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Collective Cavity QED with Mesoscopic Atom Arrays

by

Jacquelyn Jae-Yi Ho

A dissertation submitted in partial satisfaction of the

requirements for the degree of

Doctor of Philosophy

in

Physics

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Dan M. Stamper-Kurn, Chair

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Spring 2026

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Abstract

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Coupling between quantum emitters and modes of light is foundational to studies of quantum systems. Coherent interactions between matter and light have greatly advanced the development of quantum technologies. When an ensemble of atoms interacts with light that is confined within a cavity—a resonator for photons—light also mediates atom-atom interactions, leading to collective phenomena that cannot be extrapolated from microscopic behaviors. Understanding how interactions among quantum particles lead to macroscopic phenomena, such as phase transitions and breaking of symmetries, remains a challenge in modern physics. In this dissertation, I present results addressing this challenge in the context of mesoscopic arrays of optical-tweezer-trapped atoms interfaced with a high-finesse optical cavity.

In particular, the physics we investigate arises from the collective scattering of light. Utilizing the ability to place atoms with nanometer-scale precision, we engineer superradiant and subradiant states, where control over interference of the emission from multiple atoms leads to collectively enhanced or suppressed amounts of scattered light. Building on this work, we investigate the emergence of optomechanical self-organization, mediated by superradiance in the cavity electromagnetic field. By coupling atoms to the same cavity mode, the atoms exert forces on one another through the exchange of photons, leading them to autonomously order themselves spatially. We characterize mesoscopic fluctuations and their effect on symmetry-breaking dynamics associated with self-organization. Finally, we extend this framework to theoretically investigate how the phase of the atom-cavity coupling can be altered to generate higher symmetries. Our analysis reveals first-order phase transitions and instabilities induced by non-reciprocal forces. Altogether, this work establishes a path towards realizing mesoscopic systems of complex phases of matter in cavity quantum electrodynamics with unprecedented programmability.

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Chapter 1

Introduction

The ability to predict and control how quantum particles interact with light is one of the most profound successes of quantum mechanics. Since the resolution of the blackbody radiation problem [1], the field of quantum optics has evolved from describing the fundamental nature of the photon, where quantum electrodynamics unifies the wave- and particle-like descriptions of light [2, 3, 4], to engineering sophisticated systems where light and matter are inextricably linked. Example of such systems occur in quantum optomechanics [5, 6, 7, 8, 9, 10], where the coupling of light and an object’s motion allows us to not only use light as a probe of motion, but also to engineer complex motional and photonic states.

Optomechanics with ultracold atoms originates from the forces that light exerts on atoms. An atom, upon absorbing a photon, recoils due to the momentum imparted on it by the photon. Alternatively, a field of light with a spatial gradient creates a conservative potential for atoms to traverse. The study of light-induced forces has a wide range of applications, from levitating particles [11, 12] to laser cooling [13, 14, 15, 16, 17, 18] to creating gravitational wave interferometers [19, 20]. When light is confined between two mirrors, forming a cavity, the strength of light-matter interactions is enhanced. Furthermore, light inside the cavity mediates interactions among the atoms: a photon emitted by one atom exerts a force on all the intracavity atoms, allowing atoms to “tug” on one another through the exchange of photons. These light-mediated interactions give rise to collective phenomena, such as interference of the photons emitted by different atoms and self-organization of the atomic states.

In this thesis, I describe experiments performed on an apparatus that interfaces an optical tweezer array of ultracold atoms with a single cavity mode. We study the collective scattering of light from the atom array, utilizing the precise positional control afforded by the tweezers to engineer light-matter coupling. We then observe optomechanical self-organization in atom arrays, where collective scattering leads atoms to spontaneously rearrange themselves. Finally, we discuss a theoretical proposal for engineering the discrete symmetry of the cavity electromagnetic field, which also gives rise to non-reciprocal atom-atom interactions. In this chapter, I introduce basic background on ultracold atoms as a platform for quantum science experiments and on light-matter interactions in cavity quantum electrodynamics. These

concepts are applied to understand the scientific work encompassed in this dissertation.

1.1 Ultracold atoms for quantum science

Despite often being described as the “basic building blocks of matter,” atoms themselves can be quite complex. We learn to solve the textbook hydrogen atom in our first quantum mechanics classes, but treating the interactions of the electrons and nucleons in increasingly complex atoms becomes increasingly difficult. For this reason, many atomic physicists are drawn to alkali atoms, which are hydrogen-like because they only have one valence electron. For instance, the work in this dissertation was performed with the alkali atom ^{87}Rb , which contains 37 electrons, 37 protons, and 50 neutrons; being comprised of so many subatomic particles, ^{87}Rb has many bound states. Despite this complexity, we actually understand the atomic structure of ^{87}Rb quite well by considering the interactions of the single valence electron. Spin-orbit coupling gives rise to fine structure, and interaction with the nuclear spin gives rise to hyperfine structure.

Working with atoms gives us tremendous opportunities for studying fundamental physics and advancing quantum technologies. On one hand, atoms are pristine systems—each and every ^{87}Rb atom is identical. One can leverage the intrinsic, uneven energy level spacings in atoms to isolate qubits, or two-level systems, by selectively driving transitions of a certain frequency (of course, one still has to consider the effects of nearby states and transitions). We are living through an era of increasingly impeccable control over atomic degrees of freedom, precisely controlling atoms’ motion, quantum states, and interactions to build quantum simulators, computers, and sensors [21, 22].

A manifestation of this incredible control is the cooling of atoms to quantum degeneracy, leading to the production of Bose-Einstein condensates [23, 24, 25] and degenerate Fermi gases [26, 27, 28]. Scientists have loaded these quantum degenerate gases into optical lattices, performing beautiful simulations of condensed matter systems. In the past decade or so, a new type of workhorse atomic physics experiment has come online: the atomic tweezer array [29, 30, 31, 32]. By trapping single atoms in individual, programmable optical tweezer beams, we exert microscopic control over an ensemble of atoms. Though optical tweezer technology has existed for many decades, its relatively recent application to atomic physics has already led to staggering advances in quantum computation, simulation, and metrology [33, 34, 35, 36, 37, 38, 39, 40].

All of these quantum technologies are enabled by harnessing the interactions between matter and electromagnetic radiation. In atomic, molecular, and optical (AMO) physics, we often address atoms with lasers at optical frequencies to perform laser cooling and to prepare atoms in specific quantum states (e.g. through optical pumping). We may also perform readout of the atom’s quantum state through optical excitation on a cycling transition. In addition to optical frequencies, we drive transitions at microwave frequencies to perform logical operations on qubit states. Given how fundamental atom-light interactions are to the AMO community, one might take interest in directly studying interactions between atoms

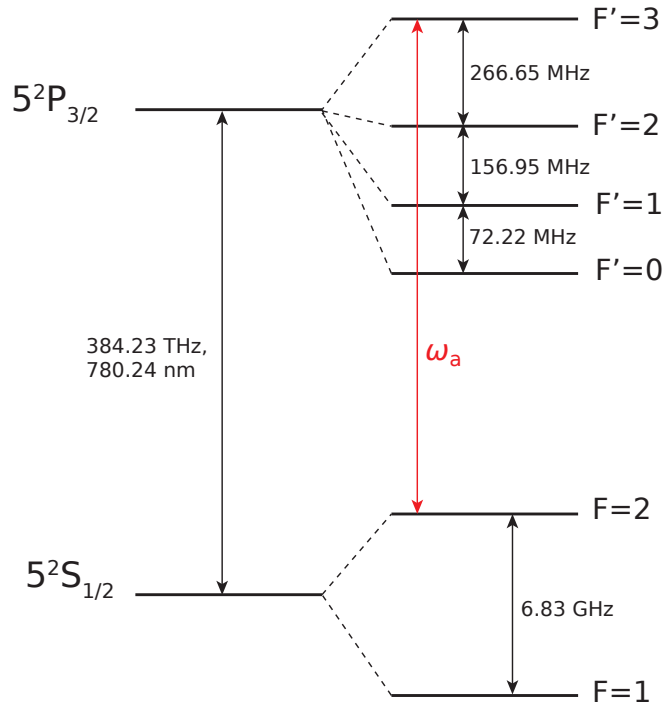


Figure 1.1: D_2 line hyperfine structure of atomic ^{87}Rb . This is the primary atomic transition addressed in this thesis. In our notation, the atomic transition frequency ω_a refers to the frequency of the $F = 2 \rightarrow F' = 3$ transition.

and light, and also light-mediated atom-atom interactions. One avenue for such studies is cavity quantum electrodynamics (QED), which is a major topic of the research presented in this dissertation.

My PhD work was done on an experiment dubbed E6, whose name derives from being the sixth apparatus built in the Stamper-Kurn group at Berkeley. The E6 apparatus houses an optical tweezer array—an array of tightly focused laser beams that trap single atoms—placed inside an optical cavity. We harness strong coupling between the atom array and the mode of light supported by the cavity to study collective behavior of the atoms mediated by light. Although not a major focus of this thesis, the cavity also presents exciting measurement opportunities with applications in quantum information science. I discuss the basic atomic structure we leverage for our experiments in Sec. 1.2 and I introduce relevant cavity QED concepts in Sec. 1.3. Details about the experimental apparatus are discussed in Ch. 2.

1.2 Rubidium atomic structure

On the E6 experiment, we primarily work with the D₂ line in the ⁸⁷Rb isotope. This is the $5^2S_{1/2} \rightarrow 5^2P_{3/2}$ transition, which occurs at a resonance frequency of approximately 384.23 THz and a wavelength of 780 nm. Note that ⁸⁷Rb also possesses a D₁ line ($5^2S_{1/2} \rightarrow 5^2P_{1/2}$) at a wavelength of 795 nm as part of its fine-structure doublet (arising from the coupling between the spin and orbital angular momentum of the outermost electron), but we do not currently utilize the D₁ line in our work. Within the D₂ line, there is additional hyperfine structure due to interaction of the outermost electron with the nucleus (nuclear spin $I = 3/2$). Thus, the ground state ($L = 0$) is split into $F = 1$ and $F = 2$ levels while the excited state ($L = 1$) is split into $F' = 0, 1, 2$, and 3. The excited state has a decay rate of $\Gamma = 2\pi \times 6.065$ MHz, corresponding to a lifetime of 26.24 ns [41]. A schematic of the D₂ line hyperfine structure is shown in Fig. 1.1.

In the work described in this thesis, we prepare the atoms in the $F = 2$ ground state. Atoms in $F = 2$ are laser cooled by excitation to the $F' = 3$ excited state, which forms a cycling transition since selection rules only permit $F' = 3$ to decay to $F = 2$. However, off-resonant excitation of the $F' = 2$ excited state leads to decay to $F = 1$ with 50% probability. Thus, we use a single repump beam on the $F = 1 \rightarrow F' = 2$ transition to pump $F = 1$ atoms back into $F = 2$. We also use excitation on the $F = 2 \rightarrow F' = 2$ transition to optically pump the atoms into a pure state; using $\sigma^\pm$ -polarized light, we pump into $|F = 2, m_F = \pm 2\rangle$ (the “stretched” states) and using π -polarized light, we pump into $|F = 2, m_F = 0\rangle$ (the “clock” state). Though we do not use the clock state for the majority of the work described in this thesis, we note that it is used as a qubit state in many works (including our own in Ref. [42]) due to its first-order insensitivity to magnetic field fluctuations (see Appendix A), leading to improved coherence times.

1.3 Atom-light interactions in cavity QED

As mentioned previously, the interaction of atoms with electromagnetic radiation is fundamental to AMO physics. Most often, we consider electric dipole interactions between an atom and an ac electric field. An atom may absorb a photon and be excited to a higher energy state if the transition between those states is allowed by selection rules. Conversely, an excited atom may emit a photon and enter a lower energy state through the same type of interaction. When approximating the atom as a two-level system, coherent interaction with a classical electromagnetic field famously gives rise to Rabi oscillations. However, even in the absence of an external driving field, atoms are *always* interacting with light in the form of electromagnetic vacuum modes. Even though vacuum states contain zero photons, they exhibit zero-point fluctuations, consistent with the uncertainty principle. Thus, atoms in free space interact with a continuum of modes. In fact, it is the interaction with these vacuum modes that gives rise to spontaneous emission [43].

The physics changes if you now place the atom in between a pair of highly reflective

mirrors facing each other. The pair of mirrors forms a cavity—a resonator for photons. Given the boundary conditions of the mirrors, light is resonant with the cavity if an integer multiple of half-wavelengths “fits” inside the cavity. The cavity modifies the vacuum density of states. If a cavity electromagnetic field mode is resonant with an atomic transition, the atom will preferentially emit into that mode with an enhanced rate (known as the Purcell effect [44]). Similarly, an off-resonant cavity suppresses emission. The cavity-enhanced spontaneous emission rate from the Purcell effect is highly useful for efficient collection of photons emitted by an atom, providing advantages for cavities as a platform for fast atomic state detection and mid-circuit readout [42, 45, 46].

Photons circulate within a cavity for a number of round trips given by the cavity’s finesse, $\mathcal{F}$, and thus interact with an intracavity atomic sample many times. When multiple atoms are placed within a cavity, a photon emitted by one atom may be absorbed by another, enabling atom-to-atom interactions via the exchange of photons. Although coherent photon exchange is possible with sufficiently high atom-cavity coupling or a large ensemble of atoms [47, 46, 48, 49, 50, 51], coherence is severely limited by dissipation due to spontaneous emission and cavity leakage [48, 46, 52]. We may also embrace the driven-dissipative nature of the atom-cavity system and study the collective photon scattering of an ensemble of atoms within a cavity. This is the approach taken in the work described in this thesis. Through careful positioning of individual atoms within the cavity, we observe interference of the photons emitted by an array of driven atoms [53]. We additionally harness collective constructive interference, known as superradiance [54], to observe atoms autonomously arranging themselves through the light they emit into the cavity [55, 56, 57, 58, 59]. In the rest of this section, I introduce cavity QED concepts that lay the foundation for understanding the results presented in this dissertation.

1.3.1 Cavity QED parameters

Vacuum Rabi frequency

The heart of cavity QED lies in the interaction between quantum emitters and quantized electromagnetic field modes. In atom-based cavity QED, the strength of this interaction is parametrized by the coupling of an atomic dipole transition to a single photon resonant with that transition. For a single mode field of frequency ω confined to a cavity of volume V , we note that the expression for the energy $\hat{H}$ bears resemblance to the energy of a harmonic oscillator:

$$\hat{H} = \frac{1}{2} \int dV (\epsilon_0 \hat{E}^2 + \frac{1}{\mu_0} \hat{B}^2) = \frac{1}{2} (\hat{p}^2 + \omega^2 \hat{q}^2) = \hbar\omega \left(\hat{c}^\dagger \hat{c} + \frac{1}{2} \right) \quad (1.1)$$

with the magnetic field $\hat{B}$ being analogous to the momentum $\hat{p}$ and the electric field being analogous to the position $\hat{q}$ (rigorous derivations of this may be found in standard quantum optics textbooks, such as Refs. [60, 43]). We have also defined $\hat{c}$ ($\hat{c}^\dagger$) as the bosonic annihilation (creation) operator. As with the quantum harmonic oscillator, we see the zero-point energy $\hbar\omega/2$ emerge. The existence of energy (equivalently, fluctuations of the electromag-

netic field) in vacuum indicates an inherently quantum source of electromagnetic noise, often called shot noise. To derive the magnitude of the vacuum electric field $\mathcal{E}$, we make use of the virial theorem and equate the energy of the mode with the harmonic oscillator zero-point energy:

$$\epsilon_0 \mathcal{E}^2 V = \frac{\hbar \omega}{2}, \quad (1.2)$$

$$\mathcal{E} = \sqrt{\frac{\hbar \omega}{2 \epsilon_0 V}}. \quad (1.3)$$

To obtain the vacuum Rabi frequency g_0 , we examine the strength of the electric dipole interaction between the vacuum electric field and an atom with a dipole moment $\hat{d}$ on a transition from state $|g\rangle$ to state $|e\rangle$:

$$g_0 = \frac{\mathcal{E} \langle g | \hat{\epsilon}_q^* \cdot \hat{d} | e \rangle}{\hbar} = \frac{d}{\hbar} \sqrt{\frac{\hbar \omega}{2 \epsilon_0 V}}. \quad (1.4)$$

Here, $d \equiv \langle g | \hat{\epsilon}_q^* \cdot \hat{d} | e \rangle$ is the dipole transition matrix element for light of polarization $\hat{\epsilon}_q$ given in the spherical basis. In this thesis, we define g_0 on the stretched transition. Because the boundary conditions of our cavity enforce a standing wave optical field, we commonly define a spatially dependent $g(\hat{z}) = g_0 \sin(k\hat{z})$, where $\hat{z}$ defines position along the cavity axis. For a Gaussian mode of waist w_0 and length L , one can integrate the cavity mode's spatial profile to obtain the mode volume $V = \pi w_0^2 L / 4$.

Cavity dissipation

Cavity mirrors are not perfectly reflective, so photons eventually leak out as a result of small amounts of transmission and loss. To understand this dissipation rate, we now derive the density of states associated with a cavity of length L . To solve this, we consider how the intracavity field $\mathcal{E}_c$ relates to the incident on ($\mathcal{E}_i$) and reflected off ($\mathcal{E}_r$) of one mirror. We define t_1, t_2 as the field transmission coefficients and r_1, r_2 as the field reflection coefficients for the two mirrors. The transmissivities ($T_1 = t_1^2, T_2 = t_2^2$) and reflectivities ($R_1 = r_1^2, R_2 = r_2^2$) obey $T_1 + R_1 + L_1 = 1$ and $T_2 + R_2 + L_2 = 1$, where L_1 and L_2 are the losses of the mirrors. Assuming there is no loss in the medium between the mirrors, the intracavity field is the sum of the transmitted incident field and the intracavity field that has been reflected off both mirrors:

$$\mathcal{E}_c = t_1 \mathcal{E}_i + r_1 r_2 \mathcal{E}_c e^{2ikL}, \quad (1.5)$$

$$\Rightarrow \mathcal{E}_c = \frac{t_1 \mathcal{E}_i}{1 - r_1 r_2 e^{2ikL}}. \quad (1.6)$$

Here, I have assumed the electric field is a plane wave. Generally, for a Gaussian beam confined between two mirrors, we not only pick up a round-trip phase of e^{2ikL} , but also

a phase contribution depending on the Gouy phase and the transverse mode order. This additional phase contribution causes the transverse mode spectrum to vary depending on the cavity geometry. However, for the sake of conveying basic cavity concepts without complicating the math, I will ignore these extra phases and assume a plane wave. Then, the intensity I_c of the intracavity light is

$$I_c \propto |\mathcal{E}_c|^2 = \frac{T_1 |\mathcal{E}_i|^2}{(1 - r_1 r_2 e^{2ikL})(1 - r_1 r_2 e^{-2ikL})} \quad (1.7)$$

$$= \frac{T_1 |\mathcal{E}_i|^2}{1 - 2\sqrt{R_1 R_2} \cos(2kL) + R_1 R_2} \quad (1.8)$$

$$= \frac{T_1 |\mathcal{E}_i|^2}{1 - 2\sqrt{R_1 R_2} (\cos^2(kL) - \sin^2(kL)) + R_1 R_2}. \quad (1.9)$$

We now consider how I_c depends on the detuning δ of the incident light from the cavity resonance frequency ω_c , which enters through the wavenumber $k = (\omega_c + \delta)/c$. The cavity boundary conditions enforce $\omega_c L/c = \pi j$ where j is an integer, so we have $kL = \pi j + L\delta/c$. We assume the incident light is near-resonant with the cavity such that δ is small. We then expand Eq. (1.9) to leading order in δ :

$$|\mathcal{E}_c|^2 \simeq \frac{T_1 |\mathcal{E}_i|^2}{1 - 2\sqrt{R_1 R_2} (1 - 2(L\delta/c)^2) + R_1 R_2} \quad (1.10)$$

$$= \frac{T_1 |\mathcal{E}_i|^2}{(1 - \sqrt{R_1 R_2})^2 + 4\sqrt{R_1 R_2} (L\delta/c)^2} \quad (1.11)$$

$$= \frac{T_1 |\mathcal{E}_i|^2}{(1 - \sqrt{R_1 R_2})^2 \left(1 + \underbrace{\frac{4\sqrt{R_1 R_2}}{(1 - \sqrt{R_1 R_2})^2}}_{4\mathcal{F}^2/\pi^2} \left(\frac{L\delta}{c} \right)^2 \right)} \quad (1.12)$$

$$= \frac{T_1 |\mathcal{E}_i|^2}{(1 - \sqrt{R_1 R_2})^2 (1 + \delta^2/\kappa^2)}. \quad (1.13)$$

From this, we see that the cavity's power spectrum is a Lorentzian in δ with a half-linewidth defined as

$$\kappa = \frac{\pi c}{2L\mathcal{F}}. \quad (1.14)$$

Here, we have also introduced the cavity finesse $\mathcal{F} = \pi(R_1 R_2)^{1/4}/(1 - \sqrt{R_1 R_2})$, which quantifies the number of round trips a photon makes within the cavity before leaking out. Now knowing that the cavity's spectrum is a Lorentzian of linewidth κ , we can define a normalized density of states $\rho_{\text{cav}}(E)$ for the cavity:

$$\rho_{\text{cav}}(E) = \frac{\rho_{\text{cav}}(\omega)}{\hbar} = \frac{1}{\pi\hbar} \frac{\kappa}{\kappa^2 + (\omega - \omega_c)^2}, \text{ where} \quad (1.15)$$

$$\int_{-\infty}^{+\infty} \rho(\omega) d\omega = \int_{-\infty}^{+\infty} \rho(E) dE = 1. \quad (1.16)$$

Dissipation into free space

While a cavity enhances the density of states near its resonance, an atom within the cavity still couples to a continuum of free space electromagnetic modes. This coupling to a continuum of vacuum states gives rise to spontaneous emission, which can be derived using the treatment by Wigner and Weisskopf [43]. The spontaneous emission rate for a dipole transition with matrix element d is given by

$$\Gamma = \frac{\omega^3 d^2}{3\pi\epsilon_0 \hbar c^3}, \quad (1.17)$$

where Γ is defined as the full-linewidth of the transition (we may also define the half-linewidth to be $\gamma \equiv \Gamma/2$). For ^{87}Rb , the D_2 line has a spontaneous emission rate of $\Gamma = 2\pi \times 6.065$ MHz.

Cooperativity

For an atom placed inside a cavity, we can compare the rate at which the atom emits photons into the cavity (Γ_{cav}) to the rate at which it emits photons into free space (Γ). Using Fermi's Golden Rule to calculate Γ_{cav} , we obtain the ratio of emission rates

$$\frac{\Gamma_{\text{cav}}}{\Gamma} = \frac{2\pi\hbar g_0^2 \rho(E)}{\Gamma} = \frac{2\pi g_0^2 \rho(\omega)}{\Gamma}. \quad (1.18)$$

On cavity resonance, we have $\rho(\omega_c) = 1/(\pi\kappa)$, leading to the ratio

$$\frac{\Gamma_{\text{cav}}}{\Gamma} = \frac{2g_0^2}{\kappa\Gamma} = \frac{g_0^2}{\kappa\gamma}. \quad (1.19)$$

We thus see that when $g_0^2/(\kappa\gamma) > 1$, the atom emits into the cavity at a higher rate than it emits into free space. This is known as the Purcell effect [44]. The enhancement of the spontaneous emission rate is thus governed by the quantity $g_0^2/(\kappa\gamma)$, which quantifies the ratio of coherent interactions to dissipation. We define this ratio as the cavity's cooperativity C :

$$C \equiv \frac{g_0^2}{2\kappa\gamma}. \quad (1.20)$$

Note that there exist different definitions of cooperativity in the literature, with various factors of 2 or 4 showing up (also depending on whether half-linewidths or full-linewidths are used). Historically, the Stamper-Kurn group has used the definition provided in Eq. (1.20), so that is the definition I use in this dissertation. Details on the parameters of the E6 science cavity can be found in Sec. 2.5.

Although the cooperativity C may seem like a very “quantum” quantity at first glance (after all, it depends on g_0 and γ , which come from field quantization), C is actually a purely geometric quantity. Assuming a Gaussian cavity mode of waist w_0 , by plugging Eqs. (1.4), (1.14) and (1.17) into Eq. (1.20), we find

$$\frac{g_0^2}{2\kappa\gamma} = \frac{3\lambda^2 \mathcal{F}}{\pi^3 w_0^2}, \quad (1.21)$$

where λ is the wavelength of the atomic transition. From this, we see that C can be increased not only with higher finesse, but also by going to larger wavelengths and smaller mode waists. Recalling that an atom's on-resonance scattering cross-section is given by $\sigma_0 = 3\lambda^2/(2\pi)$, we see that $C \propto \sigma_0 \mathcal{F}/w_0^2$, which has an interpretation as the probability of a cavity photon scattering off of the atom, enhanced by the number of round trips made inside the cavity [61].

1.3.2 The Jaynes-Cummings model and beyond

Armed with this cavity QED background, we can begin to explore basic models describing atom-cavity interactions. The simplest one of these is the Jaynes-Cummings Hamiltonian [62], which describes the coupling between a single two-level atom with transition frequency ω_a and a single cavity mode:

$$\mathcal{H}_{\text{JC}}/\hbar = \omega_c \hat{c}^\dagger \hat{c} + \frac{\omega_a}{2} \hat{\sigma}_z + g(\hat{\sigma}^+ \hat{c} + \hat{c}^\dagger \hat{\sigma}^-) \quad (1.22)$$

where we have introduced Pauli spin operators to describe a two-level atom containing a ground state $|g\rangle$ and excited state $|e\rangle$. Here, we have assumed that the atom couples to the cavity's electric field through an electric dipole interaction with vacuum Rabi frequency g , which we take to be real (this is simply a gauge choice). We have also dropped constant energy terms. The interaction terms show that a ground state atom may be excited by absorbing a cavity photon, and similarly an excited atom may transition to the ground state by emitting a photon into the cavity—processes which conserve energy. The Jaynes-Cummings model famously has hybridized atom-photon eigenstates which are linear combinations of $|g, n\rangle$ and $|e, n-1\rangle$, where the second index indicates a cavity photon Fock state. These eigenstates are separated in energy by $\hbar\sqrt{4ng^2 + \Delta_{\text{ca}}^2}$, where $\Delta_{\text{ca}} \equiv \omega_c - \omega_a$. The Jaynes-Cummings model encapsulates phenomena such as the vacuum Rabi splitting [62, 63], vacuum Rabi oscillations [64], and photon blockade [65, 66].

To arrive at this form of the electric dipole interaction, we had to make the rotating wave approximation (RWA), in which we dropped the terms that do not conserve energy (the “counter-rotating” terms). Had we not made the RWA, we would instead have the Rabi model:

$$\mathcal{H}_{\text{Rabi}}/\hbar = \omega_c \hat{c}^\dagger \hat{c} + \frac{\omega_a}{2} \hat{\sigma}_z + g(\hat{c} + \hat{c}^\dagger)(\hat{\sigma}^+ + \hat{\sigma}^-). \quad (1.23)$$

To understand how the RWA emerges, we consider the time-dependence of the interaction terms. In the Heisenberg picture, the time evolution of the operators is given by $\hat{c}(t) = e^{-i\omega_c t} \hat{c}$ and $\hat{\sigma}^-(t) = e^{-i\omega_a t} \hat{\sigma}^-$. The “co-rotating” terms then evolve as $e^{i(\omega_a - \omega_c)t} \hat{c} \hat{\sigma}^+$ and $e^{-i(\omega_a - \omega_c)t} \hat{c}^\dagger \hat{\sigma}^-$, whereas the “counter-rotating” terms evolve as $e^{i(\omega_a + \omega_c)t} \hat{c}^\dagger \hat{\sigma}^+$ and $e^{-i(\omega_a + \omega_c)t} \hat{c} \hat{\sigma}^-$. We see that the co-rotating terms evolve at the difference frequency $\pm(\omega_c - \omega_a) \equiv \pm\Delta_{\text{ca}}$ while the counter-rotating terms evolve at the sum frequency $\pm(\omega_a + \omega_c)$. When $\omega_c \approx \omega_a$, the co-rotating terms are nearly stationary while the counter-rotating terms oscillate rapidly. In the limit where $|\Delta_{\text{ca}}|, g \ll \omega_a, \omega_c$, the counter-rotating terms average out to zero to good approximation, so we neglect them and only consider the co-rotating terms (thus making the RWA).

