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Mechanism Design for Demand Response Programs

Deepan Muthirayan, Dileep Kalathil, Kameshwar Poolla, Pravin Varaiya

Abstract—Demand Response (DR) programs serve to reduce the consumption of electricity at times when the supply is scarce and expensive. Consumers or *agents* with flexible consumption profiles are recruited by an aggregator who manages the DR program. The aggregator calls on a subset of its pool of recruited agents to reduce their electricity use during DR events. Agents are paid for reducing their energy consumption from contractually established baselines. Baselines are counter-factual consumption estimates of the energy an agent would have consumed if they were not participating in the DR program. Baselines are used to determine payments to agents. This creates an incentive for agents to inflate their baselines in order to increase the payments they receive. There are several newsworthy cases of agents gaming their baseline for economic benefit. We propose a novel selfreported baseline mechanism (SRBM) where each agent reports its baseline and marginal utility. These reports are strategic and need not be truthful. Based on the reported information, the aggregator selects or calls on agents with a certain probability to meet the load reduction target D. Called agents are paid for observed reductions from their self-reported baselines. Agents who are not called face penalties for consumption shortfalls below their baselines. Under SRBM, we show that truthful reporting of baseline consumption and marginal utility is a dominant strategy. Thus, SRBM eliminates the incentive for agents to inflate baselines. SRBM is assured to meet the load reduction target. Finally, we show that SRBM is almost optimal in the metric of average cost of DR provision faced by the aggregator.

I. INTRODUCTION

The core problem in power systems operations is to maintain the fine balance of electricity supply and demand at all times. This must be done economically through markets while respecting resource and reliability constraints. Adeptly managing flexible demand is a far better alternative to increased reserve generation, since it is inexpensive, produces no emissions, and consumes no resources. While most DR programs are limited to infrequent peak shaving applications, it is recognized that demand flexibility has the potential to offer more lucrative ancillary services such as frequency regulation or loadfollowing. These applications can support balancing supply and demand to compensate for the variability of renewables. This paper, however, is concerned only with peak shaving DR applications. In DR programs, aggregators recruit residential or industrial customers who are willing to reduce their electricity consumption in exchange for financial rewards. The aggregator serves as an intermediary and represents these flexible consumers or *agents* to the local utility. The aggregator receives a payment from the utility for the ability to reduce demand at short notice, and, in-turn, pays the agents for their consumption reduction during DR events. The key difficulty is in measuring this reduction in consumption. While the actual consumption of agents is measured, their intended consumption or *baseline* is a counter-factual.

1

Commonly used baselines include historical averages of consumption on similar days (by the agent, or by a peer group of similar agents). However, there are newsworthy cases where agents have deliberately inflated their baseline to extract larger payments [1]. Inaccurate baselines can result in over-payment, compromising the cost-effectiveness of the DR program, or in under-payment, adversely affecting the ability to recruit participants into DR programs. Finally, fairness can be of concern. An agent who happens to be on vacation during a DR event receives a payment for load reduction without suffering any hardship. This can be perceived as unfair by other agents who deliberately curtail their consumption and suffer some dis-utility. Addressing these issues is essential to encourage and sustain wider use of DR programs.

A. Our Contributions

We approach DR program planning as a *mechanism design* problem. We propose a novel *self-reported* baseline mechanism (SRBM) where agents *self-report* baselines which are forecasts of their intended future consumption, and their marginal utilities to an aggregator who manages the DR program. Agents need not be truthful in their reporting. The self-reported baseline is used to determine payments in the DR program. The objective of the aggregator is to design an incentive mechanism so that (a) the DR program delivers any load reduction target, (b) each agent reports their true baseline and true marginal utility, and (c) the DR program delivers the load reduction target at minimum cost to the aggregator.

Under SRBM, we show that truthful reporting of baseline consumption and marginal utility is a dominant strategy for each agent. We show that agents are faithful to their baseline consumption when not called for DR, and that agents maximally reduce their consumption when called for DR. Under this mechanism, the aggregator can ensure adequate response, i.e. an assurance of being able to deliver the load reduction

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target D. We characterize the minimum possible average cost ϕ_{\min} of DR provision per KWh under a class of mechanisms. We then show that SRBM is nearly optimal in the metric of expected cost of DR provision, i.e. it results in a cost close to ϕ_{\min} .

There is extensive literature on network resource allocation problems and all of them consider an infinitely divisible good or service that is to be efficiently shared among distributed agents acting with self interests [2], [3]. Our setting also considers an infinitely divisible service request and selfish service providers (DR providers) but the difference being that the service provided is not measurable because of the lack of baseline against which DR service provided has to be measured. This is a challenge for mechanism design. Moreover, the literature in network resource allocation problems [2], [3] consider multishot setting and study convergence properties to a Nash Equilibrium over repeated reporting, allocation and pricing. The few works which consider a single shot setting require the consumers to report a very high dimensional bid for convergence in a single shot. Here we study convergence in a single shot with an additional challenge of lack of baseline measurement. Therefore our contribution is proposing a resource allocation dominant strategy mechanism in a single shot setting with this new challenge.

B. Related Work

Traditional DR programs reward participating agents for load reduction during peak demand periods. Agents have an incentive to deliberately inflate their baselines to increase the payments they receive [4]–[7]. Alternative baseline mechanisms (ex: aggregate baselines) which improve market efficiency are offered in [5]. These do not explicitly address baseline inflation concerns. Adverse selection and double payment effects are two other issues that arise from rewarding agents based on estimated baselines [4].

There is a substantial literature on baseline estimation methods. These can be broadly classified into (a) averaging, (b) regression, and (c) control group methods.

Averaging methods determine baselines by averaging the consumption on past days that are similar (ex: in temperature or workday) to the event day. There are many variants such as weighted averaging and using an adjustment factor to account for variations between the event day and prior similar days. A detailed comparison of different averaging methods in offered in [8]–[10]. While averaging methods are attractive because of their simplicity, they suffer from estimation biases that can be substantial [10], [11]. Also, these methods require significant data access, especially for residential DR programs [12].

Regression methods fit a load prediction model to historical data, which is then used to predict the baseline [13], [14]. They can potentially overcome biases incurred by averaging methods [12]. They often require considerable historical data for acceptable accuracy, and the models may be too simple to capture the complex behavior of individual agents.

TABLE I NOTATIONS

$\mathbb{E}[X]$	expected value of the random variable X
D	load reduction target
m	number of DR events agents must participate in
N	number of agents recruited by aggregator
u_k	utility of agent k
q_k	discretionary energy consumption of agent k
b_k	true baseline consumption of agent k
π_k	true marginal utility of agent k
π_{\max}	upper bound on marginal utilities, $\pi_{\max} \ge \max_k \pi_k$
α_k	probability that agent k is selected
f_k	baseline report of agent k
μ_k	marginal utility report of agent k
π_k^r	reward/kWh awarded to agent k
π_k^p	penalty/kWh imposed on agent k
π^{e}	retail price of energy
π^o	recruitment cost per enrolled agent
\mathbb{P}^{i}	pod <i>i</i>
\mathbb{S}^{i}	pod core <i>i</i>
\mathbb{H}^{i}	pod header i
β^i	probability that pod i is selected for DR
$ u^i$	maximum reported marginal utility in pod i
ϕ	average cost of DR provision per KWh
ψ	payout to agents per DR event

Control group methods are found to be more accurate than averaging or regression methods [16]. While they do not require large amount of historical data, they require additional metering infrastructure. This complicates and raises the costs of implementation, particularly for large numbers of recruited agents. Finally, [11] proposes a probabilistic method using Gaussian statistics to estimate baselines. We refer the readers to [17] for a discussion on these different baseline schemes.

Most of the methods mentioned above focus on baseline estimation. They often overlook the behavioral or gaming aspects of agents intentionally inflating their baselines. There are some exceptions, notably [18] which considers a linear penalty when agents deviate from their baselines and shows that the penalty induces users to report their true baselines. In [19], again under linear penalties, a centralized DR scheduling algorithm is proposed to guarantee incentive compatibility in the case of two agents. A DR market assuming known baselines is proposed in [20], [21]. The approaches in [18]-[21] either assume knowledge of utility functions or true baselines. In [22] authors propose an optimal contract mechanism for the DR aggregator. However they assume that true reduction can be observed at a later time, and the payment depends on this information. Our approach doesn't require this assumption. Instead we propose a joint design of baseline estimation and incentive design to address both problems together.

II. PROBLEM FORMULATION

A. Consumer Model

Let $u_k(q_k)$ be the utility of agent k derived by consuming q_k units of energy. We assume that the utility functions $u_k(\cdot)$ have the piece-wise linear form

$$u_k(q_k) = \begin{cases} \pi_k q_k & \text{if } q_k < b_k \\ \pi_k b_k & \text{if } q_k \ge b_k \end{cases}$$
(1)

Here b_k is the maximum possible consumption of agent k. Any additional consumption will not increase its utility. We call π_k the *true marginal utility* of agent k. We assume that π_k s are i.i.d and so are b_k s and that π_k and b_k s are independent of each other.

Let π^e be the retail price of electricity offered by the utility. The net utility $U_k(\cdot)$ of agent k is

$$U_k(q_k) = \begin{cases} \pi_k q_k - \pi^e q_k & \text{if } q_k < b_k \\ \pi_k b_k - \pi^e q_k & \text{if } q_k \ge b_k \end{cases}$$

We clearly require $\pi_k > \pi^e$, else agent k would not consume electricity. Equivalently, for every agent, the marginal utility derived from electricity consumption exceeds the retail electricity price.

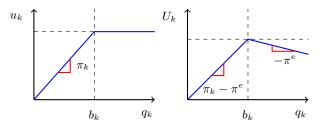


Fig. 1. (a) Utility, and (b) net utility of agent k

The optimal consumption for agent k maximizes the net utility. This is b_k as is evident from Figure 1(b). We call b_k the *true* baseline consumption of agent k.

Remark 1. We are only modeling the *discretionary* electricity consumption of an agent. This is the consumption that the agent is willing to forgo in exchange for monetary compensation. With this interpretation, b_k is the *maximum consumption reduction* that agent k is willing to provide, and π_k is the *minimum compensation* per KWh it requires to provide this reduction.

Remark 2. The agent utility functions are private information. The aggregator does not have knowledge of agent baselines and marginal utilities. We assume that the aggregator has knowledge of an upper bound on the agent marginal utilities, i.e. it knows π_{max} where

$$\pi_{\max} \ge \max_{k} \pi_k. \tag{2}$$

We note that π_{max} has the interpretation of the maximum price that the aggregator is *willing to pay* agents per KWh of demand reduction.

Remark 3. We assume that the agent parameters π_k and b_k are independent across the agents. This is reasonable because the true marginal utility and the true baseline of an agent are not dependent on the behavior of other agents.

Remark 4. The agent's utility function is chosen to be a piecewise linear concave function. This simplifies the problem setting and allows a more tractable analysis while retaining all the challenges that the general problem poses. If the agent's utility function is modeled by a general concave function, as in network resource allocation problems, then for convergence in single shot this would necessarily require the mechanism designer to elicit very high dimensional bids. This, in addition to the lack of baseline measurement, makes the general problem complex for analysis. We suggest this as a future direction for research.

B. Aggregator Model

The utility is financially motivated to reduce its procurement costs at times when supply is scarce and expensive. It purchases demand response services from aggregators in capacity markets. Suppose the aggregator commits to offer the utility D KWh of demand reduction during DR events over some contract window. It recruits N agents into its DR program from a large candidate pool. We will see later that N is a function of the consumer *selection mechanism*. The cost of recruitment is π^o per enrolled agent. *The recruited agents are obligated to participate in m DR events contractually*.

