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Quantum supergravity, supergravity anomalies and string phenomenology

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Abstract

I discuss the role of quantum effects in the phenomenology of effective supergravity theories from compactification of the weakly coupled heterotic string. An accurate incorporation of these effects requires a regularization procedure that respects local supersymmetry and BRST invariance and that retains information associated with the cut-off scale, which has physical meaning in an effective theory. I briefly outline the Pauli–Villars regularization procedure, describe some applications, and comment on what remains to be done to fully define the effective quantum field theory.
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1. Introduction

Since the first "string revolution" of 1984, starting with the Green–Schwarz discovery [1] that string theories with an $SO(32)$ or $E_8 \otimes E_8$ gauge sector are anomaly-free, there has been a considerable amount of work on orbifold compactifications of the heterotic $E_8 \otimes E_8$ string theory [2] that mimic Calabi–Yau compactification [3] on a six-dimensional manifold. In these studies, the favored mechanism for supersymmetry breaking has been gaugino condensation in

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the subgroup of the hidden sector E_8 that survives after symmetry breaking by Wilson lines, aka the Hositani mechanism. Gaugino condensation is an inherently quantum effect in the effective supergravity theory that is the large tension limit of the string theory, and, as described below, quantum anomalies play an essential role in its description. Other aspects where quantum effects play a role include the issues of vacuum stability and flavor changing neutral currents, axion physics and soft supersymmetry breaking. The last of these have contributions that are specific to supergravity, and a reliable method for computing them is imperative.

Specifically, we require a regularization procedure that respects gauge invariance and local supersymmetry. In renormalizable, globally supersymmetric theories one uses dimensional reduction. Like dimensional regularization, used for ordinary gauge theories, this procedure eliminates quadratic divergences altogether. However, in an effective theory, such as a four-dimensional supergravity theory from a ten-dimensional string theory, the quadratic divergences, or more specifically the effective cut-offs, have physical significance. For example, the effective cut-off of a few hundred GeV for the Fermi theory of weak interactions pointed the way towards relevant energies to search for new physics, and this new physics manifested itself in the form of the W and Z bosons with masses of about 100 GeV. Similarly, the need to suppress strangeness-changing neutral currents indicated a scale of a few GeV, and led to the successful prediction of the charmed quark mass. In a simple field theory one can just introduce a momentum cut-off, which generally gives correct results at one-loop, up to the precise coefficient of the quadratically divergent operators. However this procedure does not respect local symmetries, or even global supersymmetry, which is why one uses dimensional regularization or reduction in renormalizable theories with these symmetries. In the case of local supersymmetry, or supergravity, the use of a momentum cut-off can produce misleading results, as illustrated in some examples below.

The ultraviolet divergent part of the on-shell effective Lagrangian for a general supergravity theory with at most two derivatives at tree-level was determined [4] using the covariant derivative expansion [5]. As is well known, the quadratically divergent contribution is prescription-dependent. Specifically, the use of a simple cut-off or subtraction procedure for a supersymmetric theory does not yield a supersymmetric result. However, as first shown in [6], when the theory is regulated by Pauli–Villars fields embedded in a supersymmetric Lagrangian, there are additional finite terms quadratic in the PV masses that complete the one-loop action in such a way that the result is supersymmetric.

2. Pauli–Villars regularization of supersymmetry and supergravity

Pauli–Villars (PV) regularization was initially used to regulate the divergences in quantum electrodynamics. However this procedure could not be generalized to non-Abelian gauge theories because the introduction of the needed massive PV gauge bosons breaks gauge invariance, or–in the gauge-fixed version of the theory–BRST invariance [7]. However, in supersymmetric theories, the same cancellation of ultraviolet divergences that leads to the well-known nonrenormalization theorems also allows for PV regularization of these theories.

For example, some of the divergences arising from loop diagrams involving gauge boson self-couplings, that require the introduction of massive vector PV fields, are canceled in supersymmetric theories by loops involving gauginos. As a consequence a renormalizable supersymmetric theory can be regulated by introducing PV fields only in chiral supermultiplets, and BRST invariance is unbroken. In the case of supergravity, the theory can be regulated by the introduction of massive PV chiral supermultiplets and Abelian vector multiplets, and BRST invariance again remains unbroken. As a result, all the on-shell logarithmic and quadratic divergences can be

canceled [8] in a supergravity theory defined in the usual way [9] by an arbitrary holomorphic superpotential $W(Z)$, a real Kähler potential $K(Z, \bar{Z})$ and a holomorphic gauge kinetic function $f(Z)$, provided the gauge charges of matter fields have the same overall quadratic Casimirs [10] as those of some real (reducible) representation R of the Yang–Mills gauge group:

$$C_M^a = \text{Tr} T^a T^a \equiv C_R^a. \tag{2.1}$$

Here Z represents the chiral matter superfields, and the sum in the trace (2.1) runs over all the chiral fields Z^i. The condition (2.1) on the Casimirs is satisfied in the Minimal Supersymmetric Standard Model (MSSM) and in extensions thereof. In addition to two Higgs doublets, the MSSM, has $2N_f$ fundamental representations (reps) $\mathbf{n}$ of each group factor $\mathcal{G}_n = SU(n)$, $n = 2, 3$, where N_f is the number of quark flavors. Their Casimirs can be mimicked by N_f real PV reps $(\mathbf{n} + \bar{\mathbf{n}})$. Further extensions necessarily involve real representations of the Standard Model gauge group, so that the additional states can get SM gauge invariant masses. The condition (2.1) is also satisfied in the hidden sectors [11] that can accompany the Standard-Model-like $\mathbb{Z}_N$ orbifolds found in [12]. These hidden sectors also come in even numbers of representations, except for two cases. In one the hidden sector contains 3 $\mathbf{16}$'s of $SO(10)$ which contribute $C_M^{SO(10)} = 6$; this can be mimicked by a real PV rep with 6 $\mathbf{10}$'s. The other has a hidden sector with 3 $(\mathbf{5} + \mathbf{10})$'s and 6 $\bar{\mathbf{5}}$'s of $SU(5)$ with $C_M^{SU(5)} = 9$, that can be mimicked by 9 real PV reps $(\mathbf{5} + \bar{\mathbf{5}})$. Since the underlying theory is finite when all degrees of freedom are included, one would expect (2.1) to have a solution for general superstring compactifications.

The part of the resulting one-loop action that is quadratic in the PV masses is just a renormalization of the Kähler potential, while the part logarithmic in PV masses contributes to the renormalization of both the Kähler potential and the gauge kinetic function (which is no longer holomorphic at the quantum level). In addition there are new operators of dimension 6–12; those of dimension six involve the curvature of the Kähler metric and derivatives of the gauge kinetic function.

Most of the linear divergences of a generic supergravity theory can be canceled by the PV fields introduced above. Their associated chiral anomalies[1] either disappear or reappear through noninvariant PV mass terms, forming an "F-term" anomaly that incorporates the associated conformal anomaly [10]. However there are chiral anomalies associated with the affine connection in the gravitino covariant derivative and with an off-diagonal gravitino–gaugino connection that cannot be canceled by the PV fields. These form supersymmetric "F-term" anomalies together with conformal anomalies associated with total derivatives that are not canceled by the PV fields, provided the cut-off is field dependent:

$$\Lambda(Z, \bar{Z}) = \mu_0 \exp(K/4), \tag{2.2}$$

where μ_0 is a constant that can be set to infinity at the end of the calculation; the only effect of the field-dependence in (2.2) is that total derivatives with nonvanishing coefficients of $\ln \Lambda$ do not drop out of the S-matrix of the regulated theory. For example, the conformal anomaly associated with the Gauss–Bonnet term combines with the chiral anomaly proportional to the space–time curvature term $r \cdot \tilde{r}$, with an overall coefficient [10] that agrees with string loop calculations [15].

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[1] The chiral anomalies of supergravity were first evaluated in [13], including those arising from an additional connection [14] in theories with an anomalous $U(1)$ and no compensating Green–Schwarz term; these contributions are not present in the class of string-derived theories considered here.

In addition to the above-described "F-term" anomalies, there are "D-term" anomalies associated with logarithmic divergences that are not canceled by the PV regulator fields, and that have no chiral counter-parts.

The anomalies that we are concerned with here involve symmetries of the underlying string theory that are not respected at the quantum level of the effective field theory without the introduction of some cancellation mechanism that appears only at one-loop order. In compactifications of the weakly coupled heterotic string theory these are a discrete group of transformations known as "T-duality" or "target space modular invariance" [16], present in all heterotic string compactifications, and an anomalous $U(1)$ symmetry, often referred to as $U(1)_X$, that is present in most compactifications involving Wilson lines. Gauge symmetry breaking by Wilson lines is generally needed both for providing a gauge group that resembles the Standard Model and for gaugino condensation in the hidden sector, which can provide a source of supersymmetry breaking.

