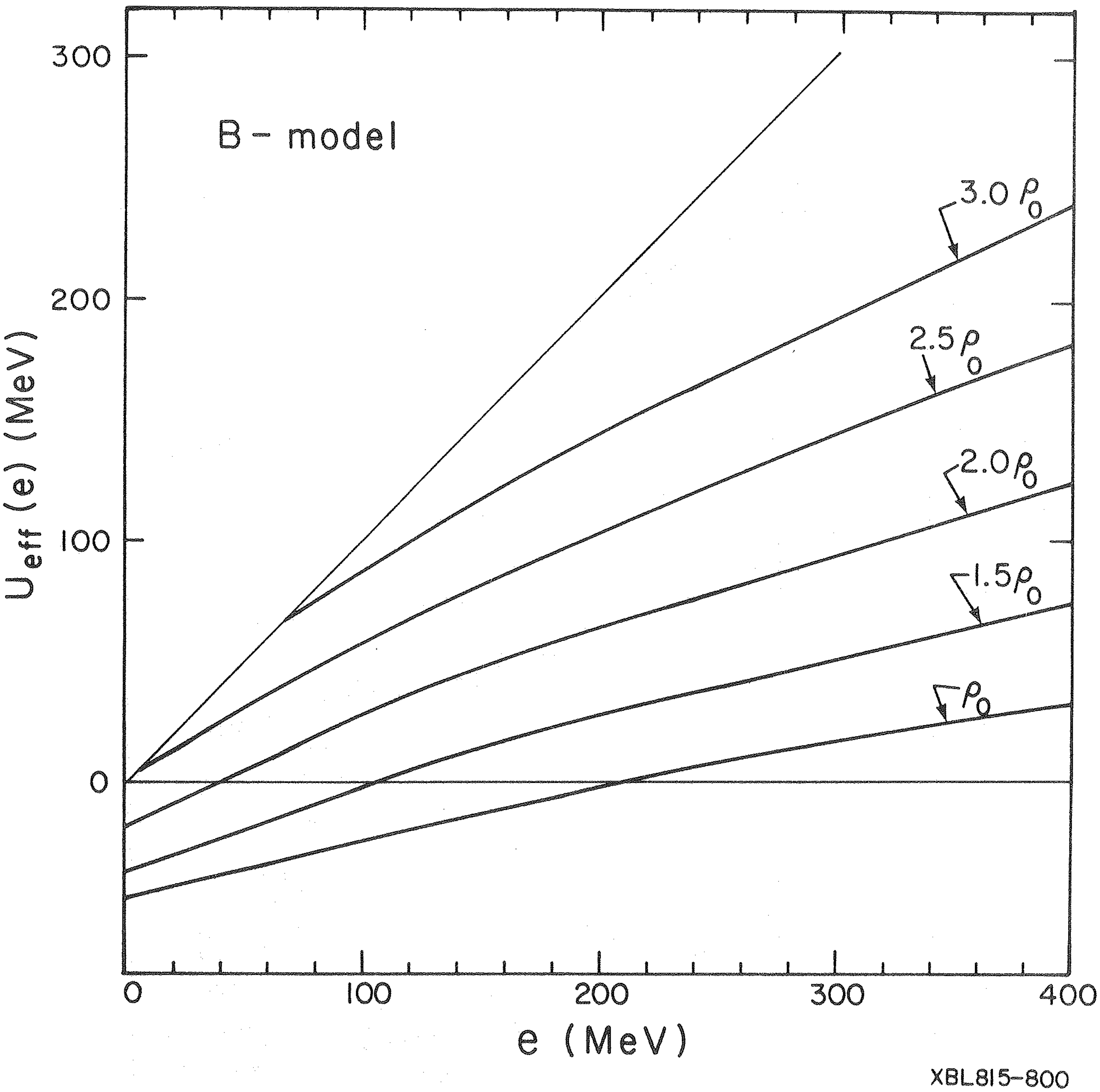

B - model
3.0 ρ_0
2.5 ρ_0
2.0 ρ_0
1.5 ρ_0
ρ_0
300
200
100
0
U_{eff} (e) (MeV)
0
100
200
300
400
e (MeV)
XBL815-800

