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Neil Fleishon

August 1978

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INELASTIC COMPTON SCATTERING IN TWO DIMENSIONAL Q. C. D.*

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ABSTRACT

The hadronic contribution to inelastic Compton scattering, $M_{\mu\nu} = \int dx e^{iq \cdot x} \langle n | T^*(J_\mu(x) J_\nu(0)) | m \rangle$, is computed to leading order in N^{-1} in two dimensional Q. C. D. Various kinematic limits are investigated. It is found that in the dual Regge region of soft processes the photons interact as purely hadronic systems. Modification of dimensional counting rules is indicated by a study of a large angle scattering analogue. In several hard inclusive non-light cone processes, parton model ideas are confirmed. The impulse approximation is valid in a Bjorken-Paschos-like limit with very virtual photons. A Drell-Yan type annihilation mechanism is found in photoproduction of massive lepton pairs, leading to identification of a parton wave function for the photon.

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Introduction

The experimental realities of hadronic physics lead to the consideration of a non-Abelian gauge theory of colored gluons interacting with fermionic quarks as a possible candidate for describing the dynamics of the strong interactions. While the proven asymptotically free¹ short distance structure of the theory allows understanding of the naive parton model successes in electron-positron annihilation and deep inelastic leptonproduction, it is hoped that the long distance problems which arise in perturbation theory are indicative of a mechanism for quark confinement.

A system which has asymptotic freedom and quark confinement and, at the same time, gives an S-matrix which is analytic and unitary on the space of physical states is clearly special, being highly constrained. It is therefore of interest to ask whether the phenomenological prejudices which have developed with regard to hadronic interaction - dual Regge ideas about soft (no large final state transverse momentum) processes and parton model ideas about hard processes - are realized in such a system.

The standard strong interaction gauge theory is quantum chromodynamics (Q. C. D.), based on the group $SU(N)$, with $N = 3$, coupled to quarks transforming as the fundamental representation. The insolubility of the theory due to infrared singularities prevents the extraction of the mass shell S-matrix elements via standard perturbation theory techniques. A likely form of attack, perturbing in N^{-1} , developed by 't Hooft,² has not led to any significant

clarification of the quantitative problem in four dimensions. In two dimensions, one space and one time, however, $(Q. C. D.)_2$ can be solved exactly to leading order in N^{-1} .³

The spectrum of $(Q. C. D.)_2$ in this approximation consists of an infinite sequence of zero width color singlet bound states. There is no continuum corresponding to free quarks. Examination of hadronic amplitudes⁴ indicates consistency with unitarity in the color singlet sector with no long range forces. Asymptotic freedom is trivial due to the fact that $(Q. C. D.)_2$ is superrenormalizable. The model is thus a "realistic" model of strong interactions, in the sense that it is a consistent structure with the required properties. As such, it has often been used as a laboratory to investigate hadron dynamics with the hope that, in the process of understanding its working mechanisms, insights into the four dimensional problem might arise.

Extrapolation to four dimensions must however be regarded with caution due to the special unphysical features arising from the lack of transverse dimensions. The usual Regge ideas are inappropriate since there is no continuous independent variable associated with the t-channel and no spin. There are no dynamical degrees of freedom associated with the gluon field, so that "glueballs" cannot appear. On the other hand, there are experimental indications of strong transverse momentum damping in high energy hadronic interactions, giving rise to the speculation that two dimensional physics may be a useful first approximation to scattering in the real world.

Previous work in the 't Hooft model, $(Q. C. D.)_2$ to leading order in N^{-1} , for the most part substantiates theoretical ideas. Examination of current form factors and two current vacuum expectation values^{4,5} confirms light cone scaling ideas and shows how underlying quark dynamics is recovered from amplitudes which show only hadronic intermediate states. The hadronic production of massive lepton pairs which, though not light cone dominated, is amenable to a naive parton model impulse approximation treatment shows parton scaling structure.⁶ Soft hadronic amplitudes show power law high energy behavior with the power determined by the properties of quanta exchanged in the crossed channel, direct and crossed channel factorization, and signature, indicative of the operation of some precursive dual Regge mechanisms.^{7,8} There is, however, no indication of a pomeron in the model. Finally, a possible modification⁵ of dimensional counting rules⁹ by small dynamically determined powers is suggested.

The purpose of this paper is to examine inelastic Compton scattering in the 't Hooft model by considering the connected part of the matrix element of the time ordered product of two vector currents between mesonic states. We first compute this amplitude and then proceed to examine it in various limits of interest. The incident virtual photon is taken spacelike and the outgoing virtual photon is taken timelike in our calculation. In general we do not consider the zero mass "on-shell" photon limit in order to avoid possible unwanted effects strongly dependent on the two dimensionality of the theory.

Several authors have investigated various aspects of the Compton amplitude in this model. Einhorn⁵ has discussed the imaginary part of the forward elastic amplitude with very spacelike photons in connection with inclusive lepton production. Brower, et al. have looked at the zero photon mass limit.⁷ In this paper we examine the full inelastic Compton scattering amplitude for arbitrary photon masses.

We show that scattering in the forward high energy limit is closely related to meson-meson scattering. The scattering amplitude shows all the Regge properties attributed to hadronic scattering. In particular, we find no conclusive evidence for fixed poles arising in the inelastic Compton amplitude which are not present in the hadronic amplitude.

We examine a part of the Compton amplitude in the forward high energy limit as an ansatz for large angle scattering for comparison with the predictions of the dimensional counting picture. After modification of the counting rules due to the lack of photon degrees of freedom is accounted for, important modifications in the asymptotic power dependence and coefficient of the amplitude remain due to non-perturbative aspects of the model.

Several inclusive limits are investigated. We show that the parton "handbag" model is realized for inclusive Compton scattering at large energy and momentum transfer with very virtual photons. In the process of photoproduction of massive lepton pairs we find that the amplitude scales in agreement with the parton model prediction¹⁰ based on the Drell-Yan parton-antiparton

annihilation mechanism. We show that the partonic structure of the photon probed in this process arises totally from couplings to negative parity mesons.

The remainder of this paper is organized as follows: In Section I, we briefly review the 't Hooft model for the purpose of establishing notation and presenting formulae which will be of use. Section II contains the computation of the Compton amplitude in two frames parity reversed with respect to each other. Exclusive scattering limits with finite mass photons are treated in Section III, while inclusive processes and comparison to parton model predictions are discussed in Section IV.

I. THE MODEL

We consider a theory of massive fermions coupled to SU(N) gauge gluons with coupling g . The Lagrangian density describing the system is

$$\mathcal{L} = \frac{1}{4} \text{Tr} (G_{\mu\nu} G^{\mu\nu}) + \bar{q} (i\not{D} - m_a I) q^a \quad (\text{I.1})$$

where

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g [A_\mu, A_\nu] \quad (\text{I.2})$$

$$D_\mu = \partial_\mu I + g \not{A}_\mu \quad (\text{I.3})$$

In the above, A_μ is an $N \times N$ matrix of anti-hermitean gluon fields transforming as the adjoint representation of the group $U(N)$ and q^a is a fundamental color multiplet of spin $\frac{1}{2}$ quark fields with flavor "a". In the quark-gluon coupling the $U(1)$ gluon component has been removed so that $\tilde{A}_\mu = A_\mu - N^{-1} \text{Tr}(A_\mu) I$. The $U(1)$ component is thus a free field. (This formulation is used because it results in Feynman rules with transparent N dependence.)

't Hooft² has shown that in the limit $g \rightarrow 0, N \rightarrow \infty, g^2 N$ fixed, it is possible to expand in powers of N^{-1} such that there is a graphical correspondence order by order between the diagrams of Q. C. D. and those of dual perturbation theory. In particular, the leading order only has contributions from planar diagrams - i. e. those diagrams with no handles and no internal quark loops. The choice of a linear gauge in two dimensions eliminates complications due to multigluon vertices and ghosts, rendering the theory soluble to leading order. This is, of course, due to the fact that the gluons in two dimensions have no dynamical degrees of freedom and can in principle be eliminated entirely from the theory in favor of a four quark Coulomb interaction.

There are two important differences between the two and four dimensional theories. First, the Coulomb potential in two dimensions is linearly rising so that quark confinement is already manifest at the level of lowest order Born exchange. It is thus impossible to gain any insight into the non-perturbative confinement mechanism in four dimensions from $(Q. C. D.)_2$. Second, there is a coherence

length scale present at the bare Lagrangian level in $(Q. C. D.)_2$ since g has dimensions of mass. As a result, the theory is well behaved in the ultraviolet and asymptotic freedom is demonstrable by power counting. Though $(Q. C. D.)_4$ can be asymptotically free, there is no analogous mass scale since the theory is just renormalizable. Typical hadronic length scales like the Regge slope cannot be motivated in perturbation theory.

The 't Hooft model is solved in the $A_- = 0$ gauge. (Lightcone coordinates are used throughout: $x^\pm = x_\mp = 2^{-\frac{1}{2}}(x^0 \pm x^1)$, $x \cdot y = x_+ y_- + x_- y_+$). While this gauge choice makes Lorentz invariance manifest, the presumed parity invariance of the gauge invariant sector is obscured. Such parity relations as can be generated by calculating the same amplitudes in parity reversed frames can hopefully be shown directly by clever manipulation of the properties of bound state wave functions. While no such proofs of these identities have been found, numerical calculations are consistent with their validity.¹¹

The Feynman rules in this gauge are derived in a straightforward manner and the theory solved using Bethe-Salpeter techniques. A kind of infinite momentum frame "time ordered" perturbation scheme results where the energy variable is the plus component of the momentum. The reader is referred to Refs. 4,5 for a discussion of these subjects. The regular cutoff method of Ref. 4 is particularly well suited to rapid calculation of the amplitudes discussed in this paper.

We proceed now to discuss the basic features of the 't Hooft model in order to lay the foundation for our calculation of the Compton amplitude. Our review is brief, and we refer the reader to Refs. 3-5 for more detail.

