Title

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Authors

Brennan, Michael J
LIU, XIAOQUAN
Xia, Yihong

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Michael J. Brennan*
Xiaoquan Liu†
Yihong Xia‡

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*Finance Department, the Anderson School, UCLA, 110 Westwood Plaza, Los Angeles, CA 90095-1481.

†Department of Accounting, Finance, and Management, University of Essex, Wivenhoe Park, Colchester, C04 38Q, UK.

‡Finance Department, The Wharton School, University of Pennsylvania, 2300 Steinberg Hall-Dietrich Hall, Philadelphia, PA 19104-6367. Phone: (215) 898-3004. Fax: (215) 898-6200. E-mail: yxia@wharton.upenn.edu.

Abstract

We estimate the parameters of pricing kernels that depend on both aggregate wealth and state variables that describe the investment opportunity set, using FTSE 100 and S&P 500 index option returns as the returns to be priced. The coefficients of the state variables are highly significant and remarkably consistent across specifications of the pricing kernel, and across the two markets. The results provide further strong evidence, which is consistent with Merton's (1973a) Intertemporal Capital Asset Pricing Model, that state variables in addition to market risk are priced.

1 Introduction

Understanding investor attitudes to risk and how these affect the pricing of assets has been a central theme of research in finance for more than forty years. The seminal work of Sharpe (1964), Lintner (1965), Mossin (1966) and, later, Breeden (1979) expresses risk premia on assets in terms of the covariances of the asset returns with aggregate payoffs, such as the return on the market portfolio (CAPM) or the growth rate of aggregate consumption (CCAPM), while Merton's (1973) intertemporal capital asset pricing model (ICAPM) expresses risk premia in terms of covariances, not only with the market portfolio, but also with state variables that describe the investment opportunity set. Most empirical work on the fundamental determinants of asset prices has concentrated on the equity and bond markets, and there is now extensive evidence that neither the simple CAPM nor extant implementations of the CCAPM are adequate for pricing equity portfolios that are constructed on criteria such as size and book-to-market ratio or industry. Brennan, Wang and Xia (2004) (hereafter BWX), Campbell and Vuolteenaho (2004), Vassalou (2003), and Lustig and Nieuwerburgh (2005), all find that covariances with non-wealth-related state variables improve the ability of asset pricing models to account for the relative returns on portfolios of common stocks.

In recent years, as the empirical limitations of the simple Black-Scholes (1973) model have become apparent, researchers have begun to pay more attention to the determinants of option prices and returns. Since asset pricing kernels succinctly summarize investors' time and risk preferences, several studies have focused on estimating the pricing kernels for options. For example, Ait-Sahalia and Lo (2000), Jackwerth (2000), Liu *et. al.* (2004), and Rosenberg and Engle (2002) estimate the pricing kernel by comparing index option prices with the distribution of returns on the underlying market indices. These studies rely on a projection of the pricing kernel onto equity return states so that the estimated pricing kernel is a function only of the equity return.[1] Other studies have estimated a pricing kernel for (index) options that includes additional variables, the most popular variables being measures of volatility and jump risks: Buraschi and Jackwerth (1999), Chernov *et. al.* (2003), Coval and Shumway (2001), Driessen and Maenhout (2003), and Pan (2002) have all found that volatility is a priced risk. For example, Coval and Shumway (2001) conclude that "something besides market risk is important for pricing the risks associated with option contracts," and suggest that stochastic volatility may be an important factor for asset pricing.

[1]The estimates of these univariate pricing kernel functions typically vary over time reflecting time variation in the underlying pricing kernel state variables.

Despite the increasing evidence that state variables other than aggregate wealth are important for pricing both equity and option returns, and despite the fact that these securities are traded in integrated capital markets, the relevance for options of pricing kernels that have been found useful in pricing equity returns has yet to be examined.[2] The objective of the current study is to fill this gap by studying the behavior of option returns in light of a simple pricing kernel which is motivated by the ICAPM and which has proven useful in pricing US bonds and stocks.

Because there is considerable debate among researchers over the state variables that enter into the pricing kernel, we start with the simple ICAPM pricing kernel that is developed by BWX to price bonds and stocks. In this parsimonious setting, time variation in the instantaneous investment opportunity set is fully described by the dynamics of the real interest rate, r, and the maximum Sharpe ratio, η, and their current values are sufficient statistics for all future investment opportunities. As a result, these are the only state variables that are priced. BWX show that this simple ICAPM outperforms the Fama-French (1995) three-factor model in pricing size and book-to-market sorted portfolios of common stocks and that both r and η risks are associated with significant risk premia. Therefore, the real interest rate and the maximum Sharpe ratio are natural variables to include in the pricing kernel, in addition to aggregate wealth.

We also include a measure of implied volatility, σ, as a state variable in the pricing kernel. This is motivated by empirical findings from option returns that volatility is a priced risk, as well as by recent theoretical models. For example, Mayfield (2004) has constructed a general equilibrium regime switching model in which volatility is priced because it is associated (negatively) with the favorableness of investment opportunities as measured by the Sharpe ratio, Brandt and Kang (2004) present evidence that stock market volatility is negatively associated with the favorableness of short run investment opportunities as measured by the (equity-market) Sharpe ratio, and Tauchen (2005) generates a two-factor structure for volatility along with time-varying risk premiums on consumption and volatility risk.

Most prior studies that extract a pricing kernel from the observed prices of index options rely on a parameterization of the stochastic process for the underlying index. For example, Rosenberg and Engel (2002) assume that the index follows a GARCH process, and both Pan (2002) and Santa-Clara and Yan (2004) assume that it follows a mixed jump-diffusion process. While a parametric assumption of the underlying asset dynamics lends power to the estimation, it also

[2] Vanden (2004) constructs an asset pricing model with wealth constraints in which option returns enter the pricing kernel for stocks.

raises the possibility of mis-specification. In this paper, we estimate the pricing kernel by applying the Generalized Method of Moment (GMM) to option returns, which avoids the need to rely on any specific option pricing model or specific assumptions about the stochastic processes for the underlying asset returns. Both Buraschi and Jackwerth (1999) and Coval and Shumway (2001) have previously applied GMM to option returns.

We estimate separate pricing kernels for the UK and US.[3] The empirical proxies that we use for the arguments of the pricing kernel are as follows. The return on aggregate wealth is taken as the return on a broad index of market returns: FTSE 250 for the UK and the CRSP value weighted index for the US. We estimate the time series of r and η, together with the expected rate of inflation, π, from panel data on zero-coupon government bond yields within an essentially affine pricing model framework. The estimate of our final state variable is a measure of return volatility implied by option prices. Our primary options data are monthly returns on 11 put and call option portfolios on the FTSE 100 index for the period April 1992 to March 2002, and on 10 put and call portfolios on the S&P 500 index for the period January 1992 to April 2002. The portfolios consist of options that have between 1 and 2 months to maturity and between 2 and 3 months to maturity at the time of portfolio formation and are formed on the basis of option type, time to maturity, and moneyness.

When the pricing kernel is constrained to be a power function of the return on aggregate wealth, the estimated coefficient of relative risk aversion for the UK (FTSE 100 options) is found to be *negative*. However, the estimated coefficient of relative risk aversion becomes positive (but not statistically significant), when the pricing kernel specification is augmented by the state variables r, η, and σ. For the US (S&P 500 options), the estimated coefficient of relative risk aversion is 2.7 when no state variables are included, and 4.8 when all the state variables are included. Only the coefficient of the innovation in the real interest rate is significant in the UK, while the coefficients of all three state variables are significant in the US. The signs of the coefficients on the state variables are the same for the US and UK.

It is possible that the significance of the state variables is induced by the restriction on the functional form of the dependency of the kernel on the return on aggregate wealth. Therefore, the kernel is next written as the product of a polynomial in the return on aggregate wealth and an exponential affine function of the state variables. When r and η are the only state variables,

[3] Brennan and Xia (2005) discuss the theoretical relation between the pricing kernels for different currencies and present empirical evidence for several currencies.

the coefficients of both variables are significant for both US and UK;[4] moreover, the signs of the coefficients are the same for the two countries, and are consistent with the estimates of the risk premia for these variables obtained by BWX (2005, Table II) using returns on size and book-to-market sorted portfolios of US common stocks. They are also consistent with the correlations between these variables and the pricing kernels that are estimated using the government bond yield data.

When an estimate of the implied volatility, σ, is added to the pricing kernel specification, its coefficient is significant and positive for both countries: the sign of the coefficient is consistent with prior studies by Coval and Shumway (2001) and Bakshi and Kapadia (2003) that have reported a negative volatility risk premium. Introducing the implied volatility makes the coefficient of η insignificant for the UK but makes no material difference for the US. As a robustness check, the expected rate of inflation is introduced into the specification. This variable is insignificant for the US and has no material effect on the other coefficients. However, the variable enters strongly and significantly for the UK, and including π makes the coefficients of r, η, and σ significant. While the signs of the coefficients of r and σ continue to be the same for the UK and US, the coefficient of η is now positive for the UK which is inconsistent with both the US results and the sign of the correlation between η and the pricing kernel estimated using the bond yield data.

While the coefficients of the state variables in the pricing kernel are not restricted by theory, for the US we find striking consistency between: first, the signs of the coefficients of the state variables in the pricing kernel estimated using the option returns in this paper; second, the signs of the correlations of the state variables with the pricing kernel estimated using the bond yield data; and third, the risk premia for the state variables estimated using equity returns by BWX. The results for the UK are less clear cut. When expected inflation π is excluded in the pricing kernel, there is complete consistency between: first, the signs of the coefficients estimated from the option returns; second, the signs of the correlations of the state variables with the pricing kernel estimated from bond yields; and third, the signs of the corresponding US estimates.[5] However, the sign of η changes when π is included in the pricing kernel.

The over-identifying restrictions of the estimation are easily rejected, but the general consistency in the estimated coefficients of the pricing kernel, not only across markets and countries, but also

[4]The finding of a significant risk premium associated with r contrasts with Pan (2002) who explicitly assumes that interest rate risk is unpriced.

[5]This form of pricing kernel has not been estimated using UK equity returns.

across different model specifications, attests to the importance of including these state variables in the pricing kernel for risky securities.