The aggregator profit is the revenue from the utility, minus the payout to the agents and recruiting costs. It may also receive penalty revenue from agents, but we will show that this is not the case under our baseline mechanisms. The total expected cost faced by the aggregator is

$$J_{\text{agg}} = m\mathbb{E}[\psi] + \pi^o\mathbb{E}[N]$$

where ψ is the payout per DR event, N is the number of recruited agents and expectation is over consumer baseline and marginal utility. The aggregator's *expected cost of demand response* ϕ , i.e. the average cost per KWh of demand reduction is then

$$\phi = \frac{J_{\text{agg}}}{D} = \underbrace{\frac{\mathbb{E}[\psi]}{D}}_{\text{payout per KWh}} + \underbrace{\frac{\pi^o \mathbb{E}[N]}{mD}}_{\text{recruitment cost}}$$
(3)

C. Event Time-line

The time-line of events in our problem formulation is shown in Figure 2. We divide these events into four periods.

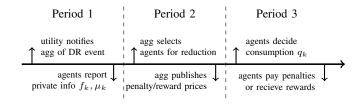


Fig. 2. Event Time-line

Period 0 (Common information): The aggregator recruits N agents into its DR program. Agents enroll based on the opportunity to receive financial rewards. The aggregator informs

participating agents of (a) agent selection mechanism, and (b) the mechanism which sets penalty and reward prices.

Period 1 (Reporting): The utility notifies the aggregator of an anticipated DR event during which it is obligated to deliver D KWh of demand reduction. All agents (indexed by k) report their baselines and marginal utilities, f_k and μ_k respectively, to the aggregator. Agents need not be truthful. We stress that the agent reports (f_k, μ_k) are strategic, i.e. agents may opt to deliberately submit incorrect reports.

Period 2 (Selection): The aggregator delivers the aggregate load reduction target D by selecting a subset of agents to call on for consumption reduction. This selection is based on the collective reports submitted by the agents. The aggregator notifies selected agents to reduce their consumption. The aggregator computes and publishes reward prices π_k^r for agents who are called, and a penalty price π_k^p for agents who are not called. These can be agent-specific.

Period 3 (Load reduction and payment): During the DR event, all agents decide on their actual consumption q_k . If agent k is called, it receives an *ex post* reward

$$R = \pi_k^r (f_k - q_k)^+.$$
 (4)

If agent k is not called, it is assessed an *ex post* penalty

$$P = \pi_k^p (f_k - q_k)^+.$$
 (5)

Thus, selected agents are rewarded for consumption reduction from their reported baselines, and agents that are not selected are penalized for consumption shortfalls below their reported baselines.

D. The Agent's Problem

We assume that the agents are rational and non-cooperative. Each agent faces a two stage decision problem. In the first stage, it has to decide the value of its reports (f_k, μ_k) . In the second stage, it has to decide on its actual energy consumption q_k during the DR event. This second stage decision depends on whether or not agent k is called for the DR event.

There are two serious complexities that arise. First, agents are selected at *random* by the aggregator in response to a DR event. The selection probability depends on the details of the aggregator's mechanism, which, in turn, depends on the submitted reports of *all* agents. Second, the reward and penalty prices faced by agent k are set by the aggregator. These vary by agent and depend on the collective reports of the other agents $i \neq k$.

We could approach the agent's decisions through a complicated game-theoretic formulation, but this is unnecessary. It happens that for our self reported baseline mechanism (detailed in Section IV), *truthful reporting is the dominant strategy*. Equivalently, agent k will choose to reveal its true baseline and true marginal cost in the first stage. This results in the lowest expected cost for agent k, regardless of the reports of other agents (see Theorem 4).

III. BASELINE-ONLY REPORTING

Before we describe our self-reported baseline mechanism, we consider a simpler scheme where agents (a) only report their baselines, and (b) face uniform reward and penalty prices independent of their reports, set by the aggregator. This intermediate analysis will inspire our more complex mechanism where agents report both their baselines and marginal utilities, and face nonuniform prices.

A. Mechanism Definition

The aggregator recruits N agents who are contractually obligated to participate in m DR events. In response to a DR event, all recruited agents are required to only report their baselines f_k . The decisions of agents are not coupled as the penalty and reward prices π^r, π^p are constants chosen by the aggregator. The aggregator selects agents independently with probability α until

$$\sum_{k} f_k > D. \tag{6}$$

Meeting (6) requires that a sufficient number N of agents be recruited. We will explore this aspect of the mechanism in Theorem 3.

The aggregator's choices under this mechanism are $(N, \pi^r, \pi^p, \alpha)$. The decision variables for agent k are its reported baseline f_k and second stage consumption q_k during the DR event.

B. Agent Decisions

The objective of each agent is to minimize its expected cost. Agent k solves a two-stage optimization problem. In the first stage, it optimally selects its baseline report f_k . In the second stage, it is informed whether or not it is selected, and then optimally selects its consumption q_k during the DR event.

Suppose agent k submits a baseline report f_k . If this agent is selected, its second stage cost function is

$$J_{s}(q_{k}, f_{k}) = \pi^{e} q_{k} - u(q_{k}) - \pi^{r} (f_{k} - q_{k})^{+}$$

If the agent is not selected, it second stage cost function is

$$J_{\rm ns}(q_k, f_k) = \pi^e q_k - u(q_k) + \pi^p (f_k - q_k)^+.$$

Define the optimal consumptions

$$q_{\mathrm{s}}^* = \arg\min_{q_k} J_{\mathrm{s}}(q_k, f_k), \quad q_{\mathrm{ns}}^* = \arg\min_{q_k} J_{\mathrm{ns}}(q_k, f_k).$$

Note that these depend on the first stage decision f_k , i.e. the reported baselines.

Let α be the selection probability. The first stage decision problem of agent k is to minimize its expected cost:

$$J(f_k) = \alpha J_{\mathrm{s}}(q_{\mathrm{s}}^*, f_k) + (1 - \alpha) J_{\mathrm{ns}}(q_{\mathrm{ns}}^*, f_k).$$

Let f_k^* be the optimal first stage decision of agent k, i.e. its reported baseline:

$$f_k^* = \arg\min J(f_k)$$

We have the following result.

Theorem 1. Suppose the mechanism prices satisfy $\pi^r \geq \pi_{max} - \pi^e$, $\pi^p \geq \pi^e$, and the probability of an agent being called satisfies $\alpha \leq \pi^e/(\pi^r + \pi^e)$. Under the baseline-only reporting mechanism,

- (a) agents truthfully report their baselines, i.e. $f_k^* = b_k$
- (b) called agents consume $q_s^* = 0$
- (c) agents that are not called consume $q_{ns}^* = b$
- (d) the aggregator receives no penalty revenue
- (e) the load reduction target D is met.

Proof: Refer Appendix

Remark 5. Result (a) assures us that there is no baseline inflation. Result (b) states that called agents maximally reduce their discretionary consumption. Result (c) implies that agents who are not called consume the same amount of electricity as they would have if they were not participating in the DR program.

C. Aggregator Cost

We now examine the aggregator's perspective. Consider the class C of *self-reporting baseline mechanisms* with *linear* reward and penalty functions as specified in (4) and (5). The reward prices can be agent-specific. Our next result offers a lower bound on the *minimum possible cost* per KWh of DR provision under *any mechanism in C*.

Theorem 2. Suppose agent true baselines $\{b_1, b_2, \dots\}$ and agent true marginal utilities $\{\pi_1, \pi_2, \dots\}$ are *i.i.d.* random variables. Assume b_is and π_ks are independent for all i, k. Let C denote the class of self-reporting baseline mechanisms with linear reward and penalty functions. Then, the expected cost for DR provision under any mechanism in C satisfies

$$\phi \ge \frac{1}{\mathbb{E}[1/\pi]} - \pi^e + \frac{\pi^o}{m\pi^e \mathbb{E}[b]\mathbb{E}[1/\pi]} = \phi_{min} \tag{7}$$

where $\mathbb{E}[b] = \mathbb{E}[b_i]$ and $\mathbb{E}[1/\pi] = \mathbb{E}[1/\pi_k]$.

Proof: Refer Appendix

Remark 6. Note that the marginal utilities are bounded and bounded away from zero as $\pi^e \leq \pi_k \leq \pi_{\text{max}}$. A standard calculation reveals that

$$1/\mathbb{E}[\pi] \le \mathbb{E}[1/\pi] \le 1/\mathbb{E}[\pi] + \mathcal{O}(\sigma^2/m^3)$$

where (m, σ^2) are the mean and variance of π . Thus, if the variance of true marginal utilities is modest, we can approximate $\mathbb{E}[1/\pi] \approx 1/\mathbb{E}[\pi]$, and the minimum expected cost of DR provision over any mechanism in C is

$$\phi_{\min} \approx \mathbb{E}[\pi] - \pi^e + \frac{\pi^o \mathbb{E}[\pi]}{m \pi^e \mathbb{E}[b]}$$
(8)

We next compute the minimal cost of DR under the baselineonly reporting mechanism. Minimizing this cost requires the aggregator to select the smallest possible reward price π^r , and the fewest number of customers N (or the largest selection probability α). We have the following: **Theorem 3.** Suppose agent true baselines $\{b_1, b_2, \dots\}$ and agent true marginal utilities $\{\pi_1, \pi_2, \dots\}$ are *i.i.d.* random variables. Assume b_is and π_ks are independent for all *i*, *k*. Under baseline-only reporting, the aggregator's profit is maximized by the optimal parameters:

- (a) reward price: $\pi^r = \pi_{max} \pi^e$
- (b) penalty price: $\pi^p \geq \pi^e$
- (c) selection probability: $\alpha = \pi^e / \pi_{max}$

The resulting expected cost per KWh of DR provision is

$$\phi_{BO} \le (\pi_{max} - \pi^e) \left(1 + \frac{\pi^e \mathbb{E}[b]}{\pi_{max} D} \right) + \frac{\pi^o \pi_{max}}{m \pi^e \mathbb{E}[b]} + \frac{\pi^o}{m D} \tag{9}$$

Proof: Refer Appendix

Remark 7. The penalty price does not affect the aggregator's cost as it derives no penalty revenue. The only constraint is that $\pi^p \ge \pi^e$ in order for agents to report truthfully.

Remark 8. For large demand reduction targets D, our upper bound (9) on the average cost of DR provision becomes

$$\phi_{\rm BO} \le \pi_{\rm max} - \pi^e + \frac{\pi^o \pi_{\rm max}}{m \pi^e \mathbb{E}[b]} \tag{10}$$

Comparing this with (8), we see that the baseline-only reporting mechanism incurs a large cost of DR provision because the aggregator does not have information about the true marginal utilities of agents. It only has access to the upper bound π_{max} . Cost increase stems from the inflation of π_{max} over $\mathbb{E}[\pi]$.

Example 1. Assume that the marginal utilities of recruited agents are uniformly distributed on [0.3, 1.3] \$/KWh. Equivalently, the mix of recruited agents demand this distribution of payments for their DR services. We use typical numbers from the PG&E jurisdiction for residential DR:

π^o	\$2/agent	recruitment cost
π_{\max}	\$1.30	max DR payment demanded by agents
$\mathbb{E}[b]$	5KWh	average reduction/DR event
m	10	max number of DR events
π^e	\$0.15	retail price of electricity
D	100KWh	DR target

This yields an average cost of DR provision of \$1.51/KWh under baseline-only reporting. This compares unfavorably with the lower bound $\phi_{\min} =$ \$0.71/KWh of Theorem 2.

The cost of DR provision per KWh under baseline-only reporting can be quite large. It is set by the *maximum* marginal utility of consumers in the recruited pool. Doing any better requires agents to reveal their true marginal utilities, allowing the aggregator to set lower reward prices while assuring that agents yield their discretionary consumption. This observation motivates us to consider a more complex mechanism which offers the promise of lower cost DR provision.

IV. SELF-REPORTED BASELINE MECHANISM (SRBM)

A. SRBM Mechanism Definition

The cost of DR provision under baseline only reporting inspires us to consider a more complex mechanism which we call Self-Reported Baseline Mechanism (SRBM). Under SRBM, all agents submit reports f_k of their baselines and μ_k of their marginal utilities. Agents may not be truthful. The key idea is to design the mechanism so that agents *reveal their true baselines and marginal utilities*. This allows the aggregator to set lower reward prices, without compromising on the delivery of the DR target. Under SRBM, the reward prices for selected agents is determined by the submitted reports of *other agents*. In this aspect, SRBM resembles classic mechanisms such as Vickrey-Clarke-Groves (VCG) auction pricing.