The effective four dimensional (4d) theory from the heterotic string includes several important "moduli" chiral supermultiplets: the dilaton supermultiplet S, whose vacuum value determines the gauge coupling constant and the θ-parameter of the 4d gauge theory, and "Kähler moduli" T^i whose vacuum values determine the size and shape of the compact six dimensional space. There are at least three of the latter in orbifold compactifications, and the group of T-duality transformations always contains an $SL(2, \mathbf{Z})$ subgroup under which these three "diagonal moduli" transform as

$$T'^i = \frac{aT^i - ib}{icT^i + d}, \qquad ad - bc = 1, \tag{2.3}$$

and which is generated by two elements: the inversion of the radii (in string units) $\mathrm{Re}\, t^i \to 1/\mathrm{Re}\, t^i$ of the three 2-tori in the compact six dimensional space, and the axionic shifts $\mathrm{Im}\, t^i \to \mathrm{Im}\, t^i + 1$. Here $t^i = T^i \big|$ is the scalar component of the chiral superfield T^i. More generally, T-duality acts as follows on chiral (antichiral) superfields $Z^p = T^i$, Φ^a ($\bar{Z}^{\bar{p}} = \bar{T}^{\bar{i}}, \bar{\Phi}^{\bar{a}}$):

$$T^i \to h(T^j), \qquad \Phi^a \to f(q_i^a, T^j)\Phi^a, \qquad \bar{T}^{\bar{i}} \to h^*(\bar{T}^{\bar{j}}), \qquad \bar{\Phi}^{\bar{a}} \to f^*(q_i^a, \bar{T}^{\bar{j}})\bar{\Phi}^{\bar{a}}, \tag{2.4}$$

where q_i^a are the modular weights of Φ^a, and, under $U(1)_X$ transformations,

$$V_X \to V_X + \Lambda_X + \bar{\Lambda}_X, \qquad \Phi^a \to e^{-q_X^a \Lambda_X}\Phi^a, \qquad \bar{\Phi}^a \to e^{-q_X^a \bar{\Lambda}_X}\bar{\Phi}^a, \tag{2.5}$$

where V_X is the $U(1)_X$ vector superfield, with Λ_X ($\bar{\Lambda}_X$) chiral (antichiral), and q_X^a are $U(1)_X$ gauge charges.

In order to faithfully represent the underlying string theory, in which both of the above symmetries are exact to all orders in perturbation theory [16], additional terms must be added to the effective supergravity Lagrangian. The terms that restore the symmetry to the coefficients of bilinears in the Yang–Mills fields and the space–time Riemann tensor at one loop were identified some time ago as a combination of four-dimensional counterparts [17] of the ten-dimensional Green–Schwarz term [1] and, for some compactifications, threshold corrections [18] that contribute to the cancellation of the modular anomaly. The implementation of these cancellations is possible only if the loop corrections in the regulated theory satisfy certain constraints. For example, in the absence of threshold corrections, the gauge charges and modular weights must satisfy:

$$8\pi^2 b = \frac{1}{24}\left(2\sum_p q_i^p - N + N_G - 21\right) \quad \forall\, i$$

$$= C^a - C_M^a + 2\sum_b (T_a^2)_b^b q_i^b \quad \forall\, i, a, \tag{2.6}$$

$$2\pi^2 \delta_X = -\frac{1}{24}\mathrm{Tr}T_X = -\frac{1}{3}\mathrm{Tr}T_X^3 = -\mathrm{Tr}(T_a^2 T_X) \quad \forall\, a \ne X, \tag{2.7}$$

where C^a is the quadratic Casimir in the adjoint of the gauge group factor $\mathcal{G}_a$, and the matter Casimir C_M^a is defined in (2.1).

The above expressions for the coefficients of one-loop generated operators that are linear in the parameters q_i^p, q_X^a of the anomalous transformations are universal. They are independent of the precise choice of PV regulator fields, provided the one-loop, on-shell quadratic and logarithmic divergences are canceled. However this is not the case for operator coefficients that are quadratic and higher order in these parameters [19,10]. We will return to this issue in Section 6.

3. Quadratic divergences

It has been pointed out [20,21] that the loop suppression parameter

$$\epsilon = \frac{1}{16\pi^2} \tag{3.1}$$

may be compensated by large coefficients, leading to significant effects from loop corrections. For example, if supersymmetry is F-term dominated with negligible vacuum energy $\langle V \rangle \approx 0$ at tree level, the quadratically divergent correction to the scalar potential reduces to:

$$V_Q = \epsilon \Lambda^2 \big[(N_\chi - 1)|M_\psi|^2 - N_G|M_\lambda|^2 - R_{i\bar{m}}F^i \bar{F}^{\bar{m}} \big], \tag{3.2}$$

where N_χ and N_G are the number of chiral and gauge supermultiplets, respectively, $M_\psi = e^K W$ is the gravitino mass, M_λ is the gaugino mass, which depends on derivatives of the kinetic function $f(z) = f(Z)|$, $z = Z|$, and $R_{i\bar{m}}$ is the Ricci tensor associated with the Kähler metric $K_{i\bar{m}}$.

3.1. Vacuum stability

Typical orbifold compactifications have many more chiral multiplets than gauge multiplets: $N_\chi \gtrsim 300$, $N_G \lesssim 65$. In addition, in many gravity mediated supersymmetry-breaking scenarios the gaugino mass M_λ is much smaller than the gravitino mass:

$$M_\lambda^2 = \frac{1}{4} f_i \bar{f}^i M_\psi^2 \ll M_\psi^2. \tag{3.3}$$

Thus the first term in (3.2) suggests the possibility of a significant *positive* contribution to the vacuum energy [20], perhaps curing the problems with classes of models that have negative vacuum energy at tree level. However, in the regulated theory (3.2) is replaced by

$$V_Q \rightarrow \epsilon \big[|M_\psi|^2 \big(N_\chi \Lambda_\chi^2 - \Lambda_{\mathrm{grav}}^2 \big) - N_G M_\lambda^2 \Lambda_G^2 - R_{i\bar{m}}F^i \bar{F}^{\bar{m}} \Lambda_\chi'^2 \big] + \cdots, \tag{3.4}$$

where the ellipsis indicates finite terms proportional to the PV squared masses such that the one-loop quadratically divergent corrections are completely absorbed into renormalizations:

$$\mathcal{L}_Q = \mathcal{L}_{\mathrm{tree}}(g_{\mu\nu}^R, K^R) - \mathcal{L}_{\mathrm{tree}}(g_{\mu\nu}, K) + O(\epsilon^2), \qquad K^R = K + \epsilon \sum_A \Lambda_A^2. \tag{3.5}$$

The effective squared cut-offs Λ_A^2 in (3.4) and (3.5) are determined by combinations of the PV masses M_I weighted by their signatures $\eta_I = \pm 1$:

$$\Lambda_A^2 = \sum_I C_A^I \eta_I M_I^2 \ln M_I^2, \qquad \sum_{I \in A} \eta_I M_I^2 = 0, \tag{3.6}$$

where C_A^I is a constant. In fact, the apparent appearance of a sizable, positive cosmological constant in (3.2) and (3.4) is misleading on several counts [22]. The sign of Λ_A^2 is, in fact, indeterminate [23] if there are five or more terms in the sum, which is generally required to eliminate all the UV divergences of SUGRA. More importantly, if $N_\chi \sim \epsilon^{-1}$ one has to sum the leading $(\epsilon \Lambda^2)^n$ terms, and supersymmetry dictates that the higher order terms complete the Lagrangian $\mathcal{L}_{\text{tree}}(g_{\mu\nu}^R, K^R)$ with K_R given by (3.5). So, for example, if the M_I^2 are field independent constants, we just get for this contribution to the loop corrected potential

$$V_Q = e^{K + \Delta K} \left[(W_i + K_i W) K^{i\bar{m}} \left(\overline{W}_{\bar{m}} + K_{\bar{m}} \overline{W} \right) - 3|W|^2 \right] + \frac{\operatorname{Re} f}{2} D^a D_a,$$

$$W_i + K_i W = e^{-K} \frac{\partial}{\partial z^i} (e^K W) = -e^{-K/2} K_{i\bar{m}} \bar{F}^{\bar{m}}, \tag{3.7}$$

where $K^{i\bar{m}}$ is the inverse of the Kähler metric, $F^m = (\bar{F}^{\bar{m}})^\dagger$ is the auxiliary field for the chiral superfield Z^m, and

$$D_a = K_i (T_a z)^i = K_{\bar{m}} (T_a^T \bar{z})^{\bar{m}}, \qquad K_i = \frac{\partial K}{\partial z_i}, \quad K_{\bar{m}} = \frac{\partial K}{\partial z_{\bar{m}}}, \tag{3.8}$$

is the auxiliary field for the vector supermultiplet V_a. If, in addition, supersymmetry is broken only by F-terms, $\langle D_a \rangle = 0$, the vacuum energy is just multiplied by a positive constant, so if it vanishes at tree level, there is no large correction of order ϵN.

