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Taking Advantage of Multiuser Diversity and Beamforming in Wireless Cellular Networks

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Abstract—This paper demonstrates that a pre-scheduled (primary) mobile user can receive information along with another user by taking advantage of transmit beamforming and multiuser diversity concepts. The motivation for combining these two approaches is to show that even in a time-sharing scheduling system (like many current systems), opportunistic users can also be scheduled without interfering with the pre-scheduled mobile users and still have good performance. The practicality of this approach is shown in the small amount of feedback required to take advantage of multiuser diversity, so long as we have full channel knowledge for the primary user. Once the channel of the primary user is known, the nullspace of its channel is used to maximize the received signal-to-noise ratio of the selected opportunistic user, given the small amount of feedback. A lower-bound for the signal-to-interference-plus-noise ratio (SINR) is derived and verified via simulation, and different approaches are then considered for the problem of selecting the appropriate secondary user conditions and ensure a logarithmic increase of SINR as a function of the number of mobile users.

I. INTRODUCTION

Multiuser diversity concept is based on the fact that when the number of mobile users in the network increases, the probability of finding a user with a good channel condition and high SINR increases. In fact in a MIMO broadcast channel with N_t base station (BS) antennas and N_r single-antenna mobile users, it has been shown that when $N_r \rightarrow \infty$, the system can achieve optimum capacity [1], [3].

In this work, we formulate a multiuser diversity approach that allows for an opportunistic mobile user (MU) scheduling within the framework of a deterministic time-sharing scheduling system by using opportunistic interference management (OIM) [1] and a simple beamforming technique. The idea is to show that by taking advantage of multiuser diversity, we can simultaneously use these two schemes to improve the SINR for the opportunistic user, while only sacrificing one antenna for transmission at the BS to this user and only receiving one integer of feedback.

The rest of the paper is organized as follows. Section II covers the previous work related to multiuser diversity and beamforming techniques. Section III outlines the basics of the system and the concepts of OIM and nullspace transmit beamforming. In section IV, we detail the framework of our approach and compute the SINR of both users. In section V,

we analyze the performance of the system by assigning the OIM conditions based on the number of mobile users and compute the average SINR performance as the number of MUs increases. Section VI compares our analysis with simulation results. We conclude the paper in section VII and outline future work.

II. PREVIOUS WORK

Random beamforming to schedule mobile users was first investigated by Viswanath et al. [2], where random beams were created in order to induce a fast-fading environment and provide more opportunities for all of the mobile users in the network to be the preferred user. The required feedback in this scheme is the SNRs of all of the mobile user in the network. This concept is extended in [3] by generating multiple random orthogonal beams and scheduling the mobile users with the highest SINRs with respect to each beam channel, and [4] showed that zero-forcing random beamforming also exhibits multiuser diversity property. It was shown that these approaches asymptotically perform similar to dirty paper coding (DPC) [5], which is the optimal approach. Other techniques in [6]–[8] use random beamforming concepts to transmit to the opportunistic user more coherently, but require more feedback, more time (latency), or both.

Recent techniques have focused on reducing the feedback required for the multiuser diversity approach. Zheng et. al. [1] showed that as the network grows large, only a small number of these users need to feedback their strongest channel to the base station, given that they have very low SNR with the remaining channels. By setting threshold conditions for strong and weak channels, this requirement leads to the SINR of each user sending feedback to be above a certain threshold, hence guaranteeing a threshold achievable rate performance. This technique is also shown to asymptotically approaches DPC [9] performance.

A notable technique similar *in concept* to our work and [9] is the work by Shen and Fitz [10], which is a cognitive radio technique that utilizes a minimum SINR threshold condition for the primary user, and allows secondary users to transmit so long as their combined interference is such that the SINR of the primary user is above the threshold condition. Note

that only the performance of the primary user is guaranteed, but not that of the secondary users. Also, [10] requires MUs with multiple receive antennas. Finally, our approach uses the OIM threshold conditions to coherently beamform to the opportunistic user only within the nullspace of the deterministically scheduled MU, and its channel will align with this beamforming vector with high probability. To the best of our knowledge, this is the first work that combines multiuser diversity and antenna index feedback to perform coherent zero-forcing transmit beamforming for two MUs using a single multi-antenna BS.

