THE WRINKLING OF A FLAME DUE TO VISCOSITY

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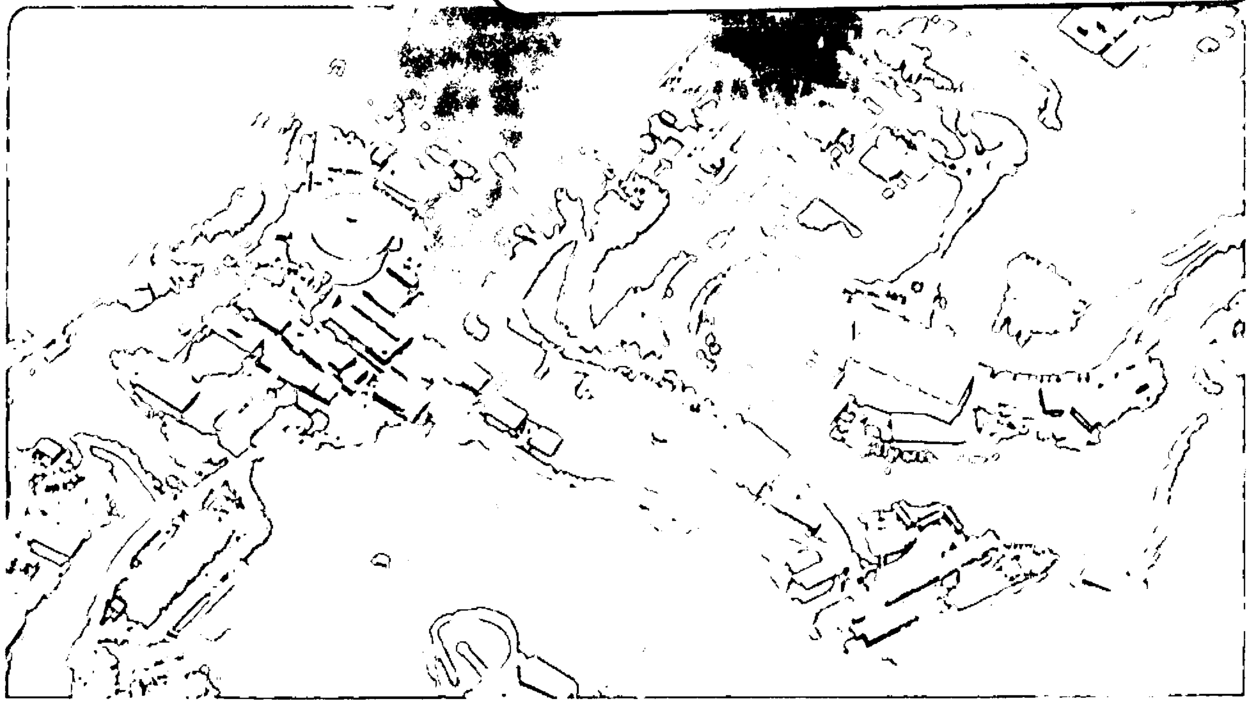

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Abstract

We present a numerical study of flame wrinkling due to viscosity. We model a flame propagating in a swirling, viscous, premixed combustible fuel inside a closed, rectangular vessel, and analyze the influence of vorticity produced by the no-slip condition along solid walls on the motion of the flame. The effect of viscosity is seen to be two-fold. First, it wrinkles the flame front, increasing the surface area available for burning and thus accelerating the combustion process. Second, the growth and decay of small eddies inhibit the ability of the flame to burn into the corners, extending the amount of time required for complete conversion of reactants to products. We perform our experiments over a variety of laminar burning velocities, and show that the slower the laminar flame speed, the greater the effect of viscosity on the rate of combustion, when compared to an inviscid calculation with the same flame speed.

One challenging and complex problem in the study of turbulent combustion is to analyze the effect of turbulence on the propagation of a flame. At high Reynolds number, turbulent eddies and recirculation zones form, due to viscous effects, which affect the position of the flame and the distribution of unburnt fuel available for combustion. In many situations, this interaction is of great importance. For example, in the design of internal combustion engines, one might attempt to direct the flow in such a way that the largest amount of fuel is burned as quickly as possible, thus maximizing the force on the piston and minimizing the amount of unburnt fuel expelled at the end of a stroke.

Questions of flame stability and the interaction of hydrodynamics with flame propagation have received considerable attention over the past few decades. We shall not attempt to review such work in this paper; a comprehensive, though now outdated, summary may be found in [9], a more recent review may be found in [12]. A standard technique in many of these investigations involves the use of perturbation analysis applied to a particular combustion model; for example, such techniques are used in [7] to investigate the effect of viscosity on the hydrodynamic stability of a plane flame front. In this work, we employ a different approach in that we develop a numerical technique to approximate the solution of a numerical model of turbulent combustion. Although out investigation assumes a fairly idealized view of flame propagation, the numerical technique employed is specifically designed for the modeling of flow at high Reynolds number. Thus, for example, the development of turbulent eddies and coherent structures in the flow is accurately portrayed in a natural and effective manner. We believe that this model and numerical technique represent a good step towards understanding tubulent combustion in situations where one is interested in the interaction between high Reynolds number flow and the speed and shape of the burning front.

