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Effect of Fluctuations on Global Analysis of Fluid Dynamical Calculations for High-Energy Nuclear Collisions

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Abstract

Composite fragment formation, finite particle number, and statistical fluctuations are shown to have important effects on the global analysis of fluid dynamical results for high-energy nuclear collisions.

Recent experimental developments[1)] promise to give us a better insight into the dynamics of high-energy nuclear collisions. A major objective of those efforts is to determine whether collective-flow effects can be identified and distinguished from expectations based on intranuclear cascade calculations[2)]. However, one expects, and recent studies[3-5)] confirm, that the formation of composite fragments, the finiteness of the multiplicity, and the statistical fluctuations in the final states affect the event analysis in important ways. In the present work we wish to elucidate this situation by analyzing events generated theoretically using fluid-dynamical results[4,6,7)] in conjunction with the statistical simulation model by Fai and Randrup[5)].

Our starting point is a series of detailed three-dimensional fluid-dynamical calculations[4,6,7)]. From these results we extract certain gross features that are used to give an approximate characterization of the final state of the fluid dynamical stage of the collision in terms of a few sources, usually one participant source and two spectator sources. These sources are then subsequently treated in two alternative ways:

i) <u>In the first</u>, each source is described as a grand canonical ensemble so that the mean fragment abundancies and spectral distributions can be calculated from the associated partition functions, as described in refs. [8,9)]. This treatment is similar to what was done in ref. [4)], except for the inclusion of fragments heavier than the α particle. Having determined the grand canonical parameters, we can then calculate directly the associated flow tensor using the method of refs. [4,9)] and subsequently the global parameters of interest, namely the flow angle θ, the aspect ratio $R_{1/3}$, etc.

ii) In the alternative treatment of the sources, we use the explosion-evaporation model of ref. [5] to generate statistically a number of actual events and make the flow analysis for each event separately, as is done experimentally. We then proceed to compare the two sets of results and achieve an impression of the effects arising from composite fragment formation, finite multiplicity, and statistical fluctuations.

Parametrization of the fluid-dynamical results in terms of sources. In refs. [6,7] fluid dynamical calculations were carried out to study the momentum transfer in nuclear collisions. In peripheral collisions the two nuclei maintain most of their velocity and are only slightly deflected and excited. As the impact parameter s is decreased, the deflection angle Θ_{CM} grows and an increasing fraction of the initial momentum is lost: the collision is inelastic, and dissipative processes like shocks and viscocity convert the translational energy into thermal excitation.

As it turns out, the deflection angle can be reasonably well approximated by the following expression at intermediate and large impact parameters,

$$\Theta_{CM}(s) \approx z_0(1 - \frac{s}{s_{max}}) \frac{\pi}{2} \quad . \qquad (1)$$

Here z_0 is the parameter governing the transverse momentum transfer. In fluid dynamical models it is of the order of unity, cf. refs. [4,6,7], and we shall adopt the value $z_0 = 1$ in our present studies.

The inelasticity can be described by the CM momentum per nucleon, p'_{CM}, of the projectile-like part of the system after the collision. For this quantity we find

$$p'_{CM} \approx p_{CM}(1 - (1 - \frac{s}{s_{max}})^2 \, y_0) \quad , \tag{2}$$

where $p_{CM} = P_0/A_0$ is the initial CM momentum per nucleon of the projectile. The detailed three-dimensional calculations in refs. [4,6,7] show that $y_0 \approx 0.5$–0.7, and we shall adopt the value $y_0 = 0.6$ in our studies.

