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TOPOLOGICAL REPRESENTATION OF LEPTONS

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I. INTRODUCTION

TOPOLOGICAL REPRESENTATION OF LEPTONS

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ABSTRACT

It is proposed that on the quantum surface of a topological expansion surface pair the lepton be associated with a 2-triangle-- $\bar{I}\bar{Y}$ --Möbius band whose perimeter is built from the same two peripheral \bar{I} -triangle edges that constitute an antiquark's contribution to a hadron-disk perimeter. Although all lepton "internal" degrees of freedom are in one to one correspondence with antiquark attributes, it is shown that the simplest representation of quark-generation (Cabibbo) mixing cannot be invoked for leptons. The proposed lepton contains a neutral \bar{Y} triangle and permits a topological representation of baryon-lepton mixing. Described in parallel is a quantum-classical surface pair to represent the (standard) electroweak leptonic current.

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How do leptons fit into the topological expansion? They are absent from the lowest complexity level--called "zero entropy", the base for strong-interaction expansion components.¹ The rules emerging for corrections to zero entropy resemble those of Feynman, which in turn relate to perturbative evaluation of Lagrangian field theory, so one expects certain features of perturbative Lagrangian theory to be found within higher orders of the topological expansion. In particular, because electroweak Lagrangian theory has revealed a need for quark-lepton pairing, it is plausible that such pairing will be required in a consistent topological theory. The recently-recognized idea of topological grand unification (TGU),² supports this expectation.

"Topological quarks" have been identified with certain triangles within the quantum-surface (Σ_Q) disks that correspond to elementary hadrons,¹ so the first guess about leptons associated them on Σ_Q with single triangles that resembled as much as possible hadron "quark" triangles. This single-triangle lepton representation was used by Ref. (3) in constructing a topological representation of electromagnetism. Subsequent developments in the topology of chirality⁴ have, however, raised doubts about the single-triangle lepton, and even before the chirality issue a lepton-generation dilemma had to be shoved under the rug. We shall explore these two puzzles and propose for both a solution that builds a lepton, using the terminology of Ref. (2), from the combination of a neutral Y-triangle with a peripheral I-triangle. We immediately discover a promising basis for the weakness of lepton generation mixing relative to quark Cabibbo mixing. Further, we are

able to formulate a topological representation of baryon-lepton mixing--
a now "standard" feature of Lagrangian theory.

II. CHIRALITY AND GENERATION DILEMMAS FOR THE SINGLE-TRIANGLE LEPTON

The form of a hadron peripheral triangle--a special case of the I-triangle discussed in Ref. (2)--is shown in Fig. 1, where the dashed line is the boundary (or belt) of the transverse classical surface (Σ_C). All indicated orientations are independently reversible, as is the charge-arc orientation and the orientation of the Σ_C area adjacent to the peripheral I-triangle. Figure 13 of Ref. (1) shows the various forms of hadron disk, in all of which every I-triangle carries this same set of independent orientations. It is explained in Ref. (1) how these orientations control I-triangle flavor, while Ref. (4) describes how the local orientation of the adjacent Σ_C controls I-triangle chirality. The collection of hadron I-triangle attributes has led to the synonym "topological quark".*

The belt subpiece** embedded in a peripheral I-triangle has at one end a vertex of the Σ_Q triangulation.¹ This vertex touches another peripheral I-triangle, usually belonging to a different hadron, that shares the two triangle edges (uncut by the belt) which meet at this vertex. At the other belt-subpiece end, where the belt cuts a triangle edge, there is located the end either of a momentum arc or a momentum-

* Reference (1) explains the sense in which "quark" triangles are "colored" by their relationship to the momentum arc. Considered individually, as in Fig. 1, all triangles are colorless.

** Σ_Q houses a "thickening" of the belt, the thickening of each belt subpiece being housed within one Σ_Q triangle. Each belt subpiece and consequently each triangle contains the end of exactly one charge arc.

copy arc, ^{4,5} shared with an adjacent triangle belonging to the same hadron.