III. PRELIMINARIES

A. System Model

Consider a system with a BS having N_t antennas that will transmit a signal to a MU who is deterministically scheduled to receive information during the current transmission period. Suppose this system can also schedule the transmission of another MU's signal during the current transmission period, such that it will not interfere with the deterministically scheduled MU, where this other MU is scheduled opportunistically using multiuser diversity. If the transmitted signals are $\mathbf{x}_i(t) \in \mathbb{C}^{N_t \times 1}$ ($i = 1, 2$), and complex Gaussian channel gain vectors from the BS to MU- i is $\mathbf{h}_i(t) = [h_{i,1}, \dots, h_{i,N_t}] \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{\beta_h})$, then the received signal at mobile user i is

$$y_i(t) = \mathbf{h}_i(t)\mathbf{x}_i(t) + \mathbf{h}_i(t)\mathbf{x}_j(t) + z_i(t), \quad (1)$$

where $z_i(t) \sim \mathcal{CN}(0, 1)$ is the receiver noise.

We will use this channel model for our analysis, dropping the time index t . The superscript $(\cdot)^T$ denotes the matrix transpose operation and $(\cdot)^H$ denotes the matrix Hermitian transpose operation. We will refer to BS antenna j as BS- j and mobile user i as MU- i .

B. Opportunistic Interference Management (OIM)

OIM is a specific multiuser diversity technique that selects a set of mobile users from a larger set of eligible mobile users that possess the OIM conditions. Define $\xi_{i,j} = |h_{i,j}|^2$ as the SNR gain of the channel from BS- j to MU- i . The OIM conditions are as follows: (1) there is a strong channel $h_{i,j}$ if the SNR gain $\xi_{i,j}$ is greater than the SNR threshold ξ_s ; (2) the remaining channels are weak if the sum of the remaining SNR gains is less than the INR threshold ξ_w . If a MU meets both of these conditions, then the MU is eligible for opportunistic scheduling. From the set of N mobile users that satisfy the OIM conditions, the number of mobile users D ($1 \leq D \leq N_t$) that will receive information will be selected such that¹

$$\begin{aligned} \xi_{i,j} &\geq \xi_s & \text{if } j = i \\ \Xi_{i \setminus i} &= \sum_{\forall j \neq i} \xi_{i,j} \leq \xi_w & \text{for } i, j \in \{1, \dots, N_t\}. \end{aligned} \quad (2)$$

¹To simplify the presentation, we use the index for mobile users from 1 to N_t .

The mobile users in the set will have satisfied the OIM conditions, and thus be eligible to receive information from one dedicated antenna at the BS. Meanwhile, this set of mobile users will not interfere significantly with each other, so long as each eligible mobile user that is selected has its information sent mostly along the strong channel.

The distribution of the multiplexing gain D was shown in [1], [9]. Since $\xi_{i,j} \sim \text{Exponential}(1/\beta_h)$, $\Pr\{\xi_{i,j} \geq \xi_s\} = e^{-\xi_s/\beta_h}$ and $\Pr\{\Xi_{i,j} \leq \xi_w\} = \frac{\gamma(N_t-1, \frac{\xi_w}{\beta_h})}{(N_t-2)!}$,² the probability that mobile user satisfies the OIM conditions is shown as $p_{\text{oim}} = N_t e^{-\xi_s/\beta_h} \frac{\gamma(N_t-1, \frac{\xi_w}{\beta_h})}{(N_t-2)!}$. The probability that $D = d$ mobile users will receive information in parallel using the OIM conditions is shown [9] to be

$$p_D(d) = \begin{cases} (1 - p_{\text{oim}})^{N_t}, & N = 0 \\ \sum_{n=1}^{N_t} \binom{N_t}{d} \sum_{l=1}^d \binom{d}{l} (-1)^{d-l} \frac{1^n}{N_t^n} \\ \quad \cdot \binom{N_t}{n} p_{\text{oim}}^n (1 - p_{\text{oim}})^{N_t-n}, & N > 0 \end{cases}$$

where N is the number of mobile users that satisfy the OIM conditions. If $N = 0$, $D = 1$ by default.

C. Transmit Beamforming

Transmit beamforming is a technique that uses multiple transmit antennas to send information to multiple users by allocating each user's information along a vector. Each beam optimizes a specified property of the desired received signal for each user, based on the constraints imposed by the signals from the other users receiving information simultaneously. Some of the techniques that exist in beamforming include maximizing received SINR, minimize transmit power, maximize sum-rate, and reduce all interference to zero (zero-forcing). We will utilize the zero-forcing method in our work to create a simple framework for generating the beamforming vectors for opportunistic users while utilizing minimal feedback.

