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ROLE OF END REGIONS IN THE STEADY STATE OF MIRROR-CONFINED PLASMAS*

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ABSTRACT

In open-ended plasma confinement at reasonable ion temperatures and relatively high densities (such as 10 keV and 10^{13} cm^{-3}, or 100 eV and 10^{10} cm^{-3}) the Coulomb-scattering loss flux alone gives rise to a charged-particle density outside the mirrors that is usually large enough to require quasi-neutrality there. Such confinement must therefore be considered as involving an external plasma even in the absence of ionization or other plasma sources in this external region. We have investigated some of the effects of this plasma on the steady-state features of mirror confinement. In particular, we find that a significant electrostatic potential must usually exist between a mirror and any external boundary (though its magnitude can be reduced by plasma sources outside the mirrors). It follows that some energetic electrons will have turning points well outside the mirrors and yet remain trapped. Any external plasma source will cause additional electrons to stream through the confinement region. An estimate has been made of the change in steady-state confinement parameters due to this streaming, in the limit of negligible unstable interaction with the confined plasma and some trapping of the streaming electrons. The effect of these properties of open-ended confinement on various instabilities has not yet been fully explored.

1. INTRODUCTION

The following analysis is applicable to open-ended confinement systems with guiding-center plasmas in the "local approximation," with particle loss times much longer than typical bounce periods, and perpendicular ion temperatures $T_{i\perp} \gg T_e$. The configuration need not be a purely poloidal mirror field, but it must have magnetic mirrors inside the vacuum chamber.

The guiding-center motion of a nonuniform plasma is described in terms of a potential for parallel motion

$$U(s,\mu) = \mu B(s) + q\phi(s), \tag{1}$$

where q is the particle charge and $\mu = mv_\perp^2/2B$ is the magnetic moment invariant. We take the maxima of B (magnetic field strength) along any field line to be at $s = \pm L$, where s is arc length measured from the midplane. We assume symmetry in s and henceforth refer only to $s > 0$. We call $s = L$ "the mirror(s)." At $s = W$ $(W > L)$ we assume there is a wall, in general accompanied by a sheath having an electrostatic potential drop ϕ_W, with the wall itself grounded. Steady-state[†] loss rates maintain the mirror-confined plasma at a positive potential, ϕ_0, at the midplane, decreasing to ϕ_L at $s = L$ and ϕ_W at $s = W^-$, a Debye length away from the wall. We always make the "local approximation" that all quantities change slowly enough with distance normal to a field line so that flux tubes can be considered independent. We call the magnetic field at $s = 0, L, W$ respectively B_0, B_L, B_W, and define the mirror ratios $R_L = B_L/B_0$ and $R_W = B_W/B_0$.

On the basis of single-particle behavior, Eq. (1) with ϕ = constant gives the familiar "loss cone" in midplane velocity space (or energy space: $\epsilon_{\parallel} \equiv 1/2\, mv_{\parallel 0}^2$, $\epsilon_{\perp} \equiv 1/2\, mv_{\perp 0}^2 = \mu B_0$). If R_M is the smallest mirror ratio accessible to particles on the given flux surface, then particles with $\epsilon_{\perp}(R_M - 1) < \epsilon_{\parallel}$ have magnetic moment too small to be confined, and they are lost in a transit time.

But at higher densities, where scattering into the loss cone becomes important, the large ratio of electron-scattering to ion-scattering rates leads to the ambipolar plasma potential ϕ_0, which balances electron and ion loss rates by trapping low-μ electrons electrostatically (KAUFMAN, 1956; POST, 1958). Thus the loss region is no longer a cone in midplane velocity space, but a pair of hyperbolas (PERSSON, 1966), one for electrons and one for ions, as shown in Fig. 1. If scattering is negligible in a bounce time, the steady-state distributions in midplane velocity, f_e and f_i, are very nearly zero in the loss regions

$$\epsilon_{\parallel} > \epsilon_{\perp}(R_M - 1) + e(\phi_0 - \phi_M) \qquad \text{(electrons)} \tag{2}$$

and

$$\epsilon_{\parallel} > \epsilon_{\perp}(R_M - 1) - e(\phi_0 - \phi_M) \qquad \text{(ions).}$$

These are then boundary conditions on the f's.

In this case one finds ϕ_0 by equating approximate electron and ion loss rates (KAUFMAN, 1956; BENDANIEL, 1961; PERSSON, 1966);‡ then in calculating the profile $\phi(s)$ one ignores the density of loss-component particles (one their way out) compared with the trapped ones,§ and also uses a better approximation to the true loss criterion. For example, BenDaniel calculates ϕ_0 using a single escape energy $e\phi_0$ for the electrons (purely electrostatic trapping). But in deriving $\phi(s)$, the electrons are assumed Maxwellian for $\epsilon_{\parallel} < \epsilon_{\perp}(R_L - 1) + e\phi_0$.** In both calculations the ion boundary is taken to be a cone of altered slope, $\epsilon_{\parallel} = \epsilon_{\perp}(R_e - 1)$, where $R_e \equiv R_L(1 + e\phi_0/T_i)^{-1}$, instead of $\epsilon_{\parallel} = \epsilon_{\perp}(R_L - 1) - e\phi_0$; and except in R_e, ϕ is ignored in the calculation of f_i. In this model the confinement region, between mirrors, is thus isolated from any exterior phenomena, as long as external sources are absent.

2. NONISOLATED MIRROR SYSTEMS

However, a further sophistication is necessary: In this paper we show that in many cases of practical interest the streaming loss alone can give rise to a charged-particle density outside the mirrors that is generally large enough to require quasineutrality there. We also show that with equal ion and electron loss fluxes, the loss-component electrons alone cannot maintain the quasineutrality, and that the ambipolar $\phi(s)$ must continue to decrease out to the wall, with the result that some energetic electrons will have turning points well outside the mirrors and yet remain trapped. These electrons, which we denote by superscript ST (streaming outside the mirrors but still trapped) assist the loss-component electrons, denoted by superscript SF (streaming, free) in balancing the ion loss-component density. The region between mirrors (henceforth called region I) is thus not isolated from the external region (region II: $s > L$).

If diffusion takes place only in region I then the streaming ion flux is independent of s in region II except for the "area factor"

$B(s)/B_L$:

$$F_i(s) = \int_0^L \frac{B(s)}{B(s_1)} \frac{dn}{dt}_{Loss} (s_1)ds_1 \approx R_s L n_0 \nu ,$$

where ν is the inverse loss time of a typical ion and n_0 is the mid-plane ion density. Since B(s) decreases with s in region II, ions are accelerated out, gaining parallel energy

$$\frac{1}{2} m_i v_{\parallel}^2 = \epsilon_{\perp}(R_L - R_W) + e(\phi_L - \phi_W)$$

by the time they reach $s = W^-$ (they creep over the barrier at $s = L$ with negligible $v_{\parallel}$).

Let $T_{\perp}^{Loss}$ be a typical value for $\epsilon_{\perp}$ among these ions. Then if

$$\frac{e|\phi_L - \phi_W|}{T_{\perp}^{Loss}(R_L - R_W)} < 1 \qquad \text{(good for } R_L - R_W \sim 1 \text{ and } T_{\perp}^{Loss} \gtrsim T_e\text{)},^{\dagger\dagger}$$

then
$$n_i(W) = \frac{F_i(W)}{v_{\parallel}} \approx n_0 \left[(T_{\perp}^{Loss}/T_{\parallel})(R_L - R_W)\right]^{-1/2} \left(\frac{R_W L \nu}{c_{\parallel}}\right) ,$$

where $c_{\parallel} \equiv \left(\frac{2}{m_i} T_{\parallel}\right)^{1/2}$ and $T_{\parallel}$ refers to the midplane. These ions, whose density cannot be matched by streaming electrons, as we will show, cause a large positive potential and attract electrons (from region I if there are no others available) until quasineutrality is established, provided that the resulting Debye length is smaller than the size of the external region.