The above characteristic features of the fluid-dynamical results permit us to approximate the final state of the fluid-dynamical stage by three sources A,B,C as follows. In the CM frame these three sources have the momenta P^A, P^B, P^C, respectively. By slightly generalizing the parametrization introduced in ref. [5], we assume the source momenta to be given in terms of the parameters y and z as

$$\begin{aligned} P^A_{\parallel} &= (1-y)\frac{A}{A_0}P_0 \;, & P^A_{\perp} &= z\frac{A}{A_0}P_0 \\ P^B_{\parallel} &= -(1-y)\frac{B}{B_0}P_0 \;, & P^B_{\perp} &= -z\frac{B}{B_0}P_0 \\ P^C_{\parallel} &= -P^A_{\parallel} - P^B_{\parallel} \;, & P^C_{\perp} &= -P^A_{\perp} - P^B_{\perp} \end{aligned} \tag{3}$$

where, as in (2), P_0 is the absolute value of the initial CM momentum of either nucleus A_0 or B_0, and A,B,C are also used to denote the baryon numbers in the sources A,B,C. Furthermore, we adopt[5] the parameter $x \in [0,1]$ to govern the leakage of excitation energy from the participant source C into the two spectator sources A and B.

The three parameters x,y,z, which determine the source characteristics, are taken as

$$
\begin{aligned}
x &= x_0 \left(1 - \frac{s}{s_{max}}\right)^2) \\
y &= 1 - [1 - (1 - \frac{s}{s_{max}})^2 \; y_0] \cos [z_0 \; (1 - \frac{s}{s_{max}}) \; \frac{\pi}{2}] \qquad (4) \\
z &= [1 - (1 - \frac{s}{s_{max}})^2 \; y_0] \sin [z_0 (1 - \frac{s}{s_{max}}) \; \frac{\pi}{2}]
\end{aligned}
$$

with $x_0 = 0.3$, so that the fluid-dynamical results (eqs. 1-2) be reproduced as well as possible.

Calculation of the flow tensor in the fluid-dynamical model. A global event analysis in the framework of a fluid-dynamical model was first performed by Kapusta and Strottman using the directed thrust[10)]. However, this calculation did not take account of the thermal smearing and the fragment formation. The neglect of the random thermal momenta leads to an overestimate of the calculated thrust and a corresponding underestimate of the sphericity of an event.

In the nonrelativistic calculations[4,6,7,9)] the thermal energy was taken into account by a method introduced in ref. [11)]. The basic idea is that the thermal velocities of the fragments should be added to the local flow velocity when calculating the flow tensor, as was suggested already by Danielewicz[12)]. Unfortunately, the statistical generation of fragments within each fluid element is very tedious and becomes prohibitively costly in realistic three-dimensional calculations.

However, since the fragments in the local rest frame of a given fluid element are characterized by a thermal distribution the expectation value of the flow tensor can be calculated analytically[9]. This remains true even when breakup into composite fragments is considered. Following ref. [4], we then evaluate the flow tensor as follows,

$$\overleftrightarrow{F} = \sum_k \frac{\overleftarrow{p}^{(k)}\overrightarrow{p}^{(k)}}{2M_k}$$

$$= \sum_S \sum_\alpha \frac{1}{2M_\alpha} \sum_{k\in S,\alpha} \overleftarrow{p}^{(k)}\overrightarrow{p}^{(k)} \tag{5}$$

$$= \sum_S \sum_\alpha \frac{1}{2M_\alpha} \int d\vec{P}\; f_\alpha \left(\frac{\vec{P}}{M_\alpha} - \vec{V}^{(S)},\; \tau_S,\; \rho_S^{(\alpha)}\right) \overleftarrow{P}\, \overrightarrow{P}$$

The first line gives the definition of the flow tensor $\overleftrightarrow{F}$ as a sum over all fragments k in a given event; a fragment has the momentum $\vec{p}^{(k)}$ and the mass M_k. The system is divided into a number of fluid elements, or sources, which are denoted by S (later to take on the values A,B,C). The sum over fragments located within a given source can be carried out separately for each particular fragment species α. This is indicated in the second line of (5). In the third line we have introduced the fact that the fragments are assumed to have a certain characteristic distribution f within a given source. This distribution is taken as a thermal Maxwell-Boltzmann distribution depending on the local temperature τ_S and the local density $\rho_S^{(\alpha)}$ of the particular species α; it is isotropic in the source frame, which moves with the velocity $\vec{V}^{(S)}$ relative to the chosen reference frame.

