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Author
Chanowitz, Michael S.

Publication Date
1975-08-01

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Invited talk presented at the Meeting of the Division of Particles and Fields of the American Physical Society, Seattle, WA, August 27 - 29, 1975

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Michael S. Chanowitz

August 20, 1975

Prepared for the U. S. Energy Research and Development Administration under Contract W-7405-ENG-48

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COLOR AND EXPERIMENTAL PHYSICS*

Invited talk presented to the 1975 Seattle
Meeting of the Division of Particles and
Fields of the American Physical Society

Michael S. Chanowitz

Lawrence Berkeley Laboratory
Univerisity of California
Berkeley, California 94720

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ABSTRACT

After a brief review of the color hypothesis and the motivations for its introduction, we discuss the experimental tests. We assume colored states have not been produced at present energies and we discuss only experimental tests which apply below the color threshold, when color is a "hidden symmetry." Some of these tests offer the possibility of distinguishing between quark models with fractional and integral quark charges.

I. INTRODUCTION

In recent years most theorists concerned with the quark hypothesis have become increasingly convinced that it is necessary to endow the quarks with a new property known as color. Color has had little impact on experimental physics and experimentalists seem to be more skeptical of the idea than most of their theoretical colleagues. This skepticism is natural and well founded for at least two reasons. The first reason is historical: color is introduced as a conceptual "fudge factor" which seems to save the quark hypothesis by complicating it. If it turns out that quarks are the aether of hadronic physics, then color is the aether drag hypothesis. The second reason is related to an essential property of color SU(3): it is a hidden symmetry, hidden either forever or until very high (unspecified) energies are attained. This limits the variety of experimental consequences which can test the validity of the color hypothesis.

At the same time it is clear that we must take the color hypothesis seriously and pursue the experimental and theoretical consequences as far as we can. Color is not just an artificial "fudge factor" but provides a very elegant basis for quark dynamics. More importantly it seems to be a necessary ingredient of the quark model, which describes a great range of data from low energy spectroscopy to high energy dynamics. Tests of the color hypothesis are therefore a crucial component of the effort to evaluate the validity of the quark model.

The "hidden" nature of color poses a challenge to theory and experiment. The experimental challenge is to perform the small number of experiments which test the color idea; almost all involve formidable technical difficulties. The challenge to theorists is to enlarge the list of experimental consequences which can clearly test the color idea.

In this talk I will sketch two versions of the color hypothesis, briefly review the motivation for the introduction of color, and then go through the short list of consequences which can be tested experimentally. I will concentrate on the energy domain in which color is a hidden symmetry; if color degrees of freedom are ever actually excited there will be many directly observable consequences.

II. TWO VERSIONS OF THE COLOR HYPOTHESIS

The two versions of the color hypothesis under discussion each require a set of nine quarks $\{p_R, p_B, p_Y, n_R, n_B, n_Y, \lambda_R, \lambda_B, \lambda_Y\}$ where R, B, Y denote the primary colors red, blue, and yellow.

There is a color SU(3) which affects rotations in the R, B, Y space just as ordinary SU(3) affects rotations in the p, n, λ space. In both versions, the familiar hadrons of "every day life" are assumed to be singlets of color SU(3). For instance, a pion which in the original model is $|\pi^+\rangle = |p\bar{n}\rangle$ becomes in the color models

$$|\pi^+\rangle = \frac{1}{\sqrt{3}}|p_R\bar{n}_R + p_B\bar{n}_B + p_Y\bar{n}_Y\rangle \quad . \tag{1}$$

Ordinary hadrons are therefore colorless "white" blends of the three colors; it is in this sense that color SU(3) is a hidden symmetry.

The versions differ in the electromagnetic charges assigned to the quarks, which are exhibited in the electromagnetic current. In the version with fractional charges,[1,2] it is

$$J^{\text{fractional}}_{\text{e.m.}} = \sum_{i=R,B,Y} \left(\frac{2}{3}\,\bar{p}_i p_i - \frac{1}{3}\,\bar{n}_i n_i - \frac{1}{3}\,\bar{\lambda}_i \lambda_i\right)$$
$$= J_{(8,1)} \quad . \tag{2}$$

The subscript (8,1) denotes the fact that this current is an octet of ordinary SU(3) and a singlet of color SU(3). So $J^{\text{fractional}}_{\text{e.m.}}$ cannot transform a white (color singlet) state into a colored (nonsinglet) state.

In the second version the quarks have integral charge:[3] two are positive, two are negative, and five are neutral. The current is

$$J^{\text{integral}}_{\text{e.m.}} = \bar{p}_R p_R + \bar{p}_B p_B - \bar{n}_Y n_Y - \bar{\lambda}_Y \lambda_Y \quad . \tag{3}$$

With a judicious application of the mathematical theorem

$$x + y - y = x \tag{4}$$

this can be written as

$$J^{\text{integral}}_{\text{e.m.}} = J_{(8,1)} - J_{(1,8)}$$
$$= J^{\text{fractional}}_{\text{e.m.}} - J_{(1,8)} \tag{5}$$

where

$$J_{(1,8)} \equiv \sum_{q=p,n,\lambda} \left(\frac{2}{3}\,\bar{q}_Y q_Y - \frac{1}{3}\,\bar{q}_B q_B - \frac{1}{3}\,\bar{q}_R q_R\right) \quad . \tag{6}$$

If we begin with $J^{\text{fractional}}_{\text{e.m.}} = J_{(8,1)}$ and interchange $p \leftrightarrow Y$, $n \leftrightarrow B$, $\lambda \leftrightarrow R$ then we obtain the new piece $J_{(1,8)}$, which is therefore a singlet of ordinary SU(3) and an octet of color SU(3).