We have

$$\underset{L<s<W}{\text{Max}}\ \lambda_D(s) = \text{Max}\left[\frac{T_e^{EXT}(s)}{4\pi e^2 n_i}\right]^{1/2} < \left(\frac{T_e}{4\pi e^2 n_0}\right)^{1/2}\left[(R_L - R_W)(T_\perp^{Loss}/T_\parallel)\right]^{1/4}$$

$$\times\left(\frac{R_W L\nu}{c_\parallel}\right)^{-1/2},$$

where we have assumed $n_e \sim n_i$ and written $T_e^{EXT}(s)$ for the local mean energy at s. In the absence of an externally supplied beam, we expect this energy to be less than or at most equal to the midplane temperature T_e because of the potential difference $e(\phi_0 - \phi_s)$. Now certainly $\nu \gtrsim 1/2\ \nu_{90\ deg}$ for $R_L - 1 \sim 1$ (the actual coefficient depends on R_L, going to 0 for $R_L \to \infty$ and ∞ for $R_L \to 1$), where

$$\nu_{90\ deg}\ (\sec^{-1}) \sim 0.7 \times 10^{-7}\ \frac{n_0 \ln \Lambda}{A^{1/2}T^{3/2}}\ (n \text{ in cm}^{-3},\ T \text{ in eV}) \qquad (4)$$

is the usual cumulative 90-deg Coulomb scattering rate for particles of mass A (amu) (SPITZER, 1962). Though f_i is not Maxwellian we assume Eq. (4) holds within a factor of 2 or so, and use $T^{3/2} = (T_\parallel T_\perp^2)^{1/2}$. If instability is the dominant loss mechanism, then we expect ν much larger than this Coulomb estimate.

Assuming $T_\perp^{Loss} \sim T_\perp$, consider two examples, both with $R_L = 3$, $R_W = 2$, $A = 1$ amu, $L = 100$ cm, and $T_\perp \gg T_e$:

(1) for $n_0 = 10^{10}$ cm^{-3}, $T_e = 10$ eV, $T_\parallel = 10$ eV, $T_\perp = 100$ eV:

$$\lambda_D(W) \lesssim 1.5 \text{ cm}.$$

(2) For $n_0 = 10^{13}\ \text{cm}^{-3}$, $T_e = 100$ eV, $T_{\|} = 1$ keV, $T_{\perp} = 10$ keV:

$$\lambda_D(W) \lesssim 0.5\ \text{cm}.$$

In general the upper bound on $\lambda_D(W)$ scales as

$$L^{-1/2} n_0^{-1} T_{\|}^{1/4} T_{\perp}^{3/4} T_e^{1/2} (\ln \Lambda)^{-1/2}$$

when the loss is dominated by ion-ion collisions. Since ordinarily $\lambda_D(W)$ is the minimum λ_D for region II, we conclude that in many mirror-machine experiments,‡‡ region II always constitutes an external plasma all the way to the wall, even in the absence of external sources.

Since electron and ion fluxes must be equal, however, the density of streaming electrons at s = W is

$$n_e^{SF}(W) \sim \frac{F_i(W)}{v_{e\|}} \sim n_0 \left[\frac{T_e(R_L - R_W) - e(\phi_L - \phi_W)}{(m_e/m_i)T_{\|}}\right]^{-1/2} \left(\frac{R_W L \nu}{c_{\|}}\right),$$

where $T_{\|}$ and $c_{\|}$ still refer to the ions. One sees from this that unless $e(\phi_L - \phi_W)/T_e \approx R_L - R_W$ the streaming electron density n_e^{SF} is insufficient for charge neutrality at s = W, and similarly at s not too near L. But if $e(\phi_L - \phi_W)/T_e \approx R_L - R_W \sim 1$, many of the electrons at $s \geq L$ would be in class ST, not in SF as assumed.

3. DENSITY AT s IN TERMS OF MIDPLANE VELOCITY DISTRIBUTION

For each point s along a field line, one can draw a curve $\epsilon_{\|} = \epsilon_s(\epsilon_{\perp}) \equiv \epsilon_{\perp}(R_s - 1) - q(\phi_0 - \phi_s)$ in midplane energy-space, such that if a particle at the midplane has $\epsilon_{\|} > \operatorname{Sup}_{s' \leq s} \epsilon_{s'}(\epsilon_{\perp})$ (where Sup indicates the maximum value), then in the absence of collisions its turning point is beyond s. Of course for those particles with $U(s',\mu) = \mu B(s') + q\phi(s')$ monotonically increasing for $s' < s$, $\mathrm{Sup}_{s' \leq s}\, \epsilon_s$ is just ϵ_s. If we assume $\phi(s)$ monotonically decreasing for all $s > 0$ (as indicated by the results of Section 2 for $s > L$, and by previous studies (BENDANIEL, 1961; PERSSON, 1966) for $s < L$), then for the electrons $U(s,\mu)$ is always increasing in region I ($s < L$) but decreases in region II ($s > L$) for large μ and increases for small μ. This is reflected in the crossing of the lines $\epsilon_s(\epsilon_\perp)$ in Fig. 2. For the ions, $U(s,\mu)$ is monotonically decreasing for $s > L$ (so that all ions passing $s = L$ are lost) and we ignore the possibility of complications arising from $\mu dB/ds < ed(-\phi)ds$ for some s, μ, $B(s)$, $\phi(s)$ in region I.[§§]

Assuming that all particles contributing to

$$n(s) = \int 2\pi v_\perp dv_\perp \int dv_\parallel f(s,v_\parallel,v_\perp)$$

include $s = 0$ in their orbits, we can use

$$\frac{\partial(v_\parallel,v_\perp^2)_s}{\partial(v_\parallel,v_\perp^2)_0} = \frac{v_{\parallel 0}R_s}{\sqrt{v_{\parallel 0}^2 - x_s^2}}, \text{ where } x_s^2 = v_{\perp 0}^2(R_s - 1) - \frac{2q}{m}(\phi_0 - \phi_s),$$

to write the integration in terms of midplane velocities $x \equiv v_{\parallel 0}$ and $y \equiv v_{\perp 0}$:

$$n(s) = 2\pi R_s \int y dy \int \frac{x dx}{\sqrt{x^2 - x_s^2}} f(0,x,y), \tag{5}$$

where particles with turning points at $s' < s$ do not contribute and are excluded by restricting the region of integration to $|x| > \mathrm{Sup}_{s'<s}\, x_{s'}(y)$.

In a perfectly confined collisionless plasma, $f = 0$ at the loss boundary $x_L(y)$.*** If $0 < |\partial f/\partial x| < \infty$ there, then $n(s) \propto (L - s)^2$ for $L - s$ small and positive, as one can see from a Taylor expansion of f about $x_L(y)$.

With imperfect confinement, however, $f(s,v_\parallel,v_\perp)$ is not mapped by the Vlasov characteristics back onto $f(0,v_\parallel,v_\perp)$ but onto some function $g(0,v_\parallel,v_\perp|s)$ which differs noticeably from f near the escape boundary. The equations governing f have been discussed by KAUFMAN and KING (1968). In the collisionless approximation to the particle dynamics our definitions of $\epsilon_\parallel$ and $\epsilon_\perp$ are obvious and are equivalent to μ,H or μ,J:

$$\epsilon_\parallel = H - \mu B_0 - q\phi_0 \qquad (H = \text{Hamiltonian} = \text{total energy}),$$

$$\epsilon_\perp = \mu B_0,$$

or equivalently

$$\epsilon_\parallel = \frac{m}{2} v_\parallel^2 \qquad \text{at } s = 0,$$

$$\epsilon_\perp = \frac{m}{2} v_\perp^2 \qquad \text{at } s = 0.$$

In the presence of scattering, we define x and y (for a particle at s) to be the values of $v_\parallel$ and $v_\perp$ that the particle would have if it returned to $s = 0$ without further scattering (i.e., along Vlasov characteristics). The definitions in terms of μ,H are the same, but now μ,H are stochastic functions whose expectation value in general depends on s. But if scattering can be neglected in the low-density region II, the dependence on s there is negligible. For this case we abbreviate $g(0,x,y|s) = g(x,y)$.

