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AND HEDONIC PRICE INDICES

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QUALITY CHANGES, CONSUMER'S SURPLUS, AND HEDONIC PRICE INDICES

W. MICHAEL HANEMANN

I. The Standard Welfare Analysis of Price Changes, - II. Consumer's Surplus Measures for Quality Changes, - III. Hedonic Price Index Theory, - IV. Conclusions, - Appendix, .

A common problem in cost-benefit analysis and other types of applied welfare analysis is to evaluate the effect of price changes on a consumer's welfare using data on observed consumption choices. There are two approaches to the solution of this problem. One is based on the Marshallian concept of consumer's surplus and the evaluation of areas under ordinary demand curves. The other is based on price index theory and the computation of weighted price ratios. Over the last 30 years, following the pioneering work of Houthakker (1951-52) and Lancaster (1966), there have been several attempts to extend the economic theory of consumer behavior to allow for differences in the quality of commodities and to explain choices among alternative qualities. The corresponding problem in applied welfare analysis is to evaluate the effect of a quality change on a consumer's welfare using data on observed consumption choices. The problem arises in many different contexts, including the estimation of the benefits to recreationists from changes in water quality at recreation sites, and the evaluation of government programs which regulate product quality. It has been approached in terms of both areas under demand curves (Stevens, 1966) and price indices calculated from "hedonically adjusted" prices (Adelman and Griliches, 1961, and, before that, Hofsten, 1952, and Court, 1939).

The purpose of this paper is to analyze systematically these two approaches to the welfare economics of quality change and to compare them with each other, and with the corresponding approaches to the welfare economics of price changes. The questions addressed here include the following. Do the conditions under which the standard Marshallian consumer's surplus measure provides an accurate

indication of the welfare effects of a price change carry over to the consumer's surplus measure of quality changes? For price changes, without imposing any restrictions on the form of the utility function, one can always obtain bounds on the magnitude of the welfare change from observable data via the Laspeyres and Paasche price indices; do similar bounds exist for hedonic price indices? Does the hedonic price index approach to measuring the welfare effects of quality change provide any significant practical advantage over the consumer's surplus approach?

In considering the consumer's surplus approach to quality change, it is useful to distinguish between the circumstances under which this provides an accurate indication of the direction in which the consumer's welfare changes, and those under which it provides an accurate indication of the magnitude of the welfare change. A basic result, established by Maler (1974) is that if there are one or more commodities which are "weakly complementary" with the quality variables which change (as defined below), the true compensating or equivalent variation for the quality change can be represented as the difference between the areas under the compensated demand functions for the complementary goods evaluated at the old and the new quality levels. It follows that the difference between the areas under the corresponding ordinary demand functions --the Marshallian measure--correctly measures the magnitude of the true welfare change if the complementary goods have zero income elasticities of demand. It is shown that, if the complementary goods have the same income elasticity of demand which is independent of their prices and the quality variables, the Marshallian measure correctly indicates the direction, but not necessarily the magnitude, of the welfare change. If the complementary goods have the same income elasticity of demand, not necessarily independent of prices or quality variables, the Marshallian quantity is path independent but it need not provide a correct indication of either the magnitude or the direction of the welfare change. These three results are shown to parallel those which hold for the standard

welfare analysis of price changes. The last result involves a condition which turns out to be equivalent to a "demand interdependence" assumption invoked by Bradford and Hildebrandt (1977) and Willig (1978), who claim that it leads to an equality between Marshallian consumer's surplus and the true willingness to pay for marginal quality changes. This claim is questioned--it is shown to hold only at the point of zero quality change. However, it is shown that if the complementary goods are all normal (inferior), this condition ensures that the Marshallian measure is an upper (lower) bound on the true measure of welfare change.

The theory of hedonic price indices has evolved independently of the consumer's surplus analysis of quality changes. However, it is shown that, as with price changes, the two approaches are in principle equivalent. There is one important difference between price and quality changes. For hedonic price indices, there is nothing similar to the Laspeyres and Paasche bounds of standard price index theory which can be applied without imposing any restrictions on the form of the utility function. The two main contributions to hedonic price index theory are those of Adelman and Griliches (1961) and Willig (1978). Adelman and Griliches develop hedonic price indices which can be calculated without restricting the form of the utility function; but it is shown that their indices do not lead to any useful bounds on the direction or magnitude of welfare change. It is shown that Willig's approach leads to a set of hedonically adjusted prices which, if they could be calculated, would provide bounds on the true welfare change without requiring any restrictions on the form of the utility function. However, these hedonically adjusted prices cannot be calculated from observable consumption data unless the utility function satisfies the "interdependent demand" condition mentioned above. Moreover, I show that with the special types of utility function for which the

hedonically adjusted prices can be easily calculated, it usually turns out that the true compensating or equivalent variations for the quality change can be computed directly from the observed demand functions. I conclude, therefore, that the hedonic price index approach does not possess any significant advantage in practice over the consumer's surplus approach to evaluating the welfare effects of a quality change.

This paper is organized as follows: Section I contains a brief summary of the standard welfare theory of price changes, emphasizing those aspects which carry over to quality changes. Section II examines the consumer's surplus approach to the welfare theory of quality changes, and Section III examines the hedonic price index approach. The conclusions are summarized in Section IV.

I. THE STANDARD WELFARE ANALYSIS OF PRICE CHANGES

In the standard neoclassical utility model an individual consumer faces a fixed vector of prices, p , has a fixed income, y , and chooses a consumption vector, x , by maximizing a quasiconcave utility function, $u(x)$, subject to a budget constraint. This yields a set of ordinary demand functions, $h^i(p, y)$; the Lagrangean multiplier, $\lambda(p, y)$; and the indirect utility function, $v(p, y) \equiv u[h(p, y)]$. The dual problem is to minimize the expenditure required to attain a given level of utility. This leads to a set of compensated demand functions, $g^i(p, u)$; the Lagrangean multiplier, $\mu(p, u)$; and the expenditure function, $m(p, u) \equiv \sum p_i g^i(p, u)$. Suppose that the consumer's income changes from y^0 to y^1 , while prices change from p^0 to p^1 . Let J be the index set for the prices which change and \bar{J} its complement, so that $p^0 = (p_J^0, p_{\bar{J}})$ and $p^1 = (p_J^1, p_{\bar{J}})$. Accordingly, the consumer's welfare changes from $u^0 \equiv v(p^0, y^0)$ to $u^1 \equiv v(p^1, y^1)$. Let $\Delta y = y^1 - y^0$ and $\Delta u = u^1 - u^0$. A monetary measure of the effect of the change on the consumer's welfare is the compensating variation, C , defined implicitly by $v(p^1, y^1 - C) = v(p^0, y^0)$ or, equivalently, by

$$C = y^1 - m(p^1, u^0) = \Delta y + C^P \quad (1)$$

where

$$C^P \equiv \int_{p^1}^{p^0} \sum g_i(p, u^0) dp_i,$$

the line integral being path independent. An alternative measure is the equivalent variation, E , defined implicitly by $v(p^1, y^1) = v(p^0, y^0 + E)$ or, equivalently, by

$$E = m(p^0, u^1) - y^0 = \Delta y + E^P. \quad (2)$$

These are two senses in which the quantities C and E can be regarded as welfare measures. As is well known, they both provide a correct indication of the direction in which the consumer's welfare changes, since

$$\text{sign} (\Delta u) = \text{sign} (C) = \text{sign} (E).$$

A more interesting question is whether the magnitude of C or E has any normative significance. The question has been answered by Willig (1976) and Mäler (1974) using two arguments. (1) A necessary condition for the Kaldor-Hicks criterion to be satisfied is that the aggregate of the individual C's be positive while, if the aggregate of the individual E's is positive, this is a sufficient condition for the Scitovsky criterion to be satisfied. If some individuals gain from the change but others lose, one needs to know the magnitude of the individual C's or E's in order to apply these criteria. (2) Suppose there is a Bergsonian social welfare function, $W = W(u^1, \dots, u^H)$, where u^h is the utility of the h th individual. This social welfare function can also be written as $W(t^1, \dots, t^H)$ where $t^h(u) \equiv m(p^0, u^h) = E^h + y^{0h}$. The change from (p^0, y^{0h}) to (p^1, y^{1h}) induces a change in social welfare amounting to $\Delta W = \sum W_h t^h = \sum W_h E^h$, where W_h is the derivative of $W(\cdot)$ with respect to its h th argument. Therefore, the magnitude of the individual E^h 's matters.¹

In applied studies, welfare evaluations are often based on the Marshallian measure of consumer's surplus involving areas under ordinary demand functions, $S = \Delta y + S^P$, where

$$S^P \equiv \int_{p^1}^{p^0} \bar{z}^h(p, y^0) dp_i. \quad (3)$$

There has been a long debate on the adequacy of S^P as a welfare measure, focusing on three main issues: (1) whether the line integral in (3) is path independent; (2) whether S^P correctly indicates the direction of welfare change--i.e., whether $\text{sign}(S^P) = \text{sign}(C^P)$; (3) whether S^P correctly measures the magnitude of welfare change--i.e., whether $S^P = C^P$.² The resolution of these issues involves three different assumptions about the nature of consumer preferences, which are described in Table I. It can be shown that assumptions (Ia), (b), and (c) are equivalent. In the case of assumptions

Table I. Assumptions Employed in the Welfare Analysis
of Price Changes

Assumption I.

- (a) The ratio (v_i/v_j) is independent of y for all $i, j \in J$.
- (b) $v(p, y) = T[\psi(p_J, p_{\bar{J}}), p_{\bar{J}}, y]$.
- (c) The income elasticities of demand for all the goods in J are the same.

