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Authors
Kolehmainen, K.
Gyulassy, M.

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Lawrence Berkeley Laboratory
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K. Kolehmainen and M. Gyulassy

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Karen Kolehmainen and Miklos Gyulassy

Nulcear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

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PION INTERFEROMETRY OF ULTRA-RELATIVISTIC HADRONIC COLLISIONS [1]

Karen Kolehmainen and Miklos Gyulassy

Nuclear Science Division, 70A-3307, Lawrence Berkeley Laboratory, Berkeley, California 94720, USA

Abstract: Pion interferometry of ultra-relativistic hadronic collisions is described in the context of the inside-outside cascade model using a current ensemble method. In contrast to the usual Gaussian and Kopylov parameterizations involving only geometrical variables, the correlation function in this model depends on a special combination of dynamical as well as geometrical parameters.

Correlations in momentum space between pairs of identical particles have long been used to obtain information about the space-time structure of the source of the particles [1,2]. The correlation function is defined as

$$C(k_1,k_2) = \frac{P_2(k_1,k_2)}{P_1(k_1)\,P_1(k_2)} \quad , \tag{1}$$

where

$$P_1(k) = E\,\frac{d^3n}{d^3k} = \frac{d^3n}{dy\,d^2k_T} \tag{2}$$

and

$$P_2(k_1,k_2) = E_1\,E_2\,\frac{d^6n}{d^3k_1\,d^3k_2} = \frac{d^6n}{dy_1\,dy_2\,d^2k_{1T}\,d^2k_{2T}} \tag{3}$$

are the invariant single particle and two-particle inclusive distributions, respectively. In eqs. (2) and (3), y is the longitudinal rapidity, $\vec{k}_T$ is the transverse momentum, and E is the particle energy. If one takes the two particle wave function to be a Bose symmetrized (or Fermi antisymmetrized) plane wave state, one finds that $C(k_1,k_2)$ is related to the Fourier transform of the space-time density distribution of particle sources. Experimental results for two pion correlations in hadronic collisions are usually fit by expressions which assume that this density of sources is given by a Gaussian in space and time or a uniformly radiating disc with exponential time dependence [3,4]. Neither of these parameterizations, however, contains the correlations between the dynamics and geometry appropriate for high energy hadronic collisions.

Such correlations arise due to the nature of the inside-outside cascade dynamics [5]. Due to Lorentz time dilation, the formation time of a secondary particle

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increases linearly with its energy. This leads to a strong correlation between the average space-time point (t, z) of particle emission and the longitudinal rapidity y. This correlation can be expressed approximately by [5].

$$
\begin{aligned}
z &= \tau_0 \sinh y \ , \\
t &= \tau_0 \cosh y \ ,
\end{aligned} \tag{4}
$$

where τ_0 is the characteristic *proper* time for particle freezeout. In nuclear collisions, we expect that $\tau_0 \gtrsim A^{1/3}$ fm. It is important to note that the final interference pattern measured by C reflects only the geometry at the proper time when the interactions cease. In refs. [6], [7], and [8], it was shown that the interplay between dynamics and geometry could significantly alter the interference patterns. The inside-outside cascade was considered in ref. [8], but with a noncovariant formalism. In this letter, we employ the current ensemble formalism developed in ref. [4] to investigate this question further. We extend this formalism to allow the sources to be moving with an arbitrary velocity distribution. In ref. [9], further applications of this formalism are discussed. Our goal here is to clarify how the inside-outside cascade nature of the dynamics influences the correlation function.

We start by considering an ensemble of currents with Fourier transform

$$
j(k) = \sum_{i=1}^{N} e^{i\phi_i} e^{ikx_i} j_0(y - y_i, k_T) \ , \tag{5}
$$

where y_i is the longitudinal rapidity of pion source i in a frame where the pion four-momentum is $k^\mu = (m_T \cosh y, \vec{k}_T, m_T \sinh y)$. We may take this frame to be the laboratory frame. The pion transverse mass, $m_T^2 = m^2 + k_T^2$, is of course the same in all frames related by boosts along the z axis. We assume, as in ref. [4], that the phases ϕ_i of the different sources are random. Source i, as characterized by a Lorentz scalar current $j_0(x)$ in its rest frame, is centered initially at space-time point x_i. The Fourier transformed current $j_0(y - y_i, k_T)$ is interpreted as the amplitude for source i to emit a pion with rapidity $y - y_i$ in the rest frame of source i. The new feature in eq. (5) which was not considered in ref. [4] is the distribution of source rapidities y_i.