We numerically model the effect of viscosity on the wrinkling of a flame front propagating in a swirling, viscous, incompressible, premixed combustible fuel inside a closed, rectangular vessel. The scheme, which uses random vortex elements coupled to a flame propagation algorithm based on Huyghen's principle, avoids the numerical smoothing present in most finite difference approximations to turbulent flow, and is free of the oscillations and instabilities that can plague front tracking methods based on marker particles. We analyze the influence of vorticity produced by the no-slip condition along solid walls, and show that the effect of viscosity is two-fold. First, it wrinkles the flame front, increasing the surface area available for burning and thus accelerating the combustion process. Second, the growth and decay of small eddies inhibit the ability of the flame to burn into the corners, extending the amount of time required for complete conversion of reactants to products. Our numerical experiments show that the smaller the laminar flame speed, the greater the effect of viscosity on the rate of combustion, when compared to an inviscid calculation with the same flame speed.

THE MODEL

We assume two-dimensional, viscous, incompressible flow inside the vessel. On solid walls, we require that the normal and tangential velocities be zero. We use the thin flame model for infinitely fast kinetics, commonly used in the analysis of premixed turbulent combustion, see [1]. This assumes an infinitely thin flame front, in which pressure fluctuations are neglected, the Mach number is assumed small, and combustion is characterized by a single step, irreversible chemical reaction taking place at a constant rate. Thus, each fluid particle exists in one of two states, burnt and unburnt, and the interface between the burnt and unburnt regions propagates at a fixed speed in a direction normal to itself into the unburnt fluid. In this model, hydrodynamics affect the motion of the flame, but there is no feedback mechanism by which the flame can influence the fluid motion. Thus, for example, we ignore wrinkling of the flame front due to exothermicity.

THE NUMERICAL APPROXIMATION

Numerical modeling of high Reynolds number flow is typically accomplished through the application of finite difference schemes to the Navier-Stokes equations. Some of the problems inherent in these techniques are

1) the necessity of a fine grid in the boundary layer region near walls where sharp gradients exist
2) the introduction of numerical diffusion; the error term associated with the approximation equation looks like a diffusion term, and this "artificial" viscosity swamps the real viscous diffusion, masking the effect of a high Reynolds number on the physics of the flow
3) the intrinsic smoothing of finite difference schemes which damps out physical instabilities
4) the need to introduce experimentally derived parameters to close turbulence relations.

The Random Vortex Method, introduced by Chorin [2], is specifically designed to deal with these problems. The flow is represented by a collection of vortex "blobs" that yield an associated velocity field. Viscous diffusion is simulated by a random walk imposed on the vortex motion. Normal boundary conditions are met through the addition of a potential flow solution, and tangential boundary conditions are satisfied by a vorticity creation algorithm (vortex sheets). By avoiding the averaging and smoothing associated with finite difference formulations, this technique allows us to follow the development of large-scale coherent turbulent structures within the flow. This technique has been applied with great success to flow past a semi-infinite flat plate [5], flow past a cylinder [6], and boundary layer instability in two and three dimensions [4].

To model the motion of the flame, one is tempted to place marker particles along the boundary between the burnt and unburnt fluid and solve an appropriate set of differential equations to update their position and hence the location of the flame front in time. Because of the difficulty involved in determining the normal direction to the front (the direction in which the flame burns) from such an approximation, the flame front usually becomes unstable and develops wild oscillations (see [10]). We avoid this problem by imposing a grid on the domain and assigning each cell a number (a "volume fraction") corresponding to the amount of burnt fluid in that cell at any given time. We allow each cell on the boundary of the burnt gas to ignite all its neighbors at the prescribed rate; this is an approximation based on Huyghen's principle, which states that the envelope of all disks centered on the front corresponds to the front displaced in a direction normal to itself, (see [3]). The motion of the flame is broken up into two stages: first, we model burning by allowing the flame to propagate in a direction normal to itself at the prescribed speed and second, we advect the burned fluid by the velocity field produced from the hydrodynamic part of the calculation. By updating these volume fractions according to the advection and burning processes, we may track the motion of the flame. This technique was first applied to model turbulent combustion over a backwards-facing step in [8].

