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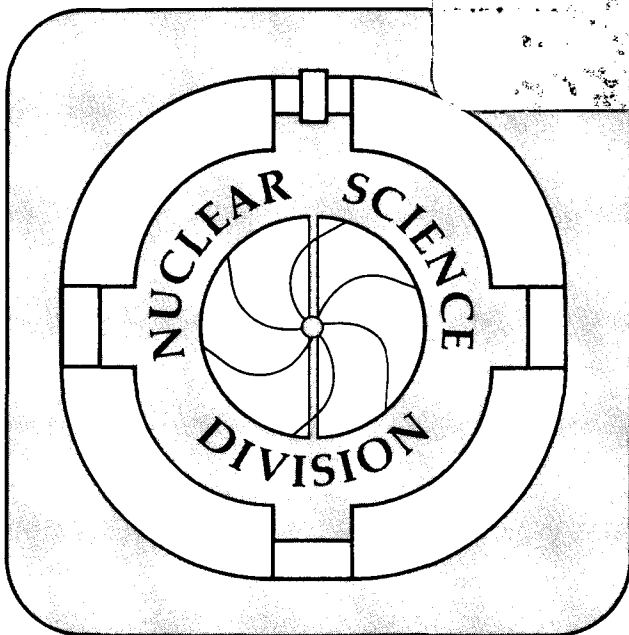
COMPLEX FRAGMENT EMISSION AT LOW
AND HIGH EXCITATION ENERGY

L.G. Moretto

August 1986

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**COMPLEX FRAGMENT EMISSION AT LOW
AND HIGH EXCITATION ENERGY**

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Division of Nuclear Physics of the Office of High Energy and Nuclear
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COMPLEX FRAGMENT EMISSION AT LOW AND HIGH EXCITATION ENERGY

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Complex fragment emission has been certified as a compound nucleus process at low energies. An extension of the measurements to heavy ion reactions up to 50 MeV/u shows that most complex fragments are emitted by highly excited compound nuclei formed in incomplete fusion reactions.

1. INTRODUCTION

In the nuclear physics community there is widespread belief that compound nuclei are entities relegated to the lower energy range of nuclear reactions. Recent developments seem to indicate that this is a myth handed down to us by our nuclear forefathers. If we were limited, as they were, to their sharp tools like protons or alpha particles, indeed we would need to stick to low energies in order to form compound nuclei. With our modern, larger, and blunter tools at our disposal like heavy ions, we are learning otherwise. As it turns out, the tendency of nuclear systems to undergo fusion seems to be all-pervasive, even in energy regions

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where one would not have expected it. Recently the energy region between 10 and 100 MeV/u has been the subject of extensive investigation. It is my conviction, borne out of our own data and by plenty more, that the tendency of nuclear systems to undergo some sort of fusion is still quite strong in this region as well.

On the evidence of abundant complex fragment production in this energy region, theories have been aired which envisage fancy non-fusion processes, like nuclear shattering¹ and liquid vapor equilibrium.² But, if I read the data correctly, Nature has already made mincemeat of those theories, having opted again in favor of compound nucleus formation and decay. I agree that this stubborn and conservative attitude on Nature's part is lamentable, and one may well reach the conclusion that Nature is not too well read. Had she but glanced at any of these fancy theories, I am sure She would not have hesitated to follow them.

Despite this strong conviction of mine, I have decided to cover all bets and do some work on one of those fancy theories to dispel rumors about my conservative attitudes and my lack of imagination. Appendix 1 is there for you, and perhaps for Nature, to peruse and to phantasize about things as they may be higher up in energy.

All joking aside, the fact remains that most of the complex fragments with $Z > 10$, and a lot of fragments with $Z < 10$, at least up to 50 MeV per nucleon bombarding energies, arise from the statistical decay of a compound nucleus formed in an incomplete fusion process. This process of incomplete fusion, or massive transfer is not well understood nor well characterized as yet. It is interesting to study the predictions of a geometrical model regarding the kinematic thresholds and the mass and momentum transfers vs.

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impact parameter. The avid reader will find an outline of such a model in Appendix 2.

Before diving head-first into the main subject of complex fragment formation at high energies, I will favor those of you who may still have a streak of incredulity by showing that very low energy compound nuclei can decay by emitting complex fragments. The comfort of this conviction will give us courage to enter the more modern energy range of 10-50 MeV per nucleon and will permit us to verify, if all goes well, that even at these higher energies, the compound nucleus mechanism for the production of complex fragments still reigns.

2. COMPLEX FRAGMENTS "DE PROFUNDIS" OR NEAR THEIR COMPOUND EMISSION THRESHOLD

Why is it so hard to believe that complex fragments can be emitted by a compound nucleus? The reason lies, I think, in the fact that we have been taught two "distinct" ways of compound nucleus decay: evaporation of light particles like n , p , ^4He on one hand, and fission on the other. Yet it should not be too difficult to appreciate that the underlying connection between the two ways is the mass asymmetry coordinate.³ Consider, in the liquid drop picture, the saddle point configuration. One of its normal modes is the mass asymmetry mode. If we choose a given mass asymmetry, we can search for the saddle point with that constraint. The locus of all these conditional saddles I have called ridge line.³ The potential energy surface looks schematically like it is shown in Fig. 1. A compound nucleus, confronted with this potential energy ridge, can choose any asymmetry it cares, provided it has enough energy to afford it. Once the ridge is surmounted, the nucleus can descend towards the product region. In most cases, except near sym-

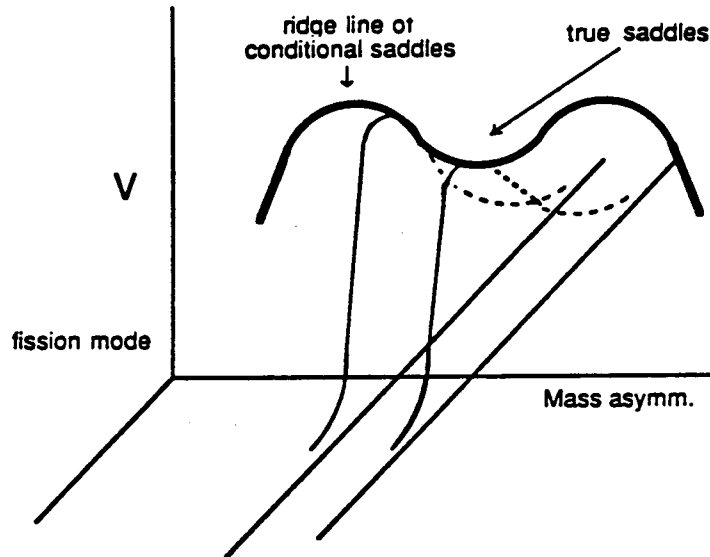


FIGURE 1

metry for heavy systems, the ridge configuration is so indented to be nearly degenerate with the scission configuration. In other words, once the ridge line is reached, the system is pretty much committed to the chosen mass asymmetry.

Why is it then, that complex fragments are not easily seen in low energy compound nucleus decay!? Since the rate of decay is approximately proportional to the level density at the ridge line $\rho(E - V(x)) \approx \rho(E) \exp - \frac{V(x)}{T}$, fragments will appear most abundantly where $V(x)$ is low. As shown in Fig. 2, in light systems this occurs at the extreme asymmetries (evaporation), while in heavier systems this can occur both at the extreme asymmetries (evaporation), and at symmetry (fission). In order to see the intermediate particles, one must look for them by beating the odds of their low cross sections. A detailed exposition of the theory on complex fragment emission, including, among other things,

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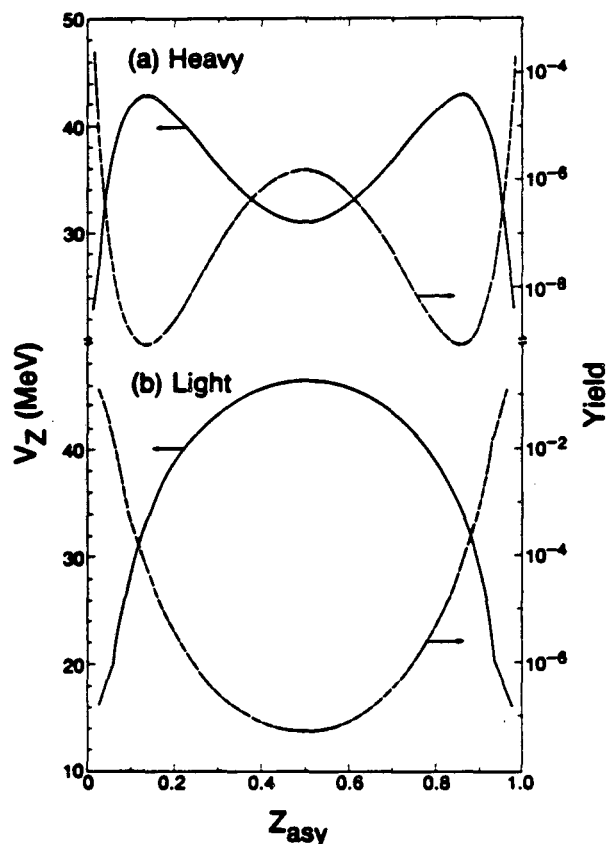


FIGURE 2 Comparison of the potential energy surfaces (solid curve) and expected yields (dashed curve) for a) a heavy CN (Au at $l = 0$ and $E^* = 97$ MeV) and b) a light CN (Ge at $l = 0$ and $E^* = 72$ MeV).

analytical expressions for kinetic energy and angular distributions is given in Ref. 3.