Fig. 2a

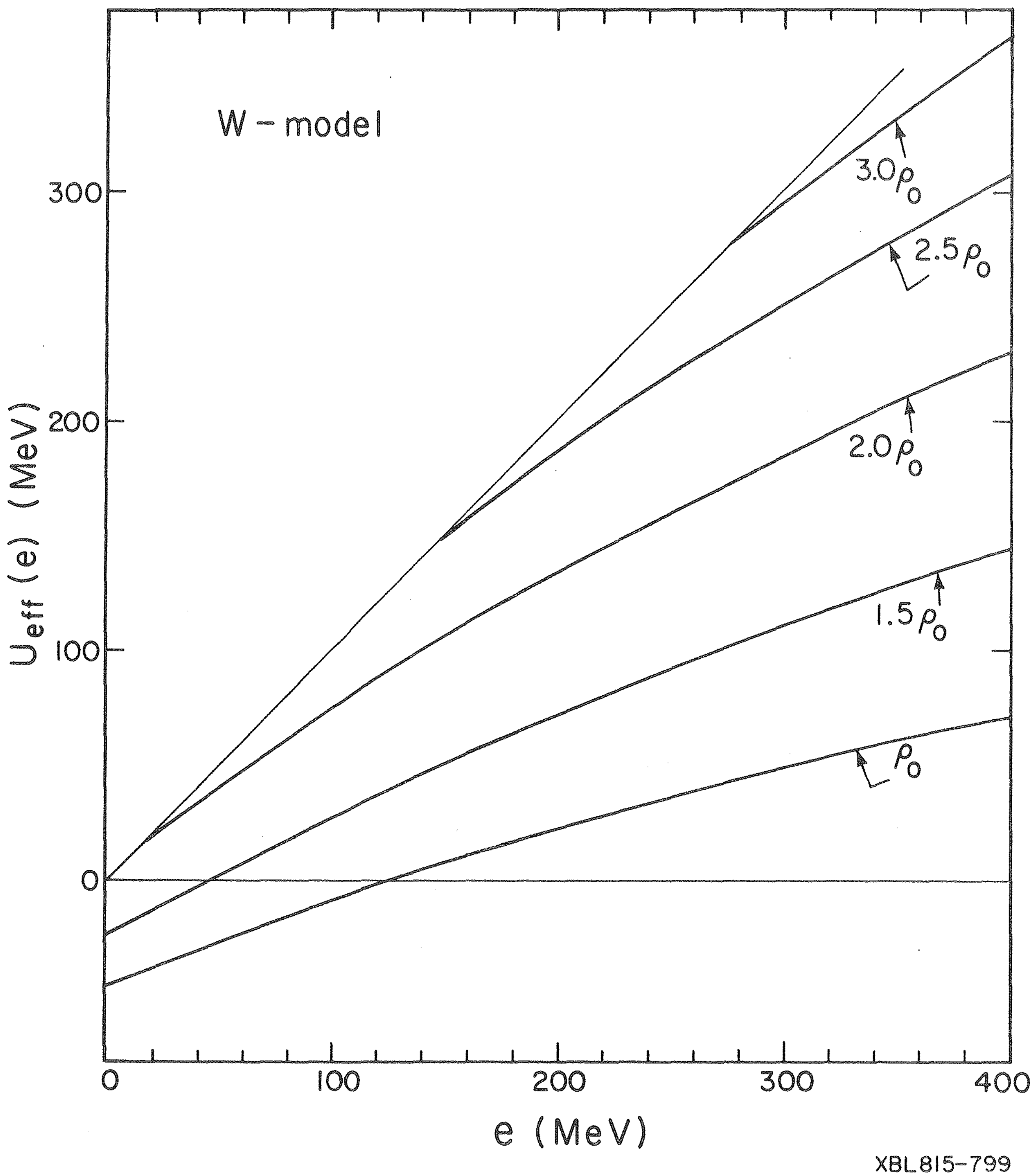

W - model
3.0 ρ_0
2.5 ρ_0
2.0 ρ_0
1.5 ρ_0
ρ_0
300
200
100
0
U_{eff} (e) (MeV)
0
100
200
300
400
e (MeV)
XBL815-799

Fig. 2b