Because of this new piece, $J^{\text{integral}}_{\text{e.m.}}$ can transform a white state into a colored state. When the threshhold for the production of colored states is reached, then according to parton and light-cone ideas $J_{(1,8)}$ should cause $\sigma(e^+e^- \to \text{hadrons})$ to increase by a factor of two and for isoscalar targets the electromagnetic deep-inelastic structure functions νW_2 and W_1 should increase by nearly a factor of two (i.e., by 9/5 if we neglect λ quarks). Since no such dramatic increase is observed in the deep-inelastic muon scattering experiment at FNAL,[4] it seems to me unlikely that $J_{(1,8)}$ is the agent of any of the present commotion in e^+e^- annihilation.

Consequently in this talk I will concentrate on the problem of experimentally investigating color when we are below the color threshhold. In this regime, $J_{(1,8)}$ cannot contribute to processes involving a single electromagnetic current (such as e^+e^- annihilation and electroproduction in the lowest order of Q.E.D.) and the two versions of color are indistinguishable from one another (though in certain processes they may be distinguished from models with no color--see Section IV). However, there are a few processes, involving two electromagnetic currents, which offer the possibility of distinguishing between the two versions of the color hypothesis even though we are below the threshhold for the production of color. These are among the experimental processes discussed in Section IV.

III. MOTIVATION

Now I want to review very briefly (and unhistorically) the principal motivations for the introduction of color. At the same time I would like to convince you that color is not just an ugly complication without redeeming features but that it provides an elegant and natural possible setting for quark dynamics.

A. The Statistics Problem

Within months of the original quark proposal by Gell-Mann and Zweig, Greenberg[1] observed that the classification of the 8 and 10 of baryons (the 56 of SU(6)) as s-wave bound states of three quarks requires the introduction of a new threefold degree of freedom (which he called parastatistics of order three and which for our purpose is essentially the three color model with fractional charges). The point is that three quarks in the 56 representation of SU(6) are symmetric under interchange of spin, spatial, and

ordinary SU(3) degrees of freedom. This violates the Fermi statistics required of spin $\frac{1}{2}$ quarks. For instance, consider the quark model assignment of the $\Delta^{++}_{s_z=3/2}$ with $I_z = S_z = +\frac{3}{2}$: $|\Delta^{++}_{s_z=3/2}\rangle = |p^\uparrow, p^\uparrow, p^\uparrow\rangle$, where the arrows denote the spin polarization $s_z = +\frac{1}{2}$. Greenberg's idea is based on the group theoretical fact that the singlet configuration of three members of an SU(3) triplet is the totally antisymmetric combination. Thus if we endow quarks with a color SU(3) degree of freedom and require that $\Delta^{++}_{s_z=3/2}$ be in a singlet of color SU(3), then

$$|\Delta^{++}_{s_z=3/2}\rangle = \frac{1}{\sqrt{6}}\Big|p^\uparrow_R, p^\uparrow_B, p^\uparrow_Y + p^\uparrow_B, p^\uparrow_Y, p^\uparrow_R + p^\uparrow_Y, p^\uparrow_R, p^\uparrow_B - p^\uparrow_R, p^\uparrow_Y, p^\uparrow_B - p^\uparrow_Y, p^\uparrow_B, p^\uparrow_R - p^\uparrow_B, p^\uparrow_R, p^\uparrow_Y\Big\rangle \qquad (7)$$

and the statistics problem is automatically resolved.

B. The Lifetime of the Neutral Pion

This subsection also begins with a problem: Sutherland and Veltman showed using current algebra and PCAC that in the limit of zero pion mass, $\pi^0 \to \gamma\gamma$ is forbidden.[5] Taking the corrections to PCAC in the amplitude to be $\sim O(m_\pi^2/1\ \mathrm{GeV}^2)$, this implies that for the real pion $\Gamma(\pi^0 \to \gamma\gamma)$ should be $\sim(m_\pi/1\ \mathrm{GeV})^4 \sim 10^{-3}$ times smaller than we would naively estimate and than it is actually measured to be. Adler, Bell, and Jackiw[6] later showed that this argument omitted a contribution ("the anomaly") due to the basic triangle diagram, and that this contribution allows $\pi^0 \to \gamma\gamma$ to proceed in the zero pion mass limit. This technical argument acquired great significance when Adler and Bardeen[7] showed that in the PCAC limit, the simple triangle diagram is the only one which contributes to any finite order in renormalizable perturbation theory. Thus within the PCAC approximation (for which there is now considerable evidence from many sources) $\pi^o \to \gamma\gamma$ becomes a unique process, since the rate can be calculated in lowest order perturbation theory despite the higher order strong interaction corrections which would ordinarily make nonsense of a lowest order calculation.

When Adler's result is used in the quark model without color the result is

$$\Gamma(\pi^o \to \gamma\gamma)_{\text{no color}} = 0.81 \text{ eV} \quad . \tag{8}$$

The experimental average[8] is

$$\Gamma(\pi^o \to \gamma\gamma)_{\text{experiment}} = 7.8 \pm 0.9 \text{ eV} \quad . \tag{9}$$

Including three colors, the prediction is increased by three in the amplitude or nine in the rate, to

$$\Gamma(\pi^o \to \gamma\gamma)_{\text{color}} = 7.3 \text{ eV} \quad . \tag{10}$$

Of all the motivations discussed here, this is the only one which tells us that the number of colors must be three. For instance, the statistics problem could be resolved by introducing any color SU(n) with $n \geq 3$. The problems of scaling and the unobservability of quarks which are discussed below only require $n \geq 2$.

I want to emphasize that it is the absence of strong interaction radiative corrections established to any finite order in perturbation theory which distinguishes this result from the multitude of other quark and parton model results and which makes it especially worthy of our attention.

