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Publication Date
2005-05-11

SIMULATIONS OF RESISTIVE-WALL INSTABILITY IN THE ILC DAMPING RINGS*

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Abstract

Options being considered for the damping rings for the International Linear Collider (ILC) include lattices with circumferences of around 16 km. The circumference, beam current and beam energy place the damping rings in a regime where resistive-wall instability is a concern, particularly as there are very demanding tolerances on the bunch-to-bunch jitter. Generally, it is possible to make good analytical estimates of the coupled-bunch growth rates in a storage ring, but particular features of the damping rings make it desirable to study the coupled-bunch instabilities using simulations. We present the results of simulations (including a bunch-by-bunch feedback system) of the transverse instabilities using a detailed lattice model.

ILC DAMPING RINGS

Damping rings are needed in the International Linear Collider (ILC) to produce ultra-low emittance and highly stable beams for acceleration in the main linac. Beam stability is critical to the luminosity and there are demanding specifications on both the longitudinal and the transverse bunch-to-bunch jitter for the beam extracted from the damping rings: in the vertical plane, the specification on the maximum bunch-to-bunch jitter is 0.01 pm. Beam jitter can come from a wide variety of sources, including: magnet vibrations; magnet power-supply ripple; pulse-to-pulse amplitude variation in the extraction kickers; phase noise on the RF system; collective effects such as instabilities driven by resistive-wall wakefields and higher-order modes in the RF cavities and vacuum chamber. Each of these sources must be carefully considered.

One option presently being considered for the ILC damping rings specifies a 16 km lattice storing 2820 bunches with charge 2×10^{10} particles, and roughly 20 ns bunch separation [1]. With these parameters, coupled-bunch instabilities driven by resistive-wall wakefields are a potential concern. In the ILC damping rings, bunch-by-bunch feedback systems will be needed to suppress both longitudinal and transverse instabilities; the jitter that the feedback systems can themselves cause (for example, because of limited resolution on the pickup, or because of amplifier noise) could be a problem, particularly in the vertical plane. It is therefore necessary to have a good understanding of the sources of the beam jitter, and the necessary performance of the feedback system.

In many cases it is possible to make reliable estimates of the growth rates expected from the resistive-wall impedance using analytical formulae. However, such estimates rely on a number of simplifying assumptions that are not necessarily valid in the case of the ILC damping rings. In particular: the fill pattern will contain frequent gaps for clearing ions or electron cloud; the internal diameter of the vacuum chamber varies widely around the ring, from 20 mm in the damping wigglers, to 80 mm in the long straight sections; the beam is strongly coupled in the long straights, to reduce space-charge effects; the average beta functions vary widely between different parts of the lattice. We have therefore developed a simulation code to study coupled bunch instabilities taking full account of all these effects, and including also the effects of synchrotron radiation damping and the feedback system. In the following sections, we compare some analytical estimates with simulation results. Since the most demanding stability specification is on the vertical beam jitter, we concentrate on instabilities in the vertical plane: attention also needs to be given to the horizontal and longitudinal degrees of freedom.

ANALYTICAL ESTIMATES

In the presence of a transverse impedance $Z_1(\omega)$, the growth rate of the $\mu-$th coupled-bunch mode is given by:

$$\frac{1}{\tau^{(\mu)}} = -\frac{ec\langle I\rangle}{4\pi\nu E_0}\,\mathrm{Re}\sum_{p=-\infty}^{+\infty} Z_1(\omega_p^{(\mu)}) \quad (1)$$

where e is the electronic charge, c is the speed of light, $\langle I\rangle$ is the average current, ν is the betatron tune, E_0 is the beam energy. The impedance is evaluated at the frequencies:

$$\omega_p^{(\mu)} = (\nu + pn_b + \mu)\,\omega_0 \quad (2)$$

where n_b is the number of bunches in the ring, and ω_0 is the revolution frequency. The resistive-wall impedance is given by:

$$Z_1(\omega) = (1 - i\mathrm{sgn}(\omega))\,\frac{Z_0 C}{2\pi b^3}\delta_{\text{skin}} \quad (3)$$

where Z_0 is the impedance of free space, C is the circumference of the ring, b is the vacuum chamber radius, and the skin depth δ_{skin} is given for a vacuum-chamber conductivity σ by:

$$\delta_{\text{skin}} = \sqrt{\frac{2c}{Z_0\sigma|\omega|}} \quad (4)$$

Using average values for the current, beta functions and the vacuum chamber radius, we can estimate the growth rates in the ILC damping rings, and find that the shortest growth time in the vertical plane is 4.5 ms, or 85 turns.