Assumption II

- (a) $\lambda(p, y)$ is independent of p_J .
- (b) $v(p, y) = \psi(p_J, p_{\bar{J}}) + \phi(p_{\bar{J}}, y)$.
- (c) The income elasticities of demand for all the goods in J are the same, and are independent of p_J .

Assumption III

- (a) $\lambda(p, y)$ is independent of (p_J, y) .
- (b) $v(p, y) = \psi(p_J, p_{\bar{J}}) + y \cdot \phi(p_{\bar{J}})$.
- (c) The income elasticities of demand for all the goods in J are zero.

(II) and (III), (a) and (b) are equivalent, and each implies (c); the converse is not true, however, if \bar{J} contains more than one element. The status of S^P as a welfare measure can be summarized as follows -- it will be shown in the next section that analogous results hold for the welfare analysis of quality changes:

PROPOSITION 1: If assumption (IIIc) holds, then $S^P = C^P$, and S^P is path independent.

PROPOSITION 2: If assumption (IIa) holds, then $\text{sign}(S^P) = \text{sign}(C^P)$, and S^P is path independent.

PROPOSITION 3: If assumption (I) holds, then S^P is path independent, but it does not follow either that $S^P = C^P$ or that $\text{sign}(S^P) = \text{sign}(C^P)$.

It should also be noted that, when assumption (II) holds, C^P can in principle be constructed directly from the observed demand functions. In this case, by Roy's lemma, the ordinary demand functions for the goods in J must have the special structure

$$h^i(p, y) = -\psi_i(p_J, p_{\bar{J}}) / \phi_y(p_{\bar{J}}, y) \quad i \in J \quad (4)$$

If one recognizes the special structure of these ordinary demand functions, by integration one can recover the functions $\Psi(p_J, p_{\bar{J}})$ and $\phi(p_{\bar{J}}, y)$, and one can then calculate C^P from the formula

$$\psi(p_J^1, p_{\bar{J}}) + \phi(p_{\bar{J}}, y^0 - C^P) = \psi(p_J^0, p_{\bar{J}}) + \phi(p_{\bar{J}}, y^0). \quad (5)$$

If assumptions (I) or (II) do not apply, there are two ways to perform empirical welfare evaluations. One is to apply Willig's (1976) well-known approximation analysis, which uses S^P to bound the true value of C^P . The other derives from price index theory. For the change from (p^0, y^0) to (p^1, y^1) , the true cost-of-living index is defined for some reference utility level u as $\pi(u) \equiv m(p^1, u) / m(p^0, u)$. Two natural indices are $\pi_C \equiv \pi(u^0)$, which involves a welfare comparison like that underlying the

compensating variation measure C , and $\pi_E \equiv \pi(u^1)$, which involves a welfare comparison like that underlying E . When used to deflate money income, these indices provide a correct indication of the direction of welfare change since

$$\text{sign}(\Delta u) = \text{sign} \left(\frac{y^1}{\pi_C} - y^0 \right) = \text{sign} \left(\frac{y^1}{\pi_E} - y^0 \right). \quad (6)$$

The magnitude of the change in real income has the same normative significance as that of C or E since, from the definitions,

$$C = y^1 - y^0 \pi_C \quad \text{and} \quad E = \frac{y^1}{\pi_E} - y^0. \quad (7)$$

Thus, welfare evaluations can be based on the compensating or equivalent variations and on the true cost-of-living indices with equal justification: all the information contained in the one concept is also encoded in the other.⁴ As is well known, without imposing any restrictions on the utility function, one can always place some bounds on the true welfare measures. This is usually done in the context of the cost-of-living indices via the Laspeyres and Paasche price indices, $L \equiv \sum p^1 x^0 / \sum p^0 x^0$ and $P \equiv \sum p^1 x^1 / \sum p^0 x^1$, where $x^0 = h(p^0, y^0)$ and $x^1 = h(p^1, y^1)$. The bounds are $\underline{\lambda} \leq \pi_C \leq L$, where $\underline{\lambda} = \min_i (p_i^1 / p_i^0)$ and $P \leq \pi_E \leq \bar{\lambda}$, where $\bar{\lambda} = \max_i (p_i^1 / p_i^0)$.⁵ In terms of the compensating and equivalent variations, the bounds are $\sum (x^1 - x^0) p^1 \leq C \leq y^1 - y^0 \underline{\lambda}$ and $(y^1 / \bar{\lambda} - y^0) \leq E \leq \sum (x^1 - x^0) p^0$. The question of whether similar bounds exist for hedonic price indices will be examined in Section III.

II. CONSUMER'S SURPLUS MEASURES FOR QUALITY CHANGES

(a) Introduction

The subject of this section is the utility maximization model,

$$\max_{x \geq 0} u(x_1, \dots, x_N, b_1, \dots, b_T) \quad \text{subject to} \quad \sum p_i x_i = y. \quad (8)$$

The utility function, $u(x, b)$, has the standard properties of a utility function with respect to x , but it also contains a vector of parameters b which affects the consumer's well-being but is not the object of his choice and does not explicitly enter his budget constraint.⁶ Several interpretations are possible. Following Mäler (1974) and Bradford and Hildebrandt (1977), one can interpret the components of b as the levels of supply of various public goods. Following Fisher and Shell (1967) and Willig (1978), one can interpret them as indicators of the quality of various goods. The latter interpretation can be elaborated as follows. There are N commodities; and associated with each commodity is a particular price, p_i , and a particular bundle of K attributes. Thus, $T = NK$ and $b = \{b_{ik}\}$, where b_{ik} is the amount of the k th attribute associated with a unit of commodity i .⁷ In this case the utility model (8) is a generalization of Lancaster's (1966) well-known model where the utility function takes the special form, $u(x, b) = u(\sum_i x_i b_{i1}, \dots, \sum_i x_i b_{iK})$. Finally, the utility model (8) can be derived from a "household production" model of the form:

$\max_{x \geq 0} u(z_1, \dots, z_Q)$ subject to $z_q = z_q(x, b)$ and $\sum p_i x_i = y$. In this case the components of b are parameters of the household production functions, $z_q(\cdot)$; a change in b could represent, for example, a change in the technology of household production. For convenience, I shall adopt the second interpretation and refer to the components of b as measures of commodity quality.

Following Mäler, Bradford and Hildebrandt, and Willig--but not Lancaster--

I assume that the solution of (8) is an interior one.⁸ This yields a set of

ordinary demand functions, $h^i(p, b, y)$, the Lagrangean multiplier, $\lambda(p, b, y)$, and the indirect utility function, $v(p, b, y) \equiv u[h(p, b, y), b]$, all of which are functions of the full set of quality variables, b , as well as of prices and income. The dual problem is $\min \sum p_i x_i$ subject to $u(x, b) = u$. Again, assume an interior solution. This yields a set of compensated demand functions, $g^i(p, b, u)$; the Lagrangean multiplier, $\mu(p, b, u)$; and the expenditure function, $m(p, b, u) \equiv \sum p_i g^i(p, b, u)$. Some properties of these functions are described in the Appendix.

Within this framework, one can analyze the welfare effects of changes in the components of b as well as in prices and income. Suppose there is a change from (p^0, b^0, y^0) to (p^1, b^1, y^1) . Accordingly, the consumer's welfare changes from $u^0 \equiv v(p^0, b^0, y^0)$ to $u^1 \equiv v(p^1, b^1, y^1)$; let $\Delta u = u^1 - u^0$. In this section I will focus on the consumer's surplus approach to measuring the change in welfare; in Section III I will take up the price index approach. By analogy with (1), I define the compensating variation measure of the welfare change, \bar{C} , by the implicit equation $v(p^1, b^1, y^1 - \bar{C}) = v(p^0, b^0, y^0)$ or, equivalently, by

$$\begin{aligned} \bar{C} &= y^1 - m(p^1, b^1, u^0) \\ &= \Delta y + \bar{C}^P + \bar{C}^b \end{aligned} \tag{9}$$

where

$$\begin{aligned} \bar{C}^P &\equiv m(p^0, b^1, u^0) - m(p^1, b^1, u^0) \\ &= \int_{p^1}^{p^0} \sum g^i(p, b^1, u^0) dp_i \end{aligned}$$

and

$$\bar{C}^b \equiv m(p^0, b^0, u^0) - m(p^0, b^1, u^0).$$

Thus, \bar{C} is the sum of a compensation for the income change, Δy ; a compensation for the price change, \bar{C}^P , which is essentially the same as C^P in the previous section; and a compensation for the quality change, \bar{C}^b , which is new. Similarly, I define the equivalent variation, \bar{E} , by $v(p^1, b^1, y^1) = v(p^0, b^0, y^0 + \bar{E})$ or by

$$\begin{aligned}\bar{E} &= m(p^0, b^0, u^1) - y^0 \\ &= \Delta y + \bar{E}^P + \bar{E}^b\end{aligned}\tag{10}$$

where

$$\begin{aligned}\bar{E}^P &\equiv m(p^0, b^0, u^1) - m(p^1, b^0, u^1) \\ &= \int_{p^1}^{p^0} \sum g^i(p, b^0, u^1) dp_i\end{aligned}$$

and

$$\bar{E}^b \equiv m(p^1, b^0, u^1) - m(p^1, b^1, u^1).$$

The quantities \bar{C} and \bar{E} are welfare measures in exactly the same way as the quantities C and E discussed in the previous section. First, their sign indicates the direction in which the consumer's welfare changes, since

$$\text{sign}(\Delta u) = \text{sign}(\bar{C}) = \text{sign}(\bar{E}).$$

Second, one can make exactly the same arguments for the normative significance of the magnitudes of \bar{C} and \bar{E} based on the Kaldor-Hicks and Scitovsky welfare criteria or the social welfare function concept.