4. ION DENSITY AT MIRROR

To determine n_i^{SF} one must either have information about the detailed loss mechanism, or else assume a model for the loss mechanism or the detailed shape of $g(0,x,y|s)$ near the escape boundary. Since the former leads to calculational difficulties, we attempt the latter in such a manner that a specified loss flux is reproduced. If all the escaping ions are placed exactly on the escape boundary (a simple assumption for g), then $n_i^{SF}(s)$ has an infinity at $s = L$ (which is rounded off, physically, by diffusion due to fluctuations). Yet if small-angle scattering predominates, with only slight scattering during a single bounce period, then (x,y) for an escaping ion cannot be too far from $x = x_L(y)$. Physically, then, a small loss rate can produce fairly large densities near $s = L$ as particles creep over the potential barrier, creating a "traffic jam" there.

Since the simplest model is inadequate, we take $g(0,x,y|L) = g_0(x,y) + g_1(x,y)$, where g_0 would be f_0 in the absence of scattering; g_1 includes a "tail" for $x > 0$ and $y \sim y_L(x)$ sufficient to give the observed loss flux. Only the width of the tail of g_1 should be important, not the exact shape.

The function $g_1(x,y)$ is chosen so that $g = g_0 + g_1$ has continuous slope and gives the same density as g_0. The loss flux, which is supplied entirely by g_1, is equated to the known loss flux at the mirror. For $g_0(x,y)$ we take a function Maxwellian at large velocities inside the trapping region, but going to zero at the loss boundary $y = y_L(x)$. Proceeding as in Appendix A we find, for small loss flux,

$$n_i(L)/n_0 \approx 2^{1/2}\pi^{3/8}R_L(P\sqrt{1+P})^{1/4}(L\nu/c_\parallel)^{3/4}, \qquad (6)$$

where $P \equiv c_\parallel^2/(R_L - 1)c_\perp^2$ and ν is as in Section 2.

Though it is difficult to calculate the transition shape, this value of n_i/n_0 goes over to that calculated in Section 2, assuming $v_\parallel = 0$ at $s = L$:

$$\frac{n_i(s)}{n_0} \approx \left[\frac{T_\perp^{\text{Loss}}(R_L - R_s) + e(\phi_L - \phi_s)}{T_\parallel}\right]^{-1/2} \frac{R_s L\nu}{c_\parallel} .$$

These expressions become equal (see Fig. 3) when

$$\frac{e(\phi_L - \phi_s)}{T_\parallel}\left[1 + \frac{T_\perp^{\text{Loss}}(R_L - R_s)}{e(\phi_L - \phi_s)}\right] = \left(\frac{R_s}{R_L}\right)^2 (P\sqrt{1+P})^{-1/2}\left(\frac{L\nu}{c_\parallel}\right)^{1/2} ,$$

which is small (e.g., of order 0.02 when $T_\perp = 10T_\parallel$ and $L\nu/c_\parallel = 0.4 \times 10^{-3}$) so that the break in Fig. 3 occurs at $e(\phi_L - \phi_s)/T_\parallel \lesssim 0.02$.

In the calculations of Appendix A and Sections 4 and 6 we treat loss processes which involve smooth diffusion in velocity space. For statistical purposes, charge exchange "moves" ions discontinuously in velocity space, replacing an ion in the trapping region by one near the origin. If charge exchange is important and if it occurs mostly in the region where $\phi \approx \phi_0$, then as the resulting cold ions are accelerated out of the plasma they produce an additional density

$$n_i^{cx}(s) \approx \frac{R_s L n_0 \nu_{cx}}{\left[(2/m_i)e(\phi_0 - \phi_s)\right]^{1/2}}$$

for s not too small. (Here ν_{cx} is an average charge-exchange collision frequency, and $R_s L n_0 \nu_{cx}$ is just the charge-exchange loss flux at s.) This additional density only strengthens the conclusions of Section 2, and in the remainder of Sections 4 and 6 it is left for the reader to add this density to the one given. Because the charge-exchanged cold ions do not accumulate near s = L as do the SF ions from smooth diffusion, $n_i^{cx}(L)/n_i^{SF}(L)$ is small until

$$\nu_{cx}/\nu_{diff} \gtrsim \left[n_{diff}(L)/n_0\right]^{-1/4}\left[e(\phi_0 - \phi_L)/T_{\parallel}\right]^{1/2},$$

which is typically about 10.

5. ELECTRON DENSITY IN TERMS OF ϕ

We take the electron distribution g(x,y) to be a "damped Maxwellian":

$$g(x,y) = G(x,y)H(x,y),$$

where G(x,y) is Maxwellian with temperatures $T_{e\parallel}$ and $T_{e\perp}$, and H(x,y) is a function which is nearly unity well inside the containment region in x,y but goes smoothly to zero well inside the loss region. We proceed as in Appendix B to integrate such a function (dimensionally) over the region $|x| > x_L(y)$ to get $n_e(s)$ as in Eq. (5). The result is found to be somewhat sensitive to the abruptness with which H(x,y) drops off, but some estimate of this is made at the end of Appendix B. The result is found to be

$$\frac{n_e(L)}{n_0} \sim 0.3\,\frac{R_L}{R_L - R_W}\,e^{-e\phi_0/T_e}\left[\kappa' c^2\left(\frac{e\phi_L}{T_e}\right)^{5/2} + \cdots\right] \qquad (7)$$

for $R_L - R_W \sim 1$, where for simplicity we have taken $T_{e\parallel} = T_{e\perp} = T_e =$

$(m_e/2)c^2$, and where κ' measures the abruptness of falloff of $H(x,y)$ near $x_L(y)$. One expects $1 \lesssim \kappa' c^2 < 4$ in order to reproduce a reasonable electron loss flux. For $s > L$ but not too near W, $n_e(s)/n_0$ is given by Eq. (7) with R_L replaced by R_s and ϕ_L replaced by ϕ_s. For $s \approx W$, we have

$$\frac{n_e(W)}{n_0} \sim 0.4 \frac{R_W}{R_L - R_W} e^{-e\phi_0/T_e}\left[\kappa' c^2 \left(\frac{e\phi_W}{T_e}\right)^{5/2} + \cdots\right] \tag{8}$$

for $R_L - R_W \sim 1$.

6. ESTIMATE OF EXTERNAL POTENTIAL

First at the mirror, from Eqs. (6) and (7) and the requirement of charge neutrality, we have

$$\frac{e\phi_L}{T_e} \sim 2.5 \left[e^{e\phi_0/T_e}(R_L - R_W)\beta/\kappa'\right]^{0.4}(P\sqrt{1 + P})^{0.1}\left(\frac{L\nu}{c_{i\|}}\right)^{0.3}, \tag{9}$$

where $\beta = c^{-2}$ (for electrons). Then at the wall, from Section 2 and Eq. (8), we have

$$\frac{e\phi_W}{T_e} \sim 2.5 \left[e^{e\phi_0/T_e}(R_L - R_W)^{1/2}(T_\|/T_\perp^{\text{Loss}})^{1/2}\beta/\kappa'\right]^{0.4}\left(\frac{L\nu}{c_{i\|}}\right)^{0.4}. \tag{10}$$

Finally we present the following numerical estimates: For $R_L = 3$, $R_W = 2$, $T_\|/T_\perp = 0.1$, $\kappa'/\beta = 2$, $T_\perp^{\text{Loss}} \approx T_\perp$, and $e\phi_0/T_e = 5$ (BENDANIEL, 1961), we have

$\frac{L\nu}{c_{i\parallel}}$	0.4×10^{-3}	0.4×10^{-4}
$\frac{e\phi_L}{T_e}$	1	0.5
$\frac{e\phi_W}{T_e}$	0.2	0.1

7. EFFECT OF EXTERNAL SOURCES ON CONFINEMENT

Finally we consider the small change in the steady state of an imperfectly confined plasma when "additional" plasma is slowly introduced into the equilibrated system by an external source (such as emission of secondaries from end walls of the chamber, or a weak electron beam running through the plasma along magnetic field lines).

