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Mapping the geometry of the F_4 group

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Abstract

In this paper we present a construction of the compact form of the exceptional Lie group F_4 by exponentiating the corresponding Lie algebra f_4. We realize F_4 as the automorphisms group of the exceptional Jordan algebra, whose elements are 3×3 hermitian matrices with octonionic entries. We use a parametrization which generalizes the Euler angles for $SU(2)$ and is based on the fibration of F_4 via a $Spin(9)$ subgroup as a fiber. This technique allows us to determine an explicit expression for the Haar invariant measure on the F_4 group manifold. Apart from shedding light on the structure of F_4 and its coset manifold $\mathbb{OP}^2 = F_4/Spin(9)$, the octonionic projective plane, these results are a prerequisite for the study of E_6, of which F_4 is a (maximal) subgroup.

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1 Introduction.

Simple Lie groups are well understood, starting from their complete classification. However, often one encounters some points which require a more detailed discussion or a new perspective. Our main interest, as an application to physics, is the construction of the E_6 group in a suitable parametrization adapted to perform non perturbative computations in GUT theories. While searching for such a construction, we have found it convenient to first determine an analog construction for its maximal subgroup F_4, which deserves a complete analysis by itself. Even though F_4 does not have a direct application to GUT theories, there are other motivations to consider F_4 separately. For example, the construction of integrable models on exceptional Lie groups and the corresponding coset manifolds could give rise to new families of integrable hierarchies. The interest for such problems is related to the fact that these groups are exceptional, which contrasts with the infinity of the classical series A_n, B_n, C_n, D_n. Of particular interest, from the mathematical point of view, is the coset manifold $\mathbb{OP}^2 = F_4/Spin(9)$, the octonionic projective plane.
However, our paper must be mainly thought of as a preparation for the construction of the E_6 group, which will be presented in a separated article. As the form of this group relevant for physics is the compact one, we need in particular the compact form of the F_4 group.
Here we realize F_4 as the group of automorphisms of the exceptional Jordan algebra. This is a 27 dimensional abelian algebra whose elements are 3×3 hermitian matrices with octonionic entries. The abelian product is obtained by symmetrizing the usual matrix product (which takes into account the octonionic product). In section 2 we describe shortly the exceptional Jordan algebra and the corresponding algebra f_4 of the infinitesimal automorphisms.
In section 3 we construct the group F_4 by exponentiating the algebra in an suitable way. The main idea is to obtain a generalized Euler parametrization of the group, in the same spirit of our previous papers [2]. However, here we also clarify the general strategy of the construction and some technical points. In particular we show the surjectivity of our map, a fact which we assumed to be true without proof in our previous papers. Some technical details are put in the appendices, including the fundamental Mathematica programs we used to compute the algebra. All other calculations can be done by hand, as we have indeed done, so we do not include the Mathematica programs we used to check them.
Some possible applications are reported in the conclusions.

2 Construction of the f_4 algebra.

The compact form of the F_4 exceptional Lie group can be realized as the automorphism group of the Jordan algebra J_3 [7, 1], that is the algebra of 3×3 octonionic hermitian matrices with product $\circ$ defined by

$$A \circ B := \frac{1}{2}(A \cdot B + B \cdot A) \ , \tag{2.1}$$

where $A, B \in J_3$ and the dot is the usual product between matrices. Note that the generic J_3 matrix has the form

$$A = \begin{pmatrix} a_1 & o_1 & o_2 \\ o_1^* & a_2 & o_3 \\ o_2^* & o_3^* & a_3 \end{pmatrix} , \tag{2.2}$$

where a_i are real numbers and o_i are octonions. Thus, in this way we obtain a 27 dimensional representation for F_4. The irreducible 26 dimensional representation can be easily obtained by restricting the 27 dimensional one to $ker(\ell)$ [1], where ℓ is the linear operator

$$\ell : J_3 \longrightarrow \mathbb{R} , \quad A \mapsto \sum_{i=1}^{3} A_{ii} . \tag{2.3}$$

However, the 27 dimensional representation is interesting because it can be extended in a natural way to the 27 dimensional irreducible representation of the exceptional Lie group E_6. We will consider this extension in a future work.
If a Lie group is realized as the automorphism group of an algebra $\mathcal{A}$, its Lie algebra is then realized as the algebra of derivations on $\mathcal{A}$. To obtain the matrix representation of the f_4 algebra we first define the linear isomorphism

$$\begin{aligned} &\Phi : J_3 \longrightarrow \mathbb{R}^{27} , \quad A \mapsto \Phi(A) , \\ &\Phi(A) := \begin{pmatrix} a_1 \\ \rho(o_1) \\ \rho(o_2) \\ a_2 \\ \rho(o_3) \\ a_3 \end{pmatrix} , \end{aligned} \tag{2.4}$$

where A is as in (2.2) and ρ is the linear isomorphism between the octonions $\mathbb{O}$ and $\mathbb{R}^8$ given by[1]

$$\begin{aligned} &\rho : \mathbb{O} \longrightarrow \mathbb{R}^8 , \quad o = o^0 + \sum_{i=1}^{7} o^i i_i \mapsto \rho(o) , \\ &\rho(o) := \begin{pmatrix} o^0 \\ o^1 \\ o^2 \\ o^3 \\ o^4 \\ o^5 \\ o^6 \\ o^7 \end{pmatrix} . \end{aligned} \tag{2.5}$$

Next we define a $\circ$ product in $\mathbb{R}^{27}$ by means of Φ:

$$x \circ y := \Phi(\Phi^{-1}(x) \circ \Phi^{-1}(y)) , \quad \forall x, y \in \mathbb{R}^{27} . \tag{2.6}$$

[1] our conventions about octonions are explained in App. A

The derivations on J_3 are then represented by matrices $M \in M(\mathbb{R}, 27)$ which must satisfy the condition

$$M(x \circ y) = (Mx) \circ y + x \circ (My) \ , \quad \forall x, y \in \mathbb{R}^{27} \ . \tag{2.7}$$

These equations can be solved by means of Mathematica which gives in fact 52 independent solutions M_i, $i = 1, \ldots 52$, which we choose to normalize with respect to the condition $-\frac{1}{6} Trace(M_i M_j) = \delta_{ij}$ and $[M_i, M_j] = -\sum_{k=1}^{3} \epsilon_{ijk} M_k$ for $i, j \in \{1, 2, 3\}$. Let $\{e_a\}_{a=1}^{27}$ be the canonical base of $\mathbb{R}^{27}$. Since the irreducible representation is realized on $ker(\ell)$, we expect the linear combination $(e_1 + e_{18} + e_{27})/\sqrt{3}$, which we will call f_{27}, to be in the kernel of all the M_i, $i = 1, \ldots, 52$, as, in fact, can be easily checked. It is then convenient to express the matrices with respect to the new base $\{f_a\}_{a=1}^{27}$ of $\mathbb{R}^{27}$ defined by

$$f_1 := (e_1 - e_{18})/\sqrt{2} \ , \tag{2.8}$$

$$f_{18} := (e_1 + e_{18} - 2e_{27})/\sqrt{6} \ , \tag{2.9}$$

$$f_{27} := (e_1 + e_{18} + e_{27})/\sqrt{3} \ , \tag{2.10}$$

$$f_a := e_a \ , \text{ in the other cases,} \tag{2.11}$$

in order to explicitly exhibit the 26 dimensional representation. We will call c_i, $i = 1, \ldots, 52$, the resulting 27×27 matrices.

In App. B we present the program used to construct these matrices. The 26×26 representation is then obtained by deleting from each matrix the last row and the last column, which, in fact, vanish. The corresponding structure constants, which characterize the algebra and also realize the adjoint representation, are shown in App.D.

To check that indeed we obtained the generators of an f_4 algebra, we computed the corresponding roots. If C_i denotes the ith matrix in the adjoint representation, we use C_1, C_6, C_{15}, C_{30} as generators of a Cartan subalgebra to calculate the roots. These turn out to be the generators of an f_4 algebra, as expected. Moreover the corresponding Killing form is negative definite and proportional to the trace product defined by

$$\langle a, b \rangle := -\frac{1}{6} Trace(ab) \ , \tag{2.12}$$

where a and b are arbitrary $\mathbb{R}-$linear combinations of the matrices c_i. Thus we have obtained a compact form of f_4.

By direct inspection of the structure constants one can easily recognize a chain of subalgebras. The first 21 matrices generate an $so(7)$ subalgebra, whose $so(i)$ subalgebras, with $i = 6, 5, 4, 3$, are generated by the first $i(i-1)/2$ matrices, respectively. Again this can be checked computing the roots of the subalgebras. A possible choice for the Cartan subalgebra is C_1 for $so(3)$, C_1, C_6 for $so(4)$ and $so(5)$ and C_1, C_6, C_{15} for $so(6)$ and $so(7)$. Adding to $so(7)$ the matrices c_i with $i = 30, \ldots, 36$ we obtain an $so(8)$ subalgebra. This corresponds to the Lie algebra of the $Spin(8)$ subgroup of F_4 which leaves invariant the three matrices J_i, $i = 1, 2, 3$, where J_i has $J_{i,ii} = 1$ as the unique non-vanishing entry. To check this, one can notice that the J_i $(i = 1, 2, 3)$ correspond to the vectors e_i of $\mathbb{R}^{27}$, which are in the kernel of the given subset of matrices.

Finally there are three evident $so(9)$ subalgebras:

1. $so(9)_1$ obtained adding $c_{45}, \ldots, c_{52}$ to $so(8)$. This corresponds to the subgroup $Spin(9)_1$ of F_4 which leaves J_1 invariant[2];

2. $so(9)_2$ obtained adding $c_{37}, \ldots, c_{44}$ to $so(8)$. This corresponds to the subgroup $Spin(9)_2$ of F_4 which leaves J_2 invariant;

3. $so(9)_3$ obtained adding $c_{22}, \ldots, c_{29}$ to $so(8)$. This corresponds to the subgroup $Spin(9)_3$ of F_4 which leaves J_3 invariant.

Again this can be checked applying the given matrices to e_1, e_2 and e_3 respectively. We will use $Spin(9)_1$ and will refer to it simply as $Spin(9)$.
To end this section let us call p the linear complement of $so(9)$ in f_4. Looking at the structure constants we find

$$[so(9), p] \subset p \ , \tag{2.13}$$

$$[p, p] \subset so(9) \ , \tag{2.14}$$

which show a structure of direct product. We don't need to look at the structure constants to discover such a structure. It follows from the fact that the trace product is ad-invariant (therefore proportional to the Killing form, F_4 being simple) and the base of matrices is orthogonal.

3 Construction of the group F_4

For connected compact Lie groups the exponential map is surjective [6]. This means that we could introduce 52 parameters x^i and simply write

$$g = g(x^1, \ldots, x^{52}) = \exp(x^i c_i) \ , \tag{3.1}$$

for any given element $g \in F_4$. However we are searching for a different kind of parametrization, in the spirit of [2, 14]. The point is that, whereas there is no difficulty in computing the volume using the exponential map parametrization, the hard problem is the determination of the range of parameters. Moreover, the difficulties increase rapidly if one needs to compute the left invariant 1-forms $g^{-1}dg$. These problems are both resolved by means of an Euler type parametrization, which gives all the quantities in terms of trigonometric functions, instead of the $\sin x/x$ functions appearing when the exponential parametrization is used.