(b) Welfare Analysis and Areas Under Demand Curves

I now take up the question of whether the welfare measures \bar{C} and \bar{E} can be approximated by areas under ordinary demand curves; for convenience, I focus on \bar{C} . Clearly, \bar{C}^P can be approximated by the Marshallian quantity \bar{S}^P , defined as in (3) but using the ordinary demand functions $h^i(p, b^1, y^0)$; Propositions 1, 2, and 3 carry over directly to \bar{S}^P . Therefore the question is whether \bar{C}^b can be

approximated by a sum of areas under ordinary demand curves, and whether analogues of Propositions 1, 2, and 3 exist for this approximation.

The first step is to relate \bar{C}^b to areas under compensated demand curves; Mäler (1974) was the first to show how this could be done. For the sake of generality, let R be the index set for the components of b which change and \bar{R} its complement--i.e., $R = \{r \mid b_r^1 \neq b_r^0\}$ and $\bar{R} = \{r \mid b_r^1 = b_r^0\}$. The vector b is partitioned

accordingly: $b = (b_R, b_{\bar{R}})$. Let $B_R = \left\{ b_r \mid b_r = \alpha b_r^0 + (1 - \alpha) b_r^1, \quad 0 \leq \alpha \leq 1 \right\}$.

Two assumptions are required. The first is that there exists at least one and possibly more commodities with the property that, if these commodities are not consumed, the marginal utility from a change in the components of b_R is zero.⁹ Let I be the index set of the commodities with this property and \bar{I} its complement. Partition the vector x accordingly: $x = (x_I, x_{\bar{I}})$. When $x_i = 0$ for all $i \in I$, the consumption vector is written $(0, x_{\bar{I}})$. The assumption is that:

(A) There exists a nonempty index set I such that

$$\frac{\partial u(0, x_{\bar{I}}, b_R, b_{\bar{R}}^0)}{\partial b_r} = 0 \quad \text{all } r \in R, b_r \in B_R.$$

The second assumption is that:¹⁰

(B) The commodities in I are nonessential.

With these assumptions, Mäler proves the following result:

LEMMA 1: If assumptions (A) and (B) hold, there exists a finite price vector, p_I^* , such that $g^i(p_I^*, p_{\bar{I}}^0, b_R, b_{\bar{R}}^0, u^0) = 0$ for all $i \in I$ and $b_R \in B_R$, and

$$\bar{C}^b = \int_{p_I^0}^{p_I^*} \sum_{i \in I} [g^i(p_I, p_{\bar{I}}^0, b^0, u^0) - g^i(p_I, p_{\bar{I}}^0, b^0, u^0)] dp.$$

A natural approximation to \bar{C}^b involving areas under ordinary demand curves is the quantity \bar{S}^b , defined by

$$\bar{S}^b \equiv \int_{p_I^0}^{\tilde{p}_I} \sum_{i \in I} [h^i(p_I, p_{\bar{I}}^0, b^1, y^0) - h^i(p_I, p_{\bar{I}}^0, b^0, y^0)] dp_i, \quad (11)$$

where \tilde{p}_I is such that $h^i(\tilde{p}_I, p_{\bar{I}}^0, b_R, b_{\bar{R}}^0, y^0) = 0$ for all $i \in I$ and $b_R \in B_R$ (assumption (B) ensures that a finite \tilde{p}_I exists). This appears to have been first suggested by Stevens (1966); it has since been widely used in empirical studies of the welfare effects of changes in environmental quality. One can ask whether the line integral in (11) is path independent, and whether it is true either that $\text{sign}(\bar{S}^b) = \text{sign}(\bar{C}^b)$, or that $\bar{S}^b = \bar{C}^b$. In order to answer these questions it is necessary to invoke the three sets of assumptions about consumer preferences which are listed in Table II and which are natural analogues of the assumptions in Table I. It can be shown that assumptions (I') (a), (b) and (c) are equivalent. In the case of assumptions (II') and (III'), (a) and (b) are equivalent, and each implies (c); the converse is not true, however, if \bar{I} contains more than one element. The status of \bar{S}^b as a welfare measure is described by the following three propositions:

PROPOSITION 4: If assumptions (A), (B) and (III'c) hold, then $\bar{S}^b = \bar{C}^b$, and \bar{S}^b is path independent.

PROPOSITION 5: If assumptions (A), (B), and (II'a) hold, then $\text{sign}(\bar{S}^b) = \text{sign}(\bar{C}^b)$, and \bar{S}^b is path independent.

PROPOSITION 6: If assumptions (A), (B), and (I') hold, then \bar{S}^b is path independent, but it does not follow either that $\bar{S}^b = \bar{C}^b$ or that $\text{sign}(\bar{S}^b) = \text{sign}(\bar{C}^b)$.

The proof of Proposition 4 follows directly from Lemma 1. The proof of Proposition 5 is presented in the Appendix. Let $\Delta\tilde{u} \equiv v(p^0, b^1, y^0) - v(p^0, b^0, y^0)$ and note that $\text{sign}(\bar{C}^b) = \text{sign}(\Delta\tilde{u})$. The proof consists of showing that, under assumption (II'a), $\bar{S}^b = \Delta\tilde{u}/\lambda$. Actually, when assumption (II') holds, \bar{C}^b can in principle be constructed directly from the observed demand functions, since their functional structure is similar to that in (4). By integration one can recover

Table II. Assumptions Employed in the Welfare Analysis
of Quality Changes

Assumption (I')

- (a) The ratio (v_r/v_i) is independent of y for all $i \in I$ and $r \in R$.
- (b) $v(p, b, y) = T[\psi(p, b), p_{\bar{I}}, b_{\bar{R}}, y]$.
- (c) The income elasticities of demand for all the goods in I are the same.

Assumption (II')

- (a) $\lambda(p, b, y)$ is independent of (p_I, b_R) .
- (b) $v(p, b, y) = \psi(p, b) + \phi(p_{\bar{I}}, b_{\bar{R}}, y)$.
- (c) The income elasticities of demand for all the goods in I are the same, and are independent of (p_I, b_R) .

Assumption (III')

- (a) $\lambda(p, b, y)$ is independent of (p_I, b_R, y) .
- (b) $v(p, b, y) = \psi(p, b) + y \cdot \phi(p_{\bar{I}}, b_{\bar{R}})$.
- (c) The income elasticities of demand for all the goods in I are zero.

the functions $\psi(p, b)$ and $\phi(p_I, b_R, y)$, and one can then calculate \bar{C}^b from a formula analogous to (5). The path independence part of Proposition 6 is straightforward, since assumption (I') implies that for $i, j \in I$

$$v_{ry} = \frac{v_r}{v_i} \quad v_{iy} = \frac{v_r}{v_j} v_{jy}.$$

Hence $v_j v_{yi} = v_i v_{yj}$ which, in turn, implies that $h_j^i(\cdot) = h_i^j(\cdot)$. The rest of Proposition 6 is somewhat less obvious. Bradford and Hildebrandt (1977) and Willig (1978) prove the following result:¹¹

LEMMA 2: If, and only if, assumptions (A), (B) and (I') hold, then

$$-\frac{v_r(p^0, b^1, y^0)}{v_y(p^0, b^1, y^0)} = \int_{\tilde{p}}^{p^0} \sum_{i \in I} h_r^i(p, b^1, y^0) dp_i \quad (12a)$$

and

$$-\frac{v_r(p^0, b^1, y^0)}{v_j(p^0, b^1, y^0)} = \frac{1}{h^j(p^0, b^1, y^0)} \int_{p^0}^{\tilde{p}} \sum_{i \in I} h_r^i(p, b^1, y^0) dp_i. \quad (12b)$$

However, this lemma has no practical implications for nonmarginal quality changes. It does not imply that, under the stated conditions, $\text{sign}(\bar{S}^b) = \text{sign}(\Delta \tilde{u})$, since

$$\begin{aligned} \bar{S}^b &= \int_{p^0}^{\tilde{p}} \sum_{i \in I} [h^i(p, b^1, y^0) - h^i(p, b^0, y^0)] dp_i \\ &= \int_{p^0}^{\tilde{p}} \sum_{i \in I} \left[\int_{b^0}^{b^1} \sum_{r \in R} h_r^i(p, b, y^0) db_r \right] dp_i \\ &= \int_{b^0}^{b^1} \sum_{r \in R} \left[\int_{p^0}^{\tilde{p}} \sum_{i \in I} h_r^i(p, b, y^0) dp_i \right] db_r \\ &= \int_{b^0}^{b^1} \sum_{r \in R} \left[\frac{v_r(p^0, b, y^0)}{v_y(p^0, b, y^0)} \right] db_r \end{aligned}$$

unless $v_y(\cdot)$ is also independent of b_R , which then implies assumption (II'a).

Does the lemma imply that $\bar{S}^b = \bar{C}^b$? No, because¹²

$$\begin{aligned}\bar{S}^b &= \int_{b^1}^{b^0} \sum_{r \in R} - \frac{v_r(p^0, b, y^0)}{v_y(p^0, b, y^0)} db_r \\ &= \int_{b^1}^{b^0} \sum_{r \in R} m_r [p^0, b, v(p^0, b, y^0)] db_r \\ &\neq m(p^0, b^0, u^0) - m(p^0, b^1, u^0) \equiv \bar{C}^b.\end{aligned}$$

This completes the proof of Proposition 6.

What, then, is the significance of Lemma 2 for applied welfare analysis?