What difficulties arise if we try to associate the single triangle of Fig. 1 with a lepton? A first consideration is that, because the lepton is a particle in contrast to the quark which is not, the lepton momentum arc should end inside the lepton quantum area--not on the perimeter where it would be shared with another particle. Figure 1 would become Fig. 2. ³ This consideration makes awkward the topology of chirality proposed in Ref. (4), which divides the classical surface into chiral areas whose boundaries are either momentum arcs, momentum-copy arcs or "vector gluon" lines that end on triangle vertices. Any classical area touching a peripheral I-triangle is locally oriented while areas touching Y triangles are not oriented. The oriented areas induce an orientation for each belt subpiece within a peripheral I-triangle (see Fig. 1), thereby controlling the "ortho" or "para" chirality of the quark. Now if the lepton momentum arc ends inside the lepton triangle, only a portion of the belt subpiece within the triangle is oriented; furthermore one must introduce for nonoriented classical-surface areas a criterion not based on Y-triangles. By introducing a Y-triangle into the lepton quantum area we shall be able to maintain a uniform chiral topology for quarks and leptons. A related unification will be that momentum arcs uniformly end on an intersection of belt with a quantum-triangle edge. Figure 1 will depict any peripheral I-triangle, whether in a hadron or in a lepton.

In Ref. (1) the notion of quark generation is associated with the edge orientations of peripheral I-triangles, orientations that must be matched in building quantum surfaces. Two edges of the peripheral I-triangle in Fig. 1 are oriented; the third is not. The reason for the distinction is that the two oriented edges always occur along the perimeter of the hadron area while the third edge lies inside the area. A principle of topological bootstrap theory is that hadron quantum numbers reside in orientations along the perimeters of elementary-hadron quantum-surface areas. This perimeter rule explains why all hadron properties may be inferred from the orientations attached to peripheral I-triangles (topological quarks); the orientations of Y triangles do not lead to additional quantum numbers. By the same token the interior edge of a peripheral I-triangle (sometimes shared with a Y triangle) is not oriented.

But if a lepton quantum-surface area is a single triangle disk, then all 3 triangle edges lie along the perimeter. Why should the third edge not be oriented? If it were, the number of distinct leptons would be double the number of quarks. Instead of 4 generations-- corresponding to the orientations of two edges¹--there would be 8. In Ref. (3) the third edge of the single triangle lepton area was given no orientation, but without topological motivation.

By including a Y-triangle in the lepton area it will be possible for the entire lepton perimeter to consist of the same two oriented I-triangle edges that build hadron perimeters. The third I-triangle edge will lie inside the lepton area, as for hadrons, and there will be a topologically-natural one to one matching of quarks with leptons.

III. NEUTRAL CORES FOR LEPTON QUANTUM AREAS

Inside every quantum triangle is the end of exactly one oriented charge arc, which lives in Σ_C with its other end inside some other triangle. The electric charge of a triangle is determined by its attached charge-arc orientation in conjunction with the triangle orientation--the latter designated +(-) if it agrees (disagrees) with Σ_Q global orientation. There are four possibilities--indicated in Table I which we reproduce from Ref. (6).

I-triangles may be either charged or neutral, but hadron Y triangles are always charged.¹ We propose that a lepton quantum area comprise one peripheral I triangle with the charged-neutral option and one invariably neutral Y triangle. Lepton quantum numbers, like hadron quantum numbers, will then be controlled by the perimeter of the particle area. Because the photon couples only to the peripheral I-triangle, we shall maintain the feature of Ref. (3) that lepton electromagnetic properties are given by the rules of quantum electrodynamics.

The proposed lepton quantum area is depicted in Fig. 3, where the two \bar{Y} -triangle edges marked "e" are to be identified with (glued to) each other. The sense of this gluing (i.e., with or without a "twist") will be discussed later in Sec. VII. The peripheral I-triangle in Fig. 3 is chosen to be the "anti" of that in Fig. 1, corresponding to the conventional definition of leptons as carrying negative charge while quarks have positive charge; the two oriented I-triangle edges constitute the complete perimeter of the lepton area. As is the case for hadrons, all three edges of the Y triangle lie inside the particle area; the lepton Y triangle is in this sense a "core"--like a baryon Y triangle.