3.1 The generalized Euler construction.

We would like to explain our general strategy for constructing a Euler type parametrization. Let G be an $n-$dimensional simple Lie group and H be one of its closed subgroups. Let λ_i be a base for $\mathcal{G} := Lie(G)$, orthonormal with respect to the Killing form. Let us assume that the first $m := dim H$ generators are a base for $\mathcal{H} := Lie H$ and let us call $\mathcal{P}$ the subspace generated by the remaining generators so that $[Lie(H), \mathcal{P}] \subset \mathcal{P}$. This means that G/H is reductive. Then it follows that any $g \in G$ can be written in the form

$$g = \exp a \exp b \ , \qquad a \in \mathcal{P} \ , \ b \in \mathcal{H} \ . \tag{3.2}$$

[2] $Spin(9)$ appears as the subgroup of F_4 which fixes a matrix J of the Jordan algebra

It is an established fact that for compact simple Lie groups such a parametrization is surjective. This fact is not, generally, well known. We give another proof of it [13] in appendix E, because it constitutes an important step in our derivation.
The next step consists in finding a subset of linearly free elements $\tau_1, \ldots, \tau_k \in \mathcal{P}$ with the following properties

- if V is the linear subspace generated by τ_i, $i = 1, \ldots, k$, then $\mathcal{P} = Ad_H(V)$, that is, the whole $\mathcal{P}$ is generated from V through the adjoint action of H;
- V is minimal, in the sense that it does not contain any proper subspaces with the previous property.

This means that the general element g of G can be written in the form

$$g = \exp(\tilde{h}) \exp(v) \exp(h) \ , \quad h, \tilde{h} \in \mathcal{H} \ , \ v \in V \ . \tag{3.3}$$

This way of writing g is surjective but redundant. The redundancy will be $r = 2m + k - n$ dimensional, where $n = dim(G)$, $m = dim(H)$ and $k = dim(V)$. The point is that in general we need less then the whole H to generate the whole V by adjunction. In fact H will contain some subgroup K generating automorphisms of V

$$Ad_K : V \longrightarrow V \ . \tag{3.4}$$

Then K must be r-dimensional and the generalized Euler decomposition with respect to H

$$G = B \exp(V) H \ , \tag{3.5}$$

where $B := H/K$.
In general the technical difficulties arise in the construction of B. In order to minimize such difficulties it is convenient to choose for H the biggest subgroup of G.

3.2 The set up for F_4.

The maximal subgroup of F_4 is $H = Spin(9)$. In section 2 we have found three $Spin(9)$ subgroups. As we said there, we choose $H = Spin(9)_1$ which we will call simply $Spin(9)$. Then $\mathcal{P}$ is the 16 dimensional real vector space generated by the matrices c_i, with $i = 22, \ldots, 29, 37, \ldots, 44$. Looking at the structure constants, we see that we can take as V any one dimensional subspace of $\mathcal{P}$. We choose c_{22} as a base for V. Thus $r = 21$. Since V is one dimensional, we expect the subgroup K to commute with c_{22} and its dimension suggests that it could be a $Spin(7)$ subgroup of $Spin(9)$. Indeed, we can check that this is true. We know that the first 21 matrices generate an $so(7)$ algebra. We will now construct a new set of 21 generators $\tilde{c}_i$, $i = 1, \ldots, 21$ commuting with c_{22} and having the same structure constants as the previous ones. To this end let us look at the $so(8)$ subalgebra: c_I, $I = 1, \ldots, 21, 30, \ldots, 36$. In particular let us start with c_α, $\alpha = 30, \ldots, 36$. Then from App.D we see that the remaining first 21 matrices can be generated as follows

$$c_{\frac{k(k-1)}{2}+i+1} = [c_{30+i}, c_{30+k}] \ , k = 1, \ldots, 6, \ i = 0, \ldots, k-1 \ . \tag{3.6}$$

Next, we notice that for $a, b \in \{22, \ldots, 29\}$ the commutator $[c_a, c_b]$ is a combination of four elements of $so(8)$, each of which has the same commutator with c_{22}. Using this, we define

$$\tilde{c}_{30+i} := -[c_{22}, c_{23+i}] \ , \quad i = 0, \ldots, 7 \ , \tag{3.7}$$

and then

$$\tilde{c}_{\frac{k(k-1)}{2}+i+1} = [\tilde{c}_{30+i}, \tilde{c}_{30+k}] \ , k = 1, \ldots, 6, \ i = 0, \ldots, k-1 \ . \tag{3.8}$$

The surprising fact, for which we have no explanation, is that the matrices $\tilde{c}_I$, with $I = 1, \ldots, 21, \ 30, \ldots, 36$ have exactly the same structure constants of c_I and $[\tilde{c}_i, c_{22}] = 0$ for $i = 1, \ldots, 21$. This is exactly the $so(7)$ we were searching for. We will call it $\widetilde{so}(7)$, so that $K = \exp(\widetilde{so}(7))$.
At this point let us note that in order to construct the Euler parametrization we proceed by induction: Together with B one needs to give the parametrization of the maximal subgroup $H = Spin(9)$. Again, this can be done applying the generalized Euler parametrization with respect to the maximal subgroup $Spin(8)$. Next, $Spin(8)$ could be decomposed with respect to $Spin(7)$ and so on. In conclusion it is convenient to start from $SU(2)$, to construct $Spin(n)$ up to $n = 9$ and finally F_4. At any step (ensured the surjectivity) the range of parameters can be determined by means of the topological method explained in [2]. To simplify our exposition we will give details only for the most interesting case F_4, and limit the $Spin(j)$ subgroups to a list. The details can be easily reproduced in the same way as for F_4.

3.3 The list of subgroups.

Here we give the results for the subgroups. The details could be considered as an exercise. Rational homology groups and roots, necessary ingredients for the Macdonald formula, are given in App.F.

3.3.1 $SU(2)$.

The generators are $c_i, \ i = 1, 2, 3$. We have

$$SU(2)[x_1, x_2, x_3] = e^{x_1 c_3} e^{x_2 c_2} e^{x_3 c_3} \ , \tag{3.9}$$

with range

$$x_1 \in [0, 2\pi] \ , \quad x_2 \in [0, \pi] \ , \quad x_3 \in [0, 4\pi] \ . \tag{3.10}$$

Note that 4π is the period of $\exp(x c_i)$ for every $i = 1, \ldots, 3$. The invariant measure is

$$d\mu_{SU(2)}[x_1, x_2, x_3] = \sin x_2 dx_1 dx_2 dx_3 \ . \tag{3.11}$$

3.3.2 $Spin(4)$.

The generators are $c_i, \ i = 1, \ldots, 6$. We take $H = SU(2)$ generated by c_5, c_6, c_3 which can be obtained from the previous one by simple substitutions. V is one dimensional and we can take c_4

as generator. $r = 1$ and $K = U(1) = e^{xc_3}$ so that $B[x, y] = H/K = e^{xc_3}e^{yc_5}$, with $x \in [0, 2\pi]$ and $y \in [0, \pi]$. Then

$$Spin(4)[x_1, \ldots, x_6] = e^{x_1 c_3} e^{x_2 c_5} e^{x_3 c_4} SU(2)[x_4, x_5, x_6] \ . \tag{3.12}$$

The invariant measure is

$$d\mu_{_{Spin(4)}} = \sin x_2 \sin^2 x_3 dx_1 dx_2 dx_3 d\mu_{_{SU(2)}}[x_4, \ldots, x_6] \ , \tag{3.13}$$

and the range of parameters

$$x_1 \in [0, 2\pi] \ , \quad x_2 \in [0, \pi] \ , \quad x_3 \in [0, \pi] \ , \tag{3.14}$$

the others being the ones of $SU(2)$.

3.3.3 *Spin*(5).

The generators are $c_i, \ i = 1, \ldots, 10$. The subgroup is $H = Spin(4)$ as before. V is one dimensional and we can take c_7 as generator. $r = 3$ and $K = SU(2)$ generated by c_α, $\alpha = 3, 5, 6$ commute with c_7 so that $B_5[x_1, x_2, x_3] = H/K = e^{x_1 c_3} e^{x_2 c_5} e^{x_3 c_4}$. Then

$$Spin(5)[x_1, \ldots, x_{10}] = B_5[x_1, \ldots, x_3] e^{x_4 c_7} Spin(4)[x_5, \ldots, x_{10}] \ . \tag{3.15}$$

The invariant measure is

$$d\mu_{_{Spin(5)}}[x_1, \ldots, x_{10}] = \sin x_2 \cos^2 x_3 \sin^3 x_4 dx_1 dx_2 dx_3 dx_4 d\mu_{_{Spin(4)}}[x_5, \ldots, x_{10}] \ . \tag{3.16}$$

and the range of parameters

$$x_1 \in [0, 2\pi] \ , \quad x_2 \in [0, \pi] \ , \quad x_3 \in [-\frac{\pi}{2}, \frac{\pi}{2}] \ , \quad x_4 \in [0, \pi] \tag{3.17}$$

the others being the ones of $Spin(4)$.

3.3.4 *Spin*(6).

The generators are $c_i, \ i = 1, \ldots, 15$. The subgroup is $H = Spin(5)$ as before. V is one dimensional and we can take c_{11} as generator. $r = 6$ and $K = Spin(4)$ generated by c_α, $\alpha = 3, 5, 6, 8, 9, 10$ commute with c_{11} so that

$$B_6[x_1, \ldots, x_4] = H/K = e^{x_1 c_3} e^{x_2 c_5} e^{x_3 c_4} e^{x_4 c_7} \ .$$

Then

$$Spin(6)[x_1, \ldots, x_{15}] = B_6[x_1, \ldots, x_4] e^{x_5 c_{11}} Spin(5)[x_6, \ldots, x_{15}] \ . \tag{3.18}$$

The invariant measure is

$$d\mu_{_{Spin(6)}}[x_1, \ldots, x_{15}] = \sin x_2 \cos^2 x_3 \cos^3 x_4 \sin^4 x_5 dx_1 dx_2 dx_3 dx_4 dx_5 d\mu_{_{Spin(5)}}[x_6, \ldots, x_{15}] \ , \tag{3.19}$$

and the range of parameters

$$\begin{aligned} &x_1 \in [0, 2\pi] \ , \quad x_2 \in [0, \pi] \ , \quad x_3 \in [-\frac{\pi}{2}, \frac{\pi}{2}] \ , \quad x_4 \in [-\frac{\pi}{2}, \frac{\pi}{2}] \ , \\ &x_5 \in [0, \pi] \ , \end{aligned} \tag{3.20}$$

the others being the ones of $Spin(5)$.

3.3.5 *Spin*(7).

The generators are $c_i,\ i = 1,\dots,21$. The subgroup is $H = Spin(6)$ as before. V is one dimensional and we can take c_{16} as generator. $r = 10$ and $K = Spin(5)$ generated by c_α, $\alpha = 3,5,6,8,9,10,12,13,14,15$ commute with c_{16} so that

$$B_7[x_1,\dots,x_5] = H/K = e^{x_1 c_3} e^{x_2 c_5} e^{x_3 c_4} e^{x_4 c_7} e^{x_5 c_{11}} \ .$$

Then

$$Spin(7)[x_1,\dots,x_{21}] = B_7[x_1,\dots,x_5] e^{x_6 c_{16}} Spin(6)[x_7,\dots,x_{21}] \ . \tag{3.21}$$

The invariant measure is

$$\begin{aligned} d\mu_{_{Spin(7)}}[x_1,\dots,x_{21}] = \sin x_2 \cos^2 x_3 \cos^3 x_4 \cos^4 x_5 \sin^5 x_6 dx_1 dx_2 dx_3 dx_4 dx_5 dx_6 \cdot \\ \cdot d\mu_{_{Spin(6)}}[x_7,\dots,x_{21}] \ , \end{aligned} \tag{3.22}$$

and the range of parameters

$$\begin{aligned} & x_1 \in [0,2\pi] \ , \quad x_2 \in [0,\pi] \ , \quad x_3 \in [-\frac{\pi}{2},\frac{\pi}{2}] \ , \quad x_4 \in [-\frac{\pi}{2},\frac{\pi}{2}] \ , \\ & x_5 \in [-\frac{\pi}{2},\frac{\pi}{2}] \ , \quad x_6 \in [0,\pi] \ , \end{aligned} \tag{3.23}$$

the others being the ones of $Spin(6)$.

3.3.6 *Spin*(8).