Willig (1978, p. 228) describes it as "characterizing conditions under which the marginal value of product quality is the quality derivative of Marshallian consumer's surplus." This may be questioned. The right-hand side of (12a) is clearly the quality derivative of Marshallian consumer's surplus, $-\partial \bar{S}^b / \partial b_r^1$. Is the left-hand side of (12a) the marginal value of product quality? Not if one means by this $-\partial \bar{C}^b / \partial b_r^1$ since

$$\begin{aligned}- \frac{\partial \bar{C}^b}{\partial b_r^1} &\equiv \frac{\partial [m(p^0, b^1, u^0) - m(p^0, b^0, u^0)]}{\partial b_r^1} \\ &= m_r(p^0, b^1, u^0) \\ &= - \frac{u_r [g(p^0, b^1, u^0), b^1]}{u(p^0, b^1, u^0)} \\ &= - \frac{u_r [h(p^0, b^1, m(p^0, b^1, u^0), b^1)]}{\lambda [p^0, b^1, m(p^0, b^1, u^0)]}\end{aligned}\tag{13a}$$

while

$$- \frac{v_r(p^0, b^1, y^0)}{v_y(p^0, b^1, y^0)} = - \frac{u_r [h(p^0, b^1, y^0), b^1]}{\lambda(p^0, b^1, y^0)}.\tag{13b}$$

The right-hand sides of (13a) and (13b) coincide only at the point $b^1 = b^0$.

Despite these negative conclusions, it turns out that assumption (I') does have a practical implication for applied welfare analysis--it permits us to use \bar{S}^b as a bound on \bar{C}^b . Define $\tilde{u} = v(p^0, b^1, y^0)$ and $\tilde{E}^b = m(p^0, b^0, \tilde{u}) - m(p^0, b^1, \tilde{u})$ and note that $\text{sign}(\bar{C}^b) = \text{sign}(\tilde{E}^b)$. The following result, which is a simple extension of one proved by Mäler (1974, pp. 130 and 131), is the analogue of the well-known proposition for the standard utility model of Section I that, when all prices change in the same direction and all goods whose prices change are normal, $C^P < S^P < E^P$:¹³

LEMMA 3: If assumptions (A), (B), and (I') hold, and if all components of b_R change in the same direction,

(a) if $m_{ru} < 0$ for all $r \in R$, then $\bar{C}^b < \bar{S}^b < \tilde{E}^b$

(b) if $m_{ru} > 0$ for all $r \in R$, then $\bar{C}^b > \bar{S}^b > \tilde{E}^b$.

The practical problem in applying this lemma is to verify the sign of m_{ru} . However, it turns out that, with assumption (I'), this sign can be deduced from that of the income elasticities of demand:

LEMMA 4: For any b_r , if there exists an index set, I_r , such that (v_i/v_r) is independent of y for all $i \in I_r$, then $\text{sign}\{h_y^i\} = -\text{sign}\{m_{ru}\}$ for all $i \in I_r$. The lemma is proved in the Appendix; when combined with Lemma 3, it leads to the following:¹⁴

PROPOSITION 7: If assumptions (A), (B), and (I') hold, and if all the components of b_R change in the same direction,

(a) if all the goods in I are normal, then $\bar{C}^b < \bar{S}^b$

(b) if all the goods in I are inferior, then $\bar{C}^b > \bar{S}^b$.

In the next section I discuss an alternative approach to bounding the magnitude of welfare changes based not on consumer's surplus measures but rather on hedonic price indices.

III. HEDONIC PRICE INDEX THEORY

(a) Introduction

It was shown in Section I that, for price changes, the consumer's surplus and price index approaches both convey the same information about the direction and magnitude of welfare changes. The same holds true for quality changes. The setup is as in Section II and is based on the utility model (8). Prices, qualities, and income change from (p^0, b^0, y^0) to (p^1, b^1, y^1) , and the consumer's utility changes from $u^0 \equiv v(p^0, b^0, y^0)$ to $u^1 \equiv v(p^1, b^1, y^1)$. I define the true cost-of-living index for some reference utility level, u , to be the ratio $\bar{\pi}(u) = m(p^1, b^1, u)/m(p^0, b^0, u)$. Two natural indices are $\bar{\pi}_C \equiv \pi(u^0)$ and $\bar{\pi}_E \equiv \pi(u^1)$ which correspond, respectively, to \bar{C} and \bar{E} as defined in (9) and (10). If one substitutes $\bar{\pi}_C$ and $\bar{\pi}_E$ for π_C and π_E , and \bar{C} and \bar{E} for C and E , equations (6) and (7) still apply.

It was also noted in Section I that one can always obtain practical bounds on π_C and π_E without making any assumptions about the form of the utility function. In this section, I investigate whether the same conclusion holds for $\bar{\pi}_C$ and $\bar{\pi}_E$. These indices can be decomposed into a pure price component and a pure quality component: $\bar{\pi}_C = \bar{\pi}_C^p \cdot \bar{\pi}_C^b$, and $\bar{\pi}_E = \bar{\pi}_E^p \cdot \bar{\pi}_E^b$, where $\bar{\pi}_C^p \equiv m(p^1, b^0, u^1)/y^0$, $\bar{\pi}_E^p \equiv y^1/m(p^0, b^1, u^1)$,

$$\bar{\pi}_C^b \equiv \frac{m(p^1, b^1, u^0)}{m(p^1, b^0, u^0)}, \text{ and } \bar{\pi}_E^b \equiv \frac{m(p^0, b^1, u^1)}{m(p^0, b^0, u^1)}.$$

In effect, $\bar{\pi}_C^p$ and $\bar{\pi}_E^p$ are the price indices one might calculate ignoring the quality change, and $\bar{\pi}_C^b$ and $\bar{\pi}_E^b$ are the respective correction factors which adjust for the quality change. The indices $\bar{\pi}_C^p$ and $\bar{\pi}_E^p$ have the same properties as the indices π_C and π_E . In particular, by adapting Pollak's (1971) proof, one obtains:

LEMMA 5: For any (b, y) and for any two price vectors, p^* and p^{**} ,

$$(a) \quad \underline{\lambda} \leq \frac{\sum [p^{**}, b, v(p^*, b, y)]}{\sum [p^*, b, v(p^*, b, y)]} \leq \frac{\sum p^{**} h(p^*, b, y)}{\sum p^* h(p^*, b, y)} \leq \bar{\lambda},$$

where $\underline{\lambda} = \min_i (p_i^{**}/p_i^*)$ and $\bar{\lambda} = \max_i (p_i^{**}/p_i^*)$; and

$$(b) \quad \underline{\lambda} \leq \frac{\sum p^* h(p^*, b, y)}{\sum p^{**} h(p^*, b, y)} \leq \frac{\sum [p^*, b, v(p^*, b, y)]}{\sum [p^{**}, b, v(p^*, b, y)]} \leq \bar{\lambda},$$

where $\underline{\lambda} = \min_i (p_i^*/p_i^{**})$ and $\bar{\lambda} = \max_i (p_i^*/p_i^{**})$.

It follows that $\bar{\pi}_C^p \leq \bar{L}$ and $\bar{\pi}_E^p \geq \bar{P}$, where \bar{L} and \bar{P} are the Laspeyres and Paasche indices

$$\bar{L} \equiv \frac{\sum p^1 \cdot h(p^0, b^0, y^0)}{\sum p^0 \cdot h(p^0, b^0, y^0)} \quad \text{and} \quad \bar{P} \equiv \frac{\sum p^1 \cdot h(p^1, b^1, y^1)}{\sum p^0 \cdot h(p^1, b^1, y^1)}$$

What can be said about the indices $\bar{\pi}_C^b$ and $\bar{\pi}_E^b$? Clearly, if $b^1 \geq b^0$, then $\bar{\pi}_C^b \leq 1$ and $\bar{\pi}_E^b \leq 1$; conversely, if $b^1 \leq b^0$, then $\bar{\pi}_C^b \geq 1$ and $\bar{\pi}_E^b \geq 1$. But there does not appear to be any result analogous to Lemma 5 which provides general bounds on $\bar{\pi}_C^b$ and $\bar{\pi}_E^b$ without requiring any restrictions on the form of the utility function.

Instead of trying to obtain bounds on $\bar{\pi}_C^b$ and $\bar{\pi}_E^b$, the literature on hedonic price indices has pursued a different course--it has sought some way to adjust prices to allow for the change in quality, so that if one uses the adjusted prices instead of p^0 and p^1 and computes a price index similar to $\bar{\pi}_C^p$ or π_C , say, one obtains a correct measure of the welfare change associated with the change from (p^0, b^0) to (p^1, b^1) . This idea was first developed systematically by Adelman and Griliches (1961); their approach is analyzed in section (c). A variant was developed by Willig (1978); this is analyzed in section (b). In both cases I examine whether the hedonic price approach provides any practical benefits as compared to the consumer's surplus approach described in Section II.