Suppose we have a mobile user MU-1 that is scheduled to receive information during the current transmission period, and has channel from the BS of \mathbf{h}_1 . Since $\mathbf{h}_1 \in \mathbb{C}^{1 \times N_t}$, the nullspace matrix $\Phi \in \mathbb{C}^{N_t \times (N_t-1)}$. Hence, if $\Phi = [\phi_1, \dots, \phi_{N_t-1}]$, $\phi_i \in \mathbb{C}^{N_t \times 1}$, then $\mathbf{h}_1 \phi_i = 0$, $\forall i$. Given the channel vector \mathbf{h}_2 from the BS antennas to MU-2, the optimum beamforming vector is given by

$$\mathbf{u}_2 = \frac{\Phi \Phi^H \mathbf{h}_2^H}{(\mathbf{h}_2 \Phi \Phi^H \mathbf{h}_2^H)^{1/2}}. \quad (3)$$

In order to fully take advantage of this approach to beamforming, the channel vectors \mathbf{h}_1 and \mathbf{h}_2 must be known. However, we will show that when a MU satisfies the OIM conditions, the full CSI is not needed.

IV. COMBINATION OF OIM AND NULLSPACE BEAMFORMING

Suppose a mobile user MU-2 is OIM eligible. If we call the BS antenna BS-2 that has the strong channel with MU-

² $\gamma(a, x) = \int_0^x t^{a-1} e^{-t} dt$

2, no part of MU-1's signal will be transmitted through BS-2. Therefore, the beamforming vector for MU-1's signal is defined as

$$\mathbf{u}_1 = \frac{\mathbf{h}_{1 \setminus 2}^H}{\|\mathbf{h}_{1 \setminus 2}\|}, \quad (4)$$

where $\mathbf{h}_{i \setminus j}$ is the channel vector for MU-1, but with entry j set to zero. This vector will maximize the received SNR at MU-1 for the case where BS- j cannot send any part of MU-1's signal.

As in the previous section, the nullspace matrix of \mathbf{h}_1 is $\Phi = [\phi_1, \dots, \phi_{N_r-1}]$ and the columns are orthonormal. If there are N mobile users satisfying the OIM threshold conditions ($N > 0$ feedback), and each OIM mobile user selected has a strong channel with an antenna at BS, then we wish to transmit as much power using the antenna with strong channel for that OIM mobile user. Let's assume one MU has OIM condition with the second BS antenna, i.e., BS-2. This means that the projection into Φ that we seek is the one that has the largest component in the second entry. Therefore, the beamforming vector in the nullspace of \mathbf{h}_1 that will maximize the transmit power at BS-2 is

$$\mathbf{u}_2 = \frac{\Phi \Phi^H \mathbf{e}_2}{\left(\mathbf{e}_2^T \Phi \Phi^H \mathbf{e}_2\right)^{1/2}}, \quad (5)$$

where \mathbf{e}_2 is the second standard basis column vector. Therefore, the transmitted signal for MU-1 is $\mathbf{x}_1 = \mathbf{u}_1 s_1$, and the transmitted signal for MU-2 is $\mathbf{x}_2 = \mathbf{u}_2 s_2$, where s_1 and s_2 are the symbols for MU-1 and MU-2, respectively. Since \mathbf{u}_2 is in the null-space of \mathbf{h}_1 , then the received signal at MU-1 is

$$y_1 = \|\mathbf{h}_{1 \setminus 2}\| s_1 + z_1, \quad (6)$$

and the received signal at MU-2 is

$$y_2 = \mathbf{h}_2 \mathbf{u}_2 s_2 + \mathbf{h}_2 \mathbf{u}_1 s_1 + z_2. \quad (7)$$

Using the beamforming vector from equations (4) and (5), the SINR for MU-1 becomes

$$\Upsilon_1 = \|\mathbf{h}_{1 \setminus 2}\|^2, \quad (8)$$

and the SINR for MU-2 becomes

$$\Upsilon_2 = \frac{\mathbf{u}_2^H \mathbf{h}_2^H \mathbf{h}_2 \mathbf{u}_2}{\mathbf{u}_1^H \mathbf{h}_2^H \mathbf{h}_2 \mathbf{u}_1 + 1}. \quad (9)$$