The generators are $c_i,\ i = 1,\dots,21,30,\dots,36$. The subgroup is $H = Spin(7)$ as before. V is one dimensional and we can take c_{30} as generator. $r = 15$ and $K = Spin(6)$ generated by c_α, $\alpha = 3,5,6,8,9,10,12,13,14,15,17,18,19,20,21$. Up to now we have proceeded in a systematical way. One could proceed in this way but technical difficulties increase with the dimension of the group. In particular the computation of the invariant measure becomes too hard for F_4. To solve this problem we have found it convenient to change the parameterization of the quotient B. The simplification consists in using as many commuting matrices as possible to realize B. This must be compatible with the fact that $B \cdot K$ must cover the whole $Spin(7)$ group. There are many possibilities. We have chosen

$$B_8[x_1,\dots,x_6] = e^{x_1 c_3} e^{x_2 c_{16}} e^{x_3 c_{15}} e^{x_4 c_{35}} e^{x_5 c_5} e^{x_6 c_1} \ . \tag{3.24}$$

The fact that it works can be checked by doing the previous analysis backward. Then

$$Spin(8)[x_1,\dots,x_{28}] = B_8[x_1,\dots,x_6] e^{x_7 c_{30}} Spin(7)[x_8,\dots,x_{28}] \ . \tag{3.25}$$

The invariant measure is

$$\begin{aligned} d\mu_{_{Spin(8)}}[x_1,\dots,x_{28}] = \sin x_4 \cos x_5 \cos x_6 \sin^2 x_6 \cos^2 x_7 \sin^4 x_7 \prod_{i=1}^{7} dx_i \cdot \\ \cdot d\mu_{_{Spin(7)}}[x_8,\dots,x_{28}] \ , \end{aligned} \tag{3.26}$$

and the range of parameters

$$x_1 \in [0, 2\pi] \ , \quad x_2 \in [0, 2\pi] \ , \quad x_3 \in [0, 2\pi] \ , \quad x_4 \in [0, \pi] \ ,$$
$$x_5 \in [-\frac{\pi}{2}, \frac{\pi}{2}] \ , \quad x_6 \in [0, \frac{\pi}{2}] \ , \quad x_7 \in [0, \frac{\pi}{2}] \ , \tag{3.27}$$

the others being the ones of $Spin(7)$.

3.3.7 *Spin*(9).

Here we used the same procedure as for $Spin(8)$. The generators are c_i, with $i = 1, \ldots, 21, 30, \ldots, 36, 45, \ldots, 52$. The subgroup is $H = Spin(8)$ as before. V is one dimensional and we can take c_{45} as generator. $r = 21$ and $K = Spin(7)$ generated by c_α, $\alpha = 3, 5, 6, 8, 9, 10, 12, 13, 14, 15, 17, 18, 19, 20, 21, 31, 32, 33, 34, 35, 36$. The choice of B can be deduced from the one for $Spin(8)$. Then

$$B_9[x_1, \ldots, x_7] = e^{x_1 c_3} e^{x_2 c_{16}} e^{x_3 c_{15}} e^{x_4 c_{35}} e^{x_5 c_5} e^{x_6 c_1} e^{x_7 c_{30}} \ . \tag{3.28}$$

and

$$Spin(9)[x_1, \ldots, x_{36}] = B_9[x_1, \ldots, x_7] e^{x_8 c_{45}} Spin(8)[x_9, \ldots, x_{36}] \ . \tag{3.29}$$

The invariant measure is

$$d\mu_{_{Spin(9)}}[x_1, \ldots, x_{36}] = \sin x_4 \cos x_5 \cos x_6 \sin^2 x_6 \cos^4 x_7 \sin^2 x_7 \sin^7 x_8 \prod_{i=1}^{8} dx_i \cdot$$
$$\cdot d\mu_{_{Spin(8)}}[x_9, \ldots, x_{36}] \ , \tag{3.30}$$

and the range of parameters

$$x_1 \in [0, 2\pi] \ , \quad x_2 \in [0, 2\pi] \ , \quad x_3 \in [0, 2\pi] \ , \quad x_4 \in [0, \pi] \ ,$$
$$x_5 \in [-\frac{\pi}{2}, \frac{\pi}{2}] \ , \quad x_6 \in [0, \frac{\pi}{2}] \ , \quad x_7 \in [0, \frac{\pi}{2}] \ , \quad x_8 \in [0, \pi] \ , \tag{3.31}$$

the others being the ones of $Spin(8)$.

3.4 Construction of F_4.

We are now ready to realize the construction of the group F_4. The maximal subgroup is a $Spin(9)$ subgroup which we choose to be $Spin(9)_1$. Then we know that the generic element of F_4 can be written formally as

$$F_4 = e^{\mathcal{P}} e^{so(9)} \ , \tag{3.32}$$

where $\mathcal{P}$ is the linear space generated by the matrices c_i, with $i = 22, \ldots, 29, 37, \ldots, 44$. Looking at the structure constants we can see that

$$\exp(-2xc_i) c_{22} \exp(2xc_i) = \cos x c_{22} \pm \sin x c_j \ , \tag{3.33}$$

where $j = 29, 25, 28, 23, 27, 26, 24$ respectively, if $i = 30, \dots, 36$ and $j = 44, 40, 43, 38, 42, 41, 39, 37$ for $i = 45, \dots, 52$. This ensures that $\mathcal{P}$ can be generated acting on c_{22} by adjunction with H.[3] Thus the generic element of F_4 is

$$g = e^a e^{x c_{22}} e^b \ , \tag{3.34}$$

with $a, b \in so(9)$. We know that the 21 dimensional redundancy of such a parametrization is due to a $Spin(7)$ subgroup of $Spin(9)$ which commutes with c_{22}. This subgroup is generated by the matrices $\tilde{c}_i$ with $i = 1, \dots 21$, which satisfy the same commutation relations of the corresponding c_i. In fact this is true also adding the $\tilde{c}_i$, $i = 30, \dots, 36$ and moreover, for the whole $Spin(9)$ subgroup, if we define $\tilde{c}_i = c_i$ for $i = 45, \dots, 52$. Thus we can use $\tilde{c}_i$ in place of c_i to construct $Spin(9)$, at least for the left factor in (3.34). Now in our construction

$$\begin{aligned} Spin(9)[x_1, \dots, x_{36}] &= B_9[x_1, \dots, x_7] e^{x_8 \tilde{c}_{45}} Spin(8)[x_9, \dots, x_{36}] \\ &= B_9[x_1, \dots, x_7] e^{x_8 \tilde{c}_{45}} B_8[x_9, \dots, x_{14}] e^{x_{15} \tilde{c}_{30}} Spin(7)[x_{16}, \dots, x_{36}] \ , \end{aligned} \tag{3.35}$$

so that

$$B_{F_4}[x_1, \dots, x_{15}] = B_9[x_1, \dots, x_7] e^{x_8 \tilde{c}_{45}} B_8[x_9, \dots, x_{14}] e^{x_{15} \tilde{c}_{30}} \ , \tag{3.36}$$

and

$$F_4[x_1, \dots, x_{52}] = B_{F_4}[x_1, \dots, x_{15}] e^{x_{16} c_{22}} Spin(9)[x_{17}, \dots, x_{52}] \ . \tag{3.37}$$

We also know that for the ranges determined for $Spin(9)$, $Spin(8)$ and $Spin(7)$, B_{F_4} covers the whole H/K, so that only the range for x_{16} remains to be determined. However we will now determine the range of all parameters.

3.5 Determination of the range of parameters.

To determine the range we will use the topological method introduced in [2]. For convenience let us recall here how it works. The first step consists in the determination of the invariant measure. It will depend explicitly on some of the parameters. One can then construct a closed variety, having the same dimension of the whole group, simply by choosing for these variables the maximal range which still allows the measure to be well defined, while for the remaining variables the range should coincide with their period.[4] If the parametrization adopted is surjective and the group is connected, then surely the variety obtained in this way covers the whole group. Here is where surjectivity is crucial!

At this point it is possible that, with this choice of parameters, we cover some points of the the group more then once. Fortunately one can check this by means of the Macdonald formula [12, 9] which gives the volume of a compact Lie group with respect to an invariant measure induced on the group by a Lebesgue measure on the Lie algebra. If the resulting number of covering is higher than 1, the range of parameters must be further reduced using some automorphism of the space of parameters which leaves the group invariant under reparametrization. See [2] for more details.

[3] More precisely this means that we could write the general element of F_4 in the form $F_4 = B[x_1, \dots, x_{15}] e^{x_{16} c_{22}} Spin(9)$, with $B = \prod_{i=1}^{15} e^{x_i c_{j_i}}$, $j_i = 30, \dots, 36, 45, \dots, 52$. However such a realization is not sufficiently simple to allow technical computations.

[4] Each of the x_i appears in the parametrization in the form $e^{x_i c_i}$ or $e^{x_i \tilde{c}_i}$, and is therefore periodic as a consequence of compactness of the group. With our normalization, we find that all periods are equal to 4π.

3.5.1 The volume of F_4.

Let us compute the volume of F_4 by means of the Macdonald formula. The Betty numbers of the exceptional Lie groups were computed in [5]. For F_4 there are four free generators for the rational homology. Their dimensions are

$$d_1 = 3,\ d_2 = 11,\ d_3 = 15,\ d_4 = 23\ . \tag{3.38}$$

The simple roots are [8]

$$r_1 = L_2 - L_3 \tag{3.39}$$
$$r_2 = L_3 - L_4 \tag{3.40}$$
$$r_3 = L_4 \tag{3.41}$$
$$r_4 = \frac{L_1 - L_2 - L_3 - L_4}{2} \tag{3.42}$$

where L_i, $i = 1, \ldots, 4$ is an orthonormal base for the Cartan algebra. The volume of the fundamental region is then

$$Vol(f_{F_4}) = \frac{1}{2}\ . \tag{3.43}$$

Furthermore there are 24 positive roots, 12 of which have length 1, and 12 have length $\sqrt{2}$ [8]. We found explicitly these roots, as explained in section 2, with $L_i = e_i$, the canonical base of $\mathbb{R}^4$. The volume of F_4 is then

$$Vol(F_4) = \frac{2^{26} \cdot \pi^{28}}{3^7 \cdot 5^4 \cdot 7^2 \cdot 11}\ . \tag{3.44}$$

3.5.2 The invariant measure on F_4.

The invariant measure on F_4 decomposes in the product of the measure on $Spin(9)$ and the one on $M = F_4/Spin(9)$. This was shown in general in [2] but let us rewrite it in terms of (3.5). If we define

$$J_H := H^{-1}dH\ , \qquad J_M := \pi_{\mathcal{P}}(e^{-x_{16}c_{22}} B_{F_4}^{-1} d(B_{F_4} e^{x_{16}c_{22}}))\ , \tag{3.45}$$

with $H = Spin(9)$, then

$$ds_M^2 = -\frac{1}{6} Trace(J_M \otimes J_M)\ , \tag{3.46}$$

is the induced invariant measure on M and

$$d\mu_{F_4} = |det(J_{Mi}^j)| d\mu_{Spin(9)} \prod_{l=1}^{16} dx_l\ , \tag{3.47}$$

where J^j_{Mi} is the 16×16 matrix defined by

$$J_M = \sum_{i,j=1}^{16} J^j_{Mi} c_{i_j} dx^i \ , \tag{3.48}$$

where c_{i_j} is the base $\{c_{22}, \ldots, c_{29}, c_{37}, \ldots, c_{44}\}$ of $\mathcal{P}$.
Let us now introduce the notation

$$M_8[x_1, \ldots, x_8] := B_9[x_1, \ldots, x_7] e^{x_8 \tilde{c}_{45}} \ , \tag{3.49}$$

$$M_7[x_9, \ldots, x_{15}] := B_8[x_9, \ldots, x_{14}] e^{x_{15} \tilde{c}_{30}} \ . \tag{3.50}$$

Then

$$\begin{aligned} J_M &= dx_{16} c_{22} + e^{-x_{16} c_{22}} M_7^{-1} dM_7 e^{x_{16} c_{22}} + e^{-x_{16} c_{22}} M_7^{-1} M_8^{-1} dM_8 M_7 e^{x_{16} c_{22}} \\ &=: dx_{16} c_{22} + e^{-x_{16} c_{22}} J_7 e^{x_{16} c_{22}} + e^{-x_{16} c_{22}} M_7^{-1} J_8 M_7 e^{x_{16} c_{22}} \ . \end{aligned} \tag{3.51}$$

Some remarks are in order now

1. $M_7 \in Spin(8)$ corresponding to the algebra of matrices c_i with $i = 1, \ldots, 21, 30, \ldots, 36$;
2. $M_8 \in Spin(9)$ corresponding to the algebra of matrices c_i with $i = 1, \ldots, 21, 30, \ldots, 36, 45, \ldots, 52$;
3. looking at commutators we see that the adjoint action of $e^{\alpha_{16} c_{22}}$ on $so(8)$ generate linear combination of the matrices of $so(8)$ itself, adding also combination of the matrices c_j with $j = 23, \ldots, 29$;
4. the adjoint action of $Spin(8)$ on $so(9)$ restricted to the linear subspace generated by $c_{45}, \ldots, c_{52}$ is a rotation, that is $Spin(8)^{-1} c_i Spin(8) = \sum_{j=45}^{52} R_i{}^j c_j$ with $i = 45, \ldots, 52$, where $R_i{}^j$ is a rotation matrix. In particular $|\det R_I{}^J| = 1$;
5. the adjoint action of $e^{x c_{22}}$ on c_i with $i = 45, \ldots, 52$ is a rotation of the form $c_i \longmapsto \cos \frac{x}{2} c_i \pm \sin \frac{x}{2} c_{\tilde{i}}$, where $\tilde{i} \in \{37, \ldots 44\}$.

