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MONOPOLY POWER CAN BE DISADVANTAGEOUS IN THE EXTRACTION OF A DURABLE NONRENEWABLE RESOURCE

by

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Monopoly Power Can Be Disadvantageous in the Extraction of a Durable Nonrenewable Resource*

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Abstract: We study a Markov equilibrium for the case where a monopolist extracts a nonrenewable resource which is converted to a durable good, which then depreciates at a constant rate. We show that in a stationary, continuous time model (infinite horizon, infinitesimal period of commitment) monopoly power can be disadvantageous. Numerical experiments confirm that this can also occur in a finite horizon, discrete model. This result is compared to previous examples of disadvantageous market power, obtained using two-period models.

JEL Classification numbers: D42, L12, Q39

Key words Disadvantageous market power, durable good, nonrenewable resources, Coase conjecture.

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1. Introduction

Standard models of the durable goods monopolist (not including Coase, 1972) assume that the potential supply of the good is unlimited. Models of monopoly resource owners typically assume that the resource (the final output) is not durable, so that current demand is not affected by previous sales, or the anticipation of future sales. We nest these two cases by modelling a monopolist who owns a nonrenewable resource which, upon extraction, is converted to a durable good. Thus, the potential supply of the good is finite; extraction (production) costs may increase with cumulative extraction; past sales affect the supply in the "second hand market" and consequently affect the demand faced by the monopolist in a given period. Here market power may be disadvantageous: *industry profits may be lower under monopoly than in the competitive equilibrium.* This result is related to the three bodies of literature concerning: (i) the Coase Conjecture; (ii) disadvantageous market power; and (iii) nonrenewable resource monopolies.

The Coase Conjecture (Coase, 1972) states that if buyers of a durable good have rational expectations and the monopolist is unable to commit to a future sales trajectory, she loses market power "in the twinkling of an eye" (as the period of commitment becomes small). Stokey (1981) and Bulow (1982) formalize and verify this Conjecture, for the case where the monopolist has constant production costs. Kahn (1986), using a Markov Perfect equilibrium (MPE), shows that if production costs are convex and the durable good is infinitely lived, market power vanishes only in the steady state. Gul et al. (1986) give sufficient conditions for the existence of a unique MPE, which matches the Coase Conjecture.

Ausubel and Deneckere (1989) show that if the buyer/seller relation is modeled as a game, and "reputational strategies" are admitted, monopoly power can be preserved even in the absence of the monopolist's ability to commit. Ausubel and Deneckere (1987) and Gul (1987) demonstrate that industry profits can be higher in a duopoly than under monopoly; these papers also assume that agents have history-dependent (non-Markov) beliefs.

We consider only Markov perfect equilibria, and thus exclude reputational strategies. In view of the extensive modeling of the durable goods monopolist using non-Markov equilibria, our restriction to MPE requires some comment. There are three reasons for the restriction.

First, we doubt that there is much to be learned using models of non-Markov equilibria for the problem of the durable nonrenewable resource. Ausubel and Deneckere (1989) have showed that without the Markov assumption, a great range of outcomes, ranging from competitive to nearly monopolistic, can emerge as equilibria for the standard durable goods monopolist. The introduction of a nonrenewable resource to the model does not change the logic of their argument, so we expect that a similar result would hold here. It does not seem particularly helpful to establish that multiple equilibria can arise for familiar reasons, in a different context. In addition, this type of result is not helpful for welfare analysis.

Second, we think that economists are still interested in Markov equilibria. This is at least partly because we rely on two- or three-period models for much of our intuition about dynamics in complicated circumstances. In these finite horizon models it is often the case that the unique perfect equilibrium is Markov[1]. We may want to know how the model behaves when the number of periods becomes large or infinite, without losing the flavor of the simpler model. In other words, we may want to require the model to be "continuous at infinity", so that our results do not rely on an infinite horizon. The Markov restriction is a natural way to impose this continuity.

Third, there are circumstances where the Markov assumption appears to describe how markets behave. Non-Markov, reputational equilibria, require that agents revise their beliefs and behavior *dramatically*, following a (noticeable) deviation by another agent. This type of discontinuous behavior may be reasonable in thin markets. Indeed, the DeBeers diamond cartel is often cited as an example of successful exercise of market power. DeBeers' reputation is probably critical to its success. If this is correct, the equilibrium is non-Markov. However, this kind of equilibrium seems less likely to emerge if one side of the market (here, buyers) consists of a continuum of agents, each of whom has negligible effect on the outcome. It is harder to believe that a continuum of buyers would dramatically change their beliefs in the event that the monopolist surprises them in a given period, and moreover that they would do so in a way that is sufficiently predictable to support an equilibrium.

Our other important assumption is that average extraction costs depend on the remaining stock but not on the rate of extraction/production. This assumption is common in resource economics. It implies that the *cumulative* cost of producing a given stock is independent of the rate at which it is produced, but the average cost may increase with

[1] There are exceptions to this, of course. For example, if there are multiple equilibria to a subgame, there may be perfect non-Markov equilibria.

previous extraction. Thus, the model is the natural extension, to a nonrenewable resource base, of Coase's model. However, the Coase Conjecture does not hold.

The second related literature shows that in some situations the inability to make future commitments does not merely inhibit the exercise of power, but can make an apparent increase in power disadvantageous. Examples of this are found in Salant et al. (1983), Ulph and Ulph (1989), and Maskin and Newbery (1990); similarly, an increase in cooperation, which is analogous to an increase in market power, can be disadvantageous [Rogoff (1985), Kehoe (1989), Gatsios and Karp (1992)]. In all of these papers, an increase in market power or cooperation without the ability to commit means that the competitive equilibrium (or the equilibrium when there is less market power, or less cooperation) may no longer constitute a feasible equilibrium. For example, the durable goods monopolist can imitate a competitive seller in the current period, but may not be able to promise to do this in the future. Without such a promise, there may be no point in imitating the competitive seller in the current period. Since the agent with market power cannot credibly promise to behave exactly as a competitive agent would, there is no reason to assume that market power need be advantageous.

In view of the existing literature, it is not surprising that market power can be disadvantageous. However, that literature has led us to expect the result in specific circumstances, where it has (with the benefit of hindsight) a fairly obvious explanation. In addition, papers on disadvantageous power rely on two-period models. In our model, however, market power is always advantageous in a two-period setting. This paper shows that the impossibility of disadvantageous power in two-period models does not imply power is advantageous in a many-period setting.

The third body of relevant literature concerns monopoly extraction of a nonrenewable resource. Until recently, this literature made no distinction between durable and nondurable resources. Monopoly extraction of oil, for which the second hand market is extremely limited, and the extraction of minerals such as bauxite (the primary component of aluminum), for which the second hand market is extensive, were modeled using standard optimal control problems. Recent papers (Chilton 1984, Karp 1993, Levhari and Pindyck 1981, Malueg and Solow 1990) recognize the difference between the two types of resources. Pindyck (1979) pointed out that the existence of a second hand market might erode the benefits of cartelization of durable nonrenewable resources.

The following section describes the continuous time (infinitesimal period of commitment) model and explains why the monopolist extracts more rapidly than competitive firms. This is the basis for the result that market power is disadvantageous. The next section specializes to linear functional forms and presents the equilibrium conditions under perfect competition and monopoly without the ability to commit. We use the equilibrium conditions to compare extraction paths and to establish the possibility of disadvantageous power. The next two sections provides further comments on the basic model and investigate the issue of exploration and strategic destruction. A conclusion follows. Technical details are collected in the Appendix.

2. *The Model and a Preliminary Result*

This section presents the model and explains why the monopolist extracts more rapidly than the competitive producer. This explanation provides the critical piece of intuition for the disadvantageous monopoly result.

Let S_t be the stock of the resource remaining at t and Q_t be the stock of the durable good. We normalize so that a unit of the resource is equivalent to a unit of the durable good; the durable good depreciates at the constant rate $\delta \geq 0$. This implies

(1) $$\dot{S} = -m$$

(2) $$\dot{Q} = m - \delta Q$$

where m is the rate of extraction (production) and $\dot{z}$ denotes dz/dt. With a discount rate of r and inverse demand for services (implicit rental rate) of F(Q), the buyers' rational (point) expectations imply that price at t is

(3) $$P_t = \int_t^\infty e^{-(r+\delta)(s-t)} F(Q_s)\, ds \Rightarrow \dot{P} = (r + \delta)P - F(Q).$$

The average cost of extraction, c(S), is non-increasing in the remaining stock of the resource: $c'(S) \leq 0$. The assumption of constant short-run average costs provides the natural extension, to an exhaustible resource framework, of Coase's model.[2]

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[2] Malueg and Solow (1990) study a discrete time version of this model, in which extraction costs are convex in the rate of extraction, and the durable good does not depreciate, and cumulative extraction is unbounded. The convexity of extraction costs generates results similar to Kahn's.