Note that the only strong term in \mathbf{h}_2 vector is $h_{2,2}$ due to OIM conditions and the rest of channel coefficients in this vector are weak. Therefore, the numerator in (9) is approximately $\xi_{2,2} |u_{2,2}|^2$. Hence, we can approximate Υ_2 with

$$\Upsilon_2 \approx \frac{\xi_{2,2} |u_{2,2}|^2}{\mathbf{u}_1^H \mathbf{h}_2^H \mathbf{h}_2 \mathbf{u}_1 + 1} = \frac{\xi_{2,2} |u_{2,2}|^2}{\alpha_1 \Xi_{2 \setminus 2} + 1}, \quad (10)$$

where α_1 is the inner-product of \mathbf{u}_1 and the unit-vector in the direction of \mathbf{h}_2 . The expected value of Υ_1 is given by

$$\mathbb{E}[\Upsilon_1] = N_t \beta_h (1 - p_{\text{oim}})^{N_r} + (N_t - 1) \beta_h (1 - (1 - p_{\text{oim}})^{N_r}). \quad (11)$$

For Υ_2 , we find a lower-bound by noting that

$$\mathbb{E}[\Upsilon_2 | N > 0] \geq \mathbb{E}[\xi_{2,2} | u_{2,2}|^2] \mathbb{E}\left[\frac{1}{\alpha_1 \Xi_{2 \setminus 2} + 1}\right]. \quad (12)$$

Using the probability of achieving the OIM condition, and calling the number of mobile users satisfying the OIM conditions N , we have that³

$$\begin{aligned} \mathbb{E}[\Upsilon_2] &= \mathbb{E}[\Upsilon_2 | N > 0] \cdot \Pr\{N > 0\} \\ &\geq \mathbb{E}[\xi_{2,2} | u_{2,2}|^2] \mathbb{E}\left[\frac{1}{\alpha_1 \Xi_{2 \setminus 2} + 1}\right] \\ &\quad \cdot (1 - (1 - p_{\text{oim}})^{N_r}) \end{aligned} \quad (13)$$

$$\begin{aligned} &\stackrel{(a)}{\geq} \frac{\mathbb{E}[\xi_{2,2} | u_{2,2}|^2]}{\mathbb{E}[\|\alpha_1 \Xi_{2 \setminus 2} + 1\|]} (1 - (1 - p_{\text{oim}})^{N_r}) \\ &= \frac{(\xi_s + \beta_h) \frac{N_r - 1}{N_r}}{\mu \frac{1}{N_r - 1} + 1} (1 - (1 - p_{\text{oim}})^{N_r}), \end{aligned} \quad (14)$$

where (a) is true because of the convexity of the function $1/x$ when $x > 0$, and $\mu = \mathbb{E}[\Xi_{2 \setminus 2}] = \frac{\beta_h \gamma(N_r, \xi_w / \beta_h)}{\gamma(N_r - 1, \xi_w / \beta_h)}$. This lower bound on the expected value of Υ_2 is used to determine how the parameters ξ_w and ξ_s can be selected in the system to obtain the best performance.

V. PERFORMANCE OF EXPECTED SINR AS A FUNCTION OF OIM THRESHOLD CONDITIONS AND NUMBER OF MOBILE USERS

In equation (14), we can see that $\mathbb{E}[\Upsilon_2]$ is a function of parameters ξ_s , ξ_w , and N_r . It is desirable to have $\mathbb{E}[\Upsilon_2]$ increases as the number of mobile users increases. This is accomplished by having the probability of $N > 0$ increases as N_r increases, or by modifying the OIM threshold conditions as the number of mobile users increases.

A. Setting an asymptotic limit for lower-bound on $\mathbb{E}[\Upsilon_2]$

There may be a situation where we wish to maintain a specific minimum performance by any mobile user that is selected opportunistically, regardless of the number of mobile users that are in the network. In other words, we select a specific strong channel threshold value ξ_s by noting that when both threshold conditions are constant, the lower-bound on $\mathbb{E}[\Upsilon_2]$ approaches asymptotically a limiting value of

$$\lambda = \frac{(\xi_s + \beta_h) \frac{N_r - 1}{N_r}}{\mu \frac{1}{N_r - 1} + 1}, \quad (15)$$

and then setting the lower-bound in (14) to $\mathbb{E}[\Upsilon_2] \geq \lambda(1 - (1 - p_{\text{oim}})^{N_r})$. We can solve equation (15) for ξ_s , and by selecting ξ_w we can determine p_{oim} and how $\mathbb{E}[\Upsilon_2]$ grows with N_r .