From these remarks one can deduce

1. dx_{16} is the only coefficient of c_{22};
2. the projection of $e^{-x_{16} c_{22}} J_7 e^{x_{16} c_{22}}$ on c_i, $i = 22, 37, \ldots, 44$, vanishes, so that it gives rise to a 7×7 diagonal block. In other words, if the columns give the projections on c_i (with ordering $i = 22, \ldots, 29, 37, \ldots, 44$) and the rows are the differentials $d\alpha_j$ (with $J = 1, \ldots, 16$ starting from below) then we must compute the determinant of the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ * & A & 0 \\ * & * & B \end{pmatrix} \tag{3.52}$$

   where A is the 7×7 diagonal block given above and B is a 8×8 block obtained by projecting $e^{-x_{16} c_{22}} M_7^{-1} J_8 M_7 e^{x_{16} c_{22}}$ on $c_{37}, \ldots, c_{44}$. The $*$ blocks are irrelevant for the computation of the determinant which in fact will be $\det A \cdot \det B$;

3. from point 5 of the remarks it follows $\det B = \sin^8 \frac{x_{16}}{2} \det \tilde{B}$, where $\tilde{B}$ is the projection of $M_7^{-1} J_8 M_7$ on $c_{45}, \ldots, c_{52}$. On the other hand, from the remaining remarks it follows $\det \tilde{B} = \det R \cdot \det \hat{B}$, where R is the orthogonal matrix introduced in remark 4 and does not contribute to the determinant, whereas $\hat{B}$ is the projection of J_8 on $c_{45}, \ldots, c_{52}$. In particular $\det \hat{B} = \sin x_4 \cos x_5 \cos x_6 \sin^2 x_6 \cos^4 x_7 \sin^2 x_7 \sin^7 x_8$.

Thus we can quite easily compute the invariant measure for M, which turns out to be

$$d\mu_M = 2^7 \cos^7 \frac{x_{16}}{2} \sin^{15} \frac{x_{16}}{2} \sin x_4 \cos x_5 \cos x_6 \sin^2 x_6 \cos^4 x_7 \sin^2 x_7 \sin^7 x_8 \cdot \\ \cdot \sin x_{12} \cos x_{13} \cos x_{14} \sin^2 x_{14} \cos^2 x_{15} \sin^4 x_{15} \prod_{i=1}^{16} dx_i \ . \tag{3.53}$$

Note that the periods of the variables are 4π so that one should take the range $x_i = [0, 4\pi]$ for $i = 1, 2, 3$ and $i = 9, 10, 11$. However it is easy to show directly from the parametrization that they can all be restricted to $[0, 2\pi]$. In fact for all $\tilde{c}_i \in so(7)$ we have that $e^{2\pi \tilde{c}_i}$ commute with $\tilde{c}_j$ and with c_{22}, so that it can be reabsorbed in the $Spin(9)$ factor of F_4. The range of x_i is then

$$\begin{array}{llll} x_1 \in [0, 2\pi] \ , & x_2 \in [0, 2\pi] \ , & x_3 \in [0, 2\pi] \ , & x_4 \in [0, \pi] \ , \\ x_5 \in [-\frac{\pi}{2}, \frac{\pi}{2}] \ , & x_6 \in [0, \frac{\pi}{2}] \ , & x_7 \in [0, \frac{\pi}{2}] \ , & x_8 \in [0, \pi] \ , \\ x_9 \in [0, 2\pi] \ , & x_{10} \in [0, 2\pi] \ , & x_{11} \in [0, 2\pi] \ , & x_{12} \in [0, \pi] \ , \\ x_{13} \in [-\frac{\pi}{2}, \frac{\pi}{2}] \ , & x_{14} \in [0, \frac{\pi}{2}] \ , & x_{15} \in [0, \frac{\pi}{2}] \ , & x_{16} \in [0, \pi] \ . \end{array} \tag{3.54}$$

Note that the firsts 15 are exactly the ones predicted by $Spin(9)/Spin(7)$. The remaining parameters x_j, $j = 17, \ldots, 52$, will run over the range for $Spin(9)$. The volume of the whole closed cycle V so obtained is then

$$Vol(V) = Vol(Spin(9)) \int_R d\mu_M = \frac{2^{26} \cdot \pi^{28}}{3^7 \cdot 5^4 \cdot 7^2 \cdot 11} \ , \tag{3.55}$$

where R is the range of parameters x_i, $i = 1, \ldots, 16$. This is the volume of F_4, so that we cover the group exactly one time.[5]

4 Conclusions.

In this paper we have considered the problem of giving an explicit construction of the F_4 simple Lie group and in particular of its compact form. The main motivation is that we are interested in studying the Lie group E_6 in a future paper, because in its compact realization it is the most promising exceptional Lie group for unification in GUT theories [3]. In particular to perform non perturbative calculations a parameterization is needed which, on the one hand should yield the most simple expression for the invariant measure on the group, while at the same time still being able of providing an explicit expression for the range of the parameters. Both these requirements are necessary in order to minimize the computation power needed for computer simulations of

[5] Obviously there is a subset of vanishing measure multiply covered.

lattice models.
It seems that the best solution to both of these problems is the determination of Euler like angles. In section 3.1 we have explained in detail the general strategy for defining such a parametrization and shown that it turns out to be surjective for each compact Lie group. It is clear from the construction that the Euler angles for a given group are not uniquely defined, but that they can depend for example on the choice of a subgroup, which can be fixed according to the requirements. The surjectivity of the map allows the use of a topological method to determine the range of parameters.
The first application of our results will be the construction of the generalized Euler parametrization of the E_6 group associated with the F_4 subgroup, along the lines of section 3.1. However another immediate interesting application could be the explicit construction of the F_4 invariant metric for $\mathbb{OP}^2 = F_4/Spin(9)$.

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A The octonionic algebra.

The octonionic algebra is obtained from the eight dimensional real vector space $\mathbb{O}$ generated by a *real unit* 1 and seven *imaginary units* i_i, $i = 1, \ldots, 7$. The structure of a non-abelian, non-associative division algebra is obtained introducing a distributive product

$$\cdot : \mathbb{O} \times \mathbb{O} \longrightarrow \mathbb{O} \ , \quad (a, b) \mapsto ab \ ,$$

by means of the following rules:

- 1 is the identity for the product;
- $i_i^2 = -1$ for $i = 1, \ldots, 7$ and $i_i i_j = -i_j i_i$ for $1 \leq i_i < i_j \leq 7$;
- the units i_1, i_2, i_3 generate a quaternionic subalgebra;
- the remaining independent products between the imaginary units are given in the next program, where $e[1] = 1$ and $e[i+1] = i_i$, $i = 1, \ldots, 7$.

B The f_4 matrices.

The matrices we found using Mathematica, and orthonormalized with respect to the scalar product $\langle a, b \rangle := -\frac{1}{6} Trace(ab)$, were computed by means of the followings programs.

B.1 Construction of the matrices.