The mass spectrum of the theory consists of a sequence of quark-antiquark bound states of alternating parity. The wave function describing the n^{th} bound state of a quark "a" and antiquark " $\bar{b}$ " (see Fig. I.A) is given by $\phi_n^{a\bar{b}}(x)$, where "x" is the Feynman scaling variable given by the ratio of the minus component of the quark momentum to the minus component of the meson momentum. The wave function satisfies a Hamiltonian equation of the form

$$H\phi_n^{a\bar{b}}(x) = (H_0 + V)\phi_n^{a\bar{b}}(x) = M_n^2 \phi_n^{a\bar{b}}(x) \quad x \in [0,1] \quad (I.4)$$

$$\phi_n^{a\bar{b}}(x) = 0 \quad x \notin [0,1] \quad (I.5)$$

where

$$H_0 \phi_n^{a\bar{b}}(x) = \left[\frac{(m_a^2 - g^2 N/\pi)}{x} + \frac{(m_b^2 - g^2 N/\pi)}{1-x} \right] \phi_n^{a\bar{b}}(x) \quad (I.6)$$

$$V\phi_n^{a\bar{b}}(x) = -\frac{g^2 N}{\pi} \int_0^1 dy \frac{\phi_n^{a\bar{b}}(y)}{(y-x)^2} \quad (I.7)$$

(Throughout, we will denote quark masses by m_i and hadron masses by M_i .) Conventionally, phases are chosen such that $\phi_n^{a\bar{b}}(x)$ is real and positive as $x \rightarrow 0$.

This implies

$$\phi_n^{a\bar{b}}(x) = (-1)^n \phi_n^{b\bar{a}}(1-x) \quad (I.8)$$

The Hamiltonian is Hermitean on the space of states with end point boundary conditions

$$\phi_n^{a\bar{b}}(x) = \begin{cases} c^{a\bar{b}} x^{\beta_a} & x \rightarrow 0 \\ (-1)^n c^{b\bar{a}} (1-x)^{\beta_b} & x \rightarrow 1 \end{cases} \quad (I.9)$$

where

$$\pi\beta_i \cot\pi\beta_i = 1 - \pi m_i^2 / g^2 N \quad (I.10)$$

The wave functions then form an orthonormal complete set

$$\sum_n \phi_n^{a\bar{b}}(x) \phi_n^{a\bar{b}}(y) = \delta(x-y) \quad (I.11)$$

$$\int_0^1 dx \phi_n^{a\bar{b}}(x) \phi_m^{a\bar{b}}(x) = \delta_{nm} \quad (I.12)$$

In the large mass limit, Eq. (I.4) can be solved approximately giving

$$\phi_n^{a\bar{b}}(x) \approx \sqrt{2} \sin n\pi x, \quad M_n^2 = \left[\frac{g^2 N}{\pi} \right] \pi^2 n, \quad n \gg 1 \quad (I.13)$$

This expression is valid except at the end points where the behavior is determined by Eq. (I.9). The wave functions obey a useful parity relation.

$$m_a \int dx \frac{\phi_n^{a\bar{b}}(x)}{x} = \pm m_b \int dx \frac{\phi_n^{a\bar{b}}(x)}{(1-x)} \quad (I.14)$$

+(-) for odd (even) parity states

A large mass scaling relation is easily derived from Eq. (I.4)

$$\lim_{n \rightarrow \infty} \phi_n^{a\bar{b}}(g^2 N \xi / \pi M_n^2) = \phi^a(\xi) \quad (I.15)$$

$$\lim_{n \rightarrow \infty} (-1)^n \phi_n^{a\bar{b}}(1 - g^2 N \xi / \pi M_n^2) = \phi^b(\xi)$$

where

$$\phi^a(\xi) \rightarrow \begin{cases} \xi^{\beta_a} & \xi \rightarrow 0 \\ \sin \xi / \pi & \xi \rightarrow 1 \end{cases} \quad (I.16)$$

The scaling function satisfies the remarkable relation¹²

$$\int d\xi \phi^a(\xi) = \pi \left[\frac{\pi}{g^2 N} \right]^{\frac{1}{2}} m_a \quad (I.17)$$

which is gotten indirectly by demanding consistency with asymptotic freedom.

The meson propagator (see Fig. I.B) is given by

$$G^{a\bar{b}}(x, y, q^2) = \sum_k \frac{\phi_k^{a\bar{b}}(x) \phi_k^{a\bar{b}}(y)}{q^2 - M_k^2 + i\epsilon} \quad (I.18)$$

and satisfies the scaling relations

$$\lim_{q^2 \rightarrow \pm\infty} G \left(\pm \frac{g^2 N \xi}{\pi q^2}, \pm \frac{g^2 N \xi'}{\pi q^2}, q^2 \right) = g_{\pm}(\xi, \xi') \quad (I.19)$$

$$\lim_{q^2 \rightarrow \pm\infty} q^2 \int_0^1 dx G \left(x, \pm \frac{g^2 N \xi}{\pi q^2}; q^2 \right) = h_{\pm}(\xi). \quad (I.20)$$

Hadronic amplitudes may be constructed from the wave functions and propagators. We now discuss amplitudes for several simple processes which appear as subprocesses in Compton scattering. The expressions given below are valid both on and off the mass shell.¹³ For simplicity, we define our units of mass so that $g^2 N / \pi = 1$.

The three point hadronic vertex (see Fig. I.C for notation) is given by

$$\begin{aligned} \mathcal{Y}_{n;km} \left(p_n, x = \frac{p_{k-}}{p_{n-}} \right) &= 2igx(1-x) \int_0^1 dw dy \phi_k^{a\bar{b}}(w) \phi_m^{b\bar{c}}(y) \\ &\cdot \left[\frac{\phi_n^{a\bar{c}}(xw) - \phi_n^{a\bar{c}}(y(1-x) + x)}{(y(1-x) + x(1-w))^2} \right]. \end{aligned} \quad (I.21)$$

The terms of $\mathcal{Y}_{n;km}$ can be represented graphically in terms of gluon exchanges between quarks as in Fig. I.C.

Einhorn⁵ has discussed the inelastic form factor for a vector current. It is given by (see Fig. I. D for notation)

$$\begin{aligned}
 F_{\mu}^{nm}(p, x = q_-/p_-) &= -\frac{(1-x)}{x} \int_0^1 dy \phi_m^{a\bar{b}}(y) \left\{ \phi_n^{a\bar{b}}(y(1-x)+x) f_{\mu}^a(q, -\frac{y(1-x)}{x}) \right. \\
 &\quad + x^2 \int dw dv G^{a\bar{a}}(w, v; q^2) f_{\mu}^a(q, v) \\
 &\quad \left. \left[\frac{\phi_n^{a\bar{b}}(xw) - \phi_n^{a\bar{b}}(y(1-x)+x)}{(y(1-x) + x(1-w))^2} \right] \right\} \\
 &\quad + (-1)^{n+m} (a \leftrightarrow \bar{b})
 \end{aligned} \tag{I.22}$$

where

$$f_{\mu}^a(q, y) = 2 e_a q \left(g_{\mu+} - \frac{m_a^2}{2y(1-y)q_-^2} g_{\mu-} \right) = f_{\mu}^a(q, 1-y) \tag{I.23}$$

describes the direct coupling of the current to quark "a". Eq. (I.22) is valid for $0 < x < 1$, so that the current has positive minus outgoing momentum component or negative minus incident momentum component. The two terms in the curly brackets of Eq. (I.22) have a simple interpretation which we have indicated in Fig. I.D. The first represents the coupling of the bare current to the mesons while the second represents the coupling of the hadronic current content to the mesons through the three point function. This separation is gauge invariant. Only negative parity mesons (n even) appear in the current channel, as can be seen by taking $v \rightarrow 1-v$ in the integral in Eq. (I.22).

We have derived the amplitude for the decay of a meson to two vector currents with positive outgoing minus momentum components. It is given by (see Fig. I.E for notation)

$$\begin{aligned}
 \chi_{\mu\nu}^{n,a}(p_n, x_1 = q_1-/p_n-, x_2 = q_2-/p_n-) &= \\
 &\frac{i}{2g} \frac{1}{x_1 x_2} (1 - (-1)^n) \int dw dy x_2^2 G(w, y; q_2^2) f_{\nu}^a(q_2, w) \\
 &\cdot \left\{ \phi_n^{a\bar{b}}(yx_2) f_{\mu}^a(q_1, \frac{-x_2}{x_1}(1-y)) - \frac{x_1^2}{2} \int dz dv G(v, z, q_1^2) f_{\mu}^a(q_1, v) \right. \\
 &\cdot \left. \left[\frac{\phi_n(x_1 z) + \phi_n(x_2 y)}{((1-y)x_2 + (1-z)x_1)^2} \right] \right\} + (\mu \leftrightarrow \nu, q_1 \leftrightarrow q_2).
 \end{aligned} \tag{I.24}$$

We show in Appendix A that the tensor is divergenceless on both indices as demanded by current conservation when the decaying meson is on the mass shell. The decay process can be thought of as the Bose symmetrized emission of one of the currents from the meson via the inelastic form factor followed by the decay of the resulting meson into the other current, as is indicated in Fig. I.E. The coefficient of the amplitude shows explicitly that only even parity (n odd) mesons can decay to two photons, as demanded by the conditions of Bose symmetry and parity invariance.