How did we choose the Y-triangle orientation in Fig. 3?

Our choice is dictated by the need to define a global Harari-Rosner orientation of Σ_C that is maintained in electroweak-boson plugs; the following section explains how this works in connection with Fig. 4(b). A less compelling but suggestive consideration is that the chosen orientation allows lepton-baryon mixing while maintaining charge conservation. Table I implies that charge is conserved if and only if the total number of (+) triangles of Σ_Q is equal to the total (-) number. Now mesons, baryoniums and electroweak bosons all have equal numbers of (+) and (-) triangles, while baryons have 3(+) triangles with 1(-) triangle. Therefore a Σ_Q containing one baryon area and one lepton area can be charge conserving, no matter how many boson areas occur, only if a lepton is built from two triangles of the same orientation.

Two remarks about the orientation pattern of Fig. 3 are in order:

- (1) The two lepton triangles constitute a single patch on Σ_Q .
- (2) The representation of lepton-baryon mixing proposed below in Sec. VI, where lepton and baryon Y triangles of the same orientation are mated, requires that if baryon Y triangles are charged then lepton Y triangles must be neutral.

IV. LEPTONIC ELECTROWEAK CURRENT

It has been proposed in Ref. (7) that hadronic electroweak currents, at the minimal level of topological complexity, belong to single-sheet classical surfaces which admit electroweak boson insertion into any hadron charge arc. We propose here as vehicle for the lepton current an analogous single-sheet Σ_C . Just as for hadrons, one may start with a lepton "propagator"; electroweak boson insertion into the propagator is proposed to follow the same pattern as for hadrons.⁷

Our candidate lepton "propagator" Σ_C is displayed in Fig. 4(a); the corresponding belt is that of Fig. 5. The three (heavy) line segments whose ends are labeled a and b in Fig. 4 are to be identified with each other and constitute a single junction line. One recognizes that Σ_C is divided into 2 areas by the momentum arc. Following the scheme of Ref. (4) one of these areas--(marked O-P in Fig. 5) containing the I-charge arc and adjacent to the I-triangle--we propose to give an (ortho-para) orientation; the other--adjacent to the junction line--is not to be oriented. Lepton chirality is then represented in exactly the same way as quark chirality.

Figure 6 shows the Σ_Q covered by the "in-out" lepton pair, each lepton area being that of Fig. 3 or its inverse. This lepton current conserves lepton generation; the following section discusses the possibility of lepton-generation mixing.

Insertion of an electroweak boson, as in the example of Fig. 4(b), follows the same pattern as for hadronic currents in Ref. (7). (Compare to Fig. 6 of Ref. (7)). If the global HR orientation is to be

consistently maintained by any electroweak-boson plug, the triangle orientations must be as shown--which correspond to Fig. 3 and Fig. 6. That is, for HR orientation to be perpetuated in electroweak-boson plugs, it must consistently induce a charge-arc orientation away from a (-) triangle and toward a (+) triangle. (Note that this induced orientation is not the charge-arc orientation referred to in Table I.)

VI. GENERATION MIXING

It is a striking empirical fact that the Cabibbo currents which mix quark generations are larger than corresponding lepton-mixing currents, whose very existence remains experimentally in doubt. In attempting to extend topological theory to represent generation mixing we have encountered a qualitative difference between quark and lepton currents that may account for the experimental asymmetry.

The simplest way to introduce generation mixing is to replace the "normal" pattern of mated (adjacent I-triangular) peripheral areas on a globally oriented Σ_Q , shown in Fig. 7(a), with the "funny" pattern of Fig. 7(b) (in both cases additional triangle complete a closed Σ_Q), where there is no global orientation. (See Appendix.) The adjacent triangle-antitriangle pair of Fig. 7(a) belong to the same generation while that of Fig. 7(b) belong to different generations. One might suppose that the configuration of Fig. 7(b) could be invoked equally well for leptons as for quarks, but such is not the case.