3.2. Flavor changing neutral currents

It was also pointed out [21] that the last term in (3.2) or (3.4) can be significant because the contracted indices of the Kähler Riemann tensor implicit in the Ricci tensor imply a sum over all the chiral supermultiplets. The Kähler potential for the twisted sector from orbifold compactification of the heterotic string is not known beyond leading (quadratic) order, and could include terms that induce flavor changing neutral current (FCNC) effects in the observable sector. Experimental limits on these effects therefore imply restrictions on the tree-level Kähler potential. A sufficient condition [22] for a "safe" Kähler potential at the quantum level is the presence of isometries of the Kähler geometry. For example, the Kähler potential for an untwisted sector (i) from orbifold compactification takes the form

$$K^{(i)} = -\ln \left(T^i + \bar{T}^{\bar{i}} - \sum_{a=1}^{N_i} |\Phi_i^a|^2 \right), \tag{3.9}$$

which has an $SU(N_i + 1, 1)$ symmetry that is necessarily also a symmetry of the Ricci tensor:

$$R_{p\bar{q}}^{(i)} = (N_i + 2) K_{p\bar{q}}^{(i)}, \qquad p, q = i, a \in (i). \tag{3.10}$$

Alternatively the suppression of FCNC effects can by achieved through a judicious choice of PV masses [22].

4. Anomaly cancellation and its implications

In $\mathbb{Z}_3$ and $\mathbb{Z}_7$ compactifications of the heterotic string, with no threshold corrections, the variation under (2.4) and (2.5) of the anomalous part of the one-loop corrected Lagrangian contains the term

$$\Delta\mathcal{L}_{\text{anom}} = \frac{1}{8} \int d^4\theta \, \frac{E}{R} \Phi H + \text{h.c.,} \tag{4.1}$$

as expressed in the Kähler $U(1)$ [$U(1)_K$] superspace formulation of supergravity [24]. Here E is the superdeterminant of the supervielbein and $R = \frac{1}{2}e^{K/2}W$, $R| = \frac{1}{2}M_\psi$ is an auxiliary field of the supergravity supermultiplet. Also

$$\Phi = \frac{1}{3} W^{\alpha\beta\gamma} W_{\alpha\beta\gamma} + W^\alpha_a W^a_\alpha \tag{4.2}$$

is a chiral superfield with $U(1)_K$ weight 2 with W^a_α and $W^{\alpha\beta\gamma}$ the Yang–Mills and spacetime curvature superfield strengths, respectively, and

$$H = -bF(T) + \frac{1}{2}\delta_X \Lambda_X \tag{4.3}$$

is a zero weight chiral supermultiplet, with b and δ_X subject to the conditions (2.6) and (2.7). The Kähler potential can be decomposed as

$$K = G(T, \bar{T}) + K_{\text{inv}} \tag{4.4}$$

with K_{inv} modular invariant, and

$$G(T, \bar{T}) \to G(T, \bar{T}) + F(T) + \bar{F}(\bar{T}) \tag{4.5}$$

under the T-duality transformation (2.4). In component notation, (4.1) reads

$$\begin{aligned}
\Delta\mathcal{L}_{\text{anom}} = &-\frac{1}{4}\sqrt{g}\left[\text{Re}\,H\left(F^{\mu\nu}_a F^a_{\mu\nu} - \frac{2}{x^2}\mathcal{D}_a\mathcal{D}^a\right) + \text{Im}\,H F_a \cdot \tilde{F}^a\right] \\
&+ \frac{\sqrt{g}}{96}\left[\text{Re}\,H\left(r^{\mu\nu\rho\sigma}r_{\mu\nu\rho\sigma} - 2r_{\mu\nu}r^{\mu\nu} + \frac{1}{3}r^2\right) + \text{Im}\,H r \cdot \tilde{r}\right] \\
&+ \frac{\sqrt{g}}{144}\left(\text{Re}\,H X_{\mu\nu} X^{\mu\nu} + \text{Im}\,H \widetilde{X}_{\mu\nu} X^{\mu\nu}\right) + \text{fermions,}
\end{aligned} \tag{4.6}$$

where $X_{\mu\nu}$ is the field strength associated with the Kähler $U(1)$ connection in the fermion covariant derivatives.

4.1. Anomaly cancellation

Anomaly cancellation is most readily implemented using the linear multiplet formulation for the dilaton [25]. A linear supermultiplet is a real superfield that satisfies

$$(\mathcal{D}^2 - 8\bar{R})L = (\bar{\mathcal{D}}^2 - 8R)L = 0, \tag{4.7}$$

where $\bar{\mathcal{D}}^2 - 8R$ is the chiral projection operator in supergravity. The superfield L has three components: a scalar, the dilaton $\ell = L|$, a spin-$\frac{1}{2}$ fermion, the dilatino χ, and a two-form $b_{\mu\nu}$ that is dual to the axion $\text{Im}\,s$; it has no auxiliary field. For the purpose of anomaly cancellation we want instead to use a real superfield that satisfies the *modified* linearity condition:

$$(\bar{\mathcal{D}}^2 - 8R)L = -\Phi, \qquad (\mathcal{D}^2 - 8\bar{R})L = -\bar{\Phi}, \tag{4.8}$$

where Φ is a chiral multiplet with $U(1)_K$ and Weyl weights [24] $w_K(\Phi) = 2$, $w_W(\Phi) = 1$, respectively. Consider a theory defined by the Kähler potential K and the kinetic Lagrangian $\mathcal{L}_{KE}$:

$$K = k(L) + K(Z, \bar{Z}), \qquad \mathcal{L}_{KE} = -3 \int d^4\theta \, E \, F(Z, \bar{Z}, V_X, L). \tag{4.9}$$

The condition for a canonical Einstein term in $U(1)_K$ superspace is give by

$$F - L\frac{\partial F}{\partial L} = -L^2 \frac{\partial}{\partial L}\left(\frac{1}{L}F\right) = 1 - \frac{1}{3}L\frac{\partial k}{\partial L}, \tag{4.10}$$

with the solution:

$$F(Z, \bar{Z}, V_X, L) = 1 + \frac{1}{3}LV(Z, \bar{Z}, V_X) + \frac{1}{3}L\int \frac{dL}{L}\frac{\partial k(L)}{\partial L}, \tag{4.11}$$

where $\frac{1}{3}V$ is a constant of integration of (4.10) over L, and is therefore independent of L. If we take

$$V = -bV(Z, \bar{Z}) + \frac{1}{2}\delta_X V_X, \qquad V(Z, \bar{Z}) = G(T, \bar{T}) + V_{inv}(Z, \bar{Z}), \tag{4.12}$$

with V_{inv} modular invariant, under an anomalous transformation we have $\Delta V = H(T, \Lambda_X) + \bar{H}(\bar{T}, \bar{\Lambda}_X)$, with H given by (4.3), and

$$\Delta \mathcal{L}_{KE} = \frac{1}{8}\int d^4\theta \frac{E}{8R}(\bar{\mathcal{D}}^2 - 8R)LH + \text{h.c.} = -\frac{1}{8}\int d^4\theta \frac{E}{R}\Phi H + \text{h.c.}, \tag{4.13}$$

since the term involving $\bar{\mathcal{D}}^2$ vanishes by partial integration [24]. The anomaly (4.1) is canceled: $\Delta \mathcal{L}_{KE} = -\Delta \mathcal{L}_{anom}$.

Now consider the following Lagrangian

$$\mathcal{L}_{lin} = -3\int d^4\theta \, E\left[F(Z, \bar{Z}, V_X, L) + \frac{1}{3}(L + \Omega)(S + \bar{S})\right], \tag{4.14}$$

where S $(\bar{S})$ is chiral (antichiral):

$$S = (\bar{\mathcal{D}}^2 - 8R)\Sigma, \qquad \bar{S} = (\mathcal{D}^2 - 8\bar{R})\Sigma^\dagger, \qquad \Sigma \neq \Sigma^\dagger, \tag{4.15}$$

with Σ unconstrained; $L = L^\dagger$ is real but otherwise unconstrained, and Ω is a real superfield that satisfies

$$(\bar{\mathcal{D}}^2 - 8R)\Omega = \Phi, \qquad (\mathcal{D}^2 - 8\bar{R})\Omega = \bar{\Phi}. \tag{4.16}$$

If we vary the Lagrangian (4.14) with respect to the unconstrained superfields Σ, $\Sigma^\dagger$, we recover the modified linearity condition (4.8). This results in the term proportional to $S + \bar{S}$ dropping out from (4.14), which reduces to (4.9), with

$$F(Z, \bar{Z}, V_X, L) = 1 - \frac{1}{3}L\big[2s(L) - V(Z, \bar{Z}, V_X)\big], \qquad s(L) = -\frac{1}{2}\int \frac{dL}{L}\frac{\partial k(L)}{\partial L}, \tag{4.17}$$

where the vacuum value $\langle s(L)|\rangle = \langle s(\ell)\rangle = g_s^{-2}$ is the gauge coupling constant at the string scale.