We complete our review of the 't Hooft model with a discussion of the meson-meson scattering amplitude represented in Fig. I.F. As indicated in this figure, the amplitude can be written in terms of three point functions and contact terms in each channel. We have made the usual separation into graphs which arise from the (s,t),(u,s), and (u,t) duality diagrams. The contribution of these duality sectors to a given scattering process is determined by the flavor content of the external mesons. The expressions given here are valid for the kinematic region which will be of interest; the case where $1 > x_m, x_n > x_k$.

$$A_{jk;mn}^{(s,t)}(p_j; x_k, x_m, x_n) = -4ig^2 x_k x_n \int dw dy \frac{\phi_j(1-x_n(1-w)) \phi_k(y) \phi_m\left(\frac{x_k y}{x_m}\right) \phi_n(w)}{(x_m + wx_n - x_k y)^2} \\ + \sum_{\ell} \mathcal{Y}_{\ell; kj} \left(\frac{x_k}{1+x_k}\right) \frac{i}{s-M_{\ell}^2 + i\epsilon} \mathcal{Y}_{\ell; mn} \left(\frac{x_m}{1+x_k}\right) + \sum_{\ell} \mathcal{Y}_{\ell; jn} (1-x_n) \frac{i}{t-M_{\ell}^2 + i\epsilon} \mathcal{Y}_{m; k\ell} \left(\frac{x_k}{x_m}\right). \quad (I.25)$$

$$A_{jk;mn}^{(u,s)} = A_{jk;mn}^{(s,t)} \quad (m \rightarrow n; t \rightarrow u) \quad (I.26)$$

$$A_{jk;mn}^{(u,t)} = 4ig^2 x_k \int_0^1 dw \phi_k(w) \int_0^1 dy \left\{ x_m \phi_m(y) \phi_n\left(\frac{x_k w}{x_n}\right) \left[\frac{\phi_j(x_m y) - \phi_j(x_m - x_k(1-w))}{(x_m(1-y) - x_k(1-w))^2} \right] \right. \\ \left. + x_n \phi_n(y) \phi_m \left(1 - \frac{x_k}{x_m} (1-w)\right) \left[\frac{\phi_j(1-x_n(1-y)) - \phi_j(x_m - x_k(1-w))}{(yx_n - wx_k)^2} \right] \right\}$$

(Equation I.27 continued on the next page)

$$+ \left[\frac{p_m^2 - M_m^2}{x_m} + \frac{p_n^2 - M_n^2}{x_n} \right] \phi_j(x_m - x_k(1-w)) \phi_m\left(1 - \frac{x_k}{x_m} (1-w)\right) \phi_n\left(\frac{x_k}{x_n} w\right) \Bigg\} \\ + \sum_{\ell} \mathcal{Y}_{j; \ell n} (1-x_n) \left[\frac{i}{t-M_{\ell}^2 + i\epsilon} \right] \mathcal{Y}_{m; \ell k} \left(\frac{1-x_n}{x_m}\right) + \sum_{\ell} \mathcal{Y}_{j; m\ell} (x_m) \frac{i}{u-M_{\ell}^2 + i\epsilon} \mathcal{Y}_{n; k\ell} \left(\frac{x_k}{x_n}\right) \quad (I.27)$$

[In all formulae involving sums over intermediate states, the sums are to be done first so that the amplitude can be written as integrals over wave functions and Green's functions.] The variables s, t , and u are the conventionally defined Mandelstam variables. The terms in the meson-meson scattering amplitude correspond to the diagrams of Fig. I. F.

The external meson flavor content may also allow contributions to the two-to-two amplitude from diagrams where the quarks circulate in the opposite direction. Denoting the amplitude represented by any given graph by A and that of the same graph with arrows reversed by $\bar{A}$, A is related to $\bar{A}$, apart from changing flavor labels, by

$$\bar{A}[\phi(x)] = A[\phi(1-x)] \quad (I.28)$$

II. THE COMPTON SCATTERING AMPLITUDE

We now turn to the calculation of the scattering process $\gamma^*(q) + n(p_n) \rightarrow \gamma^*(q') + m(p_m)$ in the 't Hooft model by considering the Lorentz tensor $M_{\mu\nu}^{nm}(q, q'; p_n, p_m) = \int d^2 x e^{-iq \cdot x} \langle n | T^*(J_{\mu}(x) J_{\nu}(0)) | m \rangle$ where γ is the quantum associated with the field whose source is the electromagnetic current J_{μ} . This current is taken to be a

Lorentz vector, flavor conserving, local bilinear in quark fields

$$J_\mu(x) = \sum_a e_a : \bar{q}^a(x) \gamma_\mu q^a(x) : \quad \text{II.1}$$

The states $|n\rangle$ and $|m\rangle$ are mesonic bound states of quark "a" and antiquark " $\bar{b}$ ". In order to facilitate comparison with experiments involving virtual Compton scattering we take the momentum of the incident current spacelike, $q^2 < 0$, and that of the final state current timelike $q'^2 > 0$.

It has been argued¹⁴ that, in two dimensions, the electromagnetic analogue, especially in hard scattering processes, is better realized by taking Lorentz scalar rather than vector currents in models where the quarks are fermions. This is because leading light cone singularities cannot be probed by vector currents.

[This is why quark masses appear in the parton model formulae of Eqs. IV.1 and IV.5 below.] The methods employed here can easily be adapted to the scalar case by changing the quark-vector current coupling $f_\mu^a(q,y)$ of Eq. (I.23) to a quark-scalar current coupling $f_s^a(q,y) = e_a \left[\frac{m_a}{y} - \frac{m_a}{1-y} \right]$.

There are two possible Lorentz-unrelated frames in which to calculate; these being related to each other by parity transformation. These frames are distinguished by the sign of the minus component of the momentum of the incident spacelike current. We call Frame I the frame where q_- is negative and Frame II the frame where q_- is positive. Since the spectrum is non-degenerate,

the bound states have definite intrinsic parity. The amplitudes computed in these frames are thus related by

$$[M_{\mu\nu}^{nm}(q_i)]_I = (-1)^{n+m} [M^{\mu\nu,nm}(q_i)]_{II} \quad \text{(II.2)}$$

where the Roman numeral subscript denotes the frame in which the tensor is calculated. In this section, the Compton scattering amplitude is derived in both frames. We do this because each amplitude has properties which are useful for demonstrating different aspects of the physical picture. In addition, comparison of the two expressions related by Eq. (II.2) brings out non-trivial features which are not evident in the amplitudes separately.

A second rank Lorentz tensor which is crossing Bose symmetric, $M_{\mu\nu}(-q, q') = M_{\nu\mu}(q', -q)$ and is divergencless on both indices, $q^\mu M_{\mu\nu} = q'^\nu M_{\mu\nu} = 0$, can be written in terms of a single structure function as

$$M_{\mu\nu}^{nm}(q, q'; p_n, p_m) = \left(-g_{\mu\nu} + \frac{q'_\mu q_\nu}{q \cdot q'} \right) \mathcal{M}^{nm}(q, q'; p_n, p_m) \quad \text{(II.3)}$$

where $\mathcal{M}^{nm}$ is symmetric under the interchange $q \leftrightarrow -q'$ and is a Lorentz scalar or pseudoscalar depending on the parities of the external mesons. We show in Appendix A that the calculable components of the Compton amplitude in the 't Hooft model do lead to the form of Eq. (II.3), so that there are no commutator anomalies present in the theory with matrix elements between different mesonic

states. The amplitude is thus specified by one component of the tensor. In our model, the minus-minus component is the simplest to calculate since $f_-^a = 2q_- e_a$ (see Eq. (I.23)).

The computation in Frame I is considerably simplified by the fact that the incident current has negative minus momentum component. The construction of the amplitude is only sensitive to "time" orderings of minus components, with the result that the incident current is automatically crossed and treated symmetrically with respect to the outgoing current. We are thus calculating, in effect, the one-to-three process $n(p_n) \rightarrow \gamma^*(-q) + \gamma^*(q') + m(p_m)$. The minus-minus component of the Compton amplitude is given in the model by the diagrams of Fig. II.A, which are characterized by simple Born exchange in each of the s, t, and u channels. The structure function is given by

$$\begin{aligned} \left[\mathcal{M}^{nm}(p_n, x_1 = -q_-/p_{n-}, x_2 = q'_-/p_{n-}) \right]_I &= \frac{q \cdot q'}{x_1 x_2 (p_{n-})^2} \\ &\cdot \sum_k \left\{ F_-^{nk}(p_n, x_1) \frac{i}{s - M_k^2 + i\epsilon} F_-^{km}(p_n + q, x_2/1 - x_1) \right. \\ &+ F_-^{nk}(p_n, x_2) \frac{i}{u - M_k^2 + i\epsilon} F_-^{km}(p_n - q, x_1/1 - x_2) \\ &+ \mathcal{Y}_{n;km}(p_n, x) \frac{i}{t - M_k^2 + i\epsilon} \mathcal{X}_{--}^{k,a}(q' - q, x_1/x, x_2/x) \\ &\left. + \mathcal{Y}_{n;mk}(p_n, 1-x) \frac{i}{t - M_k^2 + i\epsilon} \mathcal{X}_{--}^{k,\bar{b}}(q' - q, x_1/x, x_2/x) \right\} \end{aligned} \quad (II.3)$$

where s, t, and u are the usual Mandelstam variables and $x = x_1 + x_2$. The terms in curly brackets correspond one-to-one with the diagrams of Fig. II.A. In this form, it is immediately seen that unitarity is satisfied in each channel to leading order in N^{-1} .

The structure of the amplitude is more complicated in Frame II. It is very useful, however, in that it is closely related to the meson-meson scattering amplitude. We have tried to indicate this relation in Fig. II. B. Since the current is flavor conserving, each of the duality sectors of the amplitude contribute weighted by the charges of the appropriate quarks. This is indicated by the first graph of Fig. II.B, which shows that the "meson dominated" hadronic two-to-two function is a term in the Compton amplitude. Contained in this term however there is a piece arising from the off mass shell part of Eq.(I.27) which has no counterpart in hadronic scattering. The inverse propagator in this term cancels the outgoing photon channel meson propagator, resulting in a contribution which looks like the bare coupling of the outgoing current to the contact term of the (u,t) duality sector as shown in Fig. II.C. The remaining terms in the amplitude, shown in Fig. II.B have at least one bare current vertex. The Frame II scattering amplitude is given by

$$\left[M^{nm} \left(q, z = \frac{p_{n-}}{q_-}, z' = \frac{q'_-}{q_-} \right) \right]_{II} = \frac{q \cdot q'}{q_- q_-} \left\{ \frac{q - q'}{g^2} \sum_{k, \ell} \int du dv \phi_k(u) \phi_\ell(v) \right.$$

$$\cdot \frac{i}{q^2 - M_k^2 + i\epsilon} \frac{i}{q'^2 - M_\ell^2 + i\epsilon}$$

$$\cdot \left(e_a^2 \left[\bar{a}_{kn; m\ell}^{(s, t)}(q; z, 1+z-z', z') + a_{kn; m\ell}^{(u, t)}(q; z, 1+z-z', z') \right] \right.$$

$$+ e_b^2 \left[a_{kn; m\ell}^{(s, t)}(q; z, 1+z-z', z') + \bar{a}_{kn; m\ell}^{(u, t)}(q; z, 1+z-z', z') \right]$$

$$+ e_a e_b \left[a_{kn; m\ell}^{(u, s)}(q; z, 1+z-z', z') + \bar{a}_{kn; m\ell}^{(u, s)}(q; z, 1+z-z', z') \right])$$

$$- 2zq_- \int_0^1 dv \phi_n^{ab}(v) \sum_k \left(e_a \phi_k^{ab} \left(\frac{1+vy}{1+z} \right) + e_b \phi_k^{ab} \left(\frac{zv}{1+z} \right) \right) \frac{i}{s - M_k^2 + i\epsilon}$$

$$\cdot F_-^{km} \left(p+q, \frac{z'}{1+z} \right) - \frac{2q_-^2}{g} (1+z-z') \int dv dy$$

$$\cdot \sum_{k, \ell} \phi_k(v) \frac{i}{q^2 - M_k^2 + i\epsilon} \left(e_a \mathcal{Y}_{\ell; kn} \left(p_n+q, \frac{1}{1+z} \right) - e_b \mathcal{Y}_{\ell; nk} \left(p_n+q, \frac{z}{1+z} \right) \right)$$

$$\cdot \frac{i}{s - M_\ell^2 + i\epsilon} \phi_m^{ab}(y) \left(e_a \phi_\ell^{ab} \left(\frac{y(1+z-z') + z'}{1+z} \right) + e_b \phi_\ell^{ab} \left(\frac{(1+z-z')(1-y)}{1+z} \right) \right)$$

$$+ \frac{2q_-^2}{g} (1-z') \int dv dy \sum_k (1 - (-1)^k)$$

$$\cdot \left(e_a^2 G^{aa}(v, y(1-z') + z'; q^2) \phi_k^{aa}(y) \frac{i}{t - M_k^2 + i\epsilon} \mathcal{Y}_{m; kn} \left(p_m, \frac{1-z'}{1+z-z'} \right) \right. \\ \left. - (-1)^{m+n} (a \leftrightarrow b) \right) \Bigg\}. \quad (II.4)$$