The following program gives the 27×27 matrices of F_4 before and after the 26 dimensional reduction.

```
%%%% the octonionic products
QQ[1,1] = e[1];    QQ[1,2] = e[2];    QQ[1,3] = e[3];
QQ[1,4] = e[4];    QQ[1,5] = e[5];    QQ[1,6] = e[6];
QQ[1,7] = e[7];    QQ[1,8] = e[8];    QQ[2,1] = e[2];
QQ[2,2] = -e[1];   QQ[2,3] = e[5];    QQ[2,4] = e[8];
QQ[2,5] = -e[3];   QQ[2,6] = e[7];    QQ[2,7] = -e[6];
QQ[2,8] = -e[4];   QQ[3,1] = e[3];    QQ[3,2] = -e[5];
QQ[3,3] = -e[1];   QQ[3,4] = e[6];    QQ[3,5] = e[2];
QQ[3,6] = -e[4];   QQ[3,7] = e[8];    QQ[3,8] = -e[7];
QQ[4,1] = e[4];    QQ[4,2] = -e[8];   QQ[4,3] = -e[6];
QQ[4,4] = -e[1];   QQ[4,5] = e[7];    QQ[4,6] = e[3];
QQ[4,7] = -e[5];   QQ[4,8] = e[2];    QQ[5,1] = e[5];
QQ[5,2] = e[3];    QQ[5,3] = -e[2];   QQ[5,4] = -e[7];
QQ[5,5] = -e[1];   QQ[5,6] = e[8];    QQ[5,7] = e[4];
QQ[5,8] = -e[6];   QQ[6,1] = e[6];    QQ[6,2] = -e[7];
```

```
QQ[6, 3] = e[4];      QQ[6, 4] = −e[3];      QQ[6, 5] = −e[8];
QQ[6, 6] = −e[1];      QQ[6, 7] = e[2];      QQ[6, 8] = e[5];
QQ[7, 1] = e[7];      QQ[7, 2] = e[6];      QQ[7, 3] = −e[8];
QQ[7, 4] = e[5];      QQ[7, 5] = −e[4];      QQ[7, 6] = −e[2];
QQ[7, 7] = −e[1];      QQ[7, 8] = e[3];      QQ[8, 1] = e[8];
QQ[8, 2] = e[4];      QQ[8, 3] = e[7];      QQ[8, 4] = −e[2];
QQ[8, 5] = e[6];      QQ[8, 6] = −e[5];      QQ[8, 7] = −e[3];
QQ[8, 8] = −e[1];
%%%% the Jordan algebra product
Qm[x_, y_] := Sum[Sum[x[[i]]y[[j]]QQ[i, j], {i, 8}], {j, 8}]
QP[x_, y_] := {Coefficient[Qm[x, y], e[1]],  Coefficient[Qm[x, y], e[2]],
     Coefficient[Qm[x, y], e[3]],  Coefficient[Qm[x, y], e[4]],  Coefficient[Qm[x, y], e[5]],
     Coefficient[Qm[x, y], e[6]],  Coefficient[Qm[x, y], e[7]],  Coefficient[Qm[x, y], e[8]]}
o1 = {a1,  a2,  a3,  a4,  a5,  a6,  a7,  a8};
o2 = {b1,  b2,  b3,  b4,  b5,  b6,  b7,  b8};
Conj[x_] := {x[[1]],  −x[[2]],  −x[[3]],  −x[[4]],  −x[[5]],  −x[[6]],  −x[[7]],  −x[[8]]}
OctP[a_, b_] := {{Sum[QP[Part[Part[a, 1], i], Part[Part[b, i], 1]], {i, 3}] ,
     Sum[QP[Part[Part[a, 1], i], Part[Part[b, i], 2]], {i, 3}],
     Sum[QP[Part[Part[a, 1], i], Part[Part[b, i], 3]], {i, 3}]},
    {Sum[QP[Part[Part[a, 2], i], Part[Part[b, i], 1]], {i, 3}],
     Sum[QP[Part[Part[a, 2], i], Part[Part[b, i], 2]], {i, 3}],
     Sum[QP[Part[Part[a, 2], i], Part[Part[b, i], 3]], {i, 3}]},
    {Sum[QP[Part[Part[a, 3], i], Part[Part[b, i], 1]], {i, 3}],
     Sum[QP[Part[Part[a, 3], i], Part[Part[b, i], 2]], {i, 3}],
     Sum[QP[Part[Part[a, 3], i], Part[Part[b, i], 3]], {i, 3}]}}
OctPS[a_, b_] := 1/2(OctP[a, b] + OctP[b, a])
%%%% correspondence between the Jordan algebra and ℝ^27
A = {{{a[1],  0,  0,  0,  0,  0,  0,  0},  {a[2],  a[3],  a[4],  a[5],  a[6],  a[7],  a[8],  a[9]},
       {a[10],  a[11],  a[12],  a[13],  a[14],  a[15],  a[16],  a[17]}},
     {{a[2],  −a[3],  −a[4],  −a[5],  −a[6],  −a[7],  −a[8],  −a[9]},  {a[18],  0,  0,  0,  0,  0,  0,  0},
       {a[19],  a[20],  a[21],  a[22],  a[23],  a[24],  a[25],  a[26]}},
     {{a[10],  −a[11],  −a[12],  −a[13],  −a[14],  −a[15],  −a[16],  −a[17]},  {a[19],  −a[20],
       −a[21],  −a[22],  −a[23],  −a[24],  −a[25],  −a[26]},  {a[27],  0,  0,  0,  0,  0,  0,  0}}};
B = {{{b[1],  0,  0,  0,  0,  0,  0,  0},  {b[2],  b[3],  b[4],  b[5],  b[6],  b[7],  b[8],  b[9]},
       {b[10],  b[11],  b[12],  b[13],  b[14],  b[15],  b[16],  b[17]}},
     {{b[2],  −b[3],  −b[4],  −b[5],  −b[6],  −b[7],  −b[8],  −b[9]},  {b[18],  0,  0,  0,  0,  0,  0,  0},
```

```
      {b[19], b[20], b[21], b[22], b[23], b[24], b[25], b[26]}},
    {{b[10], -b[11], -b[12], -b[13], -b[14], -b[15], -b[16], -b[17]}, {b[19], -b[20],
      -b[21], -b[22], -b[23], -b[24], -b[25], -b[26]}, {b[27], 0, 0, 0, 0, 0, 0, 0}}};
FF[AA_] := {Part[{Part[Part[Part[AA, 1], 1], 1]}, 1],
    Part[{Part[Part[Part[AA, 1], 2], 1]}, 1], Part[{Part[Part[Part[AA, 1], 2], 2]}, 1],
    Part[{Part[Part[Part[AA, 1], 2], 3]}, 1], Part[{Part[Part[Part[AA, 1], 2], 4]}, 1],
    Part[{Part[Part[Part[AA, 1], 2], 5]}, 1], Part[{Part[Part[Part[AA, 1], 2], 6]}, 1],
    Part[{Part[Part[Part[AA, 1], 2], 7]}, 1], Part[{Part[Part[Part[AA, 1], 2], 8]}, 1],
    Part[{Part[Part[Part[AA, 1], 3], 1]}, 1], Part[{Part[Part[Part[AA, 1], 3], 2]}, 1],
    Part[{Part[Part[Part[AA, 1], 3], 3]}, 1], Part[{Part[Part[Part[AA, 1], 3], 4]}, 1],
    Part[{Part[Part[Part[AA, 1], 3], 5]}, 1], Part[{Part[Part[Part[AA, 1], 3], 6]}, 1],
    Part[{Part[Part[Part[AA, 1], 3], 7]}, 1], Part[{Part[Part[Part[AA, 1], 3], 8]}, 1],
    Part[{Part[Part[Part[AA, 2], 2], 1]}, 1], Part[{Part[Part[Part[AA, 2], 3], 1]}, 1],
    Part[{Part[Part[Part[AA, 2], 3], 2]}, 1], Part[{Part[Part[Part[AA, 2], 3], 3]}, 1],
    Part[{Part[Part[Part[AA, 2], 3], 4]}, 1], Part[{Part[Part[Part[AA, 2], 3], 5]}, 1],
    Part[{Part[Part[Part[AA, 2], 3], 6]}, 1], Part[{Part[Part[Part[AA, 2], 3], 7]}, 1],
    Part[{Part[Part[Part[AA, 2], 3], 8]}, 1], Part[{Part[Part[Part[AA, 3], 3], 1]}, 1]}
FFi[vv_] :=
  {{{Part[vv, 1], 0, 0, 0, 0, 0, 0, 0}, {Part[vv, 2], Part[vv, 3], Part[vv, 4], Part[vv, 5],
     Part[vv, 6], Part[vv, 7], Part[vv, 8], Part[vv, 9]}, {Part[vv, 10], Part[vv, 11],
     Part[vv, 12], Part[vv, 13], Part[vv, 14], Part[vv, 15], Part[vv, 16], Part[vv, 17]}},
   {{Part[vv, 2], -Part[vv, 3], -Part[vv, 4], -Part[vv, 5], -Part[vv, 6],
     -Part[vv, 7], -Part[vv, 8], -Part[vv, 9]}, {Part[vv, 18], 0, 0, 0, 0, 0, 0, 0},
    {Part[vv, 19], Part[vv, 20], Part[vv, 21], Part[vv, 22],
     Part[vv, 23], Part[vv, 24], Part[vv, 25], Part[vv, 26]}},
   {{Part[vv, 10], -Part[vv, 11], -Part[vv, 12], -Part[vv, 13],
     -Part[vv, 14], -Part[vv, 15], -Part[vv, 16], -Part[vv, 17]},
    {Part[vv, 19], -Part[vv, 20], -Part[vv, 21], -Part[vv, 22], -Part[vv, 23],
     -Part[vv, 24], -Part[vv, 25], -Part[vv, 26]}, {Part[vv, 27], 0, 0, 0, 0, 0, 0, 0}}}
%%%% construction of the matrices
MM = Array[mm, {27, 27}];
vaa = FF[A];
vbb = FF[B];
v1aa = MM.vaa;
v1bb = MM.vbb;
AA1 = FFi[v1aa];
BB1 = FFi[v1bb];
```

```
V1 = FF[OctPS[AA1, B]];
V2 = FF[OctPS[A, BB1]];
AB = OctPS[AB];
V = FF[AB];
VV = MM.V;
diff = VV − V1 − V2;
Do [
    Do[Do[ff[i, j, k] = Coefficient[Part[diff, k], a[i]b[j]], {i, 27}], {j, 27}], {k, 27}] ;
n = 0;
Do [Do [Do [n + +;
    If[ff[i, j, k] == 0, n = n − 1, Ff[n] = ff[i, j, k] == 0, Ff[n] = ff[i, j, k] == 0], {i, 27}] ,
  {j, 27}] , {k, 27}] ;
s[1] = {};
Do[s[i + 1] = Append[s[i], Ff[i]], {i, n}];
m = 0;
Do[Do[{m + +, gg[m] = mm[i, j]}, {i, 27}], {j, 27}];
v[1] = {};
Do[v[i + 1] = Append[v[i], gg[i]], {i, 729}];
sol = Solve[s[n], v[730]];
Do[Do[mat[i, j] = mm[i, j], {i, 27}], {j, 27}];
Do [Do [Do [If [Part[Part[Part[sol, 1, i, 1]]] == mm[j, k], mat[j, k] =
        mm[j, k]/.Part[Part[sol, 1, i]]] , {i, 677}] , {j, 27}] , {k, 27}] ;
MM = Array[mat, {27, 27}];
n = 0;
Do [Do [n + +; If [D[MM, mm[i, j]] == DiagonalMatrix [
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}] ,
   n = n − 1, Md[n] = D[MM, mm[i, j]], Md[n] = D[MM, mm[i, j]]] , {i, j}] , {j, 27}] ;
Mn[1] = Md[1];
Do [Do [
  AA[j, i] = 0, {i, 52}] , {j, 52}]
Do [Do[AA[j, i] = −Tr[Md[j].Mn[i]]/Tr[Mn[i].Mn[i]], {i, j − 1}];
 Mn[j] = Md[j] + Sum[AA[j, i]Mn[i], {i, j − 1}], {j, 2, 52}] ;
Do[c[i] = −Sqrt[6]Mn[i]/Sqrt[Tr[Mn[i].Mn[i]]], {i, 52}];
%%%% the structure constants
Do[Do[CC[i, j] = c[i].c[j] − c[j].c[i], {i, 52}], {j, 52}];
Do[Do[Do[coeff[i, j, k] = −Tr[CC[i, j].c[k]]/6, {k, 52}], {i, 52}], {j, 52}];
```

```
%%% rotation to give the irreducible 26 representation
XXX = {{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, −1, 0, 0, 0, 0, 0, 0, 0, 0, 0}/Sqrt[2],
      {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, −2}/Sqrt[6],
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},
      {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1}/Sqrt[3]};
Do[cc[i] = XXX.c[i].Transpose[XXX], {i, 52}];
```

c[i] are the matrices before the rotation, and cc[i] are the 27×27 matrices after the rotations, all having the last row and the last column vanishing. The irreducible 26 dimensional representation is obtained dropping the last row and the last column. The structure constants are provided by coeff[i,j,k].

C The f_4 matrices.

The matrices we found using Mathematica, and orthonormalized with respect to the scalar product $\langle a, b\rangle := -\frac{1}{6}Trace(ab)$, are

$$c_1 = \begin{pmatrix}
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,0,0,0 \\
0,0,-\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,0,0,0 \\
0,\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ \frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,0,0,0 \\
0,0,\ 0,\ -\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ -\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ -\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,

$$c_3 = \begin{pmatrix}
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2},0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,-\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2},0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0 & 0, & 1,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & -1, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0
\end{pmatrix} \tag{C.3}$$

$$c_4 = \begin{pmatrix}
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0
\end{pmatrix} \tag{C.4}$$

$$c_5 = \left(\begin{array}{cccccccccccccccccccccccccccccccc}
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & -\tfrac{1}{2}, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & \tfrac{1}{2}, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0, & \tfrac{1}{2}, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,1,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & -1, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0
\end{array}\right) \tag{C.5}$$

$$c_6 = \left(\begin{array}{cccccccccccccccccccccccccccccccc}
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2},0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 1, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & -1, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0,0
\end{array}\right) \tag{C.6}$$

$$c_7 = \begin{pmatrix}
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0
\end{pmatrix} \tag{C.7}$$

$$c_8 = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2}, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \tfrac{1}{2}, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\tfrac{1}{2},0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0,0,0,0,0,0,0,0
\end{pmatrix} \tag{C.8}$$

$$c_9 = \begin{pmatrix}
0,0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,-\frac{1}{2},\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,\frac{1}{2},\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\frac{1}{2},0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,0,\ 0,\ -\frac{1}{2},0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ -\frac{1}{2},0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,0,-\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ \frac{1}{2},0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,-\frac{1}{2},\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,0,\ 0,\ \frac{1}{2},\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,-\frac{1}{2},\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0
\end{pmatrix} \tag{C.9}$$

$$c_{10} = \begin{pmatrix}
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\frac{1}{2},0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,-\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\ 0,\ 0,\ -\frac{1}{2},0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,\frac{1}{2},0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\ 0,-\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\frac{1}{2},0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,-\frac{1}{2},\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,-\frac{1}{2},\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 1,\ 0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,-1,\ 0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0 \\
0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,\ 0,\ 0,0,0,0
\end{pmatrix} \tag{C.10}$$

