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Fundamental Limits of Information Dissemination in Wireless Ad Hoc Networks---Part II: Multi-Packet Reception

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Fundamental Limits of Information Dissemination in Wireless Ad Hoc Networks–Part II: Multi-Packet Reception

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***Abstract*—We present capacity and delay scaling laws for random wireless ad hoc networks under all information dissemination modalities (unicast, multicast, broadcast and anycast) when nodes are endowed with multi-packet reception (MPR) capabilities. Information dissemination modalities are modeled with an (n, m, k)-cast formulation, where n, m, and k denote the number of nodes in the network, the number of destinations for each communication group, and the actual number of communication group members that receives the information (i. e., $k \leq m \leq n$), respectively. We show that $\Theta\left(R(n)\sqrt{m}/k\right)$, $\Theta\left(1/k\right)$, and $\Theta\left(R^2(n)\right)$ bits per second constitute a tight bound for the throughput capacity of random wireless ad hoc networks under the protocol model when $m = O\left(R^{-2}(n)\right)$, $\Omega(k) = R^{-2}(n) = O(m)$, and $k = \Omega\left(R^{-2}(n)\right)$, respectively. $R(n)$ denotes the receiver range which depends on the decoding complexity of the nodes. For the minimum receiver range of $\Theta\left(\sqrt{\log n/n}\right)$ to guarantee network connectivity, a gain of $\Theta(\log n)$ for (n, m, k)-casting is attained with MPR compared to the capacity attained when receivers can decode at most one transmission at a time in [1]. Furthermore, we derive the capacity-delay tradeoff of (n, m, k)-casting when MPR is used. We show that the use of MPR can lead to both increased network capacity and reduced delays in wireless ad hoc networks.**

***Index Terms*—Scaling laws, network information theory, multi-packet reception.**

I. Introduction

THE seminal work by Gupta and Kumar [2] on the scaling laws of wireless ad hoc networks for unicasting with multi-hop communication demonstrated that when nodes can receive successfully only a single packet at a time, which we call single-packet reception or SPR, the capacity decreases as the number of nodes in the network increases. This result has motivated many studies focusing on techniques aimed at improving the unicast capacity of wireless networks (e.g., see [3], [4]), as well as the computation of the scaling laws under SPR for multicasting [5] and broadcasting [6] in wireless ad hoc networks.

Increasing the capacity of wireless networks requires increasing the concurrency with which shared channels are accessed or increasing the amount of information sent with each transmission. Multi-packet reception (MPR) [7] consists of the ability of allowing multiple nodes to transmit their packets simultaneously to the same receiver node, which can in turn decode all such packets successfully. In practice, MPR can be achieved with a variety of techniques. For example, MPR can be implemented by allowing a node to decode multiple concurrent packets using multiuser detection (MUD) or directional antennas, provided that the number of the concurrent transmissions is restricted in practice by the complexity of the nodes [8], [9]. However, practical implementation of MPR requires more realistic interference models. Recently, there has been a very useful contribution [10] on MPR. This work assumes that each node can decode at most k simultaneous transmission within node's receiver range. Such model is called k-MPR model which is a very useful contribution in considering the practical limitations of MPR and provides a good perspective on the limitations of our results in this paper. MPR has been shown [4] to increase the capacity regions of ad hoc wireless networks subject to multi-pair unicast traffic; however, no capacity results have been reported on the benefits of MPR in wireless networks subject to information dissemination classes other than unicasting.

Ahlswede, Ning, Li and Yeung [11] introduced the concept of network coding (NC) and showed that it achieves the optimal capacity for single-source multicasting in directed graphs corresponding to wired networks in which nodes are connected by point-to-point links. As a result of this work, many attempts have been made to use NC as a complementary approach to increasing the capacity of wireless networks by increasing the amount of information sent per transmission. Liu et al. [12] have shown that NC cannot increase the order capacity of wireless ad hoc networks for multi-pair unicast traffic; however, recent works [13]–[17] have shown promising results on the advantage of NC in wireless ad hoc networks subject to multicast traffic. Ramamoorthy et al. [15] proved that the throughput capacity of single source multicast is $(n-1-m)E[X]$ with high probability, where m and $n-1-m$

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are the number of destinations and relays respectively and X denotes the i.i.d. distribution of the link capacity between any two nodes in the network. Aly et al. [16] and Kong et al. [17] extended the result by Ramamoorthy et al. to more general protocol and physical models, respectively, and derived similar results. Recently, Zhang et al. [13] and Katti et al. [14] proposed analog network coding and physical-layer network coding, respectively, as ways to embrace interference. Interestingly, a careful review of these contributions reveals that ANC and PNC consist of the integration of NC with a form of MPR, in that receivers must be allowed to decode successfully concurrent transmissions from multiple senders by taking advantage of the modulation scheme used at the physical layer (e.g., MSK modulation in ANC [14]).

The motivation for the work presented in this paper is three-fold. First, while it is clear from recent work on NC that MPR may contribute to the capacity increase observed when NC is applied to wireless networks with multicast traffic, prior work does not decouple the performance gains due to NC from those resulting from MPR. Second, although Garcia-Luna-Aceves et al. [4] have shown that the order capacity of wireless ad hoc networks subject to multi-pair unicast traffic is increased with MPR, no results have been reported on the order capacity of networks with MPR subject to broadcast, multicast or anycast traffic. Third, no studies have been reported on the capacity-delay tradeoffs of wireless ad hoc networks subject to different types of information dissemination. The exception is the case of unicast traffic, for which El Gamal et al. [18] characterized the fundamental throughput-delay tradeoff for both static and mobile networks.

This paper presents a unifying approach for the computation of the order throughput of wireless networks subject to different information dissemination modalities (unicast, multicast, broadcast and anycast) and such that the nodes of the network can perform MPR. This paper extends the results we introduced in Part 1 of this work [1] for the case in which nodes perform SPR. In addition, we study the capacity-delay tradeoff in wireless ad hoc networks for different information dissemination modalities when nodes perform MPR.

Sections III presents the first results on the capacity of ad hoc networks with MPR under different forms of information dissemination other than unicast traffic based on the models in Section II. We show that the per source-destination (n, m, k)-cast throughput capacity $C_{m,k}(n)$ of a wireless random ad hoc network with MPR is tight bounded (upper and lower bounds) by $\Theta\left(R(n)\sqrt{m}/k\right)$, $\Theta\left(1/k\right)$ and $\Theta\left(R^2(n)\right)$ w.h.p.[1] when $m = O\left(R^{-2}(n)\right)$, $\Omega(k) = R^{-2}(n) = O(m)$, and $k = \Omega\left(R^{-2}(n)\right)$, respectively. n, m, and k denote the number of nodes in the network, the number of destinations for each communication group, and the number of communication group members that receive the information, respectively. For comparison purposes, we also show the (n, m, k)-cast capacity result for SPR in the first part of this work [1].

Section IV describes the capacity-delay tradeoff of wireless ad hoc networks with MPR. Section V discusses the behavior of the capacity of an ad hoc network with MPR as a function of the (n, m, k)-cast parameters and as a function of the receiver range. For the minimum value of the receiver range $(R(n) = \Omega\left(\sqrt{\log n/n}\right))$ required to guarantee network connectivity [2], the (n, m, k)-cast throughput capacity with MPR is shown to have a gain of $\Theta(\log n)$ compared to the throughput attained with SPR [1]. Furthermore, the capacity-delay tradeoff with MPR is fundamentally different than the tradeoff with SPR [1].