Since this provides a lower-bound on $\mathbb{E}[\Upsilon_2]$, an expression for the minimum number of mobile users required can

³For the rest of paper, we omit some detail derivations due to space limitations.

be found in order to achieve any desired specific value of $\mathbb{E}[\Upsilon_2]$. Suppose M is the number such that for $\rho \in (0, 1)$, $\rho\lambda = \lambda(1 - (1 - p_{\text{oim}})^M)$. Solving for M , we get

$$M = \frac{\ln(1 - \rho)}{\ln(1 - p_{\text{oim}})}. \quad (16)$$

Therefore, we know the number of mobile users so that the expected SINR is better than $\rho\lambda$. This also gives the minimum amount of feedback of $N = p_{\text{oim}}M$. Therefore, we can set a desired minimum performance for $\mathbb{E}[\Upsilon_2]$ by selecting the appropriate λ and ρ , and determine the minimum network size required for this performance.

B. Having constant $\Pr\{N > 0\}$

In order to have an SINR that increases with the number of mobile users, we can select a desired value $\epsilon \in (0, 1)$ such that $\Pr\{N > 0\} = 1 - \epsilon$ stays constant as N_r changes. In other words, we have

$$\begin{aligned} 1 - (1 - p_{\text{oim}})^{N_r} &= 1 - \epsilon \\ \Rightarrow p_{\text{oim}} &= 1 - \epsilon^{\frac{1}{N_r}}. \end{aligned} \quad (17)$$

To simplify, we can set ξ_w as constant, and solve (17) for ξ_s to find the threshold conditions for desired performance, with the important condition that $\xi_s > \xi_w$ always. Given this restriction, we can ensure that p_{oim} is a valid probability. Assigning ξ_s in this manner ensures $\mathbb{E}[\Upsilon_2]$ grows at least as

$$\mathbb{E}[\Upsilon_2] \geq \frac{\beta_h \frac{N_t - 1}{N_t} (1 - \epsilon)}{\mu \frac{1}{N_t - 1} + 1} \left(1 + \ln \left[\frac{N_t \gamma(N_t - 1, \frac{\xi_w}{\beta_h})}{(1 - \epsilon^{\frac{1}{N_r}})(N_t - 2)!} \right] \right). \quad (18)$$

This type of approach is useful for allowing users to opportunistically be scheduled with higher probability even when the number of mobile users is low. The threshold for strong channels is adjusted to allow more mobile users the likelihood of participating opportunistically. However, the average SINR when N_t is small will not be as good for the reason that over multiple transmission periods, the users are given the opportunity to transmit with lower strong channel thresholds.

VI. SIMULATION RESULTS

Figure 1 shows the performance of our approach when the asymptotic SINR lower-bound is set using the parameter λ . Notice that the smaller the required asymptotic SINR lower-bound, the easier to satisfy the required performance with smaller number of MUs. However, as the network size increases, it is feasible to have a greater asymptotic SINR lower-bound due to the higher probability that there exists at least one mobile user that satisfies the greater SINR threshold condition.

Figure 2 highlights the lower-bound on the growth of the SINR when we assign the SINR threshold condition dynamically by setting the value for ϵ . As expected, for lesser values of ϵ , the dynamic assignment of ξ_s using equation (18) can

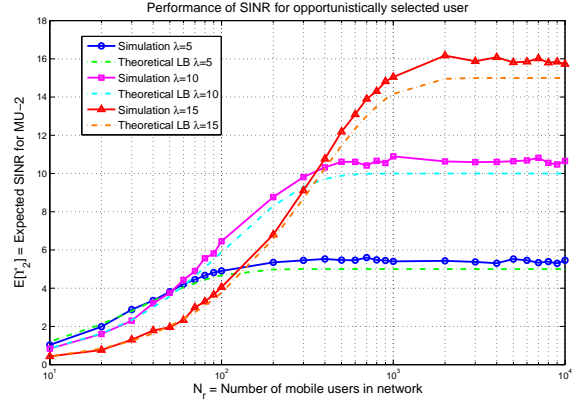


Fig. 1. Highlights the advantage of adjusting the asymptotic desired performance depending on the number of mobile users to ensure specific gains in SINR and sumrate for a specific number of mobile users; $N_t = 3$, $\beta_h = 10$, $\xi_w = 1.5$.

only occur for larger values of N_r .⁴ However, the gains that occur when it does happen become more apparent. For smaller values of N_r , the value of ϵ should be greater to allow for gains in SINR.