The fragment momentum has a global and a local (or a collective and a thermal) part, $\vec{P} = M_\alpha \vec{V}^{(S)} + (\vec{P} - M_\alpha \vec{V}^{(S)})$. The momentum integral in (8) can then easily be carried out and we obtain

$$\overleftrightarrow{F} = \sum_S \sum_\alpha \bar{\nu}_S^{(\alpha)} \left(\frac{1}{2} M_\alpha \overleftarrow{V}^{(S)} V^{(S)} + \frac{1}{2} \tau_S \overleftrightarrow{I} \right) \tag{6}$$

$$= \sum_\alpha \sum_S \bar{\nu}_S^{(\alpha)} \frac{M_\alpha}{2} \overleftarrow{V}^{(S)} \vec{V}^{(S)} + \sum_S \frac{\tau_S}{2} \bar{\nu}_S \overleftrightarrow{I}$$

where $\overleftrightarrow{I}$ is the identity tensor and $\bar{\nu}_S^{(\alpha)}$ is the mean multiplicity of the species α arising from the source S. (The total mean multiplicity from the source S is denoted by $\bar{\nu}_S$.) The first term in (6) is associated with the global collective motion of the system, while the second term arises from the local thermal motion of the fragments. This latter term is isotropic and therefore has no effect on the orientation of the principal directions of $\overleftrightarrow{F}$, hence also no effect on the extracted flow angle.

It is of interest theoretically, and often also necessitated by experimental conditions, to study the flow tensor $\overleftrightarrow{F}^{(\alpha)}$ associated with a particular fragment species. We have

$$\overleftrightarrow{F} = \sum_\alpha \overleftrightarrow{F}^{(\alpha)}$$

$$= \sum_\alpha \left[\sum_S \bar{\nu}_S^{(\alpha)} \left(\frac{M_\alpha}{2} \overleftarrow{V}^{(S)} \vec{V}^{(S)} + \frac{\tau_S}{2} \overleftrightarrow{I} \right) \right] \tag{7}$$

It is clear that the thermal contribution is most important for the flow tensors for light species since the collective term is proportional to the fragment mass M_α.

The method employed in the present study is the same as the one used in the full three-dimensional calculation[4], except that fragment species heavier than the α particle are included[5,8]. As in refs. [5,8] the grand partition function is calculated considering all nuclear states with width less than 1 MeV. This yields the temperature τ_S and the mean multiplicities $\bar{\nu}_S^{(\alpha)}$ for each source. In the calculations of ref. [4] such detailed treatment was not possible due to the large number (several thousands) of fluid cells, but in the present study we consider only three sources.

After calculating the flow tensors $\overleftrightarrow{F}^{(\alpha)}$ as described above as a function of the impact parameter, the various global variables, such as the flow angle θ and the aspect ratio $R_{1/3} = f_1/f_3$ are calculated in the standard way[2].

Due to the simplicity of the model $R_{1/3} \to 1$ when $\theta \to \pi/2$ unlike 3D fluid dynamical models where usually $R_{1/3} \to R > 1$.

<u>Microcanonical event generation</u>. The particular advantage of approximating the final fluid-dynamical stage of the collision by a few sources is the ease with which the subsequent disassembly into a finite number of physical fragments can be calculated. Indeed, all that is required is to use the appropriate parameters x,y,z (4) as input into the event generation model developed in ref. [5]. In that model, each source <u>explodes</u> statistically into a number of excited nucleides, which subsequently decay by light-particle

evaporation if sufficiently excited. The final evaporation† stage is very important since the evaporated light fragments constitute a substantial fraction of the total yield and change the emisson pattern considerably. In particular, spectator sources with low excitation give a significant contribution to the light particle yield in the microcanonical event generation via their evaporation. In the microcanonical model, a sample of final states is generated, each of which satisfy conservation of four-momentum, baryon number, and charge. The formation of composite fragments plays an important role in the event analysis, and the present study pays particular attention to the composite fragments.