The SRBM mechanism definition is as follows:

Step 1 (Pod Sorting):

Based on the submitted reports, the aggregator sorts agents into M pods, $\mathbb{P}^1, \dots, \mathbb{P}^M$. A pod is a minimal subset of recruited agents that can deliver the demand reduction target D under SRBM. We later describe a specific pod sorting algorithm that delivers the DR target cost effectively in Subsection IV-C.

Step 2 (Pod Selection):

The aggregator selects one pod from the set of M pods to deliver the demand reduction target D. Pod \mathbb{P}^i will be selected with probability β^i . The selection probability β^i and a specific pod selection mechanism is described in Subsection IV-C.

Step 3 (Agent Selection):

Agents in pod \mathbb{P}^i are sorted in increasing order of their reported marginal utility μ_k . Pods are broken into their core and their header, as sown in Figure 3. The first k^* agents form the pod *core* \mathbb{S}^i where

$$\sum_{k=1}^{k^*} f_k \ge D, \quad \sum_{k=1}^{k^*-1} f_k < D.$$
(11)

Equivalently, k^* is the smallest index such that the first k^* agents from pod \mathbb{P}^i in the sorted list of marginal utilities can deliver the target D. If pod \mathbb{P}^i is selected in response to a DR event, agents in its core \mathbb{S}^i are *called on* to provide their demand reduction. The remaining agents form the pod header \mathbb{H}^i . So, the core and header of the pod are defined as,

$$\mathbb{S}^i = \{1, \dots, k^*\}, \quad \mathbb{H}^i = \mathbb{P}^i \setminus \mathbb{S}^i$$

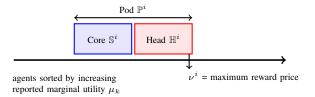


Fig. 3. Pod structure: core and header.

Step 4 (Reward Pricing):

Selected agents (i.e. in \mathbb{S}^i) receive a reward price π_k^r \$/KWh for consumption reduction $(f_k - q_k)^+$ below their self-reported baseline. Selected agents do not face penalties.

Let \mathbb{S}_{-k} be the set of agents who would have been selected from pod \mathbb{P}^i if agent k was not participating in the DR program. Define the reward price for agent k to be

$$\pi_k^r = \max\{\mu_j\} - \pi^e \,, \ j \in \mathbb{S}_{-k}.$$
(12)

We stress that the reward price π_k^r depends on the target D, and is agent-specific.

Step 5 (Penalty Pricing):

Agents who are not selected (i.e. those in the header \mathbb{H}^i) face a penalty price π^p for consumption deficits $(f_k - q_k)^+$ below their reported baselines. Under SRBM, the penalty price π^p for agent k is chosen to satisfy $\pi^p \ge \pi^e$. It is best to select the smallest penalty price, i.e. $\pi^p = \pi^e$, so as not to discourage agents from participating.

This completes the SRBM mechanism definition.

B. Agent Decisions

As with the baseline-only mechanism, agent k solves a twostage optimization problem. In the first stage, it optimally selects its baseline report f_k . In the second stage, it is informed whether or not it is selected, and then optimally selects its consumption q_k during the DR event. The complexity is that reward price π_k^r for agent k depends on its submitted reports (f_k, μ_k) and the reports of all other agents (f_{-k}, μ_{-k}) . These other reports are private information, unavailable to agent k.

Suppose agent k submits a baseline report f_k . If this agent is selected, its second stage cost function is

$$J_{s}(q_{k}, f_{k}) = \pi^{e} q_{k} - u(q_{k}) - \pi^{r}_{k}(f_{k} - q_{k})^{+}$$

If the agent is not selected, it second stage cost function is

$$J_{\rm ns}(q_k, f_k) = \pi^e q_k - u(q_k) + \pi^p (f_k - q_k)^+.$$

Selected agents are rewarded for consumption reduction from their reported baselines, and agents that are not selected are penalized for consumption shortfalls below their reported baselines.

Define the optimal consumptions

$$q_{\mathrm{s}}^* = \arg\min_{q_k} J_{\mathrm{s}}(q_k, f_k), \quad q_{\mathrm{ns}}^* = \arg\min_{q_k} J_{\mathrm{ns}}(q_k, f_k).$$

Note that these depend on the first stage decision f_k , i.e. the reported baselines.

Let β_k be the probability that agent k is selected. Its first stage decision problem is to minimize its expected cost:

$$J_k(f_k, \mu_k \mid f_{-k}, \mu_{-k}) = \beta_k J_s(q_s^*, f_k) + (1 - \beta_k) J_{ns}(q_{ns}^*, f_k)$$

We use the following notion.

Definition 1. [Dominant strategy]

Let $J_k(f_k, \mu_k \mid f_{-k}, \mu_{-k})$ be the expected cost for agent kwhen it reports (f_k, μ_k) and other agents report (f_{-k}, μ_{-k}) . The pair (f_k^*, μ_k^*) is a dominant strategy report for agent k if

$$J_k(f_k^*, \mu_k^* \mid f_{-k}, \mu_{-k}) \le J_k(f_k, \mu_k \mid f_{-k}, \mu_{-k})$$

for all reports (f_k, μ_k) and (f_{-k}, μ_{-k}) .

The agents have knowledge of only what is informed to them

as outlined in section II-C. In particular each agent is informed the following, a) that the agent is recruited with a set of agents such that their net reported capacity meets the load reduction requirement, b) that the agents in this group are ordered in increasing order of their marginal utility reports, c) that when this group is called for DR service all or some of the agents in set S which consitute the first k^* agents (as given by (11)) are selected to provide DR service, d) that the reward and penalty are determined by (12) and $\pi_k^p = \pi^e$, and d) that the probability of calling an agent k in this set S is given by, $\beta_k = \pi^e / (\pi_k^r + \pi^e)$.

Hence under SRBM, agents are not privy to the pod sorting algorithm. We call this as the *partial information* (PI) setting. Our next result establishes that truthful reporting of baselines and marginal utilities is a dominant strategy for SRBM under this partial information setting. Mechanism design for truthful reporting under complete information revelation where a consumer is also informed of the pod sorting algorithm is much more complex, and is discussed in the appendix.

We now offer our main result which establishes key properties of SRBM.

Theorem 4. Under the partial information setting (PI), SRBM has the following properties:

- (a) truthful reporting of baselines and marginal utilities is a dominant strategy
- (b) called agents consume $q_s^* = 0$, providing the maximal reduction in their discretionary consumption
- (c) agents that are not called consume $q_{ns}^* = b_k$
- (d) the aggregator receives no penalty revenue
- (e) the load reduction target D is met by each pod core.

Proof: See Appendix E.

Remark 9. Under SRBM, each consumer reports its true baseline b_k . As a result, measuring load reduction from the reported baseline coincides with the true load reduction, and called agents are paid accurately for the services they offer. This makes SRBM fair Agents who are not called consume b_k , which is exactly what they would have if they were not participating in the DR program. SRBM has a fairness property. An agent who happens to be on vacation will report their intended consumption, and must incur dis-utility to receive a DR payment. The selection process (11) meets the load reduction target D. This is because the selected agents completely yield their discretionary electricity consumption during the DR event, curtailing their consumption precisely by b_k . This is an artifact of our piece-wise linear utility function model. A more nuanced analysis with general concave utilities will yield a nonzero actual consumption. This analysis is more complex and only serves to mask the simplicity of our SRBM.

Remark 10. We have assumed that agent utility functions (and resulting true baseline consumption b_k) are deterministic. However, b_k depends on (exogenous) random parameters such as temperature and occupancy. For example, A more realistic model would accommodate dependence on exogenous random processes such as temperature and occupancy. This might result, for example, in a baseline consumption of the form

 $b_k = \bar{b}_k + a_k | \theta - \theta_0 |$. Here, θ is the realized temperature during the DR event, and θ_0 is the predicted temperature. In this case, agents can be required to report their best-effort forecast \bar{b}_k of their baseline consumption along with the temperature sensitivity a_k . Historical consumption data can be used to assist agents in making these reports. The SRBM mechanism can be easily extended to incorporate these more complex reporting scenarios. The most general scenarios with uncertain utility functions that explicitly depend on exogenous random processes θ is challenging and is an ongoing work.

C. A Pod Sorting Algorithm

We now offer a specific algorithm that sorts recruited agents into pods. From the aggregator's perspective, this sorting is attractive as it results in an expected cost of DR provision that is nearly optimal (see Theorem 6). The selection probability of a pod \mathbb{P}^i is set as,

$$\beta^{i} = \pi^{e} / \nu^{i} \le \beta^{i}_{k} = \pi^{e} / (\pi^{r}_{k} + \pi^{e}) , \ k \in \mathbb{S}^{i}$$
 (13)

Remark 11. Pod selection probability β^i may not be consistent with the individual selection probabilities of an agent k in \mathbb{S}^i , which is β_k^i . When D is large enough and b is small, one can expect $\beta_k^i \sim \beta^i$ and SRBM to be consistent with the individual seleciton probabiliy β_k^i . In order for SRBM to be consistent, an agent $k \in \mathbb{P}^i$ is made available for selection, even after \mathbb{P}^i is selected for up to β^i fraction of time, till the agent k's frequency adds up to β_k^i . It is in this sense the consumers are informed that all or some consumers belonging to the pod core would be selected when the pod is selected for DR service (Refer Section IV-B).

For a selected pod \mathbb{P}^i to deliver the DR target, the pod core must contain k^* agents, where $\sum_{1}^{k^*} b_k \ge D$. In order to compute the reward prices π_k^r , the pod header must also contain sufficiently many agents to determine the reward prices π_k^r . More precisely, we require that if any agent in the core \mathbb{S} is removed, we can still find sufficiently many agents to determine \mathbb{S}_{-k} . A sufficient condition for this is that the header \mathbb{H}^i contain k^{**} agents, where $\sum_{k^*+1}^{k^*+k^{**}} b_k \ge D$. Thus the selection criterion for pods cores and headers is identical. *Since agents in the header of a pod are not called, they can serve as the core of another pod.* This key idea is illustrated in Figure 4. It allows us to reduce the total number of agents that must be recruited which is one component of the cost of DR provision.

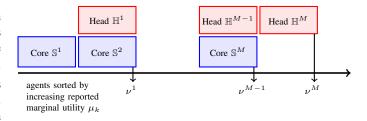


Fig. 4. Illustration of pod sorting: cores, headers, and max reward prices.

The second key idea in our pod selection algorithm is to organize agents into pods so that pods with large rewards are selected with low probability. This reduces the expected payout to agents. We begin by sorting agents in increasing order of their reported marginal utility as shown in Figure 4. The maximum reward price paid to agents in pod \mathbb{P}^i is bounded by

$$\pi_k^r \leq \nu^i - \pi^e$$
, where $\nu^i = \max_{k \in \mathbb{P}^i} \pi_k$.

Also, pod \mathbb{P}^i is selected with probability $\beta^i = \pi^e / \nu^i$. As a result, pods with larger reward prices are selected with lower probability, reducing the expected cost of DR provision. Agents with high marginal utility are called on less frequently, reducing the expected dis-utility. This simple sorting algorithm is detailed below in *pod sorting algorithm*.

1) Pod Selection: A random number $u \sim \mathcal{U}[0,1]$ is drawn. Pod core \mathbb{S}^i is selected for DR if $u \in [\sum_{j=1}^{i-1} \beta^j, \sum_{j=1}^{i} \beta^j]$. To be consistent with the selection probability of an individual consumer a consumer k in \mathbb{S}^i is selected for all draws $u \in [\sum_{j=1}^{i-1} \beta^j, \sum_{j=1}^{i-1} \beta^j + \beta_k^i]$. Consumers who are selected are rewarded for reduction from reported baseline based on unit reward announced to them. Consumers who are not selected are penalized for consuming less than the reported baseline. Because the number of pods that are recruited is such that $\sum_{i}^{M-1} \beta^{i} < 1 \leq \sum_{i}^{M} \beta^{i}$, this pod selection procedure guarantees that the required target D is delivered always.