C. Deep-Inelastic Scattering

This and the following subsection on the unobservability of quarks involve motivations for introducing color which are less direct than the preceding considerations of the statistics problem and $\pi^o \to \gamma\gamma$. They are, however, important considerations to theorists trying to understand the quark-parton puzzle and they are also essential for appreciating why color may be an especially elegant basis for quark dynamics.

The quark-parton model provides a simple, intuitive "explanation" of the scaling observed in deep-inelastic scattering of electrons, muons, and neutrinos from nucleon targets. But as an "explanation" the model raises questions which are more profound than the ones it answers: how can the constituents of hadrons, strongly interacting entities, behave as if they are free of strong binding forces? It turns out that this question has a simple answer in a class of field theoretical models, called "asymptotically free" theories,[9] in which the strength of the interaction depends on the relevant scale of distance. When the distance scale is big (we consider that ~1 fm. is "big") the interaction is strong but for the short (i.e., lightlike) distances which characterize deep inelastic scattering the interaction becomes weaker so that the parton model is approximately valid.

It was first shown[10] that theories with a color symmetry are asymptotically free and, later, that they are the only such theories.[11] More precisely it was shown that nonabelian gauge theories containing massless vector meson "gluons" are the only asymptotically free theories. The nonabelian symmetry of this theory cannot be associated with ordinary SU(3) which has no long range massless vector mesons associated with it. Thus we must assume a new, hidden symmetry and in view of the preceding subsections color SU(3) is a natural choice.

In the version of the color hypothesis with integral quark charges, the color symmetry is broken by the term $J_{(1,8)}$ in the electromagnetic current, while with fractional charges the symmetry is exact. Presumably this means that there are electromagnetic corrections to asymptotic freedom in the integral charge model. However, these corrections could probably not be disentangled from high order electromagnetic effects which will produce scaling violations regardless of the underlying strong interaction theory. Therefore I doubt that asymptotic freedom alone provides a motivation for preferring the color model with fractional quark charges.

D. Unobservability of Quarks

The failure to observe the quark constituents is of course the central paradox. It is this paradox which suggests the analogy with the unobserved aether of the last century. Like the aether, quarks may simply mark the place where we have lost the trail. However, it is also possible that the quark idea is useful and valid even though single 300 MeV quarks do not exist. Two possibilities which have been proposed are (1) that free quarks may exist though with masses much greater than 1 GeV or (2) that quarks are permanently confined within hadrons. The color hypothesis provides a natural dynamical setting for these speculations.

The possibility that quarks may be extremely massive as a consequence of color dynamics was suggested by Nambu very soon after the first formulations of the color hypothesis.[12] Nambu speculated that if the color interaction is very strong and is mediated by an octet of colored gluons (gauge vector mesons) then color nonsinglet

states and, in particular, quarks would acquire very large masses, say >> 1 GeV, as a result of the large self-energy in the colored field. Ordinary hadrons, being color singlets, would presumably not acquire large masses by this mechanism. This speculation has been verified in the semiclassical approximation to a quantum field theory[13] but the effect of quantum fluctuations has not been explored.

Nambu's speculation is easily illustrated if we imagine an SU(2) world, in which the quarks are isodoublets of ordinary SU(2) and color SU(2). We suppose quarks interact pair-wise according to a color vector potential and that they have a very large mass M >> 1 GeV. Then the mass of a composite system of n quarks is given by the sum of the quark masses plus the energy of the pair-wise color interactions:

$$E = nM + W \sum_{\substack{i,j=1 \\ i\neq j}}^{n} \vec{t}_i \cdot \vec{t}_j \ , \qquad (11)$$

where W is a very large energy to be specified in a moment. Now we may write

$$\sum_{\substack{i,j=1 \\ i\neq j}}^{n} \vec{t}_i \cdot \vec{t}_j = \sum_{i,j=1}^{n} \vec{t}_i \cdot \vec{t}_j - \sum_{i=1}^{n} \vec{t}_i^{\,2}$$

$$= \left\{ \sum_{i=1}^{n} \vec{t}_i \right\}^2 - n\vec{t}^{\,2}$$

$$= T(T+1) - \tfrac{1}{2}\left(\tfrac{1}{2}+1\right)n \qquad (12)$$

where T is the total color isospin of the composite system. Substituting (12) into (11) gives

$$E = nm + T(T+1)W \qquad (13)$$

where

$$m = M - \frac{3}{4} W_{+} . \qquad (14)$$

We choose M and W to be very large, M,W >> 1 GeV, but $m = M - \frac{3}{4} W$ to be of the order of a few hundred MeV. Then for color isosinglets (i.e., "ordinary hadrons") E is of the order ~1 GeV, but quarks and other color nonsinglet states are much heavier. The spectroscopy of the ordinary hadrons is as if quarks had the light effective mass m even though a single quark has the much larger mass M. Of course this is meant only as a simple illustration of Nambu's idea. The actual dynamics would be much more complicated.

When Nambu's speculation is implemented in a semiclassical field theory[13] the colored vector gluons are given a very large mass like the other colored states. For technical reasons this probably means that Nambu's mechanism is incompatible with asymptotic freedom.[14] If we accept this explanation of the absence of free, light quarks, we must still face the problem of understanding the deep inelastic scaling phenomena.

The dynamics of asymptotic freedom also lends itself very naturally to a speculation about the unobservability of single quarks.[15] Just as the strength of the interaction decreases at small distances, it increases at large distances. Formally at least it seems that the energy required to separate a single quark from its parent hadron becomes singular in the large distance limit. Thus quarks (and also the massless color vector gluons) might be

permanently confined to the interior of hadronic bound states. Schwinger showed several years ago that the fermions in Q.E.D. in one space dimension are confined in this sense,[16] but it is not known whether the phenomenon actually occurs in the three dimensional gauge theories which are of interest here. If this is the way the world is constructed, then quark and gluon fields will be useful and experimentally meaningful concepts (e.g., in deep inelastic phenomena) even though quarks and gluons do not exist as single particle states.