In order to characterize the compound nucleus emissions of complex particles we have studied the emission of complex fragments near threshold in the reaction $^3\text{He} + \text{Ag}$.⁴ The choice of ^3He speaks for the intense skepticism towards a compound nucleus mechanism prevailing at the time. We had to take precautions against the accusations of projectile fragmentation, deep inelastic processes and the like! To make a long story short let me show three figures. Figure 3

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70 MeV $^3\text{He} + ^{nat}\text{Ag}$

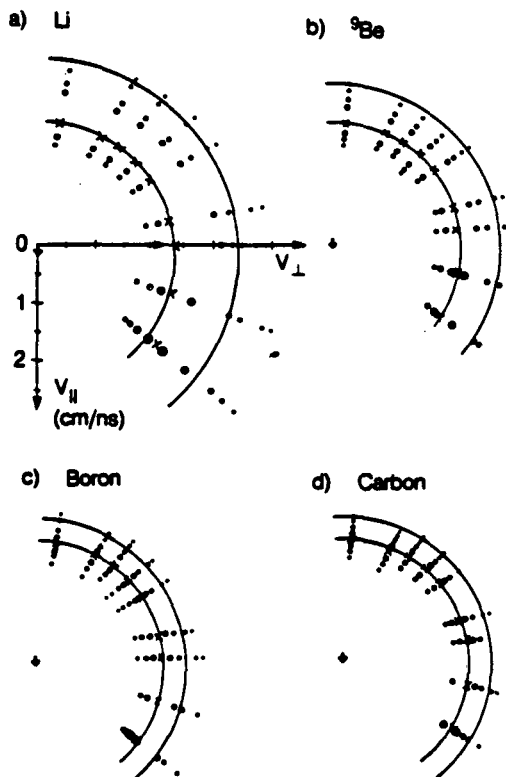


FIGURE 3 Invariant cross section plots for representative ejectiles (Li, ^9Be , B, and C). The diameter of the dots is proportional to the logarithm of the cross section and the X's indicate the peak of velocity distribution. The two large arcs are sections of circles centered on the c.m. velocity (center arrow) appropriate for complete fusion. The beam direction (0°) is indicated by the c.m. velocity vector.

shows the invariant cross section for four different fragments. Notice the presence of a single source with compound nucleus velocity, and the Coulomb-like energies of the fragments. Figure 4 shows the excitation functions of a number of fragments. Any nuclear physicist worth his or her neutrons will see in the rapid rise of their cross sections with energy the unmistakable signature of a compound nucleus

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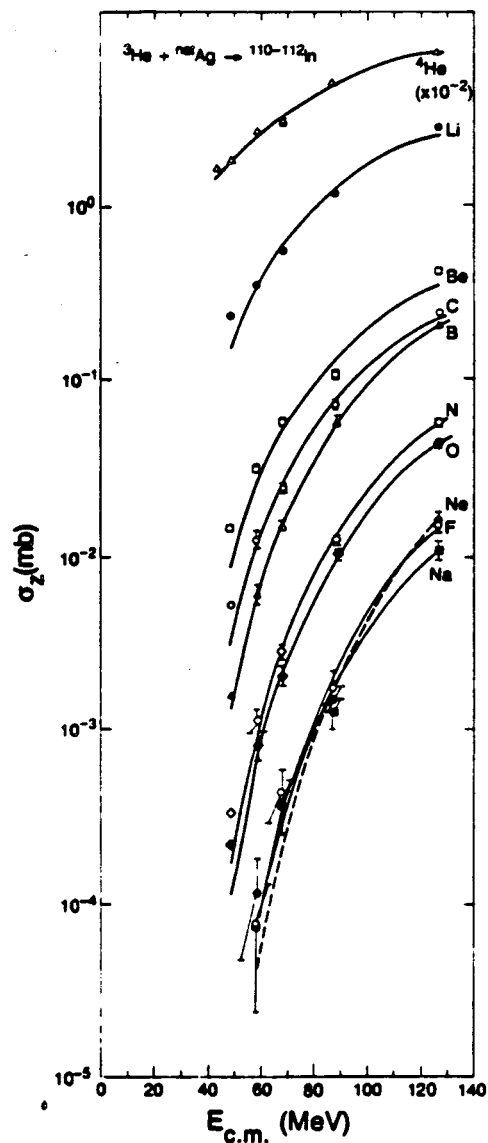


FIGURE 4 Dependence of the total integrated cross sections for emission of complex fragments on the center-of-mass energy, $E_{c.m.}$ in the reaction ${}^3\text{He} + \text{natAg}$. The points and error bars correspond to the experimental cross sections. The curves are fits with the parameters of Fig. 5.

process. The lines through the points are fits from which one can obtain the conditional barriers as well as the a_z/a_n ratios. The latter ratios turn out to be one to within a

couple of percent as expected from general considerations. Let us look at the extracted barriers in Fig. 5. The points are the experimental values, while the two lines, calculated by Sierk,⁵ represent the liquid drop predictions and the prediction of a fancier model that includes finite range effects. For the layman, these effects are due to the surface surface interaction which becomes important for strongly indented shapes. The impressive agreement between the finite range calculation and the data speaks clearly to those who remember the role of fission barriers in establishing the liquid drop parameters. But this would tempt us to stray too far. So let us return to the main theme which

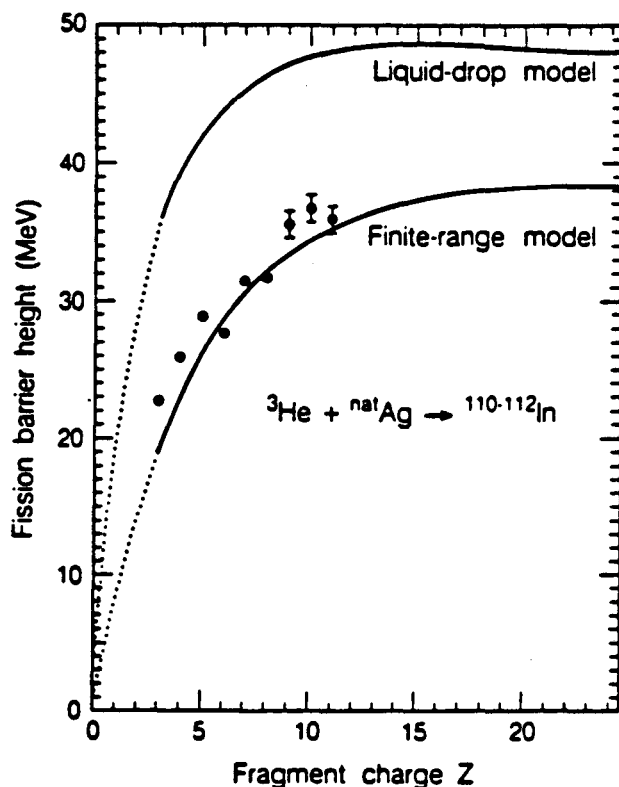


FIGURE 5 The emission barriers, B_z extracted in fitting the excitation functions for the emission of complex fragments a function of fragment charge. The liquid drop model and finite range model calculations are from Ref. 5.

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is the appreciation of the compound nucleus mechanism as the source of complex particles.

In order to satisfy ourselves that the yields do indeed reflect the topology of the ridge line, we have studied the reactions Be, C + Ge, Nb, La at 8.5 MeV per nucleon in reverse kinematics.⁶

Reverse kinematics helps a great deal in many ways. One of them is to lift the fragment kinetic energy so that, even if their rate of production is low, the products can be easily identified in a low background environment. Figure 6 shows an example of the center of mass kinetic energies which are Coulomb-like and of the source velocities which do correspond to complete fusion. Figure 7 shows the cross sections vs. Z . For the first time we see the complete Z distributions and their agreement with the compound nucleus calculations based upon the liquid drop model. Notice the sudden appearance of a maximum at symmetry for the Be + La system indicating that we are now above the Businaro-Gallone point.

3. PLUS ÇA CHANGE, PLUS C'EST LA MÊME CHOSE, OR: COMPOUND NUCLEI FOREVER?

Having established that complex fragments can be emitted by compound nuclei, and that at low energies they are only emitted by compound nuclei, the decision naturally came to see what in fact is going on at higher energies. Should any kind of compound nucleus be formed, it would decay abundantly by complex fragment emission due to its high excitation energy. This is the inescapable conclusion provided by statistical mechanics: if B_n and B_z are the neutron binding energy and the barrier for the fragment of charge Z respectively, and T the temperature, the ratio for the two decay rates is approximately

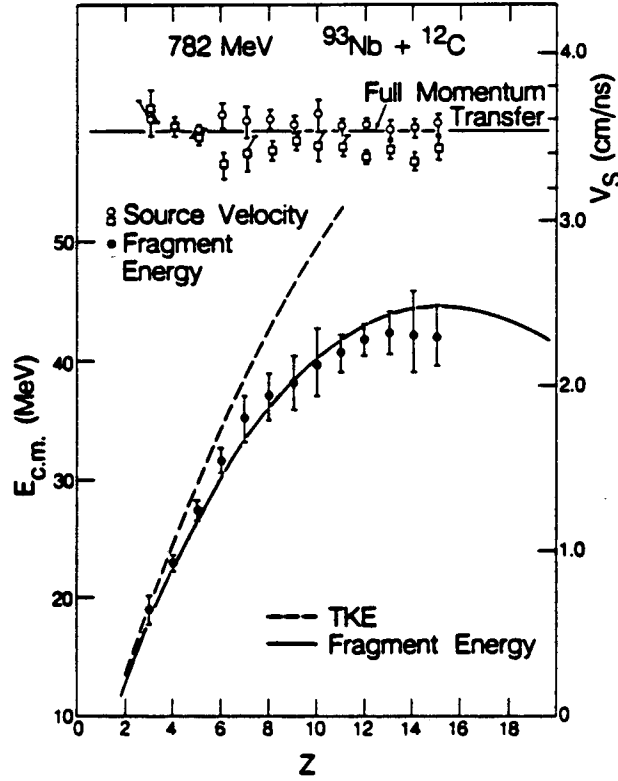


FIGURE 6 The deduced c.m. energies (filled circles) and source velocities (open symbols) for the $^{93}\text{Nb} + ^{12}\text{C}$ system. Source velocities were determined assuming that the product mass followed the line of β -stability (open circles) or the charge equilibration line (open squares). A Coulomb calculation for two spheres is shown both for the c.m. energy of the light fragment (solid line) and the total kinetic energy (dashed line). The value of the source velocity expected for full momentum transfer is indicated by the horizontal line.