(b) Willig's Hedonic Price Index Approach

Consider the price vector, p^{**} , defined by

$$v(p^{**}, b^0, y^1) = v(p^1, b^1, y^1). \quad (14)$$

This is a candidate for a quality-adjusted price vector since, if quality did not change but prices were at this level, the consumer would be just as well off as he is with (p^1, b^1) . Suppose one goes ahead and conducts the welfare evaluation as though prices had changed from p^0 to p^{**} , with no change in quality. One could calculate the equivalent variation for this change, i.e., the quantity, \bar{E}^{**} , such that $v(p^{**}, b^0, y^1) = v(p^0, b^0, y^0 + \bar{E}^{**})$; and one could calculate the corresponding true cost-of-living index $\bar{\pi}_E^{**} = m(p^{**}, b^0, u^1)/m(p^0, b^0, u^1)$. Do these really give a true indication of the change in welfare? Yes, because, by inspection, $\bar{E}^{**} = \bar{E}$ and $\bar{\pi}_E^{**} = \bar{\pi}_E$. Therefore, p^{**} is a valid hedonically adjusted price vector. A second candidate is the price vector, p^* , defined by

$$v(p^*, b^1, y^0) = v(p^0, b^0, y^0). \quad (15)$$

If quality were at the new level but prices were p^* , the consumer would be just as well off as he was originally with (p^0, b^0) . Suppose one pretends that prices had originally been p^* and had changed to p^1 , while quality had always been b^1 . Let \bar{C}^* be the compensating variation for this change--i.e., \bar{C}^* satisfies $v(p^1, b^1, y^1 - \bar{C}^*) = v(p^*, b^1, y^0)$. Let $\bar{\pi}_C^*$ be the corresponding true cost-of-living index, $\bar{\pi}_C^* = m(p^1, b^1, u^0)/m(p^*, b^1, u^0)$. By inspection, $\bar{C}^* = \bar{C}$ and $\bar{\pi}_C^* = \bar{\pi}_C$. Thus, p^* is also a valid hedonically adjusted price vector.

Whether or not these two hedonic price vectors are useful in practice depends on (i) the ease with which p^* and p^{**} can be calculated from observed data and (ii) whether or not one can obtain simple bounds on $\bar{\pi}_C^*$ or $\bar{\pi}_E^{**}$.

Ignoring the first question for the moment, I can answer the second by applying Lemma 5 to these cost-of-living indices and, hence, to $\bar{\pi}_C^*$ and $\bar{\pi}_E^{**}$. Define the

Laspeyres- and Paasche-type indices:

$$L^* \equiv \frac{\sum p^1 \cdot h(p^*, b^1, y^0)}{\sum p^* \cdot h(p^*, b^1, y^0)} \quad \text{and} \quad P^{**} \equiv \frac{\sum p^{**} \cdot h(p^{**}, b^0, y^1)}{\sum p^0 \cdot h(p^{**}, b^0, y^1)}.$$

PROPOSITION 8: Given p^* and p^{**} ,

$$(a) \quad \underline{\lambda} \leq \bar{\pi}_C \leq L^* \leq \bar{\lambda},$$

where

$$\underline{\lambda} = \min_i \left(p_i^1 / p_i^* \right) \quad \text{and} \quad \bar{\lambda} = \max_i \left(p_i^1 / p_i^* \right).$$

$$(b) \quad \underline{\lambda} \leq P^{**} \leq \bar{\pi}_E \leq \underline{\lambda},$$

where

$$\underline{\lambda} = \min_i \left(p_i^{**} / p_i^0 \right) \quad \text{and} \quad \bar{\lambda} = \max_i \left(p_i^{**} / p_i^0 \right).$$

One can use this result to obtain simple tests for the welfare effects of the change from (p^0, b^0, y^0) to (p^1, b^1, y^1) . For example, if $(y^1/L^*) > y^0$, then $\Delta u > 0$; or, if $(y^1/P^{**}) < y^0$, then $\Delta u < 0$.

So far, I have imposed no restrictions on the form of the utility function, $u(x, b)$. Willig (1978) examines the issues which I have just discussed under the assumptions of a particular restriction on the utility function, namely, assumption (I') in Table II. He argues that this assumption makes it easier to calculate the hedonically adjusted prices, p^* and p^{**} , and that it leads to a set of practical bounds--different from those in Proposition 8--which simplify welfare evaluations. I will examine both arguments, starting with the latter.

The bounds which Willig proposes are

$$L' \equiv \frac{\sum p^* h(p^0, b^0, y^0)}{\sum p^0 h(p^0, b^0, y^0)} \quad \text{and} \quad P' \equiv \frac{\sum p^1 h(p^1, b^1, y^1)}{\sum p^* h(p^1, b^1, y^1)}.$$

His proposition is that, if assumption (I') holds, then $\bar{\pi}_C \leq P'$ and $\bar{\pi}_E \geq L'$.

This proposition is actually incorrect. Willig first proves (his Lemma 1)

that $\bar{\pi}_C \leq L^{**}$ and $\bar{\pi}_E \geq P^*$ where

$$L^{**} = \frac{\sum p^{**} h(p^0, b^0, y^0)}{\sum p^0 h(p^0, b^0, y^0)} \quad \text{and} \quad P^* = \frac{\sum p^1 h(p^1, b^1, y^1)}{\sum p^0 h(p^1, b^1, y^1)}$$

with p^* and p^{**} defined by

$$v[p^*, b^1, m(p^0, b^0, u^1)] = v[p^0, b^0, m(p^0, b^0, u^1)] \quad (16)$$

and

$$v[p^{**}, b^0, m(p^1, b^1, u^0)] = v[p^1, b^1, m(p^1, b^1, u^0)]. \quad (17)$$

He points out that these bounds are unlikely to be of practical importance since, if one could calculate them, there would be sufficient information to calculate $\bar{\pi}_C$ and $\bar{\pi}_E$ directly. However, he then argues (his Theorem 4) that, if assumption (I') holds, $p^* = p^0$ and $p^{**} = p^{**}$; hence, $L' = L^{**}$ and $P' = P^*$. But this result does not follow from assumption (I'); one actually needs to invoke the more restrictive assumption (II'a) to ensure this result.¹⁵ The correct theorem is that, if (II'a) holds, then $\bar{\pi}_C \leq P'$ and $\bar{\pi}_E \geq L'$.¹⁶ But, as noted above, if (II'a) holds, \bar{C} and \bar{E} can be constructed directly from the observed ordinary demand functions and one does not need bounds on $\bar{\pi}_C$ or $\bar{\pi}_E$. If (II'a) or (I') do not hold, the more general bounds in Proposition 8 are still available.

I turn now to the problem of calculating p^* and p^{**} . If these cannot be calculated simply from observed data, the whole purpose of the hedonic price methodology is lost, and neither Proposition 8 nor Willig's theorem has any practical value. It is clear that some restrictions on the form of the utility function are required. Consider what happens when no restrictions are imposed. Since the problems of calculating p^* and p^{**} are the same, I focus on p^* . One needs to find a set of functions, $p_j(b)$, such that $v[p(b), b, y^0] = v[p^0, b^0, y^0]$; once these functions are found, $p_j^* = p_j(b^1)$. These functions satisfy

$$\frac{dp_j(b)}{db_r} = -\frac{v_r[p(b), b, y^0]}{v_j[p(b), b, y^0]},$$

which may be regarded as a system of partial differential equations with boundary conditions, $p_j(b^0) = p_j^0$. However, if one had enough information to set up and solve these differential equations, i.e., if one knew the function $v(p, b, y)$, he could calculate $\bar{\pi}_C$ and $\bar{\pi}_E$ directly; there would be no need for p^* or p^{**} .

Suppose that properties (A), (B), and (I') hold, Willig's result-- Lemma 2 above--implies that

$$\frac{dp_j(b)}{db_r} = \frac{1}{h^j [p(b), b, y^0]} \int_{p(b)}^{\infty} \sum_i h_r^i(p, b, y^0) dp_i.$$

In this case, at least, one can set up the system of differential equations from observable data, namely, the ordinary demand functions; but it will not necessarily be easy to solve them. However, Willig points out that, for some utility functions which are special cases of (I'), a simpler way to calculate p^* and p^{**} can be found based on information available from the ordinary demand functions. He deals with the special case in which $I = \{1\}$ and $\bar{R} = \phi$. He points out, for example, that, if the ordinary demand function for good 1 takes the form

$$h^1(p, b, y) = \bar{h}^1 [p_1 \psi(b), \bar{p}, y] \psi(b), \quad (18a)$$

where $\bar{p} \equiv (p_2, \dots, p_N)$, the implied indirect utility function has the form

$$v(p, b, y) = \bar{v} [p_1 \psi(b), \bar{p}, y], \quad (18b)$$

and, therefore, $p^* = (p_1^*, \bar{p}^0)$ and $p^{**} = (p_1^{**}, \bar{p}^1)$, where

$$p_1^* = \frac{\psi(b^0)}{\psi(b^1)} p_1^0 \quad \text{and} \quad p_1^{**} = \frac{\psi(b^1)}{\psi(b^0)} p_1^0. \quad (19)$$

Since the subfunction $\psi(b)$ can be recognized from the formula for the ordinary demand function, (18a), p^* or p^{**} can be calculated via (19). He obtains similar results for the ordinary demand function

$$h^1(p, b, y) = \bar{h}^1 [p_1 \psi(b, \bar{p}), \bar{p}, y] \psi(b, \bar{p}), \quad (20a)$$

which is derived from

$$v(p, b, y) = \bar{v}[p_1 \cdot \psi(b, \bar{p}), \bar{p}, y], \quad (20b)$$

and for the ordinary demand function

$$h^1(p, b, y) = \bar{h}^1[p_1 + \phi(b, \bar{p}), \bar{p}, y], \quad (21a)$$

which is derived from

$$v(p, b, y) = \bar{v}[p_1 + \phi(b, \bar{p}), \bar{p}, y]. \quad (21b)$$

In both cases the subfunctions $\psi(\cdot)$ or $\phi(\cdot)$ can be identified from the formula for the ordinary demand function, and p^* or p^{**} can be calculated from a formula analogous to (19).¹⁷