The difference relates to the meaning of +, - labels when there is no global orientation of Σ_Q , as explained in the Appendix. It is essential that quantum triangles effectively carry +, - labels so that "in" particles can be distinguished from "out" particles (+ triangles are always plugged to - triangles), but what do these labels mean if the quantum surface is not globally oriented? The appendix systematically discuss this question. Suffice it here to say that meaning is possible for (+, -) triangles on a nonorientable (closed) Σ_Q if there exists a well-defined set of "funny" edges which,

when cut, lead to a connected (bounded) orientable surface. The two oriented edges in Fig. 7(b) are "funny", but cutting along them when they belong to leptons disconnects the surface. In contrast such a cut when such edges belong to hadrons leaves a single connected surface which can be globally oriented. It is therefore possible to use Fig. 7(b) in a hadron current but not in a lepton current. Disconnection occurs in the lepton case because the two oriented edges of the lepton peripheral triangle in Fig. 3 form a cycle; such is not the case for a topological quark (within a hadron).

VI. LEPTON-BARYON MIXING

In this section we present an example of how lepton-baryon mixing might be accommodated within the topological expansion. With a single-triangle lepton there was no way to balance the baryon's core triangle and associated junction line, but the 2-triangle lepton can combine with a 4-triangle baryon to close a quantum surface--as shown by the example of Fig. 8. In Fig. 8 the two edges marked "e" are to be identified as are the two edges marked "g". (The indicated sense of identification is explained in Sec. VII.) The mating of two baryon peripheral I-triangles is the first occasion in topological particle theory where two triangles of the same orientation are proposed to be mated. Violation of electric charge conservation is avoided here because the two mated Y triangles also have the same orientation and the total number of (+) quantum triangles equals the total (-) number.

A natural adjunct to the foregoing lepton-baryon mixing proposal is a rule that, if mesons are to be added to the Σ_Q of Fig. 8, meson insertion should be made on the belt branch containing the lepton trivial vertex (the branch at the bottom of Fig. 9). The analogue of this branch (carrying topological color #1) houses all mesons in the topological theory developed so far for strong and electro-weak interactions.^{1,7}

Maintenance of HR orientation in electroweak boson plugs requires the rule that electroweak bosons may be inserted only where charge arcs connect triangles of opposite orientation. Such a rule would

mean that two of the 3 charge arcs in Fig. 9 must remain passive; electroweak boson insertion could occur only in the peripheral charge arc at the lepton trivial vertex.

The edge identifications of Fig. 8 imply that the two (anomalously) mated baryon quarks belong to the same generation, although one is charged and one neutral. The third baryon quark has quantum numbers uniformly "anti" those of the lepton (modulo meson or electroweak boson insertion). We explain in the following section why the ee and gg edge identifications of Fig. 8 are being proposed. Our explanation simultaneously deals with the ee edge identification of Fig. 3.

VII. THE LEPTON MÖBIUS BAND

In Fig. 11 we duplicate Fig. 3 with the omission of edge orientations and of momentum-arc ends but now numbering the vertices of the triangulation. It has been stated that the 12 and 13 edges are to be identified but in which sense? A rule heretofore obeyed for all Σ_Q is that each triangle is mated to exactly one other triangle--mating being defined as the sharing of all 3 vertices. If this mating rule is to be maintained, the sense of identification indicated by the arrows of Fig. 11 is required. The other type of identification would bury the vertex #1 inside the lepton area where it cannot be shared with another triangle.

With the edge identification indicated in Fig. 11 the lepton quantum area is a Möbius band; the 3 vertices labeled 1, 2, 3 become a single vertex on the perimeter. (The only other vertex--the trivial vertex #4--also lies on the perimeter.) In the lepton propagator of Fig. 6, two Möbius bands are sewed together along perimeters to form a Klein bottle. At a single point of the Klein bottle are located all vertices of both Y triangles. (There is one additional vertex of the Klein-bottle triangulation--the trivial vertex.)