IV. EXPERIMENTAL TESTS

In this section I will present the short list of experimental tests of the color hypothesis which are known to me. All are tests which apply below the threshold for the production of color nonsinglet states. Most do not distinguish between the fractional and integral charge versions of the color hypothesis. However, in subsections D and E I discuss experimental tests which can distinguish between the models with fractional and integral quark charges.

It is important to keep in mind that these predictions depend not only on the color hypothesis but also on certain hypotheses about quark-hadron dynamics. The latter hypotheses are not identical for all the predictions considered. For instance, the predictions for $\sigma(e^+e^- \to \gamma^* \to \text{hadrons})$ in Section A and for $\sigma(\gamma\gamma^* \to \text{hadrons})$ in E depend on the parton model/light-cone algebra hypothesis. The prediction for $\sigma(pp \to \mu^+\mu^- + \cdots)$ discussed in Section C depends on an extension of parton model ideas which is more speculative than the application of parton ideas in Sections A and E (for instance, this prediction cannot be obtained from the light-cone algebra). The predictions of Section D, which test the Adler anomaly, do not require the validity of the parton model at all: they are valid to any finite order in any renormalizable field theory. However, they do require extended application of the PCAC hypothesis.

Therefore if one of these predictions is verified, it tends to confirm both the color hypothesis and the additional dynamical assumptions which were introduced. If a prediction fails, the interpretation of the failure will depend on how it has failed and perhaps also on the outcome of other related tests. For example, if the scaling law discussed in Section C for $\sigma(pp \to \mu^+\mu^- + \cdots)$ is not verified, it will be evidence against the application of the parton model to this process but it will not be evidence against the color hypothesis. But if the scaling law is verified and the magnitude of the cross section agrees with the prediction of the colorless quark parton model, then the result should be counted against the validity of the color hypothesis.

A. Electron-Positron Annihilation

According to the hypothesis of scale invariance at short distances,[17] the ratio $R(s) = \sigma(e^+e^- \to \gamma \to \text{hadrons})/\sigma(e^+e^- \to \gamma \to \mu^+\mu^-)$ should approach a constant, $R(s) \to R$, for sufficiently large values of s. In the quark-parton model without color, the constant is $R = \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 = \frac{2}{3}$; in the model with color SU(3), it is $R = \frac{2}{3}\cdot 3 = 2$. The experimental data[18] shows that from $s \cong 6\ \text{GeV}^2$ to $s \cong 12\ \text{GeV}^2$ $R(s)$ is indeed constant within the ~10% systematic errors: this evidence for scaling is at least as good as the first evidence for scaling in deep-inelastic electron scattering, which covered a similar range in Q^2.

Within this range of s the average value of R is 2.45. Considering the point-to-point systematic errors of ~10% plus the 10% uncertainty in the overall normalization (which is not included in the error estimates usually displayed with the data), the experimental result is in reasonable agreement with the prediction of the color SU(3) hypothesis. Furthermore asymptotic freedom predicts that $R(s)$ should approach its asymptotic value slowly (corrections are like $\sim 1/\ln s$) and from above;[19] the $1/\ln s$ corrections are expected to be of the order of ~10% to ~25% in this range of s.[20]

The emergence of new phenomena for $s \gtrsim 12\ \text{GeV}^2$ prevents us from testing the asymptotic freedom prediction of a $1/\ln s$ approach to the asymptotic value of R. However, the new phenomena do offer a second possible testing ground for both the color hypothesis and asymptotic freedom. If we can describe the new hadronic spectroscopy in terms of new quarks and if the contribution of the new hadrons to the cross section can be disentangled from new nonhadronic degrees of freedom which may also be present, then we can again check whether the new quarks come in three colors. And if we are given a long enough respite before more new thresholds in s occur, we will also be able to determine whether the approach to scaling is logarithmic.

Although more conjectural, it is also possible that the dynamics of the new hadrons may provide insight into color and asymptotic freedom. According to the charmonium model,[21] the large mass scale of the new hadrons means that their decays are characterized by short distances and small effective coupling constants. This idea implies a variety of qualitative, semiquantitative and perhaps (if we are optimistic enough) quantitative tests. For instance, of the three predicted p-wave excitations of $\psi(3.1)$, the scalar and tensor would decay into hadrons via two color vector gluons but the pseudovector state would have to decay via four vector gluons (Yang's theorem) and would consequently have a much smaller hadronic width than its two partners.[22]

B. Violations of Scaling in Deep-Inelastic Scattering

Careful measurement of the deep-inelastic lepton-nucleon structure functions can in principal provide a test of asymptotic freedom and therefore, indirectly, of the color hypothesis. For Q^2 and ν large enough so that perturbation theory in the effective coupling constant is valid, the theory predicts logarithmic violations of Bjorken scaling. The most rigorous predictions are for the moments of the structure functions: for instance, for the proton-neutron difference in electroproduction the result is[23]

$$\int_0^1 dx\, x^{n-2}\, \nu W_2^{I=1}(x,Q^2) = C_n \left(\ln \frac{Q^2}{\mu^2}\right)^{-\lambda_n} \left[1 + O\left(\frac{1}{\ln Q^2}\right)\right] \tag{15}$$

where

$$\lambda_n = A \left\{1 - \frac{2}{n(n+1)} + 4 \sum_{m=2}^{n} \frac{1}{m}\right\} \quad . \tag{16}$$