The present discounted value (PDV) of industry profits is

$$(*) \qquad J(Q_0, S_0;\{m\}) = \int_0^\infty e^{-rt}(P_t - c(S_t))m_t dt$$

where {m} is the extraction profile and the initial time is 0.

The competitive equilibrium can be obtained by solving the social planner's problem, or by maximizing industry profits (*) subject to (1), taking as given the price trajectory, and then using (2) and imposing the rational-expectations equilibrium condition (3). If sellers take the price as given, the Hamiltonian to their maximization problem is linear in the rate of extraction, m, which can take any non-negative value. If the initial stock of the durable good is high relative to the stock of the resource (e.g. at point d^* in Figure 1), the equilibrium sales price is low and sellers set m=0 until the stock of the durable good depreciates to a low enough level; thereafter, sellers produce at a positive finite rate until the resource is exhausted. If the initial stock of the durable good is low relative to the stock of the resource (e.g. at point d' in Figure 1), sellers produce a discrete amount (set $m=\infty$ for an instant), causing the stock of the resource to jump down and the stock of the durable good to jump up; thereafter they produce at a finite rate, until the resource is exhausted. The curve in (S,Q) space, along which $0 < m < \infty$, called the singular arc, is labelled G(S) in Figure 1. (In general this curve need not be linear,

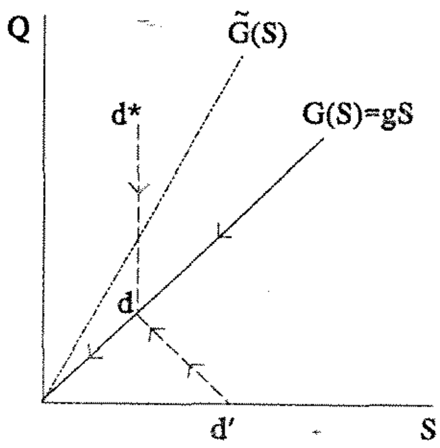

Figure 1 The Singular Arc

or intersect the origin; both of these features hold for the linear example of the next section, but this has no bearing on the discussion here.) Point d is on the singular arc. If the initial state lies above the arc, e.g. on dd*, m=0 until point d is reached. If the initial state lies below the arc, e.g. on dd', there is a jump to point d.

The singular arc describes the equilibrium. If something about the problem were changed, so that for S > 0 the singular arc were higher [e.g. in Figure 1 the singular arc were $\tilde{G}(S)$ rather than G(S)], then the resource would be extracted more rapidly. This fact, which is central to Proposition 1, below, follows immediately from the definition of the singular arc.

The equilibrium competitive rate of extraction induces an equilibrium competitive stationary price function, which we denote $P^c(S,Q)$. This price function has two important features. First, it is constant on 45° lines below the singular arc, such as the dotted line dd' in Figure 1. This is due to the fact that for any state below the singular arc, there will be an immediate jump to the arc. This result depends on the assumption of constant short run costs. The second feature, which is stated as Lema 1 in Karp (1993), is that the price function is continuously differentiable on the arc. These facts imply that $\partial P^c/\partial Q = \partial P^c/\partial S$ on or below the singular arc (where the constraint $m \geq 0$ is not binding). This equality, together with the first order conditions to the competitive firm's problem, imply that in the competitive equilibrium the following holds on the singular arc[3]

[3] In the competitive equilibrium, the usual Hotelling rule holds: $\dot{P}(S,Q) = r(P-C)$. Use $\dot{P} = P_S\dot{S} + P_Q\dot{Q}$, equations (1) and (2), and $P_Q = P_S$ to obtain equation (4).

(4) $$\phi(S,Q) \equiv P^c(S,Q) - c(S) + \frac{\partial P^c}{\partial Q}\frac{\delta Q}{r} = 0 .$$

Equation (4) implicitly defines the singular arc, in terms of the unknown function P^c and its derivative. We use the function $\phi(\cdot)$, and the following assumption in a proof below.

Assumption 1 The function $\phi(S,Q)$ is decreasing in Q: $\phi_Q < 0$.

This assumption requires that $(r+\delta)P^c_Q + \delta Q P^c_{QQ} < 0$, i.e., that the endogenous function P^c is not "very convex". This assumption is clearly met for the example in the next section (where P^c is linear), but is also satisfied much more generally. The assumption implies that $\phi < 0$ for points in the (S,Q) plane that lie above the curve defined by $\phi(S,Q) = 0$.

Before studying the monopoly equilibrium, we discuss the necessary and sufficient condition for the resource to be exhausted in the first instant in a competitive equilibrium. Consider three cases: the singular arc intersects either (i) the origin, (ii) the Q axis above 0, or (iii) the S axis to the right of 0. Figure 1 illustrates case (i) and Figure 2 illustrates case (ii). We can determine which of the three cases arises by evaluating $\phi(0,Q)$; this calculation does not require that we solve the competitive equilibrium. In case (iii), the resource is never exhausted (even asymptotically), so we need not consider this case further. For case (i) the resource is exhausted

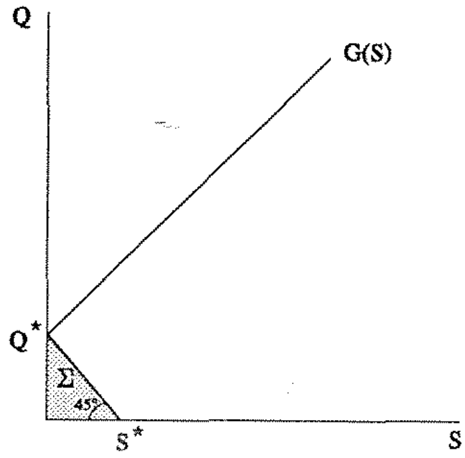

Figure 2 Exhaustion in Finite Time

asymptotically (since $Q_t \rightarrow 0$ only as $t \rightarrow \infty$). Therefore, the resource can be exhausted in the first instant only in case (ii). For cases (i) and (ii), define Q^* as the intersection of the singular arc and the Q axis. Define the set of points in the positive quadrant of the (S,Q) plane under the 45^0 line through Q^* as Σ (see Figure 2). That is, $\Sigma = \{(S,Q): S > 0, Q \geq 0$ and $S+Q \leq Q^*\}$. For case (i), where $Q^* = 0$, Σ is empty, and in case (ii) where $Q^* > 0$, Σ is non-empty. Finally, define the critical stock level, S^* as $S^* = Q^*$. From our definitions, the resource is exhausted in the first instant in a competitive equilibrium if and only if the initial condition (S_0,Q_0) ε Σ.

We now turn to the monopoly equilibrium, where we follow Stokey (1981) in assuming that buyers' expectations about future sales (and consequently about the future stock of the durable good) depend only on the current state, S,Q: $Q^e_{t+s} = q(s,Q_t,S_t)$ where Q^e_{t+s} is the buyers' point expectation of Q_{t+s} and $q(\cdot)$ is a stationary function. (We restrict attention to stationary, Markov equilibria.) These expectations (functions) generate a price function, $P^m(S, Q)$. This can be seen by replacing $Q_{t+\tau}$ by its point expectation $q(\tau,Q_t, S_t)$ in equation (3) for P. We assume that $P^m(\cdot)$ is continuous and almost everywhere differentiable. This is true for the linear example in the next section; more generally, a sufficient condition for the assumption is that the expectations functions are continuous.

The function $P^m(\cdot)$ is endogenous to the problem, but the monopolist takes it as given. The function depends on buyers' expectations of the monopolist's future sales. The monopolist at a point in time is not permitted to choose future sales (i.e., to "precommit") but can affect them indirectly, by means of her effect on the state, S, Q. If a jump does not occur, the monopolist takes the current price as given, but recognizes that she can affect future prices by means of current sales. If the monopolist sells a discrete amount Δ at an instant, i.e., causes the state to jump from S, Q to S - Δ, Q + Δ, then the sales price of Δ is $P^m(S - \Delta, Q + \Delta)$, due to buyer rationality. The monopolist maximizes (*) [with P_t replaced by $P^m(S,Q)$], subject to (1) and (2). The endogenous function $P^m(\cdot)$ must be such that, given that function, the trajectories of Q and S are optimal for the monopolist and also satisfy the equilibrium condition (3), for all values of the state $(S,Q) \geq 0$. The equilibrium function $P^m(\cdot)$ and the optimal monopoly behavior must be mutually consistent.

Except for the fact that the function P^m is unknown, this is a standard control problem. It is linear in the rate of extraction, m, as was the case for the competitive producer. Consequently, the monopolist produces a positive, finite amount only on a singular arc; above the arc production is 0 and below the arc the rate of production is unbounded, causing the state to jump.

We now demonstrate and explain the monopolist's incentive to produce more rapidly than the competitive producer. In the standard non-durable goods case, the monopolist restricts sales in order to increase the current price. This incentive is absent in the continuous time (infinitesimal period of commitment) durable goods model, where the monopolist takes the current price as given unless she sells at an infinite rate and causes the state to jump. That is, ignoring jumps, which in equilibrium might occur only in the first instant, both the monopolist and the competitive seller take the *current* price as given at all points in time. The monopolist, however, recognizes that current sales affect the evolution of the state, and thus the trajectory of future prices.