This expression is valid in the kinematic region where

$$1 > z' > z.$$

The Compton amplitudes given in Eqs. (II.3) and (II.4) are functions of Lorentz invariant "scaling" variables. In Frame I, x_1 and x_2 give the momenta of the crossed initial photon and final photon respectively as fractions of the incident meson momentum, while z and z' in Frame II give the momenta of the incident meson and final photon respectively as fractions of the incident photon momentum, where all momenta are measured in a left moving infinite momentum frame. These variables are related by

$$x_1 = -q^2 z / M_n^2, \quad x_2 = q'^2 z' / M_n^2.$$

In this section we have derived two expressions for the Compton scattering amplitude, related by Eq. (II.2), which are exact to leading order in N^{-1} in (Q. C. D.)₂. We now proceed to investigate the behavior of these amplitudes in various kinematic and current mass limits for inclusive and exclusive scattering processes. A summary of limits considered is given in Table I.

III. EXCLUSIVE LIMITS

We now study the behavior of the Compton scattering amplitude in the limit that the center of mass energy gets large while all external masses are held fixed and finite. The scattering can either occur in the forward or backward direction in this two dimensional model. This is our analogue of high energy, inelastic, exclusive scattering of real or slightly virtual photons from hadrons.

In our investigation, we take a fixed, finite mass current for the analogue of a real photon. We are particularly suspicious of any attempt to approach the lightlike photon "mass shell" limit due to the anomalous properties of zero-mass particles in two dimensional space-time. Two dimensional Abelian gauge theories, coupled either to massless or massive quarks¹⁵ contain no zero mass particles in their spectra. Zero mass particles decouple completely from hadronic amplitudes in the 't Hooft model,¹⁶ where the decoupling mechanism plays an important role in the evasion of Coleman's¹⁷ theorem regarding the impossibility of Goldstone bosons in two dimensions; an evasion not necessary in four dimensions. We thus keep the external current masses non-zero in an effort to avoid possible dimension dependent effects.

A. Forward Scattering-Regge Limit

A simple picture of Compton scattering in the Regge limit arises in Frame II. The incident current is in the positive direction in the center of mass with $q_- = \sqrt{s} \left(1 - \frac{M_m^2}{s} + O\left(\frac{1}{s^2}\right) \right)$. The initial meson state $|n\rangle$, moving in the opposite direction has vanishing minus component of momentum so that $z \approx \frac{M_n^2}{s}$. The outgoing current is scattered such that $z' \approx 1 - \frac{(M_m^2 - M_n^2)}{s}$. Thus, if $M_m^2 > M_n^2$, the kinematic region is the one to which Eq. (II.4) applies. We take this condition on the masses to be the case. Examining the amplitude in this limit, we find that the leading behavior comes entirely from the first graph of Fig. II.B. The

leading behavior of the Compton scattering is given by that part of the amplitude corresponding to the meson dominated equivalent hadronic scattering amplitude. This can be seen in a fairly straightforward way by noting that leading behavior always arises from regions of phase space where as many gluons with minus momentum components of order s^{-1} can be exchanged as possible, giving large contributions from gluon denominators. These soft gluons are exchanged in graphs of which the three point hadronic function is a subgraph (see Fig. I.C). Thus, roughly speaking, the more three point subprocesses there are in a graph, the more important that graph is in this high energy limit. It can easily be seen that all the graphs with two three point subprocesses are contained in the meson dominated hadronic amplitude. This is because a bare current vertex eliminates a three point subprocess.

The leading behavior of the Compton amplitude is thus gotten directly from that of the hadronic two-to-two scattering amplitude with the substitution $\phi^{aa}(x) \rightarrow e_a q_i G^{aa}(x; q_i^2)$ in the i^{th} current channel. The q_- terms cancel the q_- factors multiplying the brackets in Eq. (II.4). In this limit $q \cdot q' \approx \frac{1}{2}(q^2 + q'^2)$ is finite. There is thus no extra s dependence introduced by the tensor structure factor multiplying the amplitude. The Green's function in the current channels is given by $G^{aa}(x; q^2) = \int dy \sum_k \phi_k^{aa}(y) \phi_k^{aa}(x)/q^2 - M_k^2$. It can be shown using Eq. (I.13) that the contribution from high mass mesons to this sum is negligible, so that $G^{aa}(x; q^2) \approx \sum_k^\Lambda c_k(q^2) \phi_k^{aa}(x)$ for some finite Λ . Thus the Compton amplitude

is essentially a weighted sum of a finite number of hadronic two-to-two scattering functions with all external mesons of finite mass. Having established this correspondence, we have shown that the Compton amplitude has all the Regge properties of the equivalent hadronic process. We will therefore discuss some of these properties without proof, referring the reader to Ref. 7 for details.

The leading terms of the amplitude in this forward limit arise from the (s,t) and (u,t) duality sectors. The amplitude is power behaved, with power determined by the masses of the quarks in the t-channel. The power is given by $\alpha_{ij}(0) = -\beta_i - \beta_j$ where i and j denote the exchanged quarks. In the Compton scattering case $i = j = a$ or b . The amplitude factorizes in the t-channel and is consistent with obeying an unsubtracted dispersion relation. $J = -1$ fixed pole terms are present.

Although we have shown a close correspondence between photon-hadron and hadron-hadron scattering in the 't Hooft model, we can say little about the analytic structure of the amplitudes. The dependence of Eq. (II.3) and (II.4) on momentum components obscures the properties of these expressions as analytic functions of invariants. Thus, though we have been describing current channels as "meson dominated," this does not imply agreement with meson dominance models which make specific statements about the analytic behavior of the amplitude with respect to the photon mass variables. Comparison with the amplitude calculated in Frame I serves to clarify some of these issues.

The dimensionless variables x_1 and x_2 in Frame I both go to zero in the Regge limit, with $x_1 = -\frac{q^2}{s} \left[1 + \frac{M_m^2 + q^2}{s} \right]$ and $x_2 = \frac{q^2}{s} \left[1 + \frac{(M_m^2 + q^2)}{s} \right]$. The terms with two bare current vertices dominate in the limit. The terms from the (s,t) and (u,t) diagrams cancel to the two leading orders, resulting in Regge leading behavior. This cancellation points out a major difference between photon scattering and the equivalent general hadronic process. Since the electromagnetic current is flavor conserving, the (u,t) and (s,t) amplitudes cannot separately be treated as analytic functions because they cannot be physically distinguished. Only the sum of the two has any physical meaning, and it is only the sum that has the same leading behavior as in the parity reversed frame.

A non-leading contribution arising from the cancellation of these diagrams is a term which shows $J = -1$ fixed pole behavior. This fixed pole was discussed by Brower et al.⁷ in the zero mass photon limit where it becomes the leading term. It only contributes to elastic scattering and, thus, cannot be seen in the Frame II analysis where the elastic boundary is outside the applicable kinematic limit. We can conclude from the Frame II analysis, where terms involving bare current vertices are proportional to $s^{-(1+\beta)}$ in the limit, that in inelastic Compton scattering in the forward high energy region the photon acts as a purely hadronic system. In particular, the only fixed pole contributions to the inelastic amplitude are those directly related to the fixed poles of the meson-meson scattering amplitude.

Further analysis in Frame I shows that the leading order Regge behaved terms arise from diagrams with bare current vertices and from diagrams with meson dominated vertices, as opposed to the situation in Frame II. The frame dependence points out the fact that this separation of vertices does not have a general analytic meaning.

B. "Large Angle" Scattering Limit

The exclusive scattering of photons and hadrons at fixed angle in the center of mass at high energy can be studied in the model by adopting an ansatz of Brower, et al.⁷ in order to make contact with the dimensional counting rules. We consider the limiting power behavior of that analytic dual sector of the amplitude where the variables associated with the meson poles are getting large as an analogue to large angle scattering power behavior. Thus, in the forward limit considered above, the (u,s) part of the amplitude is appropriate. Note that, in accordance with this prescription, this is the only limit in which contact with large angle scattering can be reached. If we tried to study the possibly more physical limit of backward scattering, we would be led to consider the (s,t) dual sector which is, however, not analytic by itself as discussed above.

The behavior of the (u,s) sector of the Compton amplitude in the Regge limit in Frame II approaches that of the (u,s) sector of the two-to-two hadronic scattering function. Thus, from the study of the hadronic amplitude⁷ we find that $M^{(u,s)} \sim s^{-2-\beta_a-\beta_b}$.

In the standard dimensional counting picture, the fact that the Compton amplitude has the same power behavior as the equivalent hadronic amplitude would indicate that all current vertices are meson dominated, since contributions from bare photon couplings, if present, are enhanced with respect to the meson dominated ones. This enhancement, however, is due to the couplings to transverse photons only, and does not appear in longitudinal current couplings. Thus, in our two dimensional model where all currents are longitudinal, we expect the observed power law behavior equivalence between the hadronic and Compton amplitudes.