The paper is organized as follows. Section II describes the assumptions and definitions that we use throughout the paper. Section III computes a tight bound for the capacity of wireless ad hoc networks with MPR. Section IV describes the delay computation together with capacity-delay tradeoff for all information dissemination modalities when nodes are endowed with MPR. Section V discusses the results derived in the previous sections and their implications. Section VI presents our conclusions.

II. Network Model and Preliminaries

Our capacity analysis is based on the protocol model for dense networks introduced by Gupta and Kumar [2]. What we call SPR corresponds to this original protocol model, and we make an extension to account for MPR capability at the receivers.

In the model by Gupta and Kumar (i.e., SPR in our generalized model), a common transmission range $r(n)$ for all nodes is defined. Node i at position X_i can successfully transmit to node j at position X_j if for any node k at position $X_k, k \neq i$, that transmits at the same time as i, then $|X_i - X_j| \leq r(n)$ and $|X_k - X_j| \geq (1 + \Delta)r(n)$, where X_i, X_j and X_k are the cartesian position in the unit square network for these nodes. We need to define the protocol model for MPR, and in doing so we extend the MPR protocol model we first introduced in [4].

Definition 2.1: The Protocol Model for MPR: All nodes use a common transmission range $R(n)$ for all their communications. In wireless networks with MPR capability, the protocol model assumption allows simultaneous decoding of packets for all nodes as long as they are within a radius of $R(n)$ from the receiver and all other transmitting nodes have a distance larger than $(1 + \Delta)R(n)$.

Note that some transmitters outside $R(n)$ can be very strong and interfere with MPR communications, therefore it is natural to select a guard zone of $(1 + \Delta)R(n)$ around each receiver such that nodes in this region are silent during transmission. The value of Δ depends on the sensitivity of the receiver nodes and the wireless channel. It is common knowledge that once the transmitters are far from a receiver more than a certain distance, then the interference from these transmitters are so weak that can be treated as noise.

The difference between the protocol model for SPR [1], [2] and MPR is that we allow the receiver node to receive multiple packets simultaneously from different nodes within its disk of radius $R(n)$ [4]. We assume that nodes cannot transmit and receive at the same time, which is equivalent to half-duplex communication [2]. The data rate for each transmitter-receiver pair is a constant value of W bits/second and does not depend on n. Given that W does not change the order capacity of the

[1]An event occurs with high probability (w.h.p.) if its probability tends to one as n goes to infinity. Θ, Ω and O are the standard order bounds.

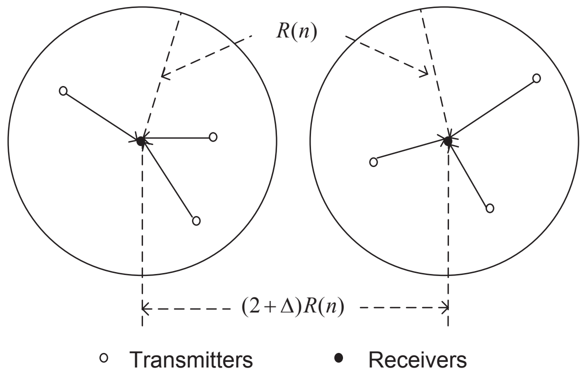


Fig. 1. MPR protocol model.

network, we normalize its value to one. The MPR protocol model is shown in Fig. 1.

We assume that the receiver range $R(n)$ of MPR is equal to transmission range $r(n)$ in SPR and to guarantee connectivity for all the nodes in the network, the communication range is defined as

$$R(n) = r(n) = \Omega\left(\sqrt{(\log n)/n}\right). \quad (1)$$

We assume a random wireless ad hoc network with n nodes distributed uniformly in the network where the area of the network is a square of unit value[2]. Our analysis is based on dense network where density of nodes is n. Hence, in our model, as n goes to infinity, the density of the network also goes to infinity.

In this paper, we study the case in which each of the n nodes in a network acts as a source for a communication group of m receivers (with $m \leq n$), with k (where $k \leq m$) closest receivers being selected to obtain the information from the source reliably. We call this characterization of information dissemination from sources to receivers (n, m, k)-casting, and use it to model all forms of one-to-one, one-to-many and many-to-many information dissemination in wireless networks. Based on the above definition of an (n, m, k)-casting, the throughput capacity for an (n, m, k)-cast simply extrapolates the original definition of feasible throughput capacity for unicasting given by Gupta and Kumar [2].

We follow the same definitions of *feasible throughput capacity of* (n, m, k)*-cast*, *order of throughput capacity*, (n, m, k)*-cast tree*, *Euclidean minimum spanning tree (EMST)*, *minimum Euclidean* $(n, m, k) -$ *cast tree*, *minimum area* $(n, m, k) -$ *cast tree*, *delay in* (n, m, k)*-cast communication* from part I of the paper [1]. For completeness of the paper, we present these definitions again here.

Definition 2.2: Feasible throughput capacity of (n, m, k)*-cast:* A throughput of $\lambda(n)$ bits per second for each node is feasible if we can define a scheduling transmission scheme that allows each node in the network to transmit $\lambda(n)$ bits per second on average to its k out of m destinations.

The per-node feasible throughput capacity of the network is defined as the number of bits per second that every node can transmit to its destination.

[2]The unit square of the network simplifies the analysis. For different shape of the network area, the result can be extended similarly.

Definition 2.3: Order of throughput capacity: $\lambda(n)$ is said to be of order $\Theta(f(n))$ bits per second if there exist deterministic positive constants c and c' such that

$$\begin{cases} \lim_{n\to\infty} \text{Prob } (\lambda(n) = cf(n) \text{ is feasible}) = 1 \\ \liminf_{n\to\infty} \text{Prob } (\lambda(n) = c'f(n) \text{ is feasible}) < 1. \end{cases} \quad (2)$$

Computing the throughput capacity of a network of n nodes requires us to consider the minimum Euclidean distance (n, m, k)-cast trees between sources and their intended receivers. Furthermore, the selection of k out of m receivers is based on choosing the closest destinations in the (n, m, k)-cast tree to the source. The rest of this section introduces additional concepts necessary for the computation of the throughput capacity of a random ad hoc network.

Definition 2.4: (n, m, k)*-Cast Tree:* An (n, m, k)-cast tree is a minimum set of nodes that connect a source node of an (n, m, k)-cast with all its intended m receivers in order for the source to send information to k of those receivers.

The construction of (n, m, k)-cast tree starts with connecting the source to m destinations using minimum number of relays or hops. After constructing this tree, we pick k out of m nodes in this tree that have minimum total Euclidean distance to the source. We refer to this selection of k nodes as "optimum" because it results in maximum throughput capacity for the network. Note that there are $\binom{m}{k}$ choices for selecting k nodes and in this paper, we have selected the above criterion for this selection.

When communicating over a broadcast channel, a transmission from a source or relay in an (n, m, k)-cast may interfere with other transmissions in the same or different (n, m, k)-casts. For a given (n, m, k)-cast to succeed, the packet from the source must reach k of the m receivers in the group reliably at least once. Furthermore, any given relay forwards a packet only once. Accordingly, one or multiple (n, m, k)-cast trees can be defined by the set of transmissions that reach each relay and destination of a given (n, m, k)-cast for the first time. When $m = k$, the resulting (n, m, m)-cast tree is also called a multicast tree. For the case in which $k \leq m$, the selection of the subset of k receivers that correctly receive the packet from the source is such that each of them is reached through a branch of the (n, m, k)-cast tree.

