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M. v. Borsig and M. Weigel

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EFFECT OF NONLOCAL FORCES ON THE INTEGRATED CROSS SECTION OF PHOTONUCLEAR REACTIONS*

M. v. Borsig†

Sektion Physik der Universität
University München
Munich, Germany

and

M. Weigel††

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

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Abstract

The modification of the dipole sum rule for the nonlocal Tabakin potential is given in the framework of Levinger's quasi-deuteron model. The included correlations cause an increase of about 60% for the value of the anomaly compared to the outcome of the independent particle model.

I. Introduction

The modification of the sum rule of Thomas-Kuhn-Reiche-Feenberg-Siegert[1] caused by nonlocal nucleon-nucleon forces has been the goal of several investigations. The simple case of exchange forces has been treated already a long time ago.[2-6] Dohnert and Rojo studied the problem up to second order for a potential with quadratic momentum dependence.[8] For the Tabakin potential[9]--consisting of sets of separable potentials--the first order correction was calculated by Weigel and Suessmann.[10] The aim of this note is to take the higher correlations in the case of the Tabakin potential into account by using the quasideuteron model proposed by Levinger et al.[11] In the second section we are going to present the general formalism needed in our approach. The results and discussion are given in Sec. III.

II. General Formalism

In the electric-dipole approximation, the integrated photonuclear cross section is given by

$$\int dE\, \sigma_\gamma(E) = \frac{2\pi^2 \hbar e}{cm} \sum_n f_{on} \quad , \tag{II.1}$$

where the dipole sum is defined as follows ($|\phi\rangle$ denotes the ground state of the system, D is the dipole operator)*

$$\sum_n f_{on} := -\frac{m}{\hbar^2} \langle \phi | [[H,D],D] | \phi \rangle \quad . \tag{II.2}$$

The matrix element of the nucleon-nucleon force has the general form:

$$\langle \underset{\sim}{r} | v | \underset{\sim}{r}' \rangle = \sum_{JST} \sum_{LL'} \sum_{T_3 M} Y^{JS}_{LM}(\hat{r}) \left| {T \atop T_3} \right\rangle \langle rJSTL | v | rJSTL' \rangle \left\langle {T \atop T_3} \right| Y^{JS}_{L'M}(\hat{r}') \ . \tag{II.3}$$

Since the contribution of the kinetic energy is well known, we are only interested in the double commutator containing the potential energy. As is in Ref. 10 we neglect the neutron excess. For the quasideuteron model we have then to deal with the following matrix element ($D_C := [[V,D],D]$):

*For more details and definitions we refer to Ref. 10.

$$F: = \langle \phi_2 | D_C | \phi_2 \rangle = \langle \phi_o | (1 + U_2^\dagger) D_C (1 + U_2) | \phi_o \rangle$$

$$= \langle \phi_o | D_C | \phi_o \rangle + 2\mathrm{Re} \langle \phi_o | D_C \, U_2 | \phi_o \rangle + O(v^2) \quad . \qquad (II.4)$$

The relevant quantities of Eq. (II.4) are defined in field theoretical formulation as follows:

$$|\phi_o\rangle : = \frac{1}{\sqrt{A!}} \prod_{j=1}^{A} a^\dagger_{\alpha_j} |0\rangle \quad , \qquad (II.5)$$

$$U_2: = \frac{1}{2} \sum_{\alpha\beta} \sum_{\gamma\mu} \frac{(1-n_\alpha)(1-n_\beta) n_\gamma n_\mu}{\varepsilon_\mu + \varepsilon_j - \varepsilon_\alpha - \varepsilon_\beta} \langle \alpha\beta | v | \gamma\mu \rangle a^\dagger_\alpha a^\dagger_\beta a_\mu a_\gamma \quad , \qquad (II.6)$$

$$D_C: = \frac{1}{2} \sum_{\alpha\beta} \sum_{\gamma\mu} \langle \alpha\beta | (vd^2 - 2vdv + d^2 v) | \gamma\mu \rangle \; a^\dagger_\alpha a^\dagger_\beta a_\mu a_\gamma \quad , \qquad (II.7)$$

with:

$$d_{12}: = (z_1 t_{13} + z_2 t_{23}) \quad .$$

With $\underset{\sim}{t}$ we denote the isospin vector operator, n_α is the occupation number in the shell-model.