We specify the ensemble of currents by the normalized probability density $D(x, y_0)$ of finding a source with rapidity y_0 at space-time point x. The invariant single pion inclusive distribution is then

$$
P_1(k) = \langle |j(k)|^2 \rangle = \langle N \rangle \int dy_0 \, D(q = 0, y_0) \, |j_0(y - y_0, k_T)|^2 \ , \tag{6}
$$

where $D(q, y_0)$ denotes the Fourier transform of $D(x, y_0)$ in x and $\langle N \rangle$ is the mean number of sources. Note that $\langle N \rangle D(q = 0, y_0) = dN/dy_0$ is the rapidity density of sources. Similarly, the invariant two pion inclusive distribution is

$$
P_2(k_1, k_2) = \langle |j(k_1)|^2 \, |j(k_2)|^2 \rangle \ . \tag{7}
$$

The angle brackets in eqs. (6) and (7) denote averages over the random phases, the number of sources, and the source density distribution in space-time and rapidity. In this model, the correlation function is given by

$$C(k_1, k_2) = 1 + \frac{| G(k_1, k_2) |^2}{G(k_1, k_1)\, G(k_2, k_2)} \ , \tag{8}$$

where

$$G(k_1, k_2) = \int dy_0\, D(k_2 - k_1, y_0)\, j_0^*(y_1 - y_0, k_{1T})\, j_0(y_2 - y_0, k_{2T}) \ . \tag{9}$$

The simplest characterization of the inside-outside cascade model, incorporating eq. (4), is given by

$$D(z, t, \vec{r}_T, y_0) = \frac{1}{\pi R_T^2} \delta(t - \tau_0 \cosh y_0)\, \delta(z - \tau_0 \sinh y_0)\, \exp\left(-\frac{r_T^2}{R_T^2}\right) \ , \tag{10}$$

where y_0 is the source rapidity, and the source distribution in the transverse direction is assumed to be a Gaussian with radius R_T. It is convenient to utilize the proper time $\tau = \sqrt{t^2 - z^2}$ and a rapidity-like variable $\eta = \frac{1}{2} \ln[(t + z)/(t - z)]$ in place of z and t. Then eq. (10) becomes

$$D(\tau, \eta, \vec{r}_T, y_0) = \frac{1}{\pi R_T^2 \tau_0}\, \delta(\tau - \tau_0)\, \delta(\eta - y_0)\, \exp\left(-\frac{r_T^2}{R_T^2}\right) \ . \tag{11}$$

More generally, eqs. (10) and (11) should contain a factor dN/dy_0, the density of sources in rapidity. We take dN/dy_0 to be a constant, thereby assuming that the central rapidity region extends to infinity in both directions. As long as we consider pions well within the central rapidity region, finite energy corrections should not be important.

From eq. (9) it is clear that G depends not only on D but also on the details of the production dynamics described by j_0. We consider two models for j_0. One, referred to as the pseudothermal model, is given by

$$j_0^{PS}(y - y_0, k_T) = \exp\left(-\frac{m_T}{2T} \cosh(y - y_0)\right) = \exp\left(-\frac{E'}{2T}\right) \ , \tag{12}$$

where E' is the pion energy in the rest frame of the pion emitter. This parameterizes the source in terms of an effective temperature T. The other model corresponds to a source in thermal equilibrium:

$$j_0^{TH}(y - y_0, k_T) = \sqrt{E'}\, \exp\left(-\frac{E'}{2T'}\right) \ . \tag{13}$$

This thermal parameterization, however, results in nonanalytic expressions for C; hence we present results here primarily for the pseudothermal case. We find, however, that the results for the two parameterizations agree closely if T' and T are related by $T \approx 1.42\, T' - 12.7$, where T' and T are in MeV.