Although this equivalence can be motivated in the dimensional counting picture, calculation of the integral power falloff by computing contributions from lowest order irreducible quark diagrams like Fig. III A in superrenormalizable (Q. C. D.)₂ where all quarks have finite fractions of hadron momentum gives an s^{-3} prediction for the "large angle" behavior of the amplitude since there are three propagators far from mass shell. Disagreement with the $s^{-2+\beta}$ 't Hooft model behavior stems from the fact that in a superrenormalizable theory it costs a factor of s^{-1} to redistribute the momentum in the bound state via one gluon exchange as in Fig. III A so that the quarks in the final state hadron all have finite momentum fraction. In the 't Hooft model this redistribution doesn't cost anything (except for nonperturbative factors $s^{-\beta}$) due to the fact that the gluon interaction is instantaneous and that there are no transverse degrees of freedom. This results in a softening of the

Bethe-Salpeter wave function kernel so that the 't Hooft wave function falls off more slowly as one of the constituent quark masses goes far off shell with the other quark mass fixed than would be assumed in the counting rule picture based on a superrenormalizable field theory.¹⁸

Aside from differences which seem to be specific to the dimensionality of the theory, modifications of counting rule ideas are indicated in the model.¹⁹ The coefficient of the power of "s" in the amplitude cannot be interpreted exclusively in terms of short distance physics. Terms involving exchanged mesons built up non-perturbatively from gluon exchange contribute. In addition, the integral power predictions of the counting rules are modified by the β 's, powers dependent on masses of constituent quarks. These powers arise from infrared singularities in the wave function and are closely related to the Regge intercepts in the model.

IV. INCLUSIVE LIMITS

We turn now to the study of the inclusive Compton process $\gamma^*(q) + n(p_k) \rightarrow \gamma^*(q') + \underline{X}$, where $\underline{X}$ represents the unobserved final hadronic state. The parton model makes definite predictions concerning the structure of the cross section in several inclusive limits. Inclusive production of large mass lepton pairs via photoproduction and deep inelastic lepton production, two non-light cone processes, are investigated in this section for the purpose of checking the validity of parton predictions in the 't Hooft model. We will show

that the amplitude for these processes scales in the 't Hooft model and that the scaling functions are interpretable in terms of free scattering of constituent quarks.

The distribution of final states in this model is extremely simple. Since it costs $N^{-\frac{1}{2}}$ to create a meson, due to the color normalization of the singlet wave function, only one particle is produced in the final state. A deep inelastic process is thus described by the exclusive production of a large mass hadron. This simplification leads to the unfortunate circumstance that scaling is impossible in the model, since the scattering process only occurs for those discrete values of s for which a final state zero width particle can be produced. An ansatz for getting scaling behavior,⁵ long used in dual models, is to smooth the meson Green's function $G(x,y;s)$ in the region where s is timelike by taking the $s \rightarrow \infty$ limit slightly off the real axis. The procedure essentially creates a final state continuum by giving the final state particles widths on the order of the resonance spacing. Whenever it is necessary in the computation, the meson propagators are smoothed in this way. In particular, this prescription allows the interpolation of the mass of the final state $|m\rangle$ using $m = s/\pi^2$ with the proviso that terms in the squared amplitude proportional to $(-1)^m$ arising from parity oscillations of the final state hadron are dropped.

A. Bjorken-Paschos Compton Limit with Very Virtual Photons

We first examine a limit of the Compton amplitude where the photon masses and momentum transfers to the hadron system are all large. This limiting case would be applicable, for example, to the deep inelastic electroproduction of massive lepton pairs.

Intuitively we expect that the large photon masses will suppress the hadronic content of the photon so that the current couples to the parton via the bare electromagnetic pointlike interaction. Further, the large momentum transfer suggests the validity of impulse approximation ideas. In this view, the two currents must interact with the same quark so as to avoid large energy in intermediate states. We are thus led to a model of the scattering process in this limit in terms of free parton "handbag" scattering amplitudes incoherently summed and folded into parton momentum distribution functions.

A description of inclusive Compton scattering at large momentum transfer with real photons in terms of a parton "handbag diagram" model has been proposed by Bjorken and Paschos.²⁰ We cannot examine this limit directly since, in two dimensions, the Mandelstam variables cannot all be made large simultaneously as long as the photon masses stay finite. This problem was encountered in the large angle exclusive scattering limit discussed above, where the power behavior of the (u,s) sector was analyzed as a large angle power behavior analogue. Here, however, we are interested in the structure of the coefficient of the power. There is no reason to associate this coefficient with the (u,s) sector

coefficient, which, in this case, consists of "crossed handbag" coherence terms which are assumed to be non-leading in the parton model.

The very virtual photon limit is achieved kinematically by taking $q^2, q'^2, s, t, v = p_n \cdot (q - q')/M_n$ large with their ratios fixed. In our two dimensional case this limit can be realized by taking $q^2 = \frac{-x_1}{1-x_1} s$, $q'^2 = \frac{x_2 s}{1-x_1} \left[1 - \frac{2M_n \sqrt{1-x_1}}{s} \right]$. The fixed dimensionless variables x_1 describe the momentum transfer across the current vertices: $x_1 = -q^2/2p_n \cdot q$, $x_2 = q'^2/2p_n \cdot q'$, $x = x_1 + x_2 = x_{B.J.} = -t/2M_n v$. They are limited by $x_2 < x < 1$ where $0 < x_2 < 1$. The Frame II scaling variables are given by $z = M_n^2(1-x_1)/s$ and $z' = 1 - \frac{2M_n v}{s}(1-x_1)$.

The naive parton model prediction is, as discussed above, gotten by summing the handbag diagrams shown in Fig. IV.A. The quark propagators in these diagrams which are attached to the wave function vertices are dressed while those between the current vertices are bare. This prescription is gauge invariant for the class of gauges where $A_- = 0$. The amplitude computed in this parton model is

$$|M|^2 = |M_{\mu\nu} M^{\mu\nu}| = \left[\frac{2\pi}{M_n v} \right]^2 \left[1 + \frac{s(1-x_2)}{u(1-x_1)} \right] \frac{1}{x_1 x_2} \cdot (e_a^4 m_a^2 |\phi_n^{a\bar{b}}(x)|^2 + e_b^4 m_b^2 |\phi_n^{a\bar{b}}(1-x)|^2) \quad (IV.1)$$

The handbag model thus predicts a scaling cross section, where the scattering function $\sqrt{s} |M|^2$ is, apart from kinematic factors, related to the deep inelastic leptonproduction structure function by making the replacement $e^2 \rightarrow e^4$. We now show that this parton description successfully gives the limiting behavior of the exact Compton amplitude in the 't Hooft model.

Frame I is, in this gauge, the natural frame in which to see simple parton behavior, as is evident in the close relation of the scaling variables in this frame to the usual parton variables. The behavior of the amplitude in this frame can be found simply by counting meson propagators, each of which contribute a factor of s^{-1} . The soft gluon mechanism, used to get leading Regge behavior, no longer operates due to the fact that x_1 and x_2 are not constrained to their edge points. This, in turn is due to the large current masses. Thus, as the currents get highly virtual, they are no longer describable in purely hadronic terms. In fact, the leading diagrams in this limit, those with the fewest meson propagators, are precisely those with bare current couplings to quarks. The other source of s dependence in the amplitude arises from the phase space pinch due to the large mass final state. Roughly speaking, this suppresses all terms in the amplitude equally. The leading contribution to the amplitude in this limit thus comes from the bare current-quark coupling terms in the s and u channel meson exchange diagrams of Fig. II.A. Included in these diagrams are graphs where the currents interact with the same quark as well as those where they interact with different quarks. These

latter "crossed" graphs cannot contribute in the handbag model. The impulse approximation is recovered, however, by looking at the amplitude in Frame II.

The limit in Frame II involves scaling variables at their edge points so that the behavior of the amplitude is less obvious. A little computation shows that the (u,s) sector of the amplitude is non-leading in the limit. These terms, proportional to $e_a e_b$, are suppressed relative to the leading $e_a e_b$ terms in Frame I. Parity conservation via Eq. (II.2) implies, therefore, cancellation of the leading Frame I terms. This cancellation implies that the quantity

$$\left| \int d\eta \phi^a(\eta) \left\{ \phi_n^{a\bar{b}}(x_1) \left[\frac{(1-x_1)}{s} h_+ \left(\frac{\eta}{1-x_1} \right) + \frac{(1-x_2)}{u} \right] + \phi_n^{a\bar{b}}(x_2) \left[\frac{(1-x_1)}{s} + \frac{(1-x_2)}{(-u)} h_- \left(\frac{\eta}{1-x_1} \left(1 - \frac{2M_N v(1-x_1)}{s} \right) \right) \right] \right\} \right|$$

is symmetric under the interchange $a \leftrightarrow b$. We have not been able to directly prove this symmetry property.

The remaining terms in Frame I are proportional to e_a^2 and e_b^2 , and arise from diagrams with bare quark current couplings. The limiting form of the amplitude is

$$\lim \left[M_{--}^{nm} \right]_I = 4p_-^2 (1-x) \left[\frac{(1-x_1)}{s} + \frac{(1-x_2)}{u} \right] \int_0^1 dw \phi_m^{a\bar{b}}(w) \left[e_a^2 \phi_n^{a\bar{b}}(w(1-x)+x) + e_b^2 \phi_n^{a\bar{b}}(w(1-x)) \right]. \quad (IV.2)$$

This is a somewhat intermediate form of the final result which we give to show that in this limit the two currents interact as if they were a single spacelike current carrying a fraction x of the minus component of the hadron momentum and interacting with the quarks via the bare vertex with an effective coupling constant

$$e_a^{\text{eff}} = -2e_a^2 \frac{q \cdot q'}{x_1 x_2} \left[\frac{(1-x_1)}{s} + \frac{(1-x_2)}{u} \right]. \quad (\text{IV.3})$$

With this coupling the invariant amplitude $\mathcal{M}^{\text{nm}}$ gotten from Eq. (IV.2) via Eq. (II.3) equals the part of the invariant form factor for the single current interaction which arises from the first, bare coupling, term in Eq. (I.22).²¹ The Bjorken-Paschos intuitive picture of the two currents giving a single large impulse to the hadronic system is realized in the 't Hooft model. This instantaneous interaction of the two currents at a single vertex in an infinite momentum frame has been proposed by Brodsky and Roy²² as a necessary condition for the application of parton model ideas to two photon processes.