Given the distribution of nodes in the plane and the protocol model we assume, the possible (n, m, k)-cast trees we need to consider include only those that render the minimum number of transmissions for a packet from the source to reach all the intended receivers (k or m) at least once. Because transmissions occur over a common broadcast channel, this implies that the (n, m, k)-cast trees in which we are interested are those that involve the minimum number of relay nodes needed to connect the source and intended receivers of an (n, m, k)-cast in order to maximize the total throughput capacity. That is, we focus on (n, m, k)-cast trees built by the aggregation of shortest paths (minimum-hop paths) between a source and all of its intended destinations. Accordingly, we adopt the following definition for (n, m, k)-cast trees in the rest of this paper.

Definition 2.5: Euclidean Minimum Spanning Tree (EMST): [19] Consider a connected undirected graph

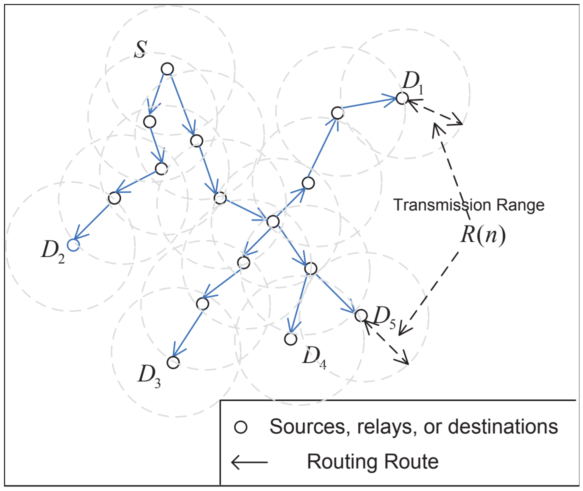


Fig. 2. Area coverage by one multicast tree.

$G = (V, E)$, where V and E are the sets of vertices and edges in the graph G, respectively. The EMST of G is a spanning tree of G with the total minimum Euclidean distance of the edges of the tree.

An (n, m, k)-cast tree is a function of the receiver range $R(n)$. Therefore, the optimum tree that has the minimum Euclidean distance is a function of $R(n)$. For this reason, changing the transmission range will change the optimum (n, m, k)-cast tree.

Definition 2.6: Minimum Euclidean (n, m, k)-Cast Tree (MEMKT$(R(n))$): The MEMKT$(R(n))$ of an (n, m, k)-cast is an (n, m, k)-cast tree in which the k destinations that receive information from the source among the m receivers of the (n, m, k)-cast have the minimum total Euclidean distance.

Definition 2.7: Minimum Area (n, m, k)-cast Tree (MAMKT$(R(n))$): The MAMKT$(R(n))$ in a (n, m, k)-cast tree with k out of m destinations for each source is a (n, m, k)-cast tree that has minimum total area. The area of a (n, m, k)-cast tree is defined as the total area covered by the circles centered around each source or relay with radius $R(n)$.

Note that EMST is spanning tree that consider only the source and destinations, while MEMKT and MAMKT are related to a real routing tree that includes the relays needed to connect the source with the destinations.

In our delay analysis, we assume that the delay associated with packet transmission is negligible and the delay is essentially proportional to the number of hops from source to destination. When the packet size is large, then the transmission delay is considerable and we no longer can ignore this delay. Our analysis does not consider this case and this is the subject of future study.

Definition 2.8: Delay of an (n, m, k)-Cast: In an (n, m, k)-cast, the delay of a packet in a network is the time it takes the packet to reach *all* k destinations after it leaves the source.

We do not take queuing delay at the source into account, because our interest is in the network delay. The average packet delay for a network with n nodes, $D_{m,k}(n)$, is obtained by averaging over all packets, all source-destination pairs, and all random network configurations.

Definition 2.9: Total Active Area (TAA$(\Delta, R(n))$): The TAA$(\Delta, R(n))$ is the total area of the network multiplied by the average maximum order of simultaneous transmissions inside a communication region of $\Theta(R^2(n))$.

It will be shown that this value has an upper bound of $O(1)$ and $O(nR^2(n))$ for SPR and MPR, respectively. In the rest of this paper, $\|S\|$ denotes the total Euclidean distance of a tree S; $\overline{\|S\|}$ is used for the statistical average Euclidean distance of the tree S.

Given that the distribution of nodes in a random network is uniform, if there are n nodes in a unit square, then the density of nodes is equal to n. Hence, if $|S|$ denotes the area of space region S, the expected number of the nodes, $E(N_S)$, in this area is given by $E(N_S) = n|S|$. Let N_j be a random variable defining the number of nodes in S_j . Then, for each random variable N_j, we have the following standard results known as the Chernoff bounds [20]:

Lemma 2.10: Chernoff bound For any $0 < \delta < 1$, we have

$$P[|N_j - n|S_j|| > \delta n|S_j|] < e^{-\theta n|S_j|}, \quad (3)$$

where θ is a variable function of δ.

Therefore, for any $\theta > 0$, there exist constants such that deviations from the mean by more than these constants occur with probability approaching zero as $n \to \infty$. It then follows that, w.h.p., a very sharp concentration on the number of nodes in an area is obtained, and the achievable lower bound can be found w.h.p., provided that the upper bound (mean) is given. In the next section, we first derive the upper bound, and then use the Chernoff bound to prove the lower bound that is achievable w.h.p.

III. The Throughput Capacity of (n, m, k)-Cast with MPR

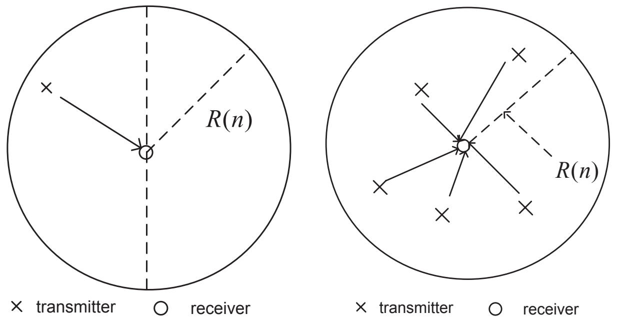
A. Upper Bound

The following lemma provides an upper bound for the per-session capacity as a function of TAA$(\Delta, R(n))$ and $S\,(\text{MAMKT}(R(n)))$. Essentially, $S\,(\text{MAMKT}(R(n)))$ equals the minimum area needed to (n, m, k)-cast a packet to k destinations out of m choices (see Fig. 2), and TAA$(\Delta, R(n))$ represents the maximum area that can be supported when MPR is used by nodes.

Lemma 3.1: In random dense wireless ad hoc networks, the per-node throughput capacity of (n, m, k)-cast with MPR is given by $O\left(\frac{1}{n} \times \frac{\text{TAA}(\Delta, R(n))}{S(\text{MAMKT}(R(n)))}\right)$.

Proof: With MPR, we observe that $S\,(\text{MAMKT}(R(n)))$ represents the total area required to transmit information from a multicast source to all its m destinations. The ratio between average total active area, TAA$(\Delta, R(n))$, and $S\,(\text{MAMKT}(R(n)))$ represents the number of simultaneous (n, m, k)-cast communications that can occur in the network. Normalizing this ratio by n provides per-node capacity, which proves the lemma. ■

Lemma 3.1 provides the upper bound for the (n, m, k)-cast throughput capacity with MPR as a function of $S\,(\text{MAMKT}(R(n)))$ and TAA$(\Delta, R(n))$. In order to compute the upper bound, we derive the upper bound of TAA$(\Delta, R(n))$ and the lower bound of $S\,(\text{MAMKT}(R(n)))$. Combining



(a) SPR (b) MPR

Fig. 3. Upper bound of total available area based on protocol model with MPR.

these results provides an upper bound for the (n, m, k)-cast throughput capacity with MPR. To compute the lower bound for $S\left(\text{MAMKT}(R(n))\right)$, we find the relationship between $S\left(\text{MAMKT}(R(n))\right)$ and the total length of Euclidean Minimum Spanning Tree (EMST), $\overline{\|\text{EMST}\|}$.