The invariant single inclusive pion distribution with the pseudothermal source is simply

$$P_1(k) = \frac{d^3 n_\pi}{dy\, d^2 k_T} = a\, K_0(m_T/T) \quad , \tag{14}$$

where a is a normalization constant. Thus T characterizes the average transverse momenta of the pions. Furthermore, the pions are uniformly distributed in rapidity as a consequence of choosing dN/dy_0 to be constant. The function G is found to be

$$G(k_1, k_2) = a\, K_0(\sqrt{u})\, \exp\left(-\frac{q_T^2 R_T^2}{4}\right) \quad , \tag{15}$$

where the dynamical and geometric effects are convoluted through the variable

$$u = 2m_{1T}m_{2T}\left(\frac{1}{4T^2} + \tau_0^2\right)\cosh\Delta y + (m_{1T}^2 + m_{2T}^2)\left(\frac{1}{4T^2} - \tau_0^2\right) + \frac{i\tau_0}{T}(m_{1T}^2 - m_{2T}^2) \quad . \tag{16}$$

In eqs. (15) and (16), $\vec{q}_T = \vec{k}_{2T} - \vec{k}_{1T}$, $\Delta y = y_2 - y_1$, and $K_0(z) \propto \int dt \exp(-z\cosh t)$ is the conventional modified Bessel function of a complex argument.

The structure of the correlation function is obviously rather different than conventional Gaussian or Kopylov parameterizations [3,4]. In this model, it is a function of four kinematic variables: the two transverse masses m_{1T}, m_{2T}, the rapidity difference Δy, and the angle ϕ between $\vec{k}_{1T}$ and $\vec{k}_{2T}$ (which enters through q_T). It depends on three physical parameters, τ_0, T, and R_T, which can be extracted by fitting experimental results. Note that there is no obvious separation of dynamical and longitudinal geometric effects.

A multi-dimensional analysis of pion correlations is neccessary to disentangle the effects of T, τ_0, and R_T. This procedure requires three steps. First, T is determined by fitting the invariant single inclusive distribution P_1 as a function of m_T by eq. (14), as shown in figure 1. Next, we examine the correlation function for the case where $\vec{q}_T = 0$, thus eliminating the dependence on R_T. Since T is known from the first step, τ_0 can now be determined. In practice, this is done by examining either C as a function of Δy for fixed m_T ($m_{1T} = m_{2T}$ for this case) or C as a function of m_T for fixed Δy. These two cases are shown in figures 2(a) and 2(b), respectively. Finally, R_T is determined by fitting the correlation function by eqs. (15) and (16) for the general case where $\vec{q}_T \neq 0$. C can be examined as a function of any of the four variables, m_{1T}, m_{2T}, Δy, or ϕ, while the other three are held fixed. An example of C vs. ϕ for fixed m_{1T}, m_{2T}, and Δy is shown in figure 3. We note that the results of Pratt [8], although noncovariant, are in good numerical agreement with ours if his expressions are evaluated in the reference frame where $k_{1z} + k_{2z} = 0$.

Since in the general case, the effects of T, τ_0, and R_T are all mixed together, this three step approach is necessary to extract meaningful parameters from correlation data. The usual procedure of analyzing the correlation function as a function of one variable, for example $|\vec{q}|$, and integrating over all other variables, is clearly inadequate. It is obvious from eq. (16) that the effective source radius thus obtained would be a nonlinear function of the three parameters in our model. Determination of the inside-outside cascade nature of the dynamics therefore requires the more complex analysis outlined above.

For practical applications, it will be important to choose a more realistic source distribution D than in eq. (11). For example, fluctuations in τ_0 and a realistic dN/dy_0 should be considered. Examples of more detailed dynamical calculations based on string models can be found in refs. [10] and [11]. In addition, final state interactions [4] must be taken into account. The main advantage of our analytic treatment is that the influence of the inside-outside cascade on $\pi\pi$ correlations is revealed in a particularly transparent way. It also suggests that the natural choice of variables and method of analysis should be significantly different than those used in pion correlation studies thus far.[2]

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[2] After completion of this work, we learned of a preprint by Hama and Padula [12] addressing this topic based on the Landau hydrodynamic model.

