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There has been considerable discussion recently on the spin assignment to the K* resonance based on the analysis of various experiments.[1,2,3] The purpose of this note is to show that if the spin of the K* is assumed to be one, good agreement is obtained for the isovector charge of the kaon,[4] while no such agreement can be obtained if the K* spin is zero. Throughout this work the approximation of retaining only the ρ meson contribution in the $I = 1$, $J = 1$ channel will be performed, and the K* will be assumed to be the only effective πK resonance.

The isovector kaon form factor satisfies the dispersion relation[5]

$$F_K(t) = \frac{1}{\pi}\int_4^\infty \frac{2q'^3\, F_\pi{}^*(t')\, B_1{}^{(-)}(t')}{\sqrt{t'}\,(t' - t)}\, dt', \tag{1}$$

where $q' = [(t'/4) - 1]^{1/2}$, $B_1{}^{(-)}(t)$ is the $I = 1$, $J = 1$ amplitude for the $\pi\pi - K\bar{K}$ process as defined by Lee,[6] and the pion mass is taken to be one. Here $F_\pi(t)$ is the pion form factor normalized to one at $t = 0$ and is given by

$$F_\pi(t) = \frac{1}{D_\rho(t)} = \frac{t_\rho}{t_\rho - t - i\gamma\,[(t-4)^3/t]^{1/2}\,\theta(t - 4)}\,, \tag{2}$$

where t_ρ and γ are the energy squared and reduced width of the ρ meson, respectively, and $\Theta(x)$ is the usual step function.

In order to calculate $B_1^{(-)}(t)$, it is convenient to define the function

$$\Gamma(t) = B_1^{(-)}(t)\, D_\rho(t) , \tag{3}$$

which has the same singularities as $B_1^{(-)}(t)$ except for the right-hand cut due to the ρ intermediate state. The left-hand cut in $\Gamma(t)$ starts at $t \cong -16$ if the lowest-energy intermediate state in the crossed channels (πK channels) is the K^* resonance at 885 MeV, and the right-hand cut starts effectively at the value of the square of the energy of the first significant state in the $J = 1$, $I = 1$ channel above the ρ meson. The dispersion relation for $\Gamma(t)$ can be evaluated in the kernel approximation first used by Balázs,[7] which provides a means of taking into account with reasonable accuracy the contribution from the left-hand cut and even some contribution from the inelastic cut. The result is a two-pole expression for $\Gamma(t)$:

$$\Gamma(t) = \frac{\alpha_1}{t - p_1} + \frac{\alpha_2}{t - p_2} \quad . \tag{4}$$

The positions of the poles in Eq. (4) are essentially determined by the approximation procedure, and the values $p_1 = -21$ and $p_2 = -200$ will be used, in agreement with the criterion used in references 7c and 8. The residues α_1 and α_2 will be determined by analytic continuation from the crossed channels in which only the K^* is retained in the δ-function approximation. For this purpose the P wave in the t channel is projected from a fixed-t dispersion relation. This gives

$$B_1^{(-)}(t) = \frac{2M^{*2}\Gamma^*}{3k^*p^2q^2} \quad (2\ell^* + 1)\; P_{\ell^*}\left(1 + \frac{t}{2k^{*2}}\right)$$

$$\times\; Q_1\left(\frac{2M^{*2} - 2M^2 - 2 + t}{4pq}\right) , \tag{5}$$

where Γ^* is the half width of the K*, ℓ^* is the spin of the K*, M* is the K* energy, M is the kaon mass, $p^2 = t/4 - M^2$, $q^2 = t/4 - 1$, and $k^{*2} = [M^{*2} - (M+1)^2][M^{*2} - (M-1)^2]/4M^{*2}$. Equations (3), (4), and (5) evaluated at t = -1 and t = -2 are used to calculate the residues α_1 and α_2 . At these values of t the improved expression of Singh and Udgaonkar for $D_\rho(t)$ is used.[8]