Here $x = Q^2/2M_p\nu$ is the scaling variable, C_n is an undetermined constant, μ^2 is a constant found experimentally to be $\sim O(1\ \mathrm{GeV}^2)$, and A is a constant which is determined by the color gauge group. For color SU(3), $A = 2/27$. Data from SLAC are compatible with the predicted logarithmic violations and with the value of A predicted

by color SU(3), though the uncertainties are too great and the kinematical domain too restricted to be conclusive.[24]

The equations for the moments, such as (15), can be inverted to give a qualitative--and perhaps even a quantitative--picture of how the structure functions change with increasing Q^2. Using positivity and the dimension of the stress tensor it is known that for interacting field theories $\lambda_2 = 0$ and $\lambda_{n+1} > \lambda_n$.[25] This implies that in a plot of νW_2 versus x, as Q^2 increases the area under νW_2 remains constant while νW_2 becomes more sharply peaked in the small x region. The most recent data from the FNAL muon experiment[4] does show this pattern of scaling violations.

With an additional dynamical assumption the equations for the moments can be inverted and solved for the structure functions.[26] The structure functions at all Q^2 are then predicted from their values at some initial $Q^2 = Q_0^2$ (and by their value at one other (x,Q^2) point to determine the parameter μ^2). The results of such analyses are again compatible with the SLAC data and with the recent FNAL data. However, it has been argued that the data is also consistent with a phenomenological treatment of nonasymptotically free theories with anomalous dimensions.[24] Enormously larger values of Q^2 would be necessary to distinguish between these two possibilities.

C. Production of Massive Muon Pairs in Hadronic Collisions

Drell and Yan have argued[27] that the parton model may be applied to the process

$$\text{proton} + \text{proton} \rightarrow \mu^+\mu^- + \ldots$$

provided that the variables

$$s = (p_{\text{proton }1} + p_{\text{proton }2})^2 \quad \text{and} \quad Q^2 = (p_{\mu_+} + p_{\mu_-})^2$$

satisfy the constraint

$$s \gg Q^2 \gg 1 \text{ GeV}^2 \qquad (17)$$

with Q^2/s finite. In this kinematical region they argue that the impulse approximation is valid and that the dominant process is the annihilation of a parton from one proton with an antiparton from the other proton. In the scaling limit in a parton model without color their expression for the inclusive muon pair cross section is

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{3Q^4} \int_0^1 dx_1 \int_0^1 dx_2\, \delta\left(x_1 x_2 - \frac{Q^2}{s}\right)$$

$$\times \sum_i e_i^{\,2} F_{2i}(x_1)\, F_{2\bar{i}}(x_2) \qquad (18)$$

where e_i is the charge of parton i and $\frac{1}{x_j} F_{2i}(x_j)$ or $\frac{1}{x_j} F_{2\bar{i}}(x_j)$ are the probabilities to find the i'th parton or anti-parton with a fraction x_j of the longitudinal momentum of proton j (in a reference frame in which the protons are moving very rapidly, with momentum $P \gg \sqrt{s}$). The $F_{2i}(x)$ may be extracted from electron, muon, and neutrino deep-inelastic scattering data.

The first experimental test of (18) is to check the scaling law: $d\sigma/dQ^2$ should be proportional to Q^{-4} times a function of the scaling variable Q^2/s. If the scaling law is verified, we may assume that the parton-antiparton annihilation mechanism is dominant and we

may proceed to study (18) quantitatively. It is at this point that we can test the color hypothesis. If the quark-partons come in n colors, then an additional factor of $1/n$ appears on the right side of (18) to account for the diminished probability that a parton of a given color in one nucleon will find its antiparton color-mate in the other nucleon. Reports[28] from a recent FNAL neutron scattering experiment appear to be consistent with (18). However, the normalization of the observed cross section is uncertain to a factor of two or three and a systematic study of the scaling law has not yet been performed.

D. Further Tests of the Chiral Anomaly

As discussed in the previous section the low energy theorem for $\pi^0 \to \gamma\gamma$ provided one of the principal motivations for the introduction of color and was the only motivation requiring that the color group be SU(3). This is a considerable theoretical weight to rest on one experimental number. Furthermore several authors have suggested that the PCAC extrapolation from zero pion mass to the mass shell may account for the missing factor of three in the amplitude and that there is no need to introduce color to explain the decay rate.[29] It is therefore important to find other tests of the physics which underlies the low energy theorem and of the strong PCAC assumption that the corrections due to the mass shell extrapolation are small.

Fortunately there is a large family of related anomalies,[30] corresponding to triangles, boxes and pentagons, which gives rise to low energy theorems which are also free of strong interaction radiative corrections. For instance, the part of the $K\ell_4$ decay amplitude $(K \to \pi\pi\ell\nu)$ which proceeds through the weak vector current is controlled by such a low energy theorem. A comparison with experiment indicates agreement with the anomaly, strong PCAC (which here includes an extrapolation to zero K mass), and color SU(3), though the uncertainties are quite large.[31] I have recently considered the low energy theorems for $\eta \to \gamma\gamma$ and $\eta \to \pi^+\pi^-\gamma$ and find[32] that the experimental results are again consistent with the anomaly, strong PCAC and color SU(3) provided that the η has a small, negative singlet-octet mixing angle as in the quadratic mass formula. These results are of additional interest because they may enable us to distinguish between the two versions of the color hypothesis even though we are below the threshold for the production of color.