In [11], we used these techniques to model the propagation of a flame in a square vessel with sides of length 1 m. The fluid motion was generated by a vortex placed in the center of the square of sufficient strength so that the velocity tangential to any wall at its midpoint was equal to 1 m/s. The calculation in [8] for turbulent flow behind a step assumed a propane-air mixture with laminar burning velocity of 12 cm/s and an inlet velocity of 6 m/s. This inlet velocity was taken as a characteristic speed scale, providing a non-dimensional laminar flame speed of $S = .02$ for a flow of inlet speed equal to unity. Since the velocity induced by a point vortex is of the form r^{-1}, where r is distance to the vortex, in such a problem it is not clear what to choose as a characteristic velocity, and hence the non-dimensional flame speed. In [10], we took a non-dimensional laminar flame speed of $S = .14$, corresponding to a characteristic velocity 7 times that of the tangential boundary velocity; this provided a balance between burning and advection with a minimum of computational labor. In the first experiment, a counterclockwise swirling, inviscid fluid is ignited halfway up the left side (Fig. 1). Since the flow is inviscid, the flame wraps smoothly around the center several times before the entire volume is burned. In the second experiment (Fig. 2), viscosity (Reynolds number of 1000) is added. So that viscous effects would have time to develop, the flow was started 2 seconds before ignition. The flame motion is strongly influenced by the counterrotating eddies that grow in the corners as a result of vorticity production along solid walls; the flame is carried around each large eddy and then dragged backwards into the corner. The front becomes jagged and wrinkled, which increases the surface area of the flame available for burning.

RESULTS

In this paper, we analyze in detail the above flame wrinkling due to viscous effects. We performed a series of numerical experiments to measure the effect of vorticity creation on the speed at which vorticity takes place in the vessel.

With laminar flame speed $S = .14$, in Figure 3 we plot the fraction of total volume that is burnt against the time elapsed since ignition. The bottom curve is the inviscid case, and the top curve is the case of ignition of a viscous, swirling fluid with Reynolds number 1000. The effect of viscosity can clearly be seen; after starting off together, the curves split apart as eddies created by the no-slip condition stretch and wrinkle the viscous flame, providing more surface area available for combustion. As the volume becomes burnt, the viscous curve flattens out faster than the inviscid curve, as the flame attempts to fill in the highly turbulent corners. After 1.85 seconds, 90% of the volume is burnt in the viscous case, compared to 41% in the inviscid case; however it takes .25 seconds for the viscous case to progress from 90% to 95%, compared to only .13 seconds for the inviscid case.

In Figures 4, 5, 6 and 7, we present similar calculations for non-dimensional laminar flame speeds .02, .06, .1 and .2, respectively. It is obvious that the slower the flame speed, the longer the time required for the vessel to become completely burnt, and the figures reflect this fact. However, one can also see the effects of viscosity; as the flame speed decreases, the gap increases between the viscous case and the inviscid case. This is because as the laminar flame speed decreases, the burning component of the flame motion is overshadowed by the advection component produced by the hydrodynamics. In the viscous case, the flow is unsteady, due to the production of vorticity to satisfy the no-slip condition on the boundary, and the eddies produced are of prime importance in bringing the flame into contact with unburnt parts of the vessel. Conversely, when the laminar flame speed is large relative to the hydrodynamic component (Fig. 7), the faster burning rate overshadows this effect and the curves are closer together.

We illustrate the above discussion in Figure 8, where we plot the percentage difference between the volume burnt in the viscous case and in the inviscid case as a function of time for the various flame speeds. Each curve changes from a solid to dotted line when over 99% of the volume is burnt in the viscous case. The smaller the flame speed, the greater the maximum of the difference between the viscous and inviscid calculation. Note that, although the peaks are comparable for $S = .06$ and $S = .02$, the graph for the slower flame speed is much wider.

CONCLUSIONS

We have shown the effect of viscosity on flame wrinkling as a function of laminar flame speed for an idealized model of turbulent combustion. We have recently extended the model to include exothermic effects along the flame front, and are now developing techniques to include such effects as temperature gradients and flame speed dependence on front curvature. We hope that as our model becomes more sophisticated, it will provide further understanding of the interaction between flame propagation and turbulent flow. All calculations were performed at the Lawrence Berkeley Laboratory. The average calculation (viscous fluid, Reynolds number 1000 with laminar flame = .1) took about three hours of CPU on a Vax 780.