Equation (IV.1) can be obtained by taking the large final state mass limit of Eq. (IV.2), using Eq. (I.17), squaring and using the tensor form given in Eq. (II.3). The incoherence results from the elimination of cross terms due to $(-1)^m$ parity oscillations. Agreement with the parton handbag model is thus confirmed. The key step in making contact with the constituent partons is Eq. (I.17). Although the final state distribution in this theory is not realistic,

the relation Eq. (I.17) stands as a prototypical example of a relation between a summation over final states and constituent parton parameters. Relations of this type must exist in order to make the transition from the bound state hadronic description to the constituent quark description of deep inelastic processes. A more complicated manifestation of the mechanism appears in the limit discussed in the next section.

We conclude by pointing out that, in Frame I in the $A_- = 0$ gauge, the lowest order quark diagrams of the "handbag" model are expected to describe the behavior of the Compton amplitude in the light cone limit on the basis of asymptotic freedom arguments. The kinematic region which probes the light cone contribution to the Compton amplitude is discussed by Iliopoulos and Paschos (I-P).²³ It is reached by a two step limiting procedure where first q^2, q'^2 and s are taken large in fixed ratio with t and v fixed, after which t and v are taken large with $x = -t/2M_N v$ fixed. This differs from the limit we have considered above where the ratio v/s is held finite throughout. This one step limit does not a priori probe the light cone due to the possible presence of large final state mass oscillations in the matrix element $\langle X | T^*(J_\mu(x) J_\nu(0)) | n \rangle$ which could invalidate the usual stationary phase analysis of leading momentum space contributions.

In the I-P limit, the second factor in the handbag amplitude of Eq. (IV.1), is of order v/s due to kinematics in two dimensions.

This factor results from the Bose symmetrization of the amplitude. The function $s^2|M^2|$ thus approaches zero as $(v/s)^2$ in the light cone limit. We have found that in the I-P limit in the 't Hooft model $s^2|M^2|$ goes to zero in agreement with the handbag prediction.

B. Photoproduction of Large Mass Lepton Pairs

The inclusive photoproduction of lepton pairs, in the limit that the mass of the pair gets large in fixed ratio to the energy in the center of mass, has been studied¹⁰ in the context of a field theoretic parton model in four dimensions with an ad hoc transverse momentum cutoff. Due to the large lepton pair mass, the cross section can be computed by considering the photon as a collection of on-shell partons, one of which annihilates with its antiparticle in the incident hadron at a bare electromagnetic vertex to form a highly virtual photon in the final state. The total cross section is given by the incoherent sum of $q\bar{q} \rightarrow \ell\bar{\ell}$ cross sections weighted by the parton momentum distribution functions of the photon and hadron. This parton model is shown in Fig. IV.B.

This photoproduction process is very similar to the Drell-Yan process, and application of the parton model is on the same theoretical footing in both cases. The major difference between the two is the presence, in photoproduction, of diagrams where the bare incident photon couples directly to quarks. In the parton model analysis, the bare coupling time ordered diagrams which

contribute in the infinite momentum frame look exactly like Fig. IV.B., allowing the bare coupling contributions to be included in the photon structure function.

The parton model prediction in the Drell-Yan scattering process is borne out in the 't Hooft model.⁶ It is thus reasonable to hope that the same is true for photoproduction scaling. We will show that this is the case, although in any frame in this gauge the final state production mechanism appears non-partonic.

In this limit we take s and q'^2 large and in fixed ratio such that $\tau = q'^2/s$, while keeping the masses of the incident current and hadron finite. The hadronic contribution to the scattering process is given, in general, by

$$H^{\mu\nu}(p_n, q, q') = \sum_{\text{pol}} \int d^2y e^{-iq' \cdot y} \langle n(p_n) \gamma(q) | j^\mu(y) j^\nu(0) | n(p_n) \gamma(q) \rangle. \quad (\text{IV.4})$$

Computing this tensor via the parton model of Fig. IV.B in two dimensions gives

$$H^{\mu\nu}_{\text{PARTON}} = \left\{ -g^{\mu\nu} + \frac{q'^\mu q'^\nu}{q'^2} \right\} \sum_a \frac{16\pi^2 e_a^2 m_a^2}{y_1 y_2 s} |\phi_Y^a(y_2)|^2 |\phi_n^a(y_1)|^2 \quad (\text{IV.5})$$

where the sum is over all flavors of quarks and antiquarks. The $\phi_1^a(x)$ are the usual wave functions describing the probability that, in an infinite momentum frame, parton "a" has momentum

fraction x of the total momentum of the i^{th} particle. The hadron wave function is measured in deep inelastic lepton production, leaving the photon wave function to be determined in this photoproduction process. The scaling variables are given by $y_1 = \sqrt{2} q'_+/\sqrt{s}$ and $y_2 = \sqrt{2} q'_+/\sqrt{s}$ in the hadron-photon center of mass frame in which the photon is moving in the positive direction (Frame I). They are restricted by $\tau \leq y_1 \leq 1$ and satisfy $\tau = y_1 y_2$. In terms of these variables we have $t = y_1(y_2 - 1)s$, $u = y_2(y_1 - 1)s$, $M_x^2 = (1 - y_2)(1 - y_1)s$ in the large s limit.

The important feature of Eq. (IV.5) is that it is a sum of terms each of which is a product of a function of y_1 and a function of y_2 . We now verify this scaling form in Frame I and Frame II separately in the 't Hooft model. By relating the expressions in each frame via parity, we extract simple expressions for the meson and photon wave function. This method is similar to the Drell-Yan analysis of Ref. 6.

In Frame I the limit is achieved by taking $x_1 = -q^2/s$, $x_2 = y_1$. The leading diagrams are those with bare current coupling at both vertices. As in the Regge limit these terms cancel in leading order. The cancellation in next order is, however, not complete and a contribution arises from the (u, t) duality sector diagram. In this order there is also a contribution from the (u, t) diagram where the incident current is meson dominated and the outgoing current is bare. These leading diagrams are shown in Fig. IV.C. There

are no other contributions. In particular, the large mass of the final current suppresses the contribution from meson dominated final current vertices. The crossed terms, proportional to $e_a e_b^-$, are also non-leading. In the limit we then have

$$\begin{aligned} \left[H^{\mu\nu} \right]_I &= \left\{ -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right\} \left\{ \frac{(4e_a^2)^2}{y_1 y_2 s} \left| \phi_n^{ab}(y_1) \right|^2 \left[\frac{1}{\sqrt{q^2}(1-y_2)} \int_0^\infty d\eta \phi^a(\eta) \right. \right. \\ &\quad \cdot \left[h_-^a(y_2 \left(\frac{\eta}{1-y_2} + q^2 \right)) - h_-^a \left(\frac{y_2 \eta}{1-y_2} \right) \right] \\ &\quad + (q^2)^2 \int_0^1 dv G(v, q^2) \left[\frac{h_-^a(q^2 y_2 v) - h_-^a(y_2 \left(\frac{\eta}{1-y_2} + q^2 \right))}{\left(\frac{\eta}{1-y_2} + q^2(1-v) \right)^2} \right] \right]^2 \\ &\quad \left. + (a \leftrightarrow b) \right\}. \end{aligned} \quad (IV.6)$$

This expression clearly gives the parton model scaling form of Eq. (IV.5). The meson probability amplitude is given by the meson wave function squared, consistent with what is found in lepton production scaling. The photon wave function is given by expression in square brackets.

The interpretation of this scattering in terms of parton annihilation arises naturally from the dominance of the (u, t) sector diagrams in the limit. This is the only sector which contains those quark diagrams where the final current arises via the annihilation of a current quark and a meson quark. All other diagrams

lead to an interpretation of final current production due to quark bremsstrahlung. The formation of the final states in this frame is, however, distinctly non-partonic, as can be seen by comparing Figs. IV.B and IV.C. The final states in the model are inextricably bound up with the incident current vertex, so that the final state production mechanism is reflected in the photon wave function.

We can get a better idea of the form of the photon wave function by computing in Frame II. Here, $z' = y_2$ and $z = M_n^2/s$ in the limit. In this frame the roles of the initial state particles are reversed, so that the amplitude is given by a simple expression for the current vertex and a complicated expression for the meson vertex. The dominant terms all arise from the (u,t) sector. The contributions come from the off-mass shell term in the contact diagram depicted in Fig. II.C and from t-channel meson exchange diagrams where the incident current is meson dominated while the outgoing current is bare, the last diagram in Fig. II.B. These contributions are represented in Fig. IV.D. The limiting amplitude is given by

$$\begin{aligned} [H^{\mu\nu}]_{II} = & \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q'^2} \right\} \left\{ \frac{(4e_a^2)^2 q^2}{y_1 y_2^s} M_n^4 |G(y_2, q^2)|^2 \right. \\ & \cdot \left[y_1 \int_0^1 dw \phi_n(w) \left[\phi^b(M_n^2(1-y_1)(1-w)) \right. \right. \\ & \left. \left. + \int dv du g_-(v M_n^2, u M_n^2) \left[\frac{\phi^b(M_n^2(1-y_1)(1-\frac{v}{y_1})) - \phi^b(M_n^2(1-y_1)(1-w))}{(y_1 w - v)^2} \right] \right] \right\} \\ & + (a \leftrightarrow b) \Bigg\} . \end{aligned} \quad (IV.7)$$

The remarks below Eq. (IV.6) apply also in this frame with the roles of the incident meson and current interchanged. The final state production mechanism is not partonic, as seen by comparing Figs. IV.B and IV.D, though the dominance of the (u,t) sector again allows a quark-antiquark annihilation interpretation. The photon wave function normalized to unity which arises from the Frame II analysis is given by:

$$\phi_Y^a(x; q^2) = G^{a\bar{a}}(x; q^2) / \int_0^1 dx (G^{a\bar{a}}(x; q^2))^2 . \quad (IV.8)$$

The partonic structure of the photon, in this model, arises from its couplings to mesons.