$$\frac{\Gamma_z}{\Gamma_n} = \exp - (B_z - B_n)/T$$

Taking $B_n \approx 7$ MeV, $B_z \approx 35$ MeV and $T = 5-6$ MeV one gets

$$\frac{\Gamma_z}{\Gamma_n} = \exp - \frac{28}{5-6} \approx 10^{-2}$$

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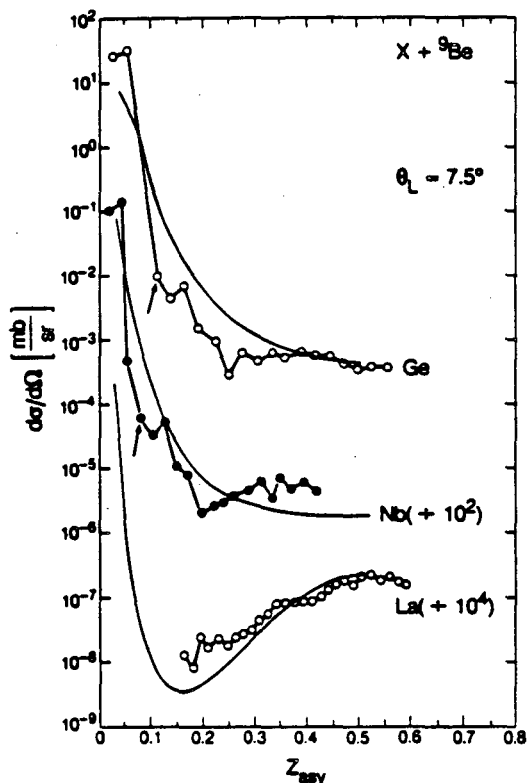


FIGURE 7 Center-of-mass cross sections for products from the ${}^{74}\text{Ge}$, ${}^{93}\text{Nb}$ and ${}^{193}\text{La} + {}^9\text{Be}$ systems detected at $\theta_{\text{Lab}} = 7.5^\circ$. The solid line is a liquid drop model calculation of the fragment yield at $\theta_{\text{c.m.}} = 30^\circ$. The arrows indicate the entrance channel asymmetry. See text. Data below $Z_{\text{asy}} = 0.15$ were not obtained for the $\text{La} + \text{Be}$ system, due to a limited dynamic range of the telescope.

In other words, allowing for the contribution of the various isotopes and for multiple chance emission, one could expect cross sections as large as several tens of millibarns per Z , or even more if we allow for angular momentum effects. The corollary of this is that any additional "fancy" mechanism should ride on top of this already substantial compound cross section.

So let me describe the results of our trek from 8.5 to 30-40 MeV/u. The reactions we chose are $\text{Nb} + \text{Be}$, C , Al in reverse kinematics. As we shall see, the choice of a rela-

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tively light target simplifies the picture crucially, because of the limitations in impact parameters and in the number of sources. While we explored first the upper part of the energy range at the Bevalac,⁷ I shall begin with the lower energy data which we collected at GSI.⁸

Let me use Fig. 8 in part to extoll the advantages of reverse kinematics. In these pictures we see the complex particles events displayed in the ΔE -E plane. The remarkable double ridge is due to a simple kinematic effect. A single source is emitting fragments in the center of mass with energies independent of direction. Because of the large center-of-mass velocity, a given lab angle intersects the kinematic circle twice giving rise to a double solution. The double solutions in velocity are shown in Fig. 9 for all Z's. This simple observation allows any person of good sense to conclude that the process is binary, especially

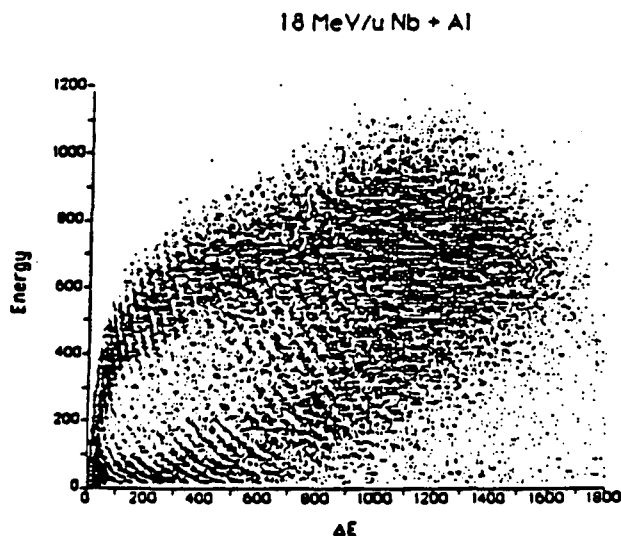


FIGURE 8 Scatter plot of E vs. ΔE for singles events produced in the reaction 18.5 MeV/u $^{93}\text{Nb} + \text{Al}$. The detector subtended angles from 4° to 12° . The two dark bands correspond to Coulomb emission from a compound nucleus forward and backward in the center-of-mass.

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18 MeV/u Nb + Be

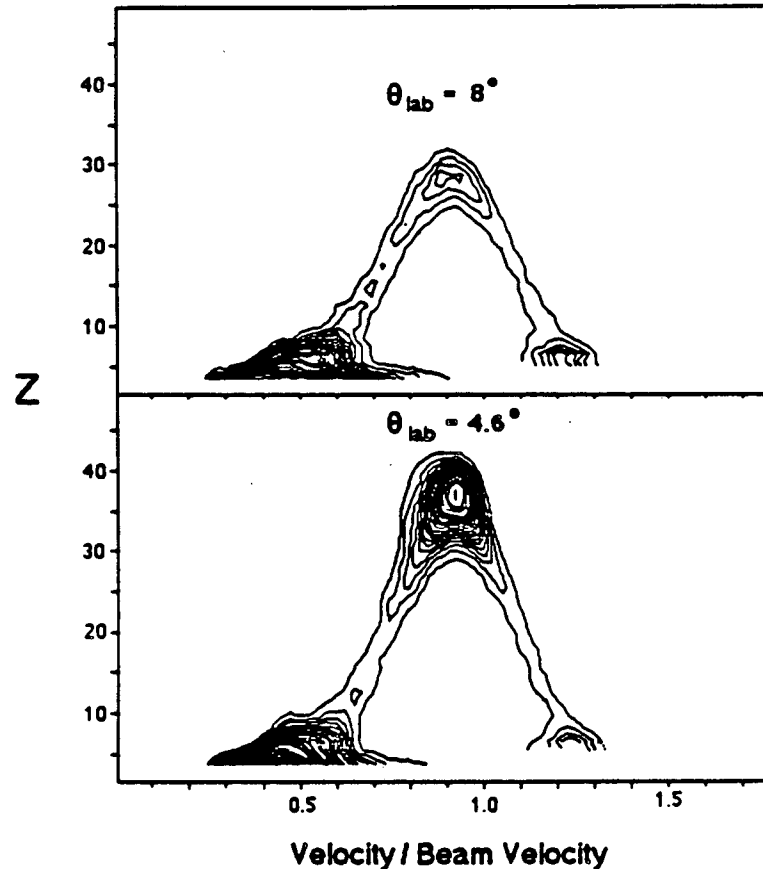


FIGURE 9 Invariant cross section plot in the Z, v for the reaction 18 MeV/u $^{93}\text{Nb} + ^9\text{Be}$.

when one obtains a Coulomb-like Z dependence for the center-of-mass velocities, shown in Fig. 10 together with their FWHM. Similarly, in Fig. 11 the velocities of the sources, shown as a function of Z indicate that we are dealing with a nearly complete fusion process. In Fig. 12 two examples of the charge distributions are given. They both show a minimum at symmetry as expected from the ridge line potential energies. For the remaining St. Thomases, who do not believe in singles, we took coincidence data with another detector placed symmetrically on the other side of

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18 MeV/u $^{93}\text{Nb} + \text{X}$

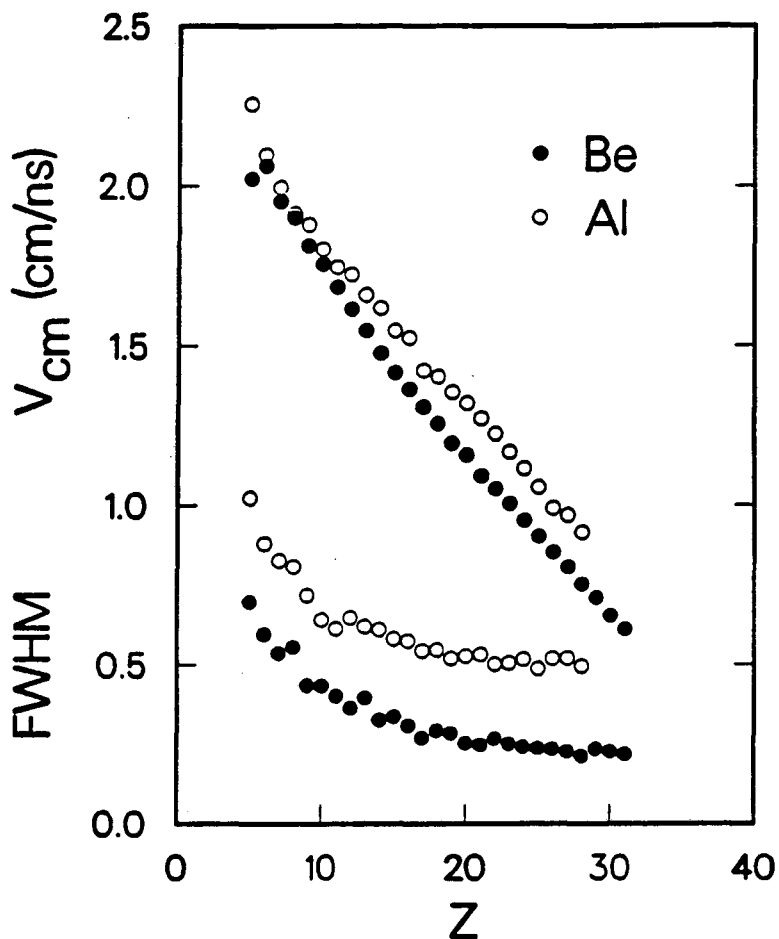


FIGURE 10 Center of mass mean velocities and widths as a function of Z for the reactions 18 MeV/u $^{93}\text{Nb} + \text{Be, Al}$.

the beam. These data are shown in Fig. 13 and emphasize the fact that the process is indeed binary. We have also verified that an upper solution fragment in one detector is in coincidence with a lower solution fragment in the other.