The Jaynes-Cummings and Rabi models may be generalized to an ensemble of N atoms symmetrically coupled to a cavity, giving rise to the Tavis-Cummings [67] and Dicke models [54, 68] respectively:

$$\mathcal{H}_{\text{TC}}/\hbar = \omega_c \hat{c}^\dagger \hat{c} + \sum_{j=1}^N \left[\frac{\omega_a}{2} \hat{\sigma}_{z,j} + g(\hat{\sigma}_j^+ \hat{c} + \hat{c}^\dagger \hat{\sigma}_j^-) \right], \quad (1.24)$$

$$\mathcal{H}_{\text{Dicke}}/\hbar = \omega_c \hat{c}^\dagger \hat{c} + \sum_{j=1}^N \left[\frac{\omega_a}{2} \hat{\sigma}_{z,j} + \frac{\alpha}{\sqrt{N}} (\hat{c} + \hat{c}^\dagger)(\hat{\sigma}_j^+ + \hat{\sigma}_j^-) \right]. \quad (1.25)$$

The Tavis-Cummings model, combined with external driving terms, serves as the theoretical starting point for the work described in this thesis, allowing us to realize effective Hamiltonians resembling $\mathcal{H}_{\text{Dicke}}$. Note that in $\mathcal{H}_{\text{Dicke}}$, we have replaced g with an atom-cavity coupling strength scaled by $\sqrt{N}$. The Dicke model is traditionally written this way so that in the thermodynamic limit $N \rightarrow \infty$, it exhibits a quantum phase transition at non-zero α . The Dicke model is discussed in detail in Ch. 4. There is theoretical debate over whether the Dicke quantum phase transition can be achieved using electric dipole coupling alone, with some arguing that the Dicke model in the form of Eq. (1.25) is incomplete because it is not invariant under gauge transformations of the field [57, 69]. Nonetheless, the Dicke model has been realized experimentally through mappings to driven-dissipative systems, which circumvent no-go theorems.

1.3.3 Including dissipation

In 1.3.1, we introduced the quantities κ and Γ , which are the decay rates of cavity photons and the atomic excited state respectively. To include dissipation in the models written in 1.3.2, we work with the Lindblad master equation [70]:

$$\dot{\hat{\rho}} = -i[\hat{H}, \hat{\rho}] + \sum_j \Gamma_j \left(\hat{L}_j \hat{\rho} \hat{L}_j^\dagger - \frac{1}{2} \{ \hat{L}_j \hat{L}_j^\dagger, \hat{\rho} \} \right), \quad (1.26)$$

where $\hat{\rho}$ is the system's density matrix, the $\hat{L}_k$ are jump operators with corresponding decay rates Γ_k , and curly brackets indicate the anti-commutator. In E6's experiments, we are often interested in measuring specific cavity-field observables, rather than the density matrix, so we instead use the master equation in the Heisenberg picture for our calculations:

$$\dot{\hat{X}} = \frac{i}{\hbar} [\hat{H}, \hat{X}] + \sum_j \Gamma_j \left(\hat{L}_j^\dagger \hat{X} \hat{L}_j - \frac{1}{2} \{ \hat{L}_j^\dagger \hat{L}_j, \hat{X} \} \right), \quad (1.27)$$

where $\hat{X}$ is the observable whose equation of motion we seek. Often in cavity QED, we deal with the Heisenberg-Langevin equations of motion to account for shot noise on the cavity

field. For example, in the Jaynes-Cummings model, the equation of motion for the cavity in the frame rotating at ω_a is

$$\dot{\hat{c}} = -i(\Delta_{ca} + \kappa)\hat{c} - ig\hat{\sigma}^- + \sqrt{\kappa}\hat{\xi}(t), \quad (1.28)$$

where $\hat{\xi}(t)$ is the Langevin operator with $\langle \hat{\xi}(t)\hat{\xi}^\dagger(t') \rangle = \delta(t - t')$ [43, 71].

Aside on adiabatic elimination

Writing down the equations of motion for system observables via Eq. (1.27) is useful for understanding the time evolution of the observables. It is also useful for making simplifications via adiabatic elimination of the atomic excited state and cavity in the limit of large detunings [72, 73]. By ignoring the fast-varying time dynamics of these observables, we reduce the number of unknowns in our equations of motion, and obtain simplified, effective equations of motion that often lend physical insight. Concrete examples of this are provided in Ch. 3 and Ch. 4.

Note that adiabatic elimination of the cavity disregards any physics resulting from cavity field dynamics, such as cavity cooling and heating (also see 3.2.3). This is because adiabatic elimination assumes that the cavity instantaneously responds to the atomic state, while in reality the response is delayed. For instance, in the case of the Jaynes-Cummings model, the delay is on a time scale of $1/|\kappa + i\Delta_{ca}|$ [74]. To account for non-adiabatic effects, one can formally integrate Eq. (1.28) and expand the obtained expression for $\hat{c}(t)$ beyond leading order in $1/(\kappa + i\Delta_{ca})$. This quasiadiabatic expansion gives [74]:

$$\hat{c}(t) \approx \frac{-ig(\hat{z})\hat{\sigma}^-}{\kappa + i\Delta_{ca}} - \frac{\frac{d}{dt}(-ig(\hat{z})\hat{\sigma}^-)}{(\kappa + i\Delta_{ca})^2} + \hat{\mathcal{F}}(t) + \mathcal{O}[(\kappa + i\Delta_{ca})^{-3}], \quad (1.29)$$

where $\hat{\mathcal{F}}(t) \propto \hat{\xi}(t)$ is the quantum noise term. As seen in Eq. (1.29), we obtain a term proportional to $\dot{g}(\hat{z})$. Recalling the spatial dependence $g(\hat{z}) \equiv g_0 \sin(k\hat{z})$, we get $\dot{g}(\hat{z}) = kg_0 \cos(k\hat{z})\dot{\hat{z}}$. The dependence on $\dot{\hat{z}}$ crucially gives rise to a damping force.

1.4 My time on E6

My time on E6 began in Fall 2020, when Professor Dan Stamper-Kurn agreed to let me rotate in his group during peak COVID. Fresh out of college (and quarantine), I admittedly had much more enthusiasm for than I had knowledge of AMO physics when I arrived at Berkeley. In fact, I came into graduate school very undecided about what research I wanted to pursue. I had primarily done research in soft matter (specifically, self-assembly of colloidal particles) as an undergraduate, but a summer in Nick Hutzler's lab at Caltech gave me a taste of AMO research that left me wanting more. Doing my rotation during those "unprecedented times" was challenging, but also a productive experience for me. In that era of social distancing, we had to sign up for shifts in the lab. Thankfully, the number of ventilation zones in Campbell

LL104 allowed me to be supervised by graduate student Justin Gerber, who was finishing up his thesis at the time. While Justin was working on the main E6 apparatus, which was mostly complete already by that time, I built an optical setup on the side for testing a digital micromirror device (DMD). At the time, we were curious about using a DMD to project arbitrary potentials onto the atoms, and perhaps even to generate tweezer arrays. This project was a crash course in Fourier optics, which was new to me at the time, and I am very thankful to Justin for patiently guiding me through it. In my test setup, I measured DMD-generated patterns with $\sim 25\%$ optical power efficiency, which we ultimately decided was not high enough for generating tweezer arrays.

At the end of my first semester of grad school, I left the group to do a rotation with Professor Naomi Ginsberg in soft matter, followed by a rotation with Professor Carlos Bustamante in biophysics. I am very grateful to then-graduate students Christian Tanner and Robert Sosa in the Ginsberg and Bustamante groups respectively; their guidance made my experiences in those labs illuminating and enjoyable. By the end of my first year, I had to decide which lab I wanted to complete my PhD in. Being an incredibly indecisive person, this was a very tough decision to make. Nonetheless, I found myself drawn to E6 for the opportunity to study and control quantum systems with interesting cavity-induced interactions. After my rotation, the DMD project had fallen to the wayside. But, there was no need to be sad about that, since there were plenty of other (more) interesting ideas to pursue.

In the summer of 2021, I started working with graduate students Emma Deist, Leon Lu, and Mary Kate Pasha on cavity-based state detection, which led to our first mid-circuit measurement result [42]. Emma, as the senior graduate student, not only showed us the ropes of running E6 (from how to use Cicero to what to do if you don't see atoms), but also taught us many fundamental AMO physics concepts and experimental techniques. Her guidance and her impressive ability to clearly define goals amid complications were crucial to our completion of the project. We were soon joined by postdoc Dr. Zhenjie Yan, whose vast knowledge and experience have been an invaluable resource. After Emma and Mary Kate left the group, Leon, Zhenjie and I began investigating how to use the programmability of the tweezer array to engineer atom-cavity interactions, which led to our work on collective scattering in collaboration with Ana Asenjo-Garcia's theory group at Columbia (see Ref. [53] and Ch. 3). This work later enabled study of cavity-mediated self-organization in mesoscopic atom arrays [55], where we probed the symmetry-breaking dynamics of an optomechanical phase transition taken out of the thermodynamic limit. It has also enabled our ongoing work on spin self-organization, which has turned out to be a very complex (but fascinating) experiment that Leon has been heroically heading.

As Leon and I turned into senior grad students and Zhenjie began to look for jobs, we were very grateful to have new grad students Tai Xiang and Tsai-Chen Lee join us on E6. I admire the work ethic and spirit they bring to our group, as well as their impressive leadership on our Rydberg and new mid-circuit measurement efforts. In the past year, we were also very fortunate to have Dr. Nathaniel Vilas join us as a postdoc. Already in his short time on E6, Nathaniel has provided incredible insight, critical thinking, and fresh ideas that have pushed E6 forward. I am excited to see E6 excel in the hands of our new generation of personnel.

1.5 List of acronyms

ac alternating current

AOM acousto-optic modulator

AOD acousto-optic deflector

APD amplified photodetector

BEC Bose-Einstein condensate

BS beamsplitter

DBR distributed Bragg reflector

dc direct current

EOM electro-optic modulator

LO local oscillator

LPF low-pass filter

MOT magneto-optical trap

NA numerical aperture

ODT optical dipole trap

PBS polarizing beamsplitter

PCA principal component analysis

PDH Pound-Drever-Hall (locking technique)

QED quantum electrodynamics

rf radio frequency

RWA rotating wave approximation

SP side probe/pump

TA tapered amplifier

TTL transistor-transistor logic

Chapter 2

The E6 apparatus

I had the privilege of joining E6 when the apparatus was producing its very first science results. I owe much to previous graduate students Emma Deist and Justin Gerber for the hard work they put into bringing E6 to life, and for showing me the ropes of running the experiment.

Much detail about the apparatus has already been written in Emma’s and Justin’s theses [75, 76]. In this chapter, I briefly summarize the main components of the apparatus, and then delve into more detail about the changes we have made to it over the course of my grad school years.

2.1 Overview of the E6 apparatus

The E6 apparatus resides in the basement of Campbell Hall. It occupies two optical tables, one housing most of our lasers (the “laser table”) and the other containing the main apparatus (the “science table”). The main apparatus consists of three chambers: a glass cell for the 2D MOT, a 3D MOT chamber, and a science chamber. The science chamber houses our optical cavity. An array of optical tweezer beams is projected in between the cavity mirrors by a microscope objective (Mitutoyo G Plan Apo 50X) with NA= 0.5. Atoms are transported from the 3D MOT chamber into the science chamber using an optical dipole trap (ODT) focused through a tunable focus lens. E6 was originally built to be able to do bulk-gas cavity QED experiments, hence the science chamber is separate from the 3D MOT chamber for better vacuum (previously measured to be $\sim 10^{-11}$ torr in our 3D MOT chamber [75]). At one point, a Bose-Einstein condensate (BEC) was made in E6. However, ever since we started working with our tweezer array, we have never returned to BEC experiments. The science we can pursue with control over single atoms in our cavity is so rich and largely unexplored that we never looked back from it.

2.2 Lasers

2.2.1 Cooling, repump, imaging, and optical pumping

On the laser table, we have our cooling laser (both are DBR lasers from Photodigm). The cooling laser is locked -210 MHz detuned of the $F = 2 \rightarrow F' = 3$ transition using saturated absorption spectroscopy in a Rb vapor cell, and the repump laser is locked to the $F = 1 \rightarrow F' = 1, 2$ crossover via modulation transfer spectroscopy in a separate vapor cell. The cooling laser is split into three paths: 2D MOT, 3D MOT, imaging, and optical pumping. The 2D MOT path goes through a single-pass acousto-optic modulator (AOM), a Toptica BoosTA pro, and then is delivered to the science table via a fiber splitter. The fiber splitter combines cooling and repump light from the laser table and delivers them to four ports on the science table to form the counter-propagating 2D MOT beams. We operate with ~ 70 mW of cooling power and ~ 1.4 mW of repump power in each of the four beams. The 3D MOT path is similar to the 2D MOT path, but uses a double-pass AOM and a home-built tapered amplifier (TA chip: EYP-TPA-0780-01000-3006-CMT03-0000). It is also delivered to the science table via a fiber splitter which combines it with 3D MOT repump light. The fiber splitter splits into six ports on the science table. In each, we typically operate with ~ 20 mW of cooling power and ~ 0.4 mW of repump power.

The repump laser light is split into three fiber ports corresponding to the 2D MOT, 3D MOT, and science chamber. The 2D MOT and science chamber repump beams are split off from the same path, sharing a common AOM. The desired frequencies are achieved by tuning the frequency of the radio frequency (rf) drive to this AOM. Since this AOM is in a single-pass configuration, each of the fibers for the 2D MOT repump and science chamber repump is only coupled when the AOM is driven at a specific rf frequency, which is different for the two paths. The 3D MOT repump path has its own AOM.

From the 3D MOT path, we split off some light for optical molasses, which performs sub-Doppler cooling in the science chamber. When I first joined E6, the molasses light went through a single AOM for switching on/off before being delivered to a fiber. Because we do not want to shift the frequency of the molasses light relative to the 3D MOT light, we used the 0th order, un-diffracted beam from the AOM. However, we later found that we could not achieve adequate extinction ratios with the 0th order, leading to leakage molasses light on the atoms if there was no shutter. This was problematic for experiments where we wished to toggle between molasses-on and molasses-off on a 10's of μ s time scale (the shutter response time is a few milliseconds). Thus, we replaced the single AOM with two AOMs which are both driven by the same rf source. The first AOM is coupled to the $+1$ diffracted order and the second is coupled to the -1 diffracted order, such that there is no net frequency shift. Although we lose on optical power in this two-AOM setup, the extinction ratio is much improved, and we often get away with stealing a bit of power from the 3D MOT path if necessary.

The imaging path contains light that we use for absorption imaging. It goes through a double-pass AOM which allows us to set its frequency to be resonant with the $F = 2 \rightarrow F' =$

3 transition. We often need to tune this frequency depending on whether we are imaging trapped or free atoms to account for the ac Stark shift from trapping light.

The optical pumping path is also derived from the cooling laser. At one point in time, optical pumping was again split into two paths, one entering the science chamber from the side and the other entering from the bottom. In recent years, we have retired the bottom port and only work with optical pumping from the side. The optical pumping light goes through a single-pass AOM coupled to the -1 order, which shifts the frequency by approximately -40 MHz to be resonant with the $F = 2 \rightarrow F' = 2$ transition for tweezer-trapped atoms.

2.2.2 Optical transport

Atoms are transported from the 3D MOT chamber to the science chamber via our transport ODT, generated by a 20 W IPG laser (wavelength of 1064 nm) that is sent through a tunable focus lens (Optotune EL-C-16-40-TC). We trap atoms at the focus of this laser, and vary the focus position by adjusting the focal length of the tunable focus lens, which takes in a control voltage. After loading atoms into the 3D MOT, we turn on the transport ODT with the diopter of the tunable focus lens set such that the laser is focused on the center of the MOT. Out of the $\sim 10^8$ atoms we load into the MOT after polarization-gradient cooling, we load $\sim 10^5$ – 10^6 atoms into the transport ODT. We transport the atoms into the science cavity with an efficiency of $\sim 10\%$. In my experience, it seems that the loading and transport efficiency of our transport ODT have deteriorated over the years. In the past (circa 2021), we could bring $\sim 10^5$ atoms into the cavity, but nowadays we see more like 10^4 . The cause for this deterioration in efficiency has remained mysterious, but we suspect it could be astigmatism in the transport laser. At one point, we also saw that replacing the AOM in the laser path temporarily improved our loading efficiency.

2.2.3 Science table 780 nm wavelength beam paths

The science table 780 nm wavelength optics layout is illustrated in Fig. 2.1. I now discuss considerations made for our molasses, side probe/pump, repump, imaging, and optical pumping paths.

Our molasses light (formerly known as “molasses 1”) does polarization-gradient cooling of the atoms. Upon emerging from its fiber port, some molasses light is picked off and sent to an APD for intensity stabilization. Mirrors then steer the beam with a slight downwards angle, so as to provide some cooling along the tweezer axis. The beam is retro-reflected by a mirror on the output side of the science chamber. Our molasses light is detuned approximately $-2\pi \times 40$ MHz from the bare $F = 2 \rightarrow F' = 3$ transition and $-2\pi \times 60$ MHz from the tweezer-shifted transition. It is polarized in a $\text{lin}\perp\text{lin}$ configuration, meaning that the polarizations of the incident and retro-reflected beams are linear and perpendicular to each other. At the time of Ref. [42], our molasses polarization used to be in a $\sigma^+ - \sigma^-$ configuration. However, we had switched to $\text{lin}\perp\text{lin}$ polarization around the time when we introduced the side probe/pump path for our work in Ref. [53] (also see the following section

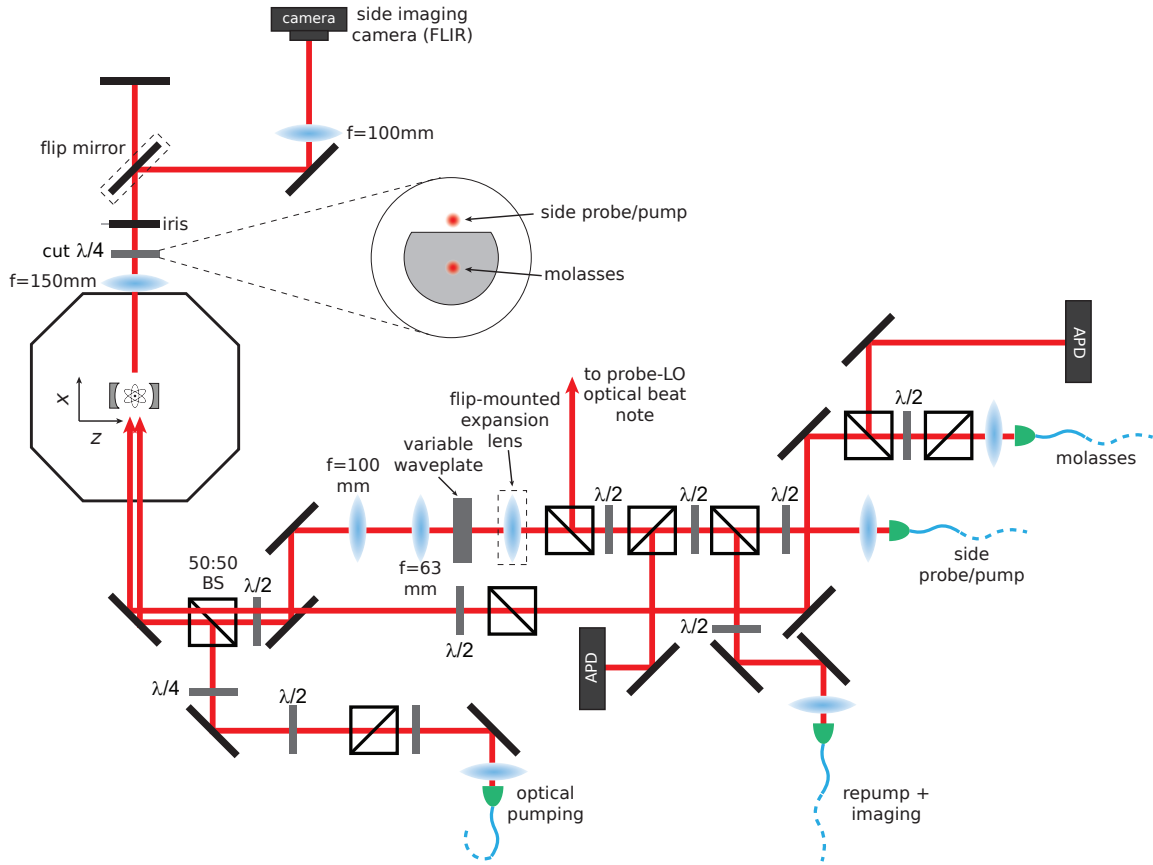


Figure 2.1: Optics layout of the science table 780 nm wavelength beam paths. Molasses, side probe/pump, repump, absorption imaging, and optical pumping lasers are all combined and enter the vacuum chamber transverse to the cavity. A retro-reflection mirror is placed on the other side of the chamber. The pointing of the side probe/pump and molasses beams is carefully aligned to ensure good overlap of the incident and retro-reflected beams. A cut quarter waveplate ($\lambda/4$) is used to create $\text{lin} \perp \text{lin}$ molasses polarization while maintaining a pure side probe polarization. Absorption imaging can be accessed by flipping up a flip-mounted mirror, which directs imaging light to our side imaging camera. All beamsplitter cubes shown are PBSs unless otherwise labeled.

for more details on the side probe/pump). We found that $\text{lin}\perp\text{lin}$ polarization seemed to cool the atoms better; perhaps this had to do with imperfect circular polarization purity in the $\sigma^+ - \sigma^-$ configuration caused by a silver mirror right before the chamber.

The molasses and side probe light share the same retro-reflection mirror on the output side of the chamber. To keep our retro-reflected side probe light vertically polarized for our work in Ref. [53], we aligned one axis of the output-side quarter waveplate with the vertically polarized side probe. We then oriented the input molasses linear polarization at 45° from the waveplate axis, such that upon passing through the waveplate twice, the retro-reflected beam would be polarized at -45° to create $\text{lin}\perp\text{lin}$ polarization at the atoms. When it came to our work in Ref. [55], we wished to use a σ^- -polarized pump. In order to have the retro-reflected side probe also be σ^- -polarized, we could not have it pass through the quarter waveplate. The solution we came up with (courtesy of Leon, who very much enjoys using the Dremel tool) was to cut the quarter waveplate such that side probe does not pass through it but molasses does. This takes advantage of molasses and side probe being spatially separated due to the vertical tilt angle of molasses. To ensure good overlap of the incident and retro-reflected side probe beams, we adjust the tilt of the retro-reflection mirror to couple the retro-reflected beam back into the side probe fiber.

The side probe/pump path is shared with the repump and imaging beams. Repump and imaging beams are prepared separately on the laser table, then combined through a fiber splitter, such that they both emerge from the same fiber port on the science table. The repump and imaging beams get combined with the side probe/pump on a PBS; the intensities of all three beams are monitored on an APD, but (as of now) none of them are intensity stabilized. At the time of Ref. [55], the side pump beam had a waist of $173 \pm 5 \mu\text{m}$. More recently, its waist has been expanded to $\sim 300 \mu\text{m}$ to accommodate atom arrays of larger span. In the past year, we have inserted a variable waveplate on this path to enable in-sequence polarization switching.

The imaging beam is used for absorption imaging of atoms trapped in the 1064 nm wavelength transport ODT. We often find it helpful for alignment to have the imaging beam large enough to span the $\sim 1 \text{ cm}$ length of the cavity. However, we would like to expand the imaging beam without also expanding the side probe, so we have inserted a lens on a flip mount in the path. When we do absorption imaging, we flip the lens into the path; when not doing absorption imaging, we flip it out of the path.

The last beam to be combined onto the side probe path is optical pumping. The optical pumping path has seen many changes over the years; it has previously been sent into the chamber from the bottom viewport and at 45° to the cavity. An additional optical pumping path has been set up in our new Rydberg cage system (not shown), which delivers beams at 45° to the cavity. Although we did not use optical pumping light in Refs. [53, 55], it was necessary in Ref. [42] to prepare atoms in the $|F = 2, m_F = 0\rangle$ clock state; it is similarly necessary for our forthcoming mid-circuit measurement and Rydberg atom works.

2.2.4 Cavity lasers

The E6 science cavity is bichromatic: it is high-finesse at wavelengths of 780 nm as well as 1560 nm. We use Toptica diode lasers for both wavelengths (LD-0780-0220-AR-1 and LD-1550-0050-AR-2) and Toptica’s DLC pro diode laser controller. These lasers are locked to an ultralow expansion (ULE) cavity. The ULE cavity spacer is from Stable Laser Systems and the mirror coatings were done by FiveNine Optics. The ULE cavity is temperature stabilized and nominally should be held at 37.9°C for zero expansion; however, in June-July 2025 we had accidentally disconnected the temperature control and used the ULE cavity at room temperature. This seemingly did not have much effect on our ability to take data, though instances like this ought to be avoided.

Some 780 nm and 1560 nm wavelength light is picked off and sent through fiber electro-optic modulators (EOMs) before being sent into the ULE cavity. We then tune the diode laser current and/or piezo scan voltage to bring one of the EO sidebands into resonance with the ULE cavity, and then lock it using a Pound-Drever-Hall (PDH) scheme through a Toptica FALC 110 unit. Thus, we tune our cavity laser frequencies by tuning the frequency of the EO side bands (controlled by a Valon digital frequency synthesizer).

The cavity 780 nm wavelength light is then split into three paths: transmission probe, side probe (or pump, we use the names interchangeably), and LO (Fig. 2.2). Each path has its own AOM for independent attenuation and transistor-transistor logic (TTL) control. The probe and side probe AOMs are driven by the same rf source, which is fed into a home-built circuit board that gives us independent amplitude and TTL control over each channel (adapted from Zhenjie Yan’s original PCB design from MIT, but bypassing the VCOs). The rf source for the LO is locked at a fixed frequency difference from the probe. This used to be an active phase lock but recently has been switched to a passive lock. See Sec. 2.6.2 for more details on the probe and LO electronics.

We reference the probe frequency to the atomic resonance frequency using the side probe to blow away atoms. As the probe frequency approaches atomic resonance, we see that a pulse of probe light depletes the atoms, whereas it does not when far off resonance. In these frequency scans, we often step the EO side band frequency by ~ 20 MHz and then do a finer scan by changing the rf frequency driving the AOM (which gives us a range of ~ 40 MHz after the double pass). It is important to check the signs of the EO side bands and the direction of the rf frequency scan—we have previously gone on a weeks-long detour because we had accidentally parked the probe on the wrong side of atomic resonance!