The paper is organized as follows. Section 2 discusses option pricing in terms of the pricing kernel. Section 3 describes the data and Section 4 discusses the estimation of the real interest rate, the maximum Sharpe ratio, and the expected rate of inflation state variables from data on bond yields. The main empirical results are presented in Section 5 and Section 6 concludes.

2 Option Prices and the Pricing Kernel

It is well known that the hypothesis of no arbitrage is equivalent to the existence of a pricing kernel, M, such that for any security return, $\tilde{R}_{t+1}$:[6]

$$E\left[\frac{M_{t+1}}{M_t}(1+\tilde{R}_{t+1)})|I_t\right] = 1 \tag{1}$$

where $\tilde{R}_{t+1}$ is the rate of return from t to $t+1$, and I_t denotes the information available at time t. In general, the pricing kernel is a function of a vector of state variables, X.

When pricing contingent claims on an underlying asset S, it is often convenient to project the pricing kernel M onto the space of the underlying asset returns. Cochrane (2001) shows that the projected pricing kernel, M^*, has the same pricing implications for payoffs that depend on S_{t+1} as does the original pricing kernel M. In a Black-Scholes economy in which the (continuously compounded) interest rate r is a constant, and the price of the underlying asset S follows a geometric Brownian Motion:

$$\frac{dS}{S} = \mu dt + \sigma_S dz_S \tag{2}$$

the (projected) pricing kernel for all contingent claims on S can be written as the product of a power function of S and a function of time t as shown in Bick (1987):

$$M^*_{t+1} = S_{t+1}^{-\gamma} \exp^{-r(t+1)}. \tag{3}$$

Rubinstein (1976) and Brennan (1979) show that a pricing kernel of the form (3) yields Black-Scholes pricing in a discrete time setting if security returns are lognormal. If the Black-Scholes assumptions are not satisfied, a projection of the true pricing kernel onto the asset return space is still feasible, but the projection may take a general and unknown functional form.

[6]Campbell et al. (1997) and Cochrane (2001) discuss the role of the pricing kernel in asset pricing.

In studying the prices of options on a market index, it is common to estimate only the projection of the pricing kernel onto the space of the underlying index returns. For example, Ait-Sahalia and Lo (2000), Jackwerth (2000), and Rosenberg and Engle (2002) use equity index option prices to estimate the projections of pricing kernels onto S&P 500 returns. Since the pricing kernel projection is a univariate function of the index returns, it does not allow for an explicit examination of other state variables that may enter the pricing kernel.

In this paper, we estimate a pricing kernel M which is motivated by the Intertemporal Capital Asset Pricing Model of Merton (1973a) and contains variables that capture the time variation in investment opportunities. BWX and Nielsen and Vassalou (2002) have argued in a diffusion setting that, if the interest rate, r, and the maximal Sharpe ratio, η, follow a joint Markov process, then they are sufficient statistics for investment opportunities and are therefore, along with the wealth of the representative agent, natural candidates as arguments of the pricing kernel in an ICAPM setting. BWX report that both state variables, r and η, command significant risk premia and play important roles in the pricing of cross-sectional size and book-to-market sorted equity portfolio returns. Note that, if the state variables, $X \equiv (r, \eta)$, are not independent of the aggregate wealth, there is no reason to expect the projection of the pricing kernel onto the aggregate wealth to retain the power function form even if the representative agent's utility of consumption is iso-elastic.

Motivated by the BWX model, we shall first assume that the pricing kernel can be written as the product of, a function of time, $f(t)$, a power function of aggregate wealth, W, and an exponential affine function of the proposed state variables, r and η:

$$M_{t+1}(W_{t+1}, r_{t+1}, \eta_{t+1}) \equiv f(t+1) W_{t+1}^{-\gamma} \exp^{c + c_2 r_{t+1} + c_3 \eta_{t+1}} . \tag{4}$$

The exponential function ensures that the pricing kernel can never be negative. We shall later consider additional potential state variables within this exponential affine framework, and consider alternatives to the iso-elastic function of wealth.

Defining the stochastic discount factor, m_{t+1}, by $m_{t+1} \equiv \frac{M_{t+1}}{M_t}$, equation (4) implies that m_{t+1} may be written in the form:

$$m_{t+1} = c_0 (1 + R_{W,t+1})^{-\gamma} \exp^{c_2 \Delta r_{t+1} + c_3 \Delta \eta_{t+1}}, \tag{5}$$

where c_j $(j = 0, 2, 3)$ are constants to be estimated together with the risk aversion parameter γ, $1 + R_{W,t+1} \equiv \frac{W_{t+1}}{W_t}$ is the gross return on the aggregate wealth, Δ is the first difference operator so that $\Delta r_{t+1} \equiv r_{t+1} - r_t$ and $\Delta\eta_{t+1} \equiv \eta_{t+1} - \eta_t$. The definition of m_{t+1} together with equation (1) implies that, for any security i, the following moment condition holds:

$$E\left[c_0(1+R_{W,t+1})^{-\gamma}\exp^{c_2\Delta r_{t+1}+c_3\Delta\eta_{t+1}}(1+\tilde{R}_{i,t+1}))|I_t\right] = 1. \tag{6}$$

If z_t is an instrumental variable in the current information set I_t, then the above Euler equation implies the unconditional moment condition:

$$E\left[c_0(1+R_{W,t+1})^{-\gamma}\exp^{c_2\Delta r_{t+1}+c_3\Delta\eta_{t+1}}(1+\tilde{R}_{i,t+1}))z_t\right] = E\left(z_t\right), \quad \forall z_t \in I_t. \tag{7}$$

We shall estimate the parameters of the pricing kernel, c_0, γ, c_2, and c_3, via GMM using moment conditions (6)-(7) where the set of asset returns, R_i, consists of returns on index option portfolios, the return on aggregate wealth, and Treasury Bill rates.

The BWX model assumes that both the real interest rate and the Sharpe ratio follow univariate Ornstein-Uhlenbeck processes. This is obviously a strong assumption, and Brandt and Kang (2004) present evidence which suggests that the Sharpe ratio (of the equity market) follows a *two-factor* process with the second factor being the market volatility, σ. Under this specification of the dynamics of the Sharpe ratio, the level of market volatility, σ, also becomes a state variable that is required to fully describe the investment opportunity set.[7] Therefore, we shall include σ in the set of state variables, X, which define the pricing kernel. However, for analytical tractability we shall treat σ as constant when we estimate the time series of the other two state variables, r and η, from panel data on Treasury Bond yields.

3 Data

The primary data are the prices of European puts and calls on the FTSE 100 index traded on the London International Financial Futures and Options Exchange (LIFFE) and the prices of European puts and calls on the S&P 500 index traded on the Chicago Board Options Exchange (CBOE). We collect prices on the second (or the closest to the second) trading day of each month from April 1992 to March 2002 for the UK and from January 1992 to April 2002 for the US,[8] on options that have maturities of less than three months. For each option that has a price on the second trading

[7] Note that σ is not necessary to describe the instantaneous myopic investment opportunity set which is fully captured by (r, η), but it may be necessary to capture the dynamics of the myopic investment opportunity set.

[8] We thank Jens Jackwerth for kindly providing us the US option data from 1992 to 1995. The data from 1996 to 2002 are collected from OptionMetrics.

day of the current month and on the second trading day of the following month, a return for the month is calculated from the proportional price change between these dates. Put and call options are assigned to portfolios on the basis of their time to maturity and moneyness. The '1-month' portfolios contain options that have between 1 and 2 months to expiration at the beginning of the month, and the '2-month' portfolios contain options with between 2 and 3 months to expiration at the beginning of the month. For each maturity, 7 portfolios of calls and of puts are formed according to moneyness. Portfolio 1 consists of options for which the ratio of the index level on the portfolio formation date to the strike price of the option is less than 0.85; for portfolio 2 the ratio is between 0.85 and 0.90; for portfolio 3, 0.90 to 0.95, portfolio 4, 0.95 to 1.0, portfolio 5, 1.0 to 1.05, portfolio 6, 1.05 to 1.10, and for portfolio 7 is greater than 1.10. Summary statistics on the portfolios are presented in Table 1.

The deep out-of-the-money call portfolios (low numbered portfolios) have negative average returns; the average return on 1-month calls that are between 5 and 10% out of the money is minus 29% per month for the UK and minus 3% per month for the US, which contrasts with a return of 1.48% per week reported by Coval and Shumway (2001) for similar (American) contracts on the SPX index for the period 1990-1995; the corresponding figures for 2-month calls are minus 15% per month and minus 7% per month. The returns to the deep out of the money call portfolios are highly skewed. The in-the-money call portfolios have slightly positive average returns and virtually no skewness. The same convention was followed in constructing the portfolios of puts so that portfolio 1 contains the deep out of the money options, and portfolio 7 the deep in the money options. As with the calls, the deep out of the money puts have strongly negative returns: the portfolio of 1-month puts that is between 10 and 15% out of the money loses an average of 29% per month for the UK and 64% per month for the US.

Not all of the moneyness buckets contain traded options each month. For example, there are 98 out of 120 months for the UK in which there is no 1-month call option that is more than 10% out of the money. Our main analysis will be conducted using the portfolios for which there are essentially no missing observations. For the UK we use 11 portfolios that have no missing observations, which include two 1-month call portfolios, two 1-month put portfolios, three 2-month call portfolios, and four 2-month put portfolios. For the US we use 10 portfolios that have no more than 2 (out of 123) missing observations, which include three 1-month call portfolios, three 1-month put portfolios, two 2-month call portfolios, and two 2-month put portfolios.

We shall use as our measure of market volatility, σ, an estimate of the implied volatility of option prices. For the UK this is obtained by first calculating the average implied volatility for each of the one month option portfolios and then averaging these values. For the US the implied volatility is taken as the VIX index.[9] The nominal interest rate, R_f, for the UK is the 3-month Treasury Bill rate reported by the Bank of England, and for the US is the 30-day Treasury Bill rates obtained from CRSP. The return on aggregate wealth for the UK is taken as the return on the FTSE 250 index which is a value weighted index of UK equities. Note that this is a broader index than the FTSE 100 on which the options are written. We choose the FTSE 250 as the market index because it is a better proxy for the return on aggregate wealth which is the theoretical variable in the pricing kernel. For the US the return on aggregate wealth is taken as the CRSP value weighted index return.