The first question facing such an analysis is whether the streaming of the externally supplied electrons (which are supplied near one edge of a potential well) causes additional instability (loss-cone modes may already be present). An intense beam or even a cold source at the end walls would produce a bump in the tail of the electron distribution and would excite two-stream modes. The analysis of this unstable, inhomogeneous system would be difficult, and is not pursued here. Instead we consider only the case in which the total energy in waves due to such interactions is small compared with $n^{SF}e\phi_0$--i.e., the case of a very weak beam--and we take two equal counterstreaming components for symmetry. We also assume that the spectrum of any preexisting instability is not changed by the new electron stream, and that the ion distribution is not altered by the new waves.

The internal source that sustains the plasma is unaffected by the new electron input from the ends. (There is no input of ions; they are all repelled by the plasma potential.) We neglect radiation changes and diffusion across the magnetic field, and assume that some fraction β of the externally supplied electrons is trapped during their transit through the plasma. One can then integrate the equations for continuity of energy and particles over a flux tube, and subtract the same equations in the absence of the external (x) source. Solutions consistent with $d/dt \equiv 0$ are then, crudely

$$\Delta(F_i\epsilon_i^L + F_e\epsilon_e^L) = 0, \qquad (11)$$
$$\Delta F_e + \beta F_x = 0,$$

where Δ indicates the change due to addition of the "x" electrons (which we take to be born with zero energy at the wall); F_e and F_i are the electron and ion loss fluxes, measured at $s = W$; the flux of x electrons at $s = W$ is F_x, which has negative sign; $\epsilon_e^L - e\phi_0$ and $\epsilon_i^L + e\phi_0$ are the mean energies of SF particles as measured at the wall (exclusive of those supplied externally).

We next observe that $\Delta F_i = 0$, since the externally supplied electrons do not affect the internal ion source balancing F_i. For the electrons, $\epsilon_e^L \approx e\phi_0$, since most of the loss-component electrons have nearly the minimum escape energy. By changing the plasma potential, one changes the loss boundary for ions in velocity space. This will, in general, induce a change in ϵ_i^L (which could in principle be calculated from the Fokker-Planck equation). But we will assume for simplicity $\Delta\epsilon_i^L \ll e\Delta\phi_0$, as would be the case if energy-conserving scattering or charge exchange were dominant. Equations (11) then become, for small changes,

$$\frac{\Delta\phi_0}{\phi_0} \approx \frac{\beta F_x}{F_e} < 0. \tag{12}$$

Steady state is not possible if the trapping rate of external electrons exceeds the electron loss rate. Again for small changes,

$$\frac{\partial F_i}{\partial n}\Delta n + \frac{\partial F_i}{\partial \phi}\Delta\phi + \frac{\partial F_i}{\partial T_i}\Delta T_i = 0,$$

and since physically $e\Delta\phi < \Delta T_i \leq 0$, we have

$$0 < \left[\frac{\phi\partial F_i/\partial\phi + e\phi\partial F_i/\partial T_i}{n\partial F_i/\partial n}\right]\left(\frac{-\beta F_x}{F_e}\right) < \frac{\Delta n}{n} \leq \frac{\phi\partial F_i/\partial\phi}{n\partial F_i/\partial n}\left(\frac{-\beta F_x}{F_e}\right) \tag{13}$$

(since $\partial F_i/\partial T_i < 0$ but $\partial F_i/\partial\phi$ and $\partial F_i/\partial n > 0$), where n, ϕ, and T are understood to refer to the midplane.

Finally we make two temperature-change estimates:

$$\frac{\partial F_e}{\partial n}\Delta n + \frac{\partial F_e}{\partial\phi}\Delta\phi + \frac{\partial F_e}{\partial T_e}\Delta T_e = -\beta F_x$$

gives

$$\frac{\Delta T_e}{T_e} \sim \frac{\phi}{T_e}\left[\left(\frac{\partial F_e/\partial n}{\partial F_i/\partial n}\right)\left(\frac{\partial F_i/\partial\phi}{\partial F_e/\partial T_e}\right) - \frac{\partial F_e/\partial\phi + F_e/\phi}{\partial F_e/\partial T_e}\right]\frac{\beta F_x}{F_e}\ , \tag{14}$$

where we have taken $\Delta T_i = 0$ for simplicity. Also

$$\Delta\epsilon_{e\|} \approx (\Delta n/n)(e\phi - \epsilon_{e\|}),$$

where $\epsilon_{e\|}$ is the mean electron parallel energy at the midplane. This comes from the fact that the new electrons are supplied at $\epsilon_{\|} = e\phi$, while any change in density due to altered loss boundary also affects mainly electrons near $\epsilon_{\|} = e\phi$. Thus for nearly isotropic electrons we have

$$\frac{\Delta T_{e\|}}{T_{e\|}} \sim \frac{\Delta n}{n} (2\Gamma - 1) > 0, \tag{15}$$

where $\Gamma \equiv e\phi/T_e$ as before and $\epsilon_{e\|} = 1/2\ T_{e\|} \approx 1/2\ T_e$.

For one estimate of the magnitudes involved in Eqs. (13) to (15), we use the simplest Coulomb-scattering model for the loss fluxes, based on the work of BENDANIEL (1961),

$$F_e,\ F_i \propto n^2,$$

$$\frac{\partial F_e}{\partial e\phi} = -\left(\frac{3}{2\Gamma} + 1\right)\frac{F_e}{T_e},$$

$$\frac{\partial F_e}{\partial T_e} = \Gamma \frac{F_e}{T_e},$$

$$\frac{\partial F_i}{\partial e\phi} = \frac{F_i}{T_i} b(R_L,\Gamma),$$

where $b(R_L,\Gamma) \sim 1$ for $\Gamma \approx 4$ and $R_L \approx 3$, which we use for typical values. With these expressions, Eqs. (13) to (15) become

$$\frac{\Delta n}{n} \sim \left(\frac{\Gamma b}{2} \frac{T_e}{T_i}\right)\left(\frac{-\beta F_x}{F_e}\right) \qquad \text{for } \Delta T_i \approx 0, \tag{16}$$

and

$$\frac{\Delta T_e}{T_e} \sim \left(\frac{T_e}{T_i} b + \frac{1}{2\Gamma} + 1\right)\left(\frac{\beta F_x}{F_e}\right), \tag{17}$$

while

$$\frac{\Delta T_{e\|}}{T_{e\|}} \sim \frac{b\Gamma}{2} (2\Gamma - 1) \frac{T_e}{T_i} \left(\frac{-\beta F_x}{F_e}\right). \tag{18}$$

the external production of electrons thus reduces the plasma potential so that the new net electron loss will still balance the unchanged ion loss rate. Predictably, T_e is reduced but $T_{e\|}$ is increased. A slightly higher density results in the new steady state, which can be reached only for $|\beta F_x| < F_e$.