However, for the demand functions (18a), (20a), and (21a), one may be able to do better than calculating p^* or p^{**} and bounding $\bar{\pi}_C$ and $\bar{\pi}_E$; it may be possible to calculate \bar{C} or \bar{E} explicitly. The key to this is the fact that the indirect utility functions (18b), (20b), and (21b) are "translations" of a standard neoclassical indirect utility function, $\bar{v}(p_1, \bar{p}, y)$. Therefore, the observed ordinary demand function, $h^1(p, b, y)$, is a corresponding translation of the ordinary demand function associated with $\bar{v}(p, y)$, denoted by $\bar{h}^1(p_1, \bar{p}, y)$. If one can identify the formula for the compensated demand function associated with $\bar{v}(p, y)$, denoted by $\bar{g}^1(p_1, \bar{p}, u)$, he can deduce the formula for the compensated demand function associated with $v(p, b, y)$ and calculate \bar{C}^p and \bar{C}^b or \bar{E}^p and \bar{E}^b directly from this. For example, given $\bar{g}^1(p_1, \bar{p}, u)$, the compensated demand function corresponding to (18a) is

$$g^1(p, b, u) = \bar{g}^1[p_1 \cdot \psi(b), \bar{p}, u] \cdot \psi(b);$$

and the formula for \bar{C}^b turns out to be

$$\bar{C}^b = \int_{\pi^0}^{\pi^1} \bar{g}^1(\pi, \bar{p}, u^0) d\pi, \quad (22)$$

where $\pi^t = p_1^0 \psi(b^t)$, $t = 0, 1$. Similar results can be obtained for (20a)

and (21a).¹⁸

I have shown that, if one could calculate the price vectors p^* and p^{**} , this would provide simple bounds on the direction or magnitude of welfare change without requiring any restrictions on the form of the utility function. However, p^* and p^{**} cannot be calculated from observable demand data without restricting the utility function. Moreover, for the special types of utility function yielding demand functions from which p^* and p^{**} can be easily computed, it usually turns out that \bar{C} and \bar{E} can be constructed directly from the demand functions. I conclude, therefore, that the hedonic price methodology outlined in this section does not possess any significant advantage in practice over the consumer's surplus approach to evaluating the welfare effects of a quality change. In the next section, I examine an alternative hedonic price methodology proposed by Adelman and Griliches. Although their quality-adjusted prices can be calculated without imposing any restrictions on the utility function, I show that they do not lead to any useful bounds on the direction or magnitude of welfare change.

(c) The Adelman-Griliches Hedonic Price Index Approach

The first task in discussing the hedonic price adjustment procedure proposed by Adelman and Griliches (1961)--henceforth, AG--is to determine the utility model in terms of which it should be evaluated. AG describe this procedure by reference to Houthakker's (1951-52) utility model, which is different from (8). The central contribution of Houthakker's model is the notion of a hedonic price function, i.e., the notion that there exists "a 'reasonably well-fitting' relation between the price of different models [of a commodity] and the level of their various but not too numerous characteristics" (Griliches, 1971, p. 4). Houthakker suggested a linear hedonic price function, $p_i = \alpha_i + \gamma_i b_i$, where b_i is a (vector or scalar) measure of the quality of good i , α_i is the "pure quantity price" of a unit of the good, and γ_i is the "quality price"--i.e., the

cost per unit of quality associated with one unit of the good. Subsequent econometric estimates of hedonic price functions have tended to support nonlinear functions, such as $p_i = \exp(\alpha_i + \gamma_i b_i)$.¹⁹ More generally, I shall write $p_i = \psi_i(\alpha_i + \gamma_i b_i)$, $\psi_i' > 0$. The chief purpose of AG's hedonic price methodology is to evaluate the effects of "variations in the quality of products available in the market place"--i.e., changes in b , in the context of the utility model (8). It can be shown that this is not a meaningful concept in the context of Houthakker's utility model.²⁰ I would argue, therefore, that the appropriate theoretical setting in which to evaluate AG's hedonic price methodology is the utility model (8), modified by the inclusion of the hedonic price function concept. This is not incompatible with my previous formulation of (8), which merely required that some particular price and set of quality characteristics be associated with each good. If one adjoins the price functions $p_i = \psi_i(\alpha_i + \gamma_i b_i)$ to (8), remembering that the maximization is performed with respect to x but not b , all the results associated with the model carry over, except that one substitutes $\psi_i(\cdot)$ for p_i in the ordinary demand functions, the expenditure function, the indirect utility function, etc.

The essence of AG's hedonic price method is that one obtains an empirical estimate of the price functions $\psi(\cdot)$, e.g., by regression analysis, and uses this to form quality-adjusted prices, which are then combined in a price-index formula. The particular formula which AG employ involves a chaining procedure in which the weights are continuously updated as in a Divisia index. Since the chaining procedure is peripheral to the main issue of the validity of the quality-adjustment method, I ignore it for the moment. The situation is this: quantity and quality prices, the set of quality levels offered to the consumer, and income all change from $(\alpha^0, \gamma^0, b^0, y^0)$ to $(\alpha^1, \gamma^1, b^1, y^1)$. Thus, the actual prices paid change from $p^0 = \psi(\alpha^0 + \gamma^0 b^0)$ to $p^1 = \psi(\alpha^1 + \gamma^1 b^1)$.

Accordingly, the consumer's utility changes from $u^0 \equiv v [\psi (\alpha^0 + \gamma^0 b^0), b^0, y^0]$ to $u^1 \equiv v [\psi (\alpha^1 + \gamma^1 b^1), b^1, y^1]$. As in the previous section, the two true cost-of-living indices on which I focus are

$$\bar{\pi}_C = \frac{m [\psi (\alpha^1 + \gamma^1 b^1), b^1, u^0]}{m [\psi (\alpha^0 + \gamma^0 b^0), b^0, u^0]} \quad \text{and} \quad \bar{\pi}_E = \frac{m [\psi (\alpha^1 + \gamma^1 b^1), b^1, u^1]}{m [\psi (\alpha^0 + \gamma^0 b^0), b^0, u^1]}.$$

AG's starting point is the decomposition of the actual price change into two parts:

$$dp = dp' + \left(\frac{\partial \psi}{\partial b} \right)_0 db. \quad (23)$$

The first term on the right-hand side "is that price movement which would have occurred in the absence of quality variations," while "the second term represents the combined effect of those price movements which are due solely to change in quality" (AG, p. 539). Let $dp = p^1 - p^0$, $dp' = p'^1 - p'^0$ and $db = b^1 - b^0$ so that (23) becomes

$$(p^1 - p^0) = (p'^1 - p'^0) + \left(\frac{\partial \psi}{\partial b} \right)_0 (b^1 - b^0). \quad (24)$$

Suppose that from a regression analysis one knows (α^0, γ^0) and (α^1, γ^1) . Using this information, one can calculate two alternative quality-adjusted price vectors

$$p^* = \psi (\alpha^0 + \gamma^0 b^1) \quad \text{and} \quad p^{**} = \psi (\alpha^1 + \gamma^1 b^0). \quad (25)$$

With p^* , one can construct a decomposition of the actual price change which, I claim, is equivalent to (23) or (24):

$$(p^1 - p^0) = (p^1 - p^*) + (p^* - p^0). \quad (26)$$

A standard price index which ignored the quality change would take the form $\sum x^t p^1 / \sum x^t p^0$, where $t = 0$ or 1 . Instead, AG propose to calculate an index

based on the "pure" price change, $p^1 - p^0$, which allows for the change in quality: $\sum x^t p^1 / \sum x^t p^0$. In forming this index, AG assert that "in the base period, p^0 is taken to be equal to p^0 by definition" (AG, p. 543), in which case one obtains from (24) and (26):

$$\begin{aligned} p'^1 &= (p^1 - p^0) - \left(\frac{\partial \psi}{\partial b} \right)_0 (b^1 - b^0) + p'^0 \\ &= p^1 - \left(\frac{\partial \psi}{\partial b} \right)_0 (b^1 - b^0) \\ &= p^1 - (p^* - p^0). \end{aligned}$$

They take base-period quantity weights for their index, which becomes:

$$L^{AG} \equiv \frac{\sum h(p^0, b^0, y^0) p'^1}{\sum h(p^0, b^0, y^0) p^0} = \frac{\sum h(p^0, b^0, y^0) [p^1 - (p^* - p^0)]}{\sum h(p^0, b^0, y^0) p^0}.$$

However, a comparison of (24) and (26) shows that AG are in error when they state that $p'^0 = p^0$. A true statement is that, by definition, $p'^1 = p^1$, whereas $p'^0 = p^*$. Thus, their index should be $\sum x^t p'^1 / \sum x^t p'^0 = \sum x^t p^1 / \sum x^t p^*$. In this case it would be natural to take the final-period quantities as weights, which leads to a Paasche-type index:

$$P^1 \equiv \frac{\sum h(p^1, b^1, y^1) p^1}{\sum h(p^1, b^1, y^1) p^*}.$$

So far, I have worked with the quality-adjusted prices p^* . If one uses p^{**} instead, this leads to an alternative decomposition of the actual price change analogous to (24):

$$(p^1 - p^0) = (p^{**} - p^0) + (p^1 - p^{**}) \quad (27)$$

or, in the AG notation,

$$(p^1 - p^0) = (p'^1 - p'^0) + \left(\frac{\partial \psi}{\partial b} \right)_1 (b^1 - b^0).$$

In this case it is true that $p^{1,0} = p^0$, while

$$\begin{aligned} p^{1,1} &= p^1 - \left(\frac{\partial \psi}{\partial b} \right)_1 (b^1 - b^0) \\ &= p^{**}. \end{aligned}$$

Here it would be natural to use the base-period quantities as weights, which leads to a Laspeyres-type index (probably the index intended by AG):

$$L' \equiv \frac{\sum h(p^0, b^0, y^0) p^{**}}{\sum h(p^0, b^0, y^0) p^0} = \frac{\sum h(p^0, b^0, y^0) [p^1 - (\partial \psi / \partial b)_1 (b_1^1 - b_0^0)]}{\sum h(p^0, b^0, y^0) p^0}.$$