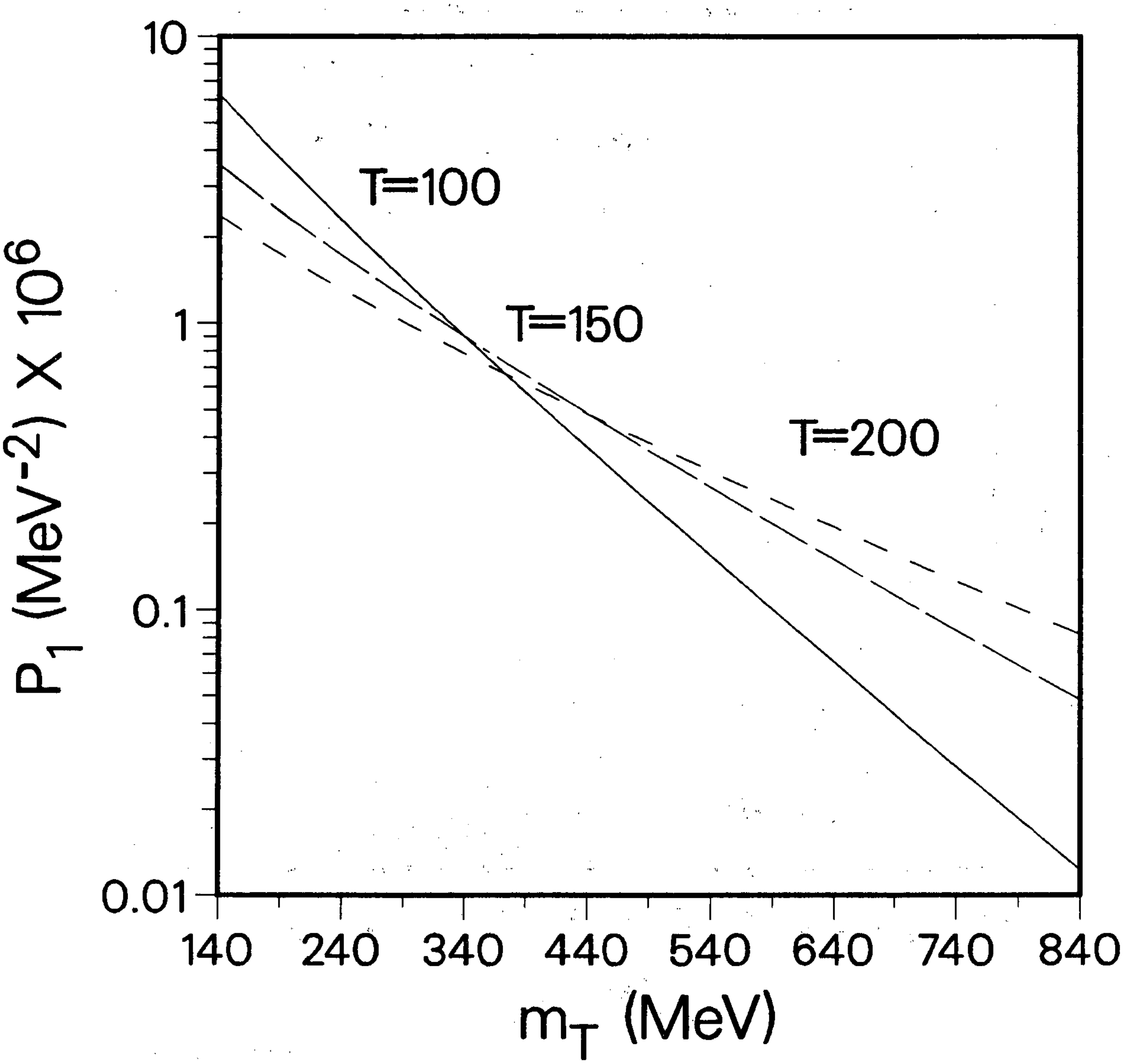
FIGURE CAPTIONS

1. Invariant single particle distribution P_1 vs. m_T for several values of the temperature parameter T (in MeV) in the pseudothermal model.