The first term of Eq. (II.4) has been calculated for nuclear matter in Ref. 10. Making the same approximation, we obtain for the second term by use of the commutator relations and insertion of momentum eigenfunctions the following result ($m = \hbar = 1$):

$$F_2 = -\frac{8}{12}\int d\underset{\sim}{p}_{12}\, d\underset{\sim}{p}'_{12}\, \frac{f(\underset{\sim}{p}_{12},\underset{\sim}{p}'_{12})}{p'^2_{12} - p^2_{12}} \sum_{\tau_1\tau_2}\ \sum_{\tau'_1\tau'_2}\ \sum_{\sigma_1\sigma_2}\ \sum_{\sigma'_1\sigma'_2} \frac{1}{4}\,\delta_{\tau_1+\tau_2,\tau'_1+\tau'_2}\,\delta_{\tau_1+\tau_2,0}$$

$$* \Big(\langle \underset{\sim}{p}_{12}\sigma_1\tau_1\sigma_2\tau_2 | (v\, r^2_{12}(t_{13}-t_{23})^2 - 2\, \underset{\sim}{r}_{12}(t_{13}-t_{23}) v \cdot \underset{\sim}{r}_{12}(t_{13}-t_{23})$$

$$+ (t_{13}-t_{23})^2\, r^2_{12}\, v) | \underset{\sim}{p}'_{12}\sigma'_1\tau'_1\sigma'_2\tau'_2 \rangle_{a.s.} * \langle \underset{\sim}{p}'_{12}\sigma'_1\tau'_1\sigma'_2\tau'_2 | v | \underset{\sim}{p}_{12}\sigma_1\tau_1\sigma_2\tau_2 \rangle \ , \qquad (II.8)$$

with:

$$f(\underset{\sim}{p},\underset{\sim}{p}'): = \frac{3A}{16\pi}\int \frac{d\underset{\sim}{K}}{p_F^3}\, \Theta\left(\frac{|\underset{\sim}{K}+\underset{\sim}{p}'|-p_F}{p_F}\right)\Theta\left(\frac{|\underset{\sim}{K}-\underset{\sim}{p}'|-p_F}{p_F}\right)\Theta\left(\frac{p_F-|\underset{\sim}{K}+\underset{\sim}{p}|}{p_F}\right)\Theta\left(\frac{p_F-|\underset{\sim}{K}-\underset{\sim}{p}|}{p_F}\right) \ . \qquad (II.9)$$

Here, we label with σ_1, τ_1 etc. the quantum numbers of the third component of the spin, isospin operator s_{13}, t_{13}. The index a.s. implies complete antisymmetrization with respect to the coordinates 1,2 and 1',2' (⟨12-21|(..)|1'2'-2'1'⟩).

In order to simplify Eq. (II.8) we use as usually the spherical average $\tilde{f}$ over the function $f(\underset{\sim}{p},\underset{\sim}{p}')$, so obtaining

$$F_2: = -\frac{2}{3}\int p^2_{12}dp_{12}\, p'^2_{12}dp'_{12}\, \frac{\tilde{f}(p_{12},p'_{12})}{p'^2_{12} - p^2_{12}} \sum_{JST}\ \sum_{LL'} \{(2f+1)*\begin{pmatrix}1/2 & 1/2 \\ 1/2 & 1/2\end{pmatrix}\Big|\begin{matrix}T\\0\end{matrix}\Big)^4$$

$$* \langle p_{12}JSTL | (v\, r^2_{12} + r^2_{12}\, v) | p'_{12}\, JSTL' \rangle \langle p'_{12}\, JSTL' | v | p_{12}\, JSTL \rangle$$

$$* (1 - (-)^{L+S+T}) * (1 - (-)^{L'+S+T})\} \qquad (II.10)$$

Since the potential matrix elements are analytical functions of p, the derivatives of $\langle p|v|p' \rangle$ with respect to p or p', respectively, are obtainable explicitly. One can then proceed with the calculation of (II.10) using standard techniques known from the second order energy.