2.2.5 Tweezer path

We use 808 nm wavelength light from a diode laser (Thorlabs LD808-SEV500) to generate our tweezers. The diode is mounted on a breadboard that sits directly on the optical table (the “lower breadboard”). The light is delivered via optical fiber to the “upper breadboard,” which houses the optical path that delivers the tweezer light into the science chamber. The tweezer path has seen a few changes throughout the course of my PhD. Prior to June 2022,

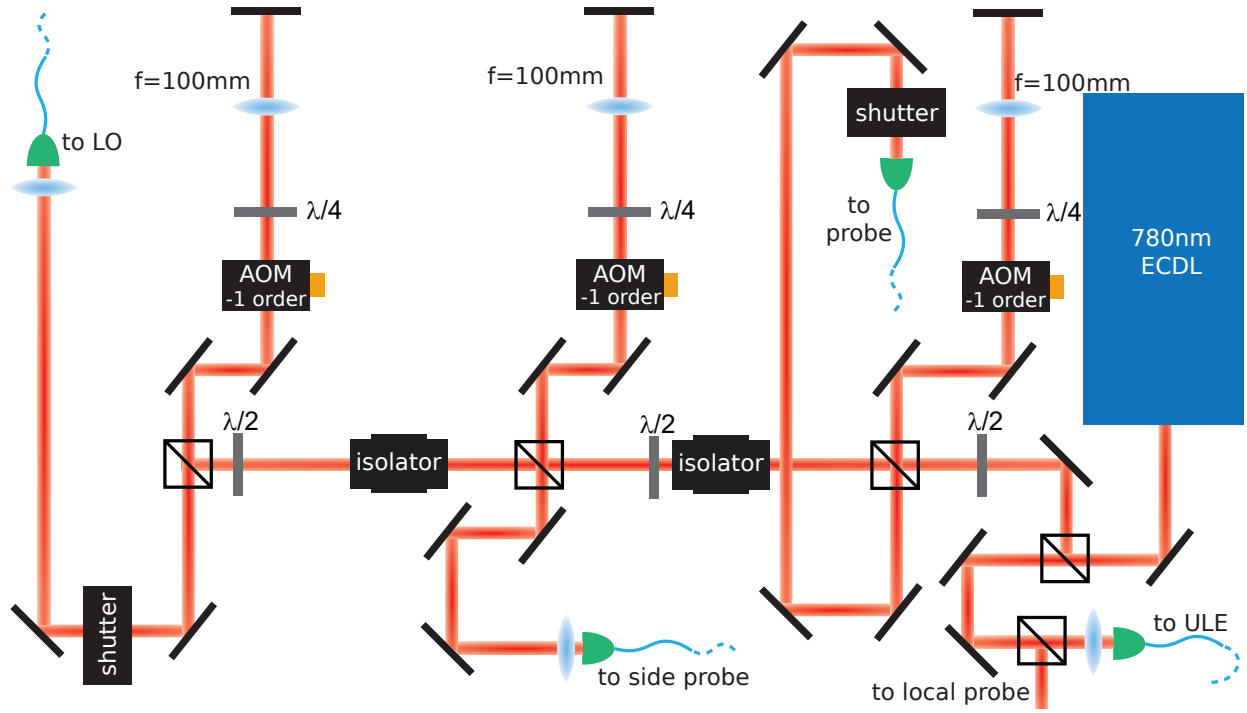


Figure 2.2: Optics table layout for the 780 nm wavelength probe laser. Light is split off from our 780 nm wavelength Topical ECDL to the ULE cavity, local probe, transmission probe, side probe, and LO. Double-pass AOMs are used to independently control the frequency and attenuation of the transmission probe, side probe, and LO. Isolators are necessary to prevent back-reflection of the LO light from entering the other paths.

the path on the lower breadboard contained an AOM before the fiber. This AOM was used for switching and intensity stabilization of the tweezer light. However, we lost about 30% of the power at this AOM, which led us to remove it in favor of more power. We instead switched to modulating the rf power going to the acousto-optic deflector (AOD) used for generating our optical tweezer beams. Although the AOD's response time is slower (takes about $5\ \mu\text{s}$ to switch on and off, and we achieve a feedback bandwidth of $\sim 20\ \text{kHz}$), this was sufficient for the experiments we were doing at the time. In January 2026, however, we have re-introduced an AOM into the tweezer path for doing Rydberg experiments, which require us to quickly pulse off the tweezers when applying Rydberg light. With the new AOM in the path, the on/off time is reduced to $\sim 300\ \text{ns}$. The light sent to the AOM is picked off using a flip mirror, so that we can switch between using and not using the AOM depending on the requirements of the experiments being performed (Fig. 2.3).

Another upgrade to the tweezer path was the insertion of a tapered amplifier (TA) in late March 2023. This TA was built and tested by Florian Zacherl, a visiting master's student at the time. With the TA in place, we could get $\sim 500\ \text{mW}$ of power on the upper breadboard,

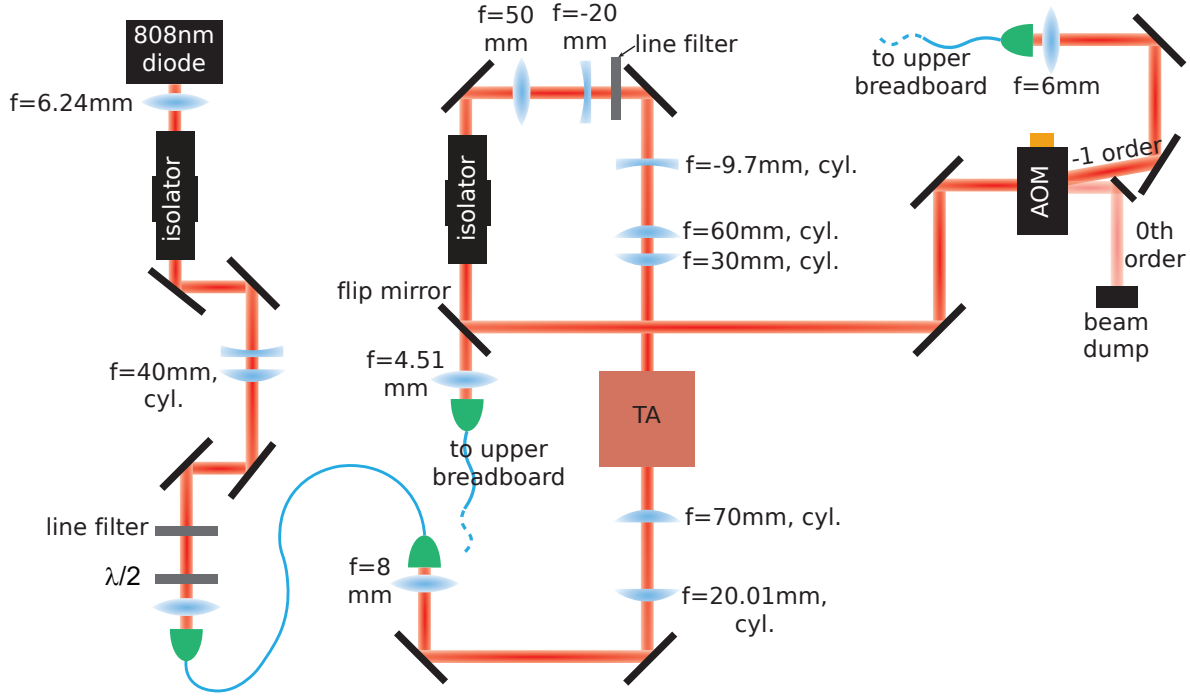


Figure 2.3: Tweezer lower breadboard optics layout. Light from the 808nm wavelength diode is beam-shaped and sent through a tapered amplifier (TA), then directed into a fiber that takes the light to the upper tweezer breadboard. In the case that we want to do rapid switching of the tweezers (required for Rydberg applications), there is a flip mirror that directs the beam to a path containing an AOM.

as opposed to the $\sim 100\text{mW}$ we had without it. This allowed us to go to larger tweezer arrays, first enabling the jump from the 16-tweezer array used in Ref. [53] to the 40-tweezer array used in Ref. [55]. Later, we found we can even load up to 70 tweezers.

The tweezer light propagates through the science chamber, emerges through the bottom bucket window, and is imaged onto a ThorCam as well as an amplified photodetector (APD) on the bottom breadboard. The photodetector is used for intensity monitoring and stabilization, while the ThorCam images the atom plane. A useful addition to the tweezer path was to insert a ThorCam on the bottom breadboard that images the Fourier plane, i.e. the plane of the AOD and the back plane of the objective lens (Fig. 2.5). The Fourier plane image is useful when aligning the tweezers. We adjust the alignment of the $4f$ -tweezer imaging system such that the different tweezer beams are collimated and centered on the Fourier plane.

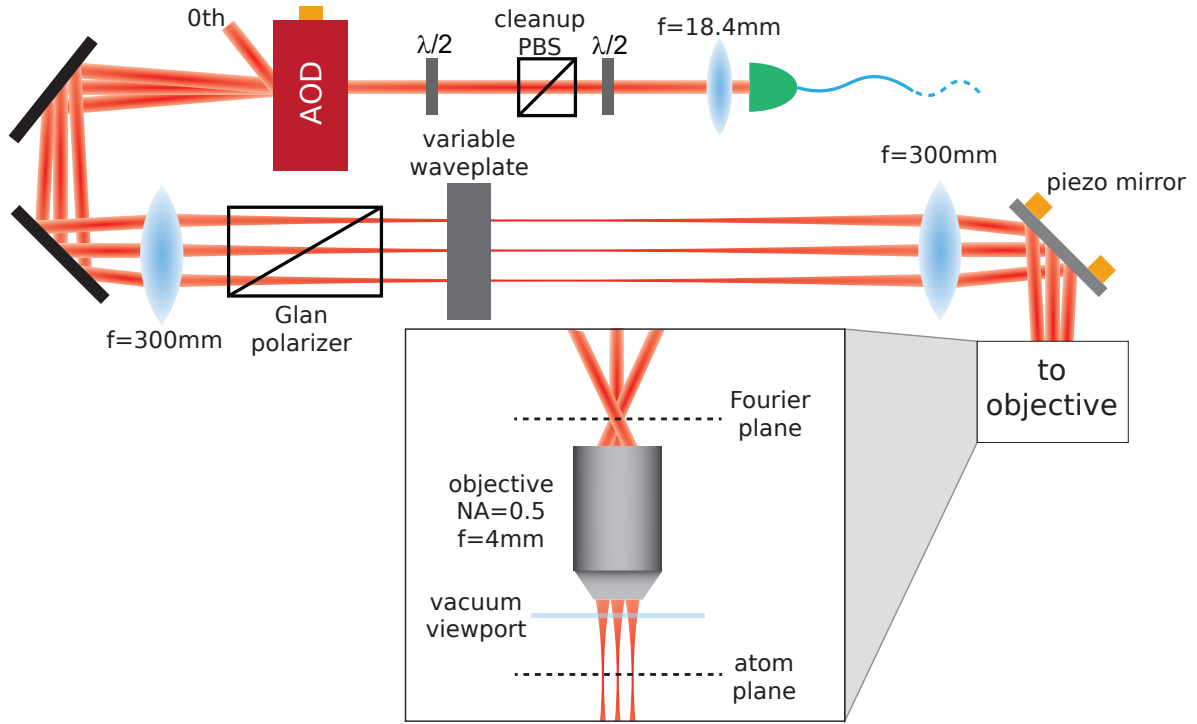


Figure 2.4: Tweezer upper breadboard optics layout. 808 nm wavelength light is routed from the lower breadboard to the upper breadboard via fiber, and then directed into the AOD, which creates multiple deflected beams. The AOD plane is imaged onto the back plane of the high-NA objective lens (the Fourier plane). The high-NA objective focuses down the beams to create an optical tweezer array at the atoms. A variable waveplate allows us to tune the tweezer polarization.

2.2.6 Local shielding and probe beams

In the past couple of years, E6 has built new laser paths for mid-circuit cavity measurement, building off work done in Refs. [42, 77]. This was an effort started by Zhenjie Yan and currently being led by Tsai-Chen Lee. For this, we acquired a 1529 nm wavelength Toptica DL pro. Because the 1529 nm wavelength light strongly shifts the $5P_{3/2}$ manifold (it couples $5P_{3/2} \rightarrow 4D_{5/2}$), we can use localized 1529 nm wavelength beams to selectively “shield” atoms from the cavity by shifting the D_2 line resonance far away from cavity resonance. The localized beams are created using an AOD and are projected from below the chamber upwards to the atoms.

The project also requires a localized 780 nm wavelength probe beam to address a single atom at a time. This beam’s position is also controlled by an AOD. The light for the local probe is picked off from our 780 nm wavelength DL pro, right before the light is sent to the ULE cavity (Fig. 2.2). Though not shown in Fig. 2.2, it requires a separate AOM that

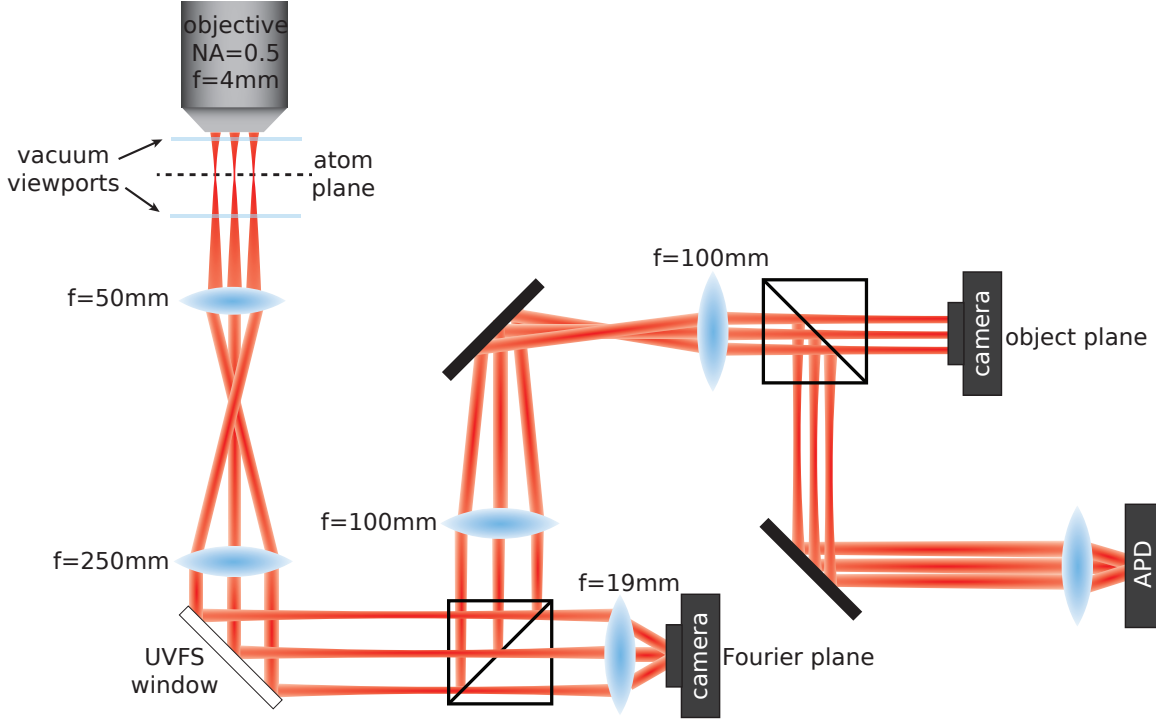


Figure 2.5: Tweezer imaging breadboard optics layout. After emerging from the vacuum chamber, the tweezer light is directed to ThorCams that image the Fourier plane and the atom (object) plane, as well as an APD. The APD is used for monitoring and active stabilization of the tweezer power in-sequence.

compensates for the frequency shift imparted by the AOD.

2.2.7 Rydberg lasers

E6's newest effort is to excite atoms to Rydberg states. Rydberg states have a high principal quantum number and, accordingly, a large dipole moment, allowing them to interact strongly with nearby Rydberg atoms via the van der Waals interaction [78]. This has led to a flood of advances in quantum computing and simulation with Rydberg atoms in recent years [35, 33]. We hope to integrate the immense capabilities of Rydberg atom arrays into our cavity experiments. This was an effort started by Leon Lu and currently being led by Tai Xiang.

We excite to the Rydberg state via two-photon excitation using wavelengths of 420 nm ($5S_{1/2} \rightarrow 6P_{3/2}$) and 1013 nm ($6P_{3/2} \rightarrow nS$). To do this, we have Moglabs Cateye ECDLs at wavelengths of 840 nm and 1013 nm. The 840 nm wavelength light is frequency doubled to obtain a 420 nm wavelength. The 1013 nm wavelength light seeds an IPG fiber amplifier that can deliver 100 W of optical power, necessary for exciting to higher lying Rydberg states.

2.3 Tweezer array characterization

Throughout my PhD, many changes have been made to our tweezer array setup, as described previously. Concurrently, we have learned how to characterize aberrations in our tweezer array, how to measure trap frequencies, and how to dynamically configure the traps.

Our tweezer array is generated by a 1D AOD. An AOD is similar to an AOM but, as its name suggests, it provides larger deflection angles than an AOM does. The AOD operating principle is that of Bragg diffraction. By driving the AOD crystal (ours is made of TeO_2) with a radio frequency tone, we generate an acoustic wave inside the crystal. When we shine laser light through the AOD, the laser light diffracts off of the acoustic wave, creating multiple orders of diffraction. Typically, we only work with the first-order diffraction, which has a deflection angle of $\Delta\theta = \lambda_O f / c_s$, where $\lambda_O = 808 \text{ nm}$ is the optical wavelength, f is the rf frequency, and $c_s = 650 \text{ m/s}$ is the speed of sound in the TeO_2 crystal. Note that $\Delta\theta$ is simply twice the Bragg angle. Seeing that $\Delta\theta \propto f$, we can drive the AOD with a superposition of rf tones f_i to create many deflected, first-order diffracted beams. These beams form the tweezer array.

2.3.1 Aberrations

While performing experiments on superradiance and subradiance, we learned that we could calibrate atom positions very precisely with respect to the standing wave cavity field (see Ch. 3 for more details). We could adjust the frequencies of the rf tones sent to the tweezer AOD to tune the spacing between tweezer sites to an integer or half-integer multiple of wavelengths. This technique allowed us to study superradiance and subradiance in the light scattered by the atom array into the cavity. We also used this method to characterize optical aberrations in the array.

In particular, we found that our tweezer array exhibits nonlinearities in the spacing between tweezer sites as well as curvature in the axis perpendicular to the array axis (Fig. 2.6). To measure this, we load a single atom into a specific tweezer site, then scan that atom along the cavity axis and along the pump laser axis (transverse to the cavity) while illuminating the atom with retro-reflected pump light. The rate at which the atom scatters light into the cavity depends on the atom's position with respect to both the pump and cavity standing waves, and reveals a sinusoidal profile in both axes. We compare the phase of the sinusoid with that obtained using an atom trapped in a reference tweezer. Keeping the reference tweezer location fixed, we repeat this process for different positions of the other tweezer. The rf frequency difference between the tones for the two tweezers is always a multiple of 0.8 MHz, which we had calibrated to correspond to a spacing of 5λ for tweezers near the center of the array. Converting the measured phase differences to displacements, we obtain Fig. 2.6(a). For tweezer positions farther away from the reference tweezer (at position 0), we see that there is a growing, nonlinear displacement in both axes. The fact that there is curvature indicates a true nonlinearity and not just a mis-calibrated tweezer spacing and AOD angle. We see that for tweezer sites $\sim 40 \mu\text{m}$ away from the reference tweezer, the

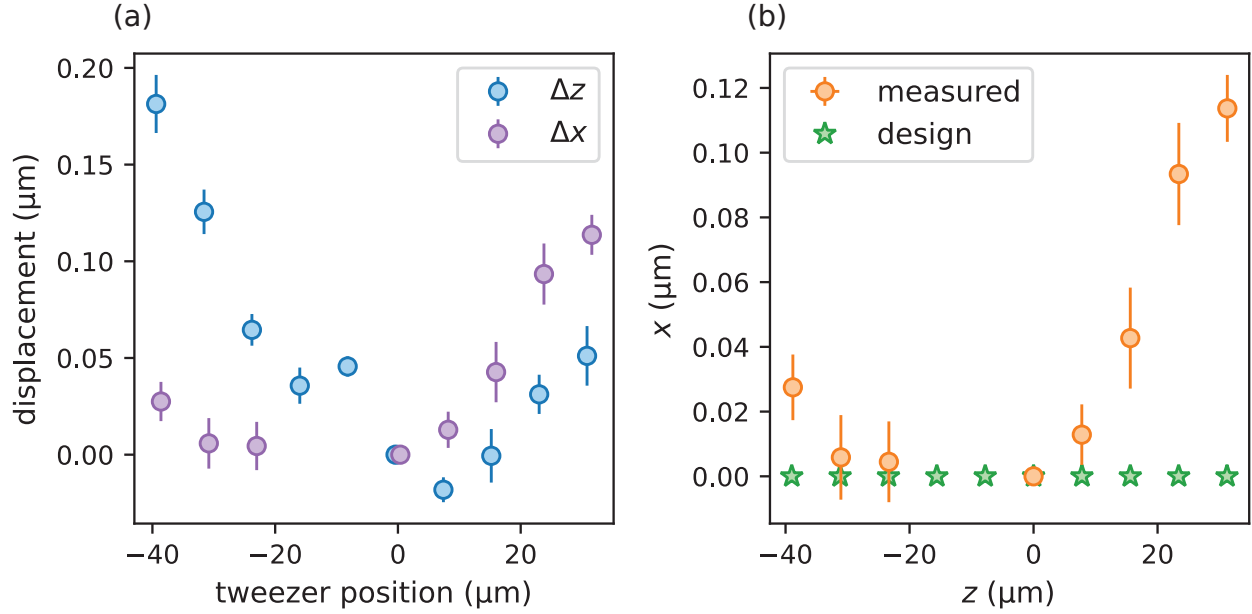


Figure 2.6: Measurement of the nonlinearity of the tweezer array. We scan the positions of single atoms across the cavity and pump axes via a piezo-actuated mirror and fit the phase of the sinusoidal variation in atom fluorescence. The phase is converted into displacements Δz and Δx from a perfectly linear array, referenced to a tweezer near the middle of the array (position 0). Note that in (a), Δx and Δz have a slight horizontal offset for visibility. In (a), tweezer position refers to the designed position of the tweezer along the array axis. In (b), we plot the measured (orange circles) z and x positions of the tweezers, compared to the designed positions (green stars).

spacing deviates by nearly $\lambda/4$ from the ideal 5λ spacing. For this reason, we are limited in the span of tweezers that we can use if we wish to only use rf waveforms with uniform spacing between tones.

In principle, we should be able to correct for the aberrations along the cavity axis (z) by simply adjusting the tweezer rf tones away from a uniform frequency difference. However, this leads to inter-modulation, where nonlinearities in the electronics lead to the formation of tones at the difference of the frequency differences. These have detrimental effects on atom survival. It is possible that we could cancel the effects of these higher-order tones by choosing the phase of each rf tone appropriately. However, we have not carefully performed such an optimization yet.

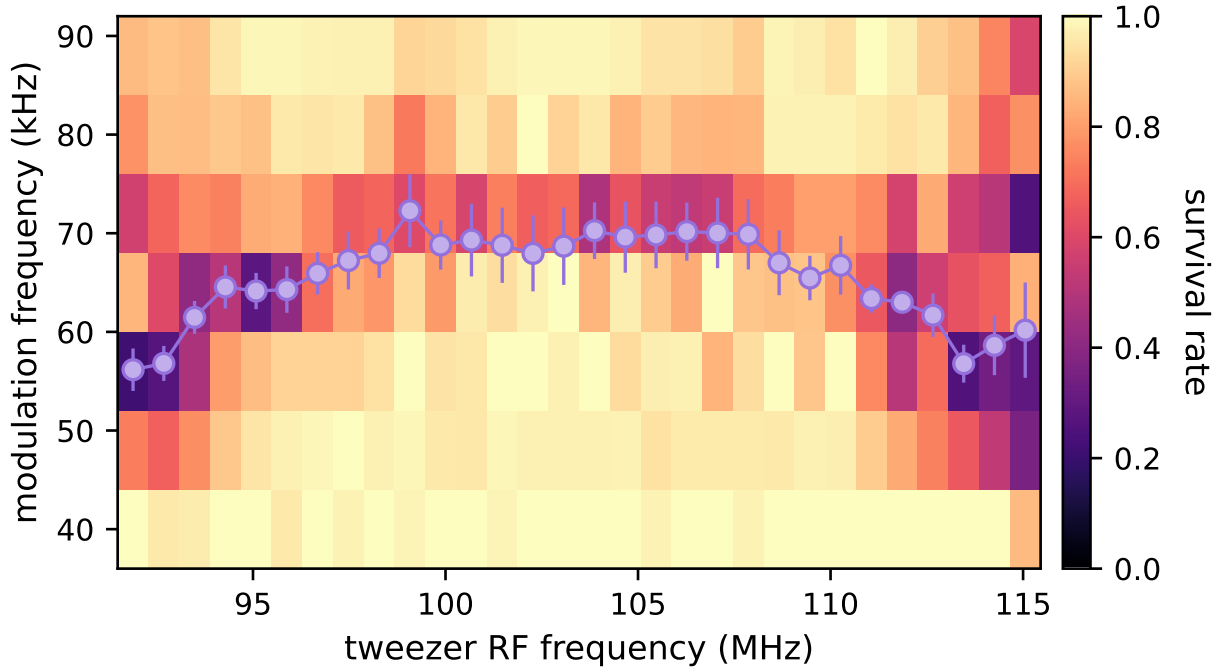


Figure 2.7: Tweezer radial trap frequency measurement via parametric heating. We measure atom survival probabilities after modulating the tweezer positions at various modulation frequencies. This measurement reveals inhomogeneity in the trap frequencies, with frequencies of ~ 70 kHz near the center of the array and frequencies near ~ 60 kHz near the edges of the array. Markers correspond to the center frequency fitted from a Lorentzian. Modulation amplitude is 5 kHz and the rf frequency difference between tweezer tones is 800 kHz (corresponding to 5λ in spacing).

2.3.2 Tweezer trap frequency measurement

We took two different approaches to measuring the tweezer radial trap frequency. The first was to do a parametric heating measurement [79], where we modulated the tweezer positions at various frequencies and observed atom loss through fluorescence imaging (Fig. 2.7). Thus, we obtain curves of atom survival as a function of modulation frequency, and fit these to Lorentzians to extract the center frequency. Doing so, we observe nonlinearity in the trap frequencies, with frequencies of ~ 70 kHz near the center of the array and frequencies of ~ 60 kHz near the edges of the array.