Lemma 3.2: In (n, m, k)-cast applications, the area of an (n, m, k)-cast tree with nodes having transmission range $R(n)$, $\overline{S\left(\text{MAMKT}(R(n))\right)}$ has the following lower bound

$$\overline{S\left(\text{MAMKT}(R(n))\right)} = \begin{cases} \Omega\left(kR(n)/\sqrt{m}\right), m = O\left(R^{-2}(n)\right) \\ \Omega\left(kR^2(n)\right), \Omega(k) = R^{-2}(n) = O(m) \\ \Omega\left(1\right), k = \Omega\left(R^{-2}(n)\right) \end{cases}. \tag{4}$$

Proof: Note that $\overline{S\left(\text{MAMKT}(R(n))\right)}$ is the same value for MPR and SPR [1] and they only depend on the communication range in the network. The proof follows by substituting $r(n)$ in Lemma 4.1 in [1] with $R(n)$. ∎

The next lemma states the upper bound for $\text{TAA}(\Delta, R(n))$ for a network using MPR.

Lemma 3.3: The total active area, $\text{TAA}(\Delta, R(n))$, has the following upper bound in networks with MPR.

$$\text{TAA}(\Delta, R(n)) = O\left(nR^2(n)\right) \tag{5}$$

Proof: As discussed earlier, $\text{TAA}(\Delta, R(n))$ for SPR equals 1, because there is only a single pair of transmitter-receiver nodes for each circle of radius $R(n)$ (see Fig. 3(a)). On the other hand, for the case of MPR, the number of transmitters in a circle of radius $R(n)$ is upper bounded by $O(nR^2(n))$. The upper bound for $\text{TAA}(\Delta, R(n))$ is achieved when the maximum number of transmitters are employed in this circle. Fig. 3(b) shows an example that this upper bound can be attained simultaneously for transmitters. Given the fact that this value also is the maximum possible number of transmitter and receiver nodes, the result follows immediately. ∎

Lemma 3.3 implies that the total active area with MPR is upper bounded by $\Theta\left(nR^2(n)\right)$. By contrast, for the case of SPR in [1], it is only $\Theta(1)$. Combining Lemmas 3.1, 3.2, and 3.3, proves the following theorem, which establishes the upper bound of (n, m, k)-cast capacity for MPR.

Theorem 3.4: In wireless ad hoc networks with MPR, the upper bound on the per-node throughput capacity of (n, m, k)-cast is

$$C_{m,k}(n) = \begin{cases} O\left(k^{-1}\sqrt{m}R(n)\right), m = O\left(R^{-2}(n)\right) \\ O\left(k^{-1}\right), \Omega(k) = R^{-2}(n) = O(m) \\ O\left(R^2(n)\right), k = \Omega\left(R^{-2}(n)\right) \end{cases}, \tag{6}$$

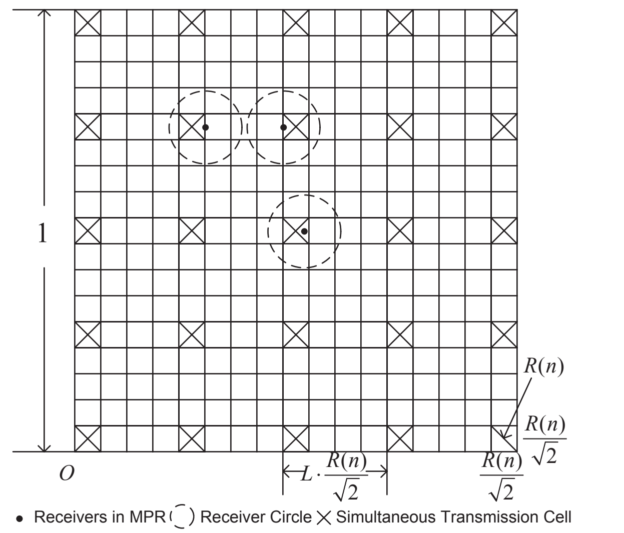


Fig. 4. Cell construction used to derive a lower bound on capacity.

B. Lower Bound

To derive an achievable lower bound, we use a TDMA scheme for random dense wireless ad hoc networks similar to the approach used in the first part of this work [1]. The difference is that we change the transmission range $r(n)$ to be the receiver range $R(n)$.

To satisfy the MPR protocol model, similarly, let $L = \left\lceil 1 + \frac{R(n)+(1+\Delta)R(n)}{R(n)/\sqrt{2}} \right\rceil = \lceil 1 + \sqrt{2}(2 + \Delta) \rceil$ represent the minimum number of cell separations in each group of cells that communicate simultaneously. If we divide time into L^2 time slots and assign each time slot to a single group of cells, interference is avoided and the protocol model is satisfied. The separation example can be shown for the upper two receiver circles in Fig. 4. For the MPR protocol model, the distance between two adjacent receiving nodes is $(2+\Delta)R(n)$. Because this distance is smaller than $(L-1)R(n)$, this organization of cells guarantees that the MPR protocol model is satisfied. Fig. 4 represents one of these groups with a cross sign inside those cells for $L = 4$. The capacity reduction caused by the TDMA scheme is a constant factor and does not change the order capacity of the network.

Next we prove that, when n nodes are distributed uniformly over a unit square area and MPR is used by nodes, we have simultaneously at least $\left\lceil \frac{1}{(LR(n)/\sqrt{2})^2} \right\rceil$ circular regions (see Fig. 4), each one containing $\Theta(nR^2(n))$ nodes w.h.p.. The objective is to find an achievable lower bound using the Chernoff bound, such that the distribution of the number of edges in this unit space is sharply concentrated around its mean, and hence the actual number of simultaneous transmissions occurring in the unit space in a randomly chosen network is indeed $\Theta(n)$ w.h.p..

Lemma 3.5: The circular area of radius $R(n)$ corresponding to the receiver range of a receiver j contains $\Theta(nR^2(n))$ nodes w.h.p. for all values of j, $1 \leq j \leq \left\lceil \frac{1}{(LR(n)/\sqrt{2})^2} \right\rceil$.

Proof: The statement of this lemma can be expressed as

$$\lim_{n\to\infty} P\left[\bigcap_{j=1}^{\left\lceil \frac{1}{(LR(n)/\sqrt{2})^2}\right\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] = 1, \quad (7)$$

where N_j and $E(N_j)$ are the random variables representing the number of nodes in the receiver circle of radius $R(n)$ centered around node j and the expected value of this random variable respectively, and δ is a positive arbitrarily small value close to zero.