* Work supported by the Director, Office of Science, High Energy Physics, U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

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COUPLED-BUNCH SIMULATION CODE

We have written a simulation code in C++ for modeling (in the time domain) the effects of long-range wakefields. The lattice is represented as a number of equally-spaced slices, and the bunches are distributed between them. A bunch is modeled as a single particle, so only coherent dipole motion is represented, and decoherence effects are not included. Tracking involves moving each bunch from one slice to the next, with the change in co-ordinates determined by the lattice functions and the wakefields at each slice. A record is maintained of the bunches moving through each slice, so the correct wakefield kick can be calculated. Each bunch can have a different charge and energy from other bunches in the ring. The processes performed by the code are: symplectic tracking; radiation damping; wakefield effects; effects of a bunch-by-bunch feedback system.

Symplectic Tracking

The betatron co-ordinates of each bunch are stored in action-angle form $(J_x, \phi_x, J_y, \phi_y)$. Symplectic tracking simply involves advancing the angle co-ordinates by the phase advance, while keeping the action co-ordinates constant.

Radiation Damping

Radiation damping is assumed to reduce the values of the action co-ordinates of a bunch, while not affecting the angle co-ordinates. If τ_{SR} is the radiation damping time, then in each time step Δt the action is reduced by:

$$J(t+\Delta t) = \exp(-2\frac{\Delta t}{\tau_{\text{SR}}})J(t) \tag{5}$$

Wakefield Effects

The wakefield kick on a bunch moving distance Δs from one lattice slice to the next is given by:

$$\Delta p_y = \frac{r_e \Delta s}{\gamma} \sum_{n=1}^{\infty} [Ny]_n \, W_1(n\Delta s) \tag{6}$$

where $W_1(z)$ is the wake function, γ is the relativistic factor of the bunch being kicked, and $[Ny]_n$ is the product of the charge and the transverse offset of the bunch that passed through the slice n time-steps previously. Each slice in the lattice has its own wake function. The resistive-wall wake function is:

$$W_1(z) = \frac{2}{\pi b^3}\sqrt{\frac{2}{Z_0 \sigma |z|}} \tag{7}$$

In practice, evaluation of the summation in Eq. (6) must be terminated at a finite upper limit. In the code, this is achieved by discarding the contribution of any bunch, once its contribution to the wakefield kick is less than some specified fraction (usually we use 10%) of the largest contribution of any bunch.

Evaluation of the wakefield kick requires conversion between Cartesian and action-angle co-ordinate systems. This can be done using a matrix $\mathbf{N}$, which relates the phase-space vectors in the two systems, $\mathbf{X}$ and $\mathbf{J}$ respectively:

$$\mathbf{X} = \mathbf{N} \cdot \mathbf{J} \tag{8}$$

$\mathbf{N}$ is the normalizing transformation of the single-turn matrix $\mathbf{M}$ at the chosen point in the lattice, and is constructed from the eigenvectors of $\mathbf{M}$. In an uncoupled lattice, $\mathbf{N}$ can also be expressed in terms of the Twiss parameters, but the eigenvector construction is more general.

Feedback System

Feedback systems are implemented by associating a pick-up with one lattice slice, and a kicker with a second lattice slice. As bunches pass the pick-up, their offsets are stored in a buffer; as they later pass the kicker, the stored offset is used to calculate a kick applied to the bunch. Any number of feedback systems can be included in the simulation, each with its own algorithm for determining the kick from the offset at the pick-up. A simple feedback system applies the kick determined by:

$$\Delta p_y|_{\text{kicker}} = g(y|_{\text{pick-up}} + \delta_y) \tag{9}$$

where g is the gain of the feedback system, and δ_y is a stochastic variable representing the limited resolution of the pick-up. Assuming a betatron phase advance $\pi/2$ between the pick-up and the kicker, the damping rate provided by the feedback system is given by:

$$\frac{1}{\tau_{\text{FB}}} = \frac{g\sqrt{\beta_1\beta_2}}{2T_0} \tag{10}$$

where β_1 and β_2 are the values of the beta function at the pick-up and kicker respectively, and T_0 is the revolution time. The presence of pick-up noise δ_y leads to the equation of motion for the action of a single bunch:

$$\frac{dJ}{dt} = \frac{\beta_2}{2T_0} g^2 \langle \delta_y^2 \rangle - \frac{2}{\tau} J \tag{11}$$

where τ is the damping time from the combined effects of synchrotron radiation and the feedback system. Clearly, the equilibrium bunch-to-bunch jitter is given by:

$$\langle J \rangle = \frac{\tau}{4T_0} \beta_2 g^2 \langle \delta_y^2 \rangle \tag{12}$$

SIMULATION RESULTS

The results of a simulation in the ILC damping rings without radiation or a feedback system, are shown in Fig. 1. The growth rates are estimated from the simulation results by fitting an exponential to the amplitude of each Fourier mode in the beam as a function of turn number. In this case, we tracked the nominal fill pattern for 30 turns, starting with a small amount of random beam-to-beam jitter.

Note that the normal modes in the presence of resistive-wall wakefields are *not* Fourier modes; the large scatter in the simulation results relative to the analytical estimate is principally a consequence of analyzing the beam motion in terms of Fourier modes, rather than the real normal modes of the system.