Alternatively, we can vary the Lagrangian (4.14) with respect to L, which determines L as a function of $S + \bar{S} + V$, subject to the condition

$$F(Z, \bar{Z}, V_X, L) + \frac{1}{3}L(S + \bar{S}) = 1 \equiv F(Z, \bar{Z}, V_X, S + \bar{S} + V), \tag{4.18}$$

which assures that once the (modified) linear multiplet is eliminated, the requirement

$$\mathcal{L}_{\mathrm{KE}} = -3 \int d^4\theta\, E F(Z, \bar{Z}, V_X, S + \bar{S}) = -3 \int d^4\theta\, E, \qquad K = K(Z, \bar{Z}, V_X, S + \bar{S}),$$

for a canonically normalized Einstein term with only chiral matter is recovered. Together with the equation of motion[2] for L, the condition (4.18) is equivalent to the condition (4.10) and the Lagrangian (4.14) becomes

$$\mathcal{L}_{\mathrm{lin}} = -3 \int d^4\theta\, E - \int d^4\theta\, E(S + \bar{S})\Omega = -3 \int d^4\theta\, E + \frac{1}{8}\left(\int d^4\theta\, \frac{E}{R} S\Phi + \mathrm{h.c.}\right). \tag{4.19}$$

Since $L = L(S + \bar{S} + V)$ is invariant under T-duality and $U(1)_X$, we require $\Delta S = -H$, so the variation of (4.19) is again given by (4.13).

For other orbifolds with T-dependent threshold corrections, the conditions (2.6) are modified somewhat, but the cancellation of the anomaly (4.1) goes through as above. In this case the modular anomaly in (4.1) is partially canceled by the threshold cancellations, and partially canceled by the "Green–Schwarz" term encoded in the terms proportional to $V(Z, \bar{Z}, V_X)$ in (4.11) and (4.18).

4.2. Gauge coupling unification

The form of $V(Z, \bar{Z})$ in (4.12) is not completely determined by the requirement of anomaly cancellation, because $V_{\mathrm{inv}}(Z, \bar{Z})$ can be any invariant function of the chiral supermultiplets. For example, in $\mathbb{Z}_N$ models with just the three "diagonal" Kähler moduli introduced in (2.3), under the $SL(2, \mathbf{Z})$ subgroup the transformations (2.4) reduce to (2.3) and

$$\Phi^a \to e^{-\sum_i q_i^a F^i} \qquad F^i = \ln(icT^i + d), \qquad \sum_i F^i = F(T), \tag{4.20}$$

and the Kähler potential takes the form

$$K = K(L) + G(T, \bar{T}) + \sum_a |\Phi^a|^2 e^{\sum_i q_i^a g^i} + O(\Phi^3), \qquad \sum_i g^i = G(T, \bar{T}), \tag{4.21}$$

with

$$g^i \to g^i + F^i + \bar{F}^i \tag{4.22}$$

under a T-duality transformation. If we took, for example,

$$V_{\mathrm{inv}} = \sum_{a=1}^{n} c_a \ln\left(|\Phi^a|^2 e^{\sum_i q_i^a g^i}\right), \qquad \sum_{a=1}^{n} c_a q_i^a = -1 \quad \forall i, \tag{4.23}$$

the Kähler moduli would drop out of $V(Z, \bar{Z})$, but the anomaly would still be canceled.

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[2] In $U(1)_K$ superspace E has an implicit dependence on the Kähler potential K such that $\partial E/\partial L = -E(\partial K/3\partial L)$. With the conventions of [24], Ω has Weyl weight $w_W(\Omega) = -w_W(E) = 2$, so $E\Omega$ is independent of K, i.e. of L, and $\delta\mathcal{L}_{\mathrm{lin}}/\delta L = 0$ together with (4.18) gives (4.10).

In fact, the T-dependence of V has been determined [26,27] by matching string theory calculations to the effective field theory. For example, in $\mathbb{Z}_3$ and $\mathbb{Z}_7$ orbifolds, with no threshold corrections, the T-dependence drops out of the coefficient of $F \cdot \tilde{F}$ when the matter fields Φ^a are set to zero. By supersymmetry, which implies a holomorphic gauge kinetic function, the coefficient of $F \cdot F$ also vanishes, which means that the contribution from the field theory loop corrections must be exactly canceled by that from the "Green–Schwarz" term in (4.11) or (4.18); this requires [26] $V_{\text{inv}}(T, \bar{T}, \Phi = 0) = 0$. This result remains true for orbifolds with threshold corrections, but the coefficient $b \to b_{\text{loop}} \neq b$ of the loop correction in (4.1), (4.3) is modified in such a way that the full anomaly is canceled in the presence of additional T-dependent threshold contributions.

In the regulated theory, the coefficient $g_{a\,\text{eff}}^{-2}$ of $F^a \cdot F_a$ at the string scale is determined by the masses of the PV fields that replace the cut-off Λ. The relevant PV masses are uniquely determined by the requirement of the cancellation of ultra-violet divergences. For orbifolds with no threshold corrections one gets

$$\frac{1}{g_{a\,\text{eff}}^2} = \frac{1}{g^2(\ell_0)} - \frac{1}{16\pi^2} \left(C^a - C_M^a \right) k(\ell_0) - \frac{2}{16\pi^2} \sum_b C_b^a \ln(1 - p_b \ell_0)$$

$$\frac{1}{g^2(\ell)} \equiv s(\ell) = -\int d\ell \frac{k'}{2\ell}, \tag{4.24}$$

where p_b is the coefficient of $|\Phi^b|^2 e^{\sum_i q_i^b g^i}$ in $V(Z, \bar{Z}, V_X)$, and $\ell_0 = \langle \ell \rangle$ is the vacuum value of the scalar component ℓ of L. The expression (4.24) is independent of the renormalization scale, and may be compared [26] with the two-loop order renormalization group invariant quantity [28]

$$\delta_a = \frac{1}{g_a^2(\mu)} - \frac{1}{16\pi^2}(3C^a - C_M^a) \ln \mu^2 + \frac{2C^a}{16\pi^2} \ln g_a^2(\mu) + \frac{2}{16\pi^2} \sum_b C_b^a \ln Z_b^a(\mu), \tag{4.25}$$

where Z_b^a are the renormalization factors for the matter fields, and μ is the renormalization scale. If we equate the scale-independent quantity δ_a with $g_{a\,\text{eff}}^{-2}$ and impose the boundary conditions

$$g(\ell_0) = g_s = g_a(\mu_s), \qquad Z_b^a(\mu_s) = (1 - p_b l)^{-1}, \qquad k(\ell_0) = \ln \mu_s^2, \tag{4.26}$$

where μ_s is the string scale in reduced Planck mass units: $m_P = (8\pi G_N)^{-\frac{1}{2}} = 1$, we obtain the renormalization group equation

$$g_a^{-2}(\mu) = g^{-2}(\mu_s) - \epsilon_a - \frac{1}{8\pi^2} \left(3C^a - C_M^a \right) \ln(\mu_s/\mu) + \frac{C^a}{8\pi^2} \ln \left[g^2(\mu_s)/g_a^2(\mu) \right]$$

$$+ \frac{1}{8\pi^2} \sum_b C_b^a \ln \left[Z_b(\mu_s)/Z_b(\mu) \right], \tag{4.27}$$

where

$$\epsilon_a = \frac{C^a}{8\pi^2} \frac{\ln g^2(\ell_0)}{k(\ell_0)} \tag{4.28}$$

is a scale-independent threshold correction. For example, in the classical limit we have

$$k(\ell) = \ln \ell, \qquad g^{-2}(\ell) = s(\ell) = -\int d\ell \frac{k'}{2\ell} = \frac{1}{2\ell}, \qquad \epsilon_a = \ln 2 \frac{C^a}{8\pi^2}. \tag{4.29}$$

This gives for the gauge unification scale in the $\overline{MS}$ scheme [29]

$$\mu_{\text{unif}}^2 = \frac{\mu_s^2}{2e} = \frac{g_s^2 m_P^2}{2e} \sim 2 \times 10^{17} \text{ GeV}. \tag{4.30}$$

This is an order of magnitude larger than what is obtained by extrapolating from low energy data [30] in the context of the minimal supersymmetric extension of the Standard Model, but in effective theories from superstrings one expects heavy states that are vector-like under the Standard Model gauge group, as well as corrections to the dilaton Kähler potential from string nonperturbative effects and/or field theory loop effects. For orbifold compactifications with threshold corrections, there are additional T-dependent terms in (4.24); these give small corrections in the weak coupling limit $T \sim 1$.

Note that the result (4.24) of the one-loop calculation incorporates the two-loop result in (4.25). This is because a supersymmetric regularization procedure necessarily gives a supersymmetric result [31]. The chiral anomaly, which is completely determined at one loop, must form a supersymmetric operator with the conformal anomaly. This two-loop "correction" to the standard one-loop form of the beta-function is encoded in the dilaton dependence of the effective cut-offs, in this case the PV masses.