Relating Eqs. (IV.6) and (IV.7) through Eq. (II.2) gives a simple expression for the amplitude which has the parton model form

$$H^{\mu\nu} = \left\{ -g^{\mu\nu} + \frac{q^\mu q^\nu}{q'^2} \right\} \left(\frac{16e_a^2}{y_1 y_2^s} C_V^a |\phi_n^a(y_1)|^2 |\phi_Y^{\bar{a}}(y_2)|^2 + (a \rightarrow \bar{b}) \right), \quad (IV.9)$$

where C_V doesn't have any y_1 dependence. We have not tried to see if $C_V^a = m_a^2 C_V^1$ as demanded by Eq. (IV.5). In the course of relating the amplitudes in each frame, highly non-trivial equalities are generated between scaling functions and wave functions. We have not tried to verify these identities but have checked consistency in power behavior at the edge points. These equalities are more complicated examples of relations between final state summations and parton descriptions than Eq. (I.13) which was used above.

CONCLUSION

The most important aspect of the results presented here is that there are no real surprises. Generally used ideas - vector meson dominance in soft processes and parton point coupling dynamics in hard processes - are found to accurately describe Compton scattering behavior in the 't Hooft model. In this model we have found that "real" photon states are effectively linear combinations of hadronic bound states, a description which becomes inapplicable as the photon is taken far off mass shell. We have shown how highly non-trivial properties of hadronic wave functions allow parton model interpretation in various deep inelastic inclusive limits.

The most interesting departure from phenomenological lore is the ubiquitous modification of dimension counting rules by small factors dependent on exchanged quarks. These factors are related to confinement singularities²⁴ and may thus be present in more realistic confining theories. This has been stressed by Brower²⁵ in the more general context of the relation between large angle scattering power behavior and Regge intercepts.

Many aspects of scattering behavior in the 't Hooft model are almost certainly dependent on the absence of transverse dimensions. The lack of real vector meson degrees of freedom, for example, probably gives rise to qualitative behavior specific to two dimensions. The failure of the pomeron to arise in hadronic scattering amplitudes is probably due to the fact that there are no real gluons in the theory. The mechanism responsible for

cancellation of the pomeron also seems to be inhibiting possible non-partonic contributions to hard processes which are leading in N^{-1} .²⁶ Parton behavior might, then, be modified by the appearance of the pomeron.

Photonic degrees of freedom in four dimensions give rise to terms in the dimensional counting picture which are responsible for leading behavior in large angle scattering. These terms do not appear in the 't Hooft model. Additionally, transverse photons might result in the presence of fixed poles²⁷ in the inelastic Compton amplitude in the soft scattering region and extra terms in the partonic description of the photon (Eq. (IV.8)).

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APPENDIX A: Divergence Properties of Current Amplitudes

Electromagnetic gauge invariance leads to the requirement that physical scattering matrix elements involving vector electromagnetic currents be conserved on all Lorentz indices

$$\partial_\mu \langle \alpha | T^* (\dots J^\mu(x) \dots) | \beta \rangle = 0 \quad (A.1)$$

As is well known, the matrix element of time ordered products of operators does not in general satisfy this divergence condition due to the presence of non-covariant Schwinger terms.²⁸ Non-covariant terms must be added to the time ordered product matrix element to produce the "T*" product matrix element, which is covariant and divergenceless.²⁹

In this Appendix we show that the current matrix elements considered - F_μ defined in Eq. (I.22), $K_{\mu\nu}^{n,a}$ defined in Eq. (I.24) and $M_{\mu\nu}^{nm}$ defined in Eq. (II.3) - are the physical "T*" product current matrix elements which are covariant and conserved on all indices when the external meson are on mass shell.

The basic mechanism for current conservation can be seen by examination of the vector current quark coupling $f_\mu(q,y)$ defined by Eq. (I.23). The coupling obeys

$$q^\mu f_\mu^a(q,y) = e_a \left(q^2 - \frac{m_a^2}{y} - \frac{m_a^2}{1-y} \right) \quad (A.2)$$

This form allows simplification of the divergence through the use of the 't Hooft Eq. (I.4) and the wave function orthogonality Eq. (I.11). A primary simplification which serves as a good example of the kinds of manipulations involved, occurs in terms where the photon is meson dominated. These terms are of the form $\int dw dy G(w,y;q^2) f_\mu^a(q,y) \chi(w)$, with χ being some given function. The divergence of this term is

$$\begin{aligned} q^\mu \int dw dy G(w,y;q^2) f_\mu^a(q,y) \chi(w) &= e_a \int dw dy \left(q^2 - \frac{m_a^2}{y} - \frac{m_a^2}{1-y} \right) G(w,y;q^2) \chi(w) \\ &= e_a \int dw dy \sum_k \left[(q^2 - \mu_k^2) + \left(\mu_k^2 - \frac{m_a^2}{y} - \frac{m_a^2}{y-1} \right) \right] \frac{\phi_k^{aa}(w) \phi_k^{aa}(y)}{q^2 - M_k^2 + i\epsilon} \chi(w) \\ &= e_a \int dw \chi(w) + e_a \int dw dy dz \frac{\phi_k^{aa}(w)}{q^2 - M_k^2 + i\epsilon} \left[\frac{\phi_k^{aa}(y) - \phi_k^{aa}(z)}{(y-z)^2} \right] = e_a \int dw \chi(w) \end{aligned}$$

We have used Eq. (A.2) in the first step, the definition of the Green's function in the second step, orthogonality and the 't Hooft equation in the third step where the second term is zero due to the antisymmetry of the integrand under $y \leftrightarrow z$. Taking the divergence thus eliminates meson propagators in the photon channels, leaving terms which can cancel with those arising from the bare current coupling terms.

The inelastic form factor, $F_\mu^{nm} = \langle n | J_\mu | m \rangle$, is conserved in the 't Hooft model when the initial and final mesons are on mass shell. The divergence is given in general by

$$F_{\mu}^{nm}(p_n, x) = \left[\frac{-q^2(1-x)}{x} - M_m^2 + (1-x)M_n^2 \right] \left[e_a \int dy \phi_m^{a\bar{b}}(y) \phi_n^{a\bar{b}}(y(1-x) + x) + (-1)^{n+m}(a \rightarrow \bar{b}) \right]. \quad (A.3)$$

The situation for the matrix elements of the time ordered product of two vector currents is complicated by the fact that, in the $A_- = 0$ gauge the plus-plus component of the tensor is not calculable by standard techniques. We will thus show that, for the matrix elements considered, there exists a Lorentz covariant tensor which is divergenceless on all indices whose minus-minus, minus-plus, and plus-minus components agree with those calculated in the model.

In the case of the amplitude $K_{\mu\nu}^{n,a} = \int d^2x e^{iq \cdot x} \langle n | T^*(J_\mu(x) J_\nu(0)) | 0 \rangle$, the whole tensor can be represented in the simple covariant form given in Eq. (I.24). The vanishing on-shell divergence of the scattering tensor depends crucially on the even parity of the decaying mesons. The off-shell divergence is given by

$$q_1^\mu K_{\mu\nu}^{n,a}(p, x_1, x_2) = \frac{ie_a x_2}{2g} (p^2 - M_n^2) (1 - (-1)^n) \int dw dy G^{a\bar{a}}(w, y; q_2^2) f_{\nu}^a(q_2, w) \phi_n^{a\bar{a}}(yx_2) \quad (A.4)$$

Finally, we turn to the more complicated case of the Compton amplitude $M_{\mu\nu}^{nm} = \int d^2x e^{iq \cdot x} \langle n | T^*(J_\mu(x) J_\nu(0)) | m \rangle$ with the meson states $|n\rangle$ and $|m\rangle$ on the mass shell. Evaluation of the divergence is simplest in Frame I with the aid of Eqs. (A.3) and

(A.4) for the off-shell divergence of the Compton subamplitudes.

To this end we define a quantity $C_{\mu\nu}^{nm}$ by

$$C_{\mu\nu}^{nm}(p_n, x_1 = -q_-/p_{n-}, x_2 = q_-'/p_{n-}) =$$

$$\begin{aligned} & \sum_k \left\{ F_{\mu}^{nk}(p_n, x_1) \frac{i}{s - M_k^2 + i\epsilon} F_{\nu}^{km}(p_n + q, \frac{x_2}{1-x_1}) \right. \\ & + F_{\nu}^{nk}(p_n, x_2) \frac{i}{u - M_k^2 + i\epsilon} F_{\mu}^{km}(p_n - q', \frac{x_1}{1-x_2}) \\ & + \mathcal{J}_{n;km}(p_n, x) \frac{i}{t - M_k^2 + i\epsilon} \mathcal{H}_{\mu\nu}^{k,a}(q' - q, x_1/x, x_2/x) \\ & \left. + \mathcal{J}_{n;mk}(p_n, 1-x) \frac{i}{t - M_k^2 + i\epsilon} \mathcal{H}_{\mu\nu}^{k,\bar{b}}(q' - q, x_1/x, x_2/x) \right\}. \quad (A.5) \end{aligned}$$

The minus-minus, plus-minus, and minus-plus components of $C_{\mu\nu}^{nm}$ coincide with the respective components of the Compton scattering tensor $\mathcal{H}_{\mu\nu}^{nm}$ in the model. Calculating $q^\mu C_{\mu\nu}$, we find a complete cancellation among crossing related terms proportional to $e_a e_{\bar{b}}$ and among the terms proportional to e_a^2 and e_b^2 which have meson propagators in the outgoing current channel in the s , t , and u channel meson exchange diagrams. The cancellation among crossing related terms in the s and u meson exchange channels

with bare coupling to the outgoing current is incomplete, however, giving

$$q^\mu C_{\mu\nu} = i e_a \frac{(1-x)}{x_2} \int_0^1 dw \phi_m^{a\bar{b}}(w) \phi_n^{a\bar{b}}(w(1-x)+x) \left[f_v^a \left(q', -\frac{w(1-x)+x_1}{x_2} \right) - f_v \left(q', -\frac{w(1-x)}{x_2} \right) \right] + (a \leftrightarrow \bar{b}). \quad (\text{A.6})$$

$C_{\mu\nu}$ thus has a non-zero divergence and thus cannot be the correct Compton amplitude. However, by Eq. (I.23), $q^\mu C_{\mu-} = 0$ and we then have proved $q^\mu M_{\mu-} = 0$.

In two dimensions it can be shown that a covariant tensor $M_{\mu\nu}(q_1, q_2)$ which satisfies $q^\mu M_{\mu-} = 0$, $M_{\mu\nu}(-q_1^\lambda, q_2^\lambda) = M_{\mu\nu}(q_2^\lambda, -q_1^\lambda)$ and the parity condition of Eq. (II.2) is divergenceless and can be written most generally as in Eq. (II.3). The M_{++} component is then defined through Eq. (II.3) in terms of one of the other tensor components. It cannot be expressed as a sum of simple products of form factors. We have thus shown that Eq. (II.3) unambiguously gives the correct Compton scattering amplitude.