Pod Sorting Algorithm

- 1 Sort agents in the increasing order of reported marginal utilities
- 2 Set pod index i = 1. Set n = 1
- 3 Place $k^*(i)$ agents indexed from n to $n + k^*(i) 1$ in pod core \mathbb{S}^i where $k^*(i)$ is the smallest number such that $n+k^{*}(i)-1$

$$\sum_{j=n} \quad f_j \ge D. \text{ Increment } n \leftarrow n + k^*(i)$$

4 Place $k^*(i+1)$ agents indexed from n to $n+k^*(i+1)-1$ in pod header \mathbb{H}^i where $k^*(i+1)$ is the smallest number such that $\sum_{j=n}^{n+k^*(i+1)-1} f_j \ge D.$

Increment $n \leftarrow n + k^*(i+1)$

- 5 Define the pod $\mathbb{P}^i = \mathbb{S}^i \cup \mathbb{H}^i$. Define the pod selection probability $\beta^i = \min \beta_k^i, \ k \in \mathbb{S}^i$. Increment $i \leftarrow i+1$
- 6 Define $\mathbb{S}^i = \mathbb{H}^{i-1}$.
- 7 If $\sum_{i} \beta^{i} < 1$, go to step 4. Else stop.

Next, we give a characterization of the number of pods M and number of recruited agents N under this pod sorting algorithm.

Theorem 5. Let N be the number of recruited agents, ψ be the payout to agents per DR event, and M be the number of pods. Under the pod sorting algorithm,

$$\mathbb{E}[N] \le (\mathbb{E}[M] + 1) (D/\mathbb{E}[b] + 1),$$

$$\mathbb{E}[M] \le \mathbb{E}[\pi] \ \pi^e + 3.$$

Proof. Refer Appendix

D. Aggregator Cost

We are now in a position to compute the expected cost of DR provision under SRBM with our pod sorting algorithm. This given in the following theorem.

Theorem 6. Suppose agent true baselines $\{b_1, b_2, \dots\}$ and agent true marginal utilities $\{\pi_1, \pi_2, \dots\}$ are *i.i.d.* random variables. Assume $b_i s$ and $\pi_k s$ are independent for all i, k. Then, the expected cost ϕ per KWh of DR provision under SRBM with the pod sorting algorithm satisfies

$$\phi_{SRBM} \le \mathbb{E}[\pi] + 2\pi^e + \left(\frac{\pi^o}{m\mathbb{E}[b]} + \frac{\pi^o}{mD}\right) \left(\frac{\mathbb{E}[\pi]}{\pi^e} + 3\right)$$
(14)

Proof: Refer Appendix

Remark 12. For large demand reduction targets D, our upper bound (14) on the average cost of DR provision becomes

$$\phi_{\text{SRBM}} \le \left(\mathbb{E}[\pi] + 3\pi^e\right) - \pi^e + \frac{\pi^o(\mathbb{E}[\pi] + 3\pi^e)}{m\pi^e \mathbb{E}[b]} \tag{15}$$

If the spread of true marginal utilities is modest, we have argued (see (8)) that the minimum expected cost of DR provision over any mechanism in the class C is

$$\phi_{\min} \approx \mathbb{E}[\pi] - \pi^e + \frac{\pi^o \mathbb{E}[\pi]}{m \pi^e \mathbb{E}[b]}$$
(16)

If the retail electricity price π^e is small compared to expected marginal utility $\mathbb{E}[\pi]$ of the recruited agents, we have

 $\phi_{\text{SRBM}} \approx \phi_{\text{min}}$

 \square

Comparing expressions (15) and (16), we conclude that SRBM is nearly optimal in the metric of expected cost of DR provision.

V. CASE STUDY AND SIMULATIONS

In this section, we illustrate the performance advantages of the SRBM mechanism compared to other standard mechanisms through numerical examples and simulations.

A. SRBM vs Baseline-Only Reporting Mechanism

Assume that the marginal utilities of recruited agents are uniformly distributed on [0.3, 1.3] \$/KWh. Equivalently, the mix of recruited agents demand this distribution of payments for their DR services. We use typical numbers from the PG&E jurisdiction for residential DR as given in the table below.

π^{o}	\$2/agent	recruitment cost				
π_{\max}	\$1.30	max DR payment demanded by agents				
$\mathbb{E}[b]$	5KWh	average reduction/DR event				
m	10	max number of DR events				
π^e	\$0.15	retail price of electricity				
D	100KWh	DR target				

This yields an average cost of DR provision of \$1.51/KWh under baseline-only reporting. This compares unfavorably with the lower bound $\phi_{\min} =$ \$0.68/KWh of Theorem 2.

However, the average cost of DR provision under SRBM in this case is is \$0.84/KWh. This is only modestly larger than the *lowest possible* average cost for DR provision of \$0.71/KWh found from Theorem 2. SRBM compares very favorably with baseline-only reporting which has an average cost of \$1.51/KWh.

B. Numerical Simulations

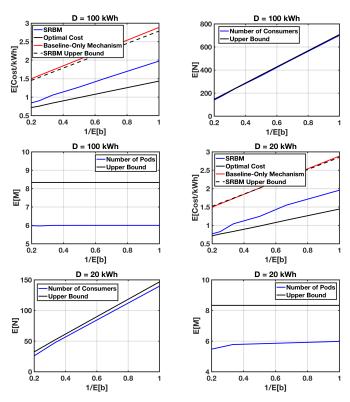


Fig. 5. Left above: \mathbb{E} [Cost/kWh] vs 1/ \mathbb{E} [b] (D = 100 kWh), right above: \mathbb{E} [N] vs 1/ \mathbb{E} [b] (D = 100 kWh), left middle: \mathbb{E} [M] vs 1/ \mathbb{E} [b](D = 100 kWh), right middle: \mathbb{E} [Cost/kWh] vs 1/ \mathbb{E} [b] (D = 20 kWh), left below: \mathbb{E} [N] vs 1/ \mathbb{E} [b] (D = 20 kWh). right below: \mathbb{E} [M] vs 1/ \mathbb{E} [b](D = 20 kWh). Upper bounds are from Theorems 5 and 6

In Fig. 5, we provide simulation results that illustrate the superior performance of SRBM mechanism. The first three plots starting from top left provides the average cost, average number of consumers recruited and average number of pods for D = 100 kWh (which is representative of a large demand reduction requirement). The next three plots are for D = 20 kWh which is representative of smaller demand reduction requirement. Each plots also contains the theoretical upper/lower bound on the quantities of interests. We use the same values for the prices as in the previous subsection.

It follows from the plots that the cost/kWh of SRBM is significantly lower compared to upper bound and the baselineonly mechanism for all the scenarios. Moreover, the cost for SRBM is close to the optimal cost (given by the lower bound). The figures also show that the actual number of consumers required for enabling the DR is very close to that predicted by Theorem (5) which further suggests the accurate modeling and analysis of the SRBM mechanism. Tables III and II summarizes the result from the simulations. In order to succinctly characterize the performance optimality, we use the notion of *competitive ratio* (CR) which is the ratio of cost of the mechanism to the optimal cost possible. The CR is close to 1, though it increases with $1/\mathbb{E}[b]$, which is expected from the upper bound in Eq. (14).

TABLE II SRBM Performance: D = 20 kWh

E[b]	5	4	3	2	1
SRBM	0.77	0.83	1.04	1.24	1.96
ϕ_{min}	0.71	0.76	0.84	0.99	1.44
CR	1.08	1.09	1.24	1.26	1.36

TABLE III SRBM Performance: D = 100 kWh

E[b]	5	4	3	2	1
SRBM	0.84	0.9	1.05	1.27	1.97
ϕ_{min}	0.72	0.76	0.84	0.99	1.43
CR	1.18	1.19	1.26	1.29	1.38

C. Self-Reported vs CAISO Baseline

California Independent System Operator (CAISO) estimates baseline by the 10/10 method. In the 10/10 method, a raw baseline \overline{b}_c is estimated by averaging the consumption of 10 non-event similar days closer to the event day. In addition an adjustment factor is calculated to account for any variation on the event day. This adjustment factor is calculated based on the consumption q_p couple of hours prior to the event and the average consumption b_p at the same time on the selected non-event days. This factor is capped at 20% both upwards and downwards. With this baseline estimation approach, the payment R_c to a consumer is given by $R_c = \pi_k^r \left(\frac{q_p}{\bar{b}_p} \bar{b}_c - q \right)$. Lets consider the same example as before, where $\pi \sim \mathcal{U}[0.3, 1.3]$ and $\pi^e = 0.15$. Lets also consider the case where the uninflated value of $q_p = b_c = b_p$ and that b_c and b_p are the true values. Here we ignore any incentives from the consumer side to alter \bar{b}_c and \bar{b}_p . From Eq. (12) it is clear that $\pi_k^r \ge \pi - \pi^e \ge \pi^e$ for each consumer. One the day of the event, if an individual consumer inflates its consumption q_p then it gains at the rate of π_k^r for each unit of increase and its loss is at the rate of retail price π^e (follows from the piecewise linear utility model Eq. (1)). Here the gain and the loss is evaluated in terms of the net utility for the hours before the DR event and during the DR event. Since $\pi_k^r \ge \pi^e$ increasing q_p results in a net positive gain for the consumer on the event day. Then each consumer will increase q_p by the maximum allowed that is 20% resulting in 20% inflation of baseline and payments, while self-reporting nullifies any such incentive to inflate (Theorem 4).

VI. CONCLUSION

In this paper, we have addressed the baseline estimation problem that is central to demand response programs. We proposed a mechanism where agents participating in a DR program selfreport their baselines and marginal utilities. Under this self reported baseline mechanism (SRBM), agents reveal their true baselines and marginal utilities. Agents that are selected for demand reduction maximally curtail their load. When agents are not selected, their actual consumption is faithful to their baselines. As a result, the aggregator is able to meet any feasible load reduction target with certainty by selecting agents whose measured reductions adds up to the total reduction. We have also derived a lower bound ϕ_{min} on the expected cost per KWh of DR provision for any mechanisms in the class C with piece-wise linear penalty and reward functions. We argue that SRBM comes close to deliver DR provision at this minimum cost.

Our work has assumed that the true baseline consumption is deterministic and that the utility function is piece-wise linear. In practice, DR mechanisms must account for uncertainty in agents intended consumption during the DR delivery window. The exploration of the DR mechanisms for general utility functions $u(q; \theta)$ where θ is an exogenous random variable remains to be explored.

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APPENDIX

A. Mathematical Preliminaries

We require two preliminary results based on the classical optional stopping theorem and the closely related Wald's equation.

Proposition 1. (Wald's equation) Let $X = \{X_k : k = 1, 2, \dots\}$ be a sequence of independent identically distributed random variables. Let $\chi = \mathbb{E}[X_k]$. Let N be a stopping time with respect to X, i.e. an integer valued random variable that depends only on $\{X_1, \dots, X_N\}$. Suppose $\mathbb{E}[N] < \infty$. Then

$$\mathbb{E}[X_1 + \dots + X_N] = \chi \mathbb{E}[N]$$
(17)

Proof: Theorem (1.6), Chapter 3 in [23]

Proposition 2. Let $X = \{X_k : k = 1, 2, \dots\}$ be a sequence of independent identically distributed random variables. Let $\chi = \mathbb{E}[X_k]$ and define the partial sums

$$S_0 = 0, S_1 = X_1, S_2 = X_1 + X_2, \cdots$$

Fix D and consider the random variable

$$N = \min\{t : S_t \ge D\}.$$

Then,

$$\frac{D}{\chi} \le \mathbb{E}[N] < \frac{D}{\chi} + 1$$

Proof: We prove the result in three steps.

a) First we show that $\mathbb{E}[N]$ is bounded.