In order to determine how this recognition affects the monopolist's behavior we perform the following thought experiment: Suppose that the monopolist solved a control problem with the competitive equilibrium price function $P^c(S,Q)$ rather than the monopoly equilibrium price function. In view of the characteristics of $P^c(S,Q)$, described above, if the monopolist were to deviate from the competitive sales trajectory by selling an extra unit over a small interval of time, this would have no first order effect on price over that interval. This is because initially the deviation increases the trajectory of Q and decreases the trajectory of S by the same amount, and these changes offset each other, leaving the price unchanged. (Recall that $\partial P^c/\partial Q = \partial P^c/\partial S$ on and below the singular arc.) The effect of the deviation on the evolution of S is permanent. However, provided that $\delta > 0$, the effect of the deviation on the evolution of Q decays over time. Therefore, the eventual effect of the deviation is to decrease the trajectory of S by more than the increase in the trajectory of Q. This implies that the deviation causes the future price trajectory to increase. (This is because $\partial P^c/\partial Q$ and $\partial P^c/\partial S$ are both negative, and increasing extraction over an interval increases Q and decreases S.) Since the deviation also increases the current flow of profits (because current price and costs are unchanged, but sales are higher) it unambiguously benefits the monopolist.

We state this conclusion formally as

<u>Proposition 1</u>: Suppose that $\delta > 0$ and Assumption 1 is satisfied. Consider the experiment in which a seller maximizes (*), with P_t replaced by $P^c(S,Q)$, subject to (1) and (2). Denote $Q = G(S)$ as the singular arc in the competitive equilibrium, and $Q = \tilde{G}(S)$ as the singular arc in the experiment. (i) $\tilde{G}(S) > G(S)$ for $S > 0$. (ii) This inequality implies that the seller in our experiment extracts more rapidly than in the competitive equilibrium, provided that $(S_0,Q_0) \notin$

Σ. If $(S_0,Q_0) \in \Sigma$ then the resource is exhausted in the first instant in the competitive equilibrium and in the experiment; in this case, the competitive equilibrium is a MPE for the monopololist.

Proof: (i) The Hamiltonian of the control problem corresponding to the experiment is $H \equiv \pi(S,Q,\lambda,\eta)\cdot m - \lambda\delta Q$, where λ and η are the costate variables associated with Q and S, respectively, and the function π is defined as $\pi \equiv P^c(S,Q) - c(S) + \lambda - \eta$. An interior maximum of H with respect to m requires that π vanish. Differentiating the singular arc $\pi = 0$ with respect to time, substituting in the costate equations, and simplifying, implies that on the singular arc the following holds:

$$(5) \qquad \beta(S,Q,\lambda,\eta) \equiv P^c(S,Q) - c(S) + \frac{\partial P^c}{\partial Q}\frac{\delta Q}{r} - \frac{\delta\lambda}{r} = \phi(S,Q) - \frac{\delta\lambda}{r} = 0.$$

The shadow value of Q, λ, must be negative. (This can be shown using the costate equation for λ, the fact that $\partial P^c/\partial Q < 0$, and the transversality condition $\lim_{t\to\infty} e^{-rt}\lambda_t = 0$.) Using Assumption 1 and equations (4) and (5), this implies that at a point in (S,Q) space where $\beta = 0$, it must be the case that $\phi < 0$. This implies that the singular arc defined by (4) lies below the arc defined by (5). (ii) This conclusion is a restatement of the characteristics of the singular arc contained in the description of Figures 1 and 2, and the definition of Σ.‖

If the monopolist faced the price function $P^c(S,Q)$ [and $(S_0,Q_0 \notin \Sigma)$], her profits would obviously be higher than in the competitive equilibrium. However, by construction, that function satisfies the rational expectations constraint (3) only when the competitive sales trajectory is followed. Provided that $\delta > 0$, the monopolist would choose a different sales trajectory, and therefore must face a different endogenous price function. To understand how the functions P^m and P^c are likely to differ, we can imagine an iterative search procedure for P^m which begins with an initial guess of the function P^c, and then modifies this guess in a way which moves us toward equilibrium. Equilibrium requires a price function and an extraction trajectory that solve the monopolist's control problem and satisfy (3). We have seen that the initial guess, P^c, results in a production trajectory which is too high to be consistent with (3). If the monopolist deviates from the competitive path in the manner described by Proposition 1, the price would have to be lower in order to satisfy (3). The next iteration should therefore use a guess that lies below P^c for all values of the state. This leads to the conjecture that P^m lies below P^c. If this conjecture is correct, as indeed it is for the linear example of the next section, it is not surprising that monopoly power can be disadvantageous.

3. *The Linear Example*

To obtain an explicit solution, needed for welfare comparisons, we restrict attention to the following example

$$F(Q) = a - bQ; \quad c(S) = k_o - kS, \quad with \quad \frac{a}{r+\delta} \leq k_o. \tag{6}$$

The last inequality means that the cost of extracting the final unit of the resource is at least as great as the PDV, to society, of an extra unit of the durable good when Q=0. This implies that the non-negativity constraint $S_t \geq 0$ is not binding, and enables us to obtain a relatively simple closed-form solution. By defining S as the stock which it is economically rational to extract, we can replace the weak inequality by an equality; hereafter we assume $a/(\delta+r) = k_o$.

Obviously we also have to assume that the initial stock satisfies $S_0 \leq k_0/k$, so that extraction costs are non-negative. Except where we state otherwise, *we assume that* $\delta > 0$.

The Appendix derives the equilibrium under competition and monopoly; the methods were outlined in the previous section. As benchmarks we also consider two other assumptions about the market structure. The "open-access" equilibrium refers to the situation where sellers are price-takers and have no property rights, so that there is free entry into the industry. The "precommitted monopoly" equilibrium refers to the case where a monopoly owner of the resource is able to make binding commitments about her future behavior [a model studied by Levhari and Pindyck (1981) and Chilton (1984)].

For the linear example in equation (6), it is obvious that the equilibrium is unique for the case of competition, "precommitted monopoly", and open access. For the case of a discrete stage, finite horizon model, the Markov equilibrium is also unique, and can be obtained using dynamic programming (solving the problem "backwards", from the final period). In the appendix we explain how we use the characteristics of the equilibrium of the discrete stage, finite horizon model to search for a particular class of equilibrium in the continuous time, infinite horizon model. We want the equilibrium of the latter model to "resemble" the limiting equilibrium (as the horizon goes to infinity and the length of each stage to 0) of the former model. This leads us to look for an equilibrium with a linear price function and a linear singular arc. However, nothing about this procedure implies that the equilibrium we have obtained is unique.

Indeed, it is often the case that there exists a continuum of Markov equilibria in infinite horizon problems, even with the additional restriction of differentiability of endogenous functions. For example, Tsutsui and Mino (1990) show that a linear-quadratic differential game has a continuum of non-linear Markov equilibria, in addition to the well-known linear equilibrium. It is natural to ask whether this is also possible in our model.

Although we do not have a proof of uniqueness, we can at least explain why the intuition provided by Tsutsui and Mino does not apply in our model. In their game, non-uniqueness arises because of an "incomplete transversality condition". There is another, more intuitive way to describe this problem. The necessary conditions to a differentiable MPE, for a game with a single state variable, can be rewritten to describe the equilibrium as a (system of) differential equation(s). However, the necessary conditions do not enable us to pin down a steady state. Unless we have a "natural boundary condition" which selects a steady state, we are left with a differential equation but no boundary condition. Obviously, there is not a unique solution to this mathematical problem, and thus not a unique MPE. In our model, however, there is a "natural boundary condition". Because the durable good decays, and because production of the good requires a non-renewable resource, the state (S,Q) must eventually approach (0,0). This is our boundary condition.[4] The fact that we have a unique steady state suggests that there probably is a unique (differentiable) MPE. However, establishing this rigorously is complicated by the fact that we have two state variables, so the equilibrium conditions lead to a system of partial differential (rather than ordinary

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[4] In order to help understand this, consider the durable good monopolist where the durable good decays, but production of the good does not require the exhaustible resource (so that aggregate production can be unbounded). In that case, there is no "natural boundary condition", since it is possible to sustain any stock as a steady state. Differentiable MPE are therefore not unique, for essentially the same reason as described by Tsutsui and Mino. This issue is studied in Karp (1994).

differential) equations. We have not established that there is a unique solution to this system, which satisfies our boundary condition, and therefore we can not be sure of uniqueness. Hereafter, we consider only the linear MPE.

The singular arc for the competitive equilibrium is linear [G(S) = gS] because of the linearity of the primitive functions, and intersects the origin because of the assumption that $a/(\delta+r)=k_0$. The slope of the arc (g) is the unique positive root of (7a). (Parameters are defined in Table 1.)