With the above results one can compute the integral in Eq. (1), which, according to the normalization used, should yield 1/2 for t = 0. Integrating with the help of the usual δ-function approximation and using 25 MeV and 50 MeV for the half widths of the K* and ρ, one obtains

$$\begin{aligned} F_k(0) &\cong \tfrac{1}{2} \times 1.6 \text{ for } \ell^* = 1 \\ &\cong \tfrac{1}{2} \times 0.06 \text{ for } \ell^* = 0 . \end{aligned} \tag{6}$$

These results[9] are directly proportional to the K* width and inversely proportional to the ρ width. Now the experimental values for these widths are not well established, and further, one does not expect the entire vector charge of the kaon to be due to the two-pion contribution. However, within these approximations it is clear from Eq. (6) that one can obtain the kaon charge if the spin of the K* is one but not if it is zero.

This work originated from a suggestion by Professor G. F. Chew to whom I am indebted. I also wish to express my gratitude to L. Balázs, B. M. Udgaonkar, and other members of the theoretical group at Berkeley for encouragement and helpful discussions. It is a pleasure to thank Dr. David L. Judd for his kind hospitality at the Lawrence Radiation Laboratory.

FOOTNOTES AND REFERENCES

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1. M. Alston, G. Kalbfleisch, H. Ticho, and S. Wojcicki have reported on a study of some angular distributions which are consistent with spin zero (in Proceedings of the 1962 International Conference on High Energy Physics at CERN, Geneva, Switzerland (in publication); also Lawrence Radiation Laboratory Report UCRL-10232, June 19, 1962).
2. R. Armenteros et al. have ruled out spin zero in studying the K* production in ($p\bar{p}$) annihilation at rest. See R. Armenteros, L. Montanet, D. R. O. Morrison, A. Shapira, S. Nilsson, J. Vandermeulen, Ch. D'Andlau, A. Astier, C. Ghesquiere, B. P. Gregory, D. Rahm, P. Rivet, and F. Solmitz, in Proceedings of the 1962 International Conference on High Energy Physics at CERN, Geneva, Switzerland (in publication); also CERN/TC/PHYSICS 62-9 (unpublished).
3. W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Lee and T. O'Halloran also ruled out spin zero by studying the process $K^+ + p \rightarrow K^* + N^*_{33}$ (UCRL-10428, August 27, 1962, to be published in Phys. Rev. Letters).
4. A qualitative argument in this direction was given by G. Frye [Nuovo Cimento 18, 282 (1960)]. S. K. Bose recently studied the isovector kaon form factor, using a subtracted dispersion relation [Nuovo Cimento 24, 970 (1962)]. The method used involves a divergency in the case of a spin-one K*, so that a cutoff is needed. No conclusion is drawn on the spin of the K*.

5. F. Ferrari, G. Frye, and M. Pusterla, Phys. Rev. 123, 308 (1961).
6. Benjamin W. Lee, Phys. Rev. 120, 325 (1960).
7. a. L. Balázs, Phys. Rev. 125, 2179 (1962); b. L. Balázs, Lawrence Radiation Laboratory Report UCRL-10026, January 19, 1962 (to be published in Phys. Rev.); c. L. Balázs, Lawrence Radiation Laboratory Report UCRL-10157, March 28, 1962 (to be published in Phys. Rev.); d. L. Balázs, Lawrence Radiation Laboratory Report UCRL-10376, July 23, 1962 (submitted to Phys. Rev.).
8. V. Singh and B. M. Udgaonkar, Lawrence Radiation Laboratory Report UCRL-10264, May 25, 1962 (submitted to Phys. Rev.).
9. In order to see how sensitive these results are to a variation in the positions of the poles p_1 and p_2, the extreme case of $p_1 = -13.5$ and $p_2 = -186$ has been considered. Although this corresponds to placing a pole outside the left-hand cut (which starts at $t \cong -16$) the value for the $\ell^* = 1$ case is only decreased by 15%, while the result for $\ell^* = 0$ is increased by a factor of two. On the other hand a displacement of the left pole to -2×10^6 increases the result for the P-wave case by 10% and decreases the S-wave result by 22%.