The data that are used for estimating the state variables r, π, and η for the UK are zero coupon bond yields for the second trading day of each month from January 1985 to May 2002 for maturities of 1, 2, 3, 5, 7, 10 and 15 years reported by the Bank of England. For the US, zero coupon bond yields are estimated from the prices of US Treasury Bonds using a cubic spline.[10]. Inflation rates were calculated from Consumer Price Index data reported by DataStream.

4 State Variables

In a diffusion setting, the process for the stochastic discount factor, m, can be written as:

$$\frac{dm}{m} = -rdt - \eta dz_m \tag{8}$$

where r is the real interest rate, and the diffusion coefficient, η, can be shown to be equal to the maximum Sharpe ratio in the economy, or the slope of the capital market line. Following BWX we assume that r and η follow Ornstein-Uhlenbeck processes:

$$dr = \kappa_r(\overline{r} - r)dt + \sigma_r dz_r, \tag{9}$$

$$d\eta = \kappa_\eta(\overline{\eta} - \eta)dt + \sigma_\eta dz_\eta. \tag{10}$$

[9] VIX is the CBOE volatility index and measures market expectations of volatility over the next 30 days as conveyed by stock index option prices. VIX estimates expected volatility from the prices of S&P 500 index options.

[10] Further details are available in Brennan and Xia (2005)

In order to estimate r and η from the panel data on *nominal* bond yields, we assume that the realized inflation follows

$$\frac{dP}{P} = \pi dt + \sigma_P dz_P, \tag{11}$$

where the expected rate of inflation, π, also follows an Ornstein-Uhlenbeck process:

$$\frac{d\pi}{\pi} = \kappa_\pi(\bar{\pi} - \pi)dt + \sigma_\pi dz_\pi. \tag{12}$$

Under these assumptions, BWX show that the nominal yield on a zero-coupon (default-free) bond of maturity τ is a linear function of the state variables, r, π, and η:

$$-\frac{\ln N}{\tau} = -\frac{\widehat{A}(\tau)}{\tau} + \frac{B(\tau)}{\tau}r + \frac{C(\tau)}{\tau}\pi + \frac{\widehat{D}(\tau)}{\tau}\eta, \tag{13}$$

where the coefficients, $\widehat{A}(\tau)$, $B(\tau)$, $C(\tau)$ and $\widehat{D}(\tau)$ are functions of the parameters of the joint stochastic process for the pricing kernel (9), (10), and (12), realized inflation (11), and the expected rate of inflation (12).

In principle, it is possible to estimate the parameters of the system (9), (10), and (12) by Maximum Likelihood using equation (13) and yields on three bonds of different maturities. However, the choice of bonds to use in the estimation is arbitrary, and there is no guarantee that the estimates will be consistent with the yields of other bonds. Therefore, to minimize the consequences of possible model mis-specification and measurement errors in the fitted bond yield data, we allow for errors in the pricing of individual bonds and use a Kalman filter to estimate the time series of the unobservable state variables r, π and η, and their dynamics, from data on bond yields.

In summary, there are three transition equations for the unobserved state variables, r, π, and η, that are the discrete time versions of equations (9), (10), and (12). There are $n+1$ observation equations. The first n of them are based on the yields (13) at time t, $y_{\tau_j,t}$, on bonds with maturities τ_j $(j = 1, \cdots, n)$, by the addition of measurement errors, ϵ_{τ_j}:

$$y_{\tau_j,t} \equiv -\frac{\ln N(t, t+\tau_j)}{\tau_j} = -\frac{\widehat{A}(t,\tau_j)}{\tau_j} + \frac{B(\tau_j)}{\tau_j}r_t + \frac{C(\tau_j)}{\tau_j}\pi_t + \frac{\widehat{D}(\tau)}{\tau}\eta_t + \epsilon_{\tau_j}(t). \tag{14}$$

The last observation equation is based on the realized inflation rate at time t:

$$\frac{\Delta P}{P} = \pi\Delta t + \epsilon_P(t).$$

This final observation equation is used to identify r and π which enter the bond yield equation (14) symmetrically.

The measurement errors, $\epsilon_{\tau_j}(t)$, are assumed to be serially and cross-sectionally uncorrelated, and to be uncorrelated with the innovations in the transition equations. To reduce the number of parameters to be estimated, the variance of the yield measurement errors was assumed to be of the form, $\sigma^2(\epsilon_{\tau_j}) = \sigma_b^2$, where σ_b is a single parameter to be estimated. This is equivalent to the assumption that the measurement error variance of the log price of the bonds is proportional to the bond maturity. In addition, it is assumed that the errors in the observation equations are uncorrelated with the innovations of the state variables, i.e., $\rho_{ir} = 0$, $\rho_{i\pi} = 0$ and $\rho_{i\eta} = 0$ $(i = \epsilon_1, \cdots, \epsilon_n, \text{ and } \epsilon_P)$.

The long run means of the state variables were set exogenously to facilitate identification and estimation. More specifically, $\bar{\pi}$ was set equal to the sample mean of the CPI inflation rate, 3.8% for the UK and 3.0% for the US; $\bar{r}$ was set equal to the difference between the sample mean of the one-month Treasury bill rate and the CPI inflation rate, which is 4.6% for the UK and 2.6% for the US. $\bar{\eta}$ was set equal to 1.2 times the sample mean of the equity market Sharpe ratio, which is 0.58 for the UK and 0.62 for the US. $\bar{\eta}$ was set 20% higher than the realized equity market Sharpe ratio to allow for the fact that the equity market is only one component of the investment opportunity set. Note that for any asset i, only the product $\rho_{im}\eta$ is identified in the estimation - therefore errors in the predetermined values of $\bar{\eta}$ will be offset by errors in the estimated correlations. Finally, σ_{ϵ_P} was set to the sample standard deviation of realized CPI inflation rates and ρ_{mP} was set to zero to reduce the number of parameters to be estimated.

Table 2 reports the estimated coefficients of the pricing kernel process. In both countries expected inflation is close to a random walk. The mean reversion for η is stronger in the US and the mean reversion for r is stronger in the UK. The estimated value of ρ_{rm} is negative for both countries. Since ρ_{rm} is defined by $\rho_{rm}dt = dz_m dz_r$ this implies a negative risk premium for assets whose returns load positively on innovations in r, and therefore a positive risk premium for long term bonds whose prices move inversely with the interest rate. In contrast, the estimated value of $\rho_{\eta m}$ is positive for both countries. Figure 1 plots the time series estimates of r, π, η, and σ for the two countries. The estimated time series of r, π and η are much more volatile for the UK than they are for the US though they have substantial co-movement. The UK real rate drops precipitously following Britain's ejection from the European Exchange Rate Mechanism in October 1992, then rises in parallel with the US rate until the first quarter of 1995 when both rates reach a local maximum. After a short decline, rates in both countries rise strongly until mid-2000, and then fall back. From 1997 on the UK rate is significantly above the US rate, reaching a maximum of over 7% in 2000. The expected rate of inflation for the US hovers in a narrow range of 2-4% throughout the sample period. In contrast, the UK rate is in the range of 4-6% until 1997 when it starts to decline rapidly, and from 1998 to the end of the sample it is close to zero. The estimated maximum Sharpe ratio fluctuates within a much narrower range for the US than it does for the UK. While they start at the same level, the UK ratio rises sharply until mid-1995 and then declines to reach a minimum at the peak of the bull market in 1999 when it is negative.[11] It recovers to around 0.7 by the end of the sample period. In contrast, the US ratio declines irregularly from the start of the sample period to reach a minimum in 1999 when it is close to the UK level; the minimum is touched again in 2001 and the two ratios converge towards the end of the sample period. The volatility measures for the two countries track each other much more closely, fluctuating in a range of 10-20% until the end of 1997, then rising sharply to reach a maximum of around 40% after the Asian crisis of October 1998, before settling down in the mid 20% range apart from a spike in the two volatility measures in October 2001 following the World Trade Center attacks.

5 Empirical Results

Buraschi and Jackwerth (2001), Bakshi and Kapadia (2003), Chernov and Ghysels (2000), Coval and Shumway (2001), and Pan (2002) among others have found evidence of a negative volatility risk premium, which would account for the negative returns associated with hedged positions that are long volatility such as puts, calls and straddles. Such a risk premium implies either that volatility is a priced state variable itself, or that innovations in volatility are correlated with innovations in other state variables that are priced.[12] To explore this latter possibility, we examine the dynamics of implied volatility, σ. In models of stochastic volatility such as Heston (1993), it is common to allow the (instantaneous) volatility to follow an AR(1) (or Ornstein-Uhlenbeck) process whose innovation is correlated with the return on the security or portfolio whose volatility is being modeled. Although σ is an *implied* volatility, it is natural to model this variable as an AR(1) process also. We allow the innovation in σ to be correlated with a polynomial (cubic) function of returns on aggregate

[11]Strictly speaking, the absolute value of η is the maximum Sharpe ratio. The negative values of η indicate negative risk premia in bond markets.

[12]Mayfield (2004) and Tauchen (2005) have constructed models in which volatility is a priced state variable. Pan (2002) shows that a reduced form model that allows for a volatility risk premium performs poorly relative to a model that allows for jump risk premia.

wealth and with the changes in the state variables estimated from bond yields, r, π, and η, so that, allowing for GARCH effects in the equation, the system to be estimated is of the form:

$$\Delta\sigma_t = a_1(a_2 - \sigma_{t-1}) + a_3 R_{W,t} + a_4 R_{W,t}^2 + a_5 R_{W,t}^3 + a_6 \Delta r_t + a_7 \Delta\eta_t + a_8 \Delta\pi_t + u_t, \quad (15)$$

$$\sigma^2(u_t) = c_1 + c_2 u_{t-1}^2 + c_3 \sigma^2(u_{t-1}). \quad (16)$$

The results of estimating various versions of equation (15) are presented in Table 3. We see that σ reverts toward a long run mean of around 0.16 to 0.22, depending on the specification, and for most specifications the mean reversion intensity is of the order of 0.20, which implies a half life for innovations of around 4 months. For the UK, only c_2, the coefficient of the ARCH term in the volatility equation, is significant in models in which the change in volatility is allowed to depend on higher powers of the market return, while for the US, only c_3, the coefficient of the GARCH term, is highly significant in all models. Therefore, there is significant but short run persistence in the volatility of the innovations in the UK, but more persistent innovation volatility in the US.[13] The negative coefficient on R_W is consistent with Black's (1976) volatility 'leverage' effect which has been widely noted.[14] Notable also is the fact that changes in volatility are strongly associated with the squared return on aggregate wealth, and even with the cubed return in the UK where the linear term becomes insignificant in the presence of the cubed return. There is no evidence that changes in implied volatility are related to changes in the theoretically motivated state variables r, and η, or in expected inflation π. Thus the negative volatility risk premium must derive either from the association of volatility with higher powers of the return on aggregate wealth,[15] from the association with other state variables that we have not identified, or because volatility is itself a priced state variable. We shall see below what evidence there is for a risk premium associated with the unspanned element of implied volatility.