(7a) $$h(g) \equiv r\alpha(r+\delta) + [(r+\delta)(r\alpha - 1) + (r\alpha + 1)\,\delta]g - (r+\delta)g^2 = 0\,.$$

As we noted above, there is also an equilibrium for the monopolist which involves a linear singular arc, the slope of which is the unique positive root of

(7b) $$f(g) \equiv -\alpha\, r\,(r+\delta) + [r\,(1-2\alpha\eta_1)]\,g + \frac{r\,(2-\alpha\eta_1)}{\eta_0}\,g^2 + \eta_1\,g^3 = 0\,.$$

Under open-access, the slope of the singular arc is the positive root of

(7c) $$-(r+\delta)\alpha + (1-\eta_1\alpha)g + g^2 = 0.$$

This equation is obtained using the fact that P = c on the singular arc, and equations (1) - (3).

In order for the equilibrium for the precommitted monopolist to be easily comparable to the previous cases, we require that the initial stock of the durable good be 0: $Q_0 = 0$. Without this assumption, the extraction path for the precommitted monopolist depends on the initial stock and calendar time as well as the current state. If $Q_0 = 0$, the slope of the singular arc here is the unique positive root of[5]

(7d) $$-r\frac{\alpha}{2} - \eta_3 g + g^2 = 0 .$$

The derivation of (7a) - (7c) assumes that $\delta > 0$. However, it is straightforward to show that in each case the equilibrium is continuous in δ at $\delta = 0$. When $\delta = 0$ the equilibrium stock level under competition or monopoly (without precommitment) equates price and marginal cost. We can verify by direct calculation that this is the level given by the point d in Figure 1 when $g = \alpha r$.

For the limiting case $\delta = 0$ the positive root is $g = r\alpha$ for each of the three equations (7a) - (7c); the roots are different when $\delta \neq 0$. Since the root of the respective equations completely characterizes the equilibrium trajectory, this result implies that the competitive equilibrium is the same with or without property rights if and only if $\delta = 0$. If $\delta = 0$ there is no reason to conserve the resource: anything that is worth extracting is extracted immediately, and there are no resource rents. More importantly, the result implies that $\delta = 0$ is necessary and sufficient for the Coase Conjecture to hold in this setting. We restate this as as

[5] The Hamiltonian to the precommitted monopolist's problem is $H = [P - c(S) - \lambda_1 + \lambda_2]m - \lambda_2\delta Q + \lambda_3[(r+\delta)P - F(Q)]$, where λ_1, λ_2, and λ_3 are, respectively, the costate variables associated with the states S, Q and P. We use the first order conditions to this problem, together with the boundary condition $\lambda_3(0) = 0$ (because P is a "jump state"), and specialize to the linear example in equation (6), to obtain (7d).

Lema 1: In the nonrenewable resource durable goods model with primitive functions given by (6), the competitive, open-access, and monopoly equilibria and corresponding profit levels are equal if and only if $\delta = 0$.[6] ‖

Given the linear model (and, for the precommitted monopolist, the assumption that $Q_0 = 0$), the equilibrium extraction paths are completely characterized by the slope of the respective singular arcs, g, a parameter which depends on $\alpha \equiv k/b$, r, δ, and the market structure. Hereafter we assume that $Q_0 = 0$, so for all market structures there is an initial jump to the singular arc. Given an initial condition (before the jump) of $S_0 > 0$, $Q_0 = 0$, the level of Q after t units of time is $Q_t = (gS_0/(1 + g))\, e^{-\rho t}$, where $\rho \equiv \delta g/(1 + g)$. A larger value of g implies a larger initial jump and a faster decay rate. Therefore, for a given initial value S_0, the resource stock remaining at $t > 0$ is a decreasing function of g. A smaller value of g implies a more conservative extraction path. We define g^c, g^m, g^o and g^p as, respectively, the positive roots of (7a) - (7d). (The mnemonic is c = competitive, m = monopoly, o = open-access, and p = precommitted monopolist.) We have

Lema 2: For the linear functions in (6) with $a/(r+\delta) = k_o$ and $\delta > 0$, (i) $g^p < g^c < g^m < g^o$, and (ii) for values of (S,Q) which satisfy $S > 0$ and $Q \leq g^c S$, $P^m(S,Q) < P^c(S,Q)$.‖[7]

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[6] Proposition 1 of Karp (1993) shows that Lema 1 holds for general functional forms, if a differentiable MPE exists; that paper states that $\delta = 0$ is necessary and sufficient for the monopoly and competitive equilibria to be equal. However, that statement should be modified, by requiring that $(S_0,Q_0) \notin \Sigma$. If $(S_0,Q_0) \in \Sigma$, the monopoly and competitive equilibria are the same, regardless of the value of δ. I thank Brian Wright for bringing this to my attention. Note that in our linear example, Σ is empty.

[7] If (S,Q) lies in the cone bounded by the two singular arcs $Q = g^c S$ and $Q = g^m S$, then the constraint $m \geq 0$ is binding for the competitive firm. We can still obtain a closed form expression for the competitive price function in this region, but it is complicated, and we are

Proof: The inequalities in (i) can be established by comparing the positive roots of (7a) - (7d). However, the only surprising inequality is the second, which implies that the monopolist extracts more rapidly than the competitive firm. We provide a geometric proof of this inequality, and in the process, we also verify part (ii). The linear example is convenient because it implies that for both the competitive and monopoly cases, the form of the equilibrium is the same. In both cases the price function is k_o - B(S+Q) when the constraint $m \geq 0$ is not binding, and the singular arc is linear, Q = gS. The comparison of equilibria therefore amounts to a comparison of equilibrium values of B and g. For an arbitrary value of g, the rational expectations constraint (3) determines an equilibrium value of B.

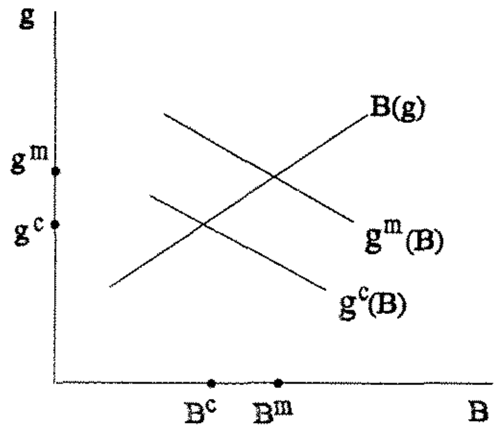

Figure 3 Equilibrium for the linear model

This relation is written in equation (A6) of the appendix, and is graphed as B(g) in Figure 3. A larger value of g means that more of the good is sold, so the price is lower, and B is larger. [Differentiation of equation (A6) confirms the fact that $B'(g) > 0$.] For an arbitrary value of B, solution of the monopoly and competitive firms' problems determine the equilibrium values of g^m and g^c. We denote the dependence on B of these values by writing

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not able to compare it with $P^m = k_o - B^m(S+Q)$. However, by continuity, the competitive equilibrium price must be less than P^m for $Q = g^cS + \varepsilon$, for small ε (i.e., slightly above the competitive singular arc). We conjecture that this relation holds at all points in the cone bounded by the two arcs. If this conjecture is correct, it is obvious that $P^m < P^c$ for $Q > g^mS$ (i.e., at points above the monopoly singular arc).

them as $g^m(B)$ and $g^c(B)$. Using equation (4) and the linear functions we can write $g^c = r(k-B)/(r+\delta)B$, so $dg^c/dB < 0$, as shown; a larger B implies lower demand which implies lower sales. Proposition 1 implies that for arbitrary B, $g^c(B)$ lies below $g^m(B)$, as is shown in Figure 3. (Our argument does not rely on the sign of dg^m/dB.) From Figure 3, it is immediate that $g^m > g^c$ (in equilibrium) and that $B^m > B^c$.

The Lema, together with the comments immediately preceding it, imply:

Proposition 2: For the linear functions in (6) with $a/(r+\delta) = k_o$, $\delta > 0$, the ordering of equilibrium extraction paths, from "most conservative" to "least conservative" is: precommitted monopolist, competitive seller, monopolist (who cannot precommit), and finally, open-access. ‖

In the previous section we showed that for general demand and cost functions which satisfy Assumption 1, the monopolist would want to deviate from the competitive equilibrium by producing more rapidly than competitive firms. Proposition 1 compared the competitive and monopoly outcomes under the counter-factual assumption that both faced the same endogenous price function. Proposition 2 compares the two equilibrium outcomes when the two types of sellers face their respective equilibrium price functions.