2. Correlation function C vs. Δy for $q_T = 0$ and fixed m_T (part a) and C vs. m_T for $q_T = 0$ and fixed Δy (part b). Labels indicate values of τ_0 in fm. Pseudothermal (PS) and thermal (TH) model results are shown separately where they differ significantly.

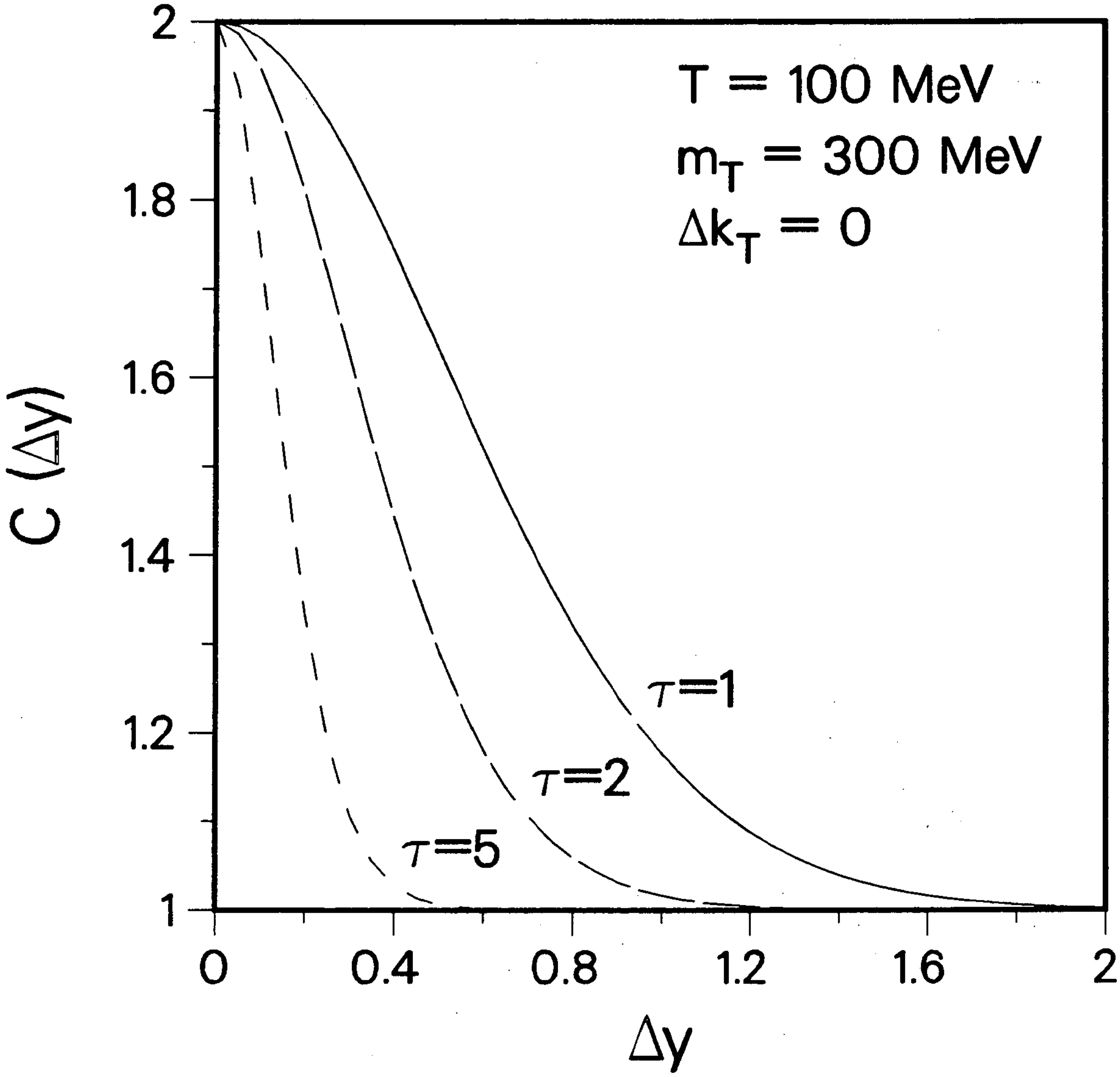
3. C vs. ϕ for fixed m_{1T}, m_{2T}, and Δy, where ϕ is the azimuthal angle between the transverse momenta $\vec{k}_{1T}$ and $\vec{k}_{2T}$. Labels indicate values of the transverse radius R_T in fm.