Two other indices which one could calculate by analogy with the indices L^* and P^{**} of the previous section are

$$L^* \equiv \frac{\sum h(p^*, b^1, y^0) p^1}{\sum h(p^*, b^1, y^0) p^*} \quad \text{and} \quad P^{**} \equiv \frac{\sum h(p^{**}, b^0, y^1) p^{**}}{\sum h(p^{**}, b^0, y^1) p^0}.$$

The quality-adjusted price vectors p^* and p^{**} are straightforward to calculate, unlike the prices p^* and p^{**} defined in (14) and (15). The key question is whether the indices formed from these prices--such as L^{AG} , L' , L^* , P' , and P^{**} --have any significance for welfare evaluations; i.e., do they provide upper or lower bounds on the true cost-of-living indices, $\bar{\pi}_C$ and $\bar{\pi}_E$? By applying Lemma 5, one can show that they do not. In the case of L' , for example, application of the lemma yields

$$\frac{m(p^{**}, b^0, u^0)}{m(p^0, b^0, u^0)} \leq L'. \quad (28)$$

If one could show that $m(p^1, b^1, u^0) \leq m(p^{**}, b^0, u^0)$, it would follow that $\bar{\pi}_C \leq L'$. But this cannot be shown. Suppose that $b^1 \geq b^0$, so that $m(p^1, b^1, u^0) \leq m(p^1, b^0, u^0)$. However, $b^1 \geq b^0$ implies $p^1 \geq p^{**}$ and, hence,

$$m(p^1, b^1, u^0) \leq m(p^1, b^0, u^0) \geq m(p^{**}, b^0, u^0).$$

If $b^1 \leq b^0$, the inequalities are reversed. It can be shown that, for similar reasons, one cannot obtain useful upper bounds on $\bar{\pi}_C$ from L^{AG} or L^* , or lower bounds on $\bar{\pi}_E$ from F' or F^{**} .

These conclusions still hold if one employs the Divisia formula in computing the hedonic price indices as Griliches (1971, p. 6) suggests. Indeed, this causes an additional complication since, as Richter (1966) and Samuelson and Swamy (1974) have shown, Divisia indices are exact if, and only if, the utility function is homothetic. Suppose, for example, that one computes the Divisia index corresponding to L' and F^{**}

$$I = \exp \int_{t_0}^{t_1} \frac{\sum x'(s) [dp'(s)/ds]}{\sum x'(s) p'(s)} ds,$$

where

$$x'(s) = h [p'(s), b^0, y(s)]$$

and

$$\begin{aligned} p'(s) &= p(s) - \left(\frac{\partial \psi}{\partial b} \right)_s [b(s) - b^0] \\ &= \psi [\alpha(s) + \gamma(s) b^0]. \end{aligned}$$

By adapting the proof in Samuelson and Swamy (1974, pp. 578 and 579), one can show that, if $u(x, b)$ is homothetic in x , this index is equal to the ratio on the left-hand side of (28). In this case, of course, $\bar{\pi}_C = \bar{\pi}_E$, but neither is equal to I and, by the argument given above, neither is bounded by it.

I conclude, therefore, that the hedonic price methodology proposed by AG has little value for welfare analysis. Obviously, it has considerable value as a descriptive tool for decomposing the sources of price changes into a quality-change component and a pure inflation component on the lines of (26) and (27). But it does not provide a practical method for assessing the direction or magnitude of welfare change.

IV. CONCLUSIONS

The purpose of this paper has been to review practical methods for evaluating the effects of a change in the quality of goods on a consumer's welfare using data on observed consumption choices. I compared two approaches to the evaluation of welfare changes--one based on the concepts of compensating and equivalent variation and the other based on the cost-of-living index concept. I showed that both concepts convey essentially the same information and are equally easy (or difficult) to implement empirically. I also showed that for quality changes, unlike price changes, it is not possible to obtain simple bounds on the welfare change without imposing restrictions on the form of the utility function. The moral of the story is that, when dealing with quality changes, it is not enough to possess data on market outcomes--i.e., prices, qualities, and quantities chosen. One must be able to deduce from the data information about the consumer's preferences, at least to the extent of fitting ordinary demand functions, in order to perform the welfare evaluation. Methodologies which do not focus explicitly on the demand side, such as AG's hedonic price technique, are inadequate for this task.²¹

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APPENDIX

I describe here some of the properties of the demand functions, expenditure function, etc., associated with the utility model (8) and prove some results left unproven in Section II.

From the definition of these functions, the following relations hold as identities:

$$g^i(p, b, u) = h^i[p, b, m(p, b, u)] \quad (\text{A.1})$$

$$h^i(p, b, y) = g^i[p, b, v(p, b, y)] \quad (\text{A.2})$$

$$\mu(p, b, u) = \lambda[p, b, m(p, b, u)] \quad (\text{A.3})$$

$$\lambda(p, b, y) = \mu[p, b, v(p, b, y)] \quad (\text{A.4})$$

$$u = v[p, b, m(p, b, u)] \quad (\text{A.5})$$

$$y = m[p, b, v(p, b, y)]. \quad (\text{A.6})$$

These identities are used to prove several of the results in the paper. The expenditure function and indirect utility functions can be shown to possess the standard properties including

$$m_i(p, b, u) = g^i(p, b, u) \quad (\text{A.7})$$

$$m_r(p, b, u) = -\frac{u_r[g(p, b, u), b]}{\mu(p, b, u)} \quad (\text{A.8})$$

$$m_u(p, b, u) = \frac{1}{\mu(p, b, u)} \quad (\text{A.9})$$

$$-\frac{v_i(p, b, y)}{v_y(p, b, y)} = h^i(p, b, y) \quad (\text{A.10})$$

$$v_r(p, b, y) = u_r[h(p, b, y), b] \quad (\text{A.11})$$

$$v_y(p, b, y) = \lambda(p, b, y). \quad (\text{A.12})$$

PROOF OF PROPOSITION 5: One can decompose $\Delta \bar{u}$ as follows:

$$\begin{aligned} \Delta \bar{u} &= \{ [v(p^0, b^1, y^0) - v(\tilde{p}, b^1, y^0)] - [v(p^0, b^0, y^0) - v(\tilde{p}, b^0, y^0)] \} \\ &\quad + v(\tilde{p}, b^1, y^0) - v(\tilde{p}, b^0, y^0) \\ &= \int_{\tilde{p}}^{p^0} \sum_{i \in I} [v_i(p, b^1, y^0) - v_i(p, b^0, y^0)] dp_i + [v(\tilde{p}, b^1, y^0) - \\ &\quad v(\tilde{p}, b^0, y^0)]. \end{aligned} \quad (\text{A.13})$$

However, the second term on the right-hand side of (A.13) vanishes since, by construction, $h^i(\tilde{p}, b_R, b_R^0, y^0) = 0$, and, by assumption (A) and (A.11),

$$v_r(\tilde{p}, b, y^0) = \partial u \left[h^I(\tilde{p}, b_R, b_R^0, y^0), x_I, b_R, b_R^0 \right] / \partial b_r = 0 \quad r \in R, \quad b_R \in B_R.$$

Thus, (A.13) becomes

$$\begin{aligned} \Delta \bar{u} &= \int_{\tilde{p}}^{p^0} \sum_{i \in I} [v_i(p, b^1, y^0) - v_i(p, b^0, y^0)] dp_i \\ &= \lambda \bar{S}^b, \text{ by assumption (II'a)}. \end{aligned}$$

A sufficient condition for the path independence of \bar{S}^b is that $h_j^i(p, b, y) = h_i^j(p, b, y)$ $i, j \in I$. Now,

$$h_j^i = \frac{\partial (-v_i/v_y)}{\partial p_j} = \frac{v_i v_{yj} - v_y v_{ij}}{(v_y)^2}, \quad \text{and} \quad h_i^j = \frac{v_j v_{vi} - v_y v_{ji}}{(v_y)^2}.$$

The continuity of $v(\cdot)$ ensures that $v_{ij} = v_{ji}$, and the independence from p_I of λ implies $v_{yi} = v_{yj} = 0$, $i, j \in I$. Thus, $h_j^i = h_i^j$, $i, j \in I$.

PROOF OF LEMMA 4: Following Mäler, I assume that $\text{sign}\{m_{ru}(p, b, u)\}$ is the same for all u . From (A.9), $m_{ru}(p, b, u) = m_{ur}(p, b, u) = -\mu_r(p, b, u) / \mu(p, b, u)^2$. Since $\mu > 0$,

$$- \text{sign} \{m_{ru}(p, b, u)\} = \text{sign} \{\mu_r(p, b, u)\}. \quad (\text{A.14})$$

Since (v_r/v_i) is independent of y , $v_{iy} = v_{ry} v_i/v_r$. By (A.10),

$$\begin{aligned} h_y^i &= (v_i v_{yy} - v_y v_{iy})/v_y^2 \\ &= (v_i/v_y) \left(v_{yy} - \frac{v_y}{v_r} v_{ry} \right) / v_y. \end{aligned} \quad (\text{A.15})$$

Since $v_r > 0$, by assumption, $v_i < 0$, $v_y > 0$, and $v_{ry} = v_{yr}$, (A.15) implies

$$\text{sign} \left\{ h_y^i(p, b, y) \right\} = \text{sign} \left\{ v_{yr}(p, b, y) - \frac{v_r(p, b, y)}{v_y(p, b, y)} v_{yy}(p, b, y) \right\}. \quad (\text{A.16})$$

From (A.3),

$$\mu_r(p, b, u) = \lambda_r [p, b, m(p, b, u)] + \lambda_y [p, b, m(p, b, u)] m_r(p, b, u)$$

or, by (A.8) and (A.11),

$$\mu_r [p, b, v(p, b, y)] = v_{yr}(p, b, y) - v_{yy}(p, b, y) \cdot \frac{v_r(p, b, y)}{v_y(p, b, y)}. \quad (\text{A.17})$$

Combining (A.14), (A.16), and (A.17) yields the desired result.