The concept of "funny" edge on Σ_Q , introduced above in Sec. V and discussed in the appendix, is relevant here. After gluing together the 12 and 13 edges of Fig. 11 one arrives at the pattern of Fig. 12 in the neighborhood of the single new edge. This pattern signals a (globally) nonorientable surface and, correspondingly,

tells us that the edge in Fig. 12 is "funny" in the sense of the appendix. By cutting along all funny edges a globally-oriented (bounded) connected surface is achieved. Figure 6, before the ee and ff gluing, constitutes an example.

The mating requirement for both I- and Y triangles leads to the gluing prescription of Fig. 8 for our candidate baryon-lepton mixing topology. Here there are 3 "funny" edges, e, f and g, on a Klein bottle with one nontrivial vertex and 2 trivial vertices of the triangulation. All triangles occur in mated pairs.

VIII. CONNECTION WITH TOPOLOGICAL GRAND UNIFICATION (TGU)

The lepton-related topological representations proposed in this paper are motivated by certain attractive features of Lagrangian field theory. We are not here operating on the bootstrap basis that characterizes the topological approach to hadrons¹ and electroweak bosons.⁸ The bootstrap theory of hadrons¹ preceding this paper nevertheless has generated almost all the ingredients that have been invoked here. These include the idea of a topological expansion based on complexity or "entropy", the idea of associating each term of the expansion with a quantum-classical surface pair, and the idea of dividing the quantum surface into oriented, mated triangular areas. The previous rules for representing momentum, spin and electric charge all have been maintained as have been the rules for chirality. We find it remarkable and encouraging that all these bootstrap-developed hadron ingredients adapt to lepton properties elucidated through Lagrangian field theory.

Particularly interesting is the use of the Y triangle in lepton representation. The Y triangle has been the most surprising and controversial aspect of the topological hadron bootstrap. The relevance to leptons, so many of whose properties are well established, should put Y triangles to a severe test as electroweak topological theory is further developed. Electromagnetic confrontation has been avoided because lepton Y triangles are electrically neutral, but neutral weak leptonic currents may have Y contributions.

Our lepton representation does invoke one topological feature not yet demanded by the bootstrap: a nonorientable quantum surface. In Ref. (2) however, it is proposed that electroweak bosons also require nonorientable Σ_Q . Global orientability of Σ_Q is thus emerging as a special topological feature confined to strong interactions.

The lepton topology described here is recognized in Ref. (2) as one element of a topological grand unification (TGU) where passage from elementary hadrons to electroweak particles is accomplished by certain Σ_Q substitutions. One of these substitutions replaces the diquark disk of Fig. 13a by the Y Möbius band of Fig. 13b, where the orientation of the Y triangle is reversed. Preserved by this substitution is "topological hypercharge"--one half the total number of clockwise minus anticlockwise triangles, a quantity identified in Ref. (4) as equal to $(B - L)$, where B is baryon number and L is lepton number. Applied to a baryon disk, the substitution of Fig. 13 leads to an antilepton Möbius band.

One of the preconditions for such a substitution rule is the fact that all elementary-particle quantum areas are divided into two pieces by a cut made along the single special edge that contains the end of the particle momentum arc. Every elementary particle area, in other words, consists of two areas sewn together along the momentum-carrying edge. The lepton representation described here is a special case of this general pattern.

ACKNOWLEDGEMENT

Important contributions to the ideas presented here have been made by J. Finkelstein. We mention in particular the electrical neutrality of the lepton's Y triangle and the relevance of "funny" edges to generation mixing. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

APPENDIX: ORIENTING NONORIENTABLE SURFACES

The following is a sequel to Appendix C of Ref. (1) and to Appendix A of Ref. (3).

We consider closed connected triangulated surfaces S. By definition an "orientation structure" for S consists of the following items:

- a) Each triangle carries independently a local orientation and a \pm sign. Hence for two adjacent triangles one can have any of the four configurations a, b, c, d from Fig. 14 (or the configurations reached by simultaneously reversing all the arrows). The common edges from Figs. 14 c and d will be called "funny". Note that an orientation structure admits two possible inversions; I'_S which everywhere changes $+\leftrightarrow-$ and I''_S which everywhere changes \leftrightarrow .