FIGURE 1



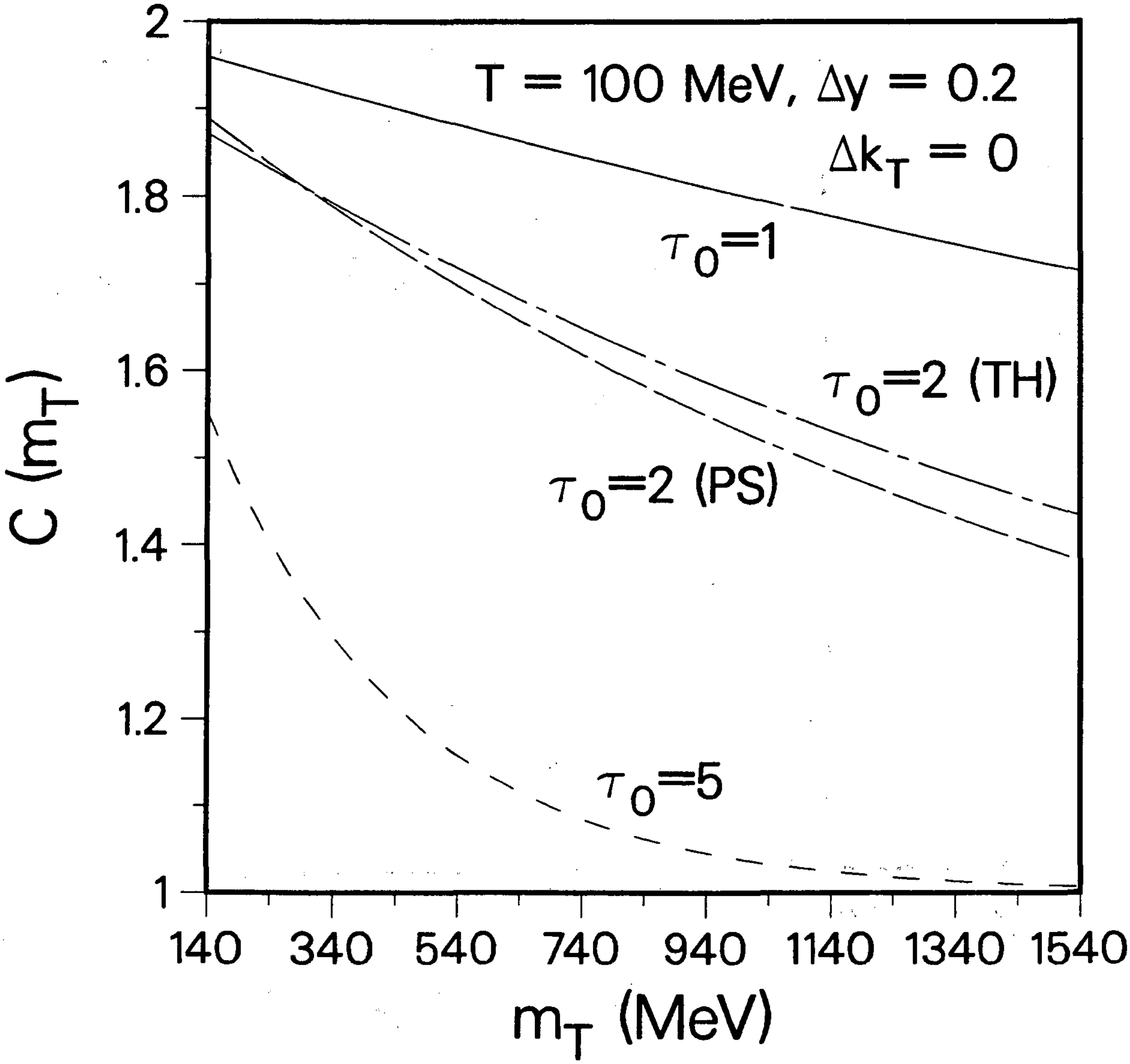
-- XBL 865-1987 --

FIGURE 2(a)