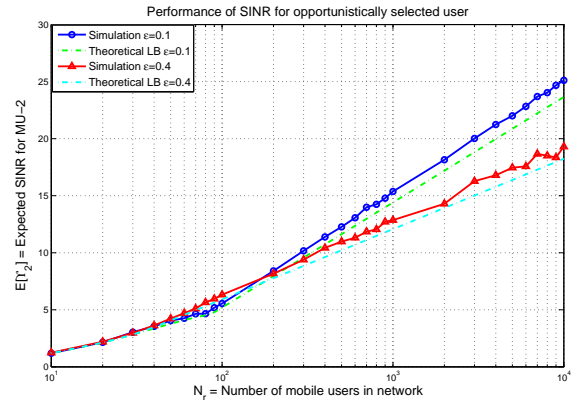


Fig. 2. Highlights the advantage of maintaining the feedback constant by setting the parameter ϵ , and finding a dynamic assignment of ξ_s that provides gains in SINR and sumrate as N_r increases; $N_t = 3$, $\beta_h = 10$, $\xi_w = 1.5$.

Finally, we compare in figure 3 how the sumrate of this technique with the case where only opportunistic MUs are scheduled. The number of MUs can be either one or two, as is the case with our current approach. As can be seen, combining the two approaches allows for much better performance when the number of MUs is small which is the practical situation. However, as the number of MUs increases, the OIM-only approach outperforms this technique but the minimum required

⁴Note that ϵ is a function of p_{oim} using equation (17) and p_{oim} is a function of ξ_s .

MUs is not practical. We did not compare our technique with beamforming-only technique because that approach does not depend on the number of users (we only need two users in the network).

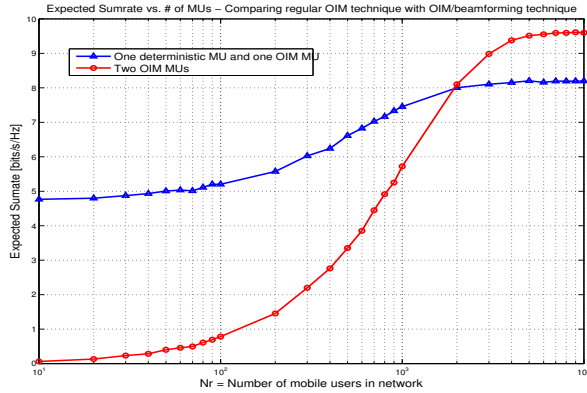


Fig. 3. Compares the sumrate performance between one deterministic MU and one OIM MU with beamforming, and regular OIM technique with at most 2 opportunistic MUs; $N_t = 3$, $\beta_h = 10$, $\xi_w = 1.5$.

VII. CONCLUSION AND FUTURE WORK

Many existing works focus on using multiuser diversity to increase the capacity of cellular networks. In this work, we demonstrate that we can take advantage of multiuser diversity while allowing the predetermined user in the system to transmit information. It has been observed that multiuser diversity gains can be made when scheduling an opportunistic mobile user together with a deterministically scheduled user, while having the interference of the opportunistic mobile user be in the nullspace of the deterministic mobile user. This allows the deterministically scheduled user to maintain a high performance in SINR and achievable rate on the average. Using the nullspace of the deterministically scheduled users channel and only the channel index of the strong channel for the opportunistic user, the gains in SINR and achievable rates for the opportunistic user can be seen as the number of total mobile users in the network increases. This allows the use of the lower-bounds on expected SINR of this user to find upper-bounds on the number of mobile users required to achieve a desired performance out of the opportunistically scheduled users. Two different approaches were formulated for setting the appropriate threshold conditions and allow the performance gains to occur.

In this work, only one opportunistic user was scheduled during each transmission period. However, as many different multiuser diversity schemes have shown, opportunistic scheduling has the best gains when more than one user is scheduled. Using the approach in this work, it can be observed that more than one opportunistic mobile user can be assigned so long as the number of antennas at the BS is $N_t > D$, where D has the distribution in equation (3). In such case, the beamforming vectors to the opportunistic users is just given

by (5) so that all of their signals are in the nullspace of MU-1. In this case, MU-1's signal can only be transmitted using $N_t - D$ BS antennas.

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