Construct the sequence of i.i.d random variables defined by

$$Y_k = \begin{cases} 0 & \text{If } X_k < a \\ a & \text{If } X_k \ge a \end{cases}$$
(18)

Call a trial k a success if $Y_k = a$. Let T denote the first success in a sequence of trials. Denote the probability of success on a given trial by p. Then $p = \mathbb{P}(Y_k = a)$ and T is a geometric random variable with probability distribution

$$\mathbb{P}(T=k) = (1-p)^{k-1}p.$$
(19)

From (19) it follows that

$$\mathbb{E}[T] = 1/p = 1/\mathbb{P}(X_k \ge a).$$

Define, $R_t = \sum_{k=1}^t Y_k$ and $M = \min\{t : R_t > D\}$. Then

$$M \leq$$
 Number of trials needed for $\lfloor D/a \rfloor + 1$ successes (20)

Denote T_{k-1}^k as the number of trials to reach k th success after the $(k-1)^{\text{th}}$ success. Then from (20)

$$\mathbb{E}[M] \le \mathbb{E}\sum_{k=1}^{\lceil D/a \rceil + 1} T_{k-1}^k = \sum_{k=1}^{\lceil D/a \rceil + 1} \mathbb{E}[T_{k-1}^k]$$
(21)

Observe that T_{k-1}^k and T are identically distributed. As a result, $\mathbb{E}[T_{k-1}^k] = \mathbb{E}[T]$ and

$$\mathbb{E}[M] \le \sum_{k=1}^{\lceil D/a \rceil + 1} \mathbb{E}[T] = \left(\lceil D/a \rceil + 1 \right) / \mathbb{P}(X_k \ge a) \quad (22)$$

From the construction of Y_n it trivially follows that $Y_n \leq X_n$ which implies that $R_t \leq S_t$. So clearly, $N \leq M$. So,

$$\mathbb{E}[N] \le \mathbb{E}[M] \le \left(\left\lceil D/a \right\rceil + 1 \right) / \mathbb{P}(X_k \ge a) < \infty$$
 (23)

b) It is straightforward to show that N is a stopping time with respect to the sigma algebra generated by $X_k, k = \{1, 2, ..., n\}$. We omit the proof.

c) We show that $\frac{D}{\chi} < \mathbb{E}[N] \leq \frac{D}{\chi} + 1$:

Since N is a stopping time,

$$\mathbb{E}[S_N] = \chi \mathbb{E}[N] \ge D \implies \mathbb{E}[N] \ge D/\chi$$

Since (N-1) is also a stopping time,

$$\mathbb{E}[S_{N-1}] = (N-1)\chi < D \implies \mathbb{E}[N] < D/\chi + 1.$$

B. Three Intermediate Results

We first explore second-stage decision of agents *under truthful* reporting of baselines. We show that if the reward price exceeds the marginal utility, selected agents completely yield their discretionary consumption. Conversely, if $\pi^r < \pi_k - \pi^e$, the reward price is insufficient to persuade agents to offer any demand reduction. If the penalty prices is non-negative, participation in the DR program does not alter the energy consumption of agents who are not selected. We have the following:

Proposition 3. Suppose agent k faces a penalty price π^p and receives a reward price π^r . Assume agent k reports its baseline truthfully, i.e. $f_k = b_k$.

(a) Suppose agent k is not selected. If $\pi^p \ge 0$, its unique optimal second-stage decision and resulting optimal cost are

$$q_{ns}^* = b_k, \ J_{ns}(q_{ns}^*) = -(\pi_k - \pi^e)b_k$$

(b) Suppose agent k is selected. If $\pi^r > \pi_k - \pi^e$, its unique optimal second-stage decision and resulting optimal cost are

$$q_s^* = 0, \ J_s(q_s^*) = -\pi^r b_k$$

(c) Suppose agent k is selected. If $\pi^r < \pi_k - \pi^e$, agent k will not yield any of its discretionary consumption, i.e. $q_s^* = b_k$.

Proof: (a) If agent k is not selected, its cost function is

$$J_{\rm ns}(q) = \pi^e q - u(q) + \pi^p (b_k - q)^{\dagger}$$

which is piece-wise linear in the decision variable q. We compute the slopes:

$$\begin{aligned} \frac{dJ_{\rm ns}(q)}{dq} &= \begin{cases} \pi^e - \pi_k - \pi^p & \text{if } q < b_k \\ \pi^e & \text{if } q > b_k \end{cases} \\ &= \begin{cases} < 0 & \text{if } q < b_k \\ > 0 & \text{if } q > b_k \end{cases} \end{aligned}$$

Thus $q_{ns}^* = b_k$ is the unique minimizer of the cost function J_{ns} . The resulting optimal cost is clearly $-(\pi_k - \pi^e)b_k$.

(b) If agent k is selected, its cost function is

$$J_{s}(q) = \pi^{e}q - u(q) - \pi^{r}(b_{k} - q)^{+}$$

This is piece-wise linear in the decision variable q. We compute the slopes:

$$\begin{aligned} \frac{dJ_{\rm s}(q)}{dq} &= \begin{cases} \pi^e - \pi_k + \pi^r & \text{if } q < b_k \\ \pi^e & \text{if } q > b_k \end{cases} \\ &= \begin{cases} > 0 & \text{if } q < b_k \\ > 0 & \text{if } q > b_k \end{cases} \end{aligned}$$

Thus $q_s^* = 0$ is the unique minimizer of the cost function J_s . The resulting optimal cost is clearly $-\pi^r b_k$.

$$\pi^{r} < \pi_{k} - \pi^{e}, \text{ we have}$$

$$\frac{dJ_{s}(q)}{dq} = \begin{cases} < 0 & \text{if } q < b_{k} \\ > 0 & \text{if } q > b_{k} \end{cases}$$

(c) If

Thus $q_s^* = b_k$ is the unique minimizer of J_s .

We next explore conditions on the penalty and reward prices, and the selection probability which *induce agents to truthfully report their baselines*.

Proposition 4. Assume the mechanism prices and selection probability satisfy $\pi^p \ge \pi^e, \alpha \le (\pi^e)/(\pi^r + \pi^e)$. Then,

- (a) an optimal first stage decision for agent k is to truthfully report its baseline, i.e. $f_k^* = b_k$.
- (b) if further, $\pi^r > \pi_k \pi^e$, truthful reporting of baselines is the unique optimal decision.

Proof: (a) Consider agent k, and fix the submitted first-stage baseline report f. Define the constant

$$c = \pi^e b_k - u(b_k) = (\pi^e - \pi_k)b_k.$$

The selection probability inequality can be re-written as

$$-\alpha \pi^r + (1 - \alpha)\pi^e \ge 0. \tag{24}$$

We first compute the optimal consumption decided by agents

in the second stage in two cases:

(i) Agent is not called. Its cost function is

$$J_{\rm ns}(q,f) = \pi^e q - u(q) + \pi^p (f-q)^+$$

Since this is a continuous piece-wise linear function, it easy to verify that the optimal consumption is either $q^* = b_k$ or $q^* = f$. The optimal cost is:

$$J_{ns}(q^*, f) = \min\{c + \pi^p(f - b_k)^+, \pi^e f - u(f)\}$$

$$= \begin{cases} \min\{c + \pi^p(f - b_k), c + \pi^e(f - b_k)\} & \text{if } f > b_k \\ \min\{c, c + (\pi_k - \pi^e)(b_k - f)\} & \text{if } f < b_k \end{cases}$$

$$= \begin{cases} c + \min\{\pi^p, \pi^e\} \cdot (f - b_k) & \text{if } f > b_k \\ c + \min\{0, (\pi_k - \pi^e)(b_k - f)\} & \text{if } f < b_k \end{cases}$$

$$= \begin{cases} c + \pi^e(f - b_k) & \text{if } f > b_k \\ c & \text{if } f < b_k \end{cases}$$

(ii) Agent is called. Its cost function is

$$J_{s}(q, f) = \pi^{e} q - u(q) - \pi^{r} (f - q)^{+}$$

Since this is a continuous piece-wise linear function, it easy to verify that the optimal consumption is either $q^* = 0$ or $q^* = b_k$. The optimal cost is:

$$J_{s}(q^{*};f) = \min\{-\pi^{r}f, c - \pi^{r}(f - b_{k})^{+}\} \\ = \begin{cases} \min\{-\pi^{r}f, c - \pi^{r}(f - b_{k})\} & \text{if } f > b_{k} \\ \min\{-\pi^{r}f, c\} & \text{if } f < b_{k} \end{cases} \\ = \begin{cases} -\pi^{r}f + \min\{0, c + \pi^{r}b_{k}\} & \text{if } f > b_{k} \\ \min\{-\pi^{r}f, c\} & \text{if } f < b_{k} \end{cases} \end{cases}$$

Combining (i) and (ii), we can write the expected cost as

$$J(f) = \alpha J_{s} + (1 - \alpha) J_{ns} = \begin{cases} -\alpha \pi^{r} f + \alpha \min\{0, c + \pi^{r} b_{k}\} \\ + (1 - \alpha)(c + \pi^{e}(f - b_{k})) & \text{if } f > b_{k} \\ \alpha \min\{-\pi^{r} f, c\} + (1 - \alpha)c & \text{if } f < b_{k} \end{cases}$$

Note that J(f) is continuous and piece-wise linear. We compute the slopes

$$\frac{dJ(f)}{df} = \begin{cases}
-\alpha \pi^r + (1-\alpha)\pi^e & \text{if } f > b_k \\
\text{either} - \alpha \pi^r \text{ or } 0 & \text{if } f < b_k
\end{cases}$$

$$= \begin{cases}
\geq 0 & \text{if } f > b_k \\
\leq 0 & \text{if } f < b_k
\end{cases}$$
(25)

Here, we have used (24). Consequently, $f^* = b_k$ is a minimizer of J(f).

(b) Next suppose $\pi^r > \pi_k - \pi^e$. Notice that

$$c + \pi^r b_k = (\pi^e - \pi_k)b_k + \pi^r b_k > 0.$$

Define $g = -c/\pi^r$, and notice that for f < g,

$$c + \pi^r f < c + \pi^r g = 0.$$

It is easy to verify that the expected cost simplifies to

$$J(f) = \begin{cases} -\alpha \pi^r f + (1 - \alpha)(c + \pi^e(f - b_k)) & \text{if } f > b_k \\ -\alpha \pi^r f + (1 - \alpha)c & \text{if } g < f < b_k \\ c & \text{if } f < g \end{cases}$$

Note that J(f) is continuous and piece-wise linear. We com-

pute the slopes

$$\frac{dJ(f)}{df} = \begin{cases} -\alpha \pi^r + (1-\alpha)\pi^e & \text{if } f > b_k \\ -\alpha \pi^r & \text{if } g < f < b_k \\ 0 & \text{if } f < g \end{cases}$$

Consequently, $f^* = b_k$ is the unique minimizer of J(f). Finally, we show that if the selection probability is too large, agents will arbitrarily inflate their baselines.

Proposition 5. Assume the selection probability satisfies $\alpha > (\pi^e)/(\pi^r + \pi^e)$. Then the optimal first stage decision is to report the maximum, i.e., $f^* = b_{\text{max}}$.

Proof: We follow the calculation of Proposition 4. The selection probability inequality can be re-written as

$$-\alpha \pi^r + (1-\alpha)\pi^e < 0.$$

It follows from (25) that

$$\frac{dJ(f)}{df} = \begin{cases} < 0 & \text{if } f > b_k \\ \le 0 & \text{if } f < b_k \end{cases}$$

Thus J(f) is monotone decreasing. The optimal first-stage decision is $f^* = b_{\text{max}}$, i.e. agents will arbitrarily inflate their baselines.

C. Proof of Theorem 1

Proof: (a) From Proposition 4, we know that an optimal first stage decision is for agents to truthfully report their baselines, i.e. $f_k^* = b_k$.

- (b) and (c) now follow immediately from Proposition 3.
- (d) is immediate on observing that $\pi^p(f_k^* q_{ns}^*) = 0$.
- (e) Recall that agents are selected at random until (see (6))

$$\sum f_k \ge D.$$

Note that (a) agents report baselines truthfully, and (c) selected agents completely yield their discretionary consumption. As a result, the set of selected agents provide $\sum_k b_k$ in demand reduction, meeting the target D.

Proof: We consider mechanisms where agents that are called receive a payment $\pi_k^r (b_k - q_k)^+$ for consumption reduction from their true baseline. The reward price can be agent specific.