Consider first the reason that $\pi^0 \to \gamma\gamma$ does not distinguish the two kinds of color. In the model with fractional charges, the amplitude is $\langle\pi^0|(J_{8,1})^2|0\rangle$ where $|0\rangle$ denotes the vacuum and $J_{(8,1)}$ was defined in Section II. With integral charges, the amplitude is $\langle\pi^0|(J_{(8,1)} - J_{(1,8)})^2|0\rangle$ and the difference between the two models is given by $\langle\pi^0|- J_{(8,1)}J_{(1,8)} - J_{(1,8)}J_{(8,1)} + J_{(1,8)}{}^2|0\rangle$. Now π^0 is a member of (8,1) (the 8 of ordinary SU(3) and the 1 of color SU(3)) but $(8,1) \times (1,8) = (8,8)$ has no projection into the 1 of color SU(3) and $(1,8) \times (1,8) = (1,1) + (1,8) + \cdots$ has no projection into the 8 of ordinary SU(3). Therefore the difference term vanishes and the amplitude is the same in both models.

However, now suppose that we consider the $\gamma\gamma$ decay of an SU(3) singlet pseudoscalar meson. Then, as was first observed by Okubo,[33] the amplitude in the integral charge model would be

$$\langle(1,1)|(J_{(8,1)} - J_{(1,8)})^2|0\rangle = \langle(1,1)|(J_{(8,1)})^2 + (J_{(1,8)})^2|0\rangle$$

$$= 2\langle(1,1)|(J_{(8,1)})^2|0\rangle \qquad (19)$$

which is exactly twice the amplitude obtained in the fractional charge model. (This is because $(8,1) \times (8,1)$ and $(1,8) \times (1,8)$ have equal projections into (1,1).)

Even for small mixing angles (e.g., $\theta = -11^\circ$), the analysis[32] of $\eta \to \gamma\gamma$ and $\eta \to \pi\pi\gamma$ indicates that the singlet component of η is responsible for about half of the observed rate. That is, for a pure octet we have $\Gamma(\eta_8 \to \gamma\gamma) = 160$ eV but with a mixing angle of $\theta = -11^\circ$ the result is nearly 300 eV. The most recent experimental value[34] is 324 ± 46 eV.

Since the singlet component makes a substantial contribution to $\eta \to \gamma\gamma$, we may hope to use the η decays to distinguish between the two versions of color. In the analysis it is crucial to consider simultaneously the low energy theorems for $\eta \to \gamma\gamma$ and $\eta \to \pi\pi\gamma$ in order to resolve the uncertainties due to our ignorance of the SU(3) singlet PCAC constant (the analogue of $F_\pi = 95$ MeV) and the mixing angle.[32] The results of this analysis are compatible with strong PCAC and color SU(3). The fractional charge version of the color hypothesis is preferred, though the differences between the predictions of the two color models are too small and the uncertainties are too great for this to be decisive.

In order to clearly distinguish between the two color models, it is necessary to consider a further step into the uncertainties of large mass PCAC extrapolations and to consider the low energy theorems for $\eta' \to \gamma\gamma$ and $\eta' \to \pi\pi\gamma$. Since the η' is taken to be dominantly in an SU(3) singlet, it is most sensitive to the differences between the two color models. Making use of the constraints obtained in the study of $\eta \to \gamma\gamma$ and $\eta \to \pi\pi\gamma$, we find that for any value of the mixing angle in the interval $0 < -\theta \leq 17^o$, the ratio of η' decays is

$$7.1 > \frac{\Gamma(\eta' \to \pi^+\pi^-\gamma)}{\Gamma(\eta' \to \gamma\gamma)} \geq 2.7 \pm 0.5$$

for fractional charges and

$$1.8 > \frac{\Gamma(\eta' \to \pi^+\pi^-\gamma)}{\Gamma(\eta' \to \gamma\gamma)} > 0$$

for integral charges. The experimental situation is confused with experimental values reported[35] between 9.1 ± 2.2 and 17 ± 3 (the confusion is due to disagreement on the branching ratio for $\eta' \to \pi\pi\gamma$). The fractional charge model is much nearer the range of experimental values and is compatible with the lowest reported value.

It is necessary to emphasize the uncertainties in this analysis: η' pole dominance may or may not be valid in these amplitudes and PCAC for the SU(3) singlet axial current involves theoretical problems[36] which are not understood. Since theoretical ignorance in these areas is nearly perfect, my attitude is by necessity purely phenomenological. To go farther we need experimental measurements of the rates $\Gamma(\eta' \to \gamma\gamma)$ and $\Gamma(\eta' \to \pi^+\pi^-\gamma)$, to see if these rates and their ratio are compatible with the low energy theorems. These rates and the total width of the η' are presently not known.

The present bounds are based primarily on strong interaction bubble chamber experiments. Primakoff photoproduction at FNAL and SPS energies and production by the two photon process in $e^\pm$ storage rings are also alternatives which should be considered.

There are also low energy theorems for the $\gamma\pi\pi\pi$ and $\gamma\gamma\pi\pi\pi$ amplitudes[37,38] which test the theory of chiral anomalies and color SU(3), though like $\pi^o \to \gamma\gamma$ they do not distinguish between the two color models. These amplitudes could be studied in e^+e^- colliding beams. In order to minimize the extrapolation to the low energy point (where all momentum invariants vanish, $p_i \cdot p_j = 0$) it is important to make the measurement as near the threshold, $s = 9m_\pi^2$, as possible. If for instance a measurement could be made at $s = 16m_\pi^2$, then the extrapolation to $s = 0$ would be comparable to the extrapolations required in the K and η decays discussed above.

The $\gamma\pi\pi\pi$ amplitude can probably be studied more favorably by using a pion beam to produce a dipion in the Coulomb field of a heavy nucleus, that is, the Primakoff effect.[39] This has a clear advantage over $e^+e^- \to \gamma \to \pi\pi\pi$ since in the Primakoff process the photon mass is very nearly zero. To minimize the other momentum invariants and the effect of the rho meson tail, the measurement should be performed as close to the dipion threshold, $s_{\pi\pi} = 4m_\pi^2$, as possible. In order to achieve a small photon mass and a pronounced Primakoff peak, the experiment would best be performed at FNAL or SPS energies. An alternative method which has been studied recently[40] is pion-electron scattering, again at very high energies. This method has the advantage that it lacks a strong interaction beckground.