From the Chernoff bound in Eq. (3), for any given $0 < \delta < 1$, we can find $\theta > 0$ such that $P[|N_j - E(N_j)| > \delta E(N_j)] < e^{-\theta E(N_j)}$. Thus, we can conclude that the probability that the value of the random variable N_j deviates by an arbitrarily small constant value from the mean tends to zero as $n \to \infty$. This is a key step in showing that, when all the events $\bigcap_{j=1}^{\left\lceil \frac{1}{(LR(n)/\sqrt{2})^2}\right\rceil} |N_j - E(N_j)| < \delta E(N_j)$ occur simultaneously, then all the variables N_j's converge uniformly to their expected values. Utilizing the union bound, we arrive at

$$\begin{aligned} & P\left[\bigcap_{j=1}^{\left\lceil \frac{1}{(LR(n)/\sqrt{2})^2}\right\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] \\ = \;& 1 - P\left[\bigcup_{j=1}^{\left\lceil \frac{1}{(LR(n)/\sqrt{2})^2}\right\rceil} |N_j - E(N_j)| > \delta E(N_j)\right] \\ \geq \;& 1 - \sum_{j=1}^{\left\lceil \frac{1}{(LR(n)/\sqrt{2})^2}\right\rceil} P[|N_j - E(N_j)| > \delta E(N_j)] \\ > \;& 1 - \left\lceil \frac{1}{(LR(n)/\sqrt{2})^2} \right\rceil e^{-\theta E(N_j)}. \end{aligned} \quad (8)$$

Given that $E(N_j) = \pi n R^2(n)$, then we have

$$\begin{aligned} & \lim_{n\to\infty} P\left[\bigcap_{j=1}^{\left\lceil \frac{1}{(LR(n)/\sqrt{2})^2}\right\rceil} |N_j - E(N_j)| < \delta E(N_j)\right] \\ \geq \;& 1 - \lim_{n\to\infty} \left\lceil \frac{1}{(LR(n)/\sqrt{2})^2} \right\rceil e^{-\theta\pi n R^2(n)} \end{aligned} \quad (9)$$

Utilizing the connectivity criterion in Eq. (1), we have $\lim_{n\to\infty} \frac{e^{-\theta\pi n R^2(n)}}{R^2(n)} \to 0$, which completes the proof. ■

The previous lemma proves that, w.h.p., there are indeed $\Theta(n)$ simultaneous potential transmitters that are in $\left\lceil \frac{1}{(LR(n)/\sqrt{2})^2} \right\rceil$ circles of radius $R(n)$ around the receivers, who can transmit simultaneously, as shown in Fig. 4. With Lemmas 3.5, we have completed the preparation for the following achievable lower bound.

Let us define $\overline{\#\text{MEMKTC}(R(n))}$ as the average total number of cells that contain all the nodes in an (n, m, k)-cast group. Also, $\overline{\#\text{MEMTC}(R(n))}$ is defined as the average total number of cells that contain all the nodes in an (n, m, m)-cast group. The following lemma establishes the achievable lower bound for the (n, m, k)-cast throughput capacity of MPR as a function of $\overline{\#\text{MEMKTC}(R(n))}$. Note that $\overline{\#\text{MEMKTC}(R(n))}$ only depends on the (n, m, k)-cast network parameters regardless of using MPR techniques. The following lemma provides a tight bound for $\overline{\#\text{MEMKTC}(R(n))}$.

Lemma 3.6: The average number of cells covered by the nodes in MEMKTC$(R(n))$, is tight bounded w.h.p. as follows:

$$\overline{\#\text{MEMKTC}(R(n))} = \begin{cases} \Theta\left(k\left(\sqrt{m}R(n)\right)^{-1}\right), m = O\left(R^{-2}(n)\right) \\ \Theta(k), \Omega(k) = R^{-2}(n) = O(m) \\ \Theta\left(R^{-2}(n)\right), k = \Omega\left(R^{-2}(n)\right) \end{cases} \quad (10)$$

Proof: The proof is similar to Lemma 4.5 in the first part [1] of this two-part series. The difference is that we substitute transmission range $r(n)$ with receiver range $R(n)$. ■

Next we discuss the routing scheme needed to achieve the lower bound capacity, which is similar to the scheme used in [21]. According to our model, each (n, m, k)-cast session creates a (n, m, k)-cast tree MEMKT$(R(n))$ to connect the source and destinations. The trees are denoted as T_is, where $i = 1, 2, \cdots, n$. The routing scheme between source and destination is such that packets are forwarded by using cells that are intersected only by T_i. There is a bound on the number of trees that each cell needs to serve, which means that we can bound the probability that the trees intersects a particular cell.

Lemma 3.7: For any $R(n) = \Omega\left(\sqrt{\log n/n}\right)$, we have (11).

Proof: For every tree T_i and cell S_{k_0,j_0},

$$\begin{aligned} p &= \text{Prob}\{\text{Tree } T_i \text{ intersects } S_{k_0,j_0}\} \\ &= \Theta\left(R^2(n)\overline{\#\text{MEMKTC}(R(n))}\right) \end{aligned} \quad (12)$$

First we bound the number of trees served by one particular cell S_{k_0,j_0}. Define i.i.d. random variables $I_i, 1 \leq i \leq n$, as follows:

$$I_i = \begin{cases} 1, \text{ if } T_i \text{ intersects } S_{k_0,j_0} \\ 0, \text{ if not} \end{cases} \quad (13)$$

Then Prob$(I_i = 1) = p, \forall i$, where p is defined in Eq. (12). Denote by Z_n the total number of trees served by S_{k_0,j_0}. Then $Z_n := I_1 + I_2 + \cdots + I_n$. By using the Chernoff bounds [20] we have, Prob$(Z_n > b) \leq \frac{E[e^{aZ_n}]}{e^{ab}}$ for all positive values of b and a. Furthermore,

$$\begin{aligned} E\left[e^{aZ_n}\right] &= (1 + (e^a - 1)p)^n \leq \exp(n(e^a - 1)p) \\ &= \Theta\left(\exp\left((e^a - 1)nR^2(n)\overline{\#\text{MEMKTC}(R(n))}\right)\right). \end{aligned} \quad (14)$$

Let us define $b = \Theta\left(nR^2(n)\overline{\#\text{MEMKTC}(R(n))}\right)$, then if a is small enough, we have

$$\begin{aligned} & \text{Prob}\left(Z_n = \Omega\left(nR^{-2}(n)\overline{\#\text{MEMKTC}(R(n))}\right)\right) \\ = \;& O\left(\exp\left(-nR^2(n)\overline{\#\text{MEMKTC}(R(n))}\right)\right). \end{aligned} \quad (15)$$

Thus, by the union bound, we have (16)

$$\lim_{n\to\infty} \text{Prob}\left(\sup_{(k,j)}\{\text{Number of trees } T_i\text{s intersecting } S_{k,j}\} = O\left(nR^2(n)\overline{\#\text{MEMKTC}(R(n))}\right)\right) = 1 \tag{11}$$

$$\begin{aligned} & \text{Prob}\left(\text{Some cell intersects } \Omega(nR^2(n)\overline{\#\text{MEMKTC}(R(n))}) \text{ trees }\right) \\ \leq\; & \sum_{k,j} \text{Prob}\left(\text{Cell } S_{jk} \text{ intersects } \Omega(nR(n)) \text{ trees }\right) \\ =\; & O\left(\frac{1}{R^2(n)} \exp\left(-nR^2(n)\overline{\#\text{MEMKTC}(R(n))}\right)\right) \end{aligned} \tag{16}$$

The right hand side tends to zero for $R(n) = \Omega\left(\sqrt{\log n/n}\right)$ as n goes to infinity for all three different regions of $\overline{\#\text{MEMKTC}(R(n))}$ from Eq. (10). ■

There exists a transmitting schedule such that in every L^2 slots (L is constant), each cell transmits at a rate W bits/second with a maximum transmission distance $R(n)$. Therefore, the rate for each cell is $\Theta\left(nR^2(n)\right)W/L^2$. From Lemma 3.7, each cell needs to transmit at a rate $O\left(C_{m,k}(n)nR^2(n)\overline{\#\text{MEMKTC}(R(n))}\right)$, with probability approaching one. In order to accommodate this requirement by all cells, we need

$$C_{m,k}(n)nR^2(n)\overline{\#\text{MEMKTC}(R(n))} = \Omega\left(\left(nR^2(n)\right)W/L^2\right) \tag{17}$$

Thus, we have proven the achievable throughput for Lemma 3.8 needed to guarantee that each cell can support this capacity.