From Proposition 3 (c), if the reward price is too low, i.e. $\pi_k^r < \pi_k - \pi^e$, agent k will not yield any of its discretionary consumption to the DR program. So, we must have $\pi_k^r \ge \pi_k - \pi^e$, and under this condition, selected agents completely yield their discretionary consumption.

If agent k knows with *certainty* that it will be called for a DR event, its first-stage cost function is

$$J(f_k, q_k) = -\pi_r^k (f_k - q_k)^+ + \pi^e q_k - u(q_k).$$

As its optimal second stage decision is $q^* = 0$, the resulting first-stage decision problem is

$$\min_{f_k} -\pi_r^k f_k.$$

This implies that agent k will arbitrarily inflate its baseline, resulting in an unbounded expected cost for DR provision. As a result, we must allow for agent k to be selected *at random* with a sufficiently small probability. From Proposition 5, the selection probability α_k for agent k must satisfy $\alpha_k \leq (\pi^e)/(\pi_k^r + \pi^e)$, else agent k will arbitrarily inflate its baseline, resulting in an unbounded expected cost for DR provision.

To minimize its DR payments to agents, the aggregator should select the smallest possible reward, i.e. it should choose $\pi_k^r = \pi_k - \pi^e$. To minimize its recruitment costs, the aggregator should recruit the smallest number of agents or maximize the selection probability. Thus we set

$$\alpha_k = \frac{\pi^e}{\pi_k^r + \pi^e} = \frac{\pi^e}{\pi_k}$$

Any other mechanism in C would result in a higher expected cost for DR provision.

With these choices, under any mechanism in the class C, the expected demand reduction from agent k is $\alpha_k b_k$, and the expected payout to agent k is $\alpha_k \pi_k^r b_k$. The expectations here are over the randomness of being selected under the DR mechanism.

Define the random variables $X_k = b_k w_k$ where $w_k = \pi^e / \pi_k$. Notice that $\{b_1, b_2, \cdots\}$ and $\{w_1, w_2, \cdots\}$ are i.i.d. Also, b_i s and w_k s are independent. Thus $\{X_1, X_2, \cdots\}$ is i.i.d. and

$$\mathbb{E}[X] := \mathbb{E}[X_k] = \pi^e \mathbb{E}[b] \mathbb{E}[1/\pi]$$
(26)

To meet the load reduction target D, we must recruit N customers where

$$\sum_{1}^{N} \alpha_k b_k = \sum_{1}^{N} \frac{\pi^e}{\pi_k} b_k = \sum_{1}^{N} X_k \ge D.$$

Using Proposition 2, we have

$$\mathbb{E}[N] \ge \frac{D}{\mathbb{E}[X_k]} = \frac{D}{\pi^e \mathbb{E}[b]\mathbb{E}[1/\pi]}.$$
(27)

The expected payout ψ to the agents (over the random variables b_k, π_k) is

$$\mathbb{E}[\psi] = \mathbb{E}\left[\sum_{1}^{N} \alpha_k \pi_k^r b_k\right] = \mathbb{E}\left[\sum_{1}^{N} \frac{\pi^e(\pi_k - \pi_e)}{\pi_k} b_k\right]$$
$$= \mathbb{E}\left[\sum_{1}^{N} (\pi^e b_k - \pi^e X_k)\right]$$
$$= \pi^e \mathbb{E}[N] \left(\mathbb{E}[b] - \mathbb{E}[X]\right)$$
$$= \pi^e \mathbb{E}[N] \mathbb{E}[b] \left(1 - \pi^e \mathbb{E}[1/\pi]\right)$$

In the final step above, we have used Wald's equation (see Proposition 1) together with (26). Combining this with (27), we obtain

$$\mathbb{E}[\psi] \ge \frac{D}{\mathbb{E}[1/\pi]} - \pi^e D.$$

We can now compute a lower bound on the expected cost of

DR provision:

$$\phi = \underbrace{\frac{\mathbb{E}[\psi]}{D}}_{\text{payout per KWh}} + \underbrace{\frac{\pi^o \mathbb{E}[N]}{mD}}_{\text{recruitment cost}}$$
$$\geq \frac{1}{\mathbb{E}[1/\pi]} - \pi^e + \frac{\pi^o}{m\mathbb{E}[b]} \cdot \frac{1}{\pi^e \mathbb{E}[1/\pi]}$$
the claim.

proving the claim.

D. Proof of Theorem 3

Proof: To minimize its DR payments to agents, the aggregator should select the smallest possible reward, i.e. it should choose (see Theorem 1) $\pi^r = \pi_{\text{max}} - \pi^e$. To minimize its recruitment costs, the aggregator should recruit the smallest number of agents or maximize the selection probability. Thus we set (see Theorem 1), $\alpha = (\pi^e)/(\pi^r + \pi^e) = (\pi^e)/(\pi_{\text{max}})$. The number of recruited agents needed to service the DR target is the smallest integer N such that

$$\sum_{1}^{N} \alpha b_k \ge D$$

From Proposition 2, we have

$$\mathbb{E}[N] < \frac{D}{\alpha \mathbb{E}[b]} + 1 = \frac{D\pi_{\max}}{\pi^e \mathbb{E}[b]} + 1.$$

If agent k is selected, it delivers b_k KWh of demand reduction. The expected payout to the agents is then From Theorem 1, the expected payout is

$$\mathbb{E}[\psi] = \mathbb{E}[\sum_{1}^{N} \pi^{r} \alpha b_{k}] = \pi^{r} \mathbb{E}[N] \alpha \mathbb{E}[b] < \pi^{r} D + \frac{\pi^{r} \pi^{e} \mathbb{E}[b]}{\pi_{\max}}$$

We can now compute an upper bound on the expected cost of DR provision:

$$\phi = \underbrace{\frac{\mathbb{E}[\psi]}{D}}_{\text{payout per KWh}} + \underbrace{\frac{\pi^{o}\mathbb{E}[N]}{mD}}_{\text{recruitment cost}}$$
$$\leq (\pi_{\max} - \pi^{e})\left(1 + \frac{\pi^{e}\mathbb{E}[b]}{\pi_{\max}D}\right) + \frac{\pi^{o}\pi_{\max}}{m\pi^{e}\mathbb{E}[b]} + \frac{\pi^{o}}{mD}$$

E. Proof of Theorem 4

Proof: Under SRBM, we have $\pi^p \ge \pi^e$. We first bound the selection probability. Suppose agent k in pod \mathbb{P}^i is selected. The reward price offered to agent k satisfies

$$\pi_k^r \le \max_{k \in \mathbb{P}^i} \mu_k - \pi^e = \nu^i - \pi^e.$$

The probability β_k that agent k is called satisfies

$$\beta_k = (\pi^e) / (\pi_k^r + \pi^e)$$

This is also independent of agent k's reports. We therefore meet all the conditions of Proposition 4. We conclude that all agents will report its baselines truthfully, or $f_k^* = b_k$ for all k. More precisely, we have

$$J(\mu_k, f_k \mid \mu_{-k}, f_{-k}) \ge J(\mu_k, b_k \mid \mu_{-k}, f_{-k})$$

We next show that agents are also best served by reporting their marginal utility truthfully, i.e.

$$J(\mu_k, b_k \mid \mu_{-k}, f_{-k}) \ge J(\pi_k, b_k \mid \mu_{-k}, f_{-k})$$

Observe that the reward price for agent k under SRBM is determined by the reports of other agents, $i \neq k$. Suppose agent k is selected. By the partial information setting this implies that the agent is part of S in its group or pod. If it remains selected when it reports its true marginal utility then it receives the same reward price. So it is no worse off being truthful.

Suppose agent k is not selected. If it remains not selected when it reports its true marginal utility, it faces the same penalty price. So again, agent k is no worse off being truthful.

The only remaining situations to consider are when agent k strategically reports its marginal utility to alter the selection decision:

• Agent k is selected, but would not selected if it is truthful. Since agent k is selected, from Proposition 3, its optimal cost is either $J_1 = -\pi_k^r b_k$ or $J_1 = -(\pi_k - \pi^e)b_k$ depending on whether $\pi_k^r \ge \pi_k - \pi^e$ or $\pi_k^r \le \pi_k - \pi^e$ resply. The reward price received by agent k under SRBM is

$$\pi_k^r = \max\{\mu_j\} - \pi^e \ , \ j \in \mathbb{S}_{-k}$$

If agent k is not selected when it is truthful, using Proposition 3, its optimal cost is $J_2 = -(\pi_k - \pi^e)b_k$. Since agent k is not selected, we can drop k from the list of agents in determining the set of selected agents. Thus, the set of selected agents in this sub-case is simply \mathbb{S}_{-k} , and since agent k is not selected, we have $\pi_k > \max_{j \in \mathbb{S}_{-k}} \mu_k = \pi_k^r + \pi^e$. As a result, $\pi_k^r < \pi_k - \pi^e$, which implies $J_1 \ge J_2$. Thus agent k is better off being truthful.

• Agent k is not selected, but would be selected if it is truthful.

Since agent k is not selected, using Proposition 3, its optimal cost is $J_1 = -(\pi_k - \pi^e)b_k$.

If agent k is selected when it is truthful, using Proposition 3, its optimal cost is $J_2 = -\pi_k^r b_k$ or $J_2 = -(\pi_k - \pi^e)b_k$. Also,

$$\pi_k^r = \max\{\mu_j\} - \pi^e \ , \ j \in \mathbb{S}_{-k}$$

Since agent k is selected when truthful, we have $\pi_k < \max\{\mu_j: j \in \mathbb{S}_{-k}\}$. As a result, $\pi_k^r > \pi_k - \pi^e$, which implies $J_1 \ge J_2$. Thus agent k is better off being truthful.

In all situations, agent k is no worse off by truthfully reporting its marginal utility and baseline, proving (a).

(b) and (c) now follow immediately from Proposition 3.

(d) is a straightforward observations.

(e) Recall that the set of selected agents from pod \mathbb{P}^i (see (11)) is $\mathbb{S} = \{1, \dots, k^*\}$ where

$$\sum_{k=1}^{k^*} f_k \ge D, \quad \sum_{k=1}^{k^*-1} f_k < D$$

Note that (a) agents report baselines truthfully, and (c) selected agents completely yield their discretionary consumption. As a result, the agents in pod core when selected provide $\sum_{k=1}^{k^*} b_k$ in demand reduction, meeting the target D.

F. Proof of Theorem 5

Proof:

(a) Let N_i be the number of agents in the pod core \mathbb{S}^i . Recall agents are sorted into \mathbb{S}^i so that

$$\sum_{k \in \mathbb{S}^i} b_i \ge D$$

Using Theorem 2, we can bound the expected number of agents in the pod core \mathbb{S}^i as

$$\frac{D}{\mathbb{E}[b]} \le \mathbb{E}[N_i] \le \frac{D}{\mathbb{E}[b]} + 1 \tag{28}$$

for all *i*. Note that this is also a bound on the number of agents in any pod header \mathbb{H}^i . Next observe from Figure 4 that the header for pod \mathbb{P}^i serves as the core \mathbb{S}^{i+1} for the subsequent pod. Thus the total number of agents recruited is

$$N = \sum_{1}^{M+1} N_i.$$

It is easy to observe that N_i s are i.i.d. The number of pods M is determined using the stopping criterion $\sum_{1}^{M} \beta^i \ge 1$ and hence M is a stopping time. Using Wald's equation, from the above equation we get,

$$\mathbb{E}[N] = (\mathbb{E}[M] + 1)\mathbb{E}[N_1] \le (\mathbb{E}[M] + 1)(D/\mathbb{E}[b] + 1)$$
(29)

(b) We now bound the number of pods M. Recall that we keep forming pods until

$$\sum_{1}^{M-1} \beta^{i} < 1 \le \sum_{1}^{M} \beta^{i}, \text{ where } \beta^{i} = \frac{\pi^{e}}{\nu^{i}}$$
(30)

Since the harmonic mean of a collection of numbers is always less than arithmetic mean, we have

$$\frac{M-1}{1/\nu^1 + \dots + 1/\nu^{M-1}} \le \frac{\nu^1 + \dots + \nu^{M-1}}{M-1}$$

As a result, we have

$$(M-1)^{2} \leq \pi^{e}(1/\nu^{1} + \dots + 1/\nu^{M-1}) \cdot \frac{\nu^{1} + \dots + \nu^{M-1}}{\pi^{e}}$$

$$< \frac{\nu^{1} + \dots + \nu^{M-1}}{\pi^{e}}$$

The final inequality above follows from (30). Note that $(M - 1)^2$ is a convex function of M. Taking expectations, using Jensen's inequality, we obtain

$$(\mathbb{E}[M-1])^2 \le \mathbb{E}[(M-1)^2] < \frac{1}{\pi^e} \mathbb{E}\left[\sum_{i=1}^{M-1} \nu^i\right]$$
 (31)

Define $\mathbb{S}^{M+1} = \mathbb{H}^M$. Agents in this $(M+1)^{\text{th}}$ pod core are never called. They are only needed to serve as the pod header \mathbb{H}^M in order to determine reward prices for pod \mathbb{P}^M under

SRBM. Define $\pi_{[k]}, k = 1, 2, \cdots$ to be the agent marginal utilities *sorted in increasing order*. Notice that (see Figure 4)

$$\nu^i \leq \pi_{[k]}$$
 for $k \in \mathbb{S}^{i+2}$.