The second approach we took to measuring the trap frequency came about when we were taking data on the optomechanical Dicke phase transition (see Ch. 4). We saw peaks emerge at the trap frequency in our heterodyne spectra. As we increased the pump laser power, the traps softened and the peaks moved to lower frequency (Fig. 2.8(a)). Knowing our pump

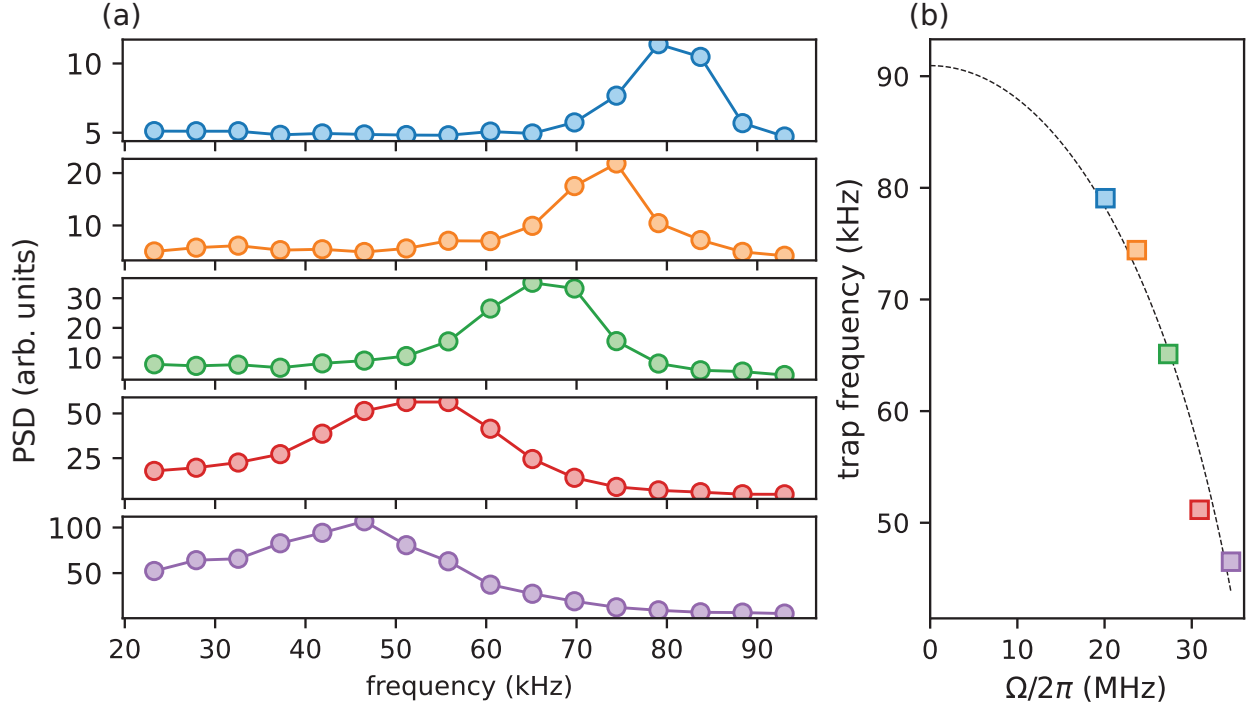


Figure 2.8: Tweezer radial trap frequency measurement via cavity fluorescence spectra. (a) We observe the softening of the trapping potential experienced by the atoms as we increase the pump strength Ω . (b) We extract the radial trap frequency ω_z at $\Omega = 0$ through a fit to the equation $\omega_{\text{eff}} = \sqrt{\omega_z^2 - a\Omega^2}$ (dashed line). We extract $\omega_z = 2\pi \times 91$ kHz. Data were taken with atom number $N = 20$ and $\Delta_{\text{pc}} = -2\pi \times 1.9$ MHz.

Rabi frequency Ω and having a model for the softened trap frequency's dependence on Ω , we fitted the data at different Ω to our model and extracted the trap frequency ω_z at $\Omega = 0$ (Fig. 2.8(b)). This revealed a trap frequency near 90 kHz. This frequency is notably higher than the ~ 70 kHz measured via parametric heating. On one hand, this is physical—the parametric heating measurement relies on atoms sampling the anharmonic portions of the tweezer traps as they escape the tweezers. This effect would cause the parametric heating measurement to suggest a trap frequency that is lower than the true trap frequency. It is also possible that we were using a higher tweezer trap depth at the time of taking the data shown in Fig. 2.8, although this was not carefully calibrated.

The measurement from our heterodyne data incidentally also gave us a measurement of the axial trap frequency. We suspect that our tweezer axis is not perfectly transverse to the cavity: as the atoms move up and down along the tweezer axis, they simultaneously move along the cavity axis. We believe that it is because of this nonzero projection onto the cavity axis that we are able to observe tweezer-axial motion in our heterodyne spectra. In our heterodyne spectra, we not only see a peak at the softened, radial trap frequency, but

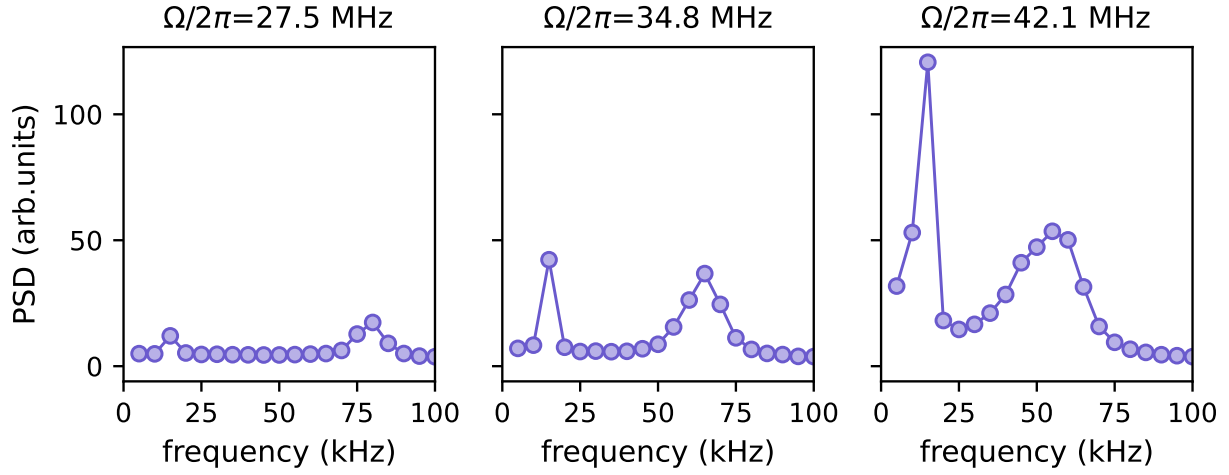


Figure 2.9: Tweezer axial trap frequency measurement via cavity fluorescence spectra. We analyze the FFT of our cavity heterodyne data and observe two peaks, corresponding to the tweezer axial and radial frequencies. We suspect that the tweezer axis is not perfectly transverse to the cavity axis, causing axial motion to have a projection onto the cavity axis. While the radial frequency is softened at higher pump strength Ω , the axial frequency remains constant at ~ 15 kHz. Data shown are taken with $N = 12$, $\Delta_{pc} = -2\pi \times 1.9$ MHz.

also a lower-frequency peak at ~ 15 kHz whose frequency does not appear to depend on the pump Rabi frequency Ω (Fig. 2.9). We interpret this ~ 15 kHz peak as corresponding to the axial trap frequency.

2.4 Atom rearrangement

One of the first major tasks I undertook as a graduate student on E6 was to figure out how to rearrange tweezers to create defect-free atom arrays. Due to stochastic loading of atoms into the tweezer traps, there is a $\sim 50\%$ probability of each tweezer site not being filled by an atom. At first glance, it may seem that stochastic loading is not an issue for E6's experiments because of the periodic nature of the cavity field. However, we actually want our atoms to be centered in the tweezer array due to the aberrations towards the edges of the array, as described in Sec. 2.3.1. We also became interested in atom-number-resolved data, so deterministic atom loading also became a must. To create defect-free arrays, we must determine which tweezer sites are not filled (which can be achieved through imaging of the atoms), and then move the atoms around until they occupy the desired configuration of tweezer sites.

Thankfully, sorting atoms into defect-free arrays in 1D is a relatively simple problem

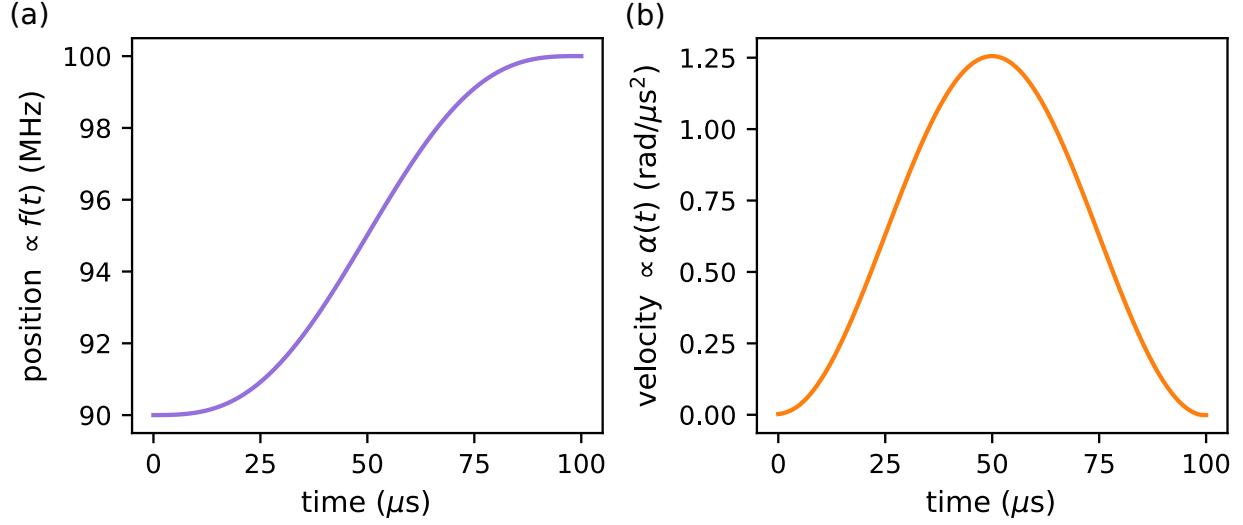


Figure 2.10: Tweezer sweep waveform example. The (a) frequency $f(t) = \frac{1}{2\pi} \frac{d\phi}{dt}$ and (b) angular acceleration $\alpha(t)$ of a sweep waveform that transports a tweezer from $f_A = 90$ MHz to $f_B = 100$ MHz in $100 \mu\text{s}$.

that others have figured out long before us. The algorithm E6 employs is based on one from the group of Manuel Endres, found in Ivaylo Madjarov’s thesis [80]. It takes advantage of the ability of the Spectrum M4i6631-x8 AWG card’s “sequence replay mode,” in which it stores waveforms in memory, and then plays them in a specified order. The algorithm involves identifying the location of a defect, say at site j , extinguishing that tweezer site and creating a new tweezer on one edge of the array, then moving all the sites on one side of j over by one site to fill the missing site: $\{1, \dots, j-1\} \rightarrow \{2, \dots, j\}$ (“left sorting”) or $\{j+1, \dots, N\} \rightarrow \{j, \dots, N-1\}$ (“right sorting”), with left and right referring to which edge of the array we re-illuminate tweezers on. Repeating this procedure for all empty sites results in a defect-free array. In E6, we create defect-free arrays of atoms in the center tweezers, which we achieve by doing an equal number of left and right sorting movements. Since the number of sorting waveforms required is linear in N , it is feasible to pre-compute the waveforms and save them to the AWG memory.

2.4.1 Sweep waveforms

We move tweezers by driving the AOD with sweep waveforms, where the frequency of the rf tones is chirped. The frequency sweeps translate to steering of the diffracted beams. Because the position of a tweezer j is proportional to the rf frequency f_j , one can write a time-dependent $f_j(t)$ that moves the tweezer in the desired trajectory. The waveform $A(t)$

sent to the AOD is then given by

$$A(t) = \sum_j A_j \sin(\phi_j(t)), \text{ where} \quad (2.1)$$

$$\phi_j(t) = \phi_j(0) + 2\pi \int_0^t f_j(t') dt'. \quad (2.2)$$

In E6, we usually just want to move a tweezer from f_A to f_B as smoothly as possible to avoid atom loss. We choose a sinusoidal velocity trajectory (Fig. 2.10), such that all time derivatives are continuous.

A sweep waveform must meet three conditions: 1) it must move from f_A to f_B in time T , 2) it must begin with phase ϕ_A , and 3) it must end with phase ϕ_B . However, when the total time T is fixed, there are not enough degrees of freedom to ensure both phase conditions are met. Therefore, we need to add an additional phase modulation to satisfy both phase conditions.

The sweep has the following velocity profile $\alpha(t)$, which satisfies $\alpha(0) = 0$ and $\alpha(T) = 0$:

$$\alpha(t) = \alpha_0(1 - \cos(2\pi t/T)). \quad (2.3)$$

Satisfying condition 1) requires

$$\int_0^T \alpha(t) dt = 2\pi(f_B - f_A) \quad (2.4)$$

$$\Rightarrow \alpha_0 = \frac{2\pi(f_B - f_A)}{T}, \text{ and} \quad (2.5)$$

$$2\pi f(t) = 2\pi f_A + 2\pi \frac{f_B - f_A}{T} \left(t - \frac{T}{2\pi} \sin\left(\frac{2\pi t}{T}\right) \right). \quad (2.6)$$

We may now integrate Eq. (2.6) to obtain the phase function $\tilde{\phi}(t)$:

$$\tilde{\phi}(t) = \phi_0 + 2\pi f_A t + 2\pi \frac{f_B - f_A}{T} \left(\frac{t^2}{2} + \left(\frac{T}{2\pi}\right)^2 \cos\left(\frac{2\pi t}{T}\right) \right). \quad (2.7)$$

Meeting condition 2) requires $\tilde{\phi}(0) = \phi_A$ so that $\phi_0 = \phi_A - (f_B - f_A)T/(2\pi)$. Now we just have to meet condition 3). There are different ways of modifying $\tilde{\phi}(t)$ to ensure that the final phase is ϕ_B . For instance, one could relax the constraint on the time T and choose the sweep time to be some time T' that is slightly shorter than T , then follow the sweep with a static waveform; in this case, T' would be chosen to ensure that $2\pi f_B(T - T')$ gives the appropriate phase shift such that $\phi(T) = \phi_B$. However, we opted for a different approach. Keeping the time T fixed, we add on an additional phase modulation term $\phi_1(t)$ and define our total phase function as $\phi(t) = \tilde{\phi}(t) + \phi_1(t)$. $\phi_1(t)$ must satisfy $\frac{d\phi_1}{dt}(0) = 0$, $\frac{d\phi_1}{dt}(T) = 0$, $\phi_1(0) = 0$ and $\phi_1(T) = (\phi_B - \tilde{\phi}(T)) \bmod 2\pi = (\phi_B - \phi_A - \pi(f_B + f_A)T) \bmod 2\pi \equiv \Delta\phi$. There are many functional forms one can pick for $\phi_1(t)$; for simplicity, we choose a cubic

polynomial. We solve for the coefficients which satisfy all four boundary conditions, and arrive at

$$\phi_1(t) = \frac{\Delta\phi}{T^3}(3T - 2t)t^2. \quad (2.8)$$

Putting it altogether, we obtain

$$\phi(t) = \phi_A - \frac{(f_B - f_A)T}{2\pi} + 2\pi f_A t + 2\pi \frac{f_B - f_A}{T} \left(\frac{t^2}{2} + \left(\frac{T}{2\pi} \right)^2 \cos\left(\frac{2\pi t}{T}\right) \right) + \frac{\Delta\phi}{T^3}(3T - 2t)t^2. \quad (2.9)$$

An example of the frequency and angular acceleration corresponding to the phase $\phi(t)$ given in Eq. (2.9) is shown in Fig. 2.10.

To sweep multiple tweezers at a time, one simply creates a superposition waveform Eq. (2.1) with each $\phi_j(t)$ having the functional form of $\phi(t)$; all that is left is to specify $f_{A,j}$ and $f_{B,j}$ for each tweezer of index j . We typically choose the amplitudes A_j to be the same for all tweezers. The specific value of A_j is chosen by maximizing the output voltage of the AWG for the given set of sweep waveforms.

2.4.2 Sorting implementation

I now describe the details of how we implement atom sorting in E6.

We first generate all the sorting waveforms (a total of $2(N - 1)$, including both left- and right-sorting) and write them to the AWG's memory, along with the "static" waveform. As its name suggests, the static waveform is just a stationary superposition of all tweezer tones, and is the default configuration of the array. After loading atoms into the tweezers, we take a fluorescence image. This image gets saved to a folder called "fluo_images_delete" on the Desktop of one of E6's main computers, CavityQED-PC. This folder is constantly monitored using Python's Watchdog library. The Watchdog "observer" is set to be triggered on file-creation events. When a new fluorescence image gets created, Watchdog runs an image analysis script that determines which tweezer sites are unoccupied. The index of each unoccupied site maps to the segment of the AWG's memory containing the sweep waveform that extinguishes that tweezer site. Half of the empty sites are assigned to left-sorting waveforms and the other half are assigned to right-sorting waveforms in order to center the atoms within the tweezer array (note that if there are an odd number of empty sites, then the atoms will be offset from the center by one site). We then write a series of steps to the AWG that tells it which sweep waveforms to play and in what order, and then tells it to return to the static waveform once all the sweeps are completed. Upon receiving an external trigger, the AWG executes the sequence of steps. We then confirm the success of the sorting by taking another fluorescence image once the sorting has been completed.

The sorting process is not perfect, as some atoms inevitably get lost as the tweezers move. We thus implement a second round of sorting after the second fluorescence image is taken to further reduce the number of defects. Recently, we have also started doing multiple rounds of data-taking within a single iteration of the Cicero sequence, and found that doing

atom sorting in between these rounds improves our statistics. To achieve a deterministic atom number $N < N_{\text{twz}}$ (N_{twz} being the number of tweezer sites), following sorting we have the AWG play a “drop” waveform, which simply extinguishes all tweezers except for the N tweezers in the center of the array.

The image analysis, rearrangement and dropping steps are all written into the Watchdog event handler, which instructs the program on what to do when an event occurs (in this case, the event is the creation of a fluorescence image file). The code for for this event handler is provided in Appendix C.

These steps form the base of E6’s tweezer capabilities. After the dropping step, we have created a deterministically loaded array of N atoms. We can then reconfigure these N atoms in whatever way we want. For instance, we may wish to switch between configurations of different tweezer spacings or positions (see Ch. 4). In recent experiments on mid-circuit measurement, we dropped to five centered tweezers and then swept these five tweezers further apart to minimize the local 780 nm and 1529 nm wavelength beams hitting multiple atoms at once.

2.5 Science cavity

The E6 science cavity was designed with three major criteria in mind: strong coupling, high output-coupling efficiency, and large transverse NA for optical access. To achieve high cooperativity with large transverse NA, the science cavity was made to be near-concentric. This reduces the cavity mode volume by creating a small waist, even though the cavity mirrors are spaced far enough apart for optical access. To enable high output-coupling efficiency, the transmissivities of the mirrors T_1 and T_2 are chosen to be asymmetric. Some cavity parameters of interest are provided in Table 2.1. For more specific details about our cavity design choices, I refer the reader to Justin Gerber’s thesis [75].

2.5.1 Locking the cavity

As mentioned in Sec. 2.2.4, laser light at wavelengths of 780 nm and 1560 nm are delivered to the science cavity. The 1560 nm wavelength light is currently only used for locking the length of our science cavity. At one point, before I joined the lab, it was intended to also be used as a lattice for trapping atoms within the cavity (hence you will find many stickers labeled with “cavity ODT” around the lab), with the advantage of being commensurate with the 780 nm wavelength probe light. However, we soon found that 1560 nm is not a good trapping wavelength for ^{87}Rb because of its proximity to the 1529 nm wavelength $5\text{P}_{3/2} \rightarrow 4\text{D}_{5/2}$ transition. Now, we lock the cavity to a 1560 nm wavelength TEM_{01} mode; this way, the atom array can be placed in the dark plane of the TEM_{01} mode and avoid being exposed to 1560 nm wavelength light. The length of the cavity is controlled using shear stack piezos (Noliac NAC2402-H2.3), one of which is placed beneath each mirror. Driving a voltage difference between the inner and outer plates of the shear piezo stack

Parameter	Definition	Value
κ	Cavity half-linewidth	$2\pi \times 0.55$ MHz
γ	Atomic half-linewidth	$2\pi \times 3$ MHz
g_0	Vacuum Rabi frequency on cycling transition	$2\pi \times 3.1$ MHz
C	Cooperativity, $C = g_0^2/(2\kappa\gamma)$	3.0
f_{FSR}	Free spectral range	15.9 GHz
$\mathcal{F}$	Finesse, $\mathcal{F} = 2\pi \times f_{\text{FSR}}/(2\kappa) = \frac{\pi(R_1 R_2)^{1/4}}{1 - \sqrt{R_1 R_2}}$	14700
L	Cavity length	9.4 mm
$\mathcal{R}$	Mirror radius of curvature	5 mm
w_0	TEM ₀₀ mode waist radius	17 μm
T_1	Input mirror transmissivity	3.5 ppm
T_2	Output mirror transmissivity	247 ppm
L_{tot}	Total mirror losses	177 ppm

Table 2.1: Parameters of the cavity QED system.

creates a shearing motion. Using the reflection of the 1560 nm wavelength light off of the cavity as our signal, we implement a PDH locking scheme that feeds back to the piezo voltage to stabilize the cavity length. In the past, we have also used the 1560 nm wavelength cavity transmission for the PDH lock; however, we found that using the reflection is more convenient, as we somewhat regularly end up blocking parts of the cavity transmission path for diagnostic purposes. The cavity can be locked by driving both piezos simultaneously or just one of them. Prior to 2025, we only drove one piezo, but started to drive both when we started trying to control the piezo-induced electric field at the atoms. As a fun fact, prior to 2025, we had accidentally always driven the outer plates of the piezo (with the inner plate grounded), rather than having the inner plate driven and the outer plates grounded (the design intention). Clearly, both ways work, but we are now sticking to the intended operation.

We set the frequency of the 1560 nm wavelength light so that the science cavity is simultaneously resonant with a 1560 nm wavelength TEM₀₁ mode and a 780 nm wavelength TEM₀₀ mode. This involves tuning the EO side band frequency for the 1560 nm wavelength light until we see the resonances of a 1560 nm wavelength TEM₀₁ mode and a 780 nm wavelength TEM₀₀ mode overlap when we scan the cavity length.

We observe the 780 nm wavelength and 1560 nm wavelength cavity resonances drift apart over time when we run our experimental sequence. We suspect that this is due to thermal cycling caused by our transport laser, which delivers 20 W of 1064 nm wavelength light into the science chamber. Although the the E6 science cavity was designed with shielding mirrors to deflect the 1064 nm wavelength light away from the cavity, it seems that some light still impinges on the cavity, causing the probe-to-cavity detuning (Δ_{pc}) to drift. The

amount of drift depends on the day and is likely coupled to the temperature stability in the room. We previously saw drifts of ~ 0.5 MHz in an hour. For about 2 years, we simply monitored the drift over the course of our data-taking runs by recording a sweep of the probe frequency across cavity resonance at the end of each iteration of the sequence, and fitting the center frequency. We then post-selected our data on the shots where the frequency drift was minimal. This passive monitoring was ultimately replaced by active feedback a Red Pitaya FPGA combined with an adder circuit, designed by Leon Lu. The Red Pitaya feedback previously adjusted the probe frequency to maintain our desired Δ_{pc} . In early 2026, we switched to adjusting the frequency of the 1560 nm wavelength light instead. Adjusting the frequency of the 1560 nm wavelength light is preferable because we want the probe to be at a fixed detuning from the atomic resonance, Δ_{pa} . When $|\Delta_{pa}| \gg \Gamma$, we can tolerate feedback to the probe frequency because the fractional change in the detuning is small. However, when working at $\Delta_{pa} \sim \Gamma$, the probe frequency varying at the MHz-level becomes problematic. We hesitated to switch to feeding back to the 1560 nm wavelength light's frequency because its optical path has a single-pass AOM, so adjusting the rf frequency to the AOM would alter the fiber coupling efficiency. However, this turned out to work in the end because we are able to lock the cavity at relatively low 1560 nm wavelength light power, and intensity stabilization of the 1560 nm wavelength light increases our locking range to ~ 14 MHz.

2.6 Cavity detection

In E6, we utilize two modalities of cavity photon detection: single photon counting via a single photon counting module (SPCM) and heterodyne detection. SPCM measurement relays information about the intensity of the cavity light but relays no phase information, while heterodyne detection yields a measurement of the cavity's electric field (both amplitude and phase) at the expense of its detection efficiency being halved. Depending on our application, we can switch between these two detection modalities. A recent upgrade, required for our work on the spins-version of the Dicke phase transition, enables switching between SPCM and heterodyne detection in-sequence via a motorized flip mirror that directs cavity and LO light into either path (see Fig. 2.12).

2.6.1 Single photon counting

Many details about our SPCM setup can be found in Emma Deist's thesis [76]. The SPCM detection path has since evolved to include two foam board "black boxes" rather than one; the two boxes detect orthogonal cavity polarizations. Note that the two SPCMs have slightly different specifications. Our newer SPCM (SPCM-AQRH-46) differs from our older SPCM (SPCM-AQR-13) in that it has a lower dark count rate (25 cts/second vs. 250 cts/second), a lower dead time (22 ns vs. 50 ns), and a higher output count rate before saturation (37 Mc/s vs. 15 Mc/s). Both SPCMs have a photon detection efficiency of $\sim 65\%$ at a wavelength of 780 nm. Each SPCM has a 780 nm wavelength line filter and lens on a lens tube that is

mounted directly onto the SPCM. It is necessary to focus the beam of light onto the SPCM sensor, since the sensor size is only 170–180 μm .

The SPCM is gated by a 5 V TTL pulse. When the SPCM detects a photon, it outputs a TTL pulse, whose width is 35 ns for the old SPCM and 10 ns for the new SPCM. The timestamps at which these pulses occur are recorded by an Opal Kelly FPGA (XEM6001), which we have programmed using code generously shared with us by Jon Simon’s group. This solution is a cost-effective and memory-efficient alternative to a recording device such as Gagescope, which we use for heterodyne detection. A caveat to the use of the Opal Kelly FPGA is that structures in the SPCM’s output pulses could be misinterpreted as multiple photons; we have seen this to be the case when the output is improperly terminated. A solution to this issue could be to have the SPCM output pulses trigger a pulse generator, which outputs clean, square pulses. However, for now we have opted to gate out these events in our data analysis by interpreting two counts time-separated by less than the SPCM dead time as a single count.

Detection efficiency

At the time of Ref. [42], the efficiency of detecting a cavity photon using our SPCM was $\sim 25\%$, which has contributions from transmission at the cavity mirrors (58%), efficiency of the optical path (81%), fiber coupling efficiency (80%), and SPCM photon detection efficiency ($\sim 65\%$). Since our rebuild of the heterodyne detection system (see Sec. 2.6.2), however, our detection efficiency has dropped to $\sim 16\%$ due to increased loss on new optics elements placed in the detection path.

2.6.2 Heterodyne receiver

To measure both the amplitude and phase of the cavity field, we set up a heterodyne detection path. This setup is different from the one described in Justin Gerber’s thesis; since we pump the atoms transverse to the cavity, we do not need or want to send local oscillator (LO) light through the science chamber. The setup that we built incorporates the ability to switch between detection via heterodyne or SPCM by flipping a flip mirror that separates the two paths. Both the balanced photodiode and the SPCM are housed in a black box to reduce stray photon counts. A later generation of the heterodyne system allows for simultaneous detection of both cavity field polarizations, and is shown in Fig. 2.11.

A simplified schematic of the heterodyne path at the time of Ref. [55] is schematically shown in Fig. 2.12. Here, we also illustrate the setup of the balanced photodiode and SPCM inside the black box. It involves interfering the leaked cavity electric field with a local oscillator (LO) field that is frequency-offset from the cavity field. The interference contains a beat note at the difference frequency, which is read on a balanced photodetector. By demodulating the measured signal at the difference frequency, one can extract the amplitude and phase of the cavity electric field. In Ref. [55], the cavity field takes on the frequency ω_p of the pump field, and the LO is detuned from the pump by $\Delta\omega = \omega_p - \omega_{\text{LO}} = 2\pi \times 20 \text{ MHz}$.