APPENDIX

A. Model Calculation of Ion Density

For $g_1(x,y)$ we assume

$$g_1(x,y) = \begin{cases} \dfrac{\partial g_0}{\partial y}(A_1 - B_1)e^{-k(y_L-y)} & \text{for } x > 0 \text{ and } y < y_L(x) \\ 0 & \text{for } x < 0 \text{ and } y < y_L(x) \\ \dfrac{\partial g_0}{\partial y}\left[A_1 e^{-a(y-y_L)} - B_1 e^{-b(y-y_L)}\right] & \text{for } y > y_L(x), \end{cases} \quad \text{(A1)}$$

where $\partial g_0/\partial y$ is to be evaluated at $y_L(x) + 0$, and where $a > b$ and $A_1 > B_1$ (see Fig. 4). On g_1 we impose three conditions:

$$\frac{\partial}{\partial y}(g_0 + g_1)\Big|_{y_{L+0}} = \frac{\partial}{\partial y}(g_0 + g_1)\Big|_{y_{L-0}} \quad \text{(continuous slope)}, \qquad \text{(a)}$$

$$\int_{-\infty}^{\infty} dx \int_0^{\infty} 2\pi y dy\, g_1(x,y) = 0 \quad \text{(density unchanged by } g_1\text{)}, \qquad \text{(b)}$$

$$\int_{-\infty}^{\infty} x dx \int_0^{y_L(x)} 2\pi y dy\, g_1(x,y) = F, \qquad \text{(c)}$$

where F denotes the fictitious flux at s = 0 which gives rise to the known flux $F_i(L) = R_L F$ at s = L. These conditions give

$$\frac{A_1 - B_1}{k} = \frac{F}{2\pi J} \quad \text{and} \quad k = -\frac{a+b}{2} + \frac{1}{2}\sqrt{(a^2 + b^2) + 4\pi J/F}\ ,$$

where

$$J \equiv \int_0^\infty \left.\frac{\partial g_0}{\partial y}\right|_{y_L(x)+0} y_L(x)x\,dx.$$

In the limit of small F, $k \to \sqrt{\pi J/F}$.

For $g_0(x,y)$ we take a function Maxwellian at large velocities in the trapping region, but going to zero at the loss boundary $y = y_L(x)$. We assume a form

$$g_0(x,y) = \begin{cases} \dfrac{n_0}{\pi^{3/2}} \dfrac{(1+P)^{3/2}}{c_\parallel c_\perp^2} e^{e\Delta\phi/T_{eff}} \left[e^{-x^2/c_\parallel^2} - e^{-x_L^2(y)/c_\parallel^2} \right] e^{-y^2/c_\perp^2}, & \text{for } x < x_L(y), \\ 0 & \text{for } x > x_L(y), \end{cases} \tag{A2}$$

where $P \equiv c_\parallel^2/(R_L - 1)c_\perp^2 = T_\parallel/T_{eff}$, $\Delta\phi \equiv \phi_0 - \phi_L$, and where the basic parameters $c_\parallel$, $c_\perp$ are related to "temperatures" by $c_\parallel^2 = 2T_\parallel/m_i$, $c_\perp^2 = 2T_\perp/m_i$, and $T_{eff} \equiv (R_L - 1)T_\perp$; and g_0 behaves like

$$\text{const} \times \exp\left[-\left(\frac{x^2}{c_\parallel^2} + \frac{y^2}{c_\perp^2} \right) \right] \text{ at large x or y for } x < x_L(y).$$

This g_0 is normalized to n_0:

$$\int_{-\infty}^{\infty} dx \int_0^\infty 2\pi y\,dy\; g_0(x,y) = n_0.$$

From this one finds

$$\left.\frac{\partial g_0}{\partial y}\right|_{y_L(x)+0} = \frac{2}{\pi^{3/2}} \frac{(1+P)^{3/2}}{P} \frac{n_0}{c_\perp^{\,4}} \frac{1}{c_\parallel} y_L(x) e^{-(1+P)x^2/c_\parallel^{\,2}}$$

and

$$J = \frac{1}{\pi^{3/2}} \frac{c_\parallel}{c_\perp^{\,2}} \frac{n_0}{(1+P)^{1/2}} \left[1 + \frac{e\Delta\phi}{T_\parallel}(1+P)\right].$$

Then

$$n_i(L) = R_L \int_0^\infty 2\pi y dy \int_{x_L(y)}^\infty \frac{xdx\ g_1(x,y)}{\sqrt{x^2 - x_L^{\,2}(y)}}$$

$$= R_L \int_0^\infty xdx \int_0^{y_L(x)} 2\pi y dy \frac{g_1(x,y)\cdot(R_L - 1)^{-1/2}}{\sqrt{y_L^{\,2}(x) - y^2}},$$

where $$y_L^{\,2} = \frac{x^2 + (2/m_i)e(\phi_0 - \phi_L)}{R_L - 1}.$$

This integral can be evaluated approximately for $ky_L(0) \gg 1$, i.e., $g(x,y)$ drops off rapidly in velocity space outside the loss boundary, falling to $e^{-1}g\left[x,y_L(x)\right]$ over a velocity small compared with $\left(\frac{2}{m_e} e\phi_0\right)^{1/2}$. We used

$$\int_0^{y_L} \frac{ydy\ e^{-k(y_L-y)}}{\sqrt{(y_L - y)(y_L + y)}} \approx \sqrt{2y_L} \int_0^{\sqrt{y_L}} e^{-kw^2} dw - \frac{3}{2}\frac{1}{\sqrt{2y_L}} \int_0^{\sqrt{y_L}} w^2 e^{-kw^2} dw,$$

replacing upper limits by ∞, and using the asymptotic form (DAVIS, 1964) of the incomplete gamma function to do the x integration,

$$\int_0^\infty (x^2 + \delta)^{3/4} e^{-hx^2} xdx = h^{-7/4} e^{h\delta} \Gamma\left[\frac{7}{4}, h\delta\right] \approx (h\delta)^{3/4}\left[1 + \frac{3}{4h\delta}\right],$$

where $h = (1 + P)/c_{\parallel}^2$ and $\delta = \frac{2}{m_i} e(\phi_0 - \phi_L)$. For small F, then, the result is

$$n_i(L)/n_0 = 2^{1/2}\pi^{3/8} R_L (P\sqrt{1 + P})^{1/4} (L\nu/c_{\parallel})^{3/4}, \tag{A3}$$

where we have used $F = n_0 L\nu$.

B. Model Calculation of Electron Density in Terms of ϕ

The region of integration for $n_e(L)$ is $x^2 > x_L^{\,2}(y^2)$, where $x_L^{\,2}(y^2) \equiv (R_L - 1)y^2 + (\varphi_0 - \varphi_L)$, with $\varphi \equiv \frac{2}{m} e\phi$. We take the electron distribution $g(x,y)$ to be a "damped Maxwellian":

$$g(x,y) = \frac{n_0}{\pi^{3/2} c_{\parallel} c_{\perp}^{\,2}} e^{-x^2/c_{\parallel}^{\,2}} e^{-y^2/c_{\perp}^{\,2}} H(x,y); \tag{B1}$$

$$H(x,y) = \begin{cases} 1 - (1 - \xi)e^{-\kappa'(x_1^{\,2} - x^2)} & \text{for } x^2 < x_1^{\,2}(y^2), \\ \xi e^{-\kappa(x^2 - x_1^{\,2})} & \text{for } x^2 > x_1^{\,2}(y^2) \text{ and } x > 0, \\ 0 & \text{for } x^2 > x_1^{\,2}(y^2) \text{ and } x < 0, \end{cases}$$

where

$$x_1^{\,2}(y^2) = \left\{ \begin{matrix} (R_L - 1)y^2 + \varphi_0 - \varphi_L \\ \\ (R_W - 1)y^2 + \varphi_0 \end{matrix} \right\} \text{for } y^2 \begin{matrix} > \\ \\ < \end{matrix} \frac{\varphi_L}{R_L - R_W} .$$

See Fig. 5. We take $\kappa, \kappa' \geq c_{\parallel}^{-2}$ and note that $\kappa' = (1 - \xi)^{-1}\xi\kappa$ for a smooth slope. (See Fig. 6.) The approximate normalization (neglecting the damping) has been used instead of the exact one, since the difference will be at most a few percent if $e\phi_0 \gtrsim 4T_e$ (as expected if