Lemma 3.8: The achievable lower bound for the (n, m, k)-cast capacity is given by

$$C_{m,k}(n) = \Omega\left(\left(\overline{\#\text{MEMKTC}(R(n))}\right)^{-1}\right). \tag{18}$$

Proof: There are $(R(n)/\sqrt{2})^{-2}$ cells in the unit square network area. With Lemma 3.7 and the fact that our TDMA scheme does not change the order capacity, it is clear that there are at most in the order of $\overline{\#\text{MEMKTC}(R(n))}$ interfering cells for any (n, m, k)-cast communication. For each cell, the order of nodes in each cell is $\Theta\left(\pi R^2(n)n\right)$. Accordingly, the total lower bound capacity is given by $\Omega\left((R(n)/\sqrt{2})^{-2} \times \left(\pi R^2(n)n\right) \times \left(\overline{\#\text{MEMKTC}(R(n))}\right)^{-1}\right)$. Normalizing this value by total number of nodes in the network, n, proves the lemma. ■

Combining Lemmas 3.6 and 3.8, we arrive at the achievable lower bound of the (n, m, k)-cast throughput capacity in dense random wireless ad hoc networks with MPR.

Theorem 3.9: The achievable lower bound of the (n, m, k)-cast throughput capacity with MPR is

$$C_{m,k}(n) = \begin{cases} \Omega\left(k^{-1}\sqrt{m}R(n)\right) & \text{for} \quad m = O\left(R^{-2}(n)\right), \\ \Omega\left(k^{-1}\right) & \text{for} \quad \Omega(k) = R^{-2}(n) = O(m), \\ \Omega\left(R^2(n)\right) & \text{for} \quad k = \Omega\left(R^{-2}(n)\right) \end{cases} \tag{19}$$

Proof: There are $(R(n)/\sqrt{2})^{-2}$ cells in the unit square network area and only $(LR(n)/\sqrt{2})^{-2}$ of these cells can communicate simultaneously because of the TDMA scheme that we described earlier. From the definition of $\overline{\#\text{MEMKTC}(R(n))}$, it is clear that there are in the order of $\overline{\#\text{MEMKTC}(R(n))}$ transmissions required in order to transfer a packet from source to all its destinations in any (n, m, k)-cast communication scheme. It is clear from Lemma 3.5 that for each of $(LR(n)/\sqrt{2})^{-2}$ simultaneous transmitting cells, there are $\Theta\left(\pi R^2(n)n\right)$ nodes transmitting packets to their respected receiver nodes using MPR. Since each one of (n, m, k)-cast group requires $\overline{\#\text{MEMKTC}(R(n))}$ transmissions, the total throughput capacity lower bound for the network is equal to $\Omega\left(\frac{(R(n)/\sqrt{2})^{-2} \times (\pi R^2(n)n)}{\overline{\#\text{MEMKTC}(R(n))}}\right)$. If we divide this value by the total number of nodes in the network, n, and substitute $\overline{\#\text{MEMKTC}(R(n))}$ with the results from Lemma 3.6, then the proof follows. ■

We have proved there is no congestion in relay nodes. Furthermore, we will prove there is not any congestion at the destinations. Assume that each source selects a destination randomly and independently, then we will prove that, w.h.p., a node can be the destination for at most $\frac{3\log n}{\log\log n}$ sources. This problem is similar to the " bins and balls problems" in [22].

Lemma 3.10: The probability of having a particular destination selected by k sources is

$$\lim_{n\to\infty} \text{Prob}[\text{destination } i \text{ has at least } k \text{ sources}] \leq \left(\frac{e}{k}\right)^k \tag{20}$$

Proof: If we look at any subset of sources of size k, then the probability that the subset of sources select destination i is $(\frac{1}{n})^k$. We then take a union bound of these probabilities over all $\binom{n}{k}$ subsets of size k. The events we are summing over, though, are not disjoint. Therefore, we can only show that the probability of a destination having at least k balls is at most $\binom{n}{k}\binom{n}{1}^k$. Using Stirling's approximation $\lim_{n\to\infty} \frac{n!}{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n} = 1$, we have

$$\lim_{n\to\infty} \frac{\binom{n}{k}\binom{n}{1}^k}{\left(\frac{e}{k}\right)^k} = 1, \tag{21}$$

which proves the lemma. ■

Lemma 3.11: There exists at most $\frac{3\log n}{\log\log n}$ sources for each destination w.h.p.

Proof: Let $k = \frac{3\log n}{\log\log n}$. From Lemma 3.10, we have

$$\begin{aligned}
&\lim_{n\to\infty} \text{Prob[destination } i \text{ has at least } k \text{ sources]} \\
\leq\ & \left(\frac{e}{k}\right)^k = \left(\frac{e\log\log n}{3\log n}\right)^{\frac{3\log n}{\log\log n}} \\
\leq\ & \exp\left(\frac{3\log n}{\log\log n}(\log\log\log n - \log\log n)\right) \\
=\ & \exp\left(-3\log n + \frac{3\log n \log\log\log n}{\log\log n}\right) \\
\leq\ & \exp(-2\log n) = \frac{1}{n^2} \qquad (22)
\end{aligned}$$

Using the union bound, we have $\lim_{n\to\infty}$ Prob[any destination has at least k sources] $\leq n\frac{1}{n^2} = \frac{1}{n}$, which implies that $\lim_{n\to\infty}$ Prob[all destinations have at most k sources] $\geq 1 - \frac{1}{n}$. ■

For MPR (n, m, k)-cast, we require that none of the destinations has a traffic load congestion larger than the total throughput it can support. This means that the maximum throughput for each destination should always be greater than the total traffic load. In the MPR case, the total throughput of each destination is $nR^2(n)$. The traffic load congestion for each destination is the multiplication of throughput per node of $C_{m,k}(n)$ and the maximum possible sources that select a node, i.e., $\frac{3\log n}{\log\log n}$. Hence,

$$nR^2(n) \geq C_{m,k}(n)\frac{3\log n}{\log\log n}, \qquad (23)$$

As long as $R(n) = \Omega\left(\sqrt{\frac{\log n}{n}}\right)$, it can be easily verified that, for all three capacity regions, $C_{m,k}(n)$ can achieve the lower bound of Theorem 3.9.

IV. Capacity and Delay Tradeoff with MPR

A. The Capacity of (n, m, k)-Cast with MPR

From Theorems 3.4 and 3.9, we can provide the tight bound for the throughput capacity of the (n, m, k)-cast when the nodes have MPR capability in dense random wireless ad hoc networks as follows.

Theorem 4.1: The throughput capacity of (n, m, k)-cast in a random dense wireless ad hoc network with MPR is

$$C_{m,k}(n) = \begin{cases} \Theta\left(k^{-1}\sqrt{m}R(n)\right) & \text{for } m = O\left(R^{-2}(n)\right) \\ \Theta\left(k^{-1}\right) & \text{for } \Omega(k) = R^{-2}(n) = O(m) \\ \Theta\left(R^2(n)\right) & \text{for } k = \Omega\left(R^{-2}(n)\right) \end{cases}. \qquad (24)$$

The receiver range of MPR should satisfy $R(n) = \Omega\left(\sqrt{\log n/n}\right)$. Note that the thresholds for different values for m and k provide various capacities for (n, m, k)-cast with MPR.