Before enlarging our list of requirements we make several observations:

- A) Grouping together those triangles with the relationship of Fig. 14(a) and those with the relationship of Fig. 14(d) yields a patch structure for S, underlying our "orientation structure".

- B) If Γ_f denotes the union of all funny edges, then for the graph Γ_f the number of edges meeting at any vertex v is even. Proof: Consider the total set of Q_v edges meeting at v and attach to each an index

$n_i^v = 1$ (or 0) according to whether the local orientation changes (or does not change) across the edge. Attach similarly an index $m_i^v = 1$ (or 0) according to whether or not there is a $+\leftrightarrow-$ change. Then

$$\sum_{i=1}^{Q_v} m_i^v, \sum_{i=1}^{Q_v} n_i^v \text{ and } \sum_{i=1}^{Q_v} (m_i^v + n_i^v) \text{ are all even. Now for each funny edge}$$

$n_i + m_i = 1$ while for all other edges $n_i + m_i$ is even (0 or 2). It follows that the number of funny edges incident or v must be even.

C) Let γ be a closed curve on S , meeting Γ_f transversally (or not at all). Then γ is orientation reversing if and only if the number of common points of γ and Γ_f is odd.

D) If S' denotes the bounded surface obtained by cutting S open along Γ_f , it follows that S' is canonically oriented in the sense that an orientation can be defined by equating + to clockwise and - to anticlockwise.

We now introduce a further requirement.

b) S' is connected.

E) S is then orientable if and only if $\Gamma_f = 0$. Further, if S is orientable and oriented, the \pm signs are redundant (or, alternatively, the orientation arrows), since + stands for clockwise and - for anticlockwise.

With the combination of requirements (a) and (b) above we may effectively speak of the global orientation of any Σ_Q and mean the global orientation of Σ'_Q where Σ'_Q is a connected orientable surface related to Σ_Q as S' is related to S in this appendix. It then suffices to attach either (+, -) signs or (\circ , \odot) arrows to triangles in order to denote local orientations. The plugging rules of Appendix C of Ref. (1) survive if applied to Σ'_Q .

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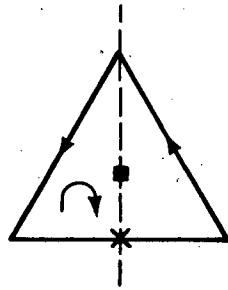
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Triangle Charge-orientation arc orientation	+	-	
"Out" of Σ_C	$Q = +1$	$Q = 0$	$Q = 0$
"Into" Σ_C	$Q = 0$	$Q = -1$	$Q = -1$

Table I: The association of electric charge with combinations of charge-arc and triangle orientations.

FIGURE CAPTIONS

1. The peripheral I triangle or "topological quark".
2. The single-triangle lepton proposed in Ref. (3).
3. The $\bar{I}\bar{I}$ lepton area on Σ_Q .
4. (a) Lepton "propagator" Σ_C .
(b) Insertion of electroweak boson into lepton "propagator" (coupling here to Y triangle).
5. The belt for Fig. 4(a).
6. Lepton "propagator" Σ_Q .
7. (a) Normal pattern for mated $\bar{I}\bar{I}$ peripheral triangles.
(b) "Funny" pattern for mated $\bar{I}\bar{I}$ peripheral triangles.
8. Σ_Q for baryon-lepton mixing.
9. Σ_C for baryon-lepton mixing.
10. Belt for baryon-lepton mixing.
11. $\bar{I}\bar{I}$ lepton area on Σ_Q .
12. Funny edge of Σ_Q within lepton area.
13. Substitution of diquark disk by Y-triangle Möbius band of the same "hypercharge".
14. The possible relationships of adjacent triangles on Σ_Q .