$$
c_{11} = \left(\begin{array}{cccccccccccccccccccccccccccccccc}
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0
\end{array}\right) \tag{C.11}
$$

$$
c_{12} = \left(\begin{array}{cccccccccccccccccccccccccccccccc}
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,0
\end{array}\right) \tag{C.12}
$$

$$c_{13} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0, & 0, & 0,0,0,0,0,0
\end{pmatrix} \tag{C.13}$$

$$c_{14} = \begin{pmatrix}
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & -\frac{1}{2}, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, \frac{1}{2}, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, -\frac{1}{2}, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, -\frac{1}{2}, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, -\frac{1}{2}, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & \frac{1}{2}, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0 \\
0, 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, & 0, & 0, & 0, & 0,0,0,0, & 0, & 0,0,0,0,0
\end{pmatrix} \tag{C.14}$$

$$c_{15} = \begin{pmatrix}
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,-\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,-\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,-\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,-\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,-\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2}
\end{pmatrix} \tag{C.15}$$

$$c_{16} = \begin{pmatrix}
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,-\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,-\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,-\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,-\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\tfrac{1}{2},0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\tfrac{1}{2},0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\tfrac{1}{2},0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\tfrac{1}{2},0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\tfrac{1}{2},0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\tfrac{1}{2},0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\tfrac{1}{2}
\end{pmatrix} \tag{C.16}$$

$$c_{17} = \left(\begin{array}{ccccccc

$$c_{19} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0,0,0
\end{pmatrix} \tag{C.19}$$

$$c_{20} = \begin{pmatrix}
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2},0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0, & 0, & 0,0,0,0
\end{pmatrix} \tag{C.20}$$

$$c_{21} = \begin{pmatrix}
0,0,\ 0,\ 0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,-\frac{1}{2},0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,\frac{1}{2},\ 0,\ 0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,-\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,\ 0,\ 0,-\frac{1}{2},0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ \frac{1}{2},0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,\ 0,\ 0,\ \frac{1}{2},0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,\ 0,\ 0,\ -\frac{1}{2},0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,\ \frac{1}{2},0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,\ 0,\ 0,\ 0,\ 0,\ 0,-\frac{1}{2},0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,-\frac{1}{2},0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,-\frac{1}{2},0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,-\frac{1}{2},0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,-\frac{1}{2},0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 1,0,0,0,0,0,\ 0,\ 1,0,0\\
0,0,\ 0,\ 0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,-1,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0\\
0,0,\ 0,\ 0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0,0,0,0,0,\ 0,\ 0,0,0
\end{pmatrix} \tag{C.21}$$

$$c_{22} = \begin{pmatrix}
0,\ 0,0,0,0,0,0,0,-\sqrt{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
\frac{1}{\sqrt{2}},\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0\\
0,\ 0,0,0,0,0,0,0,0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,0
\end{pmatrix} \tag{C.22}$$

$$c_{23} = \begin{pmatrix}
0, & 0,0,0,\sqrt{2},0,0,0,0,0, & 0, & 0, &

$$c_{25} = \begin{pmatrix}
0, & 0, -\sqrt{2}, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
\frac{1}{\sqrt{2}}, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, & 0, & 0, 0, 0, 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0
\end{pmatrix} \tag{C.25}$$

$$c_{26} = \begin{pmatrix}
0, & 0, 0, 0, 0, 0, \sqrt{2}, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
-\frac{1}{\sqrt{2}}, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0 \\
0, & 0, 0, 0, 0, 0, 0, & 0, & 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0, 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, 0, 0
\end{pmatrix} \tag{C.26}$$

$$c_{27} = \begin{pmatrix}
0, & 0,0,0,0,-\sqrt{2},0,0,0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, 0, 0, & 0, & 0, & 0, 0 \\
0, & 0,0,0,0, & 0, &

$$c_{29} = \begin{pmatrix}
0, & \sqrt{2},0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
-\frac{1}{\sqrt{2}}, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{pmatrix} \tag{C.29}$$

$$c_{30} = \begin{pmatrix}
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0,-\frac{1}{2}, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2},0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & \frac{1}{2},0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & -\frac{1}{2},0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0,0, & 0, & 0, & 0, & 0, & 0, & 0,0, & 0, & 0, & 0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0
\end{pmatrix} \tag{C.30}$$

$$c_{31} = \left(\begin{array}{cccccccccccccccccccccccccccccccc}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 1, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{array}\right) \tag{C.31}$$

$$c_{32} = \left(\begin{array}{cccccccccccccccccccccccccccccccc}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{array}\right) \tag{C.32}$$

$$c_{33} = \begin{pmatrix}
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
\end{pmatrix} \tag{C.33}$$

$$c_{34} = \begin{pmatrix}
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
\end{pmatrix} \tag{C.34}$$

$$c_{35} = \left(\begin{array}{cccccccccccccccccccccccccccccccc}
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,1,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&-1,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0
\end{array}\right) \tag{C.35}$$

$$c_{36} = \left(\begin{array}{cccccccccccccccccccccccccccccccc}
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&\frac{1}{2},&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&-\frac{1}{2},&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,1,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&-1,0,0\\
0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,&0,0,0,0,0,0,0,0,&0,&0,0,0
\end{array}\right) \tag{C.36}$$

$$c_{37} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{\sqrt{2}}, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2} \\
-\frac{1}{2\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & -\sqrt{\frac{3}{8}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \sqrt{\frac{3}{2}}, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{pmatrix} \tag{C.37}$$

$$c_{38} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{\sqrt{2}}, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
-\frac{1}{2\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & -\sqrt{\frac{3}{8}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \sqrt{\frac{3}{2}}, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{pmatrix} \tag{C.38}$$

$$c_{39} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0

$$c_{41} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & \frac{1}{\sqrt{2}}, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
-\frac{1}{2\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & -\sqrt{\frac{3}{8}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & \sqrt{\frac{3}{2}}, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{pmatrix} \tag{C.41}$$

$$c_{42} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & \frac{1}{\sqrt{2}}, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
-\frac{1}{2\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & -\sqrt{\frac{3}{8}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & \sqrt{\frac{3}{2}}, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0, & 0, & 0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{pmatrix} \tag{C.42}$$

$$c_{43} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,\frac{1}{\sqrt{2}},0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2},0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
-\frac{1}{2\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0,-\sqrt{\frac{3}{8}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,\sqrt{\frac{3}{2}},0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{pmatrix} \tag{C.43}$$

$$c_{44} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,\frac{1}{\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2},0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
-\frac{1}{2\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & -\sqrt{\frac{3}{8}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0,\sqrt{\frac{3}{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & -\frac{1}{2},0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{pmatrix} \tag{C.44}$$

$$c_{45} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{\sqrt{2}}, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0,0,0,0,0,0,0,0 \\
0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \sqrt{\frac{3}{2}}, & 0,0,0,0,0,0,0,0 \\
\frac{1}{2\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\sqrt{\frac{3}{8}}, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0
\end{pmatrix} \tag{C.45}$$

$$c_{46} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{\sqrt{2}}, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \sqrt{\frac{3}{2}}, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
\frac{1}{2\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\sqrt{\frac{3}{8}}, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0,0,0,0,0,0,0,0
\end{pmatrix} \tag{C.46}$$

$$c_{47} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
\frac{1}{2\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\sqrt{\frac{3}{8}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{pmatrix} \tag{C.47}$$

$$c_{48} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{\sqrt{2}}, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
\frac{1}{2\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\sqrt{\frac{3}{8}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{pmatrix} \tag{C.48}$$

$$c_{49} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{\sqrt{2}}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \sqrt{\frac{3}{2}}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
\frac{1}{2\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\sqrt{\frac{3}{8}}, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{pmatrix} \tag{C.49}$$

$$c_{50} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{\sqrt{2}}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \sqrt{\frac{3}{2}}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
\frac{1}{2\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\sqrt{\frac{3}{8}}, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{pmatrix} \tag{C.50}$$

$$c_{51} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{\sqrt{2}}, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & \frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & -\frac{1}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \sqrt{\frac{3}{2}}, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0

D Structure constants.

The structure constants ${s_{IJ}}^K$ are defined by $[c_I, c_J] = \sum_{K=1}^{52} {s_{IJ}}^K c_K$. We found that the coefficients $s_{IJK} := {s_{IJ}}^K$ are completely antisymmetric in the indices and the non-vanishing terms, up to symmetries, are