The pattern seen in the above figures evolves regularly and smoothly with bombarding energy and target. This indicates that indeed we are observing the same kinematic cir-

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18 MeV/u $^{93}\text{Nb} + \text{X}$

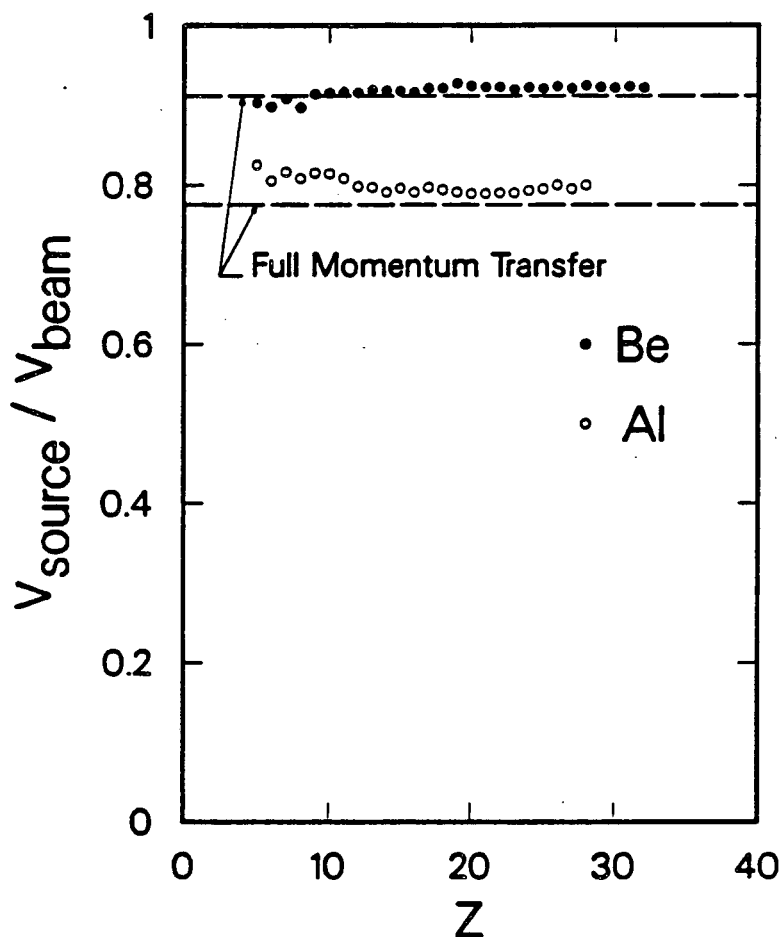


FIGURE 11 Source velocities as a function of Z for the same reactions as in Fig. 10. The two dashed lines correspond to the velocities expected from complete fusion.

cle, boosted by different velocities of the center-of-mass which are in approximate agreement with those expected from complete fusion. Furthermore, reverse kinematics allows us to verify that what you see is all there is. When we go to wider angles we lose the intersection with the kinematic circle and we see nothing. So there are no other processes

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18 MeV/u $^{93}\text{Nb} + \text{X}$

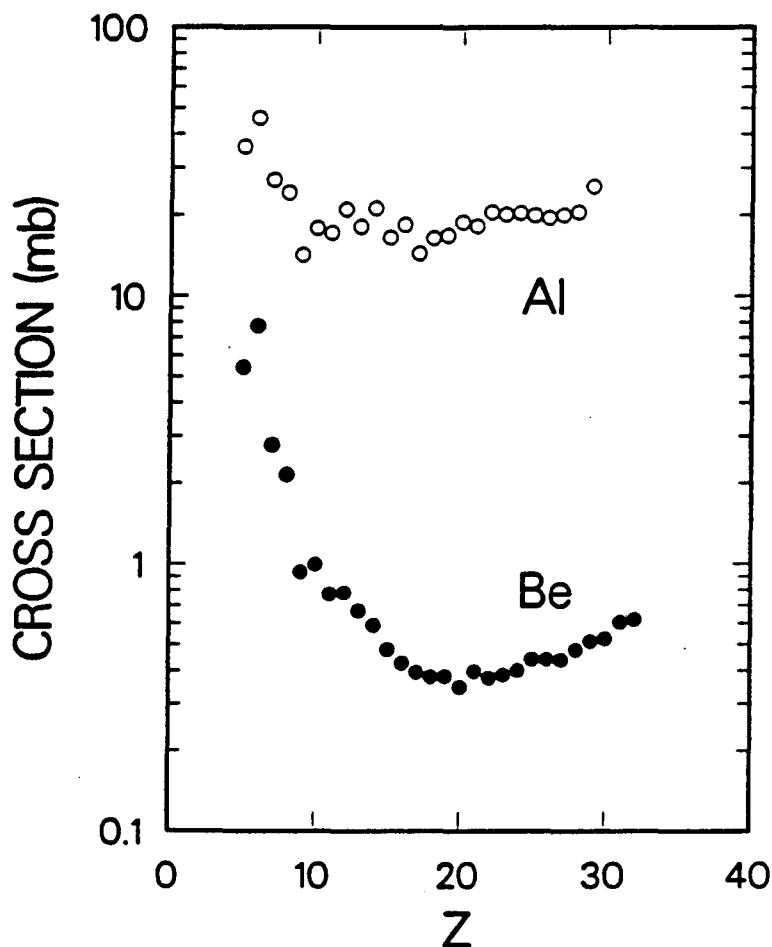


FIGURE 12 Charge distributions for the same reactions as in Fig. 10.

than the one we have described, and we can conclude that, up to 18.5 MeV/u "nihil sub sole novum," nothing new under the sun but the old compound nucleus decay.

Proceeding to the better analyzed Bevalac data, the picture we observe is very much the same. In Fig. 14 the invariant cross sections plotted in the Z-V plane show that the double solutions are retained up to 30 MeV/u. Notice

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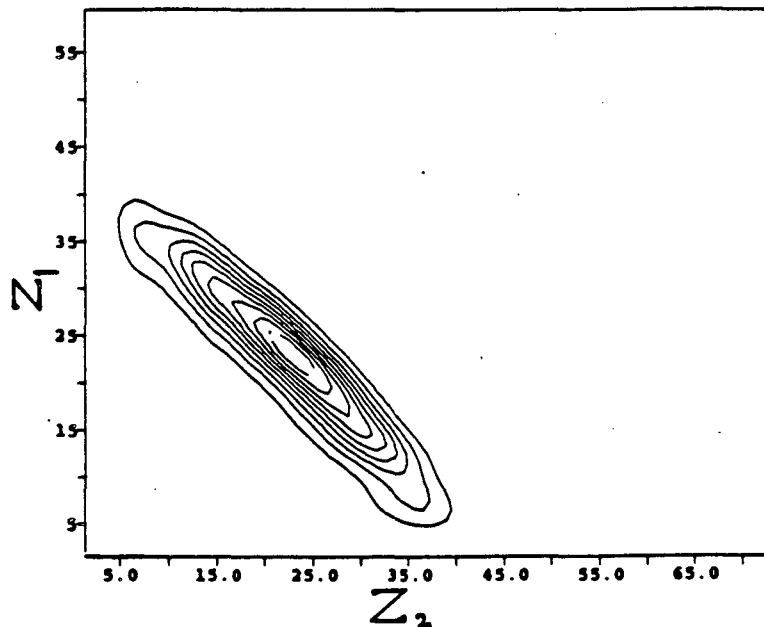


FIGURE 13. Z_1, Z_2 coincidence plot for 18 MeV/u Nb + Al.

also that, at very low Z 's, there is a trail of low velocity events which we call "big foot." This process appears to be target related, and may have to do with the onset of incomplete fusion. In this case, the events may be due to the target picking up a few nucleons from the projectile, and a corresponding fraction of its momentum.

The source velocities are shown in Fig. 15 and clearly indicate a single source for all Z 's with a velocity intermediate between the projectile velocity and the compound nucleus velocity but closer to the latter. The corresponding incomplete momentum transfer in the direct kinematic reaction is in good agreement with the standard momentum transfer systematics. The velocities in the center of mass are Coulomb-like as shown in Fig. 16 and the angular distributions are consistent with $1/\sin\theta$ as shown in Fig. 17.