- Belt
- x End of momentum arc or momentum-copy arc
- End of charge arc

Figure 1

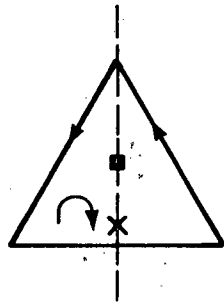


Figure 2

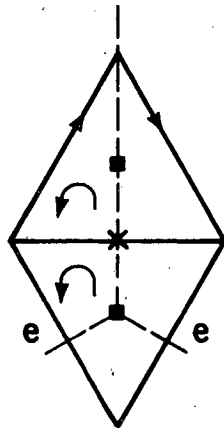


Figure 3

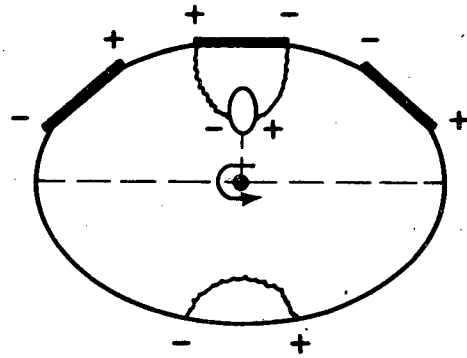
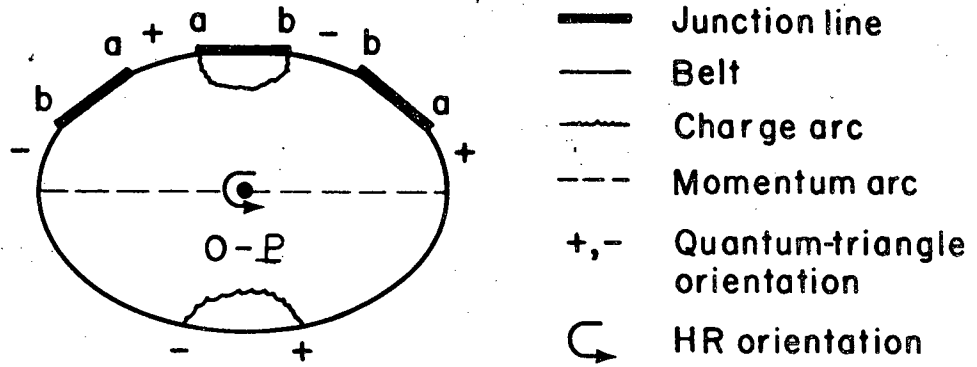
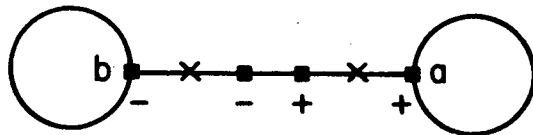