$R_L - 1 \gtrsim 1)$ and if $\kappa' \geq c_\|^{-2}$. Then

$$n_e(L) = \frac{n_0 R_L}{\pi^{3/2} c_\| c_\perp^2} \int_0^\infty 2\pi y dy\, e^{-y^2/c_\perp^2}$$

$$\times \left\{ \int_{x_L^2}^{x_1^2} \frac{d(x^2)}{\sqrt{x^2 - x_L^2}} e^{-x^2/c_\|^2} \left[1 - (1 - \xi) e^{-\kappa'(x_1^2 - x^2)}\right] \right.$$

$$\left. + \frac{1}{2} \int_{x_1^2}^\infty \frac{d(x^2)}{\sqrt{x^2 - x_L^2}} e^{-x^2/c_\|^2} \cdot \xi e^{-\kappa(x^2 - x_1^2)} \right\}.$$

Although one can evaluate this exactly by using integration by parts, the result is unnecessarily lengthy. Instead, since κ and ξ are only guessed, we give an approximate dimensional evaluation. We ignore the contributions from the "tail", i.e., $x > x_1$. The region of integration is then the pair of triangles in x^2, y^2 (one for $x > 0$ and one for $x < 0$):

$$x_L^2 < x^2 < x_1^2, \qquad y^2 < \frac{\varphi_L}{R_L - R_W} \qquad (y > 0),$$

as in Fig. 5. The area of these is $\varphi_L^2/(R_L - R_W)$; the centroid is at

$$x^2 = \varphi_0 + \frac{\varphi_L}{3}\left[\frac{R_L - 1}{R_L - R_W} - 2\right], \qquad y^2 = \frac{\varphi_L}{3(R_L - R_W)}.$$

The integrand evaluated at the centroid is

$$\frac{g(x_c,y_c)}{\sqrt{x_c^2 - x_L^2}} = \frac{n_0\beta^{3/2}}{\pi^{3/2}} e^{-\beta\varphi_0} e^{-\frac{1}{3}\beta\varphi_L(\alpha-2)}\left[1 - (1-\xi)e^{-\frac{1}{3}\kappa'\varphi_L}\right](\varphi_L/3)^{-1/2}$$

for isotropy, where $\beta \equiv 1/c_{\|}^2 = 1/c_{\perp}^2 = \frac{1}{2} m_e/T_e$, and $\alpha \equiv R_L/(R_L - R_W)$. Since

$$\int \frac{k^2 - x^2}{\sqrt{x^2 - x_L^2}} d(x^2) = 2(x^2 - x_L^2)^{1/2}(k^2 - x^2),$$

twice what one would get dimensionally, we include the extra factor of 2 in our result

$$n_e(L) \approx R_L \frac{\pi}{2} \frac{3^{1/2}\varphi_L^{3/2}}{R_L - R_W} \frac{2n_0\beta^{3/2}}{\pi^{3/2}} e^{-\beta\varphi_0} e^{-\frac{1}{3}\beta\varphi_L^{(\alpha-2)}}\left[1 - (1-\xi)e^{-\frac{1}{3}\kappa'\varphi_L}\right]$$

or

$$\frac{n_e(L)}{n_0} \approx 0.3 \frac{R_L}{R_L - R_W} e^{-\beta\varphi_0}\left[3\xi(\beta\varphi_L)^{3/2} + \frac{\kappa'}{\beta}(\beta\varphi_L)^{5/2} + \text{terms in } (\beta\varphi_L)^{7/2} \text{ etc.}\right], \qquad \text{(B2)}$$

where we have expanded $\exp\left[-\frac{1}{3}\beta\varphi_L(\alpha-2)\right]$ and $\exp\left[-\kappa'\varphi_L/3\right]$ assuming exponents $\lesssim 1$, and neglected ξ compared with κ'/β. Since the more exact calculation gives somewhat smaller coefficients for the $(\beta\varphi_L)^{7/2}$ and higher terms, we do not consider them here. We expect $\xi \ll 1$ and $\kappa' \gtrsim \beta$, so the 5/2-power term is the dominant one. Some insight into appropriate sizes for ξ and κ' is gained from the equation for electron flux at $s = W$ based on this model $g(x,y)$:

$$F_e(W) = \frac{n_0}{\pi^{1/2}} \beta^{-1/2} \xi \left(1 + \frac{\kappa}{\beta}\right)^{-1} e^{-\beta\varphi_0} \left[1 - \frac{1}{\alpha} e^{-(\alpha-1)\beta\varphi_L}\right]$$

for isotropic electrons, with α and β defined as before. This flux, depending only weakly on φ_L for $(\alpha - 1)\beta\varphi_L \gtrsim 2$, must equal $F_i(W)$. That requires

$$\frac{\xi}{1 + \kappa/\beta} = \pi^{1/2} R_W \left(\frac{m_e}{m_i} \frac{T_{i\|}}{T_e}\right)^{1/2} e^{\beta\varphi_0} \left(\frac{L\nu}{c_{i\|}}\right), \qquad \text{(B3)}$$

which is typically of order 10^{-2} or 10^{-3}. If we then choose $\kappa' = \frac{\kappa\xi}{1 - \xi}$ so that the slope of g is continuous, the left side of (B3) becomes $\beta\xi^2/\kappa'$, showing that if the "damping" of g is not too abrupt (say $\kappa'/\beta \lesssim 4$), then ξ is in fact small. The actual realistic value for κ' can be determined only from the detailed nature of the scattering process.

For $s > L$ but not too near W, Fig. 2 suggests replacing x_L^2 by $x_s^2 \equiv (R_s - 1)y^2 + (\varphi_0 - \varphi_s)$ for the left-hand boundary of the integration region.††† The function $n_e(s)/n_0$ is thus given roughly by Eq. (B2) with R_L replaced by R_s and φ_L replaced by φ_s.

The region of integration is poorly approximated near $s = W$ if the sheath potential ϕ_W is nonzero. Near the wall, then, it is better to bound the integration region by the parallel lines

$$x^2 = (R_W - 1)y^2 + (\varphi_0 - \varphi_W) \qquad \text{and} \qquad x^2 = (R_W - 1)y^2 + \varphi_0,$$

and by $y^2 = \varphi_L/(R_L - R_W)$ (again assuming $R_L - R_W$ not too small). The result for $s = W$ is then

$$\frac{n_e(W)}{n_0} \sim 0.4 \, \frac{R_W}{R_L - R_W} \, e^{-\beta\varphi_0} \left[\frac{\kappa'}{\beta} \, (\beta\varphi_W)^{5/2} + \cdots \right] \quad \text{for } R_L - R_W \gtrsim 1. \tag{B4}$$

FOOTNOTES

* This work was done under the auspices of the U. S. Atomic Energy Commission.

† Or even quasi-steady-state: true steady state in the presence of particle losses requires a source, such as neutral injection, to maintain the plasma. In the absence of such a source the problem can be treated as quasi-steady if the loss rate is sufficiently small.

‡ This requires equal electron and ion source rates. For a discussion of independent ion and electron sources one should refer to FOWLER and RANKIN (1962).

§ Under some circumstances where charge exchange is the dominant loss mechanism, one may not neglect the density of loss-component ions, since their escape time is long. See FOWLER and RANKIN (1962).

** We assume $e\phi_c$ is Eq. (46) of that paper should be $|e\phi_c|$, making some of Eq. (45) redundant.

†† In this paper the symbol ~ is used to denote "of the same order of magnitude as".

‡‡ The low-density, high-temperature ion- or neutral-injection experiments are exceptions, unless instability loss rates are quite high.

§§ If $d\phi/ds \neq 0$ at $s = M(\mu)$, the maximum of $U(s,\mu)$, then $M(\mu) \neq L$ although $M(\mu) - L$ is small for almost all trapped ions and goes to zero as $\mu \to \infty$. And if $ed(-\phi)/ds > \mu dB/ds$ for some $\mu > (B_L - B_s)^{-1} e\phi_L$ and $s < L$, then there can be ions trapped in region I away from the midplane.

*** Perfect collisionless confinement really refers to a finite time scale, and is the limiting state as the Fokker-Planck collision terms go uniformly to zero. Otherwise f could drop discontinuously at the loss boundary.

††† Although this is exact only if $\phi_{s'}(R_L - R_W) \leq \phi_L(R_{s'} - R_W)$ for all $s' \leq s$, which requires $d\phi/ds = 0$ at $s = L$.