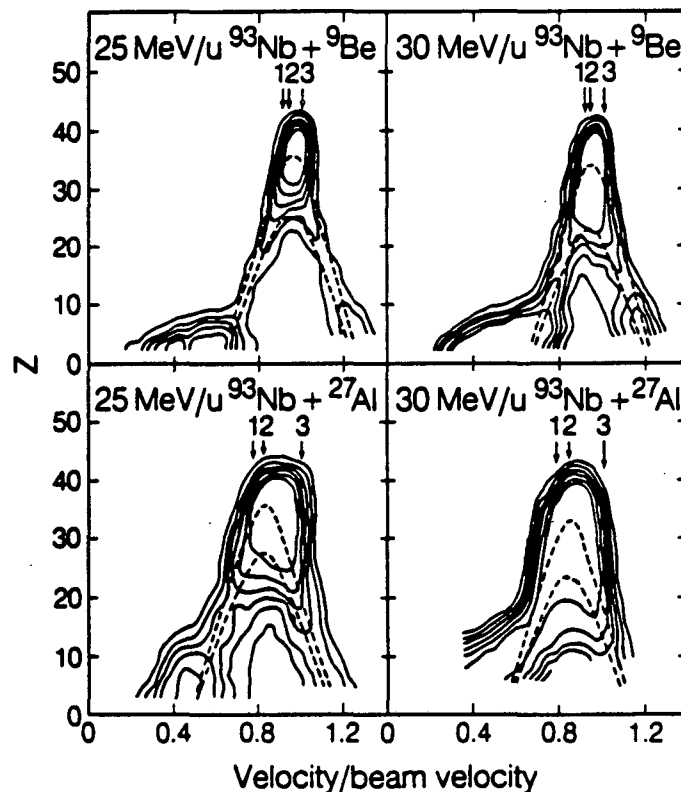


FIGURE 14 Singles distribution of reaction products plotted as logarithmic contours of invariant cross section in the Z -velocity plane. The arrows indicate the velocities for (1) full momentum transfer, (2) the experimentally determined momentum transfer and (3) the beam. Calculated (dashed lines) average velocities of complex fragments for the maximum and minimum lab angles of the telescope (3° and 8°) are indicated.

An example of charge distribution is shown in Fig. 18 together with an absolute calculation. The ability to fit the absolute cross sections vs. Z with a compound nucleus model is very significant in our eyes, because it implies a statistical branching ratio between complex fragment emission and the dominant n , p , ^4He decays.

The coincidence data are shown in Fig. 19. The hatched bands are predicted on the basis of the incomplete momentum transfer, of the resulting excitation energy, and of the

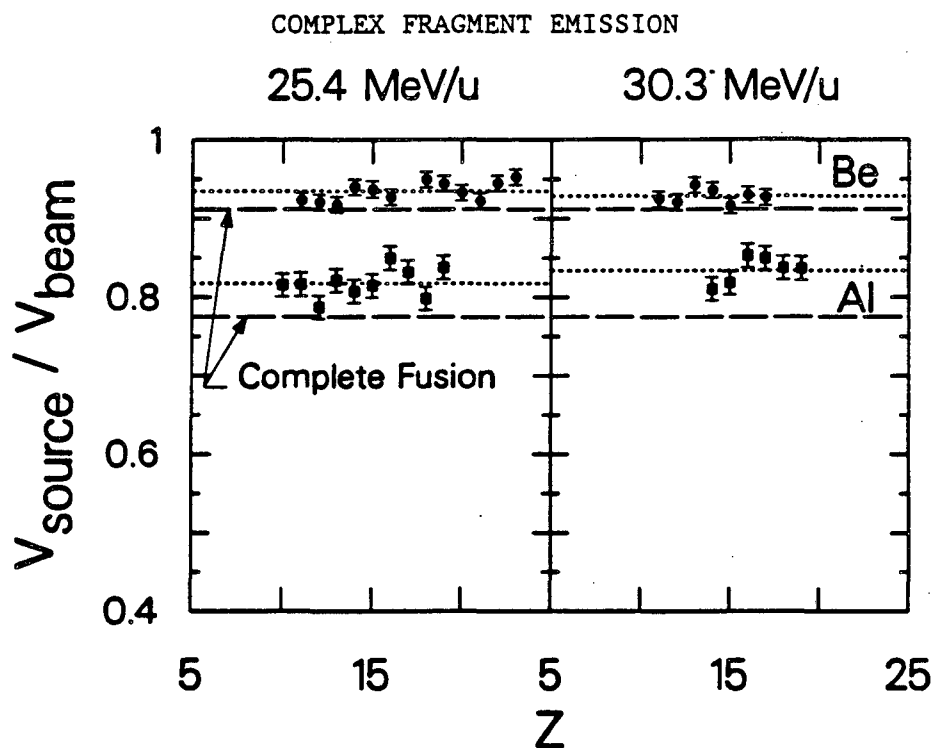


FIGURE 15 Measured source velocities vs. fragment charge for the systems 25, 30 MeV/u $^{93}\text{Nb} + \text{Al, Be}$. The dotted lines are the source velocities averaged over Z . The dashed lines are the velocities corresponding to complete fusion of projectile and target.

sequential evaporation from the binary fragments calculated from the code PACE.⁹ The overall picture is consistent with binary decay. However notice that in the case of the Al target at 30 MeV/u a small number (<5%) of events falls outside the expected band, indicating perhaps three or more body decay. A better appreciation of the coincidence data and of the calculations is given in Fig. 20 where the average sum of charges is plotted versus one of the charges. The dashed line is the primary sum inferred from the source velocity and the solid line is the calculation from PACE of the sequential charge evaporation. The excellent agreement indicates a solid understanding of the incomplete fusion process, of the energy deposition, and of the binary decay

Nb + X

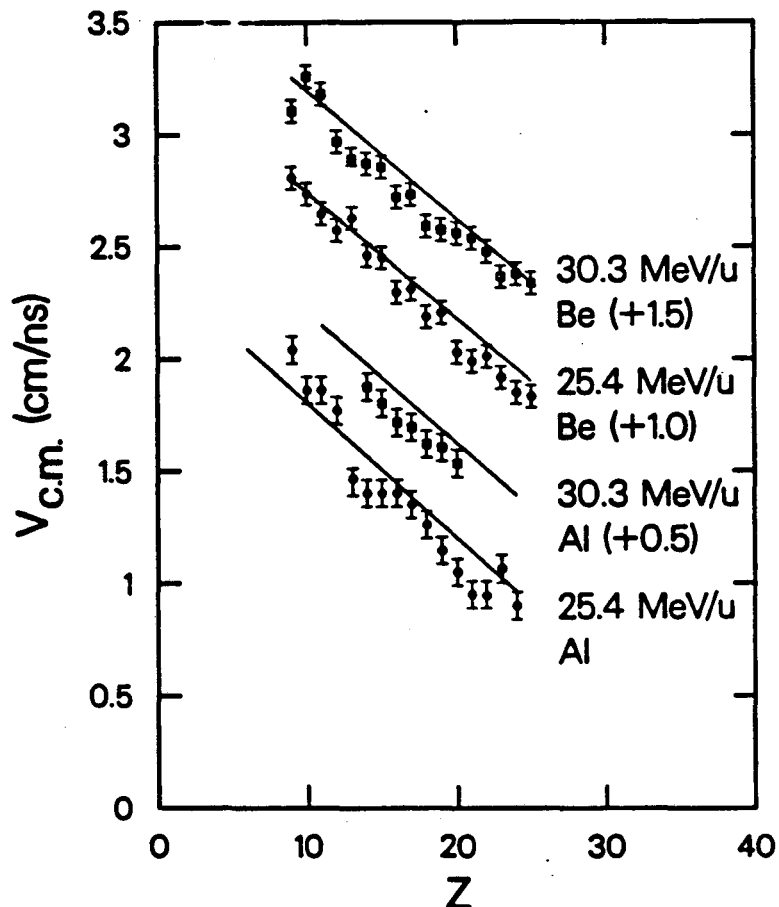


FIGURE 16 Velocities of fragments in the center-of-mass for 25 and 30 MeV/u $^{93}\text{Nb} + ^9\text{Be}, \text{Al}$. The solid lines show Coulomb calculations for two spheres with a separation given by $R = 1.224(A_1^{1/3} + A_2^{1/3}) + 2$ fm.

followed by sequential evaporation. A summary of the GSI and Bevalac data is given in Fig. 21 where the sum of the charges at symmetry is plotted vs. the energy per nucleon. The solid line gives the sum of the target and projectile charges, the long dashed line the compound nucleus charge as obtained in incomplete fusion, and the short dashed line the final sum of charges after sequential evaporation. Up to

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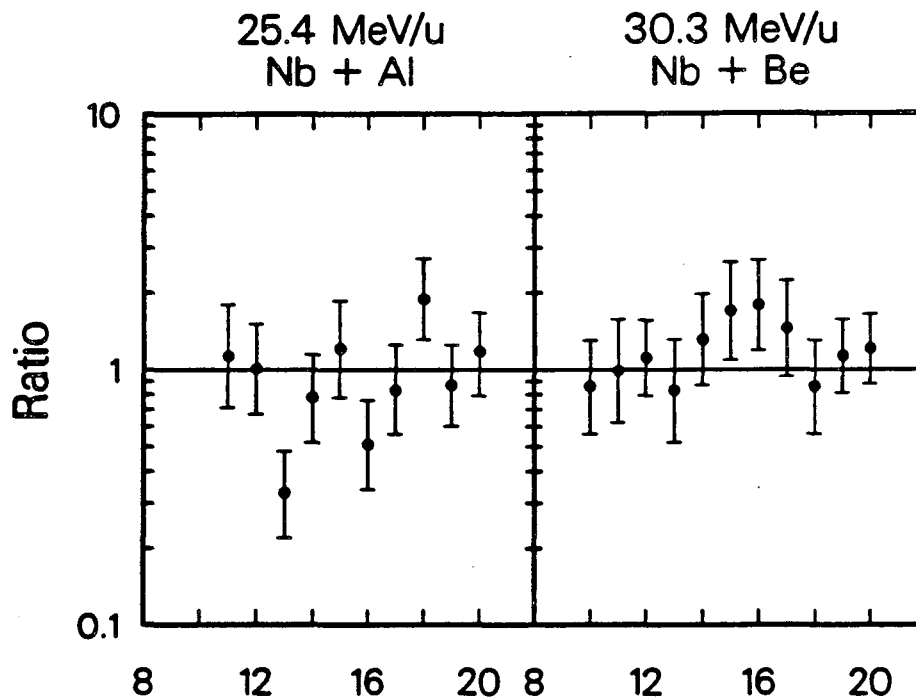


FIGURE 17 Ratios of $\frac{\partial^2 \sigma}{\partial Z \partial \theta}$ for the upper and lower kinematic solutions as a function of Z. A ratio of 1 indicates a $1/\sin\theta$ angular distribution.

here, again, the compound nucleus process appears to be the dominant if not the only mechanism of complex fragment production for $Z \geq 10$.