$$
\begin{array}{llll}
s_{1,2,3} = -1\,, & s_{1,4,5} = -1\,, & s_{1,7,8} = -1\,, & s_{1,11,12} = -1\,, \\
s_{1,16,17} = -1\,, & s_{1,22,23} = -\tfrac{1}{2}\,, & s_{1,24,26} = \tfrac{1}{2}\,, & s_{1,25,29} = \tfrac{1}{2}\,, \\
s_{1,27,28} = \tfrac{1}{2}\,, & s_{1,30,31} = -1\,, & s_{1,37,38} = -\tfrac{1}{2}\,, & s_{1,39,41} = \tfrac{1}{2}\,, \\
s_{1,40,44} = \tfrac{1}{2}\,, & s_{1,42,43} = \tfrac{1}{2}\,, & s_{1,45,46} = -1\,, & s_{2,4,6} = -1\,, \\
s_{2,7,9} = -1\,, & s_{2,11,13} = -1\,, & s_{2,16,18} = -1\,, & s_{2,22,24} = -\tfrac{1}{2}\,, \\
s_{2,23,26} = -\tfrac{1}{2}\,, & s_{2,25,27} = \tfrac{1}{2}\,, & s_{2,28,29} = \tfrac{1}{2}\,, & s_{2,30,32} = -1\,, \\
s_{2,37,39} = -\tfrac{1}{2}\,, & s_{2,38,41} = -\tfrac{1}{2}\,, & s_{2,40,42} = \tfrac{1}{2}\,, & s_{2,43,44} = \tfrac{1}{2}\,, \\
s_{2,45,47} = -1\,, & s_{3,5,6} = -1\,, & s_{3,8,9} = -1\,, & s_{3,12,13} = -1\,, \\
s_{3,17,18} = -1\,, & s_{3,22,26} = \tfrac{1}{2}\,, & s_{3,23,24} = -\tfrac{1}{2}\,, & s_{3,25,28} = -\tfrac{1}{2}\,, \\
s_{2,27,29} = \tfrac{1}{2}\,, & s_{3,31,32} = -1\,, & s_{3,37,41} = \tfrac{1}{2}\,, & s_{3,38,39} = -\tfrac{1}{2}\,, \\
s_{3,40,43} = -\tfrac{1}{2}\,, & s_{3,42,44} = \tfrac{1}{2}\,, & s_{3,46,47} = -1\,, & s_{4,7,10} = -1\,, \\
s_{4,11,14} = -1\,, & s_{4,16,19} = -1\,, & s_{4,22,25} = -\tfrac{1}{2}\,, & s_{4,23,29} = -\tfrac{1}{2}\,, \\
s_{4,24,27} = -\tfrac{1}{2}\,, & s_{4,26,28} = \tfrac{1}{2}\,, & s_{4,30,33} = -1\,, & s_{4,37,40} = -\tfrac{1}{2}\,, \\
s_{4,38,44} = -\tfrac{1}{2}\,, & s_{4,39,42} = -\tfrac{1}{2}\,, & s_{4,41,43} = \tfrac{1}{2}\,, & s_{4,45,48} = -1\,, \\
s_{5,8,10} = -1\,, & s_{5,12,14} = -1\,, & s_{5,17,19} = -1\,, & s_{5,22,29} = \tfrac{1}{2}\,, \\
s_{5,23,25} = -\tfrac{1}{2}\,, & s_{5,24,28} = \tfrac{1}{2}\,, & s_{5,26,27} = \tfrac{1}{2}\,, & s_{5,31,33} = -1\,, \\
s_{5,37,44} = \tfrac{1}{2}\,, & s_{5,38,40} = -\tfrac{1}{2}\,, & s_{5,39,43} = \tfrac{1}{2}\,, & s_{5,41,42} = \tfrac{1}{2}\,, \\
s_{5,46,48} = -1\,, & s_{6,9,10} = -1\,, & s_{6,13,14} = -1\,, & s_{6,18,19} = -1\,, \\
s_{6,22,27} = \tfrac{1}{2}\,, & s_{6,23,28} = -\tfrac{1}{2}\,, & s_{6,24,25} = -\tfrac{1}{2}\,, & s_{6,26,29} = -\tfrac{1}{2}\,, \\
s_{6,32,33} = -1\,, & s_{6,37,42} = \tfrac{1}{2}\,, & s_{6,38,43} = -\tfrac{1}{2}\,, & s_{6,39,40} = -\tfrac{1}{2}\,, \\
s_{6,41,44} = -\tfrac{1}{2}\,, & s_{6,47,48} = -1\,, & s_{7,11,15} = -1\,, & s_{7,16,20} = -1\,, \\
s_{7,22,26} = -\tfrac{1}{2}\,, & s_{7,23,24} = -\tfrac{1}{2}\,, & s_{7,25,28} = -\tfrac{1}{2}\,, & s_{7,27,29} = \tfrac{1}{2}\,, \\
s_{7,30,34} = -1\,, & s_{7,37,41} = -\tfrac{1}{2}\,, & s_{7,38,39} = \tfrac{1}{2}\,, & s_{7,40,43} = -\tfrac{1}{2}\,, \\
s_{7,42,44} = \tfrac{1}{2}\,, & s_{7,45,49} = -1\,, & s_{8,12,15} = -1\,, & s_{8,17,20} = -1\,, \\
s_{8,22,24} = -\tfrac{1}{2}\,, & s_{8,23,26} = -\tfrac{1}{2}\,, & s_{8,25,27} = -\tfrac{1}{2}\,, & s_{8,28,29} = -\tfrac{1}{2}\,, \\
s_{8,31,34} = -1\,, & s_{8,37,39} = -\tfrac{1}{2}\,, & s_{8,38,41} = -\tfrac{1}{2}\,, & s_{8,40,42} = -\tfrac{1}{2}\,, \\
s_{8,43,44} = -\tfrac{1}{2}\,, & s_{8,46,49} = -1\,, & s_{9,13,15} = -1\,, & s_{9,18,20} = -1\,, \\
s_{9,22,23} = \tfrac{1}{2}\,, & s_{9,24,26} = -\tfrac{1}{2}\,, & s_{9,25,29} = \tfrac{1}{2}\,, & s_{9,27,28} = \tfrac{1}{2}\,, \\
s_{9,32,34} = -1\,, & s_{9,37,38} = \tfrac{1}{2}\,, & s_{9,39,41} = -\tfrac{1}{2}\,, & s_{9,40,44} = \tfrac{1}{2}\,, \\
s_{9,42,43} = \tfrac{1}{2}\,, & s_{9,47,49} = -1\,, & s_{10,14,15} = -1\,, & s_{10,19,20} = -1\,, \\
s_{10,22,28} = \tfrac{1}{2}\,, & s_{10,23,27} = \tfrac{1}{2}\,, & s_{10,24,29} = -\tfrac{1}{2}\,, & s_{10,25,26} = -\tfrac{1}{2}\,, \\
s_{10,33,34} = -1\,, & s_{10,37,43} = \tfrac{1}{2}\,, & s_{10,38,42} = \tfrac{1}{2}\,, & s_{10,39,44} = -\tfrac{1}{2}\,, \\
s_{10,40,41} = -\tfrac{1}{2}\,, & s_{10,48,49} = -1\,, & s_{11,16,21} = -1\,, & s_{11,22,27} = -\tfrac{1}{2}\,, \\
s_{11,23,28} = -\tfrac{1}{2}\,, & s_{11,24,25} = \tfrac{1}{2}\,, & s_{11,26,29} = -\tfrac{1}{2}\,, & s_{11,30,35} = -1\,, \\
s_{11,37,42} = -\tfrac{1}{2}\,, & s_{11,38,43} = -\tfrac{1}{2}\,, & s_{11,39,40} = \tfrac{1}{2}\,, & s_{11,41,44} = -\tfrac{1}{2}\,, \\
s_{11,45,50} = -1\,, & s_{12,17,21} = -1\,, & s_{12,22,28} = \tfrac{1}{2}\,, & s_{12,23,27} = -\tfrac{1}{2}\,, \\
s_{12,24,29} = -\tfrac{1}{2}\,, & s_{12,25,26} = \tfrac{1}{2}\,, & s_{12,31,35} = -1\,, & s_{12,37,43} = \tfrac{1}{2}\,, \\
s_{12,38,42} = -\tfrac{1}{2}\,, & s_{12,39,44} = -\tfrac{1}{2}\,, & s_{12,40,41} = \tfrac{1}{2}\,, & s_{12,46,50} = -1\,, \\
s_{13,18,21} = -1\,, & s_{13,22,25} = -\tfrac{1}{2}\,, & s_{13,23,29} = \tfrac{1}{2}\,, & s_{13,24,27} = -\tfrac{1}{2}\,,
\end{array}
$$

$s_{13,26,28} = -\frac{1}{2}$, $s_{13,32,35} = -1$, $s_{13,37,40} = -\frac{1}{2}$, $s_{13,38,44} = \frac{1}{2}$,
$s_{13,39,42} = -\frac{1}{2}$, $s_{13,41,43} = -\frac{1}{2}$, $s_{13,47,50} = -1$, $s_{14,19,21} = -1$,
$s_{14,22,24} = \frac{1}{2}$, $s_{14,23,26} = -\frac{1}{2}$, $s_{14,25,27} = -\frac{1}{2}$, $s_{14,28,29} = \frac{1}{2}$,
$s_{14,33,35} = -1$, $s_{14,37,39} = \frac{1}{2}$, $s_{14,38,41} = -\frac{1}{2}$, $s_{14,40,42} = -\frac{1}{2}$,
$s_{14,43,44} = \frac{1}{2}$, $s_{14,48,50} = -1$, $s_{15,20,21} = -1$, $s_{15,22,29} = \frac{1}{2}$,
$s_{15,23,25} = \frac{1}{2}$, $s_{15,24,28} = \frac{1}{2}$, $s_{15,26,27} = -\frac{1}{2}$, $s_{15,34,35} = -1$,
$s_{15,37,44} = \frac{1}{2}$, $s_{15,38,40} = \frac{1}{2}$, $s_{15,39,43} = \frac{1}{2}$, $s_{15,41,42} = -\frac{1}{2}$,
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$s_{16,39,44} = -\frac{1}{2}$, $s_{16,40,41} = \frac{1}{2}$, $s_{16,45,51} = -1$, $s_{17,22,27} = -\frac{1}{2}$,
$s_{17,23,28} = -\frac{1}{2}$, $s_{17,24,25} = -\frac{1}{2}$, $s_{17,26,29} = \frac{1}{2}$, $s_{17,31,36} = -1$,
$s_{17,37,42} = -\frac{1}{2}$, $s_{17,38,43} = -\frac{1}{2}$, $s_{17,39,40} = -\frac{1}{2}$, $s_{17,41,44} = \frac{1}{2}$,
$s_{17,46,51} = -1$, $s_{18,22,29} = \frac{1}{2}$, $s_{18,23,25} = \frac{1}{2}$, $s_{18,24,28} = -\frac{1}{2}$,
$s_{18,26,27} = \frac{1}{2}$, $s_{18,32,36} = -1$, $s_{18,37,44} = \frac{1}{2}$, $s_{18,38,40} = \frac{1}{2}$,
$s_{18,39,43} = -\frac{1}{2}$, $s_{18,41,42} = \frac{1}{2}$, $s_{18,47,51} = -1$, $s_{19,22,26} = -\frac{1}{2}$,
$s_{19,23,24} = -\frac{1}{2}$, $s_{19,25,28} = -\frac{1}{2}$, $s_{19,27,29} = -\frac{1}{2}$, $s_{19,33,36} = -1$,
$s_{19,37,41} = -\frac{1}{2}$, $s_{19,38,39} = -\frac{1}{2}$, $s_{19,40,43} = -\frac{1}{2}$, $s_{19,42,44} = -\frac{1}{2}$,
$s_{19,48,51} = -1$, $s_{20,22,25} = \frac{1}{2}$, $s_{20,23,29} = -\frac{1}{2}$, $s_{20,24,27} = -\frac{1}{2}$,
$s_{20,26,28} = -\frac{1}{2}$, $s_{20,34,36} = -1$, $s_{20,37,40} = \frac{1}{2}$, $s_{20,38,44} = -\frac{1}{2}$,
$s_{20,39,42} = -\frac{1}{2}$, $s_{20,41,43} = -\frac{1}{2}$, $s_{20,49,51} = -1$, $s_{21,22,23} = \frac{1}{2}$,
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$s_{21,37,38} = \frac{1}{2}$, $s_{21,39,41} = \frac{1}{2}$, $s_{21,40,44} = \frac{1}{2}$, $s_{21,42,43} = -\frac{1}{2}$,
$s_{21,50,51} = -1$, $s_{22,23,33} = -\frac{1}{2}$, $s_{22,24,36} = -\frac{1}{2}$, $s_{22,25,31} = \frac{1}{2}$,
$s_{22,26,35} = -\frac{1}{2}$, $s_{22,27,34} = -\frac{1}{2}$, $s_{22,28,32} = \frac{1}{2}$, $s_{22,29,30} = \frac{1}{2}$,
$s_{22,37,52} = -\frac{1}{2}$, $s_{22,38,48} = -\frac{1}{2}$, $s_{22,39,51} = -\frac{1}{2}$, $s_{22,40,46} = \frac{1}{2}$,
$s_{22,41,50} = -\frac{1}{2}$, $s_{22,42,49} = \frac{1}{2}$, $s_{22,43,47} = \frac{1}{2}$, $s_{22,44,45} = -\frac{1}{2}$,
$s_{23,24,35} = -\frac{1}{2}$, $s_{23,25,30} = -\frac{1}{2}$, $s_{23,26,36} = \frac{1}{2}$, $s_{23,27,32} = \frac{1}{2}$,
$s_{23,28,34} = -\frac{1}{2}$, $s_{23,29,35} = \frac{1}{2}$, $s_{23,37,48} = \frac{1}{2}$, $s_{23,38,52} = -\frac{1}{2}$,
$s_{23,39,50} = -\frac{1}{2}$, $s_{23,40,45} = -\frac{1}{2}$, $s_{23,41,51} = \frac{1}{2}$, $s_{23,42,47} = \frac{1}{2}$,
$s_{23,43,49} = -\frac{1}{2}$, $s_{23,44,46} = \frac{1}{2}$, $s_{24,25,34} = \frac{1}{2}$, $s_{24,26,33} = -\frac{1}{2}$,
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$s_{24,38,50} = \frac{1}{2}$, $s_{24,39,52} = -\frac{1}{2}$, $s_{24,40,49} = \frac{1}{2}$, $s_{24,41,48} = -\frac{1}{2}$,
$s_{24,42,46} = -\frac{1}{2}$, $s_{24,43,45} = -\frac{1}{2}$, $s_{24,44,47} = \frac{1}{2}$, $s_{25,26,32} = \frac{1}{2}$,
$s_{25,27,36} = -\frac{1}{2}$, $s_{25,28,35} = \frac{1}{2}$, $s_{25,29,33} = \frac{1}{2}$, $s_{25,37,46} = -\frac{1}{2}$,
$s_{25,38,45} = \frac{1}{2}$, $s_{25,39,49} = -\frac{1}{2}$, $s_{25,40,52} = -\frac{1}{2}$, $s_{25,41,47} = \frac{1}{2}$,
$s_{25,42,51} = -\frac{1}{2}$, $s_{25,43,50} = \frac{1}{2}$, $s_{25,44,48} = \frac{1}{2}$, $s_{26,27,30} = -\frac{1}{2}$,
$s_{26,28,31} = \frac{1}{2}$, $s_{26,29,34} = \frac{1}{2}$, $s_{26,37,50} = \frac{1}{2}$, $s_{26,38,51} = -\frac{1}{2}$,
$s_{26,39,48} = \frac{1}{2}$, $s_{26,40,47} = -\frac{1}{2}$, $s_{26,41,52} = -\frac{1}{2}$, $s_{26,42,45} = -\frac{1}{2}$,
$s_{26,43,46} = \frac{1}{2}$, $s_{26,44,49} = \frac{1}{2}$, $s_{27,28,33} = -\frac{1}{2}$, $s_{27,29,35} = \frac{1}{2}$,
$s_{27,37,49} = -\frac{1}{2}$, $s_{27,38,47} = -\frac{1}{2}$, $s_{27,39,46} = \frac{1}{2}$, $s_{27,40,51} = \frac{1}{2}$,
$s_{27,41,45} = \frac{1}{2}$, $s_{27,42,52} = -\frac{1}{2}$, $s_{27,43,48} = -\frac{1}{2}$, $s_{27,44,50} = \frac{1}{2}$,
$s_{28,29,36} = \frac{1}{2}$, $s_{28,37,47} = -\frac{1}{2}$, $s_{28,38,49} = \frac{1}{2}$, $s_{28,39,45} = \frac{1}{2}$,
$s_{28,40,50} = -\frac{1}{2}$, $s_{28,41,46} = -\frac{1}{2}$, $s_{28,42,48} = \frac{1}{2}$, $s_{28,43,52} = -\frac{1}{2}$,
$s_{28,44,51} = \frac{1}{2}$, $s_{29,37,45} = -\frac{1}{2}$, $s_{29,38,46} = -\frac{1}{2}$, $s_{29,39,47} = -\frac{1}{2}$,
$s_{29,40,48} = -\frac{1}{2}$, $s_{29,41,49} = -\frac{1}{2}$, $s_{29,42,50} = -\frac{1}{2}$, $s_{29,43,51} = -\frac{1}{2}$,
$s_{29,44,52} = -\frac{1}{2}$, $s_{30,37,44} = -\frac{1}{2}$, $s_{30,38,40} = \frac{1}{2}$, $s_{30,39,43} = \frac{1}{2}$,