Therefore

$$N_{i+2}\nu^i \le \sum_{k \in \mathbb{S}^{i+2}} \pi_{[k]}$$

where N_i is the number of agents in pod core \mathbb{S}^i .

As a result, we have

$$\sum_{i=1}^{M-1} N_{i+2} \nu^{i} \leq \sum_{i=1}^{M-1} \sum_{k \in \mathbb{S}^{i+2}} \pi_{[k]} = \sum_{k \in \mathbb{S}^{3} \cup \dots \cup \mathbb{S}^{M+1}} \pi_{[k]}$$
$$\leq \sum_{k \in \mathbb{S}^{1} \cup \dots \cup \mathbb{S}^{M+1}} \pi_{[k]} = \sum_{k=1}^{N} \pi_{[k]} = \sum_{k=1}^{N} \pi_{k}$$
(32)

The final equality follows because the sum of all of the *sorted* random variables $\pi_{[k]}$ is identical to the sum of the unsorted random variables π_k . Taking expectations of both sides of (32),

$$\mathbb{E}\left[\sum_{i=1}^{M-1} N_{i+2}\nu^{i}\right] \leq \mathbb{E}\left[\sum_{k=1}^{N} \pi_{k}\right] = \mathbb{E}[N]\mathbb{E}[\pi]$$
$$= \mathbb{E}[N_{1}]\mathbb{E}[(M+1)]\mathbb{E}[\pi].$$
(33)

where we used (29) to get the last equality. Notice that N_{i+2} and ν^i are independent, and since N_i s are i.i.d., by a simple conditioning argument,

$$\mathbb{E}\left[\sum_{i=1}^{M-1} N_{i+2}\nu^{i}\right] = \mathbb{E}[N_{1}]\mathbb{E}\left[\sum_{i=1}^{M-1}\nu^{i}\right]$$
(34)

From (34) and (33),

$$\mathbb{E}\left[\sum_{i=1}^{M-1}\nu^i\right] \le \mathbb{E}[(M+1)]\mathbb{E}[\pi]$$
(35)

Combining (31) and (35), we obtain

$$\left(\mathbb{E}[M]+1\right)\left(\mathbb{E}[M]-3\right]\right) \le \left(\mathbb{E}[(M-1)]\right)^2 \le \frac{\mathbb{E}[M+1]\mathbb{E}[\pi]}{\pi^e}$$

This simplifies to

$$\mathbb{E}[M] < \frac{\mathbb{E}[\pi]}{\pi^e} + 3$$

proving the claim.

G. Proof of Theorem 6

Agent k in Pod \mathbb{P}^i is selected with probability $\beta_k^i = \pi^e / (\pi_k^r + \pi^e)$. Selected agents deliver D KWh of demand reduction. Thus, the expected payout to agents per DR event is

$$\mathbb{E}[\psi] \leq \mathbb{E}[\sum_{i=1}^{M} \sum_{k \in \mathbb{S}^{i}} \beta_{k}^{i} \pi_{k}^{r} b_{k}] \leq \mathbb{E}[\sum_{i=1}^{M} \pi^{e} D]$$
$$= \mathbb{E}[\sum_{i=1}^{M} \pi^{e} D] \leq E[M] \pi^{e} D.$$
(36)

From (3), the average cost of DR provision per KWh of demand reduction is

$$\phi = \frac{\mathbb{E}[\psi]}{D} + \frac{\pi^o \mathbb{E}[N]}{mD}$$

Using the bound on $\mathbb{E}[\psi]$ from (36) and the bound on $\mathbb{E}[N]$ from Theorem 5, we get the desired result.

H. SRBM for Complete Information (CI) Setting

The main challenge in the mechanism design is that the selection probabilities of individual consumers has to be limited (see Theorem 1) so that the reported baselines are not inflated. This is unlike traditional resource allocation problems where the baseline is available. This necessitates the mechanism designer to recruit more than the minimum number of agents required to meet the target D. SRBM recruits multiple such sets of consumers and sorts them in to pods, where each pod has a core that contains the minimum number of agents required to meet the target D. Supposing that the agents reveal the true values of their baseline and marginal utility, then the mechanism can minimize its payout by selecting pod cores of higher marginal utilities by a lower proability. The reward design ((12)), would then imply that pod cores with higher rewards are selected less often.

In the complete information setting this can couple the selection probabilities with the agent reporting. As a result, a characterization like Thm. 1 might not be feasible. So the challenge here is to design a *pod sorting* algorithm, combined with a *pod selection* and *reward pricing* mechanism such that the incentive for the agents to inflate their baseline reports or their marginal utility reports to increase payments is nullified while pod sorting is cost effective.

Below we discuss the mechanism for this complete information setting. Agents are sorted in to pods based on their reported marginal utility as in Fig. 4. Agent k's selection probability and unit reward is given by,

$$\beta_k^i = c_k^i \pi^e / \nu_{-k}^{i+1}; \ \pi_k^r = \nu_{-k}^i - \pi^e \tag{37}$$

Where,

$$\begin{split} \mu_k^{j+} &= \nu_{-k}^j \frac{\nu_{-k}^{j+2} - e_k^j \left(\nu_{-k}^{j+1}\right)^2 / \nu_{-k}^j}{\nu_{-k}^{j+2} - e_k^j \nu_{-k}^{j+1}} \ , \ c_k^1 = 1 \ , \ e_k^j = \frac{c_k^{j+1}}{c_k^j} \\ \mu_k^{j+} &= \nu_{-k}^{i-1} \ , \ \text{if} \ j > i \ , \ \mu_k^{j+} = \nu_{-k}^{j-1} \ \text{if} \ j \leq i \ , \end{split}$$

And $\nu_{-k}^{j} =$ highest report in j th pod if k was excluded And i is the pod allotted to consumer k (38)

The above equation can be rewritten as,

$$e_k^j = \frac{\nu_{-k}^{j+2}(\mu_k^{j+} - \nu_{-k}^j)}{\nu_{-k}^{j+1}(\mu_k^{j+} - \nu_{-k}^{j+1})} > 0$$
(39)

Remark 13. There are three differences from mechanism for PI setting, the reward, the selection probability and the inclusion

of an additional factor in the calculation of the selection probability.

The number of pods M in the mechanism is chosen such that $\sum_{j=1}^{M-1} \beta^j < 1 \leq \sum_{j=1}^M \beta^j$, where a \mathbb{P}^i 's core \mathbb{S}^i is such that $\sum_{k=1}^{N-1} f_k < D \leq \sum_{k=1}^N f_k$ and N is the number of consumers in \mathbb{S}^i . Here the difference is that $\beta^i \neq \min \beta_k$. Selection: a random number $u \sim \mathcal{U}[0,1]$ is drawn. Pod core \mathbb{S}^i is selected for DR if $u \in [\sum_{j=1}^{i-1} \beta^j, \sum_{j=1}^i \beta^j]$. To be consistent with the selection probability of an individual consumer a consumer k in \mathbb{S}^i is selected for all draws $u \in [\sum_{j=1}^{i-1} \beta^j, \sum_{j=1}^{i-1} \beta^j + \beta_k]$. Agents who are selected are rewarded for reduction from reported baseline based on unit reward announced to them. Agents who are not selected are penalized for consuming less than the reported baseline.

Difference between SRBM for CI and SRBM for PI: Unlike SRBM for PI, the selection probability of agent k has an additional jump factor c_k^i . This guarantees that the selection probability of an individual agent decays at a sufficient rate that the agent does not have the incentive to misreport and jump to a higher pod just to gain higher reward. Also note that the reward and the probability without the factor are also different. In SRBM for PI, the reward is $\pi_k^r = \nu_{-k}^{i-1} - \pi^e$ while in SRBM for CI the reward is $\pi_k^r = \nu_{-k}^i - \pi^e$. Similarly the selection probability in SRBM for PI is $\beta_k^i = \pi^e / \nu_{-k}^i$ while in SRBM for CI the selection probability without the jump factor is $\beta_k^i / c_k^i = \pi^e / \nu_{-k}^{i+1}$. This change in the selection probability and the reward enables design of a mechanism where the jump factors are computable using the reported information.

Theorem 7. *The modified SRBM for the complete information setting has the following properties,*

- (a) truthful reporting of baselines and marginal utilities is a dominant strategy
- (b) called agents consume $q_s^* = 0$, providing the maximal reduction in their discretionary consumption
- (c) agents that are not called consume $q_{ns}^* = b_k$
- (d) the aggregator receives no penalty revenue
- (e) the load reduction target D is met by each pod core.

Proof. The jump factor c_k^i ((38)), that is used in the selection probability of agent k, is only dependent on $\mu_k^{(i-1)+}, \nu_{-k}^{i-1}, \nu_{-k}^{i+1}, \nu_{-k}^i$ and c_k^{i-1} . Hence the recursive scheme for computation of c_k^i is independent of the agent report μ_k and that the jump factor c_k^i is independent of the reports of agent k. This implies that the selection probability of a agent $k, \beta_k = c_k^i \pi^e / \nu_{-k}^{i+1}$ is independent of its reports.

Now all we need to check is that whether $\beta_k \leq \pi^e/(\pi_k^r + \pi^e)$. When e_k^j is zero on the right of (38), we get,

$$\mu_{k}^{j+} \leq \nu_{-k}^{j-1} < \nu_{-k}^{j} \frac{\nu_{-k}^{j+2} - e_{k}^{j} \left(\nu_{-k}^{j+1}\right)^{2} / \nu_{-k}^{j}}{\nu_{-k}^{j+2} - e_{k}^{j} \nu_{-k}^{j+1}} \bigg|_{e_{k}^{j} = 0} = \nu_{-k}^{j}$$

$$(40)$$

Calling right hand side of (38) as $g(\nu_{-k}^j, \nu_{-k}^{j+1}, \nu_{-k}^{j+2}, e_k^j)$, it is easy to check using chain rule for differentiation that $\frac{dg}{dz^j}$

0. And
$$g(\nu_{-k}^{j}, \nu_{-k}^{j+1}, \nu_{-k}^{j+2}, e_{k}^{j}) = 0$$
 when $c_{k}^{j+1} = \frac{c_{k}^{j} \nu_{-k}^{j+2} \tilde{\nu}_{-k}^{j}}{(\nu_{-k}^{j+1})^{2}}$.
Combining these two observations it is clear that there is a c_{k}^{j}

such that,

$$0 < c_k^i < \frac{c_k^{i-1} \nu_{-k}^{i+1} \nu_{-k}^{i-1}}{(\nu_{-k}^i)^2}$$
(41)

Similarly it follows that,

$$0 < c_k^{i-1} < \frac{c_k^{i-2} \nu_{-k}^i \nu_{-k}^{i-2}}{(\nu_{-k}^{i-1})^2}$$
(42)

Applying recursively we get that,

$$0 < c_k^i < \frac{\nu_{-k}^1 \nu_{-k}^{i+1}}{\nu_{-k}^2 \nu_{-k}^i} < \frac{\nu_{-k}^{i+1}}{\nu_{-k}^i}$$
(43)

This implies the selection probability of agent k satisfies,

$$\beta_k = c_k^i \pi^e / \nu_{-k}^{i+1} < \frac{\nu_{-k}^{i+1}}{\nu_{-k}^i} \pi^e / \nu_{-k}^{i+1} = \frac{\pi^e}{\nu_{-k}^i} = \frac{\pi^e}{\pi_k^r + \pi^e}$$
(44)

From the above observation and the fact that β_k and π_k^r are independent of agent k's reports. Then the proof of Thm. 1 applies here and it follows that the agent k reports its *true baseline* i.e. $f_k = b_k$.