Preliminary data on the reaction C + La in reverse kinematics obtained at the Bevalac at 50 MeV/u also indicate the persistence of binary decay at such high energies as shown in Fig. 22.

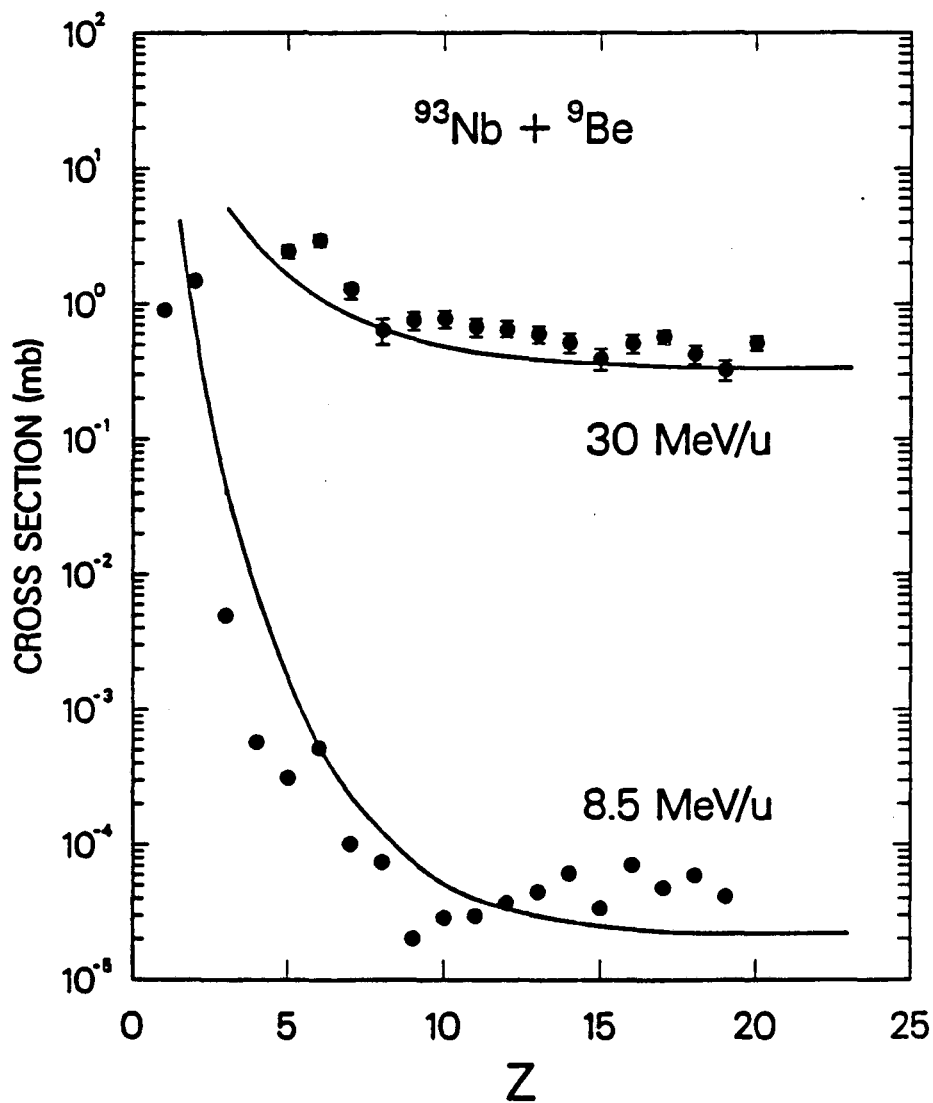


FIGURE 18 Angle-integrated cross sections (symbols) for complex fragments emitted from the reaction $^{93}\text{Nb} + ^9\text{Be}$ at 30 and 8.5 MeV/u. Liquid-drop model calculations (solid line) of the fragment yield for the two reactions.

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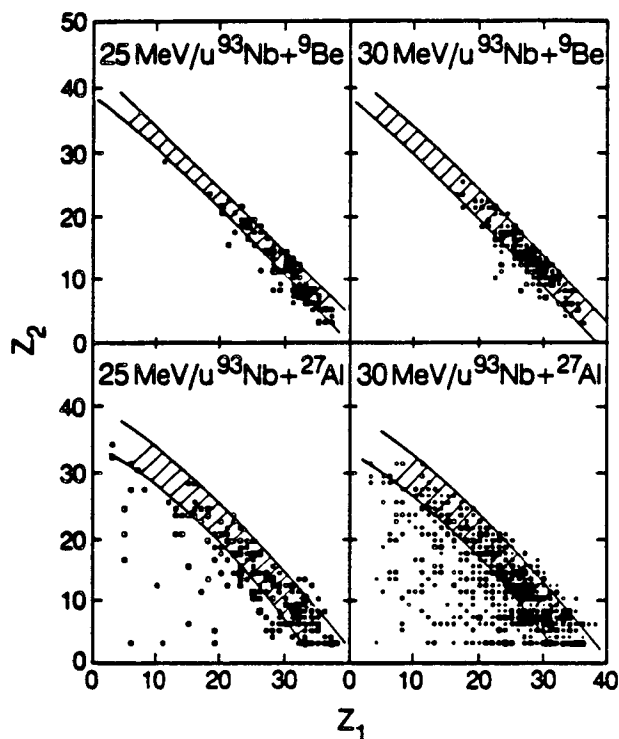


FIGURE 19 Scatter plots of coincidence events between the 5.5° telescope (Z_1) and the -11° telescope (Z_2). The shaded areas represent an estimation of regions where binary events should lie following sequential evaporation from the primary fragments.

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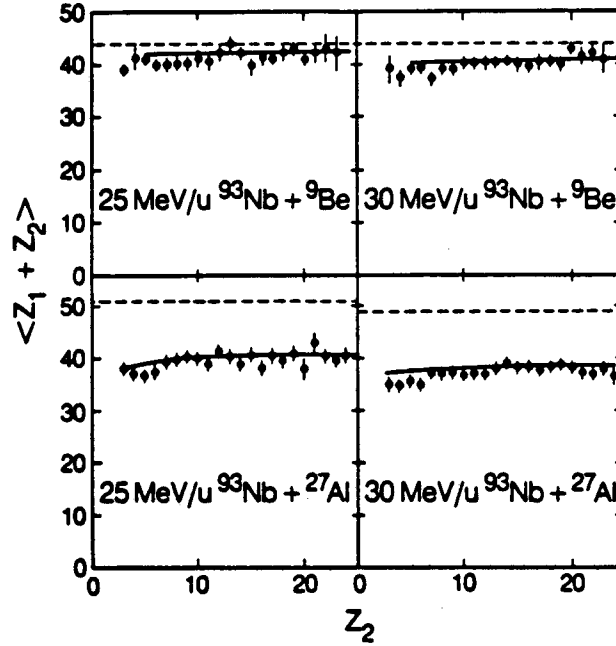


FIGURE 20 The mean sum $\langle Z_1 + Z_2 \rangle$ of coincidence events (solid symbols) plotted as a function of Z_2 . The dashed lines indicate the average charge of the compound system as estimated from the mass transfer. The charge loss of binary events, due to sequential evaporation, was estimated using the PACE⁹ code and the residual $\langle Z_1 + Z_2 \rangle$ values are indicated by the solid curves.

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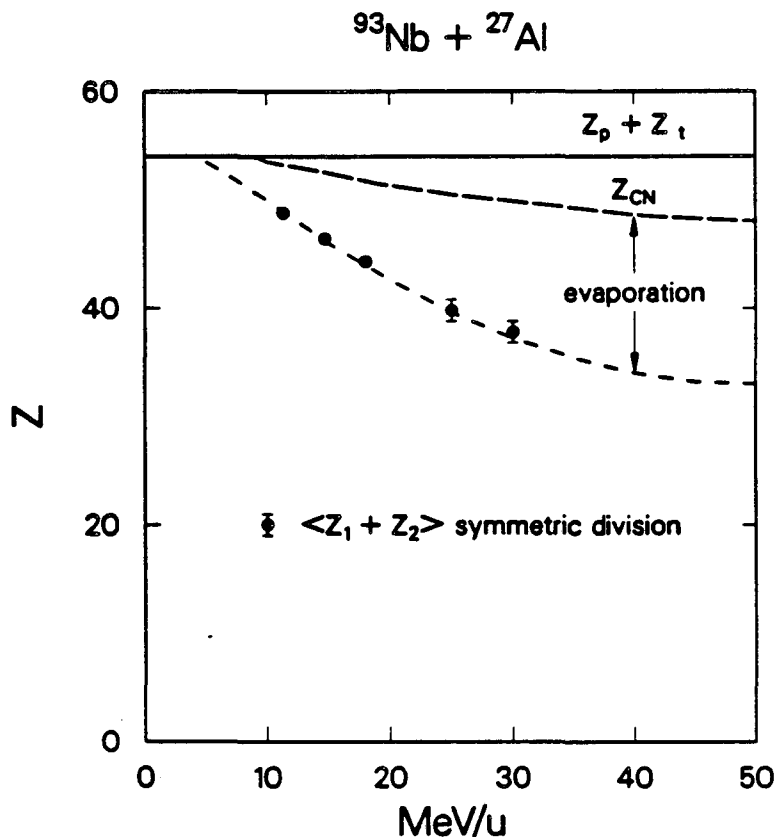


FIGURE 21 The solid circles show the average sum of the charges ($\langle Z_1 + Z_2 \rangle$) for nearly symmetric complex fragments measured in coincidence from the reaction $^{93}\text{Nb} + ^{27}\text{Al}$ as a function of bombarding energy. The curve labelled Z_{CN} represents the average charge of a compound system formed in an incomplete fusion reaction. It was calculated using the known systematics of momentum transfer. The charge lost due to evaporation from the primary fragments was calculated with PACE. The calculated residual charge ($Z_{\text{CN}} - Z_{\text{evaporated}}$) of the two fragments (short dashed curve) is in excellent agreement with the data.