FOOTNOTES

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¹Not only does this argument explain the normative significance of the magnitude of E, as opposed to its sign, but it also provides an empirical procedure for aggregating equivalent variations across individual consumers. Atkinson (1970) and others have proposed certain functional forms for social welfare functions which can be used to obtain the weights, W_h , for aggregating the individual equivalent variations.

²Recent contributions include Chipman and Moore (1976) and Dixit and Weller (1979). Note that analogous results for E^P can be obtained by substituting y^1 for y^0 in (3).

³As an example, consider the case where $J = \{1\}$, $\bar{J} = \{2\}$, and $h^1(p, y) = A p_1^{-\alpha} p_2^{\alpha-\eta} y^\eta$. One recognizes that $-\psi_1(p_1, p_2) = A p_1^{-\alpha} p_2^{\alpha-1}$ and $\phi_y(p_2, y)^{-1} = p_2^{1-\eta} y^\eta$. By integration,

$$\psi(p_1, p_2) = A \left(\frac{1}{\alpha - 1} \right) \left(\frac{p_1}{p_2} \right)^{1-\alpha} \quad \text{and} \quad \phi(p_2, y) = \left(\frac{1}{1 - \eta} \right) \left(\frac{y}{p_2} \right)^{1-\eta}.$$

⁴The idea of using approximations of π_C and π_E to approximate C and E via (7) is exploited in Lake, Hanemann, and Oster (1979).

⁵For a proof, see Pollak (1971, pp. 12-19).

⁶I adopt the convention that the components of b are measured in such a way that $u_r \equiv \partial u / \partial b_r \geq 0$. Throughout this paper, the subscripts r and s will denote the partial derivations of a function with respect to the components, b_r and b_s , of b . The subscripts i and j will denote the partial derivatives with respect to p_i and p_j or x_i and x_j , depending on the context. The subscripts y and u will denote partial derivatives with respect to y and u .

⁷Under this interpretation, each brand of a good is treated as a separate

commodity. A consumer chooses a particular level of quality of a good by choosing a particular brand.

⁸The case of a corner solution to (8) is treated in Hanemann (forthcoming).

⁹This implies an interdependence in utility between some components of b and some components of x . Bradford and Hildebrandt, who regard the components of b as public goods and the components of x as private goods, give several examples of such an interdependence, e.g., between public highways and private vehicles. If one thinks of the components of b as characteristics of commodities, the assumption of interdependence is still likely to be satisfied: the marginal utility of an attribute is likely to be zero if the commodity with which the attribute is associated is not consumed.

¹⁰The implications of assumptions (A) and (B) for the indirect utility function are spelled out by Willig [1978, equations (46) and (47)].

¹¹For the derivation of (12a), see Bradford and Hildebrandt (1977, pp. 122 and 123); for (12b), see Willig (1978, Theorem 1). Note that (12b) can be obtained from (12a) by dividing both sides by $x_j = -v_j/v_y$.

¹²I make use here of (A.3), (A.5), (A.6), (A.8), (A.11), and (A.12).

¹³Mäler proves that, if $m_{ru} < (>) 0$ for all $r \in R$, $\bar{C}^b < (>) \bar{E}^b$. Assumptions (A), (B), and (I') are needed in order to relate \bar{S}^b to \bar{C}^b and \bar{E}^b .

¹⁴Lemmas 3 and 4 also explain why empirical estimates of \bar{C}^b and \bar{E}^b obtained from bidding games and willingness-to-pay surveys are often different by an order of magnitude. Willig (1976) shows that, for a wide range of income elasticities of demand, C^P and E^P should be of approximately the same order of magnitude. Several writers--for example, Rowe, d'Arge, and Brookshire (1980, p. 8)--appear to suggest that the same should be true of \bar{C}^b and \bar{E}^b . In fact, as Lemmas 3 and 4 imply, this is not so. It can be shown that Willig's approximation formulas do not carry over to \bar{C}^b , \bar{E}^b , and \bar{S}^b .

¹⁵ Property (II'a) states that v_y is independent of the prices and quality levels which change. This is clearly required to ensure that p^* and p^{**} , which satisfy (14) and (15), also satisfy (16) and (17).

¹⁶ It should be noted that Willig actually considers the case where only one price changes, i.e., $p^0 = (p_1^0, \bar{p})$ and $p^1 = (p_1^1, \bar{p})$ where $\bar{p} = (\bar{p}_2, \dots, \bar{p}_N)$. In this case, one can go beyond Paasche- and Laspeyres-type bounds on $\bar{\pi}_C$ and $\bar{\pi}_E$ since then $p^* = (p_1^*, \bar{p})$, $p^{**} = (p_1^{**}, \bar{p})$, and, by Propositions 8(a) or 8(b), $\underline{\lambda} = \bar{\lambda} = \bar{\pi}_C$ or $\bar{\pi}_E$. Hence, $\text{sign}(\Delta u) = \text{sign}[(y^1/y^0) - \underline{\lambda}]$.

¹⁷ For (20a, b), $p_1^* = p_1^0 \cdot \psi(b^0, \bar{p}^0)/\psi(b^1, \bar{p}^0)$; for (21a, b), $p_1^* = p_1^0 + \phi(b^0, \bar{p}^0) - \phi(b^1, \bar{p}^0)$.

¹⁸ For (20a), \bar{c}^b is given by (22) with $\pi^t = p_1^0 \psi(b^t, \bar{p}^0)$; for (21a), \bar{c}^b is given by (22) with $\pi^t = p_1^0 + \phi(b^t, \bar{p}^0)$.

¹⁹ For a summary of these studies, see Griliches (1971).

²⁰ This is discussed in Hanemann (forthcoming), where the differences between the utility model (8) and Houthakker's model are examined in some detail.

²¹ This conclusion also applies to the "property value" method of assessing the worth of environmental and locational amenities. The implications of my analysis for this methodology will be analyzed in a separate paper.

REFERENCES

- Adelman, I., and Z. Griliches, "On an index of quality change," Journal of the American Statistical Association, LVI (1961), 535-48.
- Atkinson, A.B., "On the measurement of inequality," Journal of Economic Theory, II (1970), 244-63.
- Bradford, D.F., and G. Hildebrandt, "Observable preferences for public goods," Journal of Public Economics, VIII (1977), 111-32.
- Chipman, J.S., and J.C. Moore, "The Scope of Consumer's Surplus Arguments," in Evolution, Welfare and Time in Economics (eds. A.M. Tang, F.M. Westfield, and J.S. Worley; Lexington: D.C. Heath and Company, 1976).
- Court, A.T., "Hedonic price indexes with automotive examples," in The Dynamics of Automobile Demand (New York: General Motors Corporation, 1939).
- Dixit, A.K., and P.A. Weller, "The Three Consumer's Surpluses," Economica, XLVI (1979), 125-35.
- Fisher, F.M., and K. Shell, "Taste and quality change in the pure theory of the true cost-of-living index," in Value, Capital, and Growth: Essays in Honour of Sir John Hicks (ed. J.N. Wolfe; Edinburgh: University of Edinburgh Press, 1967).
- Griliches, Z (ed.), Price Indexes and Quality Change (Cambridge: Harvard University Press, 1971).
- Hanemann, W.M., "Quality and Demand Analysis," in New Directions in Econometric Modeling and Forecasting in U.S. Agriculture (ed. Gordon C. Rausser; New York: Elsevier/North-Holland Publishing Company, forthcoming).
- Hofsten, E. Von, Price Indexes and Quality Changes (Stockholm: Bokforlaget Forum AB, 1952).
- Houthakker, H., "Compensated changes in quantities and qualities consumed," Review of Economic Studies, XIX (1951-52), 155-64.
- Lake, E., M. Hanemann, and S. Oster, Who Pays for Clean Water? The Distribution of Water Pollution Control Costs (Boulder: Westview Press, 1979).
- Lancaster, K., "A new approach to consumer theory," Journal of Political Economy, LXXIV, No. 2 (1966), 132-57.

- Mäler, K.G., Environmental Economics: A Theoretical Enquiry (Baltimore: The Johns Hopkins University Press, published for Resources for the Future, Inc., 1974).
- Pollak, R.A., The Theory of the Cost of Living Index (U.S. Department of Labor, Bureau of Labor Statistics, Office of Prices and Living Conditions, Working Paper 11, 1971).
- Richter, M.K., "Invariance axioms and economic indices," Econometrica, XXXIV (1966), 749-55.
- Rowe, Robert D., Ralph C. d'Arge, and David S. Brookshire, "An experiment on the economic value of visibility," Journal of Environmental Economics and Management, VII, No. 1 (March 1980),
- Samuelson, P.A., and S. Swamy, "Invariant economic index numbers and canonical duality: survey and synthesis," American Economic Review, LXIV, No. 4 (1974), 566-93.
- Stevens, J.B., "Recreation benefits from water pollution control," Water Resources Research, II (1966), 167-82.
- Willig, R., "Consumer's surplus without apology," American Economic Review, LXVI, No. 4 (1976), 589-97.
- _____, "Incremental consumer's surplus and hedonic price adjustment," Journal of Economic Theory, XVII (1978), 227-53.