Table 4 reports summary statistics for the variables that we shall consider as possible arguments of the pricing kernel. Note that the correlations of changes in the state variables with the return on aggregate wealth are quite significant. A regression of the aggregate wealth return on changes in r, π, η, σ yields a value of R^2 of around 29% for the UK and 27.5% for the US. Therefore, if the true

[13]Pan (2002) and Chernov *et al.* (2003) among others have documented the stochastic volatility of the volatility process.

[14]For a summary of the evidence see Bekaert and Wu (2000).

[15]Note that hedge portfolios that have been used to identify the volatility risk premium hedge only against the direct effect of market returns on option prices and not the indirect effect arising from the association between market returns and volatility.

pricing kernel is of the form (4), then it is unlikely that the projection of the stochastic discount factor onto the space of aggregate wealth returns will yield a simple iso-elastic function. This may account for the non-monotonicity (absence of local risk aversion) as well as the time variation in the pricing kernel projections estimated by Rosenberg and Engle (2002) and Jackwerth (2000) among others. We shall examine this issue below.

Table 5 reports GMM estimates of the coefficients of the generalized iso-elastic pricing kernel. These are estimated from the system of (empirical) moment conditions:

$$\frac{1}{T}\sum_{t=1}^{T}\left[c_0\left(1+R_{W,t}\right)^{c_1}\exp^{c_2\Delta r_t+c_3\Delta\eta_t+c_4\Delta\sigma_t}(1+\tilde{R}_{i,t)})-1\right]z_{t-1}=0 \qquad (17)$$

where $c_1 \equiv -\gamma$ is the negative of the coefficient of relative risk aversion, $R_{i,t}$ is the return on each of the test assets (portfolios), and z_{t-1} is the instrumental variable. The test assets are the option portfolios, aggregate wealth, and the Treasury Bill. The instruments are $(1, R_f, r, \pi, \eta, \sigma)$. Separate estimations are reported for all the options portfolios together, for the 1 and 2 month option maturity portfolios separately, and then for the call and put option portfolios separately.[16] For each sample we estimate three models. In the first, the pricing kernel is assumed to be an iso-elastic function of the aggregate wealth return *only*; this corresponds to the pricing kernel projections of Rosenberg and Engle (2002), Jackwerth (2000) and others for the S&P 500, and of Liu *et. al.* (2004) for the FTSE index. The second model introduces the ICAPM state variables r and η, and the third model includes, in addition, the implied volatility. The $t-$ratios and $J-$statistics reported in the estimations should be treated with some caution because of the extreme non-normality of the return data and the limited sample size. In the discussion that follows we shall take this caution as given.[17] The robustness of the results can be assessed by considering the results for the different subsamples.

Consider first the estimates for the UK reported in Panel A. For the first model of pricing kernel which excludes any state variables, the estimate of $c_1 = -\gamma$ is *positive* (around 0.3) and highly significant (t-ratio of 10.5) when all the option portfolios are included in the test assets, and the point estimates are also positive for all of the subsamples except the calls-only subsample. The positive estimates of c_1 imply a negative risk aversion parameter γ, and are therefore inconsistent

[16]The estimation was carried out in Eviews v5 updating the weighting matrix once.

[17]Ferson and Foerster (1994) discuss small sample biases in GMM estimates of an asset pricing model. Lynch and Wachter (2004) introduce GMM-based estimation methods that uses all available data for each moment condition, but their methods do not lend themselves to our context where the missing observations occur at random points in each series.

with risk aversion if the FTSE 250 index is taken as a proxy for aggregate wealth and if there are no variables other than the aggregate wealth return in the pricing kernel. In contrast, all of the estimates of c_1 reported in Panel B for the US are negative and imply coefficients of relative risk aversion ranging from 0.7 to 3.0.

Consider next the effect of introducing the state variables r and η into the pricing kernel. Now the estimate of c_1 for the UK is negative in every case, although the estimated t-statistics are not large. The estimates of the risk aversion parameter $\gamma = -c_1$ range from 1.6 to 4.4, depending on the particular sample of option portfolios used in the estimation. The corresponding estimates for the US range from 2.1 to 6.0.[18] The estimated coefficient of Δr is positive in every case and usually highly significant for both the US and the UK; this is consistent both with the signs of the estimated correlations ρ_{rm} estimated from the bond yield data and reported in Table 2, and with the negative risk premium for interest rate risk found by BWX (2004, Table 2) for size and book-to-market sorted US stock portfolios. The sign of the parameter implies that there is a negative risk premium associated with being long 'r' risk, or a positive risk premium for securities such as bonds whose returns load negatively on the interest rate. The coefficient of $\Delta\eta$ is negative and highly significant in almost all the US estimations; this is again consistent with the sign of the estimate of $\rho_{\eta m}$ for the US in Table 2, as well as with the BWX findings. However the estimated coefficients of $\Delta\eta$ for the UK, while generally negative and therefore consistent with the sign of $\rho_{\eta m}$ estimated from UK government bond yields, are not significant.

When the implied volatility is added to the pricing kernel, its coefficient is positive in all the samples but one (US puts where it is statistically insignificant), and it does not change our qualitative remarks about the other coefficients. The coefficient is significant in all the US estimations that include both call and put portfolios, but is not significant in any of the estimations for the UK. The positive coefficient on $\Delta\sigma$ implies a negative risk premium associated with a long position in volatility, which is consistent with the findings of Bakshi and Kapadia (2003) and Coval and Shumway (2001) who find that returns on market neutral straddles and other positions that are long volatility are negative.

[18] Mayfield (2004) reports estimates of the coefficient of relative risk aversion for the US of around unity in a model with changing volatility. The coefficient of risk aversion estimated in Coval and Shumway (2001) ranges from -6.68 to 6.0 depending on which US S&P 500 option straddle is used in the estimation. Both Aït-Sahalia and Lo (2000) and Rosenberg and Engle (2002) use the US S&P 500 index options to estimate empirical pricing kernels and the estimated risk aversion coefficient has an average value of 12.7 in Aït-Sahalia and Lo (2000) and of 7.36 in Rosenberg and Engle (2002).

In summary, we have found that if no state variables are included in the pricing kernel the estimated iso-elastic kernel for the UK is inconsistent with risk aversion, but this inconsistency is eliminated by the inclusion of the state variables. All the US estimates are consistent with risk aversion. We have found that interest rate risk, the 'η' risk, and the volatility risk are priced consistently in the two countries. In all of the estimations reported in Table 5, the $J-$statistic for the over-identifying restrictions rejects the null hypothesis. Despite this, we can conclude that if the pricing kernel is constrained to be of the iso-elastic form in the aggregate wealth return, then there is strong evidence that there are additional state variables in the pricing kernel in addition to aggregate wealth. The evidence is strongest for the interest rate, and is also strong for the implied volatility. However, it is possible that these variables attain their significant role because of the constraint that we have imposed on the functional dependence of the pricing kernel on the aggregate wealth return. Therefore, we shall examine the effect of allowing more general functional forms for the return related element of the pricing kernel.

Bansal and Viswanathan (1993) argue that high-dimension non-linear pricing kernels are necessary to price the non-linear payoffs of derivative assets, and use a neural networks approach to approximate the unknown pricing kernel. Chapman (1997) and Rosenberg and Engel (2002) use orthogonal polynomials to approximate a nonlinear pricing kernel. We follow the latter and allow the return dependent element of the pricing kernel to be approximated by a sum of orthogonal polynomials in the aggregate wealth return. Then the pricing kernel is written as the product of a sum of orthogonal polynomials in the aggregate wealth return and an exponential affine function of the state variables:

$$m = \wp^n(R_W) \exp^{c_2 \Delta r + c_3 \Delta \eta + c_4 \Delta \sigma} \tag{18}$$

where

$$\wp^n(R_W) = \theta_0 C_0 \left(1 + R_W\right) + \sum_{k=1}^{n} \theta_k C_k (1 + R_W)$$

is an $n-$term generalized Chebyshev polynomial[19] expansion in the gross return on aggregate wealth, $(1+R_W)$, and $C_k(1+R_W)$ stands for the k^{th} order Chebyshev polynomial. The Chebyshev polynomial is defined over the domain [-1,1] with terms $C_k(x) = \cos(k \cos^{-1}(x))$. In the generalized

[19]There are several families of orthogonal polynomials. Chapman (1997) estimates the pricing kernel as a function of aggregate consumption using a four-term Legendre polynomial expansion, and Rosenberg and Engel (2002) estimate the empirical pricing kernel as a function of the S&P 500 index return using a three-term generalized Chebyshev polynomial expansion.

Chebyshev polynomial, the gross return $1 + R_W$, which is defined over the interval $[a, b]$, is first transformed into x via $x = \frac{2(1+R_W)-a-b}{b-a}$ before $C_k(x)$ can be calculated. Following Rosenberg and Engel (2002), we set the return domain for the Chebyshev polynomial to be $[0.9, 1.1]$, and gross returns below (above) 0.9 (1.1) are set to 0.9 (1.1). We consider approximations with $n = 3, 4, 5$ terms of the gross return on aggregate wealth, $1 + R_W$. While we do not restrict the pricing kernel to be strictly positive in our estimation, the estimated pricing kernels all turn out to be positive in the range of gross returns $[0.94, 1.06]$.