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FIGURE LEGENDS

Fig. 1. Electron and ion loss boundaries in midplane velocity space when $\phi_0 \neq 0$.

Fig. 2. Lines $\epsilon_{\|} = \epsilon_s (\epsilon_{\perp})$ for $L < s < W$.

Fig. 3. Approximate density outside the main confinement region.

Fig. 4. Shape of g_1 for given x.

Fig. 5. Electron-loss boundaries in midplane energy space.

Fig. 6. g (with Maxwellian shown dashed) and H vs x for fixed y.

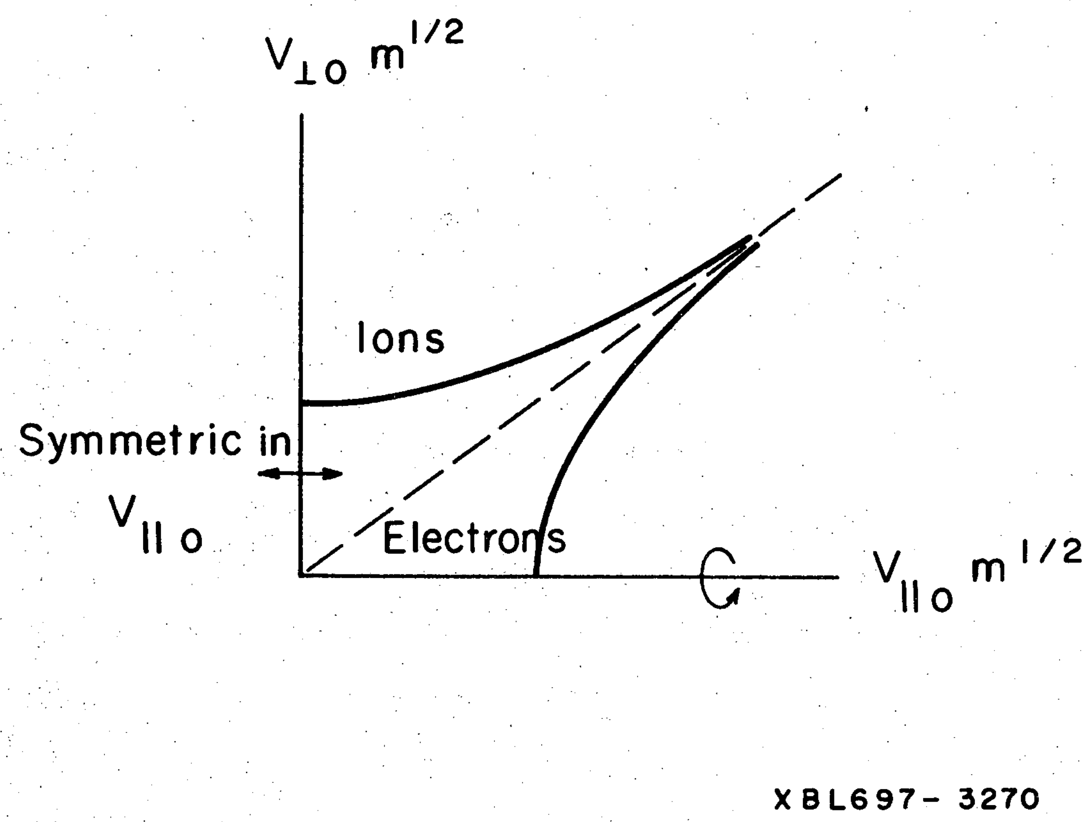
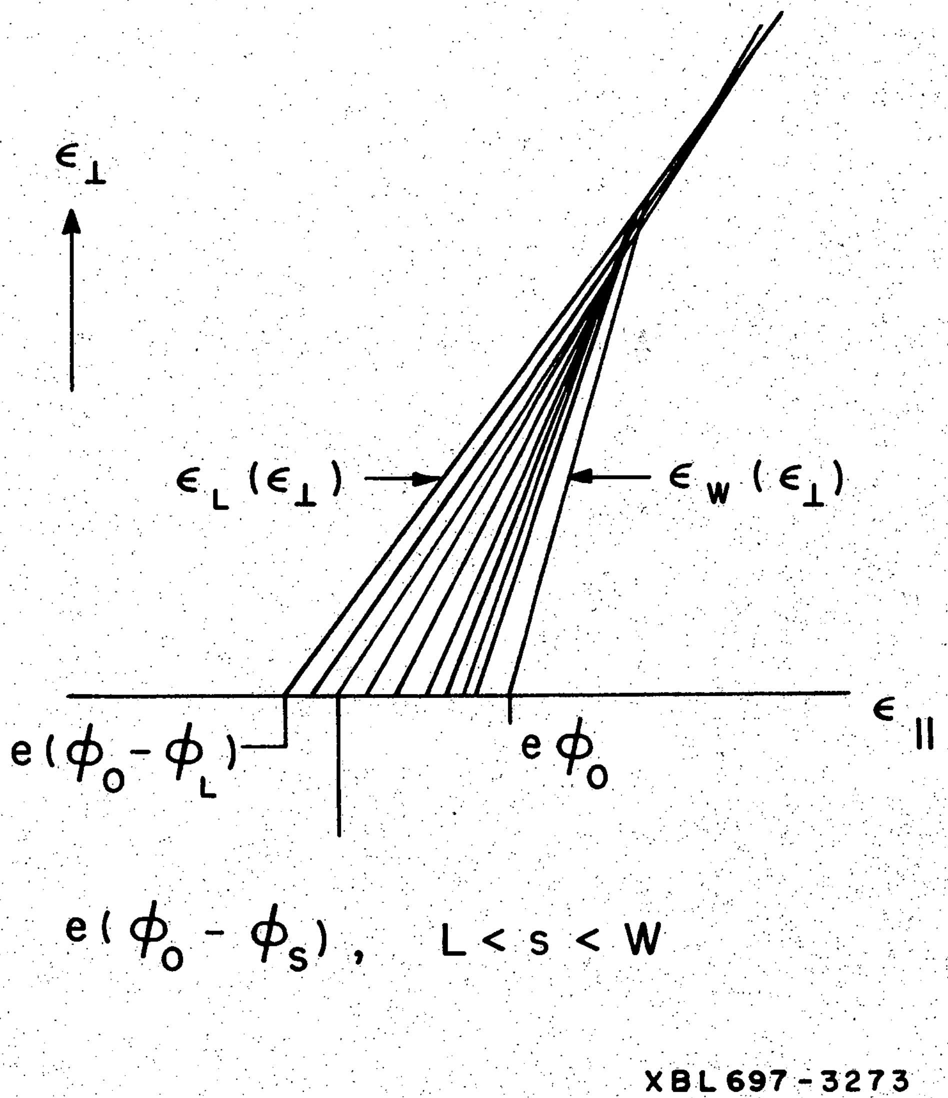
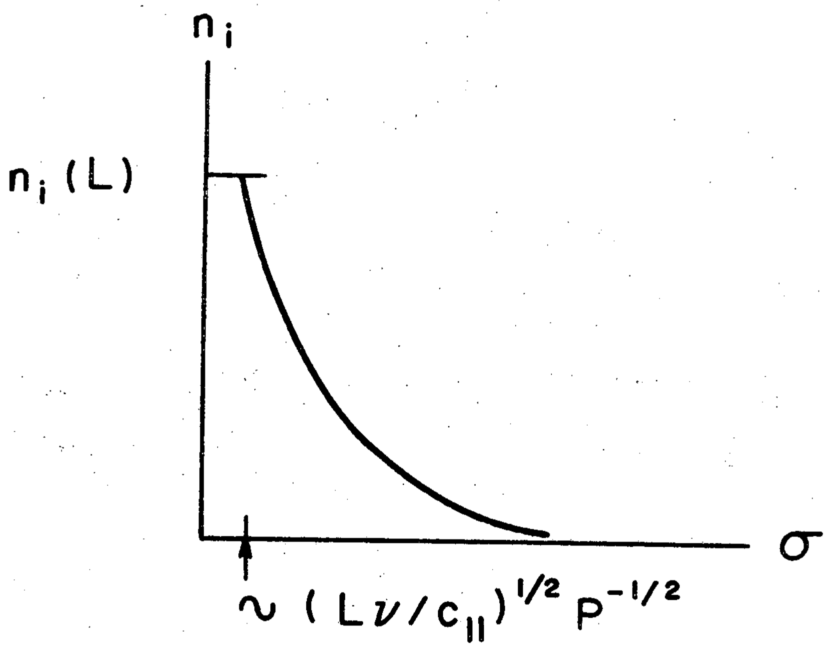