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50 MeV/u La + C

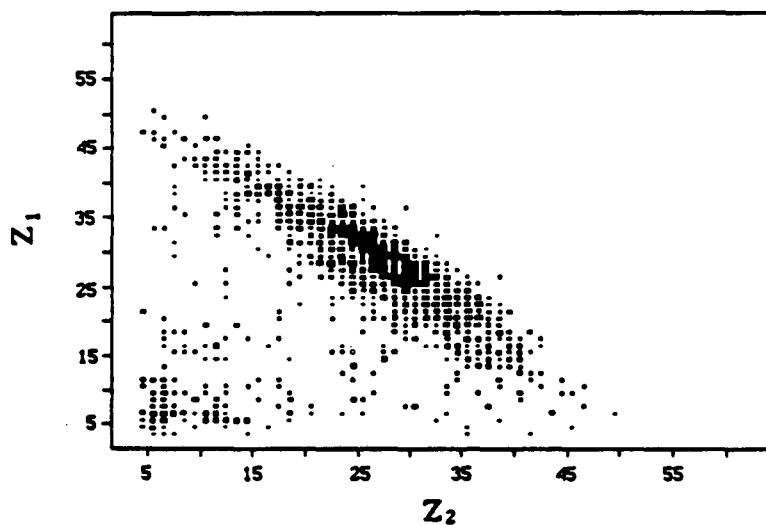


FIGURE 22 Z_1, Z_2 coincidence plot for the reaction 50 MeV/u La + C.

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4. CONCLUSION

We can state with a good degree of confidence that up to the highest explored energy, compound nucleus decay is by far the main source of complex fragments for $Z > 10$ and a very important one for $Z < 10$. An additional source at low A's is the "big foot" which is target related and seems to be consistent with the target picking up a few nucleons from the projectile and decaying in its turn. The very thorough exploration of angle and energy "phase space" allowed by reverse kinematics does not leave much room for any other processes. Can we conclude then that they are not there? I think we must wait for ternary and quaternary events which will undoubtedly dominate the scene at higher energies. However even with these events one needs to be cautious. Ternary, quaternary and higher multiplicity events can originate from sequential binary decays. In fact, once one has a good excitation function for the binaries, it is a "simple" exercise to predict the rate of sequential ternaries and quaternaries. This will be the background, and it will not be small, on top of which we shall have to look in search of fancier mechanisms.

ACKNOWLEDGEMENT

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APPENDIX 1
THE ROLE OF SURFACE IN NUCLEAR SHATTERING

Aichelin and Huefner¹ have postulated that fragment formation in the intermediate energy region can be viewed as a shattering process. They attempted to calculate the fragment mass distribution in terms of a least biased approach that was later shown¹¹ to be a saddle point approximation to the Euler number partition.

Among the many shortcomings of this approach is its lack of energy dependence and its inability to connect the mass distributions to other observables. A possible way to introduce an energy dependence in the problem is suggested by the fact that it takes energy to produce the extra surface associated with fragment formation. In what follows, a way is shown to evaluate the mass distribution with the constraint of a fixed amount of generated surface.

As in Ref. 1 we define a probability $P(m,a)$ of producing a fragment of a mass with multiplicity m . The constraints are

$$\sum_m P(m,a) = 1 \quad (1)$$

for each a

$$\sum_m \sum_a ma P(m,a) = N \quad , \quad (2)$$

N being the mass of the object fragmented

$$\sum_m \sum_a kma^{2/3} P(m,a) = S \quad , \quad (3)$$

S being the surface produced. The information associated with P modified by the constraints is

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$$I = \sum_m \sum_a P(m,a) \ln P(m,a) - K(a) P(m,a) + Dma P(m,a) + Ama^{2/3} P(m,a) \quad (4)$$

where $K(a)$, D , and A arise from the introduction of the constraints. Minimization of the information I gives

$$P(m,a) = \exp[K(a) - 1] \exp[-m(Da + Aa^{2/3})] = C(a) \exp[-m(Da + Aa^{2/3})] \quad (5)$$

Applying (1)-(3) to (5) one obtains:

$$C(a) = 1 - \exp[-(Da + Aa^{2/3})] \quad (6)$$

$$\sum_a \frac{a}{a \exp(Da + Aa^{2/3}) - 1} = N \quad (7)$$

$$k \sum_a \frac{a^{2/3}}{a \exp(Da + Aa^{2/3}) - 1} = S \quad (8)$$

Summing $P(m,a)$ over m one has

$$P(a) = \frac{1}{\exp(Da + Aa^{2/3}) - 1} \quad (9)$$

By solving Eqs. (7) and (8) simultaneously for D and A and substituting the values so obtained in Eq. (9) one arrives at the derived distribution. If no restriction is imposed upon the surface, then $A = 0$ which defines the unconstrained surface S_0 . If a restriction is imposed by fixing S near S_0 one can linearize the problem and obtain an analytical result.

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$\Delta A =$

$$\frac{\Gamma(5/3) \zeta(5/3)}{[-4/3\Gamma(4/3)\zeta(4/3) + \{5/3\Gamma(5/3)\zeta(5/3)\}^2 / \{2\Gamma(2)\zeta(2)\}]} D_o^{2/3} \frac{\Delta S}{S_o}$$

$$\Delta D = -\Delta A \frac{5/3\Gamma(5/3)\zeta(5/3)}{2\Gamma(2)\zeta(2)}$$

$$D_o = \frac{\sqrt{\zeta(2)}}{\sqrt{N}}$$

Numerically one has

$$\Delta A = -1.618305 D_o^{2/3} \frac{\Delta S}{S_o}$$

$$\Delta D = -0.971156 \Delta A$$

$$D_o = \frac{1.2825}{\sqrt{N}}$$

As an example, Fig. 23 shows the resulting mass distribution assuming $N = 200$. The three curves correspond to $\Delta S/S_o = 0$; 0.2; -0.2.

One can readily see that by requiring more surface area one favors the formation of light fragments and by requiring less surface area one enhances the production of heavy fragments.

Lastly, it remains to be established how much energy is invested in surface production in any given reaction. This may not be easy to determine. However it may be possible to infer that from the determination of the total fragment kinetic energy in the center of mass of the fragmenting nucleus. If the virial theorem can be applied, then a relation must exist between the average total kinetic energy and

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N=200

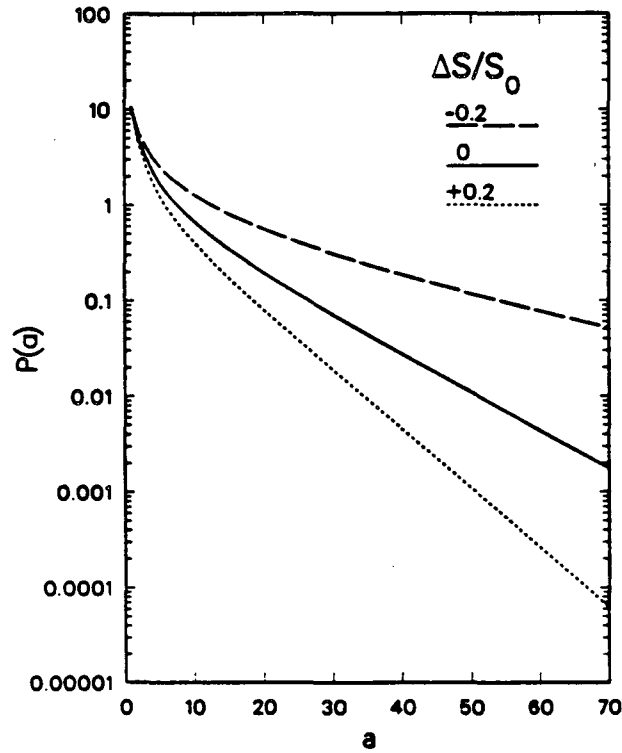


FIGURE 23

the average potential energy which is approximately proportional to the average produced surface. One could try $\bar{E}_{kin} = a_s \bar{S}$ where a_s is the liquid drop surface energy coefficient.

APPENDIX 2

MOMENTUM TRANSFER IN INTERMEDIATE ENERGY COLLISIONS

The extensive measurements of momentum transfer in intermediate energy heavy ion collisions demand an elucidation of the essential kinematic and dynamic features of the associated interaction.

Let us consider the following greatly simplified model. Two nuclei of mass A and B collide in such a way that the nucleus B occludes a portion α of A and, correspondingly, A occludes a portion β of B. Let $\Delta\alpha$ and $\Delta\beta$ be the separation energies of the occluded parts α and β from their respective nuclei A and B.

The following questions arise:

I. If $\Delta\beta = \infty$

- 1) What is the minimum energy with which B must strike A so that the occluded portion α is shaved off and attaches itself to B?
- 2) At any higher energy than calculated in 1) what is the momentum of A- α and B+ α ? Similarly, if $\Delta\alpha = \infty$ one can ask the questions symmetric to the ones above.

II. If neither $\Delta\alpha$ nor $\Delta\beta = \infty$

- 1) At what energy will any of the pieces α, β be shaved off?
- 2) At what energy will both of the pieces α, β be shaved off?

It is simple to show that the questions based in I have an answer that is purely kinematical and independent on the dynamics while the questions posed in II do require a solution of the dynamical problem. In other words the answers to I require solely the knowledge of A, B, α, β , $\Delta\alpha, \Delta\beta$, while the answers to II require additional information like restoring forces etc. Unfortunately this additional informa-

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tion is strongly model-dependent. However some illumination is already provided by answering the easier questions in I.