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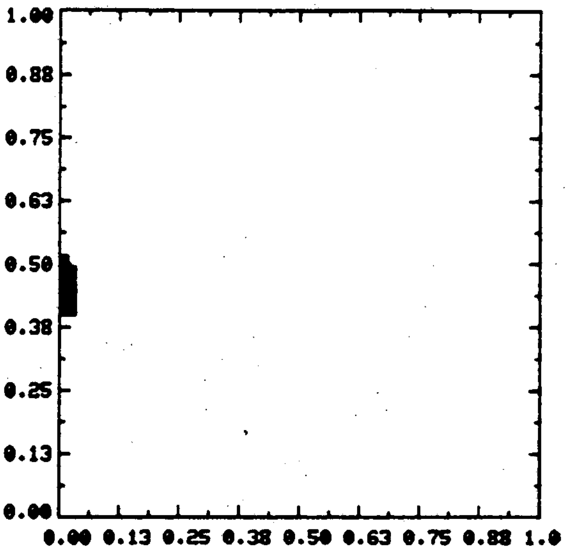
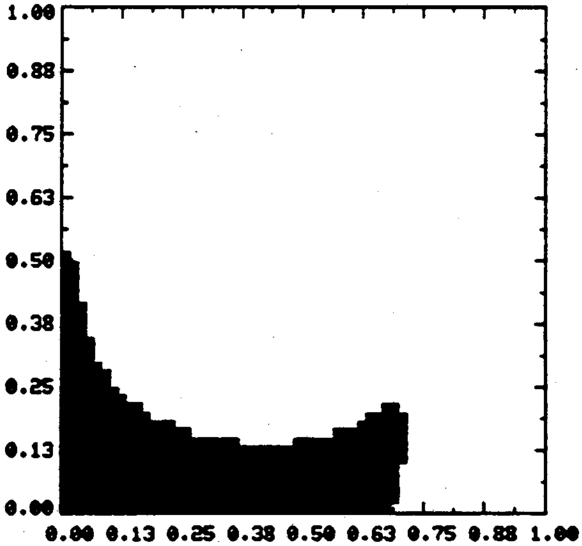


Time = .1



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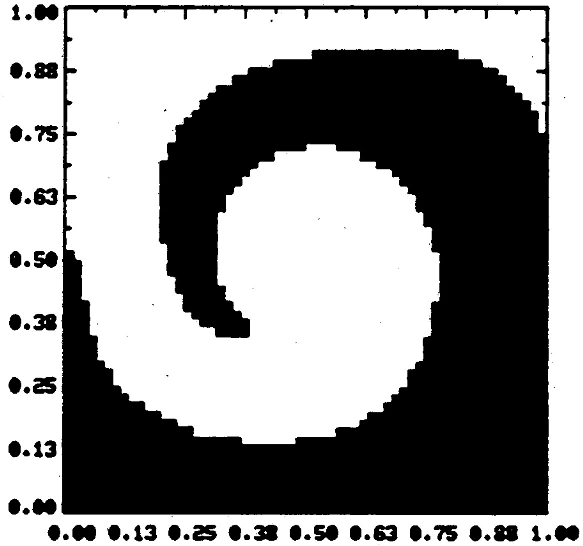

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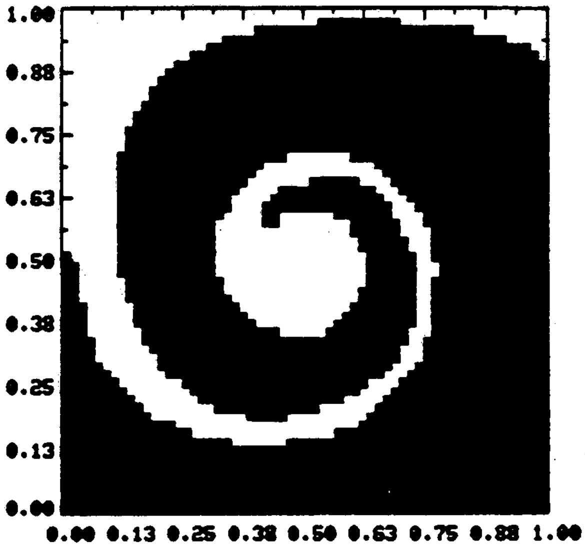