Figure 4



- x End of momentum arc
- End of charge arc

Figure 5

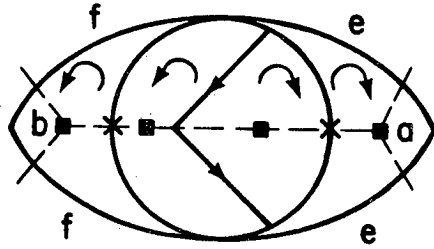


Figure 6

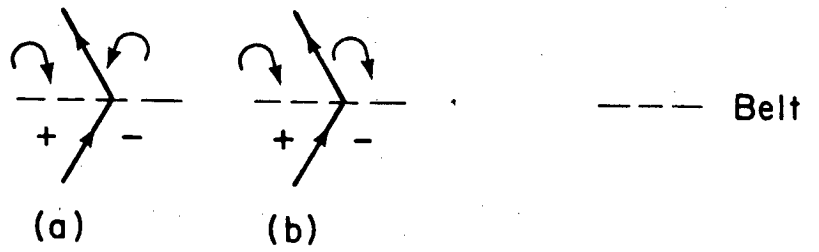


Figure 7

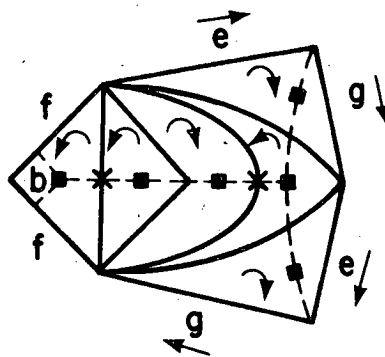


Figure 8

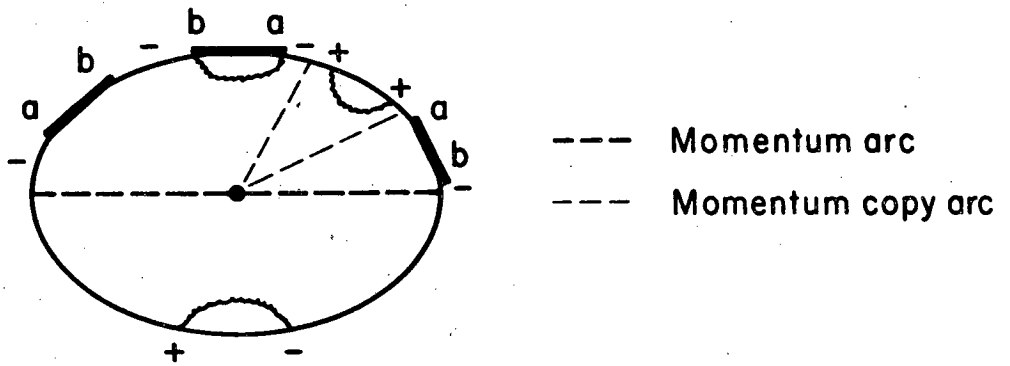


Figure 9

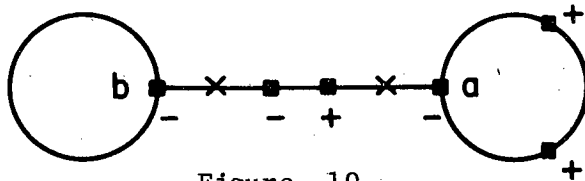


Figure 10

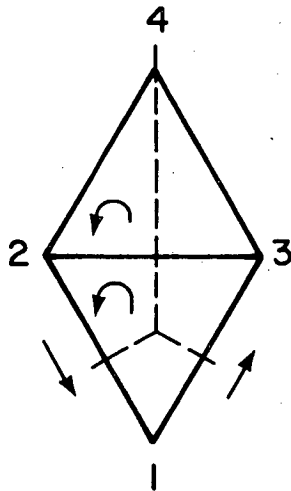


Figure 11

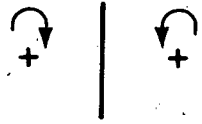


Figure 12

29

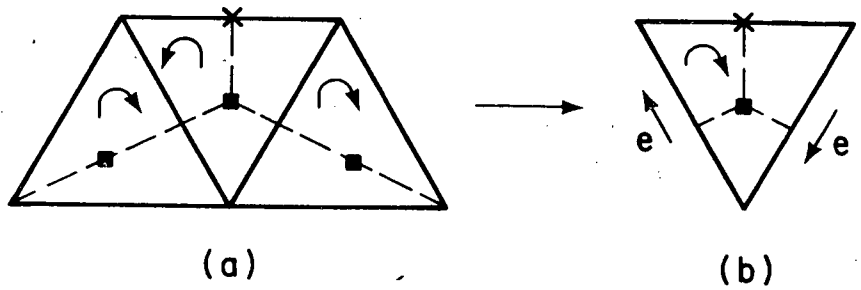
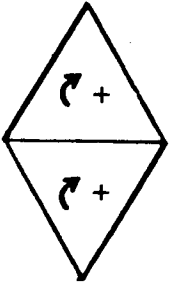
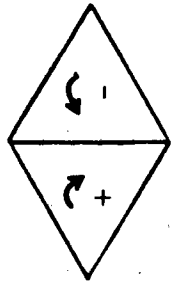


Figure 13

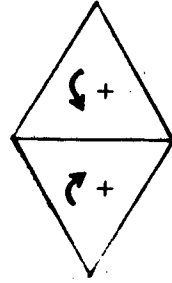
XBL 823-175



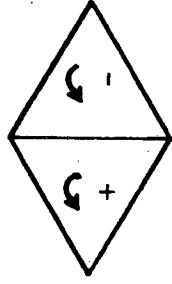
(a)



(b)



(c)



(d)

Figure 14

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