For each specified impact parameter, the model of ref. [5)] is employed to generate a sample of complete events. The microcanonical event generation is implemented as a Monte Carlo computer code and it yields exclusive events that incorporate the full complexity associated with fragment formation, finite multiplicity and statistical fluctuations. This advantage necessitates the handling of large sets of data much in the same way as in the experimental case. The calculation of the flow tensor, on the other hand, simply amounts to a straightforward application of the definition.

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†In the fluid dynamical calculations[4,6,7,9,11] the term "evaporation" corresponds to the first stage of our present explosion-evaporation description, while the final decay of the produced excited fragments has so far not been considered in full scale hydrodynamical calculations.

The random element enters in the model through the fact that for each source S we generate a statistical representation of the corresponding exculsive probability, i.e., a sample of multifragment final states distributed statistically. The sample of complete events generated in this manner is subsequently subjected to the flow analysis.

Discussion. In Figs. 1 and 2 we compare the resulting fluid-dynamical and statistically generated flow diagrams (flow angle θ versus aspect ratio $R_{1/3}$ [2]) in the collision ^{93}Nb + ^{93}Nb at 400 MeV/nucleon. The continuous curves represent the fluid-dynamical expectation with three different species selections: 1) protons only, 2) all hydrogen and helium fragments, i.e, p, d, t, ^{3}He, α and 3) all fragments, including neutrons. The parameter values have been fixed to $x_0 = 0.3$, $y_0 = 0.6$ and $z_0 = 1.0$ for simplicity. In the fluid-dynamical description a given impact parameter yields a well-defined point in the flow diagram (in the actual generation of the figures the impact parameters $s/s_{max} = 0.1, 0.3, 0.5, 0.7$ have been used). Nevertheless, at each impact parameter we included the full fluid-dynamical curve to guide the eye. As should be expected, exclusion of the heavier fragments leads to more spherical events. Since the fluid-dynamical description does not include the secondary evaporation-type deexcitation of the spectators, we terminated the continuous lines for selections 1) and 2) at the impact parameters where the spectators no longer explode, thereby leading to the fluid-dynamical treatment of one source only, which, by definition, results in complete sphericity.

In Fig. 1 a sample of ten statistically generated events are displayed as dots (if $\theta \leq \pi/2$) or open squares ($\theta > \pi/2$, $\pi - \theta$ plotted) for each impact parameter considered. In Fig. 2 the results for all different impact parameters are shown for selection 2, which corresponds to the experimental situation. To further simulate experimental conditions the number of events plotted for a given impact parameter is proportional to s.

It is immediately clear that fluctuations play an important role here: the spread of the points is very large and they follow only very loosely the fluid-dynamical expectation. As a word of caution we recall that the parameterization (1,2) of the fluid-dynamical results was meant for intermediate and large impact parameters, so that good correspondence should not be expected for central collisions.

When comapring selections 1), 2), and 3) one should remember that the flow tensor, although ideally coalescence invariant, depends on the specific species selection which must always be made in a real experiment. Thus, in contrast to naive expectations, the observed flow tensor is not coalescence invariant, as is clearly borne out by Fig. 1. Due to the final evaporation there is an observable shift for light fragments (selections 1 and 2) towards more elongated emission patterns compared to fluid-dynamical expectations. The fluctuations associated with these selections are also considerably larger than those of selection 3. Especially the fluctuation of the flow angle θ is very large for small impact parameters, because the events are rather close to spherical.

In conclusion we have shown that the total flow tensor $\overleftrightarrow{F}$ (selection 3) is fairly close to the fluid-dynamical expectation. However, in the experimental situation, where only light fragments are considered, the measured flow tensor depends strongly on the cut off on the fragment mass considered. It is very important to take account of this fact when comparing experimental and theoretical results.