$$
\begin{array}{cccc}
s_{30,41,42}=\frac{1}{2}\ , & s_{30,45,52}=-1\ , & s_{31,37,40}=-\frac{1}{2}\ , & s_{31,38,44}=-\frac{1}{2}\ , \\
s_{31,39,42}=\frac{1}{2}\ , & s_{31,41,43}=-\frac{1}{2}\ , & s_{31,46,52}=-1\ , & s_{32,37,43}=-\frac{1}{2}\ , \\
s_{32,38,42}=-\frac{1}{2}\ , & s_{32,39,44}=-\frac{1}{2}\ , & s_{32,40,41}=-\frac{1}{2}\ , & s_{32,47,52}=-1\ , \\
s_{33,37,38}=\frac{1}{2}\ , & s_{33,39,41}=\frac{1}{2}\ , & s_{33,40,44}=-\frac{1}{2}\ , & s_{33,42,43}=\frac{1}{2}\ , \\
s_{33,48,52}=-1\ , & s_{34,37,42}=-\frac{1}{2}\ , & s_{34,38,43}=\frac{1}{2}\ , & s_{34,39,40}=-\frac{1}{2}\ , \\
s_{34,41,44}=-\frac{1}{2}\ , & s_{34,49,52}=-1\ , & s_{35,37,41}=\frac{1}{2}\ , & s_{35,38,39}=\frac{1}{2}\ , \\
s_{35,40,43}=-\frac{1}{2}\ , & s_{35,42,44}=-\frac{1}{2}\ , & s_{35,50,52}=-1\ , & s_{36,37,39}=\frac{1}{2}\ , \\
s_{36,38,41}=-\frac{1}{2}\ , & s_{36,40,42}=\frac{1}{2}\ , & s_{36,43,44}=-\frac{1}{2}\ , & s_{36,51,52}=-1\ .
\end{array}
$$

E Surjectivity of the product parametrization.

We will follow [4]. First note that $M \equiv G/H$ is a compact homogeneous manifold. Let $\pi : G \longrightarrow G/H$ and $\pi_{\mathcal{P}} : \mathcal{G} \longrightarrow \mathcal{P}$ be the natural projections. If G is provided with a bi-invariant Riemannian metric (as it happens for simple compact Lie groups) then M can also be provided with such an invariant metric. In particular for compact semisimple Lie groups we can use the Killing metric. The Levi-Civita connection is then exactly the connection induced by the horizontal distribution defined by taking $(L_g)_*\mathcal{P}$ as horizontal space at any $g \in G$, where L_g is the left multiplication on G and the lower $*$ indicate the push-forward.[6] The invariant metric on M is then obtained by requiring for $\pi_* : T_gG \longrightarrow T_{\pi(g)}M$ to be an isometry between the horizontal component of T_gG and $T_{\pi(g)}M$ for any $g \in G$. Thus M is geodesically complete and π becomes a Riemannian submersion. From this, if $o := \pi(H)$, $Exp_o : T_oM \longrightarrow M$ is the exponential map generated by the geodesic flow from o and $\exp : \mathcal{G} \longrightarrow G$ is the exponential map of the Lie group, then it can be shown that $Exp_o(a) = \pi \exp(a)$ for any $a \in \mathcal{P}$ ([4], p.47 exercise 1.21). But Exp_o is surjective, as follows from the Hopf-Rinow theorem, ([4], theorem 1.9). This completes the proof.

F The orthogonal subgroups.

In this section we collect the volumes of the orthogonal subgroups obtained by means of the Macdonald formula. The rational homology groups of all simple Lie groups are known to be the homology of the product of odd dimensional spheres: $H_*(G) = H_*(S^{d_1} \times \ldots \times S^{d_r})$, r being the rank of the group [10]. For the subgroups of F_4 we find

$$
\begin{array}{ll}
SO(3): & d_1 = 3\ ; \\
SO(4): & d_1 = 3,\ d_2 = 3\ ; \\
SO(5): & d_1 = 3,\ d_2 = 7\ ; \\
SO(6): & d_1 = 3,\ d_2 = 5,\ d_3 = 7\ ; \\
SO(7): & d_1 = 3,\ d_2 = 7,\ d_3 = 11\ ; \\
SO(8): & d_1 = 3,\ d_2 = 7,\ d_3 = 7,\ d_4 = 11\ ; \\
SO(9): & d_1 = 3,\ d_2 = 7,\ d_3 = 11,\ d_4 = 15\ .
\end{array}
$$

The roots of the subgroups are the ones given in [8], with $L_i = e_i$ identified with the usual orthonormal bases of an Euclidean space. The volumes can then easily computed by mean of the

[6] Here we are using the fact that $G \xrightarrow{\pi} G/H$ is a principal bundle over M, see [11]. In particular if $T_eG \simeq \mathcal{G}$ is the tangent space to the identity e of G, then $\mathcal{P}$ is its horizontal component.

Macdonald formula, giving

$$
\begin{aligned}
SO(3): &\qquad V = 2^4 \cdot \pi^2 \ ; \\
SO(4): &\qquad V = 2^5 \cdot \pi^4 \ ; \\
SO(5): &\qquad V = \frac{2^8 \cdot \pi^6}{3} \ ; \\
SO(6): &\qquad V = \frac{2^8 \cdot \pi^9}{3} \ ; \\
SO(7): &\qquad V = \frac{2^{12} \cdot \pi^{12}}{3^2 \cdot 5} \ ; \\
SO(8): &\qquad V = \frac{2^{12} \cdot \pi^{16}}{3^3 \cdot 5} \ ; \\
SO(9): &\qquad V = \frac{2^{17} \cdot \pi^{20}}{3^4 \cdot 5^2 \cdot 7} \ .
\end{aligned}
$$

G More on the subalgebras.

In section 2 we observed that our 27 dimensional representation of F_4 has a decomposition $\mathbf{26} \oplus \mathbf{1}$ in irreducible representations. Similar decompositions can be obtained for the subgroups simply by a direct computation of the weights. For example we computed the decomposition of $so(i)$ for $i = 9, 8, 7, 6$ finding:

- for $so(9)$
$$so(9) = \mathbf{16} \oplus \mathbf{9} \oplus \mathbf{1}^2$$
where **16** is the spin representation and **9** the vector representation;

- for $so(8)$
$$so(8) = \mathbf{8}_v \oplus \mathbf{8}_+ \oplus \mathbf{8}_- \oplus \mathbf{1}^3 \ .$$
Here $\mathbf{8}_v$ is the vector representation and $\mathbf{8}_\pm$ are the spin representations with positive and negative chirality;

- for $so(7)$
$$so(7) = \mathbf{8}^2 \oplus \mathbf{7} \oplus \mathbf{1}^4 \ ,$$
where **8** is the spin representation and **7** is the vector one;

- for $so(6)$
$$so(6) = \mathbf{6} \oplus \mathbf{4}_+^2 \oplus \mathbf{4}_-^2 \oplus \mathbf{1}^5 \ ,$$
where $\mathbf{4}_\pm$ are the chiral spin representations and **6** the vector one.

References

[1] J. F. Adams, *Lectures on Exceptional Lie Groups*, The University of Chicago Press, 1984.

[2] S. L. Cacciatori, B. L. Cerchiai, A. Della Vedova, G. Ortenzi and A. Scotti, "Euler angles for G(2)," J. Math. Phys. **46** (2005) 083512 [arXiv:hep-th/0503106];
S. L. Cacciatori, "A simple parametrization for G2," J. Math. Phys. **46** (2005) 083520 [arXiv:math-ph/0503054];
S. Bertini, S. L. Cacciatori and B. L. Cerchiai, "On the Euler angles for SU(N)" J. Math. Phys. **47** (2006) 043510 [arXiv:math-ph/0510075].

[3] F. Caravaglios and S. Morisi, *Gauge boson families in grand unified theories of fermion masses:* $E_6^4 \times S_4$, hep-ph/0611068 ;
F. Caravaglios and S. Morisi, *Fermion masses in* E_6 *grand unification with family permutation symmetries*, hep-ph/0510321 .

[4] Isaac Chavel, *Riemannian Geometry: A modern Introduction*, Cambridge University Press, 1997.

[5] C. Chevalley, *The Betti numbers of the exceptional Lie groups*, Proceedings of the international congress of Mathematicians, Cambridge, Mass., 1950, Vol.2, Amer. Math. Soc., Providence, R. I. , 1952, pp. 21,24.

[6] J. J. Duistermaat, J. A. C. Kolk, *Lie Groups*, Springer (1991): Corollary (3.1.4), page 138.

[7] Hans Freudenthal, *Lie groups in the foundations of geometry*, Adv. Math. 1 (1964), 145-90.

[8] W. Fulton, J. Harris, *Representation Theory: A First Course*, Springer (1999).

[9] Y. Hashimoto, *On Macdonald's formula for the volume of a compact Lie group*, Comment. Math. Helv. 72 (1997) 660-662.

[10] Heinz Hopf, *Über Die Topologie der Gruppen-Manningfaltigkeiten und Ihre Verallgemeinerungen*, The Annals of Mathematics, Vol.42, No. 1 (1941), pp. 22-52.

[11] S. Kobayashi, K. Nomizu, *Foundations of differential geometry*, Vol. 1, Wiley Classics Library, 1996.

[12] I. G. Macdonald, *The volume of a compact Lie group*, Invent. math. **56**, 93-95 (1980).

[13] S. Pigola, private communication.

[14] T. E. Tilma and G. Sudarshan, "Generalized Euler Angle Parametrization For U(N) With Applications To SU(N) Coset Volume Measures," J. Geom. Phys. **52** (2004) 263;
T. E. Tilma and G. Sudarshan, "Generalized Euler Angle Parametrization For SU(N)," J. Phys. A **35** (2002) 10467.