The low energy theorem for the two photon decay of the scalar resonance, $\varepsilon \to \gamma\gamma$, also provides a test of the color hypothesis.[41] With the usual assignment of the ε to an SU(3) singlet, this decay is also sensitive to the difference between fractional and integral charge color models. The result is

$$\Gamma(\varepsilon \to \gamma\gamma) = 0.2\, R^2 \text{ keV} \tag{20}$$

where $R = 2$ for the color model with fractional charges and $R = 4$ with integral charges. This low energy theorem is not free of radiative corrections in perturbation theory. Rather its validity depends on the same assumptions about short distance behavior which are implicit in the parton model and the light cone algebra. The measurement of $\varepsilon \to \gamma\gamma$ could be performed using the two photon process in $e^{\pm}$ storage rings or perhaps by Primakoff photoproduction.

Preliminary indications based on a very few events are consistent with the rate predicted by the low energy theorem with color, though the uncertainties are too great to distinguish color models.[42] Furthermore, equation (20) is based on a narrow-width pole dominance extrapolation from the low energy point; for quantitative comparisons it is important to do a more careful treatment of the extrapolation which takes account of the large width of the ε.

E. Deep Inelastic Scattering From a Photon Target

Electron-positron storage rings can be used to study photon-photon scattering by selecting events in which the electron and positron do not annihilate.[43] This cross section is smaller than the annihilation cross section by a factor α^2 but when the photons are near the mass shell the cross section is enhanced by a factor $\sim(\ell n\, E/m_e)^2$. This together with the strong forward peaking of the $\gamma\gamma$ events means that it should be possible to measure $\sigma(\gamma\gamma \to \text{hadrons})$, probably in the next generation of storage ring experiments.

In the parton model (or, equivalently, using the light cone algebra) the cross section for two highly virtual photons to scatter into hadrons is given by the easily computed diagram for the two photons to annihilate and create a quark-antiquark pair.[44] This cross section contains a factor $\sum_i e_i^4$ where the e_i are the quark charges, allowing us in principle to distinguish between the following possibilities:

$$\sum_i e_i^4 = \begin{cases} \left(\frac{2}{3}\right)^4 + \left(\frac{1}{3}\right)^4 + \left(\frac{1}{3}\right)^4 = \frac{2}{9} & \text{No Color} \\ \frac{6}{9} & \text{Color-Fractional Charges} \\ \frac{12}{9} & \text{Color-Integral Charges (below color threshold)} \\ \frac{36}{9} & \text{Color-Integral Charges (above color threshold)} \end{cases}$$

For the same group theoretical reasons discussed in connection with the η and η' decays in Section D, the scattering of two highly virtual photons allows us to distinguish between the two versions of the color hypothesis at energies below the color threshold. Unfortunately this is of purely theoretical interest, since when both photons are very virtual the enhancement factors $[\ell n(E/m_e)]^2$ are replaced by suppression factors due to the photon propagators and the cross section is far too small to observe.

However, R. P. Worden has shown in a beautiful analysis[45] that it may be possible to measure $\sum_i e_i^4$ in processes in which only one photon is virtual while the second is near its mass shell. Worden obtains this result by using analyticity together with the light-cone parton model results of Ref. 44.

The transverse structure function, W_T, is controlled by the scattering of the virtual photon from the hadronic (vector dominated) components of the on-mass-shell photon; therefore W_T is not sensitive to the presence of color. But the longitudinal structure function W_L and a second structure function W_3 (which appears because the photons tend to be polarized) are dominated by parton-antiparton creation and are therefore proportional to $\sum_i e_i^4$. The structure functions may be separated experimentally by characteristic dependences on the photon polarizations, and in the case of W_3, by a characteristic angular dependence.

Since only one photon is virtual, the possibility of observing this process is greatly improved. It would be interesting to explore the feasibility of measuring this process either at present high energy storage rings or in the next generation of storage rings with beam energies of ~ 15 GeV.

V. CONCLUDING REMARKS

Though it has not been established that color is an absolutely necessary element of the quark model, it is a very natural element. If the color hypothesis is shown to be inconsistent with experiment, it will reflect on the validity of the quark model itself unless alternative explanations of the problems discussed in Section III can be obtained. Present experimental knowledge is consistent with the color hypothesis but more experimental information should be forthcoming as discussed in Section IV. Some of the experiments discussed in Section IV--e.g., those involving storage rings or deep-inelastic scattering--will soon be performed since they are part of large on-going experimental programs. Other experiments, equally worthy of study, have not yet, to my knowledge, been placed on an experimental program. In this category I have in mind especially the Primakoff experiments discussed in Section IV which test the physics underlying the low energy theorem for $\pi^0 \to \gamma\gamma$.

ACKNOWLEDGMENT

I want to thank Herb Steiner for some discussions about color which in part motivated this talk. I also wish to thank John Ellis for helpful conversations.

FOOTNOTES AND REFERENCES

* This work was supported by the U. S. Energy Research and Development Administration.