Table 6 reports the estimated coefficients of the state variables for three different specifications of the state variables, as well as for three different degrees of the orthogonal polynomial in the aggregate wealth return. Columns (i) and (ii) of the table report the results when r and η are the only state variables. The coefficient estimates are fairly insensitive to the degree of the approximating polynomial, and for the US the coefficients are very close to those reported for the iso-elastic kernel in Table 5. For both countries the coefficients of Δr ($\Delta\eta$) are positive (negative) and significant, which is consistent with the signs of ρ_{rm} ($\rho_{\eta m}$) as discussed above. When the implied volatility is introduced into the kernel, its coefficient is positive and significant for both countries, which is also consistent with the results reported above for the iso-elastic kernel. However, for the UK the inclusion of implied volatility makes the coefficient of η insignificant and even changes its sign when the degree of the approximating polynomial is 5. Columns (vi)-(x) of the table show the effect of introducing the expected rate of inflation into the pricing kernel as a robustness check. First, for the US the coefficient of $\Delta\pi$ is always insignificant and the other coefficient estimates are essentially unchanged; this is consistent with the insignificant value for $\rho_{\pi m}$ reported in Table 2: there is no evidence that inflation risk is priced in the US. For the UK, however, the coefficient of $\Delta\pi$ is positive and highly significant. This implies that there is a negative risk premium associated with a long position in expected inflation: since the returns on long term nominal bonds have a negative loading on changes in expected inflation, the coefficient implies that they will earn a positive risk premium for this inflation exposure. The difference between the risk premia for inflation exposure in the US and UK can perhaps be accounted for by the greater variability of expected inflation in the UK - as seen in Figure 1b and Table 4, its standard deviation is more than four times as great as that in the US. Moreover, with the introduction of expected inflation, the coefficients of Δr, $\Delta\eta$ and $\Delta\sigma$ for the UK all become positive and significant. However, as noted above, the positive coefficient on $\Delta\eta$ for the UK is inconsistent with the sign of $\rho_{\eta m}$.

Figure 2 plots the aggregate wealth-related component of the pricing kernel, $\phi(R_W) \equiv \wp^n(R_W)$, for $n = 3$ for the UK; this component of the kernel is the value that the kernel attains when changes in the state variables are set to zero. Five estimates of $\phi(R_W)$ are shown. The first, $\phi^1(R_W)$, is calculated from the parameters estimated from the model with r, η, and σ as state variables, using returns on the 11 option portfolios with complete data. The second, $\phi^2(R_W)$, is calculated from the parameters estimated from the same model but using data on all 28 option portfolios, including those with large numbers of missing observations. We report the estimated kernel derived from all 28 portfolios because the larger number of strike prices helps to identify the return related component of the pricing kernel. The third and fourth estimates, $\phi^3(R_W)$ and $\phi^4(R_W)$, are calculated from parameters estimated using the same two data sets but a model of the pricing kernel that excludes the state variables: it can thus be regarded as a projection of the kernel onto the space of aggregate wealth returns. The fifth estimate is $\phi^5(R_W) \equiv (1 + R_W)^{-\gamma}$ with $\gamma = 1.17$, which is the return related component of the generalized iso-elastic pricing kernel when the state variables (r, π, η) are included. Note that the estimates of the pricing kernels are least precise in the tails where there are relatively few return observations: only 10% of the return observations are below minus 4% and only 10% of the observations are above 6.5%. When the state variables are not included, the wealth related component of the generalized polynomial kernel takes on an inverted 'U' shape for both the large and the small data sample, which is clearly inconsistent with risk aversion. When the state variables are included, the kernel estimated from the small sample takes on a rotated 'S' shape which is downward sloping for returns between minus 4% and plus 3.5%, but is increasing at the extremities. When all 28 option portfolios are used in the estimation, the kernel is closer to being monotonically decreasing but still has an increasing region below minus 3.5%. We suspect that this portion of the curve is not well identified because of the paucity of return observations in this region.

Figure 3 plots the corresponding kernel estimates for the US. These tend to be much more monotonic than the UK estimates. The kernel estimates when no state variables are included are virtually identical whether derived using the 10 option portfolios with 'complete' data or using all 28 option portfolios: the estimates are increasing only slightly for returns below minus 4%. When the state variables are included, the wealth related element of the kernel estimated using the 28 portfolios is virtually coincident with the estimate without state variables, while the estimate of the wealth related element of the kernel estimated using the 10 portfolios with 'complete' data is monotonically decreasing which is consistent with universal risk aversion.

6 Conclusion

In this paper we have used returns on portfolios of FTSE 100 and S&P 500 index options to estimate pricing kernels that depend, not only on the aggregate wealth return, but also on up to four state variables, the real interest rate, r, the maximum Sharpe ratio, η, the implied volatility of option prices, σ, and the expected rate of inflation, π. The first two state variables are derived from the BWX (2004) version of the ICAPM and are estimated from data on government bond yields following Brennan and Xia (2005). The implied volatility is included as a state variable because previous studies have found evidence that hedged option portfolios that are formed to have a positive loading on volatility have negative excess returns, and because of evidence that it is necessary to include this variable to capture the dynamics of the maximum Sharpe ratio, η. Preliminary regressions show that innovations in implied volatility are strongly related to the first, second, and third (for the UK) moments of the aggregate wealth return, but are not significantly related to innovations in the state variables. This raises the possibility that the risk premium that has been found to be associated with volatility is due to the association of this variable with the return on aggregate wealth. This is investigated by including the implied volatility in the pricing kernel along with a flexible functional form of the aggregate wealth return.

We estimate the pricing kernels by GMM using the returns on 11 (10) portfolios of UK (US) index options formed according to type, moneyness, and time to expiration. We express the pricing kernel as the product of a function of the aggregate wealth return and an exponential affine function of the changes in the state variables. In the first estimations the pricing kernel is constrained to depend on a power function of the aggregate wealth return. When no state variables are included in the kernel, the estimated power is positive and highly significant for the UK, which is inconsistent with risk aversion. However, when the changes in the state variables are included in the kernel, the estimate of the power becomes negative, although usually it is not significant. The state variable r appears significantly in the pricing kernel, but neither the maximum Sharpe ratio, η, nor the implied volatility, σ, are significant. For the US, the estimated coefficient of relative risk aversion is 3.2 when no state variables are included and 4.9 when r, η, and σ are included. The coefficients of all three state variables are significant and their signs are consistent with previous findings of a negative volatility premium, with the estimated correlations between the pricing kernel and r

and η estimated from bond yields, and with the risk premia estimated using portfolios of common stocks by BWX (2004).

It is possible that the significance of the state variables is due to the constrained functional form of the dependence of the kernel on the aggregate wealth return. Therefore we repeat the estimations by substituting a polynomial expression in the wealth relative for the power function. The coefficients on the state variables for the US are essentially unaffected by the functional form of the dependence of the kernel on the aggregate wealth return, and the coefficients of Δr, $\Delta\eta$, and $\Delta\sigma$ are consistent and significant across specifications. For the UK, the coefficients of Δr and $\Delta\sigma$ are also consistent and significant across specifications. While the coefficient on $\Delta\eta$ in the US is negative and significant in all specifications and is unaffected by the addition of $\Delta\pi$, whose coefficient is not significant in the US, the $\Delta\eta$ coefficient in the UK is not significant in the presence of $\Delta\sigma$ and becomes significantly positive when the specification includes $\Delta\pi$ which enters the pricing kernel highly significantly in the UK. Finally, the signs of the coefficients of the state variables in the pricing kernel are generally consistent across countries, consistent with the signs of their correlations with the pricing kernel estimated from bond yields, and consistent with the BWX risk premium estimates. The exception is η for the UK when π is included as a state variable.

The results in the paper provide further evidence that the failure of traditional asset pricing models is at least in part due to the failure to include in the pricing kernel non-wealth-related variables that are important to investors because they describe future investment opportunities. This of course is the insight underlying Merton's (1973) classic analysis. However, the rejection of the over-identifying restrictions implied by the model suggests that there are other variables than those we have considered that are important to investors. Longstaff (1995) suggests that transactions costs and liquidity effects are also important for the pricing of index options, and Santa-Clara and Saretto (2005) point to the importance of margin requirements as obstacles to the achievement of equilibrium in options markets.

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Table 1 Summary Statistics of Returns for Option Portfolios

Moneyness is defined by $(I - K)/K$ for call and $(K - I)/K$ for puts where I is the level of the index on the portfolio formation date and K is the option exercise price. Returns are calculated as the proportional monthly price change on an equally weighted portfolio of options of the given maturity and moneyness characteristics. The 1-month portfolios consist of options whose time to maturity at the beginning of the month is between 1 and 2 months; the 2-month portfolios consist of options with maturities of between 2 and 3 months. The sample period is from 1992.04 to 2002.03 for FTSE index option returns and from 1992.01 to 2002.04 for S&P 500 index option returns.