4.3. Hidden gaugino condensation

A popular candidate for supersymmetry breaking in the context of superstring theory is through gaugino condensation in a hidden sector, that is, a Yang–Mills sector that couples to the Standard Model only through gravitational strength couplings. Effective theories for gaugino and matter condensates were first constructed in globally symmetric theories [32], by matching the anomalies of the effective condensate Lagrangian to those of the underlying Yang–Mills Lagrangian. This can be generalized [33] to the supergravity case by introducing chiral superfields with $U(1)_K$ weight 2 and 0, respectively, for gaugino condensates U_a and matter condensates Π_a^α:

$$U_a \simeq (W_a^\alpha W_\alpha^a)_{\text{hid}}, \qquad \Pi_a^\alpha \simeq \prod_b (\Phi_a^b)_{\text{hid}}^{n_b^\alpha}, \tag{4.31}$$

where the elementary chiral field Φ_a^b is charged under the strongly coupled hidden sector gauge group $\mathcal{G}_a$. The effective Lagrangian for these fields is

$$\mathcal{L}_{\text{eff}}(U_a, \Pi_a^\alpha) = \frac{1}{8} \int d^4\theta \frac{E}{R} \sum_a U_a \left[b_a' \ln \left(e^{-K/2} U_a \right) + \sum_\alpha b_a^\alpha \ln \Pi_a^\alpha \right] + \text{h.c.} \tag{4.32}$$

with the constant coefficients

$$b_a' = \frac{1}{8\pi^2} \left(C^a - \sum_b C_b^a \right), \qquad b_a^\alpha = \sum_{b \in \alpha} \frac{C_b^a}{4\pi^2 d_a^\alpha}, \qquad d_a^\alpha = \dim \left(\Pi_a^\alpha \right), \tag{4.33}$$

determined [33,34] by requiring that the variation of (4.32) reproduce the variation (4.1), (4.3) of the underlying theory, with U_a identified as in (4.31), and by matching the other anomalies of the effective theory to those of the underlying theory, including the anomalies under $U(1)_K$ (R-symmetry) and conformal transformations. Since the right hand side of the modified linearity constraint (4.8) now has $W^a W_a$ in (4.2) replaced by U_a for the strongly coupled hidden gauge groups, overall modular and $U(1)_X$ invariance are restored as before.

Adding a gauge invariant superpotential to the effective theory

$$W(\Pi) = \sum_{a,\alpha} C^a_\alpha(T) \Pi^\alpha_a, \tag{4.34}$$

where the T-dependence of the coefficients C assures invariance under T-duality, leads to a solution to the equations of motion with nonvanishing condensate vacuum values and masses of order of the condensate scale or larger. Integrating out these heavy condensates gives a potential for the scalar moduli t^i, s. This potential always has a minimum at the vanishing coupling limit $\langle s \rangle = g^{-2}(\mu_s) \to \infty$, with no gaugino condensate and no supersymmetry breaking. In fact, this is the only minimum in the absence of the symmetry-restoring Green–Schwarz term in (4.11) or (4.18) if the classical form $k(L) = \ln L$, or equivalently $k(S, \bar{S}) = -\ln(S + S)$, of the dilaton Kähler potential is used. This was known as the "runaway dilaton problem". However, when the Green–Schwarz term is included, there is a second runaway direction, this time in the direction of strong coupling, where string nonperturbative effects cannot be ignored. Including these effects provides a mechanism [35] for dilaton stabilization, known as Kähler stabilization, at finite coupling and with nonvanishing condensates and supersymmetry breaking.

4.4. Axion physics

The last term in the Lagrangian (4.19), with $\Phi = W^\alpha_a W^a_\alpha$ has a classical R-symmetry, under which the Yang–Mills fields strengths W_α and the condensates U transform, respectively, as

$$W^a_\alpha(\theta) \to W'^a_\alpha(\theta') = e^{\frac{i}{2}\alpha} W_\alpha(\theta'), \qquad U_a(\theta) \to U'_a(\theta') = e^{i\alpha} U(\theta') \tag{4.35}$$

where α is a constant parameter, and θ' is related to θ in such a way that the integral over θ in

$$\int d^4\theta \frac{E'(\theta')}{R'(\theta')} e^{i\alpha} \Phi(\theta') = \int d^4\theta' \frac{E(\theta')}{R(\theta')} \Phi(\theta') = \int d^4\theta \frac{E(\theta)}{R(\theta)} \Phi(\theta), \tag{4.36}$$

is invariant.[3] For an arbitrary chiral superfield Φ such that

$$\Phi'(\theta') = e^{i\beta} \Phi(\theta'), \tag{4.37}$$

under R-symmetry with gauge superfields transforming as in (4.35), the component fields transform as

$$\left. \frac{\partial^n}{\partial \theta^n} \Phi(\theta) \right| \to e^{i(\beta - \frac{n}{2}\alpha)} \left. \frac{\partial^n}{\partial \theta^n} \Phi(\theta) \right|, \qquad n = 0, 1, 2. \tag{4.38}$$

The symmetry under (4.35) is anomalous at the quantum level. For example, under $U(1)_K$ the gauge supermultiplets transform as in (4.35), while matter chiral supermultiplets transform simply as

$$\Phi^A_a(\theta) \to \Phi^A_a(\theta'), \qquad \Pi^\alpha_a(\theta) \to \Pi^\alpha_a(\theta'), \tag{4.39}$$

and the shift in the Yang–Mills coupling in (4.19) is given by

$$\Delta\mathcal{L}_{\mathrm{YM}}(\alpha) = \frac{i\alpha}{8} \sum_a b'_a \int d^2\theta\, W^\alpha_a W^a_\alpha + \mathrm{h.c.} = -\frac{\alpha}{4}\sqrt{g} \sum_a b'_a F_a \cdot \tilde{F}^a, \tag{4.40}$$

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[3] The integral $\int d^4\theta E/R$ in local supersymmetry transforms the same way as $\int d^2\theta \to e^{-i\alpha} \int d^2\theta'$ in global supersymmetry.

with b'_a given in (4.33). In the absence of nonperturbative effects, the right-hand side of (4.40) is a total derivative, and has no effect on the S-matrix. This is no longer true when condensation occurs, and $W_a^\alpha W_\alpha^a$ is replaced by the condensate U_a for one or more gauge group factors $\mathcal{G}_a$. Then R-symmetry is generally broken, just as quark condensation in QCD breaks chiral symmetry. However, if the Lagrangian is independent of the axion $a = \mathrm{Im}\, s$ except for its coupling to Φ in (4.19), there is a residual R-symmetry [36] in the case of just one condensate U_c. The variation of the condensate term in, e.g., (4.40) can be compensated by for a shift in the axion:

$$\mathrm{Im}\, s \rightarrow \mathrm{Im}\, s - \alpha b'_c. \tag{4.41}$$

In the class of models for gaugino condensation discussed above, the "classical" condensate Lagrangian (i.e. the part without the logs) is not invariant under $U(1)_K$ in the presence of the superpotential (4.34), which gives a superpotential Lagrangian term of the form (4.36) with $\Phi = \frac{1}{2} e^K W(\Pi)$. In this case the classical R-symmetry has, instead of (4.39)

$$\Phi_a^{b \in \alpha}(\theta) \rightarrow e^{i\alpha/d_a^\alpha} \Phi_a^{b \in \alpha}(\theta'), \qquad \Pi_a^\alpha(\theta) \rightarrow e^{i\alpha} \Pi_a^\alpha(\theta'), \tag{4.42}$$

and, for the strongly coupled condensate U_c, the shifts (4.40), (4.41) are replaced by

$$\Delta \mathcal{L}_{\mathrm{YM}}(\alpha) = \frac{i\alpha}{8} b''_c \int d^2\theta\, U_c + \mathrm{h.c.}, \qquad \Delta\, \mathrm{Im}\, s = -\alpha b''_c, \qquad b''_c = b'_c + \sum_\alpha b_c^\alpha. \tag{4.43}$$

If the condensates Π_c^α have dimension 3, from (4.33) we have simply

$$b''_c = b_c = \frac{1}{8\pi^2}\left(C^c - \frac{1}{3}\sum_A C_A^c\right), \tag{4.44}$$

which is related to the β-function by the one-loop order RGE

$$\frac{\partial g_a(\mu)}{\partial \ln \mu} = -\frac{3b_a}{2} g_a^3(\mu). \tag{4.45}$$

The dilaton potential[4] is dominated by the gauge group $\mathcal{G}_c$ with the largest beta-function coefficient b_c and the largest condensation scale

$$\Lambda_c \sim e^{-1/3b_c g_s^2} \mu_s, \qquad \langle |u_c| \rangle \sim \Lambda_c^3, \qquad u_c = U_c|. \tag{4.46}$$

The dilaton acquires a mass of order $\langle |u_c| \rangle$ in reduced Planck units, but the axion remains massless if there is a single condensate. In this case it is a prime candidate for the QCD axion.