B. The Delay of (n, m, k)-Cast with MPR and its Relationship with The Capacity

In this section, we present the tradeoff between delay and capacity. As we defined in [1], packet delay is proportional to the total number of hops required from each source to reach all its destinations. In order to compute this delay, we first prove the following lemma.

TABLE I
Relationship between Capacity and Delay with MPR

	$D_{m,k}(n)$	$C_{m,k}(n)$
$m = O\left(R^{-2}(n)\right)$	$\Theta\left(\frac{k}{\sqrt{m}R(n)}\right)$	$\Theta\left(\frac{\sqrt{m}R(n)}{k}\right)$
$\Omega(k) = R^{-2}(n) = O(m)$	$\Theta(k)$	$\Theta\left(\frac{1}{k}\right)$
$k = \Omega\left(R^{-2}(n)\right)$	$\Theta\left(R^{-2}(n)\right)$	$\Theta\left(R^2(n)\right)$

Lemma 4.2: The delay of (n, m, k)-cast in a random dense wireless ad hoc network with MPR is

$$D_{m,k}(n) = \Theta\left(\overline{\#\text{MEMKTC}(R(n))}\right) \qquad (25)$$

Proof: From the definition of $\overline{\#\text{MEMKTC}(R(n))}$ and Lemma 3.6, we conclude that $\overline{\#\text{MEMKTC}(R(n))}$ is proportional to the minimum number of hops in which the information is routed from source to all its destinations. Given that we are using a TDMA scheme to achieve the lower bound for the capacity, it is clear that in order to transport the information from one cell to the next adjacent cell, we need between one to two hops (see Fig. 4). Therefore, $\overline{\#\text{MEMKTC}(R(n))}$ is also in the same order as the total number of hops. Based on the definition of delay, it is clear that $\overline{\#\text{MEMKTC}(R(n))}$ is also the same order bound as the total delay which proves the Lemma. ■

Theorem 4.3: The relationship between capacity and delay for (n, m, k)-cast with MPR is given below and shown in Table I.

$$C_{m,k}(n)D_{m,k}(n) = \Theta(1) \qquad (26)$$

Proof: The results can be easily derived by comparing Theorem 4.1 with Lemmas 4.2. ■

The relationship between capacity and delay in (n, m, k)-cast with SPR is presented in [1].

V. Discussion of Results

Theorems 4.1 provides capacity results for MPR whose fundamental difference from the SPR in [1] is the fact that the MPR scheme embraces interference, while SPR is based on avoiding interference by limiting transmission range. Below we will discuss the ramifications of these two strategies in terms of the capacity-delay tradeoff.

A. $C_{m,k}(n)$ as a Function of Group Size (m)

Comparing the capacities attained with MPR and SPR for unicast capacity region (see Fig. 5 for $M = k$ case), the ratio is equal to $\Theta\left(R(n)\sqrt{n\log n}\right)$. The same ratio is equal to $\Theta\left(R^2(n)n\right)$ in the broadcasting capacity region. By considering the connectivity criterion, i.e., $R(n) = \Omega\left(\sqrt{\log n/n}\right)$, then it is easy to show that the capacity gain for MPR compared to SPR is larger in broadcast capacity region than in the unicast region. The larger gains attained with MPR for the broadcast region are a consequence of the fact that, as the number of destinations increases, more copies of the same packets must be sent to a larger number of nodes. In a network using MPR, concurrent broadcast transmissions can

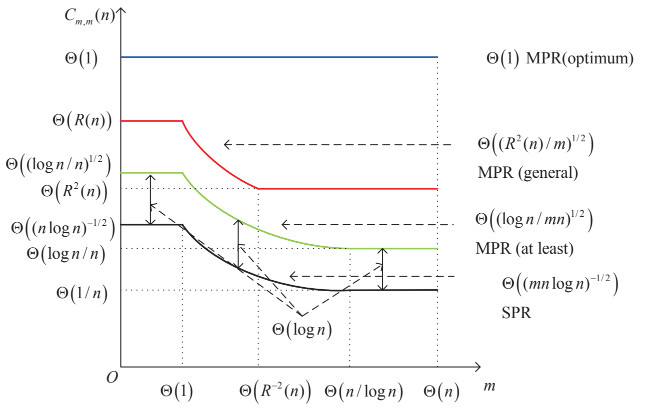


Fig. 5. Order throughput capacity of (n, m, m)-cast with SPR and MPR as a function of number of destinations m and receiver range $R(n)$.

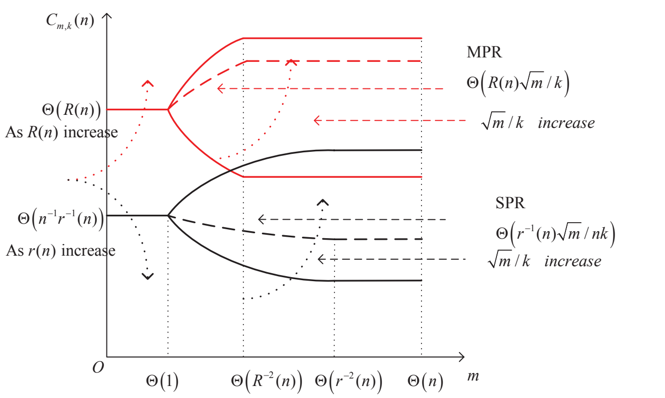


Fig. 6. Order throughput capacity of (n, m, k)-cast with SPR and MPR.

be decoded by the receivers while at most one broadcast transmission can succeed when SPR is used.

Fig. 6 compares the throughput capacity of MPR to that of SPR [1]. Comparing the results for both cases when the number of destinations for each session is smaller than $\Theta\left(R^{-2}(n)\right)$, it appears that they both have the same term as $\sqrt{m}/k$. However, for MPR this term is multiplied by $R(n)$, while for SPR this term is divided by $r(n)$. If we assume $R(n) = r(n)$, it appears that increasing the receiver range increases the capacity for the MPR scheme, while it decreases the capacity for SPR. This fundamental difference is due to the fact that the MPR scheme embraces interference, while SPR is based on avoiding it by limiting transmissions around receivers.

We note that the capacity of anycast or manycast is greater than the capacity of unicast for $k = O\left(\sqrt{m}\right)$, even if each node requires to transmit its packets to more than one destination. This result shows that, as long as $k = O\left(\sqrt{m}\right)$, the total number of hops required to transmit packet to k destinations is always, on average, less than sending the packet from the same source to a single randomly selected destination in unicast communications. Equivalently, the total Euclidean distance for a manycast tree is on average less than the Euclidean distance between any randomly selected source and destination in unicast communication. However, these Euclidean distances become the same, on average, when $k = \Theta\left(\sqrt{m}\right)$. As it can be seen from this figure, the total Euclidean distance in a manycast tree increases as k increase and for $k = \Omega\left(\sqrt{m}\right)$, the capacity of manycast becomes less than that of unicast because of the total Euclidean distance in the manycast tree is larger than the Euclidean distance between any randomly selected source and destination in unicast communication.