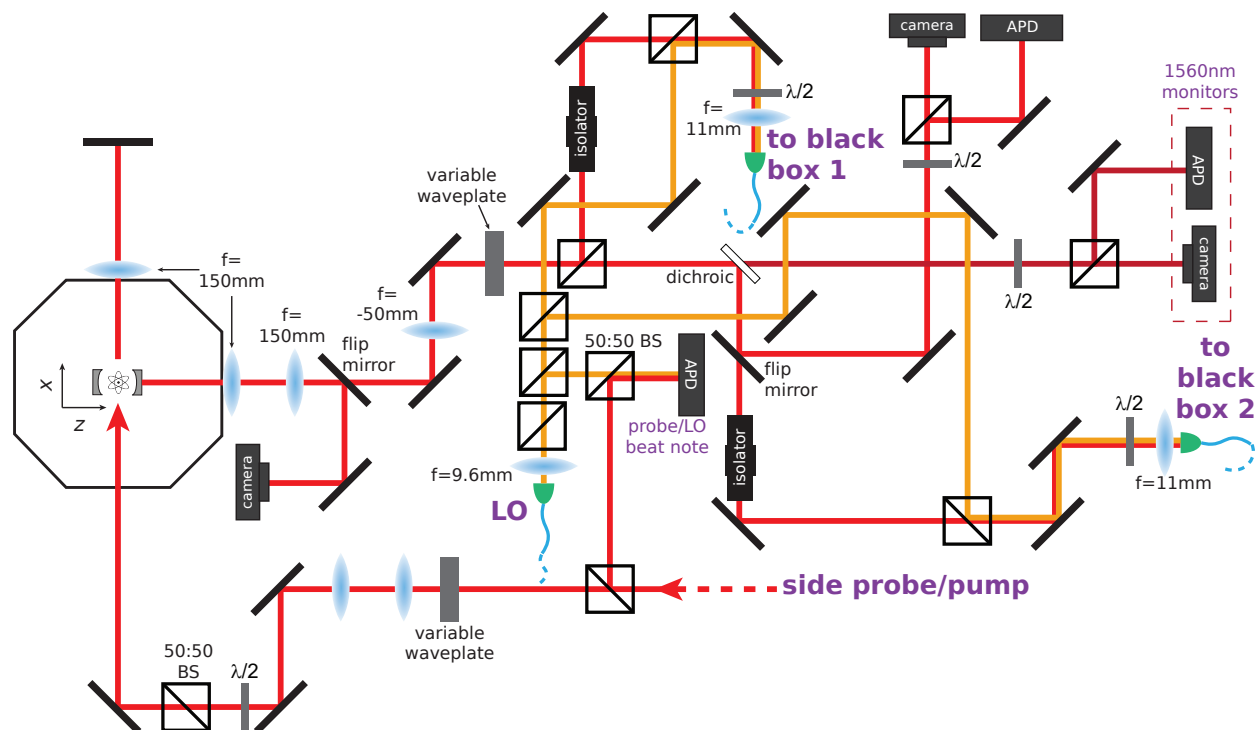


Figure 2.11: Cavity detection path optics layout. Light leaking out of the cavity is split by polarization and directed into two “black boxes.” The cavity light is also combined with LO light for heterodyne detection. An APD monitors the relative phase between the LO and the side probe/pump; we pick off some of each beam and interfere them to create an optical beat note. We also monitor the cavity transmission of the 1560 nm wavelength light used for locking the cavity length. All beamsplitter cubes are PBSs unless otherwise noted.

The LO is chosen to be red-detuned from the pump so as to be farther detuned from atomic resonance. This is taken into account because even though LO light never hits the atoms directly, it can enter the cavity through back-reflections on the cavity detection optics path. We have indeed seen that this LO reflection onto the atoms does occur, and so it is necessary to place optical isolators in the cavity detection path to attenuate any LO back reflection. We have measured an isolation of ~ 26 dB for one of the isolators in our detection path.

The frequencies of the pump and LO fields are set by the rf drive to an AOM in each optical path. The rf drives were originally phase-locked to each other using a circuit designed by Emma Deist. This involved using a phase-frequency detector (Hittite HMC439QS16G) and difference circuit that takes in a signal and reference. The signal was the beat note of the rf drives for the pump and LO, obtained by mixing the coupled outputs from their respective voltage-controlled oscillators (VCOs) and applying a low-pass filter to isolate the difference-frequency tone at $2\pi \times 10$ MHz. Note that because the pump and LO optical paths

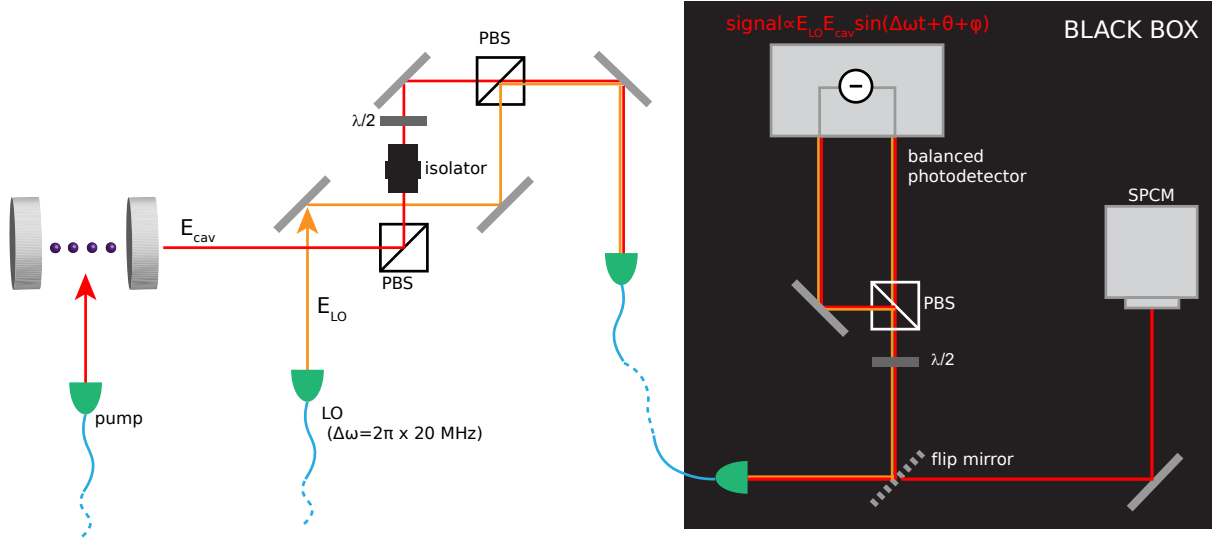


Figure 2.12: Heterodyne and black box schematic. On the left is a simplified schematic of the cavity heterodyne detection path at the time of Ref. [55] (see Fig. 2.11 for a more recent, complete schematic). Cavity and LO light are directed into a fiber, which takes the beams into a “black box” containing a balanced photodiode for heterodyne detection and an SPCM for photon counting. We switch between heterodyne detection and photon counting using a flip mirror. The cavity and LO lights interfere on a PBS inside the black box to generate a heterodyne signal.

utilize double-pass AOM setups, the frequency difference on the optical signals is double that of the rf. The 10 MHz reference signal was generated by a Red Pitaya FPGA. The circuit generated an error signal that was input to an analog proportional integral controller (a design by Jiraphat Tiamsuphat at the University of Maryland, Joint Quantum Institute), which zeros out the error signal by feeding back to voltage control of the LO VCO. However, we found this phase lock to not be very stable; we would often need to reboot the phase-frequency detector and tweak the lockbox parameters. Example heterodyne power spectra from our optomechanical Dicke phase transition experiments are shown in Fig. 2.13. After completion of our optomechanical Dicke phase transition work, we opted to replace the active probe-LO phase lock with a passive lock, where two frequency-offset outputs from the same digital frequency synthesizer are mixed with the same rf tone (see Fig. 2.14).

Heterodyne setup design considerations

Our heterodyne detection setup has gone through many iterations. Here, I discuss some of our original ideas, their drawbacks, and the reasons why we ultimately settled on the design we have now.

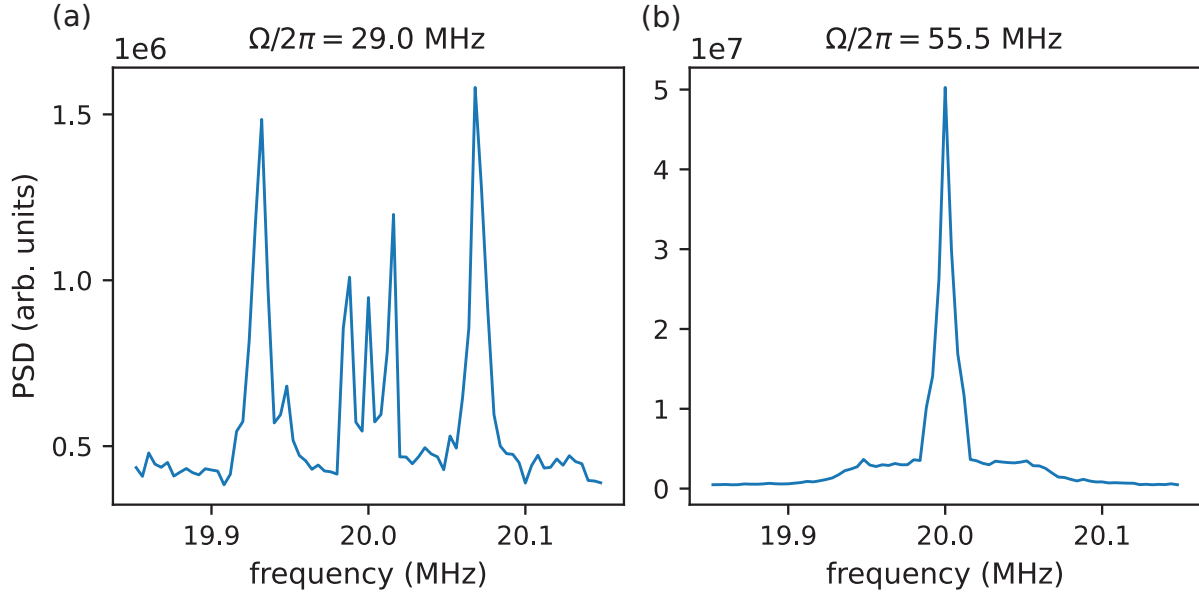


Figure 2.13: Heterodyne power spectral density data from the optomechanical Dicke phase transition experiment taken at a pump strength (a) below Ω_c , and (b) above Ω_c . In (a), one clearly sees the motional sidebands corresponding to the tweezer radial and axial trap frequencies, whereas in (b) the signal is dominated by spontaneous symmetry breaking.

In our original heterodyne design, the LO light was picked off from the side probe/pump *on the science table*. The LO light was then sent through a 40 MHz AOM driven at 30 MHz, to provide a frequency offset, before being combined with the cavity light on a PBS. The reasoning for having the LO light derive from the pump light on the science table was to minimize relative phase drifts between the pump and LO. By having them be delivered through the same fiber as well as sharing much of the same optical path, we reasoned that much of the phase noise and mechanical drifts would be common-mode.

There were a few issues with this setup, however. A minor inconvenience was that the 30 MHz heterodyne frequency necessitated a higher Gagescope sampling rate (we did not have any AOMs that worked at lower rf frequencies). Because we were already on the edge of the AOM's operating rf frequency range, the coupling efficiency was also unideal. The main issue was that we could not sufficiently block the 0th order diffraction from the AOM, so our LO light always carried some modulation at 30 MHz. This drowned out the cavity signal at low light levels. Nonetheless, for moderate light levels, the detection worked as expected, which we tested by sending transmission probe light through the cavity while sweeping its frequency; we could resolve the lineshape and phase shift as probe the frequency crossed cavity resonance (Fig. 2.15). A solution to the residual modulation issue was to use two AOMs, one right after the other, and have the heterodyne frequency be the difference of the two AOM rf drive frequencies. However, with limited space on the optical table, alignment

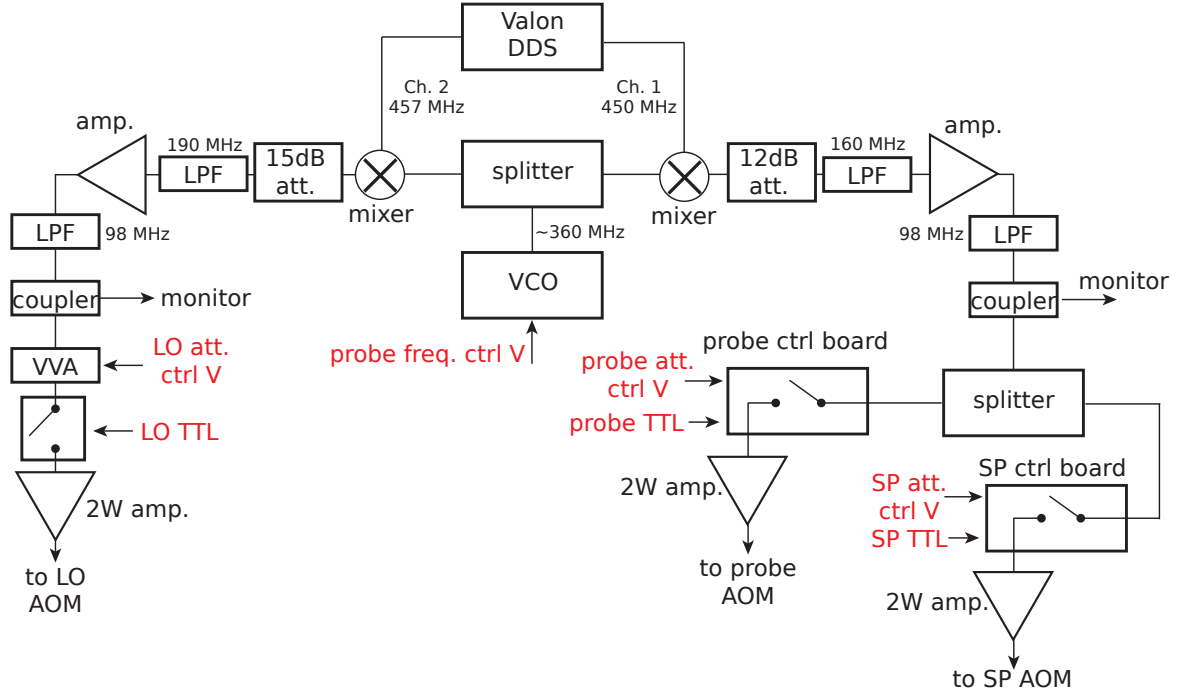


Figure 2.14: The probe and LO rf circuits at the time of the writing of this thesis. Cicero analog and digital controls are shown in red text. Note that we currently operate with a heterodyne frequency of 14 MHz, while in Ref. [55] we operated at 20 MHz.

of the laser into each AOM proved to be difficult. We also found that this setup led to the creation of additional harmonics. Ultimately, we decided to circumvent these issues by putting the LO on a separate double-pass AOM from the probe on the laser table (see Fig. 2.2) and removing the AOM(s) on the science table.

Another choice we made with our heterodyne design was to couple the cavity and LO lights into the same fiber, before having them be sent to the PBS and balanced photodetector for heterodyne detection. Although the fiber introduces unwanted phase shifts (see the following sections), the fiber acts as a spatial mode filter, ensuring perfect mode-matching between the cavity and LO beams—an important consideration for maximizing the heterodyne signal.

Heterodyne theory

I shall review the theory of heterodyne detection. As represented in Fig. 2.12, the cavity field $E_{\text{cav}} \cos(\omega_p t)$ and LO field $E_{\text{LO}} \cos(\omega_{\text{LO}} t + \phi)$ are combined on a polarizing beamsplitter (PBS). The phase ϕ represents the relative phase between the cavity and LO fields, which changes over time due to temperature-induced drifts in fiber and optical path lengths. Defin-

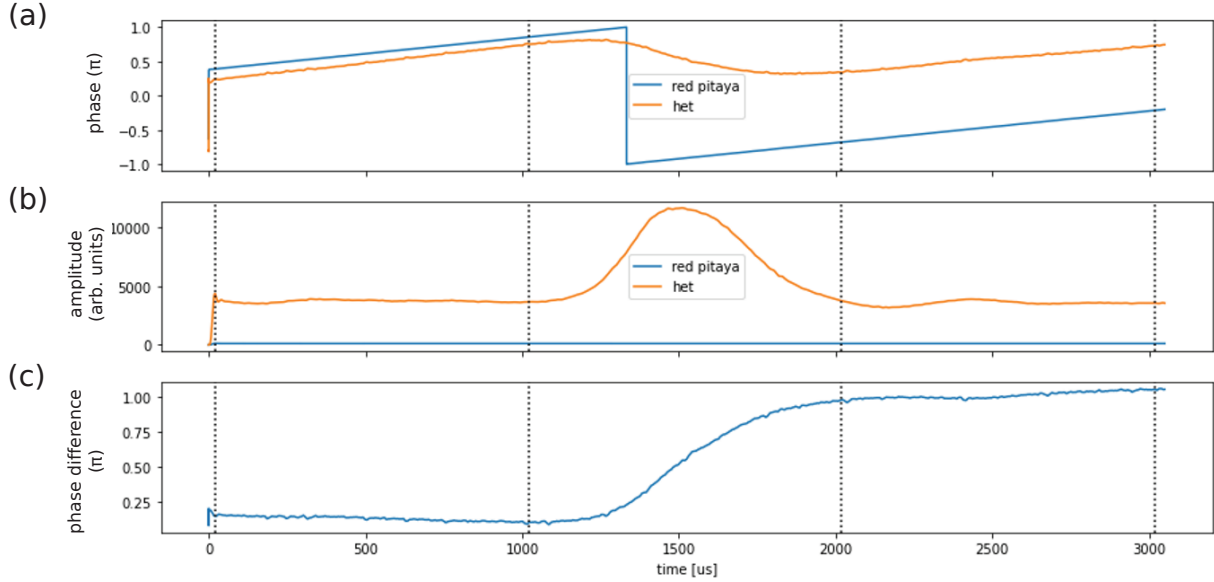


Figure 2.15: Early heterodyne test, July 2023. We sent transmission probe light into the cavity, swept the probe frequency across cavity resonance, and recorded the heterodyne signal from this sweep. Also shown is the amplitude and phase of the 10 MHz rf drive (generated by a Red Pitaya), to which the probe and LO were phase-locked. This is just a screenshot because the raw data were deleted to free up space on our server!

ing the horizontal and vertical axes of the PBS as $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ respectively, the state of the field that is then coupled into a fiber is $E_{\text{cav}} \cos(\omega_{\text{p}}t) \hat{\mathbf{x}} + E_{\text{LO}} \cos(\omega_{\text{LO}}t + \phi) \hat{\mathbf{y}}$. Upon emerging from the fiber, a half-waveplate rotates the field by 45° , and the field is then split again on a second PBS. The fields emerging on the two output ports of the second PBS are given by

$$\frac{1}{\sqrt{2}}(E_{\text{cav}} \cos(\omega_{\text{p}}t) - E_{\text{LO}} \cos(\omega_{\text{LO}}t + \phi)) \hat{\mathbf{x}}, \quad (2.10)$$

$$\frac{1}{\sqrt{2}}(E_{\text{cav}} \cos(\omega_{\text{p}}t) + E_{\text{LO}} \cos(\omega_{\text{LO}}t + \phi)) \hat{\mathbf{y}}. \quad (2.11)$$

The light from each output port of the PBS is sent to the balanced photodetector, which subtracts the intensities of the two signals. The voltage output V of the balanced photodetector represents the product of the cavity and LO fields through

$$V \propto \frac{1}{2}(E_{\text{cav}} \cos(\omega_{\text{p}}t) + E_{\text{LO}} \cos(\omega_{\text{LO}}t + \phi))^2 - \frac{1}{2}(E_{\text{cav}} \cos(\omega_{\text{p}}t) - E_{\text{LO}} \cos(\omega_{\text{LO}}t + \phi))^2 \quad (2.12)$$

$$= 2E_{\text{cav}}E_{\text{LO}} \cos(\omega_{\text{p}}t) \cos(\omega_{\text{LO}}t + \phi) \quad (2.13)$$

$$= E_{\text{cav}}E_{\text{LO}} \cos(\Delta\omega t - \phi) + E_{\text{cav}}E_{\text{LO}} \cos((\omega_{\text{p}} + \omega_{\text{LO}})t + \phi). \quad (2.14)$$

V is recorded using “Gagescope” (GaGe CSE161G4-4GS CompuScope Digitizer), which samples at a rate of 200 megasamples/second. The first term in Eq. (2.14) is the signal of interest, which occurs at the beat frequency. The second term in Eq. (2.14) occurs at the sum frequency, which can be ignored by applying a low-pass filter. Taking the Fourier transform of V , we extract the amplitude (proportional to $E_{\text{cav}}E_{\text{LO}}$) and the phase ϕ .

A caveat to this detection scheme is that the phase we read out from V is sensitive to any relative phase drift between the $\hat{\mathbf{x}}$ - and $\hat{\mathbf{y}}$ -polarized fields in the fiber, which is polarization-maintaining. The $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ axes are aligned to the fast and slow axes of the fiber, which acts like a waveplate and puts a relative phase θ between the two axes. When we demodulate V at $\Delta\omega$, the phase we extract is not just ϕ , but rather $\phi + \theta$. Since θ drifts with thermal fluctuations in the lab, we had to develop a way to calibrate the phase drift using the atoms (see Ch. 4 for details).

Heterodyne system rebuild

The heterodyne setup described above served us well enough to complete the work in Ref. [55], but it had its limitations. The major limitation was the relative phase drift between the pump and LO, which constrained the amount of time we could take data for in between antinode reference frames. If this phase drifted by more than π , we would not be able to tell in our data analysis. When we rebuilt the heterodyne path in late 2024, we implemented two improvements.

The first improvement was to install a 50:50 beamsplitter that interferes pump and LO light on the science table. We monitor the optical beat note on a photodiode whose signal is also recorded by Gagescope. The phase of this beat note measures the path length fluctuations in the optical fibers that relay light from the laser table to the science table. We could then subtract the measured phase of the optical beat note from our data. This greatly improved the phase stability of our heterodyne detection.

The second improvement was to also eliminate phase fluctuations from the fiber that takes the cavity and LO light into the dark box. This simply involved removing the half-waveplate from the dark box and moving it to be before the fiber. We can consider what this does to the optical field.

As before, the field before the half-waveplate is $E_{\text{cav}} \cos(\omega_{\text{p}}t)\hat{\mathbf{x}} + E_{\text{LO}} \cos(\omega_{\text{LO}}t + \phi)\hat{\mathbf{y}}$. The half-waveplate then rotates this field by 45° to obtain $((E_{\text{cav}} \cos(\omega_{\text{p}}t) - E_{\text{LO}} \cos(\omega_{\text{LO}}t + \phi))\hat{\mathbf{x}} + (E_{\text{cav}} \cos(\omega_{\text{p}}t) + E_{\text{LO}} \cos(\omega_{\text{LO}}t + \phi))\hat{\mathbf{y}})/\sqrt{2}$. This is the field that enters the fiber. Upon emerging from the fiber, the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ components pick up an indeterministic relative phase θ . The fields on the output ports of the second PBS then become

$$\frac{1}{\sqrt{2}}(E_{\text{cav}} \cos(\omega_{\text{p}}t) - E_{\text{LO}} \cos(\omega_{\text{LO}}t + \phi))\hat{\mathbf{x}}, \quad (2.15)$$

$$\frac{1}{\sqrt{2}}(E_{\text{cav}} \cos(\omega_{\text{p}}t + \theta) + E_{\text{LO}} \cos(\omega_{\text{LO}}t + \phi + \theta))\hat{\mathbf{y}}. \quad (2.16)$$

After subtracting the intensities of the two ports, the output signal of the balanced photodetector is

$$V \propto \frac{1}{2}(E_{\text{cav}} \cos(\omega_{\text{p}}t + \theta) + E_{\text{LO}} \cos(\omega_{\text{LO}}t + \phi + \theta))^2 - \frac{1}{2}(E_{\text{cav}} \cos(\omega_{\text{p}}t) - E_{\text{LO}} \cos(\omega_{\text{LO}}t + \phi))^2 \quad (2.17)$$

$$= \frac{1}{2}E_{\text{cav}}^2(\cos^2(\omega_{\text{p}}t + \theta) - \cos^2(\omega_{\text{p}}t)) + \frac{1}{2}E_{\text{LO}}^2(\cos^2(\omega_{\text{LO}}t + \phi + \theta) - \cos^2(\omega_{\text{LO}}t + \phi)) \quad (2.18)$$

$$+ E_{\text{cav}}E_{\text{LO}}(\cos(\omega_{\text{p}}t) \cos(\omega_{\text{LO}}t + \phi) + \cos(\omega_{\text{p}}t + \theta) \cos(\omega_{\text{LO}}t + \phi + \theta))$$

$$= -\frac{1}{2}(E_{\text{cav}}^2 \sin(\theta) \sin(2\omega_{\text{p}}t + \theta) + E_{\text{LO}}^2 \sin(\theta) \sin(2\omega_{\text{LO}}t + 2\phi + \theta)) \quad (2.19)$$

$$+ E_{\text{cav}}E_{\text{LO}} \left[\cos(\Delta\omega t - \phi) + \frac{1}{2} \cos(\omega_{\text{p}}t + \omega_{\text{LO}}t + \phi) + \frac{1}{2} \cos(\omega_{\text{p}}t + \omega_{\text{LO}}t + \phi + 2\theta) \right]$$

After applying a low-pass filter to get rid of the terms at $\{2\omega_{\text{p}}, 2\omega_{\text{LO}}, \omega_{\text{p}} + \omega_{\text{LO}}\}$, we see that the only term that survives in Eq. (2.19) is $E_{\text{cav}}E_{\text{LO}} \cos(\Delta\omega t - \phi)$, which has no dependence on θ . The phase drift from the fiber affects both the cavity and LO fields equally and cancels out in the beat note.

The last upgrade we made to the heterodyne system was to implement polarization-resolved heterodyne detection. The cavity light is split on a PBS, and then each output port of the PBS is directed to its own heterodyne detector. The current setup now contains two black boxes, each housing a balanced photodetector and SPCM.

Gagescope

As mentioned earlier, we use a PC-based oscilloscope known as Gagescope for recording our heterodyne signal. The output voltage of the balanced photoreceiver (New Focus Model 1807) is first sent through a bias tee. We monitor the dc output of the bias tee on an oscilloscope, as this tells us about how well-balanced the LO signal is on the photodetector. The rf output is sent through a combination of rf filters as well as a low-noise amplifier before reaching Gagescope. More recently, we have added additional rf circuitry to down-sample our heterodyne beat note down to 3 MHz before it reaches Gagescope. This was done so that we can use a lower Gagescope sampling rate (20 MHz rather than 200 MHz), so as to reduce the size of the data files we save. This was necessary for our recent spins work because of how long-lived the self-organized state is; we now record cavity field data for tens (sometimes even hundreds) of milliseconds within a single iteration of our Cicero sequence, as opposed to the mere 250 μs used in Ref. [55].

Gagescope has the capability of recording signals on four channels simultaneously. In our optomechanics work, we only needed to monitor two channels: the heterodyne signal and the probe-LO rf beat note. We monitored the rf beat note so that any phase drifts in the rf could be subtracted off from our data. In our more recent work on the spins version of the Dicke model we use three channels: two record heterodyne signals for both cavity polarizations, and the third monitors the newly implemented pump-LO optical beat note (rather than the rf beat note).

Gagescope also has the ability to record multiple data segments of specified length within a single data file. This capability is particularly useful to us because we often take “reference frames” in addition to “data frames” for the purpose of using the atoms to measure the relative phase between the pump and LO. Previously, we would take a single data frame that included the reference data, and then gate it in software. However, this meant that most of the data we were saving was shot noise, because we needed to cool the atoms between the reference and data frames. Once we started implementing the multiple segment capability, we could just trigger Gagescope when we wished to take data segment. I found Gagescope’s Python software development kit to be very helpful when I was writing our data acquisition code. We had previously tried to use E3’s version of Gagescope acquisition code, but theirs turned out to be incompatible with newer versions of Gagescope.