If there is more than one term in the sum over a in (4.32), the axion acquires a small mass m_a. For example if there are two strongly coupled gauge groups $\mathcal{G}_c$, $\mathcal{G}_d$, both with $\dim\left(\Pi_{c,d}^\alpha\right) = 3$, we get [33]

$$m_a \sim (b_c - b_d)\left\langle \sqrt{|u_1 u_2|} \right\rangle. \tag{4.47}$$

This two-condensate system has a point of enhanced symmetry where the β-functions are equal, and R-symmetry remains unbroken. If the string a axion is to play the role of the QCD, or Peccei–Quinn, axion its mass due to symmetry-breaking must be much smaller than the QCD

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[4] In Kähler stabilization models the T-moduli are generically stabilized at self-dual points with vanishing F-terms, and supersymmetry breaking is dilaton dominated. We fix $t^i = T^i\big|$ at these self-dual points in what follows.

condensation scale. The only realistic possibility in the heterotic string context is that of a single hidden sector gaugino condensate.[5]

The Peccei–Quinn symmetry was introduced to eliminate the CP violating term in the QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \frac{\theta}{32\pi^2} F^a \cdot \tilde{F}_a, \tag{4.48}$$

where $F^a_{\mu\nu}$ is the gluon field strength, that is expected to contribute to the S-matrix in the presence of nonperturbative strong coupling effects. The term in (4.48) can be rotated away by a chiral transformation on quarks because the associated anomaly generates a term of the same form. However, unless there is at least one massless quark, any chiral symmetry is broken by quark masses, and CP violation reappears in the form of phases in the quark mass matrix. CP conservation is preserved only if there is a nonanomalous symmetry involving chiral transformations on the quarks such that θ in (4.48) can be set to zero in the basis in which the quark mass matrix is real. If this symmetry is broken only by quark masses much smaller than the QCD condensation scale, there will be a small violation of CP that is acceptable as long as $\bar{\theta}$, the value of θ in the real quark mass basis, is less than 10^{-10} as required by the stringent limits on the neutron dipole moment.

A convenient toy model for studying the axion mass is supersymmetric $SU(N_c)$ with N flavors, i.e. N quark and N antiquark chiral superfields Q^A, Q^c_A. The effective theory for a condensate in this case can be constructed as above, except that the matter condensate

$$\Pi^\alpha_Q = \det \mathbf{\Pi}, \qquad \mathbf{\Pi}^A_B = Q^A Q^c_B, \qquad \dim \Pi^\alpha_Q = 2N, \tag{4.49}$$

is determined by the requirement of invariance under the nonanomalous symmetry $SU(N)_L \otimes SU(N)_R$. The condensate Lagrangian takes the form

$$\mathcal{L}(U_Q) = \frac{1}{8} \int d^4\theta \frac{E}{R} U_Q \big[S + b'_Q \ln U_Q + b^\alpha_Q \ln \det \mathbf{\Pi} \big] + \text{h.c.},$$
$$b'_Q = \frac{1}{8\pi^2} (N_c - N), \qquad b^\alpha_Q = \frac{1}{8\pi^2}. \tag{4.50}$$

If we add a superpotential for $\mathbf{\Pi}$,

$$W(\mathbf{\Pi}) = \text{Tr}\big[\mathbf{C}(T) \mathbf{\Pi} \mathbf{C^c}(T) \mathbf{M} \big], \tag{4.51}$$

where $\mathbf{M}$ is the quark mass matrix and $\mathbf{C}$, $\mathbf{C^c}$ are matrix-valued functions of the Kähler moduli that assure T-duality invariance of the "classical" condensate Lagrangian. The classical Lagrangian is also invariant under an R-symmetry if U_Q transforms as in (4.35) and

$$\mathbf{\Pi} \to e^{i\alpha} \mathbf{\Pi}, \qquad \det \mathbf{\Pi} \to e^{iN\alpha} \det \mathbf{\Pi}. \tag{4.52}$$

Then (4.50) transforms as

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[5] A possibly dangerous contribution to the axion mass is from higher dimension operators [36] such as $\mathcal{L} \ni \sum_n \int d^4\theta (E/R) c_n(Z) U^{n+1}_c m_P^{-3n}$. However, the dimension of these operators is severely restricted by T-duality [37]. The minimal T-duality $SL(2, \mathbf{Z})$ group of (2.3) requires $n \geq 4$ which gives a contribution that could be comparable to the axion mass generated by the QCD condensate [38] if $\langle |u_c| \rangle / m_P^3 \sim 10^{-12}$, and therefore problematic. However any group larger than the minimal one should result in a negligible contribution. For example $SL[(2, \mathbf{Z})]^2$ and $SL[(2, \mathbf{Z})]^3$ require $n \geq 8$ and $n \geq 12$, respectively.

$$\Delta\mathcal{L}(U_Q) = \frac{1}{8}\int d^4\theta \frac{E}{R}U_Q\left[\Delta S + i\alpha\left(b'_Q + Nb^\alpha_Q\right)\right] + \text{h.c.,} \tag{4.53}$$

and is invariant provided

$$\Delta S = i\,\Delta\,\mathrm{Im}\,s = -i\alpha\left(b'_Q + Nb^\alpha_Q\right) = -i\frac{\alpha N_c}{8\pi^2}. \tag{4.54}$$

In the global supersymmetry limit, $m_P \to \infty$, $\mathrm{Re}\,s \to g^{-2}$, the results found for supersymmetric Yang–Mills theories using holomorphic arguments [39] are recovered in this effective theory [38].

If a confined sector with dimension-three matter condensates is also present, there is a nonanomalous R-symmetry in the absence of quark masses. It is defined by (4.35), (4.42)–(4.44) and

$$\mathbf{\Pi} \to e^{i\beta}\,\mathbf{\Pi}, \qquad \det\mathbf{\Pi} \to e^{iN\beta}\det\mathbf{\Pi}, \qquad \beta = \alpha\left(\frac{8\pi^2 b_c - N_c}{N} + 1\right). \tag{4.55}$$

In the presence of quark masses, $\det\mathbf{M} \neq 0$, the R-symmetry is broken, except at the point of enhanced symmetry:

$$\beta = \alpha, \qquad b_c = \frac{N_c}{8\pi^2}, \tag{4.56}$$

and the axion acquires a mass [38]

$$m_a = \frac{F_\pi}{F_a}\frac{|8\pi^2 b_c - N_c|}{\sqrt{2n}b_c}m_\pi, \tag{4.57}$$

where n is the number of flavors with quark masses below the QCD condensation scale (here taken to be degenerate), m_π is the common mass of the corresponding light pseudoscalars, F_π is the pion decay constant (93 MeV in QCD),

$$a = \left\langle\sqrt{2\ell/k'(\ell)}\right\rangle\mathrm{Im}\,s = F_a\,\mathrm{Im}\,s \tag{4.58}$$

is the canonically normalized axion, and F_a is its coupling to the Yang–Mills sector at the string scale in reduced Planck units:

$$\mathcal{L}_{\text{st}} \ni -\frac{\mathrm{Im}\,s}{4}\sum_a F^a\cdot\tilde{F}_a = -\frac{a}{4F_a}\sum_a F^a\cdot\tilde{F}_a. \tag{4.59}$$

In the case that we are actually interested in, QCD condensation occurs well below the scale of supersymmetry breaking, and one must find the correct effective pion–axion theory by first integrating out the heavy superpartners of Standard Model particles, as well as the heavy quarks. However the result in (4.57) is essentially unchanged; for just two light quarks u, d, it is simply multiplied by a function of the quark mass ratio:

$$m_a|_{n=2} \to \frac{2\sqrt{z}}{1+z}\,m_a|_{n=2}, \qquad z = \frac{m_u}{m_d}. \tag{4.60}$$

The result (4.57) appears to be at odds with the well-known relation [40] between the axion mass and its coupling strength. However F_a is the axion coupling at the *string scale*. When the gaug-

inos are integrated out, their coupling to the axion generates new terms in the couplings of the axion to gauge field strengths. For the axion coupling to QCD gluons, one gets a contribution [38]

$$\Delta\mathcal{L}_{\text{QCD}} = \frac{a}{4F_a} \frac{N_c}{8\pi^2} (F \cdot \tilde{F})_Q. \tag{4.61}$$

Combining this term with the QCD term in (4.59) one gets for the total axion–gluon coupling at low energy

$$\mathcal{L}_{\text{QCD}} \ni -\frac{a}{4F_a} \left(1 - \frac{N_c}{8\pi^2}\right) (F \cdot \tilde{F})_Q \equiv -\frac{na}{32\pi^2 f_a} (F \cdot \tilde{F})_Q, \tag{4.62}$$

where we have introduced an alternative normalization f_a for the axion coupling that is often found in the literature. In terms of this parameter, the axion mass for $n = 2$ takes the familiar form

$$m_a = \frac{2\sqrt{z}}{1+z} \frac{F_\pi}{f_a} m_\pi. \tag{4.63}$$

The axion mass vanishes at the point of enhanced symmetry (4.56) and one loses a potential solution to the "strong CP problem". This is because, under (4.37), (4.38) and (4.55) the quark superfields Q transform with phase $\frac{1}{2}\beta$ and the quarks $q = \partial Q / \partial\theta|$ with phase

$$\frac{1}{2}(\beta - \alpha) = \alpha \frac{8\pi^2 b_c - N_c}{2N}, \tag{4.64}$$

which vanishes at the symmetry point, so the nonanomalous R-symmetry does not affect the quark mass matrix and cannot be used to set θ to zero in the basis where the masses are real.

