Title

OPTIMIZING SUBROUTINE

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PROGRAM — PROJECT — JOB HIGH-FIELD MAGNET DEVELOPMENT				
ANALYSIS				
TITLE OPTIMIZING SUBROUTINE				

Traditionally, we have designed the cross-sections of multipole magnets to make certain higher-order field aberations exactly zero. For example, for a dipole with three free variables - block angles, say - we can make C_3(sextupole), C_5(decapole), and C_7(14-pole) exactly zero.

This does not necessarily result in a magnet producing the best field. (What we mean by "best field" is not often well defined, which makes getting it, or knowing we've got it when we have, a bit difficult. Nevertheless...). We could, in the above example, reduce C_9 and C_{11}, in principle, by letting C_3, C_5, and C_7 be slightly non-zero, and we might end up with a better magnet.

One systematic way of doing this is to minimize the sum of the weighted squares of a specified number of higher order multipoles. A scheme for doing that is developed in this report. A basic subroutine (Mincoef) has been built into several minicomputer programs. It is more complex, and therefore slower, than the simple Newton's method previously used, but even on the disgustingly slow HP9845, convergence takes perhaps 20 sec. for even a rather complex design.

(Just before Christmas of '80 it came to my attention that Palmer at BNL was doing this sort of thing, using an optimizing program called MINUITL, and so I gave it a whack. I later found that MINUITL is an obsolete, but still available, LBL program, and it was added to an LBL magnet design program. MINUITL is a rather complex affair - the write-up is a quarter inch thick. So far, experience with it has not been very successful.)

Scheme For Making the Sum of the Weighted Squares of Errors a Minimum

We let y_i, $i = 1$ to p represent the set of dependendent variables to be optimized — the certain higher-order field aberration coefficients in the case at hand.

We let x_j, $j = 1$ to q represent the set of independent variables which we adjust to optimize the y's — certain mechanical-dimension parameters, if the currents are fixed, or the currents if the dimensions are fixed.

For the scheme to work, $p \geq q$.

Then

$$y_i = y_i(x_1, x_2, \ldots, x_j, \ldots, x_q) \qquad (1)$$

and

$$dy_i = \frac{\partial y_i}{\partial x_1} dx_1 + \frac{\partial y_i}{\partial x_2} dx_2 + \ldots \qquad (2)$$

We represent $\partial y_i / \partial x_j$ by a_{ij}

and so for a linear relationship

$$y_i = \sum_{j=1}^{q} a_{ij} x_j \qquad (3a)$$

$$\Delta y_i = \sum_{j=1}^{q} a_{ij} \Delta x_j \qquad (3b)$$

Let $y'_i = y_i + \Delta y_i$

The error in y_i, which we call ϵ_i, is

$$\epsilon_i = y'_i - Y_i = y_i + \Delta y_i - Y_i \qquad (4)$$

where Y_i is the set of desired values of y_i; usually, but not necessarily, zero.

We wish to minimize S, where

$$S \equiv \sum_{i=1}^{p} K_i \epsilon_i^2 \qquad (5)$$

where K_i is a set of weighting factors,

by finding a new set of x_i values, namely

x_i', where

$$x_i' = x_i + \Delta x_i$$

To minimize S, we let $\partial S / \partial x_j = 0$, $j = 1$ to q. Partial differentiation of the equation for S, then, gives

$$\sum_{i=1}^{p} K_i \epsilon_i \frac{\partial \epsilon_i}{\partial x_j} = 0 \,, \quad j = 1 \text{ to } q \qquad (6)$$

For illustration, we expand (6) for the case

$$p = 3, \; q = 2$$

$$K_1\epsilon_1 a_{11} + K_2\epsilon_2 a_{21} + K_3\epsilon_3 a_{31} = 0$$
$$K_1\epsilon_1 a_{12} + K_2\epsilon_2 a_{22} + K_3\epsilon_3 a_{32} = 0 \qquad (8)$$

or

$$K_1(y_1 - Y_1 + a_{11}\Delta x_1 + a_{12}\Delta x_2)\,a_{11} + K_2(y_2 - Y_2 + a_{21}\Delta x_1 + a_{22}\Delta x_2)\,a_{21} + K_3(y_3 - Y_3 + a_{31}\Delta x_1 + a_{32}\Delta x_2)\,a_{31} = 0$$
$$K_1(\quad \downarrow \quad)\,a_{12} + K_2(\quad \downarrow \quad)\,a_{22} + K_3(\quad \downarrow \quad)\,a_{32} = 0$$