Aside on SNR

The data in Ref. [55] were taken with an LO power of 0.57 mW. We later learned that the signal-to-noise ratio (SNR) is higher at higher LO powers. More recently, we have been operating at a power of 1–2 mW in each black box, which gives an SNR of roughly 12. At the time of writing of this thesis, we have not confirmed whether this SNR is limited by shot noise, though this could be easily checked by measuring the scaling of the SNR with cavity probe power. A caveat to working at higher LO powers is that one has to be careful to not saturate the difference signal on the balanced photodetectors. We believe the difference signal drifts due to the fibers not being perfectly polarization-maintaining. This is corrected for by feeding back to the angle of the half-waveplate before each fiber port. For a while, this feedback was done by hand, but Leon has recently installed Red Pitaya active feedback on motorized rotation mounts.

Chapter 3

Collective light scattering in atom arrays

In his seminal work, R.H. Dicke put forth the concepts of superradiance and subradiance [54]—that for N excited emitters confined to a space much smaller than the wavelength of light, the photons emitted by different emitters are indistinguishable, leading to interference in their emission processes. When the emitters are symmetrically coupled to the radiation field, the interference is constructive, leading to N^2 scaling of the emitted intensity. There also exist states where asymmetric couplings lead to destructive interference in the emission, causing the ensemble not to radiate at all (subradiance).

We set out to investigate Dicke’s N^2 -scaling prediction in ensembles comprised of few to many atoms. Though previous experiments observed collective effects in systems of two [81, 82, 83] to five [84] atoms, arbitrary control over the amplitude and phase of individual atoms was lacking in these studies. We demonstrated superradiance and subradiance in the regime of a single excitation among an array of emitters within an optical cavity [53]. Thus, our setup differs from the description of superradiance in Dicke’s original work, which considered the decay of a fully inverted ensemble. Nevertheless, we are able to engineer various amplitudes and signs of coupling between individual atoms and a single cavity mode by utilizing the nanometer-scale positional control of our optical tweezer array. In this chapter, I shall detail the theoretical principles and experimental techniques we developed that enabled this work.

3.1 Collective scattering theory

I now describe the E6 setup in detail. Consider a 1D tweezer array containing N atoms and placed along the axis of an optical cavity (frequency ω_c), as shown in Fig. 3.1. Each atom j has optical transition frequency ω_a and is positioned at $(\hat{x}_j, \hat{y}_j, \hat{z}_j)$. The cavity axis is taken to be $\mathbf{z}$. The tweezer potential $V(\hat{x}, \hat{y}, \hat{z}, s)$ provides confinement of the atoms in three dimensions and is assumed to be identical for each atom (in reality there is some aberration—see Ch. 2). V also depends on the atom’s internal state $s \in \{|g\rangle, |e\rangle\}$, where V is an attractive potential for ground state atoms and a repulsive potential for excited state atoms. The array is pumped transverse to the cavity with coherent light propagating along

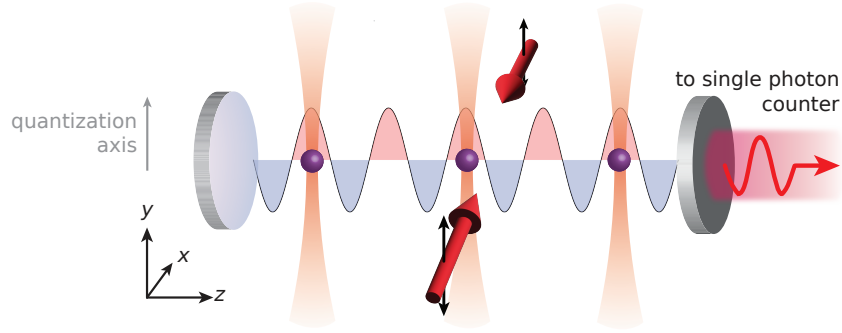


Figure 3.1: Schematic of the collective scattering experiment. Atoms are held in optical tweezers with an adjustable spacing. To see superradiance, we require the atoms to be spaced by an integer-number of wavelengths, as shown in the schematic. The atoms are driven by a retro-reflected, y -polarized pump laser (red arrows). The light emitted by the atoms into the cavity leaks out of one mirror and is directed to the SPCM.

$\mathbf{x}$ (frequency ω_p and Rabi frequency $\Omega(\hat{x}_j)$). The extent of the pump beam along $\mathbf{z}$ is taken to be much larger than the span of the tweezer array such that Ω only depends on $\hat{x}$ for the purpose of the relevant physics. Taking the waist of each tweezer and the cavity mode to be centered about $y = 0$, we ignore any motion in y and only consider the $(\hat{x}, \hat{z})$ plane defined by the cavity axis and pump propagation axis. The Hamiltonian describing this system is

$$\hat{H} = \hbar\omega_c \hat{c}^\dagger \hat{c} + \sum_{j=1}^N V_j(\hat{x}_j, \hat{z}_j, s_j) + \frac{\hbar\omega_a}{2} \hat{\sigma}_j^z - \frac{\hbar^2 \nabla_j^2}{2m} + \hbar g(\hat{z}_j) \hat{\sigma}_j^+ \hat{c} + \frac{\hbar\Omega(\hat{x}_j) e^{-i\omega_p t}}{2} \hat{\sigma}_j^+ + \text{H.c.} \quad (3.1)$$

where $\hat{c}$ is the annihilation operator for cavity photons and H.c. refers to the Hermitian conjugate. Here, $g(\hat{z}_j) \equiv g_0 \cos(k\hat{z}_j)$ is the coupling strength between each atom and the cavity mode. In the frame rotating at ω_p , the Hamiltonian becomes

$$\hat{H}_{\text{rot}} = -\hbar\Delta_{\text{pc}} \hat{c}^\dagger \hat{c} + \sum_{j=1}^N V_j(\hat{x}_j, \hat{z}_j, s_j) - \frac{\hbar\Delta_{\text{pa}}}{2} \hat{\sigma}_j^z - \frac{\hbar^2 \nabla_j^2}{2m} + \hbar g(\hat{z}_j) \hat{\sigma}_j^+ \hat{c} + \frac{\hbar\Omega(\hat{x}_j)}{2} \hat{\sigma}_j^+ + \text{H.c.} \quad (3.2)$$

where we have defined the detunings $\Delta_{\text{pa}} \equiv \omega_p - \omega_a$ and $\Delta_{\text{pc}} \equiv \omega_p - \omega_c$. In the limit $|\Delta_{\text{pa}}| \gg \Omega, g_0$, we may adiabatically eliminate the atomic excited state to obtain an effective single-atom Hamiltonian [85]

$$\hat{H}_1 = -\frac{\hbar^2 \nabla^2}{2m} + V(\hat{x}, \hat{z}) + \underbrace{\frac{\hbar\Omega^2(\hat{x})}{4\Delta_{\text{pa}}}}_{\text{ac Stark shift}} - \hbar \underbrace{\left(\Delta_{\text{pc}} - \frac{g_0^2}{\Delta_{\text{pa}}} \cos^2(k\hat{z}) \right)}_{\text{dispersive shift}} \hat{c}^\dagger \hat{c} + \underbrace{\frac{\hbar g_0 \Omega(\hat{x})}{2\Delta_{\text{pa}}} (\hat{c} + \hat{c}^\dagger) \cos(k\hat{z})}_{\text{lattice potential}}. \quad (3.3)$$

We see the emergence of terms corresponding to the ac Stark shift from the pump, the dispersive shift of the atoms on the cavity resonance frequency, and a lattice potential resulting from interference of the pump and cavity fields. We shall see that this lattice term is crucial to our observation of collective scattering effects.

From Eq. (3.1), we can obtain an expression for the steady state cavity field operator $\hat{c}$ through the master equation:

$$\dot{\hat{c}} = \frac{i}{\hbar} [\hat{H}, \hat{c}] - \kappa \hat{c} = 0 \quad (3.4)$$

$$\implies \hat{c} = i \sum_{j=1}^N g^*(\hat{z}_j) \hat{\sigma}_j^- / (\kappa - i\Delta_{\text{pc}}), \quad (3.5)$$

where we have introduced cavity photon decay with half-linewidth κ and we ignore stochastic noise on the cavity field. We see that $\hat{c}$ depends on the steady-state value of $\hat{\sigma}_j^-$. We similarly solve for $\hat{\sigma}_j^-$:

$$\dot{\hat{\sigma}}_j^- = \frac{i}{\hbar} [\hat{H}, \hat{\sigma}_j^-] - \gamma \hat{\sigma}_j^- = 0 \quad (3.6)$$

$$\implies \hat{\sigma}_j^- = - \frac{i \left(\frac{\Omega(\hat{x}_j)}{2} + g(\hat{z}_j) \hat{c} \right) (\gamma + i\Delta_{\text{pa}}) \hat{\sigma}_j^z}{\Delta_{\text{pa}}^2 + \gamma^2} \quad (3.7)$$

$$\simeq \frac{i \left(\frac{\Omega(\hat{x}_j)}{2} + g(\hat{z}_j) \hat{c} \right) (\gamma + i\Delta_{\text{pa}})}{\Delta_{\text{pa}}^2 + \gamma^2} \quad (3.8)$$

where in the last line we assume low atomic saturation ($\langle \hat{\sigma}_j^z \rangle \simeq -1$). Plugging Eq. (3.8) into Eq. (3.5), we obtain

$$\hat{c} = - \frac{\sum_j g^*(\hat{z}_j) \Omega(\hat{x}_j) (\gamma + i\Delta_{\text{pa}}) / 2}{(\kappa - i\Delta_{\text{pc}}) (\gamma^2 + \Delta_{\text{pa}}^2) + \sum_j |g(\hat{z}_j)|^2 (\gamma + i\Delta_{\text{pa}})} \quad (3.9)$$

$$\simeq - \frac{i \sum_j g^*(\hat{z}_j) \Omega(\hat{x}_j) / (2\Delta_{\text{pa}})}{-i(\Delta_{\text{pc}} - \sum_j |g(\hat{z}_j)|^2 / \Delta_{\text{pa}}) + (\kappa + \gamma \sum_j |g(\hat{z}_j)|^2 / \Delta_{\text{pa}}^2)} \quad (3.10)$$

$$= - \frac{i \sum_j \eta_j(x_j, z_j)}{-i(\Delta_{\text{pc}} - \sum_j |g(\hat{z}_j)|^2 / \Delta_{\text{pa}}) + (\kappa + \gamma \sum_j |g(\hat{z}_j)|^2 / \Delta_{\text{pa}}^2)}. \quad (3.11)$$

In the second line, we have assumed $|\Delta_{\text{pa}}| \gg \gamma$. We also define the quantity $\eta_j(x_j, z_j) \equiv g^*(\hat{z}_j) \Omega(\hat{x}_j) / (2\Delta_{\text{pa}})$ for convenience. We see that the numerator of Eq. (3.10) is proportional to the sum of the lattice potentials in Eq. (3.3) for each atom. In the denominator, we see two effects of the atoms on the cavity. One is the dispersive shift $\sum_j |g(\hat{z}_j)|^2 / \Delta_{\text{pa}}$ of the cavity resonance. The other is the absorptive broadening $\gamma \sum_j |g(\hat{z}_j)|^2 / \Delta_{\text{pa}}^2$ of the cavity.

From Eq. (3.10), one can see how interference comes about. If $\arg(g^*(\hat{z}_j) \Omega(\hat{x}_j))$ is the same for all j , then the cavity field contribution from each atom adds in phase with that

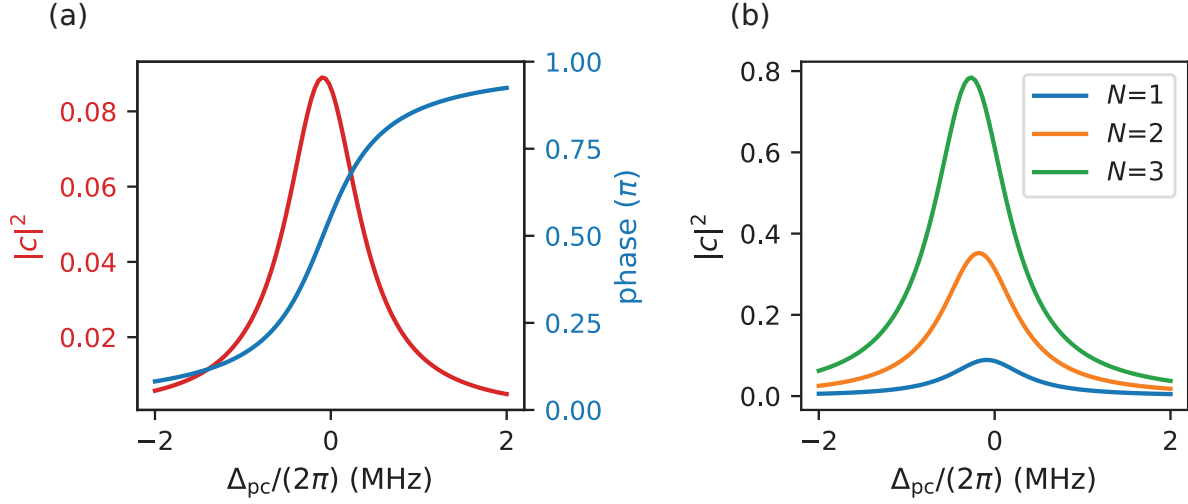


Figure 3.2: Mean-field simulation of the cavity field. (a) Mean-field cavity photon number and phase of c as a function of Δ_{pc} for a single atom in the cavity. (b) Mean field cavity photon number for $N = 1, 2, 3$ atoms superradiantly emitting into the cavity. Plots correspond to Eq. (3.10) using the parameters $(x_j = 0, z_j = 0)$, $g_0 = 2\pi \times 3$ MHz, $\Omega = 2\pi \times 10$ MHz, $\Delta_{pa} = -2\pi \times 100$ MHz, $\kappa = 2\pi \times 0.5$ MHz, $\gamma = 2\pi \times 3$ MHz.

of the other atoms, leading to constructive interference (Fig. 3.2). Conversely, if half of the atoms scatter π out of phase with the other half, then $\sum_j g^*(\hat{z}_j)\Omega(\hat{x}_j) = 0$ as a result of destructive interference, assuming that all atoms are pinned in place (in reality, there are fluctuations). In the E6 experiment, we separately tune the phase and amplitude of the field contribution for each atom through the tweezer positions. Though it is not required to observe the relevant physics, we retro-reflect our pump beam to create a standing wave, $\Omega(\hat{x}) = \Omega \cos(k\hat{x})$, and make our gauge choice such that Ω is real-valued.

In Fig. 3.2, we have plotted expectation values, ignoring noise in how the atoms couple to the cavity and pump fields. In reality, we see that thermal noise and polarization dynamics have a significant effect on the signals we observe. These effects are described below.

3.1.1 Thermal averaging

Consider that we take a single atom centered at $x = 0$ and scan its position along the cavity, z . As we pump the atom and monitor the photons leaking out of the cavity, we expect to see full contrast oscillations if the atom were fully localized. However, thermal and quantum noise broaden the atom's wavefunction. In E6, the broadening is primarily due to thermal noise, since we expect our optical molasses only cools the atoms down to a temperature T of $30 - 50 \mu\text{K}$. As the atoms scatter pump photons, their temperature may continue to increase. An atom confined in the (approximately harmonic) tweezer trap has a thermal

standard deviation in position of $\sigma = \sqrt{k_B T / (m\omega_z^2)}$, where ω_z is the tweezer's radial trap frequency.

Above, we saw the average cavity field $c = \langle \hat{c} \rangle \propto \sum_j \eta_j(x_j, z_j)$. The photon number $|c|^2 \propto |\sum_j \eta_j(x_j, z_j)|^2 = \sum_j |\eta_j|^2 + \sum_{i \neq j} \eta_i^* \eta_j$. Now consider that the atoms are spaced by an integer number of wavelengths and all within the same pump wavefront. Taking the thermal average $\langle \rangle_{\text{th}}$, symmetry enforces $\langle \eta_i \rangle_{\text{th}} = \langle \eta_j \rangle_{\text{th}} \equiv \langle \eta \rangle_{\text{th}}$. Then, the average photon number $\langle |c|^2 \rangle_{\text{th}} \propto N \langle |\eta|^2 \rangle_{\text{th}} + (N^2 - N) |\langle \eta \rangle_{\text{th}}|^2 = N \langle \Delta \eta^2 \rangle_{\text{th}} + N^2 |\langle \eta \rangle_{\text{th}}|^2$. We immediately see that if the atoms are perfectly localized, then the variance $\langle \Delta \eta^2 \rangle_{\text{th}} = 0$ and we recover N^2 scaling. In reality, there is also a linear term.

We now consider how temperature affects the contrast C_1 of the photon signal when we scan a single atom across the cavity axis. For this, we need only calculate $\langle \eta^2 \rangle_{\text{th}}$. Assuming a Gaussian thermal distribution $\rho(x, z) = \frac{\exp\{-(x^2+z^2)/(2\sigma^2)\}}{2\pi\sigma^2} = \rho_{1D}(x)\rho_{1D}(z)$, where $\rho_{1D}(x)$ is the one-dimensional Gaussian distribution, we find

$$\begin{aligned} C_1 &= \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cos^2(kz) \cos^2(kx) \rho(x, z) dx dz - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sin^2(kz) \cos^2(kx) \rho(x, z) dx dz}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cos^2(kz) \cos^2(kx) \rho(x, z) dx dz + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sin^2(kz) \cos^2(kx) \rho(x, z) dx dz} \\ &= \frac{\int_{-\infty}^{\infty} \cos^2(kz) \rho_{1D}(z) dz - \int_{-\infty}^{\infty} \sin^2(kz) \rho_{1D}(z) dz}{\int_{-\infty}^{\infty} \cos^2(kz) \rho_{1D}(z) dz + \int_{-\infty}^{\infty} \sin^2(kz) \rho_{1D}(z) dz} \\ &= e^{-2k^2\sigma^2}. \end{aligned} \tag{3.12}$$

Thus, the single-atom contrast is a direct measure of σ , which in turn allows us to infer the atom temperature. In E6, we observe a typical single-atom contrast of $\sim 30\%$ when pumping the atoms well below saturation, corresponding to $\sigma \sim 100$ nm and a temperature $T \sim 34 \mu\text{K}$ (assuming a tweezer trap frequency of $\omega_z = 2\pi \times 91$ kHz; see Sec. 2.3). Note that this is considerably larger than the quantum-limited uncertainty we expect from our tweezers, which is ~ 25 nm.

Integer spacing

We can perform a similar calculation of how positional spread affects $\langle |c|^2 \rangle_{\text{th}}$ for an integer-wavelength-spaced array of N atoms. With each atom centered at a position $(x_0 = 0, z_0)$, we have $\langle |c|^2 \rangle_{\text{th}} \propto \langle (\sum_{i=1}^N \cos(kx_i) \cos(k(z_i - z_0)))^2 \rangle_{\text{th}}$, where we calculate:

$$\begin{aligned} \langle (\sum_{i=1}^N \cos(kx_i) \cos(k(z_i - z_0)))^2 \rangle_{\text{th}} &= N \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cos^2(kx) \cos^2(k(z - z_0)) \rho(x, z) dx dz \\ &\quad + N(N-1) \left(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cos(kx) \cos(k(z - z_0)) \rho(x, z) dx dz \right)^2 \\ &= aN + bN(N-1). \end{aligned} \tag{3.13}$$

The coefficients a and b are defined as

$$a = \frac{1}{8}e^{-2k(iz_0+2k\sigma^2)}(1 + e^{2k^2\sigma^2})(1 + e^{4ikz_0} + 2e^{k(2iz_0+2k\sigma^2)}), \quad (3.14)$$

$$b = e^{-2k^2\sigma^2} \cos^2(kz_0). \quad (3.15)$$

Half-integer spacing

In the previous section, we saw that with an integer-wavelength-spaced array, we obtain $|c|^2 \propto (\sum_j \eta_j(x_j, z_j))^2 = \sum_j \eta_j^2 + \sum_{i \neq j} \eta_i \eta_j = N\langle \eta^2 \rangle_{\text{th}} + (N^2 - N)\langle \eta \rangle_{\text{th}}^2$, where we have assumed all η_j to be real-valued. The N -dependence of $\langle |c|^2 \rangle_{\text{th}}$ differs when the atoms are spaced by a half-integer number of wavelengths, since neighboring atoms see opposite signs of the cavity field. With half-integer spacing, however, the coefficient in front of the $\langle \eta \rangle_{\text{th}}^2$ term is a bit trickier to calculate, since the product $\eta_i \eta_j$ can have different signs for different i and j .

We want to know how many terms in $\sum_{i \neq j} \eta_i \eta_j$ are positive and how many are negative. Let us first consider the case where N is even, such that $N = 2n$. Then, there are n positive η_i terms and n negative η_i terms. For the product $\eta_i \eta_j$ to be positive, η_i and η_j need to have the same sign. There are N ways to pick one of the η_i , and $n - 1$ ways to pick an η_j with the same sign. Therefore, there are $N(n - 1) = N(N/2 - 1)$ positive terms. For $\eta_i \eta_j < 0$, there are $Nn = N^2/2$ ways to pick η_i and η_j with opposite signs. Thus, for even N we get

$$\langle |c|^2 \rangle_{\text{th}} \propto aN + b(N(N/2 - 1) - N^2/2) = (a - b)N \quad (3.16)$$

If instead N is an odd number, the counting is slightly different. Suppose we have n positive η_i and $n - 1$ negative η_i , so $N = 2n - 1$. There are n ways to first pick a positive η_i and $n - 1$ ways to pick a second η_j that is also positive. There are $n - 1$ ways to first pick a negative η_i and $n - 2$ ways to pick a second η_j that is also negative. So, there are $n(n - 1) + (n - 1)(n - 2)$ positive $\eta_i \eta_j$ terms. To count the negative terms, there are n ways to pick the first η_i to be positive and $n - 1$ ways to pick η_j to be negative. Similarly, there are $n - 1$ ways to pick the first η_i to be negative and n ways to pick η_j to be positive. Putting it all together:

$$\langle |c|^2 \rangle_{\text{th}} \propto aN + b(n(n - 1) + (n - 1)(n - 2) - 2n(n - 1)) = aN - (N - 1)b. \quad (3.17)$$

In Fig. 3.3, we plot $\langle |c|^2 \rangle_{\text{th}}/N$ for various N in integer and half-integer spacing configurations for $\sigma = 100$ nm. With half-integer spacing, we notice that for $N > 1$, destructive interference suppresses the photon signal at the cavity antinodes relative to the at the nodes ($z_0 = \lambda/4$ and $z_0 = 3\lambda/4$). For integer-spacing, the signal at the antinodes is superradiant, scaling super-linearly with N . For both the integer and half-integer cases, the signal at the nodes scales linearly with N , since those are the locations where $\langle \eta_i \rangle_{\text{th}} = 0$.

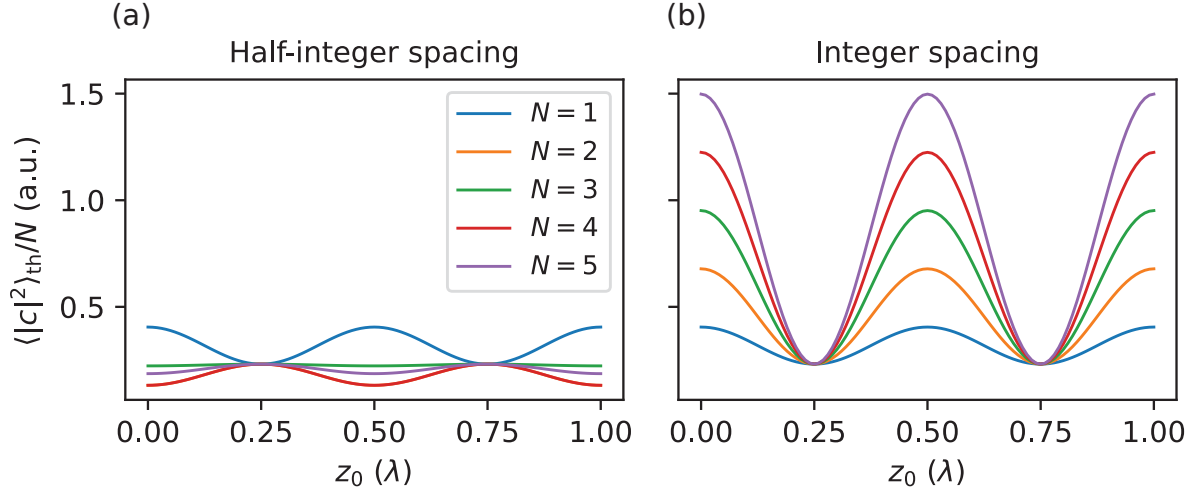


Figure 3.3: Effect of thermal averaging on superradiance and subradiance. Given a thermal positional standard deviation of $\sigma = 100$ nm in the pump-cavity plane, we analyze the scaling of cavity photon number $\langle |c|^2 \rangle$ with atom number N when the atoms are spaced by (a) a half-integer number of wavelengths or (b) an integer number of wavelengths. In (a), we see that subradiance suppresses the photon number at the antinodes ($z_0 = 0, 0.5\lambda, \lambda$). In (b), superradiance leads to a superlinear scaling of the photon number at the antinodes. In both (a) and (b), we see that the photon number at the nodes ($z_0 = 0.25\lambda, 0.75\lambda$) is the sum of the incoherent scattering of all the atoms, and thus scales linearly with N .

3.1.2 Polarization dynamics

In atomic physics, we often treat atoms as two-level systems. While the two-level approximation often provides intuition and qualitative understanding, it often falls short when attempting to make quantitative comparisons between experiment and theory. Indeed, E6 ran up against this problem in our first collective scattering measurements. We saw a scattering rate that scaled faster than N , but not at N^2 . We have already seen that this is in part due to thermal fluctuations in the atom positions, but it is not the full picture. We must also consider polarization dynamics.

Given that we drive the atoms transverse to the cavity, the only pump polarization that fully couples to the cavity is linear and along the y -axis. However, our cavity supports two nearly-degenerate polarization modes¹, and thus an atom could scatter light into either the y -polarized cavity mode (Rayleigh scattering) or the x -polarized cavity mode (Raman scattering). Rayleigh scattering preserves the initial m_F state of the atom while Raman scattering does not. Not only do we need to consider the scrambling of the m_F state, but we also need to account for the fact that the excited state manifold contains multiple F levels.

¹We have actually measured the cavity's linear birefringence to be $2\pi \times (75 \pm 15)$ kHz.