5. Soft supersymmetry breaking at one-loop

When supersymmetry is broken in a hidden sector of a generic supergravity theory, the Lagrangian for the "observable" (i.e. Standard Model) sector acquires "soft" supersymmetry-breaking terms. These are terms of dimension two or three that do not affect the cancellations of ultraviolet divergences that are present in the supersymmetric theory. They include gaugino masses, holomorphic functions of chiral scalars that are cubic (A-terms) and quadratic (B-terms), and "soft" scalar masses, that, is scalar squared mass terms $m^2_{j\bar{i}}\bar{\phi}^{\bar{j}}\phi^i$ that have no fermionic counterparts. However, there are cases where these terms are absent at tree level. For example, if supersymmetry is F-term mediated, $\langle D_a \rangle = 0$, $\langle F^T \rangle \neq 0$, the so-called "no-scale" Kähler potential of (3.9) leads to vanishing soft terms at tree level if the superpotential is independent of the Kähler moduli T^i. In such cases loop-induced soft terms become important, as they do if some tree-level soft terms are suppressed. For example, in the class of models outlined in Section 4.3, the gaugino masses and A-terms are much smaller than soft scalar masses [41] if the β-function coefficient b_c of the dominant condensing gauge group is an order of magnitude or so smaller than the parameter b appearing in the Green–Schwarz term (4.12).

The loop corrections to soft supersymmetry-breaking terms include the so-called "anomaly mediated" contributions that are present even when there are no soft terms at tree-level; they arise from the super-Weyl anomaly of standard supergravity. Some of these are model independent in the sense that they are determined only by the β- and γ-functions appearing in the RGE's of the low energy theory, and are independent of the mechanism by which supergravity is broken. That is, they are independent of the vacuum values of the auxiliary fields except for the supergravity auxiliary field, whose vacuum value $\langle R \rangle = M_\psi$ signals the breaking of *local* supersymmetry.

The "model-independent" contribution to gaugino masses m_a was first identified in [42,43]. This result was subsequently confirmed and completed [44–46], with the result

$$m_a^{\text{1-loop}} = -g_a^2(m_a)\left(\frac{3}{2}b_a M_\psi + \frac{1}{2}b_a'\langle F^i K_i\rangle + \frac{1}{8\pi^2}\sum_b C_a^b\langle F^i \partial_i \ln K_{b\bar{b}}\rangle\right), \tag{5.1}$$

where the first term is the "model independent" one alluded to above. In the models of interest here, only the auxiliary fields F^S, F^{T^i} have nonvanishing vacuum values at the hidden sector supersymmetry scale, and the class of condensation models described in Section 4.3 generally have $\langle F^{T^i}\rangle = 0$. When the 4d Green–Schwarz term and threshold effects are included, there are additional contributions to the gaugino masses; these modify only the coefficients of $\langle F^{T^i}\rangle$. The full expressions for the gaugino masses as well as other soft terms are given in [47].

The result (5.1) was obtained by analyses [44,45] of the loop-induced operator that transforms as in (4.1), by using [48] spurion techniques [49], and by an explicit PV calculation [44]. In the last case, the "anomaly-induced" gaugino mass results from a B-term insertion on the squark lines in PV squark–quark ($\phi^P - \chi^P$) loop contributions to the gaugino masses:

$$\mathcal{L}_{\text{PV}}\left(\phi^P\right) \ni -e^K W_{\text{PV}}(\phi)\langle\overline{W}\rangle + \text{h.c.} = -e^K \mu_{PQ}\phi^P\phi^Q \bar{M}_\psi + \text{h.c.} \tag{5.2}$$

Model independent contributions to A-terms were also found using spurion techniques [48,49], namely:

$$A_{abc}^{\text{1-loop}} \ni (\gamma_a + \gamma_b + \gamma_c) M_\psi, \tag{5.3}$$

where γ_a is the anomalous dimension for the light supermultiplet ϕ^a, χ^a. However, this technique failed to yield an analogous contribution to soft scalar masses, for which a such a contribution appeared only at two-loop order, proportional to the derivative of the anomalous dimension.

Pauli–Villars calculations [46] of these effects confirmed the contribution (5.3), and a similar B-term contribution

$$B_{ab}^{\text{1-loop}} \ni (\gamma_a + \gamma_b) M_\psi, \tag{5.4}$$

but in addition yielded a one-loop model-independent contribution to soft scalar masses:

$$(m_a^2)_{\text{soft}}^{\text{1-loop}} \ni \gamma_a |M_\psi|^2. \tag{5.5}$$

The source of this discrepancy can be traced to the fact that in the earlier calculations a holomorphic form for the supersymmetry-breaking spurion was assumed. This corresponds, in PV language, to PV B-terms, but no soft PV masses. If only B-terms were present soft scalar masses could result from a double B-term insertion, but these contributions in fact cancel; the result in (5.5) instead arises from a PV soft (squared) mass insertion. Repeating the spurion analysis without the assumption of holomorphism indeed reproduces [46] the term in (5.5).

The compete expressions for the soft supersymmetry breaking terms in the scalar potential are quite complicated. They depend on the tree-level soft terms, as well as on unknown mass parameters in the Pauli–Villars sector. This is in contrast to the result (5.1), which is completely determined by the low energy theory. The PV masses depend on the PV Kähler metric. In the case of gauge couplings, *all* PV chiral multiplets that are charged under $\mathcal{G}_a$ contribute both to the ultraviolet divergences associated with the loop-induced Yang–Mills operator containing $W_a^\alpha W_\alpha^a$ and

to the soft masses m_a; their gauge-charge weighted masses are constrained by the requirement of ultraviolet finiteness. On the other hand, only a subset Φ^P of PV chiral multiplets contribute to the renormalization of the Kähler potential through their couplings to light fields ϕ^p in superpotential terms $W \sim \Phi^P \Phi^Q \phi^r$. The Kähler metric of the Φ^P is fixed by the finiteness requirement. However each PV field Φ^P has a PV mass coupling to some other field $\Phi^{P'}$ which need have no coupling to light fields and no restriction on its Kähler metric. As a result the masses

$$m_P = e^{K/2} (K_{PP} K_{P'P'})^{-\frac{1}{2}} \mu_{PP'} \tag{5.6}$$

of the fields Φ^P that contribute to the scalar soft terms are not fixed by finiteness alone, and depend on the details of string/Planck scale physics.

6. Unfinished business

The F-term anomalies of the form (4.1) that are linear in the parameters of the anomalous transformations are well understood and can be canceled by a combination of the 4d Green–Schwarz term and string loop threshold effects. However the anomalous terms that are higher order in these parameters depend on the details of the regularization procedure [10,19]. It appears likely that if a regularization procedure can be found that allows for the implementation of full anomaly cancellation in the context of the weakly coupled heterotic string, it will entail some constraints on higher order terms in the Kähler potential (4.21) for the untwisted sector fields Φ^a of orbifold compactifications. This is reminiscent of the constraint (2.1) on gauge charges, and could have important implications for flavor changing neutral currents discussed in Section 3.2. Such a procedure would certainly entail restrictions on the PV masses, which, as discussed in the previous section, play a role in soft supersymmetry breaking parameters in the case that loop corrections to these are important. Therefore the solution to the problem of full anomaly cancellation will have direct implications for phenomenology.

Finally, as mentioned Section 2 there are also D-term anomalies, and as yet there has been no serious attempt to determine how they might be canceled in a string theory context. The resolution of this issue may also have a bearing on the effective supergravity Lagrangian.

7. Final word

Raymond Stora was a cherished friend and colleague who was always very supportive. I had the great pleasure of organizing with Raymond a very successful summer school at Les Houches in 1981; most of the participants in that session are still active in particle physics today. Bruno Zumino and I shared many pleasurable occasions with Raymond and Marie-France. I will miss him.