	A. UK FTSE 100 Index Option Portfolios							**B. US S&P 500 Index Option Portfolios**						
Portfolio	1	2	3	4	5	6	7	1	2	3	4	5	6	7
	Out of Money				In Money			Out of Money				In Money		
Moneyness (%)	≤ -15	$(-15, -10]$	$(-10, -5]$	$(-5, 0]$	$(0, 5]$	$(5, 10]$	> 10	≤ -15	$(-15, -10]$	$(-10, -5]$	$(-5, 0]$	$(0, 5]$	$(5, 10]$	> 10
	1 Month Calls							1 Month Calls						
Average	-0.18	-0.31	-0.29	-0.15	-0.02	0.02	0.04	-0.24	-0.26	-0.03	-0.02	0.02	0.03	0.03
Std. Dev.	0.61	0.67	1.06	1.00	0.68	0.47	0.27	0.51	1.50	1.72	1.10	0.66	0.44	0.24
Skewness	2.24	2.11	2.91	1.39	0.56	0.11	0.14	1.09	5.36	3.21	1.41	0.37	-0.03	0.01
Missing Obs.	98	71	10	0	0	1	1	88	71	24	2	1	1	5
Number of Options	110	192	480	595	557	497	2290	217	162	309	636	617	460	1091
	1 Month Puts							1 Month Puts						
Average	-0.17	-0.29	-0.26	-0.26	-0.15	-0.06	-0.03	-0.65	-0.64	-0.57	-0.38	-0.18	-0.06	-0.03
Std. Dev.	1.96	1.48	2.25	1.26	0.77	0.52	0.31	0.36	0.53	0.67	0.81	0.73	0.51	0.29
Skewness	5.88	4.26	7.10	3.85	1.53	0.78	0.07	1.83	3.03	2.92	2.11	1.25	0.49	-0.06
Missing Obs.	25	6	2	0	0	3	32	35	16	1	1	2	19	57
Number of Options	931	529	567	574	569	543	1494	667	305	471	618	640	335	397
	2 Months Calls							2 Months Calls						
Average	-0.07	-0.26	-0.15	-0.05	0.01	0.02	0.04	-0.18	-0.11	-0.07	0.03	0.03	0.03	0.03
Std. Dev.	1.60	1.10	1.01	0.77	0.55	0.40	0.26	0.36	1.46	1.03	0.79	0.53	0.38	0.22
Skewness	3.83	2.89	2.04	1.18	0.56	0.19	0.06	1.99	3.46	1.72	0.84	0.33	0.04	-0.21
Missing Obs.	94	43	1	0	0	0	2	92	70	23	2	2	8	7
Number of Options	160	348	573	573	524	465	2151	193	152	307	554	504	360	844
	2 Months Puts							2 Months Puts						
Average	-0.07	-0.16	0.18	0.16	-0.11	-0.06	-0.03	-0.39	-0.37	-0.32	-0.23	-0.14	-0.04	-0.01
Std. Dev.	2.43	1.55	1.26	0.88	0.61	0.44	0.29	0.57	0.56	0.61	0.62	0.56	0.45	0.31
Skewness	7.12	5.19	4.65	3.10	1.48	0.85	0.29	1.74	1.78	1.90	1.54	1.16	0.62	0.40
Missing Obs.	24	3	0	0	0	0	31	13	23	6	1	2	32	63
Number of Options	1346	529	542	536	545	516	1397	600	240	370	524	543	286	355

Table 2 Term Structure Model Parameter Estimates

This table reports estimates of the parameters of the stochastic process of the investment opportunity set, equations (9) to (12), obtained from a Kalman filter applied to inflation rates and bond yields. The state variables are r, the real interest rate, π, the expected rate of inflation, and η, the volatility of the pricing kernel or the Sharpe ratio of the economy. In the table, m denotes the pricing kernel and P is the price level. Asymptotic t-ratios are in parentheses.

A. The United Kingdom							
	σ_b	σ_r	σ_π	σ_η	κ_r	κ_π	κ_η
Estimate	0.44%	0.63%	0.92%	0.207	0.143	0.000	0.104
t-ratio	(49.74)	(10.35)	(12.94)	(2.45)	(4.68)	(0.76)	(1.89)
	$\rho_{r\pi}$	$\rho_{r\eta}$	ρ_{rm}	$\rho_{\pi\eta}$	$\rho_{\pi m}$	$\rho_{\eta m}$	
Estimate	-0.104	-0.234	-0.714	-0.191	0.178	0.833	
t-ratio	(0.75)	(1.14)	(3.49)	(0.78)	(1.93)	(4.38)	
	$\bar{r}$	$\bar{\pi}$	$\bar{\eta}$	σ_P	ρ_{Pm}	ML	
Pre-set Value	4.56%	3.78%	0.58	1.63%	0.00	8,522.2	

B. The United States							
	σ_b	σ_r	σ_π	σ_η	κ_r	κ_π	κ_η
Estimate	0.48%	2.77%	0.81%	0.193	0.290	0.002	0.292
t-ratio	(52.82)	(10.92)	(5.15)	(1.88)	(1.97)	(0.70)	(3.84)
	$\rho_{r\pi}$	$\rho_{r\eta}$	ρ_{rm}	$\rho_{\pi\eta}$	$\rho_{\pi m}$	$\rho_{\eta m}$	
Estimate	0.027	-0.413	-0.801	-0.199	-0.276	0.919	
t-ratio	(0.11)	(0.54)	(6.22)	(0.69)	(1.66)	(2.57)	
	$\bar{r}$	$\bar{\pi}$	$\bar{\eta}$	σ_P	ρ_{Pm}	ML	
Pre-set Value	2.62%	3.00%	0.62	0.77%	0.00	9,334.6	

Table 3 The Behavior of At-the-Money Option Implied Volatility

This table reports the results from regressions of innovations of at-the-money option implied volatility on the polynomials of the market return, and the innovations in the instantaneous real interest rate r, the maximum Sharpe ratio η, and the expected inflation π:

$$\begin{aligned} \Delta\sigma_t &= a_1(a_2 - \sigma_{t-1}) + a_3 R_{W,t} + a_4 R_{W,t}^2 + a_5 R_{W,t}^3 + a_6 \Delta r_t + a_7 \Delta\eta_t + a_8 \Delta\pi_t + u_t \\ \sigma^2(u_t) &= c_1 + c_2 u_{t-1}^2 + c_3 \sigma^2(u_{t-1}). \end{aligned}$$

The Bollerslev-Wooldrige robust $t-$ratios are reported in the parenthesis.

United Kingdom									United States									
A. Linear Model																	
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	$R^2(\%)$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	$R^2(\%)$
0.11	0.22	-0.29			-0.75	-0.05		23.9	0.13	0.21	-0.36			-0.23	-0.12		31.3
(3.05)	(8.86)	(6.58)			(1.20)	(1.10)			(2.16)	(.7.87)	(4.84)			(0.28)	(1.18)		
c_1	c_2	c_3							c_1	c_2	c_3						
0.00002	-0.05	1.05							0.0002	0.25	0.53						
(9.29)	(0.99)	(20.90)							(1.77)	(1.77)	(2.34)						
B. Quadratic Model																	
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	$R^2(\%)$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	$R^2(\%)$
0.16	0.20	-0.40	4.19		0.80	-0.08		32.9	0.21	0.16	-0.40	4.93		-0.10	-0.08		38.7
(3.58)	(8.99)	(4.75)	(2.85)		(0.89)	(1.34)			(2.91)	(11.80)	(5.44)	(3.87)		(0.12)	(0.71)		
c_1	c_2	c_3							c_1	c_2	c_3						
0.0006	0.35	0.12							0.00008	0.10	0.81						
(2.55)	(2.13)	(0.57)							(1.32)	(1.18)	(6.23)						
C. Cubic Model																	
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	$R^2(\%)$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	$R^2(\%)$
0.16	0.17	-0.06	5.29	-49.52	0.80	-0.08		38.3	0.22	0.16	-0.46	5.31	12.67	-0.21	-0.08		39.6
(3.42)	(8.52)	(0.61)	(4.13)	(3.79)	(0.96)	(1.38)			(2.98)	(12.73)	(4.75)	(4.33)	(1.16)	(0.23)	(0.71)		
c_1	c_2	c_3							c_1	c_2	c_3						
0.0004	0.51	0.18							0.00007	0.07	0.85						
(3.47)	(1.95)	(1.12)							(1.31)	(0.98)	(7.73)						
D. Cubic Model with Expected Inflation																	
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	$R^2(\%)$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	$R^2(\%)$
0.16	0.17	-0.11	5.13	-44.47	-0.03	-0.06	-1.83	38.0	0.22	0.16	-0.46	5.33	12.63	-0.25	-0.08	0.27	39.6
(3.80)	(8.19)	(1.08)	(4.08)	(3.46)	(0.03)	(1.19)	(1.62)		(2.97)	(12.75)	(4.75)	(4.26)	(1.16)	(0.28)	(0.72)	(0.13)	
c_1	c_2	c_3							c_1	c_2	c_3						
0.0004	0.52	0.18							0.00007	0.07	0.85						
(3.17)	(2.09)	(1.13)							(1.30)	(0.98)	(7.72)						

Table 4 Summary Statistics of Pricing Kernel Arguments and Instruments

Panel A reports the mean, the standard deviation, and the skewness of the risk free rate, the market return, the level and the innovations of the state variables r, π, η, and σ. Panel B reports the contemporaneous correlations between the market return and the innovations in r, π, η and σ. All variables are annualized.

Panel A: Mean, Standard Deviation, and Skewness

	r	Δr	η	$\Delta\eta$	σ	$\Delta\sigma$	π	$\Delta\pi$	R_f	R_W
1. United Kingdom										
Mean	0.034	-0.000	0.728	-0.002	0.217	0.022	0.027	-0.000	0.062	0.125
Stdev	0.024	0.003	0.882	0.069	0.072	0.035	0.023	0.003	0.013	0.148
Skew	-0.045	-2.051	0.120	0.416	1.054	0.800	-0.172	0.102	1.313	-0.109
2. United States										
Mean	0.023	-0.000	-0.063	-0.001	0.193	0.000	0.028	-0.000	0.052	0.117
Stdev	0.014	0.003	0.338	0.030	0.060	0.036	0.005	0.001	0.005	0.146
Skew	0.397	-0.338	-0.105	0.955	0.699	-0.196	-0.204	0.230	0.280	-0.818

Panel B: Contemporaneous Correlations

	1. United Kingdom						2. United States				
	Δr	$\Delta\eta$	$\Delta\sigma$	$\Delta\pi$	R_W		Δr	$\Delta\eta$	$\Delta\sigma$	$\Delta\pi$	R_W
Δr	1.0					Δr	1.0				
$\Delta\eta$	-0.08	1.0				$\Delta\eta$	-0.24	1.0			
$\Delta\sigma$	0.22	-0.08	1.0			$\Delta\sigma$	-0.05	-0.08	1.0		
$\Delta\pi$	-0.45	0.18	-0.02	1.0		$\Delta\pi$	0.23	0.13	-0.04	1.0	
R_W	-0.17	-0.09	-0.44	-0.14	1.0	R_W	0.09	-0.16	-0.49	-0.02	1.0

Table 5 GMM Estimates of Iso-Elastic Pricing Kernels

The table reports GMM estimates of iso-elastic pricing kernels for monthly returns on option portfolios, the market index returns, and the Treasury bill rate. The general pricing kernel equation is of the form:

$$m = c_0(1 + R_W)^{c_1} e^{c_2 \Delta r + c_3 \Delta \eta + c_4 \Delta \sigma}$$

where R_W is the return on aggregate wealth proxied by the FTSE 250 index return in the UK and the CRSP value weighted market returns in the US; Δr is the change in the estimated real interest rate; $\Delta \eta$ is the change in the estimated maximal Sharpe ratio; and $\Delta \sigma$ is the change in the estimated at-the-money volatility of the index options. Instruments are the unit vector, the Treasury Bill rate, the estimated real interest rate, the estimated expected rate of inflation, the estimated Sharpe ratio, and the estimated at the money option volatility. In Panel A, the test assets are portfolios of options on the FTSE 100 index, the FTSE All Share Index, and the 3-month Treasury Bill. In Panel B, the test assets are portfolios of options on the S&P 500 index, the CRSP value weighted market Index, and the US 30-day Treasury bill. The option portfolios contain options with maturities of between 1 and 2 months, and between 2 and 3 months and with different moneyness. The sample period is 1992.04 to 2002.03 for Panel A and 1992.01 to 2002.04 for Panel B.