In view of the fact that the equilibrium trajectory under the monopolist lies between the trajectories under competition and open-access, we expect the PDV of industry profits under monopoly to lie between the PDV of profit levels under competition and open-access. Given a value of g and an initial condition $Q_0 = 0$, $S_0 > 0$, the equilibrium PDV of producer profits, under all market structures, equals $\gamma(g,\alpha,r,\delta,b)\cdot S_0^2$. The function $\gamma(\cdot)$ is very complicated, but for Proposition 3 below, it is sufficient to note that it is analytic at $\delta = 0$. The complexity of $\gamma(\ )$ makes it difficult to establish that market power is disadvantageous in general; the problem is that we can not show that $\gamma(\cdot)$ is globally concave in g.[8] However, we have the following local result, for small positive values of δ.

Proposition 3: For sufficiently small positive δ and for the linear functions in (6) with $Q_0 = 0$, monopoly profits are strictly less than industry profits in the competitive equilibrium and strictly greater than industry profits in the open access equilibrium.‖

To verify that power is disadvantageous for large δ, we performed several hundred simulations for r in the range of (.01, .1), δ in the range of (.001, 3) and α in the range of (.1,3). In *all* cases, monopoly power was disadvantageous. In a few cases, profits under monopoly are less than half of competitive profits. For small values of δ, however, the loss due to market power is negligible; this is consistent with the discussion of the limiting case $\delta = 0$, and Lema 1.

In order to determine whether disadvantageous market power is a peculiarity due to the use of an infinite horizon, infinitesimal period of commitment model, we compared equilibrium payoffs under monopoly and under competition in an N-stage model with a finite period of commitment, ε, for the functions given in (6). We found that for large but finite N and small but positive ε, the monopolist receives a lower payoff.

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[8] We know that industry profits are maximized under the precommitted monopolist, and that those profits equal $\gamma(g^p)S_0^2$. Therefore g^p maximizes $\gamma(g)$. If we knew that γ were globally concave in g, we could use the fact that $g^p < g^c < g^m$ to establish that $\gamma(g^c) > \gamma(g^m)$.

4. Discussion of the Results

In the discussion of Proposition 1 we emphasized that $P^c(S,Q)$ is constant on 45° lines below the singular arc; the derivation of (4), which was used in the proof of the Proposition, relies on this fact. This characteristic of P^c is due to the assumption that costs are linear in the rate of extraction. If costs were non-linear in extraction, the first order effect of a decrease in S would not exactly offset the effect of an increase in Q, so the intuition which preceded the Proposition would not hold.

The importance of a small period of commitment, which was verified by numerical experiments, can also be explained by reference to Proposition 1. In the discrete time analog to the model, the seller with a period of commitment of ε chooses sales of $m\varepsilon$ at the beginning of a period. If the endogenous price function is P(S,Q), and the stock at the beginning of period t is S_t,Q_t, the seller obtains the price $F(Q_t+m_t\varepsilon)\varepsilon + e^{-(r+\delta)\varepsilon}\cdot P[S_t - m_t\varepsilon, e^{-\delta\varepsilon}(Q_t + m_t\varepsilon)]$. The effect of m_t on the current price is of the same order of magnitude as ε, so for $\varepsilon > 0$, the monopolist seller has the usual incentive to *restrict* sales, which opposes the incentive to increase sales identified in Proposition 1. For a sufficiently large period of commitment, the usual incentive dominates, and market power must be advantageous.

The intuition for Proposition 1 also relied on the assumption that the future horizon was substantial. For $\delta > 0$, a positive deviation from the competitive sales path eventually has a larger affect on the trajectory of S than on the trajectory of Q, and therefore increases price. This would not happen if $\delta = 0$ or if the time horizon were negligible. Again, the numerical experiments confirm the importance of a long time horizon.

In a two-period version of our model market power can never be disadvantageous. It is worth stating the reason for this because it illustrates how the intuition from two-period models can be misleading. At the beginning of the first stage, the monopolist is capable of selling the same amount as her competitive counterpart. This leaves the monopolist at the same state as her counterpart; the value of being at that state in the second stage is greater for the monopolist. In addition, the first-period profits are greater for the monopolist if she sells the same amount as her competitive counterpart: the monopolist receives a higher price since buyers anticipate that future sales will be lower under the monopolist than under the competitive counterpart.

This line of reasoning does not extend to a model with many periods. The above argument is based on the assumption (which is valid for two periods) that, if the monopolist were to imitate her competitive counterpart in the current stage, buyers would expect her to sell less in subsequent stages than her counterpart does. With a sufficiently large number of periods, however, control over current sales does not enable the monopolist to insure that future sales, and the future stock of the durable good, will always be lower under monopoly. The competitive producer will eventually exhaust the nonrenewable resource, and leaving some (economically viable) resource in the ground cannot be part of a Markov equilibrium under the monopolist. Therefore, for the infinite-horizon model when $\delta > 0$, the trajectories of the stock of durable good under the monopolist and the competitive producer are either identical or cross. (When $\delta > 0$, the stock of the durable good approaches 0 and the monopolist will eventually want to sell more of the good.) Therefore, the monopolist and the competitive producer extract the same amount of the resource and produce the same amount of the durable good asymptotically. This means that the monopolist cannot credibly commit to having a lower stock of the durable good present in the market at each point in the future.

We explained why the intuition from the two period model can be misleading. However, that model does help us understand why mimicking competitive behavior in each period, although feasible, is not an equilibrium strategy. Selling at the competitive level in the current period is not a signal (in a MPE) that this strategy will be followed in the future. Since sales price depends to a large extent on future sales, selling at the competitive level in the current period does not give the monopolist the competitive level of profits. This is true in a two period or a many-period model. The surprising result, of course, is that mimicking competitive sales early in the program does not enable the monopolist to do at least as well as the competitive firm, if there are many periods remaining. The monopolist may have lower profits than the competitive firm.

5. Exploration and Strategic Destruction[9]

Thus far we have assumed that the initial stock of the resource is exogenous, and that the stock can be decreased only by selling it, i.e., by adding to the stock of the durable good. In this section we consider the monopolist's incentives for exploration and/or strategic destruction of the resource. We also compare these incentives with those of a competitive firm.

For a given level of Q, a reduction in S increases the equilibrium price, because buyers know that future sales must be reduced. If the monopolist were able to destroy the

[9] I thank an anonymous referee for suggesting the topic of this section.

stock, she would benefit from the price increase. Of course she would lose because the volume of future sales must fall, and because extraction costs increase. If the benefit ever exceeds the loss, then the monopolist has an incentive to destroy part of the stock for strategic reasons. We show that for the linear example in equation (6), strategic destruction does not occur, but that in other cases it can occur.

Consider the monopolist's incentives in the first instant, when the initial stock is at a point like d' in Figure 1, where $S_0 > 0$ and $Q_0 = 0$[10]. As we stated in Section 3, for our linear example, the monopolist's value function is $\gamma^m S^2$, which is increasing in S. Clearly, the monopolist would have no incentive to destroy any of the resource before the jump to the singular arc. Now suppose that the monopolist is on the singular arc, at a point like $d \equiv (S_d, gS_d)$ in Figure 1. If buyers expect the monopolist to remain on the singular arc (as the equilibrium predicts) and if the monopolist actually follows this path, then we can again show (using tedious but straightforward calculations) that the payoff is quadratic: θS_d^2. The function θ depends on the exogenous parameters, but not on the state (S,Q), provided that $Q = g^m S$. At point d, the monopolist does not want to deviate by destroying some of the resource, e.g. causing the state to jump to a point $d'' \equiv (S_d - \Delta, gS_d)$ to the left of d ($\Delta > 0$). The point d'' is above the singular arc, so from that state the monopolist's best policy is to set m = 0 until the stock of durable good decays to $g(S_d - \Delta)$. Suppose this takes τ units of time; during this interval the monopolist receives no revenue and incurs no costs. The payoff from destroying Δ units of stock is $e^{-r\tau}\theta(S_d - \Delta)^2 < \theta S_d^2$. The same kind of argument shows that

[10] The assumption that $Q_0 = 0$ results in tremendous simplifications. If $Q_0 > 0$, the value of the jump to the singular arc includes cross-product terms in Q and S.

it is not optimal to destroy the stock when the state is above the singular arc. In summary, (for our linear example) the monopolist has no incentive to destroy the stock, either before a jump, or while on the singular arc, or if, for some reason, the state is above the singular arc.

We noted that the monopolist incurs two types of costs from raising the price by destroying the stock: (i) the reduction in future sales and (ii) the increase in extraction costs. The first cost necessarily occurs, but the existence of the second cost is merely an assumption. We can imagine situations where extraction costs do not depend on the remaining stock. To show that strategic destruction can occur, we therefore consider the model where $c(S) \equiv 0$. In this case, it is easy to show, using equation (4), that a sufficient condition for the competitive singular arc to intersect the Q axis at a point $Q^* > 0$, is $F'(0) > -\infty$. Suppose that this inequality holds, so that Σ is not empty.