B. $C_{m,k}(n)$ as a Function of Receiver Range ($R(n)$)

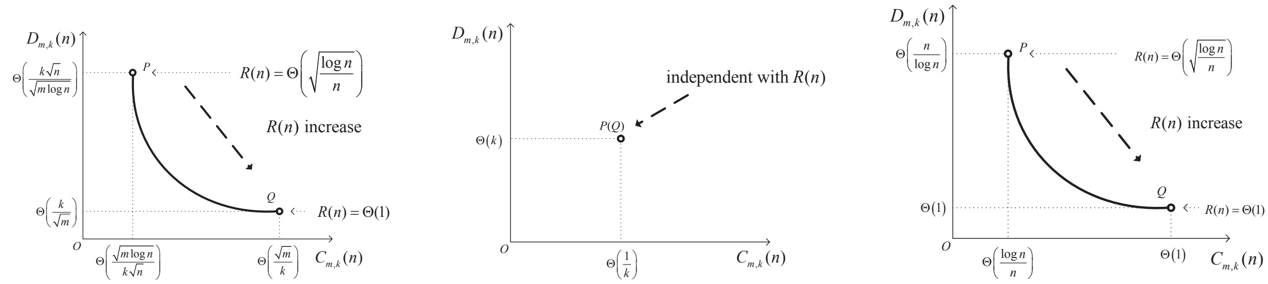
Eq. (24) show that the throughput capacity of wireless ad hoc networks do increase with the increase in the receiver range $R(n)$ when the receivers decode more than one packet at a time. Similar result for MPR was shown in [4] for the case of unicasting. This result is in sharp contrast to results attained with SPR in [1], with which increasing the communication range decreases the capacity. In networks with MPR, by increasing the receiver range in the network we actually increase the total number of simultaneous transmissions at any given time. In contrast, for networks with SPR, a larger transmission range leads to increased interference at larger number of nodes, which forces these nodes to be silent during a communication session.

Clearly, the capacity of the network is maximized if we maximize the number of simultaneous transmissions in the network. Ideally, if the receiver range can be made $\Theta(1)$, then a network using MPR can scale linearly with n. Obviously, the receiver range is restricted in practice by the complexity of the nodes. However, even with the receiver range is assumed to have the minimum value, which is the connectivity criterion in Eq. (1), MPR still renders a capacity gain compared to SPR. Furthermore, this gain is still an order gain equal to $\Theta(\log n)$ compared to the capacity attained with SPR for (n, m, k)-casting. Our result corroborates and extends the capacity gain result reported in [4] for unicast.

C. Capacity-Delay Tradeoff

Theorem 4.1 presents the capacity information for MPR. There are three different capacity regions depending on the values of k and m in (n, m, k)-cast. Figs. 7(a), 7(b) and 7(c) compare the tradeoff between throughput capacity and delay for MPR (SPR results are reported in [1]) for all these three regions of capacity. By observing the capacity for MPR and SPR, we notice that the receiver range $R(n)$ is multiplied for capacity computation in MPR in two regions in Eq. (24) and in one region is independent of $R(n)$ while the transmission range $r(n)$ is divided for capacity computation in SPR in the first two regions of capacity. These behaviors are shown in Figs. 7(a), 7(b) and 7(c). This fundamental difference is due to the fact that the MPR scheme embraces interference, while SPR is based on avoiding interference by limiting transmission range.

The above result indicates that large capacity increases can be attained by embracing interference with MPR and embracing opportunism by appropriate use of in-network storage and information dissemination from the nearest site(s) of a communication group, rather than from pre-defined origins hosting the content. If the communication group is the entire network ($m = n$), information flows from the closest neighbor(s) to each node and the maximum capacity gain is attained. If the group size is independent of the size of the



(a) $m = O\left(R^{-2}(n)\right)$ (b) $\Omega(k) = R^{-2}(n) = O(m)$ (c) $k = \Omega\left(R^{-2}(n)\right)$

Fig. 7. The tradeoff between capacity and delay with MPR.

network ($m = \Theta(1)$), the order capacity is the same as for unicast.

Fig. 7(a) is the first region in capacity for MPR. Interesting observation is the fact that unlike SPR in [1] that increasing capacity results in increasing delay, we can increase capacity and decrease delay simultaneously with MPR. This is a significant advantage of using MPR and stems from the fact that MPR embraces interference and consequently, we do not need to sacrifice capacity or delay to improve the other parameter.

Fig. 7(b) shows the capacity-delay tradeoff in the second capacity region. For the case of MPR, the capacity or delay is not a function of $R(n)$ and therefore, there is no tradeoff. For this case in SPR, increasing $r(n)$ decreases capacity but has no effect on the delay as shown in [1].

Fig. 7(c) is the third region of capacity for MPR. This is the broadcasting region of capacity and it is clear that SPR does not provide any tradeoff [1]. In general, by increasing the transmission range we can decrease delay while the capacity remains constant. The reason for this behavior is the fact that all nodes in broadcasting region are receiving the packet and increasing transmission range does not create any interference. On the other hand, when we use MPR and increase the receiver range, again both capacity and delay are improved similar to the first case. Clearly, the capacity of the network with MPR is maximized if we maximize the number of simultaneous transmissions in the network. Ideally, if the receiver range can be made $\Theta(1)$, then a network using MPR can scale linearly with n. Obviously, the receiver range is restricted in practice by the complexity of the receivers. However, even with the minimum value for the receiver range, which is the connectivity criterion, MPR still renders a capacity gain compared to SPR. Furthermore, this gain is still an order gain equal to $\Theta(\log n)$ compared to the capacity attained with SPR for (n, m, k)-casting.

In summary, the tradeoff between capacity $C_{m,k}(n)$ and delay $D_{m,k}(n)$ with MPR is in sharp contrast to SPR [1]. The results in this paper provide new directions and opportunities for future research activities in wireless ad hoc networks. Another important aspect that we did not discuss in this paper is related to practical limitations and decoding complexity that we can have with MPR scheme. This aspect is important and its investigation is the subject of future studies.

VI. Conclusion

We showed that the throughput capacity of (n, m, k)-cast with multi-packet reception/transmission is $\Theta\left(R(n)\sqrt{m}/k\right)$ when $m = O\left(R^{-2}(n)\right)$, $\Theta\left(1/k\right)$ when $\Omega(k) = R^{-2}(n) = O(m)$ and $\Theta\left(R^2(n)\right)$ when $k = \Omega\left(R^{-2}(n)\right)$. When $R(n) = \Omega\left(\sqrt{\log n/n}\right)$ to satisfy the connectivity criterion, MPR leads to the minimum throughput capacity gain of at least $\Theta(\log n)$ compared to the (n, m, k)-cast throughput capacity with SPR. When $R(n) = \Theta(1)$, which is the maximum receiver range for MPR, the network is linearly scalable. However, this case is not practical in real systems, and simply provides a guideline for designing networks. It suggests that, in order to increase the capacity of wireless ad hoc networks, we must embrace interference at the physical layer by using MPR. This result is in sharp contrast with traditional interference-dominated networks based on SPR. Finally, when the number of destinations is greater than $\Theta\left(R^{-2}(n)\right)$, or equivalently when the receiver range is larger than $\Theta\left(\sqrt{\log n/n}\right)$, there are higher throughput capacity gains with MPR. This is the case in broadcasting or multicasting with larger numbers of destinations, because MPR schemes can inhibit the negative effects of interference compared to SPR [1].

We can decrease the average concurrent transmissions with MPR by a factor of $f(n)$. Therefore, the total number of simultaneous transmissions in a circle of radius $R(n)$ reduces into $\frac{\pi n R^2(n)}{f(n)}$. The reduction in capacity as a result of decreasing the average concurrent transmissions with MPR for each node is straightforward. It is important in the future work to study practical constraints for MPR and derive the constant capacity gain utilizing this approach when the decoding complexity of nodes only allows a finite number of packets to be decoded.

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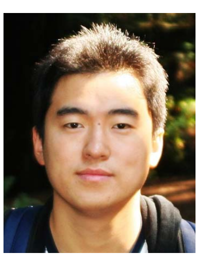

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