No.	Option Maturity&Type	c_o	c_1	c_2	c_3	c_4	GMM $J-$statistic	Portfolios Observations
	Panel A. UK FTSE 100 Option Portfolios							
1	1 and 2 months Calls and Puts	1.005	0.276				0.92	13
		(2235.82)	(10.49)					1,560
2	1 and 2 months Calls and Puts	0.910	-2.265	189.805	-0.141		0.92	13
		(38.36)	(1.53)	(4.89)	(0.28)			1,560
3	1 and 2 months Calls and Puts	0.886	-1.171	204.052	-0.065	1.520	0.92	13
		(32.71)	(0.51)	(5.18)	(0.13)	(0.89)		1,560
4	2 months Calls and Puts	0.998	0.360				0.74	9
		(3342.14)	(20.37)					1080
5	2 months Calls and Puts	0.848	-3.718	230.689	-0.492		0.69	9
		(20.20)	(2.04)	(4.93)	(0.95)			1080
6	2 months Calls and Puts	0.824	-1.097	240.379	-0.254	2.634	0.68	9
		(17.82)	(0.41)	(5.04)	(0.46)	(1.33)		1,080
7	1 month Calls and Puts	1.002	0.081				0.45	6
		(712.76)	(1.02)					720
8	1 month Calls and Puts	0.836	-4.367	247.413	-0.876		0.42	6
		(12.67)	(2.07)	(4.17)	(1.50)			720
9	1 month Calls and Puts	0.760	0.243	276.007	-0.258	5.121	0.41	6
		(10.15)	(0.08)	(4.19)	(0.34)	(1.85)		720
10	1 and 2 months Calls	1.058	-3.610				0.59	7
		(301.70)	(20.90)					840
11	1 and 2 months Calls	1.020	-1.589	21.090	0.278		0.55	7
		(122.86)	(2.71)	(2.16)	(1.30)			840
12	1 and 2 months Calls	0.986	-0.584	33.477	0.286	1.570	0.56	7
		(59.66)	(0.35)	(2.34)	(0.94)	(1.18)		840
13	1 and 2 months Puts	1.043	5.445				0.67	8
		(137.17)	(16.72)					960
14	1 and 2 months Puts	0.828	-3.523	246.116	-1.256		0.78	8
		(15.54)	(1.73)	(4.38)	(1.44)			960
15	1 and 2 months Puts	0.075	-12.514	1024.38	1.271	5.896	0.99	8
		(4.15)	(3.70)	(10.22)	(1.05)	(1.79)	960	

Table 5 (continued)

No.	Option Maturity&Type	c_o	c_1	c_2	c_3	c_4	GMM $J-$statistic	Portfolios Observations
				Panel B. US S&P 500 Option Portfolios				
1	1 and 2 months Calls and Puts	1.064	-2.688				0.96	12
		(585.00)	(15.73)					1,473
2	1 and 2 months Calls and Puts	0.793	-5.994	112.137	-23.734		0.90	12
		(11.98)	(3.03)	(3.29)	(4.06)			1,473
3	1 and 2 months Calls and Puts	0.670	-4.758	131.648	-29.034	3.342	0.92	12
		(9.63)	(2.20)	(3.38)	(4.52)	(1.89)		1,473
4	2 months Calls and Puts	1.022	-1.675				0.61	6
		(344.05)	(7.10)					737
5	2 months Calls and Puts	0.792	-2.752	77.156	-23.123		0.42	6
		(9.91)	(1.14)	(1.79)	(3.37)			737
6	2 months Calls and Puts	0.688	-0.055	105.850	-24.584	6.655	0.38	6
		(7.68)	(0.02)	(2.07)	(3.13)	(2.24)		737
7	1 month Calls and Puts	1.067	-2.984				0.91	8
		(425.79)	(13.73)					984
8	1 month Calls and Puts	0.781	-5.661	109.509	-25.528		0.80	8
		(9.49)	(2.39)	(2.84)	(3.82)			984
9	1 month Calls and Puts	0.636	-2.928	150.856	-29.760	5.790	0.79	8
		(7.04)	(1.01)	(3.36)	(4.12)	(2.05)		984
10	1 and 2 months Calls	1.014	-2.527				0.53	7
		(1458.51)	(38.88)					860
11	1 and 2 months Calls	0.851	-2.134	36.060	-16.326		0.33	7
		(12.21)	(0.90)	(0.77)	(2.33)			860
12	1 and 2 months Calls	0.808	-0.663	54.284	-16.754	3.025	0.33	7
		(11.06)	(0.26)	(1.10)	(2.27)	(1.17)		860
13	1 and 2 months Puts	1.133	-0.666				0.93	7
		(173.03)	(1.22)					861
14	1 and 2 months Puts	1.080	-2.870	83.709	1.828		0.85	7
		(45.12)	(1.96)	(3.73)	(0.82)			861
15	1 and 2 months Puts	1.085	-3.103	84.130	1.797	-0.618	0.85	7
		(44.14)	(2.02)	(3.78)	(0.78)	(0.55)		861

Table 6 GMM Estimates of Polynomial Pricing Kernels

The table reports GMM estimates of polynomial pricing kernels for monthly returns on option portfolios, the market index returns, and the Treasury bill rate. The general pricing kernel equation is of the form:

$$m = \wp^n(R_W)e^{c_2\Delta r + c_3\Delta\eta + c_4\Delta\sigma + c_5\Delta\pi}$$

where $\wp^n(R_W)$ is a Chebyshev polynomial of order n in the aggregate wealth return R_W proxied by the FTSE 250 index return in the UK and the CRSP value weighted market returns in the US; Δr is the change in the estimated real interest rate; $\Delta\eta$ is the change in the estimated maximal Sharpe ratio; $\Delta\sigma$ is the change in the estimated at-the-money volatility of the index options; and $\Delta\pi$ is the change in the expected rate of inflation. Instruments are the unit vector, the Treasury Bill rate, the estimated real interest rate, the estimated expected rate of inflation, the estimated Sharpe ratio, and the estimated at the money option volatility. In Panel A, the test assets are portfolios of options on the FTSE 100 index, the FTSE All Share Index, and the 3-month Treasury Bill. In Panel B, the test assets are portfolios of options on the S&P 500 index, the CRSP value weighted market Index, and the US 30-day Treasury bill. The option portfolios contain options with maturities of between 1 and 2 months, and between 2 and 3 months and with different moneyness. The sample period is 1992.04 to 2002.03 for Panel A and 1992.01 to 2002.04 for Panel B.

	I		II			III				
	$m = \wp^n(R_W)e^{c_2\Delta r + c_3\Delta\eta}$		$m = \wp^n(R_W)e^{c_2\Delta r + c_3\Delta\eta + c_4\Delta\sigma}$			$m = \wp^n(R_W)e^{c_2\Delta r + c_3\Delta\eta + c_4\Delta\sigma + c_5\Delta\pi}$				
n	c_2	c_3	c_2	c_3	c_4	c_2	c_3	c_4	c_5	Portfolios/ Observations
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(ix)	(x)	
					Panel A. UK FTSE 100 Option Portfolios					
3	251.83	-1.25	395.97	-0.63	6.43	1161.71	3.91	11.46	357.63	13
	(5.91)	(2.20)	(8.66)	(0.98)	(3.30)	(15.80)	(4.87)	(4.47)	(6.95)	1560
4	274.36	-1.55	395.71	-0.69	6.17	965.19	3.05	9.97	296.61	13
	(6.45)	(2.70)	(8.65)	(1.05)	(2.32)	(15.24)	(3.74)	(4.26)	(6.52)	1560
5	268.06	-1.69	888.86	2.88	8.46	913.44	3.07	9.91	239.63	13
	(6.28)	(2.86)	(15.87)	(3.42)	(3.14)	(14.86)	(3.76)	(3.28)	(5.41)	1560
					Panel B. US S&P 500 Option Portfolios					
3	75.48	-20.48	127.76	-59.28	7.16	135.96	-58.80	7.17	60.03	12
	(2.34)	(3.72)	(2.84)	(6.49)	(2.80)	(2.86)	(6.58)	(2.45)	(0.58)	1473
4	82.71	-110.25	149.97	-42.54	9.73	144.13	-40.98	9.30	-4.35	12
	(1.97)	(9.52)	(3.23)	(5.36)	(3.66)	(2.87)	(5.15)	(3.03)	(0.04)	1473
5	42.53	-36.47	156.54	-43.68	10.99	146.59	-41.78	9.87	18.21	12
	(1.12)	(5.05)	(3.34)	(5.38)	(3.84)	(2.89)	(5.16)	(3.06)	(0.17)	1473

Figure 1

Time Series of Estimated r, π, η, and σ for UK and US

The figure plots the time series of the real interest rate r, the expected inflation π, the maximum Sharpe ratio η, and the at-the-money option implied volatility σ in UK for the sample period of 1992.04 to 2002.03 and in US for the sample period of 1992.01 to 2002.04. For the UK, the time series of r, π, and η are estimated from the zero-coupon nominal UK government bond yield, and σ is estimated from 1-month FTSE 100 option prices. For the US, r, π, and η are estimated from the zero-coupon nominal US government bond yields, and σ is estimated from the 30-day S&P 500 index option prices.