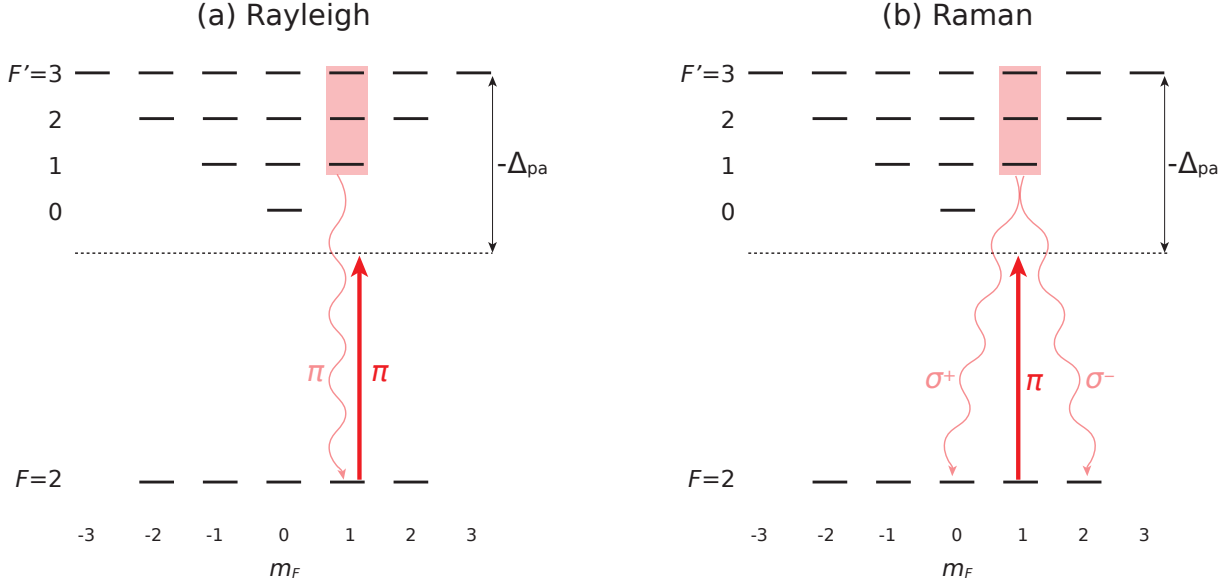


Figure 3.4: Rayleigh and Raman scattering in magnetic sub-levels. (a) Rayleigh scattering preserves the $|F = 2, m_F\rangle$ state. This corresponds to the process where the atom absorbs a π -polarized pump photon and then emits a π -polarized photon. (b) Raman scattering results in a change between the initial and final $|F = 2, m_F\rangle$ states. For an atom that starts out in $|F = 2, m_F\rangle$, this corresponds to the atom absorbing a π photon and then emitting either a σ^+ or σ^- photon, such that the final state is $|F = 2, m_F \pm 1\rangle$. Selection rules allow $F = 2$ to couple to $F' = 1, 2, 3$, so we must consider the scattering amplitudes to these three excited state hyperfine manifolds.

Rayleigh and Raman scattering

As discussed previously, $\langle |c|^2 \rangle = N \langle \Delta \eta^2 \rangle + N^2 \langle \eta \rangle^2$. We have seen the effects when the angle brackets indicate a thermal average, $\langle \rangle_{\text{th}}$. We now consider what happens when we take an average over m_F states $\langle \rangle_{m_F}$, accounting for the full hyperfine structure of the $5P_{3/2}$ manifold.

In Rayleigh scattering, an atom absorbs a pump photon and emits a photon into the cavity with the same polarization; thus the atom returns to its initial m_F state (Fig. 3.4(a)). In our setup, preservation of the spin state enables the observation of collective enhancement because the state does not depend on which atom emitted a photon, thus allowing the scattering processes for different atoms to interfere. In contrast, Raman decay into the cavity means that an atom emits a photon polarized orthogonal to the pump photon, which necessarily leads to a change in the atom's m_F state (Fig. 3.4(b)). Because the final state of the system now depends on which atom emitted a photon, Raman scattering processes add incoherently. Note that this assumes that the atom's state is collapsed before scattering

a photon onto a single m_F state; should coherence build up among the m_F states, it is possible to have coherent Raman scattering (this is the subject of our forthcoming work on spin self-organization). Below, I explain how we calculate the rates for Rayleigh and Raman scattering for all $|F = 2, m_F\rangle$ states at various detunings Δ_{pa} from $F' = 3$. We find that in ^{87}Rb , there is a “magic detuning” at approximately $\Delta_{\text{pa}} = -2\pi \times 507 \text{ MHz}$ where Raman scattering is nearly completely suppressed and all $|F = 2, m_F\rangle$ states couple to the cavity equally such that $\langle \Delta\eta^2 \rangle_{m_F} \simeq 0$ [53, 86].

The two-photon scattering amplitude $\eta(x, z) = g(z)\Omega(x)/\Delta_{\text{pa}}$ can be generalized to include dependence on m_F state. For Rayleigh scattering processes, η takes the form

$$\eta_{\text{Rayleigh}}(x, z, m_F) = \sum_{F'=1,2,3} \frac{g(z, m_F, F', \Delta m_F = 0)\Omega(x, m_F, F', \Delta m_F = 0)}{\Delta_{\text{pa}, F'}}. \quad (3.18)$$

Here, we define $\Delta_{\text{pa}, F'}$ as the detuning of the probe from the F' level. Note that g and Ω now depend on the Clebsch-Gordan coefficients for π transitions from $|F = 2, m_F\rangle$ to each $|F', m_F\rangle$ state, where the π -polarization enforces $\Delta m_F \equiv m_{F'} - m_F = 0$. Assuming that our atoms are prepared un-polarized, the Rayleigh scattering amplitude averaged over m_F states for a fixed Δ_{pa} is

$$\langle \eta_{\text{Rayleigh}}(x, z) \rangle_{m_F} = \frac{1}{5} \sum_{m_F} \sum_{F'=1,2,3} \frac{g(z, m_F, F', \Delta m_F = 0)\Omega(x, m_F, F', \Delta m_F = 0)}{\Delta_{\text{pa}, F'}}. \quad (3.19)$$

The average number of cavity photons emitted by a symmetrically coupled N -atom array is then proportional to $N^2 \langle \eta_{\text{Rayleigh}}(x, z) \rangle_{m_F}^2 + N \langle \Delta\eta_{\text{Rayleigh}}^2 \rangle_{m_F}$ where $\langle \Delta\eta_{\text{Rayleigh}}^2 \rangle_{m_F} \equiv \langle \eta_{\text{Rayleigh}}(x, z)^2 \rangle_{m_F} - \langle \eta_{\text{Rayleigh}}(x, z) \rangle_{m_F}^2$. A non-zero $\langle \Delta\eta_{\text{Rayleigh}}^2 \rangle_{m_F}$ indicates that different $|F = 2, m_F\rangle$ states scatter different numbers of photons into the cavity on average; similar to thermal averaging, these fluctuations contribute linearly in N to the total cavity photon number.

For Raman scattering processes, we define

$$\eta_{\text{Raman}}(x, z, m_F, q) = \sum_{F'=1,2,3} \frac{g(z, m_F - q, F', \Delta m_F = q)\Omega(x, m_F, F', \Delta m_F = 0)}{\Delta_{\text{pa}, F'}}. \quad (3.20)$$

where $q = \pm 1$ corresponds to $\sigma^\pm$ polarization. Pump photons couple the initial state $|F', m_F\rangle$ to $|F', m_F\rangle$, and x -polarized cavity photons couple $|F', m_F\rangle$ to $|F = 2, m_F - q\rangle$.

We assume that our molasses prepares our spins in an unpolarized, incoherent mixture of $|F = 2, m_F\rangle$ states. Because of this, atoms in different m_F states Raman scatter into the cavity with a spin-dependent phase, which averages to 0 for an equal mixture. Therefore, we assume $\langle \eta_{\text{Raman}} \rangle_{m_F} = 0$ and the only Raman scattering that occurs is incoherent, in the form of $\langle \eta_{\text{Raman}}^2 \rangle_{m_F}$.

To understand why $\langle \eta_{\text{Raman}} \rangle_{m_F} = 0$, we look at how the cavity’s y -polarized field couples to the atoms. For the sake of gaining intuition, I shall treat the case of a $F = 1/2 \rightarrow F' = 3/2$ transition, rather than deal with all the complexities of $F = 2 \rightarrow F' = 3$.

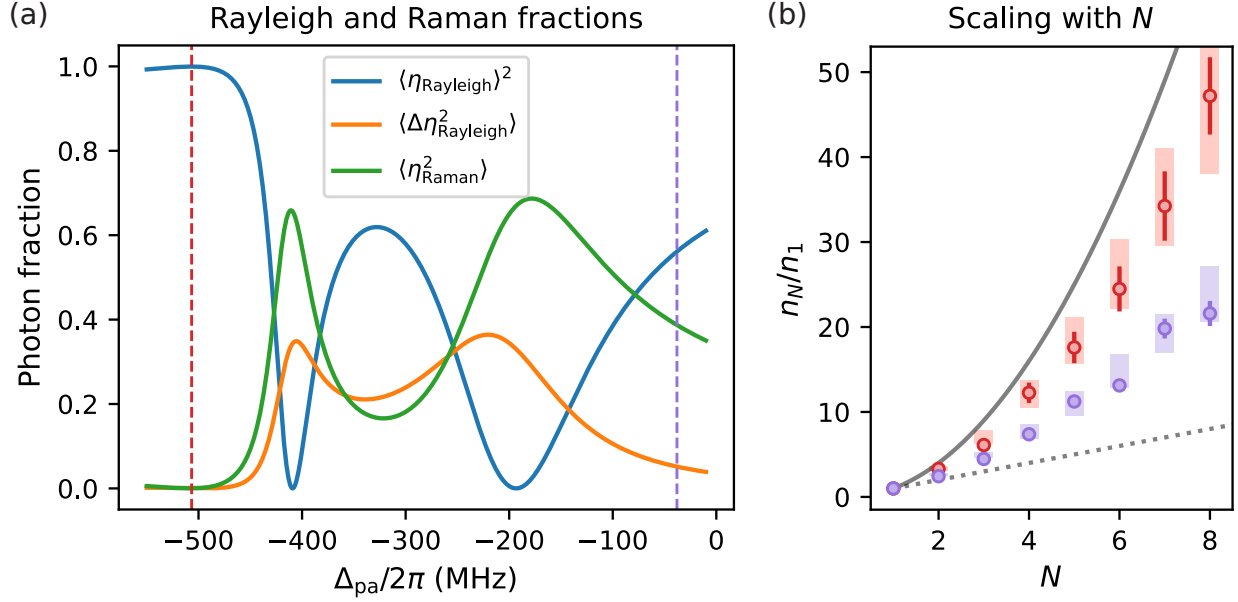


Figure 3.5: The magic detuning. (a) Fractions of Rayleigh- and Raman-scattered photons as a function of the detuning Δ_{pa} . We see that at the magic detuning $\Delta_{\text{pa}} \simeq -2\pi \times 507$ MHz, the coherent Rayleigh photon fraction reaches a maximum, while incoherent Rayleigh and Raman scattering are negligible. (b) Number of cavity photons emitted by N atoms, normalized by the number of photons emitted by a single atom, at the magic detuning (red) and at $\Delta_{\text{pa}} = -2\pi \times 38$ MHz (purple). The solid gray curve shows the N^2 -scaling for ideal superradiance and the dashed gray curve shows incoherent N -scaling. At the magic detuning, the data are consistent with the scaling expected from thermal averaging alone, given our estimated thermal positional spread of $\sigma = 100$ nm (red shading). Meanwhile, the photon number data taken at $\Delta_{\text{pa}} = -2\pi \times 38$ MHz do not scale as rapidly with N , due to the combined effects of thermal and m_F -state averaging. The purple shading corresponds to a Monte Carlo simulation assuming equal population of all the m_F states.

Recall the effective single-atom Hamiltonian, Eq. (3.3). Let us only examine the terms that couple to the cavity:

$$-\hbar \left(\Delta_{\text{pc}} - \frac{g(\hat{z})^2}{\Delta_{\text{pa}}} \right) \hat{c}^\dagger \hat{c} + \frac{\hbar g(\hat{z}) \Omega(\hat{x})}{2\Delta_{\text{pa}}} (\hat{c} + \hat{c}^\dagger). \quad (3.21)$$

In writing down these terms, we previously assumed the cavity only supports a single mode with the same polarization as the pump. For ease of notation in writing down spin matrices, I now choose the quantization axis to be along the cavity, such that π polarization is along $\mathbf{z}$ and the polarization of the pump is $\mathbf{y} = i(\sigma_+ + \sigma_-)/\sqrt{2}$. To incorporate two degenerate polarization modes of the cavity, we introduce cavity photon operators $\hat{c}_+$ and $\hat{c}_-$ for the σ_+ and σ_- cavity modes. We also introduce atomic dipole operators $\hat{d}_+$ and $\hat{d}_-$, which correspond to taking a ground state atom to the excited state via a σ_+ or σ_- photon. For the $F = 1/2 \rightarrow F' = 3/2$ toy model, these operators are the matrices

$$\hat{d}_+ = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1/3} \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \hat{d}_- = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \sqrt{1/3} & 0 \\ 0 & 1 \end{pmatrix}, \quad \hat{d}_y = \begin{pmatrix} i/\sqrt{2} & 0 \\ 0 & i/\sqrt{6} \\ i/\sqrt{6} & 0 \\ 0 & i/\sqrt{2} \end{pmatrix}, \quad (3.22)$$

where the basis is ordered by descending m_F for both the ground and excited state manifolds. The new cavity interaction terms read

$$-\hbar \Delta_{\text{pc}} (\hat{c}_+^\dagger \hat{c}_+ + \hat{c}_-^\dagger \hat{c}_-) + \frac{\hbar g(\hat{z}) \Omega(\hat{x})}{2\Delta_{\text{pa}}} (\hat{c}_+^\dagger \hat{d}_+^\dagger \hat{d}_y + \hat{c}_-^\dagger \hat{d}_-^\dagger \hat{d}_y + \text{H.c.}) + \frac{\hbar g(\hat{z})^2}{\Delta_{\text{pa}}} \sum_{i,j \in \{+, -\}} \hat{c}_i^\dagger \hat{c}_j \hat{d}_i^\dagger \hat{d}_j. \quad (3.23)$$

The second term here corresponds to an atom being excited by a $\mathbf{y}$ pump photon, and then getting de-excited by a cavity photon with either σ_+ or σ_- polarization. The last term is again the dispersive shift, incorporating the processes of excitation and de-excitation by cavity photons. Multiplying out the dipole operators, we obtain

$$\begin{aligned} & -\hbar \Delta_{\text{pc}} (\hat{c}_+^\dagger \hat{c}_+ + \hat{c}_-^\dagger \hat{c}_-) + \frac{\hbar g(\hat{z}) \Omega(\hat{x})}{2\Delta_{\text{pa}}} \left(\frac{2}{3} (\hat{c}_y + \hat{c}_y^\dagger) + \frac{i}{3} (\hat{c}_x - \hat{c}_x^\dagger) \hat{\sigma}_z \right) \\ & + \frac{\hbar g(\hat{z})^2}{\Delta_{\text{pa}}} \left(\frac{1}{3} (\hat{c}_+^\dagger \hat{c}_+ - \hat{c}_-^\dagger \hat{c}_-) \hat{\sigma}_z + \frac{2}{3} (\hat{c}_+^\dagger \hat{c}_+ + \hat{c}_-^\dagger \hat{c}_-) \right). \end{aligned} \quad (3.24)$$

Finally, we see the emergence of the Raman scattering term, proportional to $i(\hat{c}_x - \hat{c}_x^\dagger) \hat{\sigma}_z$. It is $\pi/2$ out of phase with the Rayleigh scattering term (proportional to $\hat{c}_y + \hat{c}_y^\dagger$), and, importantly, also depends on $\hat{\sigma}_z$, unlike the Rayleigh scattering term. Different populations of m_F levels will cause $\langle \hat{\sigma}_z \rangle$ to take on different values and different signs. For an equal distribution of m_F states, $\langle \hat{\sigma}_z \rangle_{m_F} = 0$ and there is no coherent Raman scattering. In Eq. (3.24), we also see the emergence of circular birefringence (depending on $\hat{c}_+^\dagger \hat{c}_+ - \hat{c}_-^\dagger \hat{c}_-$) in addition to the usual dispersive shift.

Treating the $F = 2 \rightarrow F' = 3$ problem is much more difficult because there are many higher-order terms in the spin expansion. I shall not treat that problem here, as this will be discussed in Leon Lu’s forthcoming thesis.

In Fig. 3.4, we plot the fraction of coherent Rayleigh-, incoherent Rayleigh-, and Raman-photons scattered by a single ^{87}Rb atom at various pump detunings from $F = 2 \rightarrow F' = 3$ resonance. At $\Delta_{\text{pa}} = -2\pi \times 507 \text{ MHz}$, the coherent Rayleigh fraction reaches a maximum of 99.96%, with incoherent Rayleigh and Raman scattering near vanishing. At this “magic detuning,” the atoms scatter as if they were ideal two-level systems.

3.2 Experimental observations

3.2.1 Using atoms to resolve the 780 nm lattice

The idea behind E6’s experiments was simple—place atoms an integer number of wavelengths apart and shine a laser at them, and they should scatter superradiantly into the cavity. Place them a half-integer number of wavelengths apart and we should see subradiance. However, getting to the stage of being able to observe such collective effects required careful calibrations of the atoms’ positions with respect to the cavity and pump fields. There are three crucial components to performing this calibration. The first is the ability to tune the tweezer spacing, which we do in our 1D tweezer array by adjusting the rf tones generated by the Spectrum AWG, which drives the AOD. The second is the ability to finely rotate the tweezer array about the tweezer axis ($\mathbf{y}$), so the array can be oriented at different angles within the $\mathbf{xz}$ -plane. For this, we have the AOD on a rotation mount. Initially this was a standard, Thorlabs rotation mount, which was later replaced by a motorized rotation stage (Thorlabs PRM1Z8). Third is the ability to translate the array in two dimensions within the $\mathbf{xz}$ -plane. While this could also be done with a 2D AOD, E6 has achieved this with a 1D AOD by adding to the optical path a mirror on a Polaris kinematic mount with two piezoelectric adjusters, which can be used to translate the center-of-mass of the array in-sequence. I describe the process of performing such calibrations below.

When we started out, we got a rough idea of our atom spacing by analyzing fluorescence images and then converting camera pixels to distance. To precisely know the imaging system’s magnification, we relied on fluorescence imaging to measure the ac Stark shift induced by the 1560 nm standing wave in our cavity. 1560 nm light strongly shifts the $5\text{P}_{3/2}$ state by coupling to the $4\text{D}_{5/2}$ state. For relatively low powers, the 1560 nm light shifts the atomic resonance closer to resonance with our molasses light (red detuned from bare atomic resonance by $\sim 2\pi \times 40 \text{ MHz}$), thus causing the atoms to fluoresce more strongly when they are in an antinode of the 1560 nm standing wave. Scanning the position of the array across the cavity axis using the piezo mirror, we observed a sinusoidal variation in the atomic fluorescence, whose periodicity we knew to be $1560 \text{ nm}/2 = 780 \text{ nm}$ [87]. This method was also useful for doing initial alignment of the AOD angle. With the array oriented perpendicular to the cavity axis, we generated two tweezers $\sim 2.4 \mu\text{m}$ apart (well within the waist of the

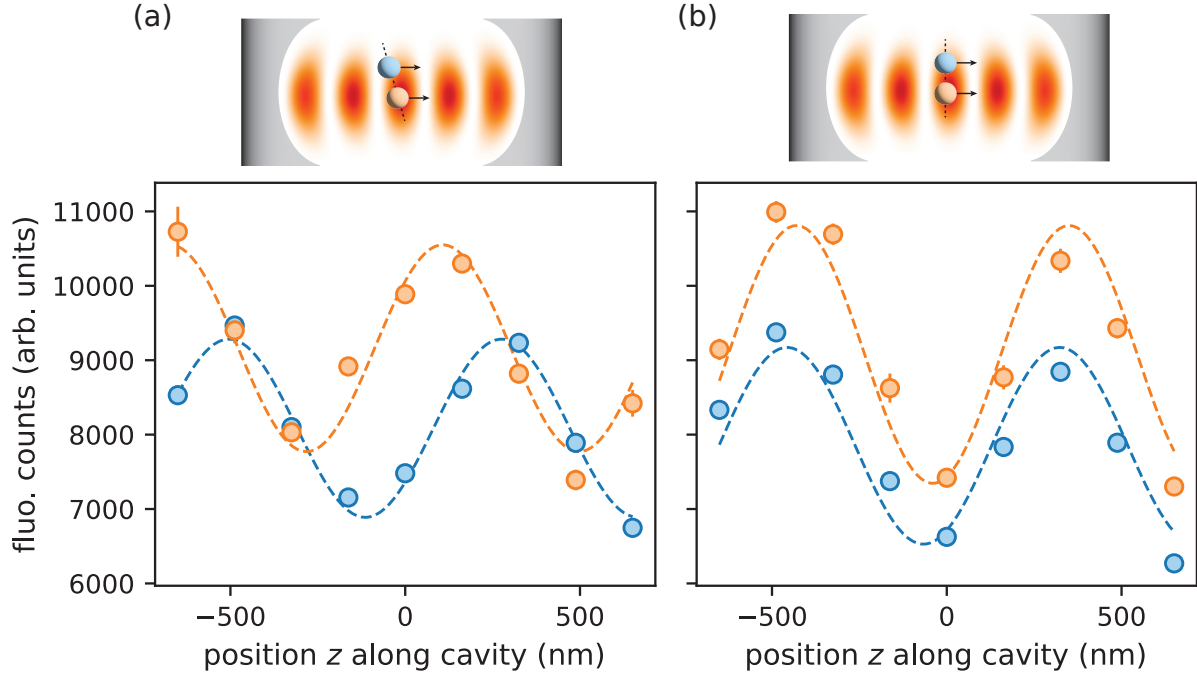


Figure 3.6: Calibration of the AOD angle and tweezer spacing using the 1560 nm standing wave. (a) Fluorescence imaging data of two atoms scanned along the cavity axis. We see a phase offset in the sinusoidal curves traced out by each atom. (b) After adjustment of the AOD angle, the fluorescence signals of the atoms are in phase. Knowing that the periodicity of the standing wave is 780 nm, we used this measurement for initial calibrations of the tweezer spacing and of the scan distance of the piezo mirror.

cavity) and scanned them across the cavity axis. We observed a clear phase difference in the sinusoidal fluorescence pattern traced out by the atoms in each tweezer. We could then rotate the AOD to zero-out this phase difference (Fig. 3.6). Although the 1560 nm light was immensely helpful for performing initial alignment, we ultimately learned that the cavity photon signal from the atoms was the most accurate indicator of their positions.

To perform fine calibration of the tweezer spacing, we rotated the AOD axis to be along the cavity axis and generated a two-tweezer waveform. We choose the tweezer spacing to be an integer number of wavelengths apart, e.g. 10λ . We then use the piezo mirror to scan the tweezers across the cavity axis while illuminating them with pump light, and measure the cavity photon signal on our SPCM. Since the atoms are probabilistically loaded into the array, we collect data where single tweezer sites are loaded as well as where both sites are loaded. Analyzing the cavity photon data where a single atom loaded, we fit a sinusoid to the signal from each atom and compare the phase. If there is a phase difference, then we adjust the tweezer spacing. Previously, this was done by changing the frequency difference

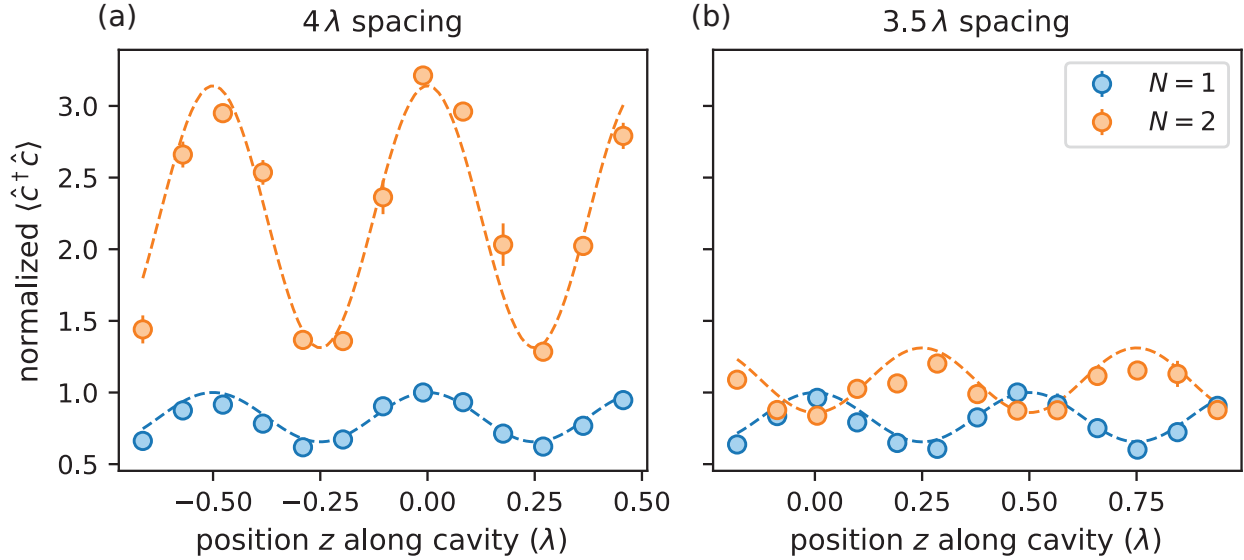


Figure 3.7: Superradiant and subradiant cavity scattering by two atoms. The positions of either one (blue) or two (orange) atoms are scanned along the cavity axis. (a) When the atoms are spaced by 4λ , the two-atom signal exhibits superradiance at the antinodes of the cavity field. (b) When the atoms are spaced by 3.5λ , the two-atom signal is subradiant at the cavity field antinodes, with the two-atom signal being even lower than the single-atom signal. Data are taken at the magic detuning, $\Delta_{\text{pa}} \simeq -2\pi \times 507$ MHz. Dashed lines are zero-parameter fits assuming a thermal standard deviation $\sigma = 110$ nm.

of the rf tones that drive the AOD. However, this method has the drawback that at the level of frequency precision we require, the tweezer waveforms become very long due to the requirement that the waveforms obey periodic boundary conditions for the superposition of rf tones. We later opted for a more convenient method of adjusting the tweezer spacing, which simply involves clocking the AWG with a function generator and adjusting its frequency. This way, we are able to adjust the tweezer spacing without having to change the waveforms, and we can choose the rf frequencies to be convenient numbers to minimize the waveform length. We adjust the spacing until the photon signals from the atoms are in-phase with one another, then we increase the tweezer spacing by a factor of 2 and repeat the same measurement. Going to a larger tweezer spacing allows us to calibrate more precisely the conversion between rf frequency and distance at the atom plane. We repeat this procedure until the atoms are spaced by the maximal extent of the array that we use in our experiments. While performing adjustments to the tweezer spacing, we also check the two-atom scattering signal to ensure we have not accidentally gone from an integer-spacing to a half-integer-spacing configuration. For integer spacing, we see that the two-atom signal is superradiant (maximal signal exceeds the sum of the single atom signals, Fig. 3.7(a)) and for half-integer

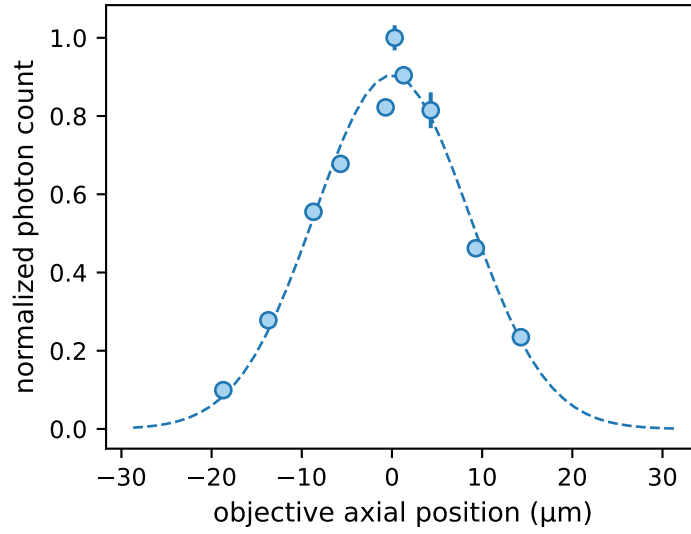


Figure 3.8: Measurement of w_0 using cavity fluorescence. We scan the position of the objective lens along the tweezer axis and record the number of photons emitted by an array of $N = 8$ atoms at each position. From a Gaussian fit, we extract a waist of $w_0 = 17.2 \pm 0.8 \mu\text{m}$.

spacing it is subradiant (signal is maximally suppressed at the antinodes, Fig. 3.7(b)).