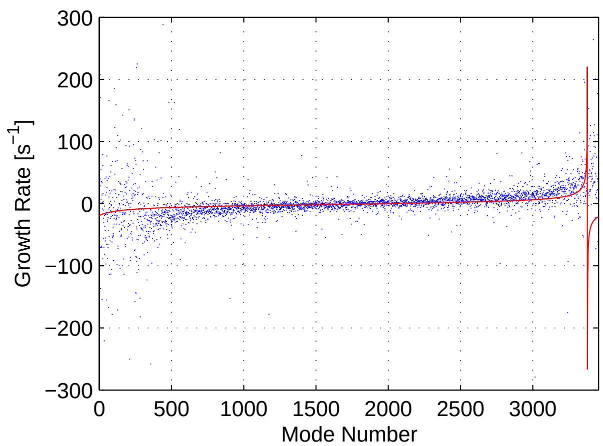

Figure 1: Growth rates of coupled-bunch modes driven by resistive-wall wakefields in the ILC damping rings. The points show simulation results; the line shows an analytical estimate.

Fig. 2 shows the results of a simulation including a feedback system with a gain adjusted to produce a damping rate of 300 s^{-1}, and with synchrotron radiation damping that provides an additional damping of 37 s^{-1}. The feedback system and radiation damping have the expected effect, and all modes are now damped.

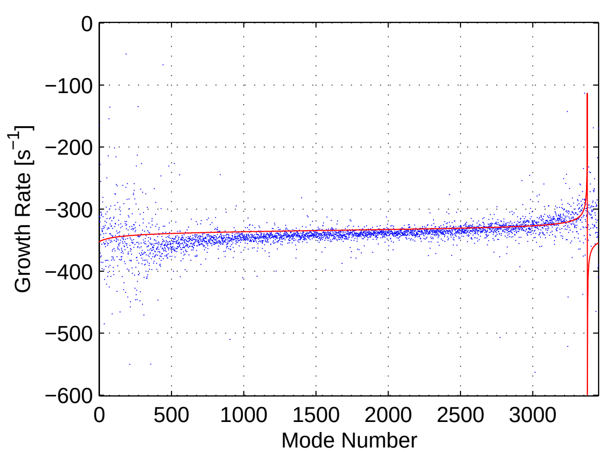

Figure 2: As Fig. 1, but with feedback system and radiation damping.

The specification on the beam stability is that the bunch-to-bunch jitter should be less than 10% of the beam size; in the vertical plane, the specified vertical emittance is 2 pm, so the bunch-to-bunch action jitter must be less than 0.01 pm. The feedback system is set for a gain of $g = 0.00315$ and the beta function at the kicker is $\beta_2 = 8.8$ m. The revolution period is $T_0 = 53.2\ \mu$s, and the total damping rate (from synchrotron radiation and the feedback system) is $1/\tau \approx 330\ s^{-1}$; so from Eq. (12), the pick-up needs a resolution of better than 3 μm to avoid inducing jitter on the beam outside the stability specification.

To test the effects of noise from feedback system, we tracked a beam starting with zero betatron action for each bunch. The parameters used in the simulation were as given above, with gaussian noise (rms 3 μm) added to the pick-up signal. The average action over all the bunches in the beam as a function of the turn number in the simulation is shown in Fig. 3. The simulation results agree well with the prediction from Eqn. (11).

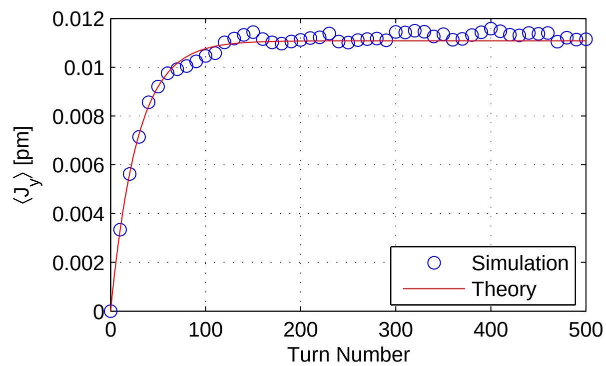

Figure 3: Growth of the betatron action averaged over all bunches in the beam, over 500 turns.

CONCLUSIONS

Analytical estimates of coupled-bunch growth rates in one possible design for the ILC damping rings have been confirmed using time-domain simulations. The simulations include variations in fill pattern, chamber radius and beta functions, and also include synchrotron radiation and bunch-by-bunch feedback systems. The simulation code is ready to handle other sources of long-range wakefields (for example, higher-order modes in the RF cavities) and betatron coupling; these effects will be included in future studies.

Studies so far have indicated the parameter regime in which the transverse bunch-by-bunch feedback system will need to operate. Growth times of the resistive-wall instability are likely to be less than 100 turns. Jitter induced on the beam from the feedback system is a concern, but can be kept within specification if the feedback system pick-up has a resolution better than a few microns.

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