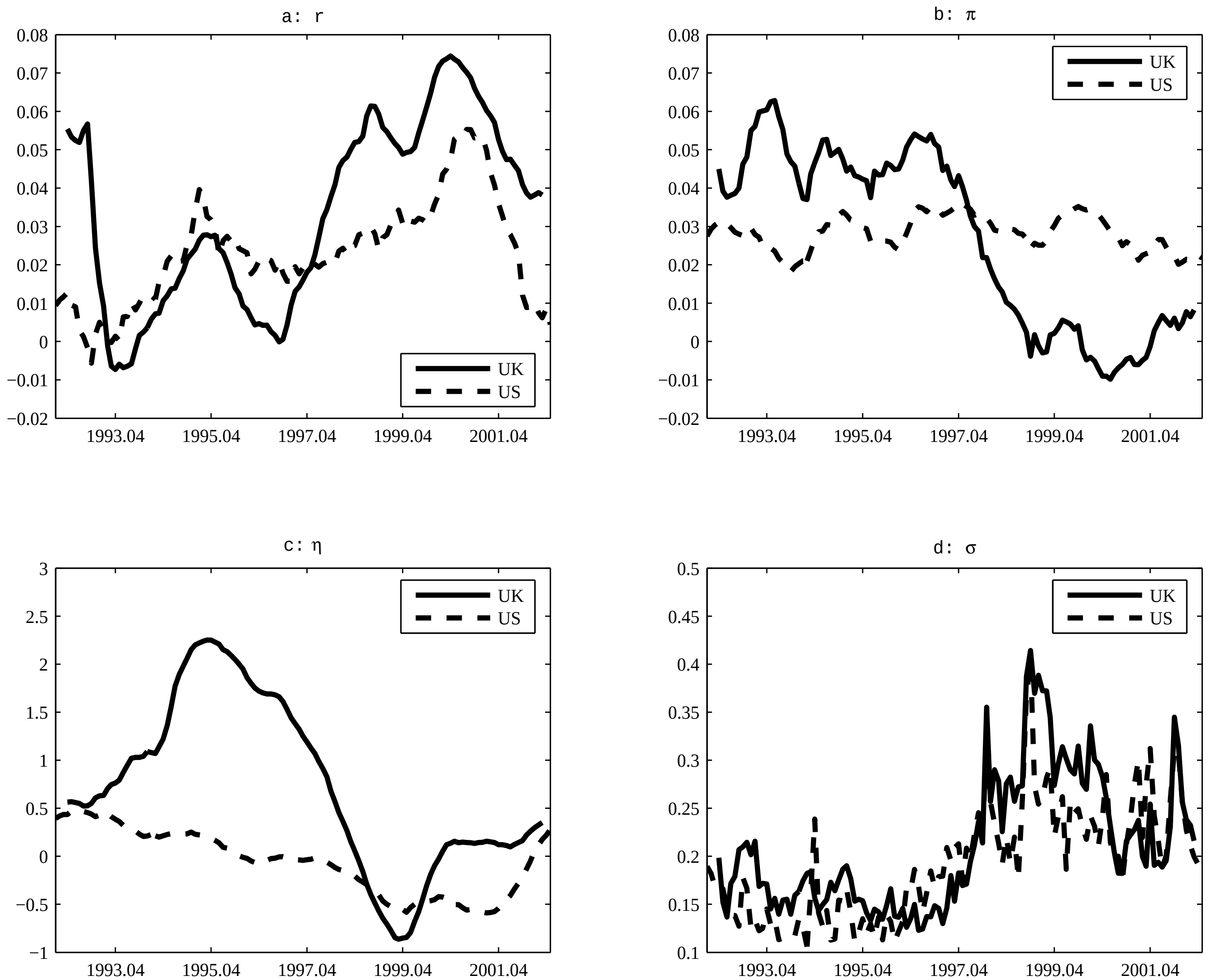

Figure 2

Return Related Element of Estimated Iso-Elastic and Polynomial Pricing Kernels in UK

The figure plots the market return related component of the pricing kernel, $\phi(R_W) \equiv \wp^n(R_W)$, for $n = 3$, which is obtained by setting the state variable innovations to zero. Five $\phi(R_W)$ are plotted: 1) $\phi^1(R_W)$ is calculated from parameters which are estimated from the model with the three state variables (r, η, σ) using 11 option portfolios; 2) $\phi^2(R_W)$ is calculated from parameters which are estimated from the model with the three state variables (r, η, σ) using 28 option portfolios; 3) $\phi^3(R_W)$ is calculated from parameters which are estimated from the model without any state variables using 11 option portfolios; 4) $\phi^4(R_W)$ is calculated from parameters which are estimated from the model without any state variables using 28 option portfolios, and 5) $\phi^5(R_W) \equiv (1 + R_W)^{-\gamma}$ is the market return related component of the iso-elastic pricing kernel for $\gamma = 1.17$, which is estimated from the model (17) with the three state variables (r, η, σ) using 11 option portfolios and reported in line 3 of Panel A in Table 5 for the UK data.

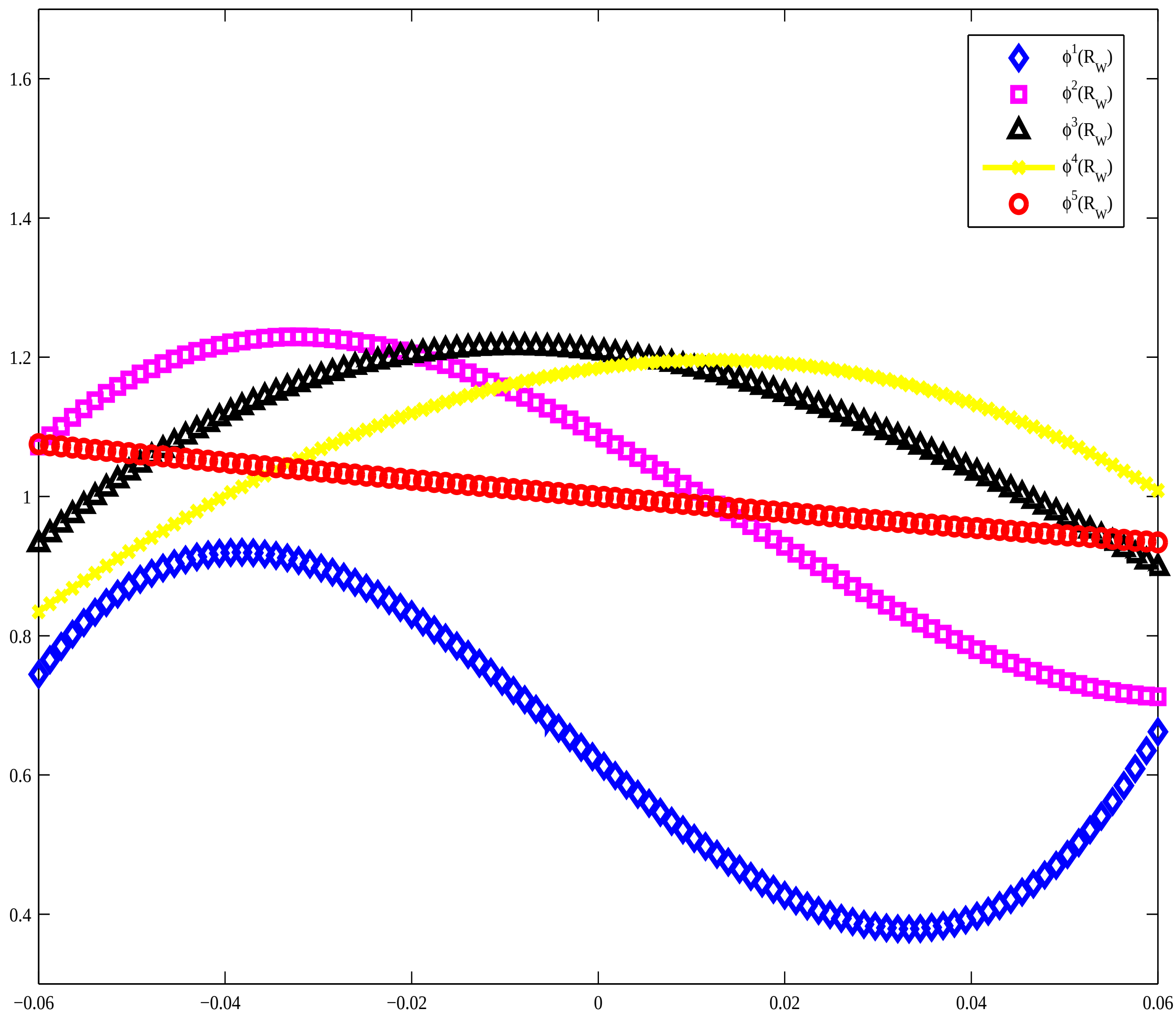

Figure 3

Return Related Element of Estimated Iso-Elastic and Polynomial Pricing Kernels in US

The figure plots the market return related component of the pricing kernel, $\phi(R_W) \equiv \wp^n(R_W)$, for $n = 3$, which is obtained by setting the state variable innovations to zero. Five $\phi(R_W)$ are plotted: 1) $\phi^1(R_W)$ is calculated from parameters which are estimated from the model with the three state variables (r, η, σ) using 10 option portfolios; 2) $\phi^2(R_W)$ is calculated from parameters which are estimated from the model with the three state variables (r, η, σ) using 28 option portfolios; 3) $\phi^3(R_W)$ is calculated from parameters which are estimated from the model without any state variables using 10 option portfolios; 4) $\phi^4(R_W)$ is calculated from parameters which are estimated from the model without any state variables using 28 option portfolios, and 5) $\phi^5(R_W) \equiv (1 + R_W)^{-\gamma}$ is the market return related component of the iso-elastic pricing kernel for $\gamma = 4.88$, which is estimated from the model (17) with the three state variables (r, η, σ) using 10 option portfolios and reported in line 3 of Panel B in Table 5 for the US data.