We collect coefficients of Δx_j, put everything else on the right, and express in matrix form:

$$\begin{pmatrix} K_1 a_{11} a_{11} + K_2 a_{21} a_{21} + K_3 a_{31} a_{31} & K_1 a_{12} a_{11} + K_2 a_{22} a_{21} + K_3 a_{32} a_{31} \\ K_1 a_{11} a_{12} + K_2 a_{21} a_{22} + K_3 a_{31} a_{32} & K_1 a_{12} a_{12} + K_2 a_{22} a_{22} + K_3 a_{32} a_{32} \end{pmatrix} \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \end{pmatrix}$$

$$= \begin{pmatrix} K_1 (Y_1 - y_1) a_{11} + K_2 (Y_2 - y_2) a_{21} + K_3 (Y_3 - y_3) a_{31} \\ K_1 (Y_1 - y_1) a_{12} + K_2 (Y_2 - y_2) a_{22} + K_3 (Y_3 - y_3) a_{32} \end{pmatrix} \quad (8')$$

We represent this by

$$B \, \Delta X = C$$

The terms of the B and C matrixes are evidently

$$b_{jk} = \sum_{i=1}^{p} K_i a_{ik} a_{ij} \,, \quad c_j = \sum_{i=1}^{p} K_i (Y_i - y_i) a_{ij} \,, \quad i = 1 \text{ to } p,\ j = 1 \text{ to } q,\ k = 1 \text{ to } q$$

We invert the B matrix, calculate $\Delta X = B^{-1} C$, and get new values of x,

$$x_j' = x_j + \Delta x_j \,, \quad j = 1 \text{ to } q$$

If (1) is really linear, then S will be the desired minimum. If (1) is not linear we will obtain the first approximation.

Scheme For Making Higher-Order Multipoles Exactly Zero

The development through eq. 5 is as before, except that $p = q$

We set the errors ϵ_i to zero, so

$$\Delta y_i = Y_i - y_i$$

For $q = 3$, for example, equations 3b become, in matrix form

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{pmatrix} = \begin{pmatrix} Y_1 - y_1 \\ Y_2 - y_2 \\ Y_3 - y_3 \end{pmatrix} \qquad (9)$$

or $\quad A \, \Delta X = D$

We invert the A matrix, calculate $\Delta X = A^{-1}D$, and get new values of x, $x_j' = x_j + \Delta x_j$

It can be shown that equation 8' of the previous discussion is equivalent to the above for the case $p = q$.

Equation 9 can be derived in a more direct fashion, as follows

For $\quad y_i = f(x_1, x_2, \ldots, x_j, \ldots, x_q) \qquad i = 1 \text{ to } q$

$$dy_i = \sum_{j=1}^{q} \frac{\partial y_j}{\partial x_j} dx_j$$

It is evident that $\partial y_j / \partial x_j \equiv a_{ij}$. We replace dy_i by Δy_i, dx_j by Δx_j, and make the error zero by setting $\Delta y_i = Y_i - y_i$, which yields Eq. 9.

```
3740  SUB Mincoef(A(*),Wf(*),Y(*),Bigy(*),P,Q,Delx(*),B(*),C(*),D(*),Flag)
3750  MAT B=ZER
3760  MAT C=ZER
3770  FOR I=1 TO P
3780  FOR J=1 TO Q
3790  C(J)=C(J)+Wf(I)*A(I,J)*(Bigy(I)-Y(I))
3800  FOR K=1 TO Q
3810  B(J,K)=B(J,K)+Wf(I)*A(I,K)*A(I,J)
3820  NEXT K
3830  NEXT J
3840  NEXT I
3850  IF Q=1 THEN 3880
3860  CALL Simeq(B(*),C(*),D(*),Delx(*),Q,Flag)
3870  SUBEXIT
3880  Delx(1)=C(1)/B(1,1)
3890  SUBEND

3920 SUB Simeq(A(*),C(*),D(*),X(*),Order,Flag)
3930  OPTION BASE 1
3940 REDIM A(Order,Order),C(Order),D(Order,Order),X(Order)
3950 Flag=0
3960 IF DET(A)=0 THEN Flag=1
3970 IF DET(A)=0 THEN SUBEXIT
3980 MAT D=INV(A)
3990 MAT X=D*C
4000 SUBEND
```

Variables in argument list

In program	In analysis
A(I, J)	$a_{i,j}$
Wf(I)	K_i
Y(I)	y_i
Bigy(I)	Y_i
P	p
Q	q
Delx(J)	Δx_j
B (J, K)	b_{jk}
C(J)	c_j
D(J , K)	Used in simultaneous equation sub.
Flag	= Ø normally; = 1 if determinant of A is Ø.

I = 1 TO P
J = 1 TO Q
K = 1 TO Q

No variables are in COMMON, none are dimentioned in subroutines.

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