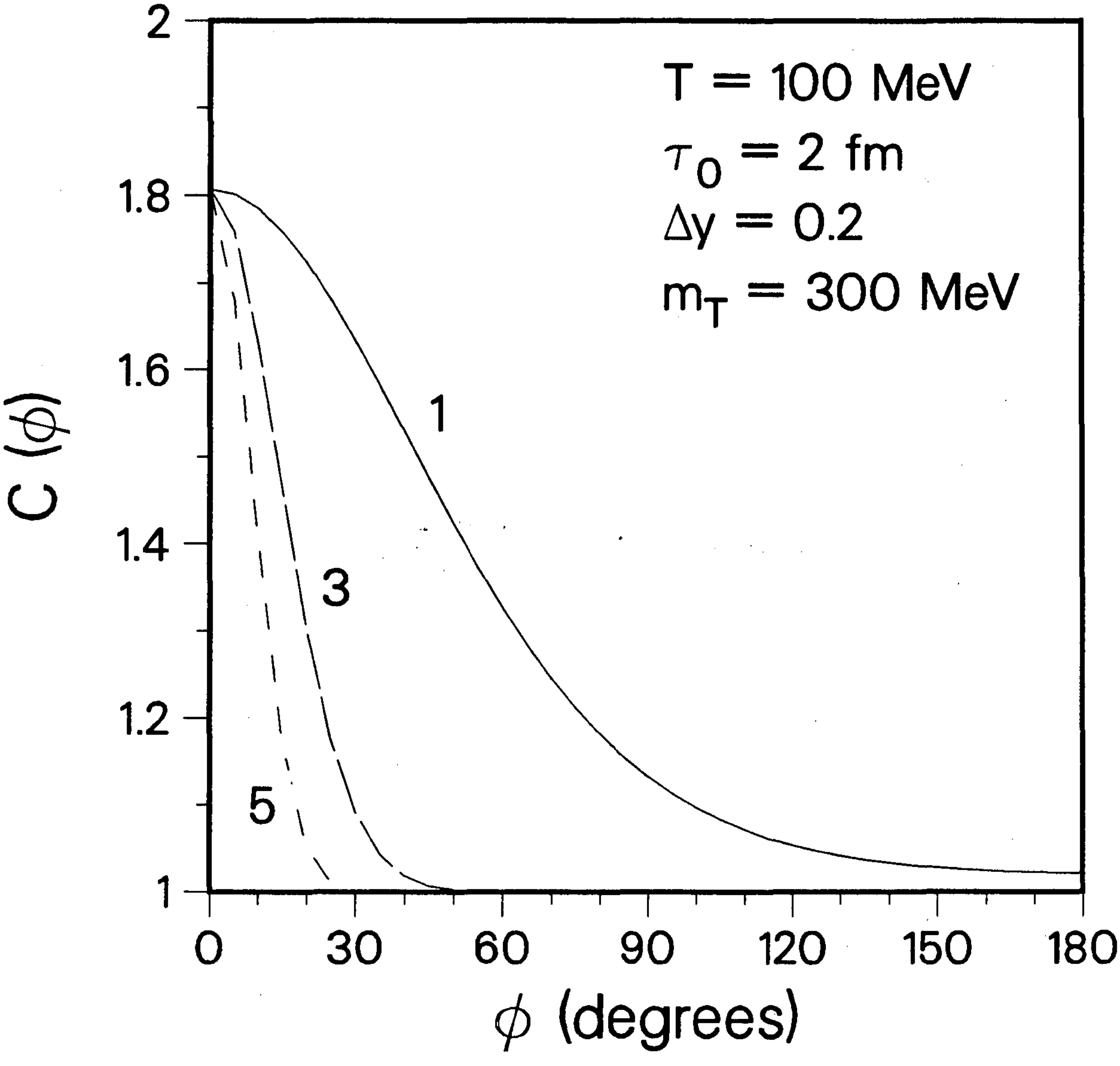
-- XBL 865-1990 --

FIGURE 2(b)



-- XBL 865-1989 --

FIGURE 3



-- XBL 865-1988 --