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References

[1] M.B. Green, J.H. Schwarz, Phys. Lett. B 149 (1984) 117.
[2] D.J. Gross, J.A. Harvey, E.J. Martinec, R. Rohm, Nucl. Phys. B 256 (1985) 253.
[3] P. Candelas, G.T. Horowitz, A. Strominger, E. Witten, Nucl. Phys. B 258 (1985) 46.
[4] M.K. Gaillard, V. Jain, Phys. Rev. D 49 (1994) 1951;
    M.K. Gaillard, V. Jain, K. Saririan, Phys. Rev. D 55 (1997) 833;
    M.K. Gaillard, V. Jain, K. Saririan, Phys. Lett. B 387 (1996) 520.
[5] M.K. Gaillard, Nucl. Phys. B 268 (1986) 669;
    O. Cheyette, M.K. Gaillard, Phys. Lett. B 197 (1987) 205;
    P. Binetruy, M.K. Gaillard, Nucl. Phys. B 312 (1989) 341;
    V. Jain, Phys. Rev. D 40 (1989) 1213;
    J.W. Burton, M.K. Gaillard, V. Jain, Phys. Rev. D 41 (1990) 3118;
    For a recent description of the method see B. Henning, X. Lu, H. Murayama, arXiv:1412.1837 [hep-ph].
[6] P. Binetruy, M.K. Gaillard, Phys. Lett. B 220 (1989) 68.
[7] C. Becchi, A. Rouet, R. Stora, Ann. Phys. 98 (1976) 287.
[8] M.K. Gaillard, Phys. Lett. B 342 (1995) 125;
    M.K. Gaillard, Phys. Lett. B 347 (1995) 284;
    M.K. Gaillard, Phys. Rev. D 58 (1998) 105027;
    M.K. Gaillard, Phys. Rev. D 61 (2000) 084028.
[9] E. Cremmer, S. Ferrara, L. Girardello, A. Van Proeyen, Phys. Lett. B 116 (1982) 231;
    E. Cremmer, S. Ferrara, L. Girardello, A. Van Proeyen, Nucl. Phys. B 212 (1983) 413;
    J.A. Bagger, Nucl. Phys. B 211 (1983) 302.
[10] D. Butter, M.K. Gaillard, Phys. Lett. B 679 (2009) 519;
    M.K. Gaillard, Pramana 78 (2012) 875;
    D. Butter, M.K. Gaillard, Phys. Rev. D 91 (2) (2015) 025015.
[11] Joel Giedt, Ann. Phys. 289 (2001) 251;
    Joel Giedt, Ann. Phys. 297 (2002) 67.
[12] L.E. Ibàñez, H.-P. Nilles, F. Quevedo, Phys. Lett. B 187 (1987) 25;
    A. Font, L. Ibàñez, D. Lust, F. Quevedo, Phys. Lett. B 245 (1990) 401;
    A. Font, L.E. Ibàñez, F. Quevedo, A. Sierra, Nucl. Phys. B 331 (1990) 421.
[13] D.Z. Freedman, B. Kors, J. High Energy Phys. 0611 (2006) 067;
    H. Elvang, D.Z. Freedman, B. Kors, J. High Energy Phys. 0611 (2006) 068.
[14] P. Binetruy, G. Dvali, R. Kallosh, A. Van Proeyen, Class. Quantum Gravity 21 (2004) 3137.
[15] I. Antoniadis, E. Gava, K.S. Narain, Phys. Lett. B 283 (1992) 209;
    I. Antoniadis, E. Gava, K.S. Narain, Nucl. Phys. B 383 (1992) 93.
[16] A. Giveon, N. Malkin, E. Rabinovici, Phys. Lett. B 220 (1989) 551;
    E. Alvarez, M. Osorio, Phys. Rev. D 40 (1989) 1150.
[17] G.L. Cardoso, B.A. Ovrut, Nucl. Phys. B 369 (1992) 351;
    J.-P. Derendinger, S. Ferrara, C. Kounnas, F. Zwirner, Phys. Lett. B 271 (1991) 307;

M. Dine, N. Seiberg, E. Witten, Nucl. Phys. B 289 (1987) 589;
J.J. Atick, L.J. Dixon, A. Sen, Nucl. Phys. B 292 (1987) 109.
[18] L.J. Dixon, V.S. Kaplunovsky, J. Louis, Nucl. Phys. B 355 (1991) 649;
I. Antoniadis, K.S. Narain, T.R. Taylor, Phys. Lett. B 267 (1991) 37.
[19] C.A. Scrucca, M. Serone, J. High Energy Phys. 0102 (2001) 019.
[20] K. Choi, J.E. Kim, H.P. Nilles, Phys. Rev. Lett. 73 (1994) 1758.
[21] K. Choi, J.S. Lee, C. Munoz, Phys. Rev. Lett. 80 (1998) 3686.
[22] M.K. Gaillard, B.D. Nelson, Nucl. Phys. B 751 (2006) 75.
[23] P. Binetruy, M.K. Gaillard, Phys. Lett. B 220 (1989) 68.
[24] P. Binétruy, G. Girardi, R. Grimm, M. Muller, Phys. Lett. B 189 (1987) 83;
P. Binétruy, G. Girardi, R. Grimm, Phys. Rep. 343 (2001) 255.
[25] P. Binétruy, G. Girardi, R. Grimm, M. Müller, Phys. Lett. B 265 (1991) 111;
P. Adamietz, P. Binétruy, G. Girardi, R. Grimm, Nucl. Phys. B 401 (1993) 257.
[26] M.K. Gaillard, T.R. Taylor, Nucl. Phys. B 381 (1992) 577.
[27] P. Mayr, S. Stieberger, Nucl. Phys. B 412 (1994) 502;
V. Kaplunovsky, J. Louis, Nucl. Phys. B 422 (1994) 57.
[28] M.A. Shifman, A.I. Vainshtein, Nucl. Phys. B 277 (1986) 456.
[29] V.S. Kaplunovsky, Nucl. Phys. B 307 (1988) 145;
V.S. Kaplunovsky, Nucl. Phys. B 382 (1992) 436 (Erratum);
M.K. Gaillard, R.l. Xiu, Phys. Lett. B 296 (1992) 71.
[30] U. Amaldi, W. de Boer, P.H. Frampton, H. Furstenau, J.T. Liu, Phys. Lett. B 281 (1992) 374.
[31] S. Ferrara, N. Magnoli, T.R. Taylor, G. Veneziano, Phys. Lett. B 245 (1990) 409;
P. Binétruy, M.K. Gaillard, Nucl. Phys. B 358 (1991) 121.
[32] G. Veneziano, S. Yankielowicz, Phys. Lett. B 113 (1982) 231;
T.R. Taylor, G. Veneziano, S. Yankielowicz, Nucl. Phys. B 218 (1983) 493.
[33] P. Binetruy, M.K. Gaillard, Y.-Y. Wu, Nucl. Phys. B 493 (1997) 27.
[34] M.K. Gaillard, J. Giedt, A.L. Mints, Nucl. Phys. B 700 (2004) 205;
M.K. Gaillard, J. Giedt, A.L. Mints, Nucl. Phys. B 713 (2005) 607 (Erratum).
[35] P. Binetruy, M.K. Gaillard, Y.Y. Wu, Nucl. Phys. B 481 (1996) 109;
J.A. Casas, Nucl. Phys. Proc. Suppl. 52A (1997) 289.
[36] T. Banks, M. Dine, Phys. Rev. D 50 (1994) 7454.
[37] D. Butter, M.K. Gaillard, Phys. Lett. B 612 (2005) 304.
[38] M.K. Gaillard, B. Kain, Nucl. Phys. B 734 (2006) 116.
[39] A.C. Davis, M. Dine, N. Seiberg, Phys. Lett. B 125 (1983) 487;
N. Seiberg, Phys. Rev. D 49 (1994) 6857.
[40] M. Srednicki, Nucl. Phys. B 260 (1985) 689.
[41] P. Binetruy, M.K. Gaillard, Y.Y. Wu, Phys. Lett. B 412 (1997) 288.
[42] L. Randall, R. Sundrum, Nucl. Phys. B 557 (1999) 79.
[43] G.F. Giudice, M.A. Luty, H. Murayama, R. Rattazzi, J. High Energy Phys. 9812 (1998) 027.
[44] M.K. Gaillard, B.D. Nelson, Y.Y. Wu, Phys. Lett. B 459 (1999) 549.
[45] J.A. Bagger, T. Moroi, E. Poppitz, J. High Energy Phys. 0004 (2000) 009.
[46] M.K. Gaillard, B.D. Nelson, Nucl. Phys. B 588 (2000) 197.
[47] P. Binetruy, M.K. Gaillard, B.D. Nelson, Nucl. Phys. B 604 (2001) 32.
[48] A. Pomarol, R. Rattazzi, J. High Energy Phys. 9905 (1999) 013.
[49] L. Girardello, M.T. Grisaru, Nucl. Phys. B 194 (1982) 65;
G.F. Giudice, R. Rattazzi, Nucl. Phys. B 511 (1998) 25;
N. Arkani-Hamed, G.F. Giudice, M.A. Luty, R. Rattazzi, Phys. Rev. D 58 (1998) 115005.