Fig. 1



$\epsilon_\perp$
$\epsilon_L(\epsilon_\perp)$
$\epsilon_W(\epsilon_\perp)$
$\epsilon_\parallel$
$e(\phi_0 - \phi_L)$
$e\phi_0$
$e(\phi_0 - \phi_s)$, $L < s < W$
XBL 697-3273

Fig. 2



$$\sigma = \frac{T_{\perp}^{Loss}\,(R_L - R_s) + e\,(\phi_L - \phi_s)}{T_{\parallel}}$$

XBL697-3271

Fig. 3

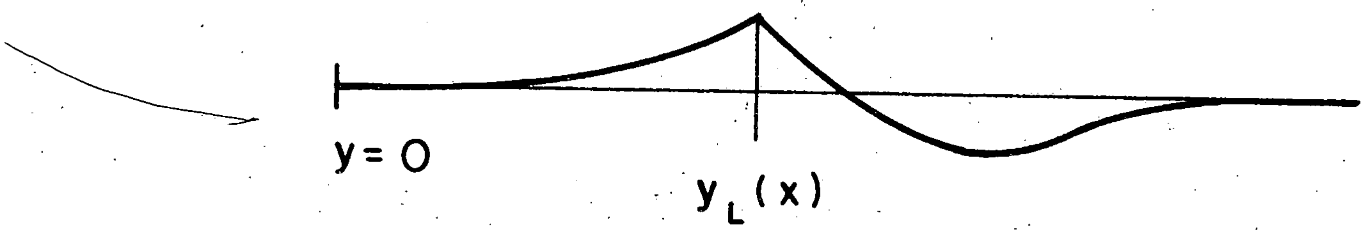


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Fig. 4

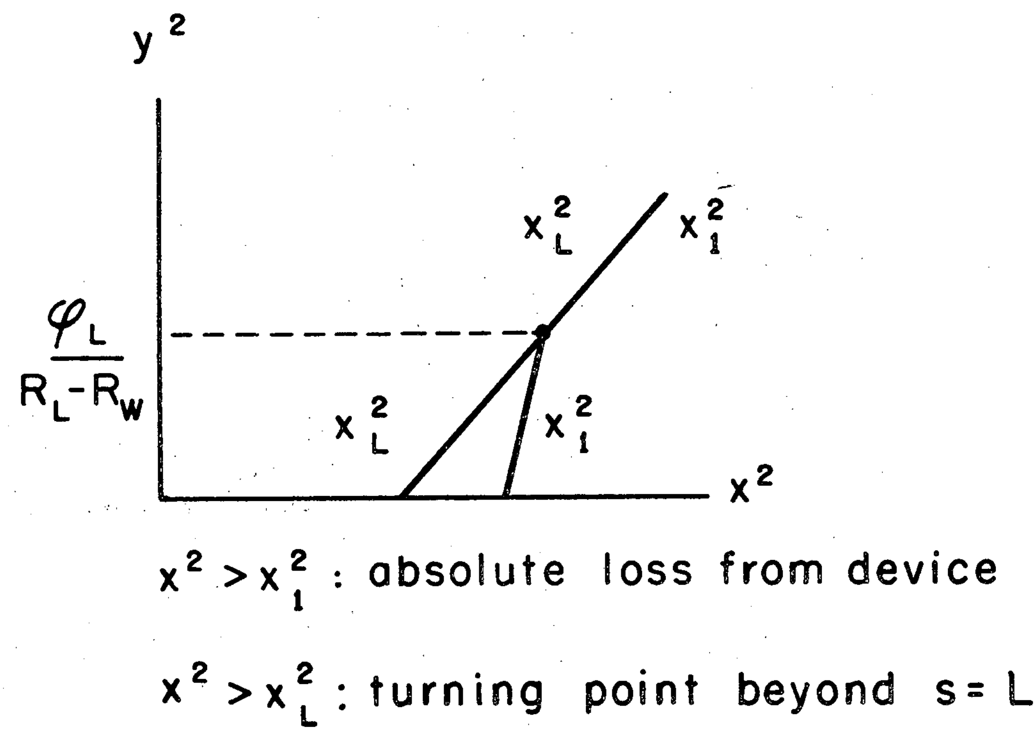

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Fig. 5

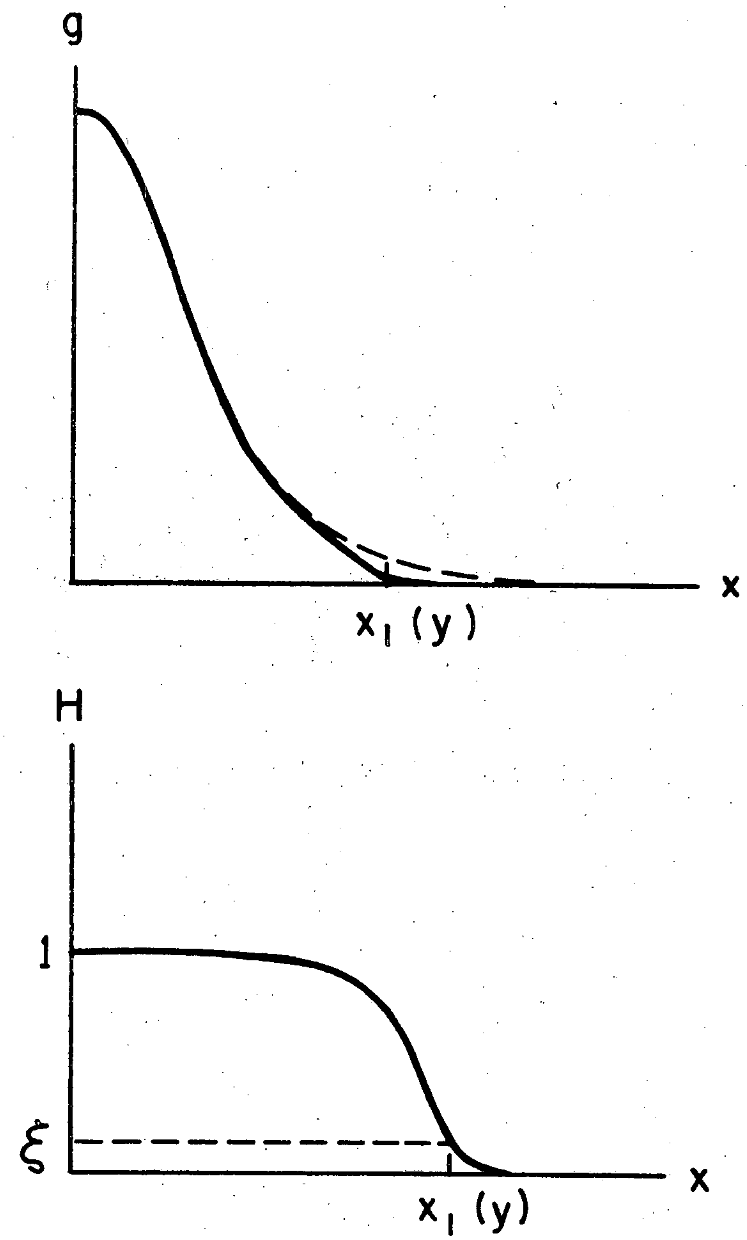

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Fig. 6

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