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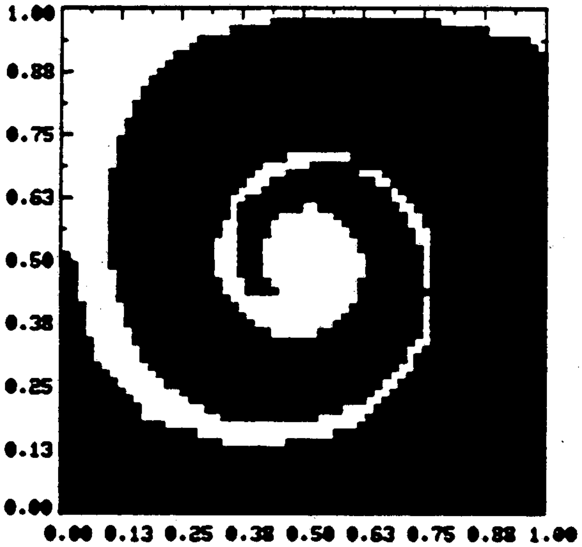
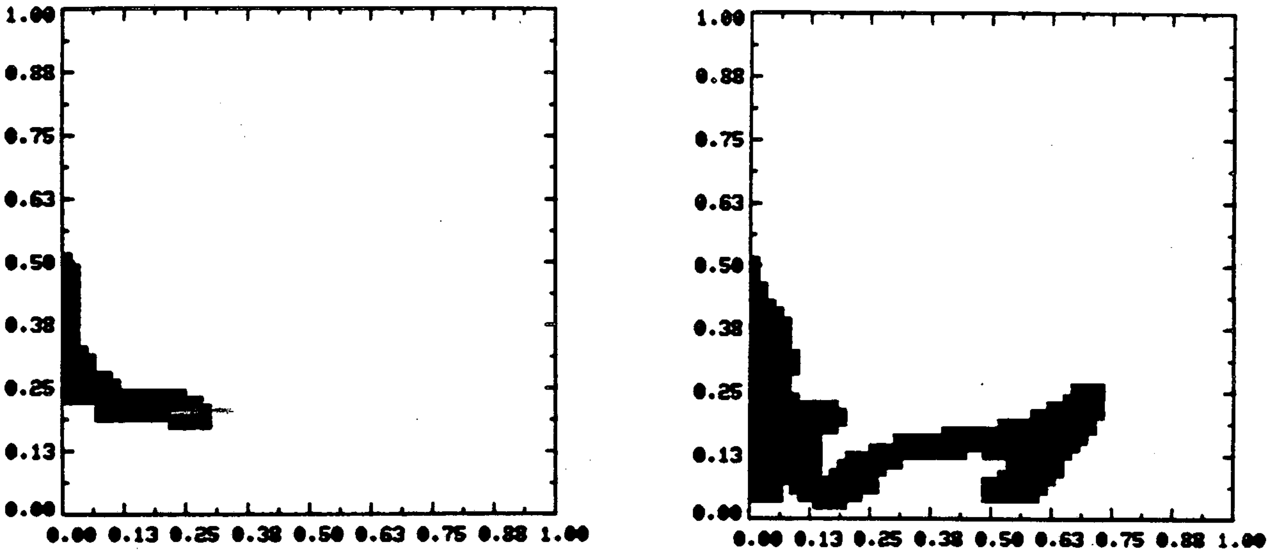