Note that it is trivial to show that the agent does not gain by changing its marginal utility report such that it is still within \mathbb{S}^i (pod core allotted if agent k reports truthfully) but at a different position. This is because the rewards, selection probabilities for a agent k does not change once its pod core is fixed. Next we show that the agent does not gain either by jumping to a lower or a higher pod core.

Denote by *i* the pod core number that is allotted to agent *k* on reporting truthfully and denote by U_k^i the net utility of agent *k* in \mathbb{S}^i . Then,

$$U_{k}^{i} = \beta_{k}^{i} \pi_{k}^{r} b_{k} + (1 - \beta_{k}^{i})(\pi_{k} - \pi^{e}) b_{k} = \frac{c_{k}^{i} \pi^{e}}{\nu_{-k}^{i+1}} (\nu_{-k}^{i} - \pi^{e}) b_{k} + \left(1 - \frac{c_{k}^{i} \pi^{e}}{\nu_{-k}^{i+1}}\right) (\pi_{k} - \pi^{e}) b_{k}$$

$$(45)$$

Define \tilde{U}_k^j as the utility of agent k in \mathbb{S}^j , but using the computed jump factor c_k^j when k is allotted \mathbb{S}^i . Then,

$$\tilde{U}_{k}^{j} = \beta_{k}^{j} \pi_{k}^{r} b_{k} + (1 - \beta_{k}^{j})(\pi_{k} - \pi^{e}) b_{k} \\
= \frac{c_{k}^{j} \pi^{e}}{\nu_{-k}^{j+1}} (\nu_{-k}^{j} - \pi^{e}) b_{k} + \left(1 - \frac{c_{k}^{j} \pi^{e}}{\nu_{-k}^{j+1}}\right) (\pi_{k} - \pi^{e}) b_{k} \\$$
(46)

Case $j \ge i$: Here we first show that $\tilde{U}_k^{j+1} \le \tilde{U}_k^j$ and then show that $U_k^j \le \tilde{U}_k^j \le \tilde{U}_k^i = U_k^i$. Taking the difference of

$$\begin{split} \tilde{U}_{k}^{j+1} & \text{and } \tilde{U}_{k}^{j} \text{ we get,} & \text{and } \tilde{U}_{k}^{i-1} \text{ we get,} \\ \tilde{U}_{k}^{j+1} - \tilde{U}_{k}^{j} &= \frac{c_{k}^{j+1}\pi^{e}}{\nu_{-k}^{j+2}} \nu_{-k}^{j+1} b_{k} + \left(1 - \frac{c_{k}^{j+1}\pi^{e}}{\nu_{-k}^{j+2}}\right) \pi_{k} b_{k} & \tilde{U}_{k}^{i} - \tilde{U}_{k}^{i-1} &= \frac{c_{k}^{i-1}\pi^{e}b_{k}\pi_{k}}{\nu_{-k}^{i+1}\nu_{-k}^{i}} \\ & - \frac{c_{k}^{j}\pi^{e}}{\nu_{-k}^{j+1}} \nu_{-k}^{j} b_{k} - \left(1 - \frac{c_{k}^{j}\pi^{e}}{\nu_{-k}^{j+1}}\right) \pi_{k} b_{k} & - \nu_{-k}^{i-1} \left(\nu_{-k}^{i+1} - e_{k}^{i-1} \frac{(\nu_{-k}^{i})}{\nu_{-k}^{i+1}}\right) \\ & = \frac{c_{k}^{j+1}\pi^{e}}{\nu_{-k}^{j+2}} \nu_{-k}^{j+1} b_{k} - \frac{c_{k}^{j}\pi^{e}}{\nu_{-k}^{j+2}} \pi_{k} b_{k} - \frac{c_{k}^{j}\pi^{e}}{\nu_{-k}^{j+1}} \nu_{-k}^{j} b_{k} & = \frac{c_{k}^{i-1}\pi^{e}b_{k}\pi_{k}}{\nu_{-k}^{j+1}} \left(\nu_{-k}^{i-2} \left(\nu_{-k}^{i+1}\right) + \frac{c_{k}^{i-1}\pi^{e}}{\nu_{-k}^{j+1}}\right) \\ & = \frac{c_{k}^{j}\pi^{e}b_{k}}{\nu_{-k}^{j+2}\nu_{-k}^{j+1}} \left(e_{k}^{j}(\nu_{-k}^{j+1})^{2} - \nu_{-k}^{j+2}\nu_{-k}^{j} - \pi_{k}(e_{k}^{j}\nu_{-k}^{j+1} - \nu_{-k}^{j+2})\right) \text{ hen from (38) it follows that,} \\ & = \frac{c_{k}^{j}\pi^{e}b_{k}\pi_{k}}{\nu_{-k}^{j+2}\nu_{-k}^{j+1}} \left(\pi_{k} \left(\nu_{-k}^{j+2} - e_{k}^{j}\nu_{-k}^{j+1}\right) - \nu_{-k}^{j}\left(\nu_{-k}^{j+1} - e_{k}^{j-1}\frac{(\nu_{-k}^{j+1})^{2}}{\nu_{-k}^{j+1}}\right) \\ & -\nu_{-k}^{j}\left(\nu_{-k}^{j+2} - e_{k}^{j}\frac{(\nu_{-k}^{j+1})^{2}}{\nu_{-k}^{j}}\right) \right) \tag{47}$$

Then from (38), (47) and the fact that $\mu_k^{j+} = \nu_{-k}^{i-1} \ge \pi_k$, because agent k's true marginal utility π_k satisfies $\nu_{-k}^{i-2} \le$ $\pi_k \leq \nu_{-k}^{i-1}$, we get,

$$\begin{split} \tilde{U}_{k}^{j+1} &- \tilde{U}_{k}^{j} \leq \frac{c_{k}^{j} \pi^{e} b_{k} \pi_{k}}{\nu_{-k}^{j+2} \nu_{-k}^{j+1}} \left(\mu_{-k}^{j+} \left(\nu_{-k}^{j+2} - e_{k}^{j} \nu_{-k}^{j+1} \right) \right. \\ &- \nu_{-k}^{j} \left(\nu_{-k}^{j+2} - e_{k}^{j} \frac{(\nu_{-k}^{j+1})^{2}}{\nu_{-k}^{j}} \right) \right) = 0 \\ &\Rightarrow \tilde{U}_{k}^{j+1} \leq \tilde{U}_{k}^{j} \end{split}$$
(48)

Next we show that $c_{k\in\mathbb{S}^r}^j \leq c_{k\in\mathbb{S}^i}^j$, when r > i, and then it trivally follows that $U_k^j \leq U_k^j$. If the agent k jumps to higher pod core r (by reporting high) then the value $\mu_{k\in\mathbb{S}^r}^{j+1} = \min\{\nu_{-k}^{j-1}, \nu_{-k}^{r-1}\} \geq \nu_{-k}^{i-1} = \mu_{k\in\mathbb{S}^i}^{j+1} \forall j > i$ and $\mu_{k\in\mathbb{S}^r}^{j+1} = \nu_{-k}^{j-1} = \mu_{k\in\mathbb{S}^i}^{j+1} \forall j \leq i$. Also, (38) becomes $\mu_{k\in\mathbb{S}^r}^{(j-1)+} = g(\nu_{-k}^{j-1}, \nu_{-k}^j, \nu_{-k}^{j+1}, e_{k\in\mathbb{S}^r}^{j-1})$. For a particular j > i, every argument of g is fixed except e_k^j , which depends on every argument of g is fixed except e_k^i , which depends on the core r agent k is in. We showed before that the right hand side of (38) i.e. $g(\nu_{-k}^j, \nu_{-k}^{j+1}, \nu_{-k}^{j+2}, e_k^j)$ is decreasing in e_k^j . Then it follows that $e_{k\in\mathbb{S}^r}^j \leq e_{k\in\mathbb{S}^i}^j$ r > i. Since $c_k^1 = 1$ always it follows that $c_{k\in\mathbb{S}^r}^j \leq c_{k\in\mathbb{S}^i}^j$ when r > i. Setting j = r it follows that $c_{k\in\mathbb{S}^j}^j \leq c_{k\in\mathbb{S}^i}^j$. This implies that $U_k^j \leq \tilde{U}_k^j$. From here it follows that $U_k^j \leq \tilde{U}_k^j \leq \tilde{U}_k^i = U_k^i$.

Case j < i: First the agent does not have any incentive to move to a pod core \mathbb{S}^j where $j \leq i - 2$. This is because the reward for agent k when it is in a core $j \leq i-2$ is $\pi_k^r = \nu_{-k}^j \leq \nu_{-k}^{i-2} \leq \pi_k$. This implies that the incentive to reduce is not sufficient in any of these pod cores. Hence the agent's utility $U_k^j = (\pi_k - \pi^e)b_k < U_k^i$. From (38) it follows that $c_{k\in\mathbb{S}^j}^j = c_{k\in\mathbb{S}^i}^j$ and $c_{k\in\mathbb{S}^j}^i = c_{k\in\mathbb{S}^i}^i$ when j = i - 1. This implies $U_k^j = \tilde{U}_k^j = \tilde{U}_k^{i-1}$. Finding the difference between \tilde{U}_k^i and \tilde{U}_{k}^{i-1} we get,

$$\tilde{U}_{k}^{i} - \tilde{U}_{k}^{i-1} = \frac{c_{k}^{i-1}\pi^{e}b_{k}\pi_{k}}{\nu_{-k}^{i+1}\nu_{-k}^{i}} \left(\pi_{k}\left(\nu_{-k}^{i+1} - e_{k}^{i-1}\nu_{-k}^{i}\right)\right) \\
- \nu_{-k}^{i-1}\left(\nu_{-k}^{i+1} - e_{k}^{i-1}\frac{(\nu_{-k}^{i})^{2}}{\nu_{-k}^{i-1}}\right)\right) \\
\geq \frac{c_{k}^{i-1}\pi^{e}b_{k}\pi_{k}}{\nu_{-k}^{i+1}\nu_{-k}^{i}} \left(\nu_{-k}^{i-2}\left(\nu_{-k}^{i+1} - e_{k}^{i-1}\nu_{-k}^{i}\right)\right) \\
- \nu_{-k}^{i-1}\left(\nu_{-k}^{i+1} - e_{k}^{i-1}\frac{(\nu_{-k}^{i})^{2}}{\nu_{-k}^{i-1}}\right)\right) \qquad (49)$$

$$\tilde{U}_{k}^{i} - \tilde{U}_{k}^{i-1} \geq \frac{c_{k}^{i-1} \pi^{e} b_{k} \pi_{k}}{\nu_{-k}^{i+1} \nu_{-k}^{i}} \left(\nu_{-k}^{i-2} \left(\nu_{-k}^{i+1} - e_{k}^{i-1} \nu_{-k}^{i} \right) \right)
- \nu_{-k}^{i-1} \left(\nu_{-k}^{i+1} - e_{k}^{i-1} \frac{\left(\nu_{-k}^{i} \right)^{2}}{\nu_{-k}^{i-1}} \right) \right)
= 0 \Rightarrow \tilde{U}_{k}^{i} \geq \tilde{U}_{k}^{i-1} \Rightarrow U_{k}^{i} \geq U_{k}^{i-1} = U_{k}^{j}$$
(50)

This establishes the first point. The other points follow from here. Hence proved.

PLACE РНОТО HERE

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