Let us consider first the case of A at rest being hit by B with momentum P. If the piece α sticks to B we have for the momentum of A - α and B + α

$$\frac{P_{A-\alpha}}{P} = \frac{m_{\alpha}^*}{B + \alpha} \left[1 - \sqrt{1 - \frac{2\Delta\alpha}{m_{\alpha}^* v_0^2}} \right]$$

$$\frac{P_{B+\alpha}}{P} = 1 - \frac{m_{\alpha}^*}{B + \alpha} \left[1 - \sqrt{1 - \frac{2\Delta\alpha}{m_{\alpha}^* v_0^2}} \right]$$

where

$$m_{\alpha}^* = \frac{(B + \alpha)(A - \alpha)}{A + B} ;$$

$$v_0 = \frac{B}{B + \alpha} v ;$$

$$v = \frac{P}{B}$$

The complementary case involves again A being at rest, but, this time, picking up a piece β from the projectile B. We have

$$\frac{P_{A+\beta}}{P} = \frac{\beta}{B} \left[1 + \frac{A(B - \beta)}{(A + B)\beta} \left[1 - \sqrt{1 - \frac{2\Delta\beta}{m_{\beta}^* v_0^2}} \right] \right]$$

$$\frac{P_{B-\beta}}{P} = \frac{B - \beta}{B} \left[1 - \frac{A}{A + B} \left[1 - \sqrt{1 - \frac{2\Delta\beta}{m_{\beta}^* v_0^2}} \right] \right]$$

$$\text{where } m_{\beta}^* = \frac{(A + B)(B - \beta)}{A + B} ;$$

$$v_0 = \frac{A}{A + \beta} v \quad ;$$

$$v = \frac{P}{B} \quad .$$

It is interesting to notice that full momentum transfer (no shaving off of α or β) occurs when the square roots vanish, or when

$$\frac{2\Delta\alpha}{m_\alpha v_0^2} = 1 \quad ; \quad \frac{2\Delta\beta}{m_\alpha v_0^2} = 1 \quad .$$

This occurs when the energy of B is

$$\frac{E}{B} = \Delta\alpha \frac{(B + \alpha)(A + B)}{(A - \alpha)B^2} \quad ;$$

$$\frac{E}{B} = \Delta\beta \frac{(A + \beta)(A + B)}{(B - \beta)A^2}$$

Notice also that at asymptotically large energies the momentum transfers tend to those one would have without binding, or $\Delta = 0$.

In the spirit of the geometrical model one can attempt to calculate $\alpha, \beta, \Delta\alpha, \Delta\beta$ from the impact parameter. The quantities α, β are given by Swiatecki as reported in Ref. 10. For the quantities $\Delta\alpha, \Delta\beta$ we can take the energy of the new surface created by the abrasion process. For this we need to calculate the area of lateral surface defined by a cylinder and a sphere of radii r and R at an impact parameter b . If S is such a lateral area the total new surface created is $2S$. The area S can be obtained analytically

$$S = 8r \sqrt{R^2 - r^2 - b^2} + 2br E\left(\frac{\pi}{2}, k\right)$$

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$$k = \frac{2br}{\sqrt{R^2 - r^2 - b^2 + 2br}}$$

for $R^2 - r^2 - b^2 > 2br \geq 0$, and

$$S = 4r \sqrt{\frac{1}{br}} \left\{ (R^2 - r^2 - b^2 - 2br) \right. \\ \left. \times F\left(\frac{\pi}{2}, \frac{1}{k}\right) + 4br E\left(\frac{\pi}{2}, \frac{1}{k}\right) \right\}$$

for $|R^2 - r^2 - b^2| < 2br$, where E, F are the elliptic integrals of 1st and 2nd kind. As an example let us consider the reaction Be + Nb.

Figure 24 shows the impact parameters at which incomplete fusion processes become energetically possible for Be and Nb breakup. The area below each curve corresponds to the fragmentation region; above each curve is the complete fusion region. At impact parameters less than 3.5 fm the Be projectile is completely occluded by the Nb target; hence, Be fragmentation does not occur. Note that at all impact parameters less energy is required to shatter Be due to the smaller surface area created.

Figure 25 shows the fraction of the initial momentum in the target-like (Nb) fragment as a function of bombarding energy per nucleon. The momentum transfer was calculated assuming projectile (Be) breakup at various impact parameters. Figure 26 is the complement of Fig. 25 showing the fractional momentum in the projectile residue. As would be expected, central collisions lead to larger momentum transfer. In the limit of large bombarding energies the momentum transfer at any given impact parameter tends to a constant as $\frac{\Delta\alpha}{E_{lab}} \rightarrow 0$.

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$^{93}\text{Nb} + ^9\text{Be}$
Onset of Incomplete Fusion

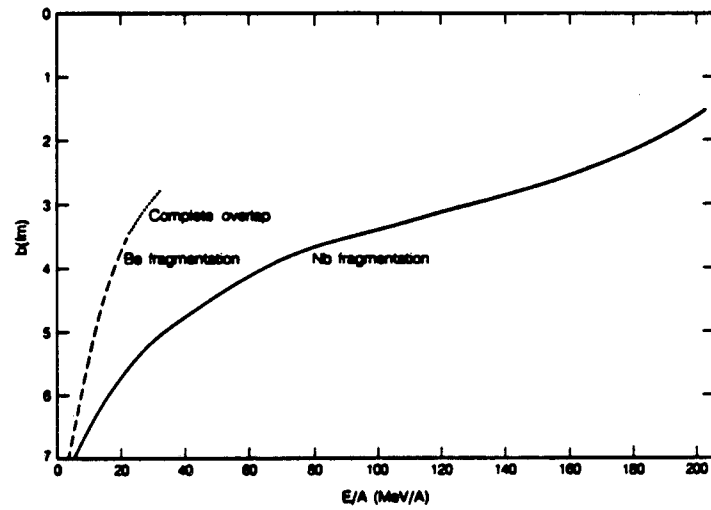


FIGURE 24

$^{93}\text{Nb} + ^9\text{Be}$
Momentum Transfer at Selected Impact Parameters
- Projectile Breakup
Target-like Fragment

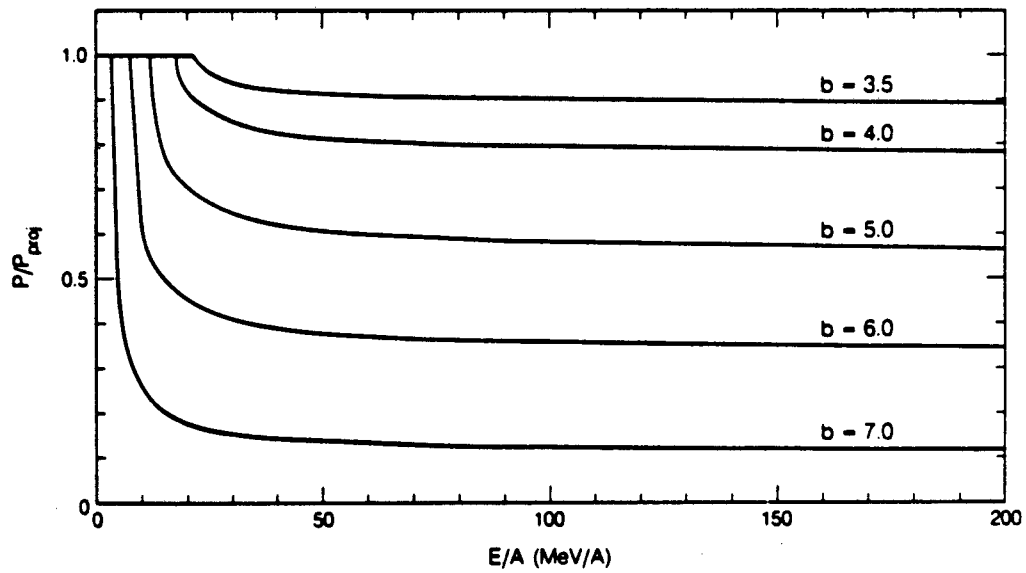


FIGURE 25 Momentum of Nb-like fragment after Be breakup.

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$^{93}\text{Nb} + ^9\text{Be}$
Momentum Transfer at Selected Impact Parameters
- Projectile Breakup
Projectile Residue

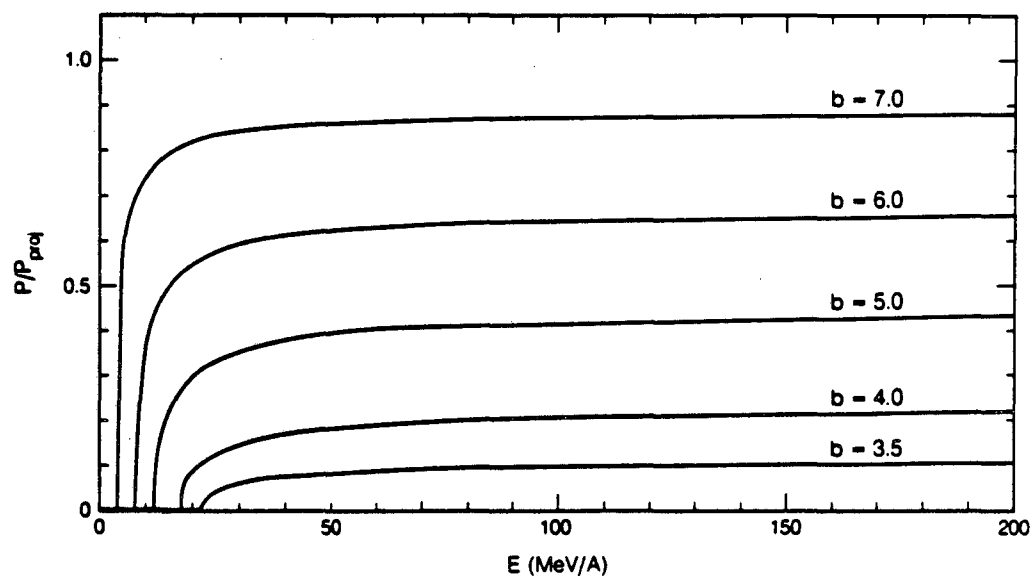


FIGURE 26 Momentum of Be-like fragment after Be breakup

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