III. Results and Discussion

For the relevant dipole anomaly defined by

$$C = 4\, A^{-1} \sum_{n} f_{on} - 1 = C_1 + C_2 \quad , \tag{III.1}$$

the obtained results are shown in Table I. An important correction arises from the fact, that one should use in (II.6), (II.8), and (II.10) consistent single particle energies, which can be taken into account by the effective mass approximation. This causes an additional factor m^* for C_2. Tabakin gives the value $m^* = 0.6$. In Table I we have given both alternatives ($m^* = 1.$, $m^* = 0.6$). A comparison with the results of Dohnert and Rojo shows, that we get slightly higher values for C ($m^* = 1.$). Due to the uncertainties in the experimental determination of the integrated cross section the theoretical result remains within the experimental limits.[11] A dark point is the order of C_2 compared with C_1. Since both terms are—except for S-waves—of the same magnitude, the convergence remains an open question.

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†Present Address: Max-Planck-Institut zur Erforschung der Lebensbedingungen der wissenschaftlich-technischen Umwelt, D-813 Starnberg, Germany.

††Present Address: Sektion Physik der Universität, Munich, Germany.

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Table I. Dipole anomaly as a function of the Fermi momentum ($p_F = 1.52/r_o$). The contributions of the different partial waves are displayed explicitly. For each quantum number set the first row corresponds to the IPM, the second gives the change due to the correlations. The total outcome is given on the bottom ($m^* = 0.6$).

	r_o[F]	1.00	1.07	1.15	1.24
	$p_F[F^{-1}]$	1.52	1.42	1.32	1.22
$^1S_0$		0.22 0.04	0.19 0.04	0.18 0.04	0.15 0.04
$^3S_1$		0.61 0.12	0.56 0.13	0.51 0.13	0.45 0.13
$^3S_1$ - $^3D_1$		-- 0.19	-- 0.19	-- 0.19	-- 0.19
$^3D_1$		-0.08 0.06	-0.08 0.05	-0.08 0.03	-0.07 0.02
$^1P_1$		0.29 0.36	0.27 0.30	0.23 0.24	0.20 0.19
$^3P_0$		0.04 -0.01	0.03 -0.01	0.01 -0.01	0.01 -0.01
$^3P_1$		0.13 0.17	0.14 0.15	0.14 0.13	0.13 0.11
$^3P_2$		-0.10 0.05	-0.09 0.04	-0.09 0.03	-0.08 0.03
$^1D_2$		-0.09 0.01	-0.09 0.01	-0.08 0.01	-0.08 0.01
$^3D_2$		-0.07 0.13	-0.06 0.11	-0.05 0.09	-0.04 0.07
$^3D_3$		-0.18 0.02	-0.18 0.02	-0.17 0.02	-0.16 0.01

(continued)

Table I. (continued)

	$r_o[F]$	1.00	1.07	1.15	1.24
	$p_F[F^{-1}]$	1.52	1.42	1.32	1.22
$C_1 = \Sigma\, C_1^{(j)}$		0.94	0.88	0.81	0.13
$C_2 = \Sigma\, C_2^{(j)}$		1.14	1.03	0.91	0.80
$C = C_1 + C_2$		2.08	1.91	1.72	1.53
$C^* = C_1 + m^* C_2$		1.62	1.50	1.36	1.21