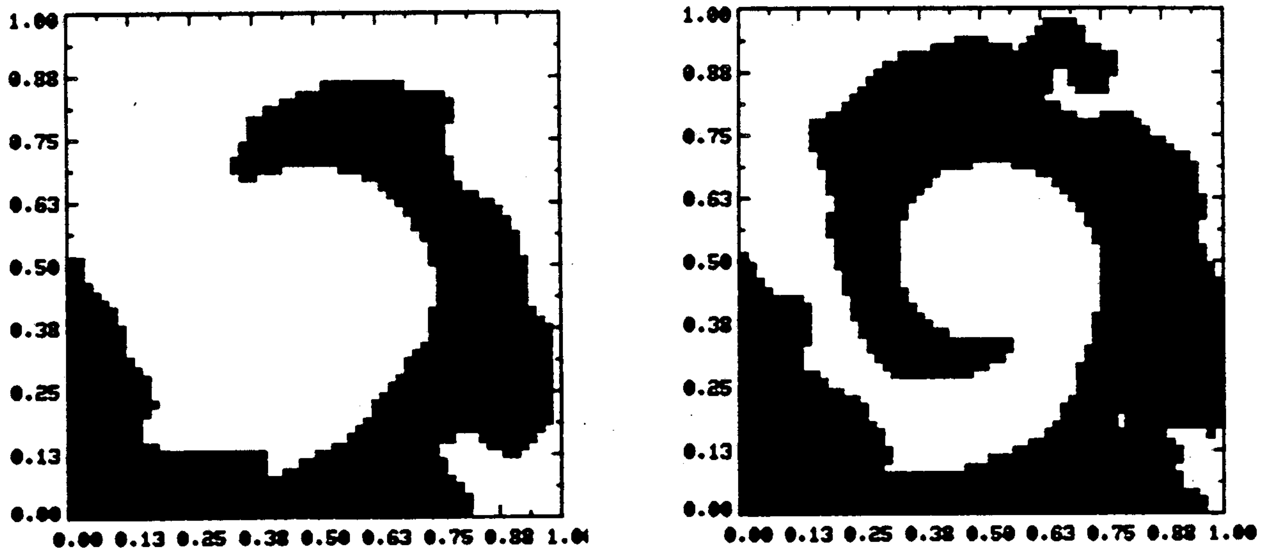
Time = 2.25

Figure 1: Inviscid Flow, Flame Speed S = .14



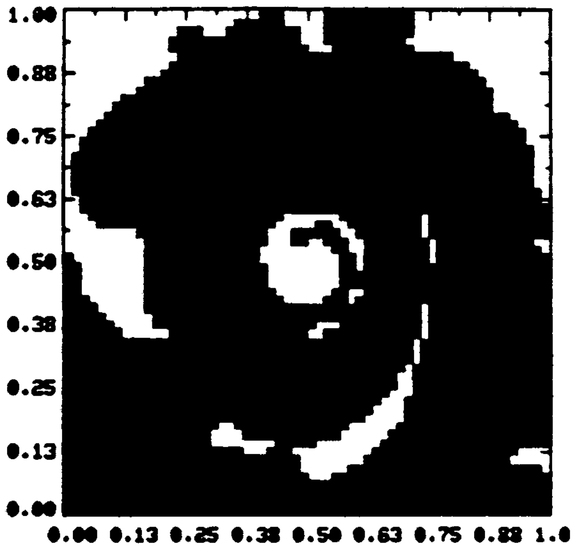
Time = 2.45

Time = 2.81



Time = 3.36

Time = 3.62



Time = 3.86

Figure 2: Viscous Flow, Re = 1000, S = .14

Figure 3

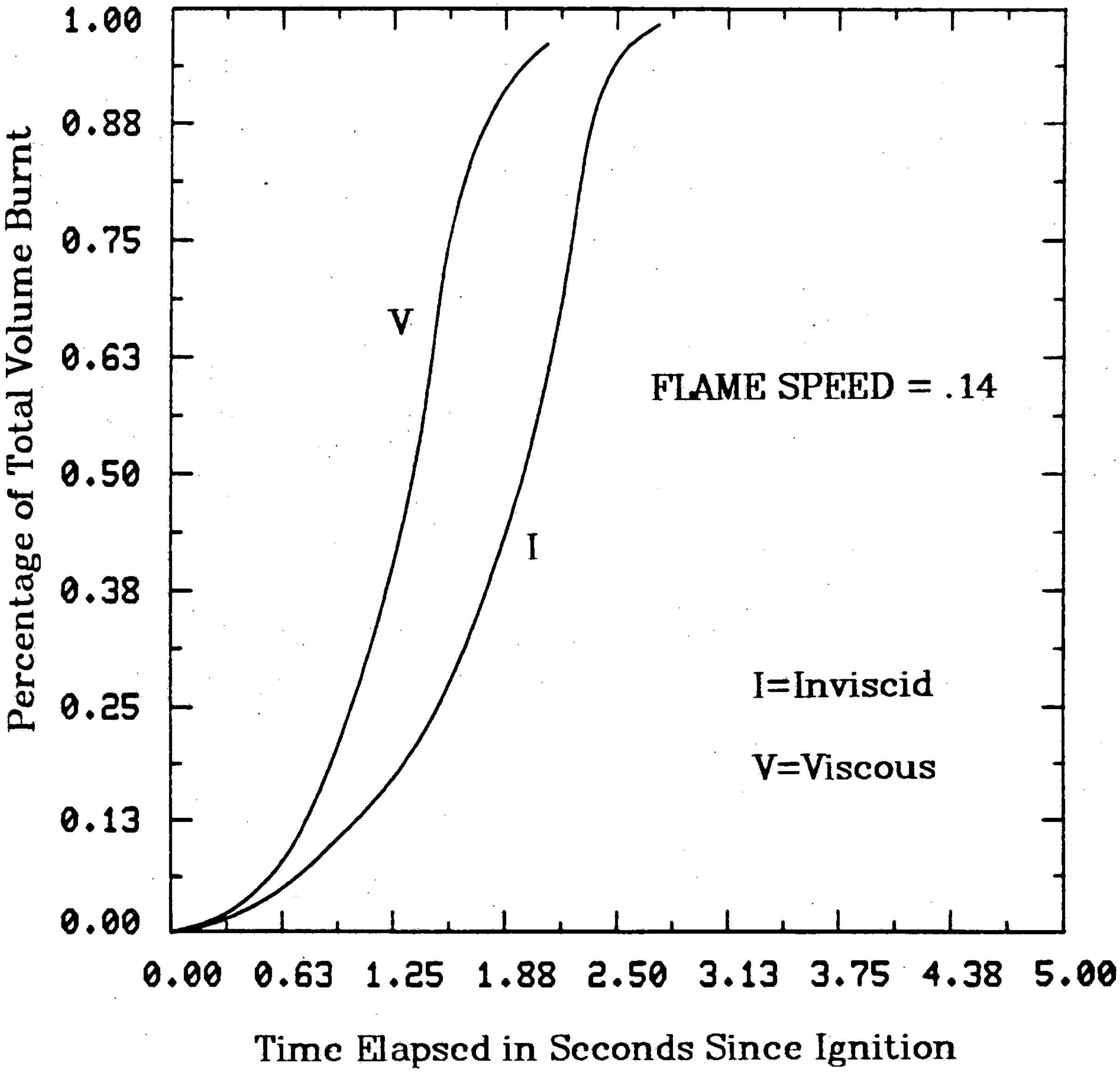

1.00
0.88
0.75
0.63
0.50
0.38
0.25
0.13
0.00
Percentage of Total Volume Burnt
V
I
FLAME SPEED = .14
I=Inviscid
V=Viscous
0.00
0.63
1.25
1.88
2.50
3.13
3.75
4.38
5.00
Time Elapsed in Seconds Since Ignition