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FIGURE CAPTIONS

Fig. 1 Fluid-dynamical (continuous curves with open circles) and statistically generated (dots if $\theta < \pi/2$ and open squares if $\theta > \pi/2$) flow diagrams: flow angle θ versus the aspect ratio $R_{1/3}$ of the flow tensor $\overleftrightarrow{F}$ for the reaction $^{93}Nb + {}^{93}Nb$ at 400 MeV/nucleon energy. Four different impact parametes s = 0.1, 0.3, 0.5, 0.7 s_{max} and three different fragment species selections (p, p-α, all fragments) are shown. At each impact parameter the full fluid-dynamical curve is plotted. Both the fluid-dynamical expectation and the statistically generated $\overleftrightarrow{F}$ depend strongly on the selection.

Fig. 2 Flow diagram obtained by superimposing diagrams of the type in Fig. 1 for the "experimental" selection 2 (p-α). In order to further simulate the experimental situation, the number of events with a given impact parameter s has been chosen as 10 s/s_{max}; the appropriate value of 10 s/s_{max} is indicated at the open circles. The solid dots are the fluid-dynamical results corresponding to all three selections.

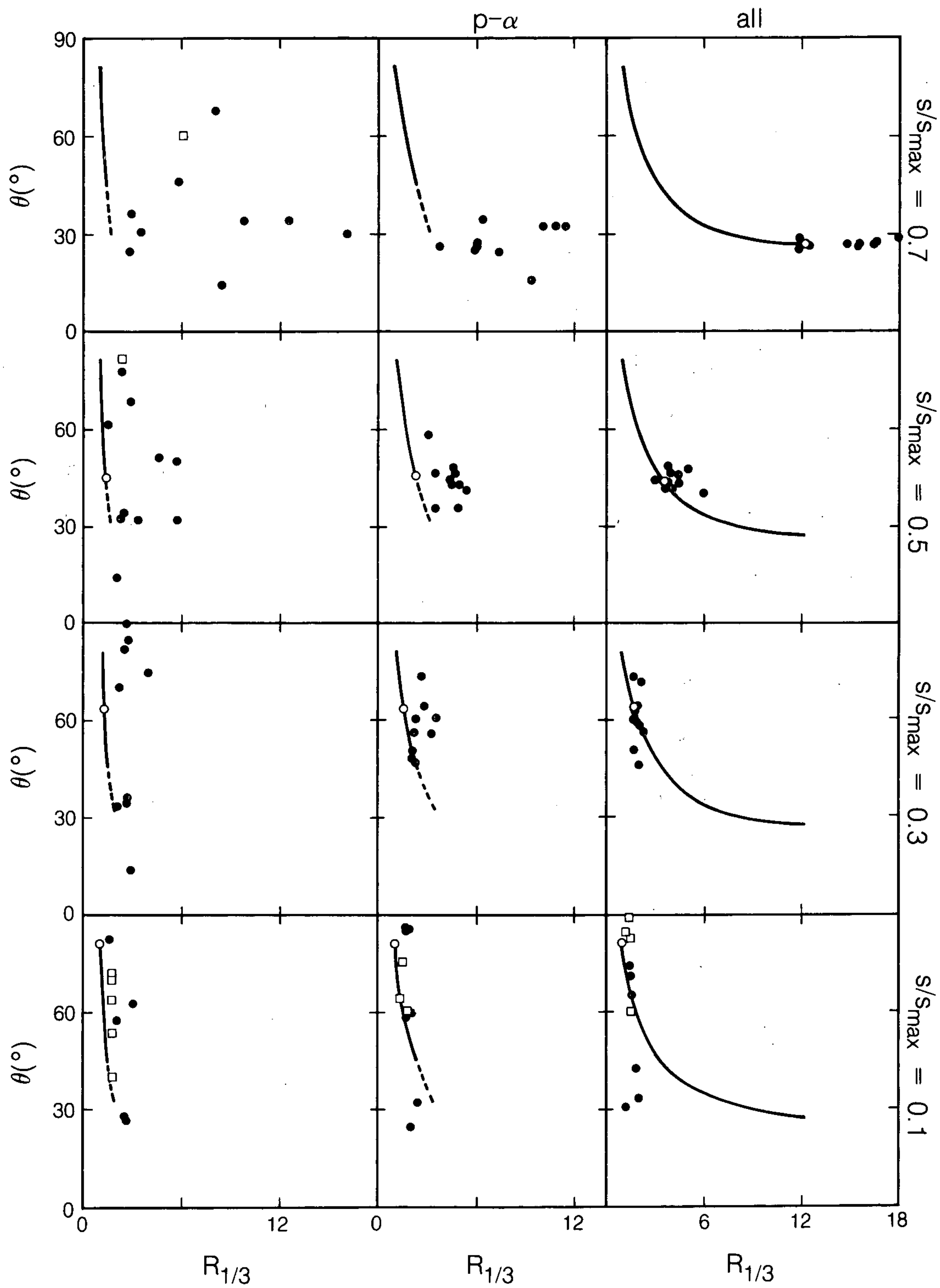

Fig. 1

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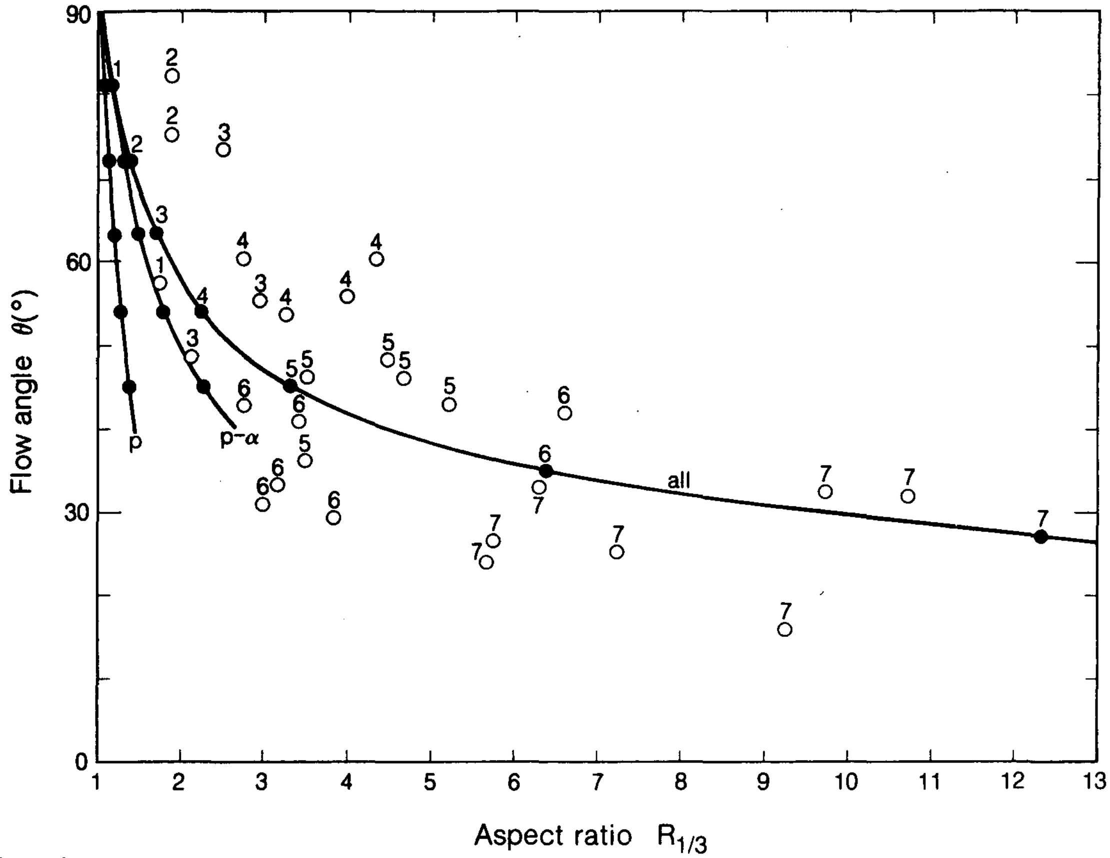

Fig. 2

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