1. O. W. Greenberg, Phys. Rev. Lett. 13, 598 (1964).
2. W. A. Bardeen, H. Fritzsch, and M. Gell-Mann, Scale and Conformal Symmetry in Hadron Physics, ed. R. Gatto (Wiley, New York, 1973).
3. M. Y. Han and Y. Nambu, Phys. Rev. B139, 1006 (1965); Y. Miyamoto, Prog. Theo. Phys. Sup. Extra No., 187 (1965); A. Tavhelidze, Proceedings of the Seminar on High Energy Physics and Elementary Particles, I.A.E.A., Vienna, 763 (1965).
4. Y. Watanabe et al., Lawrence BErkeley Laboratory Report LBL-3870 (1975); C. Chang et al., Lawrence Berkeley Laboratory Report LBL-3886 (1975).
5. For a complete review with references to the original literature see, S. L. Adler, Lectures on Elementary Particles and Quantum Field Theory, ed. S. Deser et al. (M.I.T. Press, Cambridge, Mass., 1971).
6. S. L. Adler, Phys. Rev. 177, 2426 (1969); J. S. Bell and R. Jackiw, Nuovo Cimento 60A, 47 (1969).
7. S. L. Adler and W. A. Bardeen, Phys. Rev. 182, 1517 (1969).
8. Particle Data Group, Phys. Lett. 50B, 1974.
9. For a recent review, see H. D. Politzer, Physics Reports 14, 129 (1974).
10. G. 't Hooft, 1972 (unpublished); D. Gross and F. Wilczek, Phys. Rev. Lett. 26, 1343 (1973); H. D. Politzer, Phys. Rev. Lett. 26, 1346 (1973).
11. S. Coleman and D. Gross, Phys. Rev. Lett. 31, 851 (1973).

12. Y. Nambu, Preludes in Theoretical Physics, ed. A. de Shalit et al. (North Holland, Amsterdam, 1966).
13. W. Bardeen, M. Chanowitz, S. Drell, M. Weinstein, and T.-M. Yan, Phys. Rev. D11, 1094 (1975).
14. See reference 9, p. 148, and references therein.
15. See, for instance, A. Casher, J. Kogut, and L. Susskind, Phys. Rev. Lett. 31, 792 (1973).
16. J. Schwinger, Phys. Rev. 128, 2425 (1962).
17. K. Wilson, Phys. Rev. 179, 1499 (1969).
18. J. E. Augustin et al., Phys. Rev. Lett. 34, 764 (1975).
19. T. Appelquist and H. Georgi, Phys. Rev. D8, 4000 (1973); A. Zee, Phys. Rev. D8, 4038 (1973).
20. The leading corrections are $\frac{4}{9}/\ln\frac{s}{\mu^2}$, where the scale $\mu^2 \sim 1\ \mathrm{GeV}^2$ is suggested by studies of the electroproduction data (see Section IV.B).
21. T. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 43 (1975).
22. This remark is due to Ken Lane.
23. D. Gross and F. Wilczek, Phys. Rev. D8, 3633 (1973); H. Georgi and H. Politzer, Phys. Rev. D9, 416 (1974).
24. See, for instance, the analysis of W.-K. Tung, Univ. of Chicago preprint EFI 75/36, July, 1975, and references therein.
25. O. Nachtman, Nucl. Phys. B63, 237 (1973).
26. G. Parisi, Phys. Lett. 43B, 107 (1973) and 50B, 367 (1974). See also Ref. 9, p. 167.
27. S. Drell and T.-M. Yan, Phys. Rev. Lett. 25, 316 (1970).
28. W. Y. Lee, reported to the 1975 SLAC Summer Institute on Particle Physics, to be published.

29. S. Drell, Phys. Rev. D7, 2190 (1973); R. Delbourgo and M. Scadron, Univ. of Arizona preprint (July, 1975).

30. W. Bardeen, Phys. Rev. 184, 1848 (1969); R. Brown, C.-C. Shih, and B.-L. Young, Phys. Rev. 186, 1491 (1969).

31. Calcualtion by J. Wess and B. Zumino, reported in L.-M. Chounet, J.-M. Gaillard, and M. K. Gaillard, Phys. Reports 4C, 201 (1972).

32. M. Chanowitz, Radiative Decays of η and η' as Probes of Quark Charges, Lawrence Berkeley Laboratory report LBL-4200 (July, 1975).

33. S. Okubo, Symmetries and Quark Models, ed. R. Chand (Gordon and Breach, New York, 1970). See also H. Suura, T. Walsh, and B.-L. Young, Lettere al Nuovo Cimento 4, 505 (1972).

34. A. Browman et al., Phys. Rev. Lett. 32, 1067 (1974).

35. See Aguilar et al., Phys. Rev. D6, 29 (1973); A. Rittenberg, Lawrence Radiation Laboraotry report UCRL-18863, Thesis (1969), and Ref. 8.

36. See S. Weinberg, Proceedings of the XVII International Conference on High Energy Physics, ed. J. Smith (Science Research Council, London, 1974).

37. M. V. Terent'ev, JETP Letters 14, 140 (1971) and Physics Letters 38B, 419 (1972).

38 S. Adler, B. Lee, S. Treiman, and A. Zee, Phys. Rev. D4, 3897 (1971).

39. This has been studied by A. Zee, Phys. Rev. D6, 900 (1972). See also Ref. 37.

40. R. Crewther and S. Rudaz, unpublished.

41. R. Crewther, Phys. Rev. Lett. 28, 1421 (1972); M. Chanowitz and J. Ellis, Physics Letters 40B, 397 (1972) and Phys. Rev. D7, 2490 (1973).

42. S. Orito, M. Ferrer, L. Paoluzi, and R. Santonico, Physics Letters 48B, 380 (1974).

43. For a review with references see H. Terazawa, Rev. Mod. Phys. 45, 615 (1973).

44. D. Gross and S. Treiman, Phys. Rev. D4, 2105 (1971); T. Walsh and P. Zerwas, Nucl. Phys. B41, 551 (1972). For a more complete list, see Ref. 45.

45. R. P. Worden, Phys. Lett. 51B, 57 (1974).

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