For any initial condition $(S_0,Q_0) \varepsilon \Sigma$, exhaustion is instantaneous in the competitive equilibrium, and the price in the first instant is $P^{\sigma c}(S_0,Q_0) = \int_0^\infty e^{-(r+\delta)\tau}F([S_0+Q_0]e^{-\delta\tau})d\tau$ (obtained using equation (3) and the fact that future sales must be 0, since the resource is exhausted). We use the superscript σc to remind the reader that this is the competitive price function for points in Σ. Assumption 1 holds for points in Σ if F(Q) is not "very convex"; we adopt this assumption. From Proposition 1, we then know that if the monopolistic seller faces this price function, she will want to exhaust the resource immediately, provided that strategic destruction is not an option. Therefore, if strategic destruction is impossible and $(S_0,Q_0) \varepsilon \Sigma$, the function $P^{\sigma c}(S,Q)$ is a Markov equilibrium price function, and monopoly profits are $P^{\sigma c}(S_0,Q_0)S_0$, the same as competitive profits. (Proposition 1.ii.)

If the monopolist can destroy Δ units of the resource in the first instant, monopoly profits in the ensuing Markov equilibrium are $\Pi(S_0,Q_0,\Delta) \equiv P^{oc}(S_0-\Delta,Q_0)[S_0-\Delta]$. Therefore, a sufficient condition for strategic destruction to be an equilibrium strategy is that Π is an increasing function of Δ for $(S_0,Q_0) \in \Sigma$.[11] We can show by example that this sufficient condition may or may not be satisfied, depending on the parameters of the problem. We do this using the case of linear rental demand, $F(Q) = a - bQ$ and zero extraction costs. Substituting the linear $F(Q)$ into P^{oc}, setting $S = 0$, and using equation (4), implies that $Q^* = ar(r+2\delta)/b(r+\delta)^2$. For simplicity, suppose that $Q_0 = 0$, so that exhaustion is immediate if and only if $S_0 \leq Q^*$. Define $\hat{S}$ as the value of S that maximizes $P^{oc}(S,0)S$. For linear F(Q), $\hat{S} = a(r+2\delta)/2b(r+\delta)$. Consequently, the monopolist's optimal policy is to destroy $\Delta = S_0 - \hat{S}$, for $S_0 \in (\hat{S}, S^*]$. In other words, for $Q_0 = 0$, there are initial value of S_0 such that the monopolist would want to engage in strategic destruction, provided that $\hat{S} < S^*$. A necessary and sufficient condition for the last inequality is $\delta < r$. In summary, under the maintained assumption that $Q_0 = 0$, strategic destruction occurs if $\delta < r$ whenever $\hat{S} < S_0 \leq S^*$; if $\delta > r$, strategic destruction never occurs when $S_0 \leq S^*$.

We now briefly consider the monopolist's incentives to add to the stock, e.g. by costly exploration. Again, we begin with the example in equation (6) and assume that $Q_0 = 0$. At the initial instant before a jump, the payoff is $\gamma^m S^2$. Since this function is not only increasing, but is also convex, we see that the incentives to add to the stock are higher, the larger is the stock. The reason for this is that (by assumption) an increase in the stock decreases

[11] For points outside of Σ, instantaneous exhaustion does not occur in the competitive equilibrium, so the price function we identified is not the competitive price function.

extraction costs. This saving is more valuable, the greater is future extraction, i.e., the greater is the stock. Thus, if it ever pays the monopolist to incur the exploration cost necessary to increase the stock, she will want to do so immediately. From our example with zero extraction costs, is obvious that in some cases the monopolist would not engage in exploration.

Finally, consider the relative incentives of the monopolist and the competitive firm to engage in exploration. As we noted in Section 3, for all our simulations, $\gamma^c > \gamma^m$, and Proposition 3 states that this inequality must hold when δ is small. For these cases, at least, the marginal value of a unit of stock is greater for the competitive firm for the example in equation (6). Therefore, although the monopolist has an incentive to increase its stock size, this incentive is greater for the competitive firm. Our discussion of the case where extraction costs are 0, showed that the marginal value of a unit of stock might be negative for the monopolist. The marginal value for the competitive firm, which takes price as given, is always non-negative. This suggests that the incentives for exploration tend to be lower for the monopolist under quite general circumstances.

6. Conclusion

If (i) a monopolist produces a durable good which depreciates at a constant, positive rate, (ii) average costs are independent of the rate of production, (iii) the monopolist is unable to precommit, and sells to nonstrategic buyers with rational expectations, and (iv) production requires a nonrenewable resource, also owned by the monopolist, then the natural extension of the Coase Conjecture fails, and may do so in a surprising way: monopoly power, instead of disappearing, can be disadvantageous. This may provide an explanation for the paucity of cartels selling durable resources; not only is it difficult to create and maintain a cartel, but the reward to doing so may be negative.

Our results do not imply that a durable nonrenewable resource monopoly is necessarily disadvantageous. There are three types of circumstances where monopoly would be advantageous. First, the equilibrium may be non-Markov, as appears to be the case with the DeBeers cartel. Second, the equilibrium may be Markov, but the time horizon may be sufficiently short, or the period of commitment sufficiently long, that the result conforms to the intuition we obtain from two-period models. Third, there may be something about the technology that provides a substitute for commitment in a Markov equilibrium. Examples of this include an upper limit on the rate of extraction, or costs which are convex in the rate of extraction, or the possibility of strategic destruction.

In addition to providing insight into nonrenewable resource durable goods monopolies, the analysis is of more general interest. We provided an example of a situation where market power cannot be disadvantageous in a two-period setting. When we examine the reason for this, we recognize that the assumption of two (or a small number of) periods is critical to the result. Previous examples of disadvantageous market power used two-period models. These examples may have suggested that, in situations where market power can be disadvantageous, this possibility can be established in a simple manner. Our counter-example suggests that the possibility of disadvantageous market power may be very hard to detect, since it may be due to factors which are assumed away when we work with two-period models. Disadvantageous market power may be more prevalent than is commonly thought.

Appendix: Derivation of (7a) and (7b) and Proof of Proposition 3

Derivation of (7a): An interior solution ($0 < m < \infty$) to the competitive firm's maximization problem requires $dP/dt = r[P - c(S)]$. Setting this equation equal to the second part of (3) implies

(A1) $$P = [F(Q) - rc(S)]/\delta .$$

This equation holds only on the singular arc, $Q = G(S)$. Differentiating this function with respect to time, using (1) and (2), implies

(A2) $$m = \frac{\delta G(S)}{1 + G\prime(S)}$$

on the singular arc. Differentiating (A1), using (3), (A1) and (A2), results in

(A3) $$\begin{aligned} rF(G(S)) - r(r + \delta)\, c(S) = \\ -F\prime(G(S))\delta G(S) + \frac{(rc\prime(S) + F\prime(G(S))\, \delta G(S)}{(1 + G\prime(S))}. \end{aligned}$$

We substitute the linear functions from (6) into (A3) and use the "trial solution" $G(S) = g_o + gS$ in the result. Equating coefficients of S and 1 implies that g solves (7a), and $g_o = 0$.

Derivation of (7b): The derivation proceeds in three steps. We first explain why we restrict attention to a particular form for the Markov equilibrium. We then show that the positive root to (7b) defines the equilibrium. Finally, we show that the positive root is unique.

Step 1. We look for an equilibrium with the following two characteristics: (i) There is a linear singular arc $Q = gS$; for $Q < gS$ the state jumps to the singular arc, and for $Q > gS$ extraction is 0 until the arc is reached. (ii) The endogenous price function is linear, with $P = k_o - B(S+Q)$ when the constraint $m \geq 0$ is not binding. Solving for this equilibrium thus requires finding the two constants g and B. "Step 1" of our argument explains why these two characteristics of the equilibrium are reasonable.

We view the continuous time model as a means for studying the behavior of a discrete time model in which the period of commitment (the length of a stage) is small, and the horizon is large. Therefore we restrict attention to equilibria of the continuous model that have the same characteristics as the limit (as the period of commitment vanishes) of the equilibrium of the discrete time model. This discrete time model is, in turn, taken to be the stationary limit, as the horizon becomes infinite, of a finite horizon model. Thus, we can use dynamic programming and backward induction for the discrete model. In other words, we begin with a discrete stage model with the length of each stage ε and a finite number of stages, N. We let $N \rightarrow \infty$ to obtain a stationary solution. We then let $\varepsilon \rightarrow 0$ to obtain a continuous time model.[12]

I. Over a period during which the rate of extraction is m, the amount extracted is $m\varepsilon$. The discrete versions of equations (1) and (2) are $S_{t+\varepsilon} - S_t = -m\varepsilon$, and $Q_{t+\varepsilon} = e^{-\delta\varepsilon}(Q_t + m\varepsilon)$. To simplify notation, we shall assume that the amount $m\varepsilon$ becomes available instantly at the beginning of a stage, and depreciation occurs at the end of the stage. That is, if the stock is Q_t at the start of a stage before sales, then it is $Q_t + m\varepsilon$ as soon as sales have occurred, and it remains at that level until the end of the period, when it decays to $e^{-\delta\varepsilon}(Q_t + m\varepsilon)$. This assumption has no effect on the limiting case where $\varepsilon \rightarrow 0$, but it simplifies the description of the discrete model. The cost of extracting $m\varepsilon$ given an initial stock of S_t is $\int_0^\varepsilon (k_o - kS_\tau)m\,d\tau =$

[12] The remaining paragraphs in Step 1 are preceded by Roman numerals, in order to facilitate communication with referees. These numerals will be dropped in future versions of the paper.