Figure 4

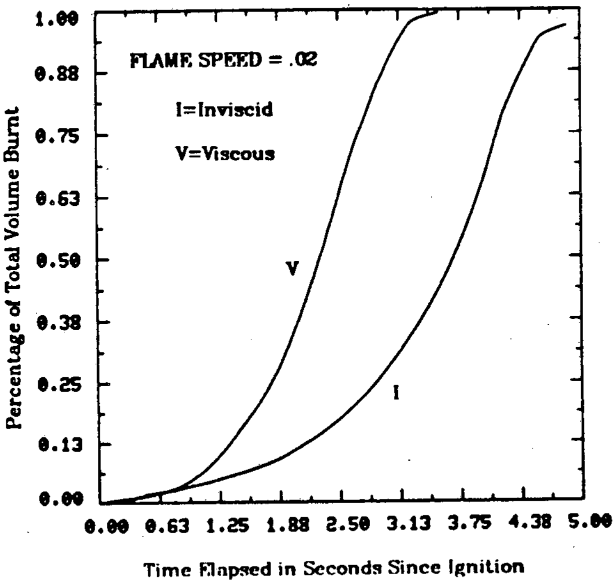


Figure 5

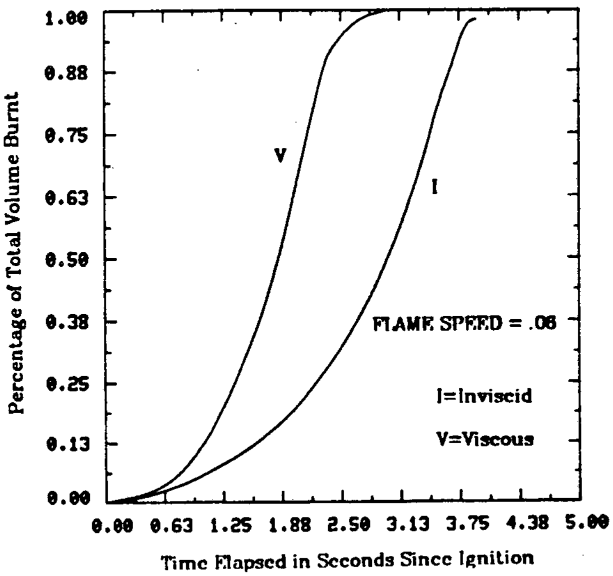


Figure 6

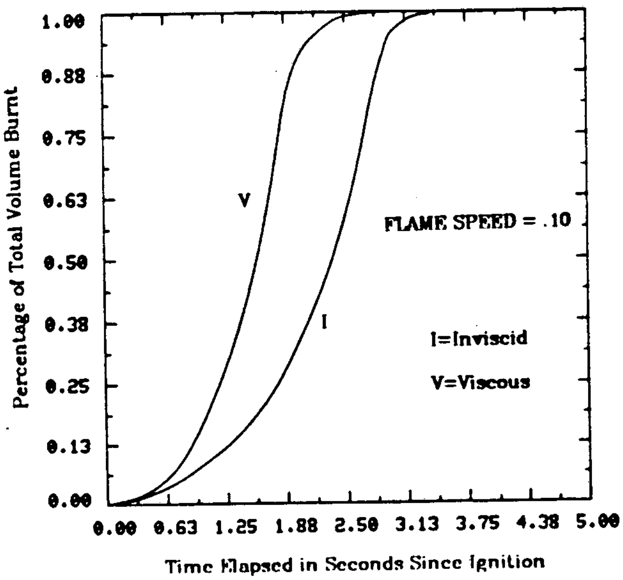


Figure 7

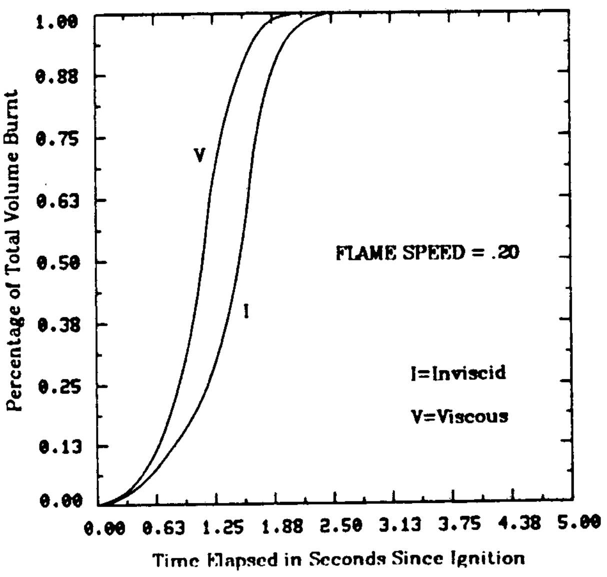
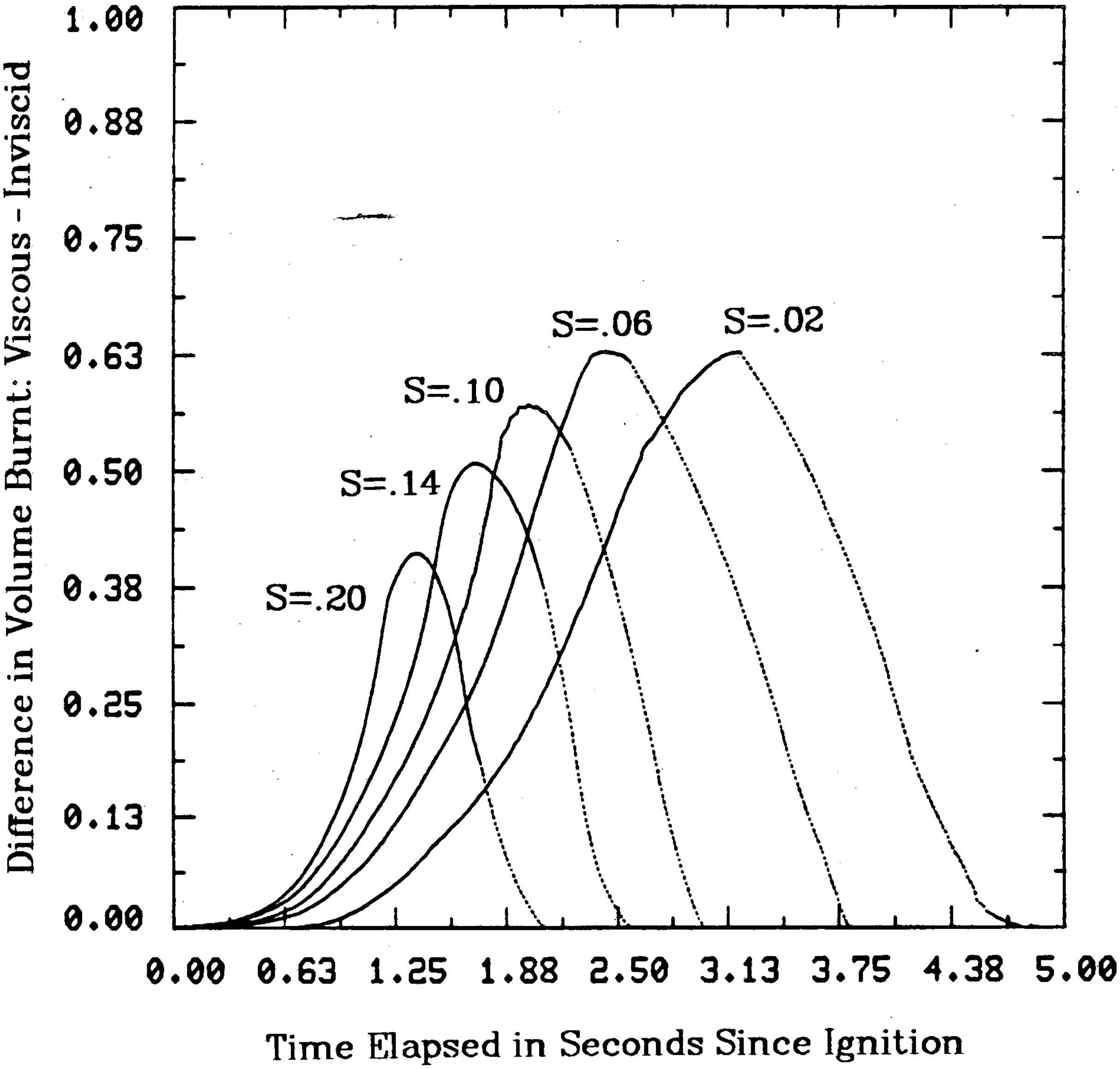


Figure 8


Difference in Volume Burnt: Viscous - Inviscid
1.00
0.88
0.75
0.63
0.50
0.38
0.25
0.13
0.00
S=.06
S=.02
S=.10
S=.14
S=.20
0.00
0.63
1.25
1.88
2.50
3.13
3.75
4.38
5.00
Time Elapsed in Seconds Since Ignition

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