REFERENCES

1. Thorough treatment of the ideas of asymptotic freedom is given by H. D. Politzer, Phys. Rep. 14C, 129 (1974) and by D. J. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973).
2. G. 't Hooft, Nucl. Phys. B72, 461 (1974).
3. G. 't Hooft, Nucl. Phys. B75, 461 (1974).
4. C. G. Callan, Jr., N. Coote, and D. J. Gross, Phys. Rev. D 13, 1649 (1976).
5. M. B. Einhorn, Phys. Rev. D 14, 3451 (1976).
6. J. Kripfganz and M. G. Schmidt, CERN TH-2266 (1977).
7. R. C. Brower, J. Ellis, M. G. Schmidt, and J. H. Weis, Nucl. Phys. B128, 131 (1977).
8. M. B. Einhorn and E. Rabinovici, Nucl. Phys. B128, 421 (1977).
9. S. J. Brodsky and G. R. Farrar, Phys. Rev. D 11 1309 (1975). Dimensional counting is also discussed by V. Mateev, R. Muredyan and A. Tavkhelidze, Nuovo Cimento Lett. 7, 719 (1973).
10. R. L. Jaffe, Phys. Rev. D 4, 1507 (1971).
11. Numerical verification of a complicated parity relation is claimed in Ref. 6, for example.
12. This relation was first derived in Ref. 5 and correctly given in Ref. 6.
13. The expressions given define our off meson mass shell continuation of the amplitude in the $A_- = 0$, regular cutoff gauge. It is a useful definition a posteriori for construction of complicated amplitudes from a simple set of diagrams in this gauge. The continuation does not result from any field theoretic, L. S. Z. type of scheme.

14. A. Casher, J. Kogut and L. Susskind, Phys. Rev. D 10, 732 (1974).
15. Massless Q. E. D. in two dimensions is solved by J. Schwinger, Phys. Rev. 125, 397 (1962) and Phys. Rev. 128, 2425 (1962). Operator solutions are discussed by J. H. Lowenstein and J. A. Swieca, Ann. Phys. 68, 172 (1971). The spectrum of massive Q. E. D. is treated by S. Coleman, Ann. Phys. 101, 239 (1976).
16. A. W. Smith, DAMTP 76/17 (1976).
17. S. Coleman, Comm. Math. Phys. 31, 259 (1973).
18. The power fall off of the 't Hooft wave function is coincidentally exactly that used in counting rules based on renormalizable perturbation theory. As a result, the "large angle" 't Hooft model behavior is well described in many cases by the four dimensional counting rules.
19. The dimensional counting modifications discussed here all appear at the simpler level of large q^2 form factor behavior. See Ref. 5, Sect. IV.
20. J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).
21. We define the invariant form factor $F^{nm}(q^2)$ by

$$F_{\mu}^{nm} = \left(p_{n\mu} + p_{m\mu} - \frac{(M_n^2 - M_m^2)}{q^2} q_{\mu} \right) F^{nm}(q^2).$$
22. S. J. Brodsky and P. Roy, Phys. Rev. D 3, 2914 (1971).
23. J. Iliopoulos and E. A. Paschos, Phys. Rev. D 6, 1340 (1972).
24. The connection between confinement singularities and high energy behavior is discussed in R. C. Brower, W. L. Spence, and J. H. Weis, Phys. Rev. Lett. 40, 674 (1978).

25. R. C. Brower, private communication.
26. Such terms are discussed by M. B. Einhorn, Phys. Rev. D 15, 3037 (1976).
27. Fixed poles associated with bare photon couplings appear, for example, in the model of S. J. Brodsky, F. E. Close, and J. F. Gunion, Phys. Rev. D 5, 1384 (1972).
28. J. Schwinger, Phys. Rev. Lett. 3, 296 (1959).
29. See, for example J. D. Bjorken, Phys. Rev. 148, 1467 (1966).

FIGURE CAPTIONS

- Fig. I.A: The bound state wave function $\phi_n^{a\bar{b}}(x)$.
- Fig. I.B: The meson Green's function $G(x,y,q^2)$.
- Fig. I.C: The three-point hadronic vertex $\gamma_{n;km}(p_n, x)$ and its representation in terms of gluon exchange.
- Fig. I.D: The inelastic form factor $F_\mu(p, x)$ and its decomposition into bare and meson dominated parts.
- Fig. I.E: The meson $\rightarrow$ current + current amplitude $\gamma_{\mu\nu}^{n,a}(p_n; x_1, x_2)$.
- Fig. I.F: The hadronic two-to-two scattering amplitude $A_{jk;mn}(p_j, x_k, x_m, x_n)$. In (a), (b), (c), we show the schematic decomposition of the (s,t), (u,s) and (u,t) duality amplitude in terms of three point functions and contact terms.
- Fig. II.A: The structure of the Compton Amplitude M_I^{nm} in Frame I. The incident current is drawn to the left as a remnant of the time ordering of the diagrams. The diagrams correspond one-to-one to the terms of Eq. (II.3).
- Fig. II.B: The structure of the Compton Amplitude M_{II}^{nm} in Frame II. The diagrams correspond one-to-one to the terms of Eq. (II.10).
- Fig. II.C: A schematic representation of the term arising from the off-mass shell piece of the hadronic two-to-two amplitude which contributes to $[M_{II}^{nm}]$.

- Fig. III.A: An irreducible quark scattering diagram contributing to Compton scattering.
- Fig. IV.A: The parton handbag diagrams. The dotted propagators are dressed. The dashed line indicates that the discontinuity is to be taken.
- Fig. IV.B: Parton model of photoproduction of large mass lepton pairs via parton-antiparton annihilation.
- Fig. IV.C: Contributions to the photoproduction of large mass lepton pairs in 't Hooft model Frame I.
- Fig. IV.D: Contributions to the photoproduction of large mass lepton pairs in 't Hooft model Frame II.

Fig. I.A

Fig. I.B

XBL 789-1693A

Fig. I.C

Fig. I.D

Fig. I.E

Fig. I.F(a)

Fig. I.F(b)

Fig. I.F(c)

$$\begin{aligned}
 & \text{Diagram 1: A circle labeled } M_1^{nm} \text{ with external lines } x_1 p_n \text{ and } x_2 p_n. \\
 & = \sum_k \left[\text{Diagram 2: A chain of two circles with external lines } x_1 \text{ and } x_2 \text{ and internal lines } k \text{ and } m \text{ on the right.} \right] \\
 & + \sum_k \left[\text{Diagram 3: A chain of two circles with external lines } x_1 \text{ and } x_2 \text{ and internal lines } k \text{ and } n \text{ on the left.} \right] \\
 & + \sum_k \left[\text{Diagram 4: A chain of two circles with external lines } x_1 \text{ and } x_2 \text{ and internal lines } k \text{ and } m \text{ on the right, with a different internal connection.} \right] \\
 & + \sum_k \left[\text{Diagram 5: A chain of two circles with external lines } x_1 \text{ and } x_2 \text{ and internal lines } k \text{ and } n \text{ on the left, with a different internal connection.} \right]
 \end{aligned}$$

Fig. II.A

$$\begin{aligned}
 & \text{Diagram 1: A circle labeled } M_2^{nm} \text{ with external lines } q_1 \text{ and } z'q_1. \\
 & = \sum_{k, l} \left[\text{Diagram 2: A chain of two circles with external lines } q_1 \text{ and } z'q_1 \text{ and internal lines } k \text{ and } l \text{ on the top.} \right] \\
 & + \sum_k \left[\text{Diagram 3: A chain of two circles with external lines } q_1 \text{ and } z'q_1 \text{ and internal lines } k \text{ and } m \text{ on the right.} \right] \\
 & + \sum_{k, l} \left[\text{Diagram 4: A chain of two circles with external lines } q_1 \text{ and } z'q_1 \text{ and internal lines } k \text{ and } l \text{ on the bottom.} \right] \\
 & + \sum_{k, l} \left[\text{Diagram 5: A chain of two circles with external lines } q_1 \text{ and } z'q_1 \text{ and internal lines } k \text{ and } l \text{ on the bottom, with a different internal connection.} \right]
 \end{aligned}$$

Fig. II.B

XBL 769-1689A

Fig. II.C

Fig. III.A

XBL 789-1691A

Fig. IV.A

Fig. IV.B

Fig. IV.C

XBL 789-1709 A

Fig. IV.D

LIMIT	t	u	q ²	q' ²	M _m ²	x ₁	x ₂	z	z'
FORWARD HI ENERGY (SECT. III.A) LARGE ANGLE (SECT III.B)	$\frac{(M_n^2 - M_m^2)(q'^2 - q^2)}{s}$	-s	finite fixed	finite fixed	finite fixed	-q ² /s	q' ² /s	M _n ² /s	1
ELECTRO-PRODUCTION OF MASSIVE PAIRS "BJORKEN-PASCHOS LIMIT" (SECT. IV.A)	-2M _n v(x ₁ + x ₂)	$\frac{-s(1-x_2)}{1-x_1} \left(1 - \frac{2M_n v(1-x_1)}{s}\right)$	$\frac{-x_1}{1-x_1} s$	$\frac{x_2 s}{1-x_1} \left(1 - \frac{2M_n v(1-x_1)}{s}\right)$	2M _n v(1-x ₁ -x ₂)	0 < x ₁ < 1-x ₂ fixed	0 < x ₂ < 1 fixed	M _n ² (1-x ₁)/s	1 - $\frac{2M_n v}{s} (1-x_1)$
PHOTOPRODUCTION OF MASSIVE PAIRS (SECT. IV.B)	y ₁ (y ₂ -1)s	y ₂ (y ₁ -1)s	finite fixed	y ₁ y ₂ s	(1-y ₁)(1-y ₂)s	-q ² /s	y ₁ y ₁ y ₂ < y ₁ < 1 fixed	M _n ² /s	y ₂ y ₁ y ₂ < y ₂ < 1 fixed

TABLE I: LEADING BEHAVIOR AS $s \rightarrow \infty$ OF THE KINEMATIC VARIABLES IN THE LIMITS CONSIDERED

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