$(k_o - kS_t)m\varepsilon + [k(m\varepsilon)^2]/2 \equiv C(S_t,m;\varepsilon)$. Note that the portion of costs that is quadratic in m, is $o(\varepsilon)$ (it is proportional to ε^2).

II. For the finite horizon model, the seller has N opportunities to sell the good. After the N'th sale, consumers continue to use the purchased stock, until it erodes to 0, but no further sales are made. Given the linear rental and cost functions defined in equation (6), then in period N, if the constraint $m \geq 0$ is not binding, the equilibrium has three important characteristics, stated in the following Remarks. Remark (i): The equilibrium sales rule is a linear function of the state vector, (S_N,Q_N). Remark (ii): The monopolist's value function is quadratic in the state. Remark (iii): The equilibrium price is linear in the state (i.e., $P_N = A_N - B_{1N}S_N - B_{2N}Q_N$ for some numbers A_N, B_{1N}, B_{2N}). Remark (iii) requires some explanation. Given our assumptions about timing, buyers are willing to pay $[a - b(Q_N + m\varepsilon)]\{1 - e^{-r\varepsilon}\}/r + e^{-(r+\delta)\varepsilon}P_{N+1}(Q_{N+1})$ for a unit of the stock at stage N. The function P_{N+1}, which is derived from the linear rental function F(Q), is linear, and by Remark (i) m is a linear function of the state vector. This establishes Remark (iii).

III. We can now use a standard inductive proof to establish that Remarks (i), (ii) and (iii) hold at stage $t \leq N$, provided that the constraint $m \geq 0$ is not binding. This is true for all finite N. We can then consider the infinite horizon limit, as $N \to \infty$. In order for the stationary equilibrium to be the limit of the equilibrium to the finite stage problem, Remarks (i) - (iii) must hold.[13] In particular, when the constrain $m \geq 0$ is not binding, the control rule

[13] We have not established uniqueness in the infinite horizon model, so we can not rule out that there are other stationary Markov equilibria which have different properties, e.g., non-linear endogenous price functions. The issue of uniqueness is addressed in the text.

is linear, the value function is quadratic, and the endogenous price function is linear ($P(S,Q) = A - B_1S - B_2Q$).

IV. We now want to establish that the discrete stage dynamic programming problem is linear-quadratic in the control, m, and more importantly, that the quadratic terms in m are $o(\varepsilon)$. This means that as $\varepsilon \to 0$, the maximization problem becomes linear in m, so that the solution involves a singular arc and bang-bang controls. The discrete stage DPE is

$$J(Q_t,S_t) = \max_m \left[\left[a - b\left(Q_t + m\varepsilon\right)\right] \frac{1-e^{-r\varepsilon}}{r} + e^{-(r+\delta)\varepsilon} P(Q_{t+\varepsilon},S_{t+\varepsilon}) \right] m\varepsilon - C(S_t,m;\varepsilon)$$
$$+ e^{-r\varepsilon} J(Q_{t+\varepsilon},S_{t+\varepsilon}) \ .$$

The first term on the right side in square brackets is the current price. Note that this function is $0(\varepsilon)$ in the control, m. That is, for non-infinitesimal ε, the monopolist can affect the current price, but her ability to do so diminishes as ε shrinks. Since the equilibrium price in the next period is a linear function of the state, which is a linear function of m, revenue in the current period is a linear-quadratic function of m. However, from the previous comments, the revenue terms which are quadratic in m are $o(\varepsilon)$. We mentioned above that current extraction cost $C(S,m;\varepsilon)$, is quadratic in m, with the quadratic term being $o(\varepsilon)$. By Remarks (i) and (ii), $J(Q,S)$ is a quadratic function of the state. Since the state is linear in $m\varepsilon$ $\partial^2 J(Q_{t+\varepsilon},S_{t+\varepsilon})/\partial m^2$ is proportional to ε^2. In summary, the terms in the DPE which are quadratic in m are proportional to ε^2. If we Taylor expand the discrete stage DPE around $\varepsilon = 0$ and take limits to obtain the continuous time control model, we obtain a problem which is linear in m, as we set out to show. We summarize this as Remark (iv): The continuous time control problem is linear in m, and therefore involves a singular arc and bang-bang controls.

V. We now use Remarks (ii) and (iv) to establish that the singular arc (for the continuous time limit) is linear: $Q = gS$. To this end, define the state vector $y \equiv (S,Q)$. Using Remark (ii), for the discrete stage problem, m is a linear function of the state on a two dimensional set in the plane S,Q (where $m \geq 0$ is not binding). In this set we can substitute the linear control rules into the state system, and write the dynamic system as $y_{t+\varepsilon} = W(\varepsilon)y_t$, or $[y_{t+\varepsilon} - y_t]/\varepsilon = [W(\varepsilon) - I]y_t/\varepsilon$. The matrix W is obtained by substitution of the equilibrium control rule into the state equation; I is the identity matrix. We have also used the fact that the system must converge to the point $(S,Q) = (0,0)$, so the linear system does not contain an intercept. Define $\tilde{W} = \lim_{\varepsilon \to 0}[W(\varepsilon) - I]/\varepsilon$, so that the limiting form of the linear difference equation is $\dot{y} = \tilde{W}y$. By Remark (iv), m takes interior values only on the singular arc (which is of dimension 1) in the continuous time problem. Consequently, $\tilde{W}$ must have a non-negative eigenvalue. (Otherwise, a jump would not be required at a state off the singular arc.) Therefore, the solution to $\dot{y} = \tilde{W}y$ implies $Q/S = n_1e^{ht}/n_2e^{ht} \equiv g$, where h is the negative eigenvalue of $\tilde{W}$ and n_1, n_2 determine the singular arc.

VI. Finally, we use Remark (iii) and (iv) to explain why we require $P = k_o - B(S+Q)$. By Remark (iii) we want a linear price function when $m \geq 0$ is not binding: $P = A - B_1S - B_2Q$. By Remark (iv) buyers anticipate a jump (in the continuous time problem) when the state is below the singular arc. Therefore, price must be constant on 45° lines such as dd' in Figure 1, so $B_1 = B_2 = B$. We noted in the previous paragraph that the singular arc must intersect the origin. (When $Q = 0$, the monopolist would not stop extracting if $S > 0$. Given the assumption that $a/(r+\delta) = k_o$, it would not be optimal to exhaust the resource while $Q >$

0.) When S = Q = 0 buyers would be willing to pay $a/(r+\delta) = k_o$ for a unit of the resource, so $A = k_o$.

Step 2. Equations (A2) and (1) imply $\dot{Q} = -\delta gQ/(1+g)$. Use this expression in the time derivative of $P = k_0 - B(S + Q) = k_0 - B(1 + g)\,Q/g$ to obtain

(A4)
$$\dot{P} = \delta BQ.$$

Substitute the linear expression for P into (3) to obtain

(A5)
$$\dot{P} = (r + \delta)\left(k_0 - \frac{(1+g)\,B}{g}Q\right) - (a - bQ)$$
$$= \left[b - \frac{(r+\delta)\,B(1+g)}{g}\right]Q.$$

Equating (A4) and (A5) implies

(A6)
$$b - \frac{(r+\delta)\,B(1+g)}{g} = \delta B.$$

Define $\sigma \equiv k/B$, $\eta_0 \equiv (r + \delta)/(r + 2\delta)$, $\eta_1 \equiv r + 2\delta$ and use the definition of α ($\equiv k/b$) to rewrite (A6) as

(A7)
$$\sigma = \frac{\alpha\eta_1}{g}(g + \eta_0).$$

We now turn to the monopolist's maximization problem. We use the fact that the endogenous price function is differentiable on the singular arc, $Q = gS$, with

(A8)

$$\left.\frac{\partial P}{\partial Q}\right|_{S,\, gS} = -B.$$

Differentiability, on the singular arc, of the price function, can be established using the same type of calculations which are used to establish Lema 1 in Karp (1993).

The Hamiltonian and first order conditions for the monopolist's problem are

(A9)

$$H = [P - c + \lambda - \eta]m - \lambda\delta Q$$

(A10)

$$\dot{\lambda} = (r + \delta)\lambda - \frac{\partial P}{\partial Q}m$$

(A11)

$$\dot{\eta} = \left(- \frac{\partial P}{\partial S} + \frac{dc}{dS}\right)m + r\eta$$

(A12)

$$P - c + \lambda - \eta = 0 \ .$$

The variables λ and η are the costate variables associated with Q and S, respectively.

We now differentiate with respect to time equation (A12) and substitute (1), (2), (A10) and (A11) into the result; eliminate η using (A12), to obtain

(A13)

$$\delta\left(\lambda - \frac{\partial P}{\partial Q}Q\right) = r(P - c)$$

which holds on the singular arc. We now differentiate (A13) with respect to time, using (A8), (A10), and the linear price function, to obtain

(A14) $$\delta[(r + \delta)\lambda + Bm + B(m - \delta Q)] = r(-B(1 + g) + k)\,\dot{S},$$

which implies $\delta(r + \delta)\,\lambda = [r(k - B(1 + g)) + 2\delta B]\,\dot{S} + \delta^2 BgS$. Use (A13) to eliminate λ and rearrange to obtain

(A15) $$\gamma_0 S = \gamma_1\,\dot{S}$$

where $\gamma_0 \equiv (r + \delta)\,rk - Bg((r + \delta)^2 + \delta^2) - r(r + \delta)\,B$ and $\gamma_1 \equiv rk - B(r + rg - 2\delta)$. Use $-\dot{m} = \dot{S} = -\delta gS/(1 + g)$ in (A15) to obtain $\gamma_0 = -\gamma_1\delta g/(1 + g)$. Rearrange this equation, using previous definitions and $\eta_2 \equiv (r + \delta)/r$ to obtain

(A16) $$\eta_0(1 - \sigma) + (2 - \sigma)g + \eta_2\,g^2 = 0.$$

Substitute (A7) into (A16) and rearrange to obtain (7b) in the text.

Table 1: Definitions of Parameters Used to Derive (7)

$\alpha = k/b$	$\sigma = k/B$
$\eta_0 = (r+\delta)/(r+2\delta)$	$\eta_1 = r + 2\delta$
$\gamma_0 = (r+\delta)rk - Bg[(r+\delta)^2 + \delta^2] - r(r+\delta)B$	$\gamma_1 = rk - B(r+rg-2\delta)$
$\eta_2 = (r+\delta)/r$	$\eta_3 = r[\alpha + (\alpha\delta - 2)/(r + \delta)]/2$

Step 3. We now show that there is a unique positive root to this cubic. To this end, note that $f(0) = -\alpha\eta_1\,\eta_0^2 < 0$ and $\lim_{g\to\infty} f(g) = \infty$, where f(g) is defined in (7b). Therefore there exists a positive root of (7b). To show that the positive root is unique, consider the

following cases: (i) $0 < \alpha\eta_1 < 1/2$, (ii) $1/2 < \alpha\eta_1 < 2$, and (iii) $2 < \alpha\eta_1$. For each of these cases we apply Descartes' Rule of Signs to verify that there exists at most one positive root.

<u>Proof of Proposition 3</u>: We will prove only that for small positive δ, $\gamma(g^c, \cdot) > \gamma(g^m, \cdot)$. The second inequality in the Proposition, $\gamma(g^m, \cdot) > \gamma(g^o, \cdot)$, can be shown by a parallel argument. We first note that because g^c, g^m and g^o are analytic functions of δ, lemas 1 and 2 imply

(A17) $$\frac{\partial g^c}{\partial \delta} < \frac{\partial g^o}{\partial \delta} \; ; \; \frac{\partial g^c}{\partial \delta} < \frac{\partial g^m}{\partial \delta}$$

evaluated at $\delta = 0$.

We now establish that $\gamma(g^c) > \gamma(g^o)$ for small δ. From Lema 1, $\gamma(g^c, \cdot) = \gamma(g^o, \cdot)$ iff $\delta = 0$. Moreover, for small positive δ it must be the case that $\gamma(g^c, \cdot) > \gamma(g^o, \cdot)$. This follows because profits on the singular arc are positive with property rights (where rent is positive) and 0 without property rights; therefore, if it were true that $\gamma(g^c, \cdot) \leq \gamma(g^o, \cdot)$ for any $\delta > 0$, it would have to be the case that profits in the initial jump were higher in the open access than in the competitive equilibrium. However, at the initial jump, Price = Marginal Cost under open access, so a marginal decrease in the initial supply would increase profits from the initial jump. Since a marginal increase in δ (from 0) makes the initial supply (from the jump) marginally smaller under competition than under open access, it follows that $\gamma(g^c, \cdot) > \gamma(g^o, \cdot)$ for sufficiently small positive δ.

Define γ^* as the value of industry profits under monopoly, competition, and open access when $\delta = 0$; γ^* is a function of α and r, but we suppress those arguments. Since γ is an analytic function of g^i and δ, and g^i is an analytic function of δ, we have, in view the previous paragraph, that for small positive δ:

$$\gamma(g^c,\cdot\,) = \gamma^* + \left[\gamma_g \frac{dg^c}{d\delta} + \gamma_\delta\right]\delta + o(\delta) >$$

(A18)

$$\gamma^* + \left[\gamma_g \frac{dg^o}{d\delta} + \gamma_\delta\right]\delta + o(\delta) = \gamma(g^o,\cdot\,)$$

$$\Rightarrow \gamma_g\left[\frac{dg^c}{d\delta} - \frac{dg^o}{d\delta}\right] > 0\,.$$

The last line of (A18) and the first inequality in (A17) imply that $\gamma_\delta < 0$ evaluated at $\delta = 0$. This inequality, together with the second inequality in (A17) imply that for small γ

$$\gamma(g^c,\cdot\,) - \gamma(g^m,\cdot\,) = \gamma_g\left[\frac{dg^c}{d\delta} - \frac{dg^m}{d\delta}\right] + o(\delta) > 0\,.$$

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REFERENCES

Ausubel, L., and R. Deneckere: "One is Almost Enough for Monopoly" *Rand Journal of Economics* 18 (2) 255 - 274.

------------ (1989): "Reputation and Bargaining in Durable Goods Monopoly," *Econometrica*, 57(3), 511-532.

Bulow, J. (1982): "Durable Goods Monopolists," *Journal of Political Economy*, 90, 314-332.

Chilton, J (1984): "The Pricing of Durable Exhaustible Resources: Comment," *Quarterly Journal of Economics* 99 (3) 629 - 637.

Coase, R. (1972): "Durability and Monopoly," *Journal of Law and Economics*, 15, 143-149.

Gatsios, K. and L. Karp (1992) "The Welfare Effects of Imperfect Harmonization of Trade and Industrial Policy," *Economic Journal* 102: 107 - 116.

Gul F. (1987): "Noncooperative Collusion in Durable Goods Oligopoly" *Rand Journal of Economics* 18 (2) 248 - 254.

Gul, F., H. Sonnenschein, and R. Wilson (1986): "Foundations of Dynamic Monopoly and the Coase Conjecture," *Journal of Economic Theory*, 39, 155-190.

Kahn, C. (1986): "The Durable Goods Monopolist and Consistency with Increasing Costs," *Econometrica*, 54, 274-294.

Karp, L. (1993) "Monopoly Extraction of a Durable Nonrenewable Resource: Failure of the Coase Conjecture," *Economica*, 60, 13-26.

------- (1994) "Depreciation Erodes the Coase Conjecture", Giannini Foundation Working Paper, U.C. Berkeley.

Kehoe, P. J. (1989), "Policy Cooperation Among Benevolent Governments may be Undesirable" *Review of Economic Studies* 56: 289-296.

Levhari, D., and R. Pindyck (1981): "The Pricing of Durable Exhaustible Resources," *Quarterly Journal of Economics*, 96, 365-377.

Maskin, E., and D. Newbery (1990) "Disadvantageous Oil Tariffs and Dynamic Consistency: the Paradox of the Disadvantageous Monopolist" *American Economic Review* 80:143-156.

Malueg, D., and J. Solow (1990): "Monopoly Production of a Durable Exhaustible Resource," *Economica* 57:29-47.

Pindyck, R. (1979): "The Cartelization of World Commodity Markets," *American Economic Review* 69, 154 - 158.

Rogoff, K. (1985), "Can International Monetary Policy Cooperation be Counterproductive?" *Journal of International Economics* 18, 199-217.

Salant, S., S. Switzer, and R. Reynolds (1983): "Losses Due to Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium," *Quarterly Journal of Economics*, 48, 185-200.

Stokey, N. (1981): "Rational Expectations and Durable Goods Pricing," *Bell Journal of Economics*, 12, 112-128.

Tsutsui, S. and K. Mino (1990): "Nonlinear Strategies in Dynamic Duopolistic Competition with Sticky Prices," *Journal of Economic Theory* 52: 136 - 161.

Ulph, A., and D. Ulph (1989) "Gains and Losses from Cartelization in Markets for Exhaustible Resources in the Absence of Binding Future Contracts" In F.

van der Pleog and A. J. de Zeeuw (eds.) Dynamic Policy Games in Economics, (Elton) Elsevier Science Publisher, North Holland.