A similar procedure is used to align the AOD angle to the wavefront of the pump beam. Instead of scanning the two atoms across the cavity, we scan them along the pump axis and we adjust the angle of the AOD using the rotation mount. This procedure became vastly easier once we switched to the motorized rotation mount.

3.2.2 Measuring w_0 using cavity fluorescence

Cavity fluorescence can also be used to resolve the Gaussian radial profile of the cavity field. We used this method to measure the waist w_0 of our cavity (Fig. 3.8). To do so, we stepped the axial position of our high-NA objective (which is on a motorized translation stage). At each step, we then scanned an array of $N = 8$ atoms (spaced by 4λ) across the pump and cavity axes and recorded the maximal photon count rate; this lattice scan was done to ensure that we record the photon count rate with the atoms at the antinodes of the lattice. Plotting the maximal photon count rate as a function of the objective height, we observed a Gaussian profile with $w_0 = 17.2 \pm 0.8 \mu\text{m}$. From this, we infer a cavity mode volume of $V = \pi w_0^2 L / 4 = 2.13 \times 10^{-12} \text{ m}^3$ and $g_0 = 2\pi \times 3.1 \text{ MHz}$.

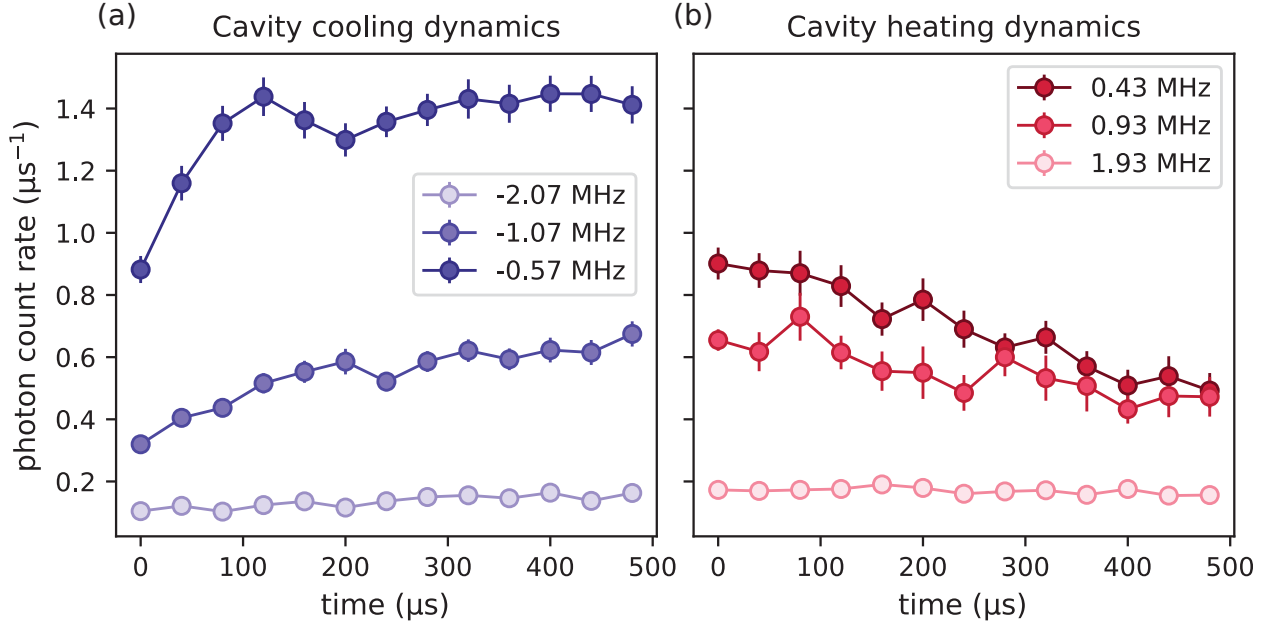


Figure 3.9: Cavity photon dynamics at various Δ_{pc} . Depending on the sign of Δ_{pc} , the rate of photon scattering by an array of $N = 8$ atoms at the cavity field antinodes either grows or diminishes over time. (a) When $\Delta_{pc} < 0$, we see the photon count rate increase over a time scale of $500 \mu s$. This seems to indicate better confinement of the atoms to the cavity field antinodes, consistent with cavity cooling. (b) When $\Delta_{pc} > 0$, we see the opposite: the photon count rate decreases over time, consistent with the atoms delocalizing due to cavity heating.

3.2.3 Cavity cooling and heating dynamics

We were able to observe dynamics in the photon time traces that indicated the presence of cavity cooling when the pump was red-detuned from the cavity, and the presence of cavity heating when the pump was blue-detuned from the cavity. With $N = 8$ atoms situated on the antinodes of the cavity-pump lattice (spaced by 4λ), we observed the initial photon dynamics for different pump frequencies, as seen in Fig. 3.9. This was done by dividing each time trace into $40 \mu s$ bins and measuring the average photon count rate in each bin. When the pump is far-detuned on either side of cavity resonance ($|\Delta_{pc}| \gg \kappa$), the system appears to reach steady state within the first $40 \mu s$ and we do not observe dynamics. As Δ_{pc} approaches $-\kappa$, we observe the photon count rate increasing over time; the initial rate of increase is maximal when $\Delta_{pc} \simeq -\kappa$. This is consistent with cavity cooling dynamics, where cavity cooling helps to localize the atoms at the antinodes of the field, thus reducing thermal averaging. On the other side of cavity resonance, when $\Delta_{pc} \sim +\kappa$, we instead see the cavity photon count rate decrease over time, consistent with the atoms heating up and

thermally sampling a wider spread of the lattice.

3.2.4 Experimental challenges

Drift of the tweezers and lattice

A caveat to our tweezer alignment procedure is that the measured contrast diminishes over time; the oscillations get smeared out as the tweezer array drifts with respect to the cavity-pump lattice. Although our high-NA objective is mounted onto the vacuum chamber (in which the cavity is mounted), the rest of the tweezer optics are not. Similarly, the side pump optics are not common-mode with either the cavity or tweezers. Thus, all three are able to mechanically drift with respect to each other. We noticed that the drift of the pump field seems to be worse than that of the cavity. If there is a future iteration of E6, I would suggest fixing these optics to the chamber as part of the design. We have seen drifts of $\lambda/4$ occur on the scale of tens of minutes, with some days being worse than others depending on the temperature stability of the experiment. At some point, the drift was making data-taking increasingly untenable, so we sought a workaround. The idea was to scan the atoms across the pump axis in a single iteration of the Cicero sequence, while the tweezer position along the cavity axis was stepped between iterations of the sequence. In this way, we were effectively always doing 2D position scans, but now in the time that it normally took to do a 1D scan. Being able to scan in 2D in a reasonable amount of time was crucial for being able to track the drift over time, so that we could always identify where the atoms were with respect to the antinodes. Eventually, this method of scanning in-sequence was extended to both the cavity and pump axes and evolved into shot-to-shot feedback done by a Red Pitaya FPGA. In this way, we could ensure that the array position was stabilized to the cavity-pump lattice.

Pump polarization

In our first attempts at observing superradiance, we opted for using our molasses beam as our pump (at the time, we did not have a “side pump” setup). While we were able to see some coherent enhancement and suppression as we varied the inter-atom spacing, these factors were much smaller than expected. We suspected that this originated from the polarization gradient of the molasses, which leads to increased Raman scattering. This prompted us to set up the side pump beam in a purely linear polarization, which ultimately enabled the observation of coherent enhancement and suppression in agreement with theory.

Side pump homodyne

At some point, after realigning the side pump onto the atoms, we noticed an asymmetric oscillation as we scanned a single atom across the cavity (Fig. 3.10)—the antinode peaks would alternate in height, as if another field was in the cavity and interfering with the atom signal. We confirmed this by checking for a cavity photon signal without any atoms

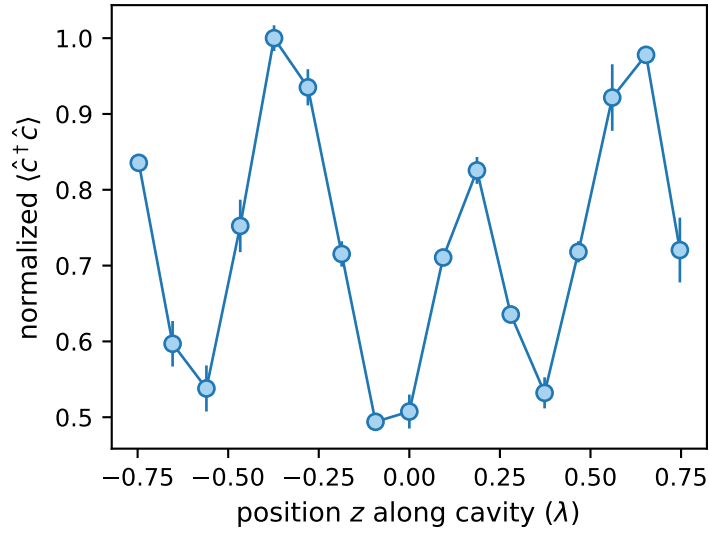


Figure 3.10: Side pump homodyne signal. We scanned a single atom along the cavity axis and observed an asymmetric oscillation in the cavity fluorescence signal. This asymmetric signal seemed to originate from a stray reflection of the side pump beam, which somehow made its way into the cavity and interfered with the light emitted by the atom.

loaded but with the side pump still on. As we scanned the side pump frequency across cavity resonance, a clear peak emerged, indicating that side pump light was somehow being reflected into the cavity. In our first attempt to get rid of this stray reflection, we adjusted the pointing of the beam to try to minimize the homodyne signal. Though we saw some improvement with this, the signal did not go away. Ultimately, we decided to shrink down the beam by a factor of ~ 3 , achieving a new beam waist of $173 \pm 5 \mu\text{m}$ at the atoms. While this still did not get rid of the homodyne signal, it allowed us to do our experiments at ~ 9 times lower pump power. Since the strength of the homodyne signal appeared to depend on the pump power, this reduced the effect significantly.

Chapter 4

Optomechanical self-organization

The interaction between atoms and light can create startling effects when the atoms and light are confined to an optical cavity. Atoms preferentially emit light back into the electromagnetic mode defined by the cavity. These photons simultaneously exert mechanical forces on the atoms, pulling them into ordered configurations—a phenomenon known as self-organization. This effect is amplified when the atoms emitting light in phase with one another. Constructive interference leads to a superlinear scaling of the intracavity intensity of light, known as superradiance [54, 57, 88]. Optomechanical self-organization has been observed in bulk gases in the context of the Dicke phase transition [56, 89, 90]. In our work, we explored the Dicke phase transition in a mesoscopic system of atoms held in optical tweezers [55]. The tweezer array allowed precise control over atom number, enabling us to build up the self-organization phase transition literally atom-by-atom. We studied how the critical point and dynamics of this system depend on the atom number with unprecedented control.

In the rest of this chapter, I will introduce the concepts explored in our work [55], provide technical details about how we performed these experiments, discuss lessons learned, and end with prospects for future work.

4.1 The Dicke Model

There are several nice review papers on the Dicke model [57, 72], so I shall only give a brief introduction here. Dicke first formulated the concept of superradiance in the context of a molecular gas confined within a wavelength of light, such that photons emitted by individual molecules are indistinguishable [54]. He found that as a gas of excited molecules radiates, the system passes through superposition states that emit coherently, and the emission rate scales with the square of the total number of molecules. Dicke termed the coherently enhanced emission rate as “super-radiant.” Building on Dicke’s idea, Hepp and Lieb [68] formulated a Hamiltonian that displays steady-state superradiance, as opposed to the transient effect

described by Dicke. Known as the Dicke model, this Hamiltonian has the form

$$\begin{aligned}\hat{H}/\hbar &= \omega_c \hat{c}^\dagger \hat{c} + \frac{\omega_z}{2} \sum_{i=1}^N \hat{\sigma}_i^z + \frac{\alpha}{\sqrt{N}} (\hat{c} + \hat{c}^\dagger) \sum_{i=1}^N \hat{\sigma}_i^x \\ &= \omega_c \hat{c}^\dagger \hat{c} + \omega_z \hat{S}_z + \frac{2\alpha}{\sqrt{N}} (\hat{c} + \hat{c}^\dagger) \hat{S}_x.\end{aligned}\tag{4.1}$$

This Hamiltonian simply describes an ensemble of N spin-1/2 systems with resonance frequency ω_z symmetrically coupled (with strength α) to a cavity mode with bosonic annihilation operator $\hat{c}$ and resonance frequency ω_c . In the second line, we define total spin operators $\hat{S}_\alpha = \frac{1}{2} \sum_i \hat{\sigma}_i^\alpha$. Note that the final term in Eq. (4.1) does not make the rotating wave approximation, in contrast to related models such as the Jaynes-Cummings and Tavis-Cummings models [62, 67].

One might ask how the ground state of the Dicke model evolves as one tunes the coupling strength λ between the spins and the cavity. At one extreme, consider $2\alpha/\sqrt{N} \ll \omega_z, \omega_c$. In this case, minimizing the average energy $\langle \hat{H} \rangle$ favors $\langle \hat{\sigma}_i^z \rangle = -1$, i.e. all emitters in their ground state, and $\langle \hat{c}^\dagger \hat{c} \rangle = 0$, meaning no photons in the cavity. This is called the normal phase. Now consider the other extreme, $2\alpha/\sqrt{N} \gg \omega_z, \omega_c$. We see that the final term in Eq. (4.1) dominates; this term can provide a negative-energy contribution that overcomes the positive-energy penalties from creating excited state polarization ($\langle \hat{S}_z \rangle > -N/2$) and from generating cavity photons. This is achieved when the spins collectively point along $\mathbf{x}$ and when the cavity field, proportional to $\hat{c} + \hat{c}^\dagger$, takes on non-zero amplitude and the opposite sign of $\langle \sum_i \hat{\sigma}_i^x \rangle$. This is the superradiant regime. We also see that superradiance is accompanied by $\mathbb{Z}_2$ symmetry, since Eq. (4.1) is invariant under simultaneous sign flips of the cavity field and the projection of the spins along $\mathbf{x}$ ($\hat{c} \rightarrow -\hat{c}$, $\hat{\sigma}_i^x \rightarrow -\hat{\sigma}_i^x$).

Holstein-Primakoff transformation

An alternative form of the Dicke Hamiltonian involves mapping the spin-based version of the Dicke model to a bosonic formalism. The mapping of spins to harmonic oscillators is known as the Holstein-Primakoff transformation [91]. This transformation is valid for large spins when the spin state is close to its ground state. In this limit, the spin raising and lowering operators ($\hat{S}_+$ and $\hat{S}_-$) act analogously to bosonic creation and annihilation operators ($\hat{b}^\dagger$ and $\hat{b}$). The mapping is as follows:

$$\hat{S}_- \rightarrow \hat{b} \sqrt{N - \hat{b}^\dagger \hat{b}},\tag{4.2}$$

$$\hat{S}_z \rightarrow \hat{b}^\dagger \hat{b} - \frac{N}{2}.\tag{4.3}$$

For large N , we map $\hat{S}_x \rightarrow \sqrt{N}(\hat{b} + \hat{b}^\dagger)/2$. Then, the Dicke Hamiltonian becomes

$$\hat{H}/\hbar = \omega_c \hat{c}^\dagger \hat{c} + \omega_z \hat{b}^\dagger \hat{b} + \alpha(\hat{b} + \hat{b}^\dagger)(\hat{c} + \hat{c}^\dagger).\tag{4.4}$$

4.1.1 The superradiant phase transition

Above, I have given a qualitative description of what one might expect the ground state of the Dicke Hamiltonian to be for small and large values of the spin-cavity coupling strength α . Now we would like to quantitatively determine the critical point α_c at which the transition from normal to superradiant occurs.

Perhaps the simplest way to approach this is to use mean field theory and assume that the spins are in a product state. In this approximation, we take the cavity field to be a classical number c and the spin state $|\psi\rangle$ to be in a superposition $|\psi\rangle = (x/2)|\uparrow\rangle + \sqrt{1-x^2/4}|\downarrow\rangle$, where x is a real number. The choice to neglect a complex relative phase between $|\uparrow\rangle$ and $|\downarrow\rangle$ assumes that $\langle\hat{\sigma}_i^y\rangle = 0$, which we expect because Eq. (4.1) does not involve $\hat{\sigma}_i^y$. We obtain an energy functional $E(x, c)$ given by

$$E(x, c) = \omega_c c^2 + \frac{N\omega_z}{4}x^2 + 2c\alpha\sqrt{N}x\sqrt{1-x^2/4} \quad (4.5)$$

$$\simeq \omega_c c^2 + \frac{N\omega_z}{4}x^2 + 2c\alpha\sqrt{N}x \quad (4.6)$$

where we ignore constant energy offsets and in the second line we assume x to be small, which we expect to be the case near the critical point. Minima of this energy functional correspond to equilibrium solutions. Setting $dE/dc = 0$ and $dE/dx = 0$, we find

$$\underbrace{\begin{pmatrix} 2\omega_c & 2\alpha\sqrt{N} \\ 2\alpha\sqrt{N} & N\omega_z/2 \end{pmatrix}}_{\mathbf{M}} \begin{pmatrix} c \\ x \end{pmatrix} = 0. \quad (4.7)$$

Eq. (4.7) has non-trivial solutions if $\det(\mathbf{M}) = 0$. Solving for α gives the critical coupling strength

$$\alpha_c = \frac{\sqrt{\omega_c\omega_z}}{2}. \quad (4.8)$$

This derivation assumed that the physics is described wholly by energy. It neglects the temperature T of the spins, which would be captured by free energy. A derivation of the critical point at thermodynamic equilibrium is given in Ref. [57]. The resulting $\alpha_c(T)$ is given by

$$\alpha_c(T) = \frac{1}{2}\sqrt{\omega_c\omega_z\coth\left(\frac{\hbar\omega_z}{2k_B T}\right)}. \quad (4.9)$$

As experimentalists working with optical cavities, however, Eqs. (4.8-4.9) raise an alarm. For ω_c and ω_z at optical frequencies (for instance, $2\pi \times 384$ THz for the D₂ transition in ⁸⁷Rb), α_c also occurs at optical frequencies. Achieving a vacuum Rabi frequency $g_0 \sim \alpha_c/\sqrt{N}$ is unfathomable with the mode sizes achieved through diffraction-limited optics for optical transitions in atoms, and led to debate over no-go theorems for the Dicke model [57, 69, 92]. Although this prevents realization of the Dicke model in its original form, experimentalists have nonetheless found ways to map the Dicke model to driven-dissipative systems that bring the critical point into an experimentally accessible regime.

4.1.2 Driven-dissipative realization with tweezers

As an illustrative example of how the Dicke model may be mapped to a driven-dissipative system, I shall discuss the tweezer-based optomechanical realization that we have in E6.

Consider a 1D tweezer array of atoms placed at the nodes of the cavity standing wave. For simplicity, we take the tweezers to be spaced by an integer-number of wavelengths λ , such that all the atoms are coupled to the cavity with the same sign. The atoms are pumped from the side with a pump laser of frequency ω_p and Rabi frequency Ω . Without loss of generality, we shall take Ω to be a real number. In the frame rotating at ω_p , the Hamiltonian describing this system is

$$\hat{\mathcal{H}} = -\hbar\Delta_{\text{pc}}\hat{c}^\dagger\hat{c} + \sum_{j=1}^N \frac{p_j^2}{2m} + \frac{1}{2}m\omega_z^2\hat{z}_j^2 - \hbar\Delta_{\text{pa}}\hat{\sigma}_j^+\hat{\sigma}_j^- + \frac{\hbar\Omega}{2}(\hat{\sigma}_j^+ + \hat{\sigma}_j^-) + \hbar g_0 \sin(k\hat{z}_j)(\hat{c}^\dagger\hat{\sigma}_j^- + \hat{\sigma}_j^+\hat{c}) \quad (4.10)$$

where $\Delta_{\text{pa}} \equiv \omega_p - \omega_a$, $\Delta_{\text{pc}} \equiv \omega_p - \omega_c$, and ω_a is the frequency of the atomic transition. Here, we have assumed that the tweezer traps are harmonic with trap frequency ω_z . For simplicity, we focus on the 1D motion along the cavity axis, ignoring atomic motion along the pump and tweezer axes. Assuming $\Delta_{\text{pa}} \gg \{\Omega, g_0, \kappa, \gamma\}$, we may adiabatically eliminate the atomic excited state and obtain

$$\hat{\mathcal{H}}' = -\hbar\Delta_{\text{pc}}\hat{c}^\dagger\hat{c} + \sum_{j=1}^N \frac{p_j^2}{2m} + \frac{1}{2}m\omega_z^2\hat{z}_j^2 + \hbar\frac{g_0\Omega}{2\Delta_{\text{pa}}}(\hat{c} + \hat{c}^\dagger)\sin(k\hat{z}_j) + \hbar\frac{g_0^2}{\Delta_{\text{pa}}}\sin^2(k\hat{z}_j)\hat{c}^\dagger\hat{c} \quad (4.11)$$

We see two terms that describe the atom-cavity coupling emerge. There is a term proportional to $\hat{c} + \hat{c}^\dagger$ and one proportional to $\hat{c}^\dagger\hat{c}$. As we shall see, the $\hat{c} + \hat{c}^\dagger$ term describes a Dicke-like interaction between the atoms and the cavity field. The $\hat{c}^\dagger\hat{c}$ is the dispersive shift of the atoms on the cavity. For $|\Delta_{\text{pc}}| \gg |Ng_0^2/\Delta_{\text{pa}}|$, the effect of this term is negligible and we ignore it for the present discussion. However, as we shall see in Sec. 4.3.1, the dispersive shift creates a Kerr nonlinearity that results in bistability and hysteresis.

We now show how Eq. (4.11) maps to the Dicke model. For this, we shall make use of the Holstein-Primakoff representation Eq. (4.4). First, we make the Lamb-Dicke approximation $\sin(k\hat{z}_j) \simeq k\hat{z}_j$, which assumes small displacements of the atoms away from the cavity nodes. Ignoring the dispersive shift, we notice that the cavity field only couples to the center-of-mass position of the atoms through $(\hat{c} + \hat{c}^\dagger)\sum_j \hat{z}_j$. Thus, we expect that the underlying physics is captured by just considering the center-of-mass mode, where all the atoms move in phase with one another. In general, we refer to the mechanical mode that couples linearly to the cavity as the dominant mode. In the Hamiltonian Eq. (4.10), the center-of-mass mode is the dominant mode because of our choice to write the cavity coupling terms symmetrically for all the atoms. We re-write the Hamiltonian in terms of the center-of-mass operators to

obtain

$$\hat{\mathcal{H}}_{\text{com}} = -\hbar\Delta_{\text{pc}}\hat{c}^\dagger\hat{c} + \frac{\hat{p}_{\text{com}}^2}{2M} + \frac{1}{2}M\omega_z^2\hat{z}_{\text{com}}^2 + \hbar\frac{Ng_0\Omega}{2\Delta_{\text{pa}}}(\hat{c} + \hat{c}^\dagger)\sin(k\hat{z}_{\text{com}}) \quad (4.12)$$

$$\simeq -\hbar\Delta_{\text{pc}}\hat{c}^\dagger\hat{c} + \frac{\hat{p}_{\text{com}}^2}{2M} + \frac{1}{2}M\omega_z^2\hat{z}_{\text{com}}^2 + \hbar k\frac{Ng_0\Omega}{2\Delta_{\text{pa}}}(\hat{c} + \hat{c}^\dagger)\hat{z}_{\text{com}} \quad (4.13)$$

where $\hat{z}_{\text{com}} = \sum_j \hat{z}_j/N$, $\hat{p}_{\text{com}} = \sum_j \hat{p}_j$, and $M = Nm^1$. We then write $\hat{p}_{\text{com}}$ and $\hat{z}_{\text{com}}$ in terms of bosonic ladder operators $\hat{b}$ and $\hat{b}^\dagger$. This yields

$$\hat{\mathcal{H}}'_{\text{com}} = -\hbar\Delta_{\text{pc}}\hat{c}^\dagger\hat{c} + \hbar\omega_z\hat{b}^\dagger\hat{b} + \hbar k\frac{Ng_0\Omega}{2\Delta_{\text{pa}}}\sqrt{\frac{\hbar}{2M\omega_z}}(\hat{c} + \hat{c}^\dagger)(\hat{b} + \hat{b}^\dagger). \quad (4.14)$$

In going from Eq. (4.13) to Eq. (4.14), we have also substituted $\hat{z}_{\text{com}} = \sqrt{\hbar/(2M\omega_z)}(\hat{b} + \hat{b}^\dagger)$. From Eq. (4.14), the connection to the Dicke model in the Holstein-Primakoff representation is immediately clear. Upon the mappings $\omega_c \rightarrow -\Delta_{\text{pc}}$ and $\alpha \rightarrow \hbar k\frac{Ng_0\Omega}{2\Delta_{\text{pa}}}\sqrt{\frac{\hbar}{2M\omega_z}}$, one recovers Eq. (4.4).

Including cavity decay

So far, we have only investigated the Hamiltonian physics of this driven system. However, we know that light leaks out of the cavity at rate κ , providing dissipation. We can consider the effect of κ through the Lindblad master equation with jump operator $\sqrt{2\kappa}\hat{c}$, which yields the following equations of motion for $\hat{\mathcal{H}}_{\text{com}}$:

$$\dot{\hat{c}} = (i\Delta_{\text{pc}} - \kappa)\hat{c} - i\frac{N\Omega}{2\Delta_{\text{pa}}}g_0\sin(k\hat{z}_{\text{com}}) + \sqrt{\kappa}\hat{c}_{\text{in}} \quad (4.15)$$

$$\dot{\hat{p}}_{\text{com}} = -M\omega_z^2\hat{z}_{\text{com}} - \frac{N\Omega}{2\Delta_{\text{pa}}}\hbar k g_0 \cos(k\hat{z}_{\text{com}})(\hat{c} + \hat{c}^\dagger) \quad (4.16)$$

$$\dot{\hat{z}}_{\text{com}} = \frac{\hat{p}_{\text{com}}}{M} \quad (4.17)$$

For the sake of rigor, in Eq. (4.15), we have included a vacuum noise input to the cavity in the form of $\hat{c}_{\text{in}}$. However, let us now appeal to mean field theory and replace all operators by their expectation values, in which case $\langle\hat{c}_{\text{in}}\rangle = 0$.

Mechanical equations of motion

We can simplify Eqs. (4.15-4.17) further by adiabatically eliminating the cavity field. This assumes the cavity field to be in its steady state, $\dot{\hat{c}} = 0$. Generally, adiabatic elimination

¹There is some subtlety to the approximations made here, since $\sum_j \sin(k\hat{z}_j)/N \neq \sin(k\sum_j \hat{z}_j/N)$ in general. We assume that the approximation is reasonable in the Lamb-Dicke regime.

of the cavity is valid when Δ_{pc} dominates over other timescales in the system [73]; while this is not strictly true in our experiment, it holds at low pump powers, or when the atomic positions do not stray far from the cavity nodes. The adiabatic cavity field is given by

$$c = \frac{iN\Omega g_0}{2\Delta_{\text{pa}}} \frac{1}{i\Delta_{\text{pc}} - \kappa} \sin(kz_{\text{com}}) \quad (4.18)$$

Plugging the expression for c into Eq. (4.16), we obtain a purely mechanical expression for the force on the center-of-mass:

$$\dot{p}_{\text{com}} = -M\omega_z^2 z_{\text{com}} - \frac{\hbar N^2 \Omega^2 g_0^2 k}{4\Delta_{\text{pa}}^2} \frac{\Delta_{\text{pc}}}{\Delta_{\text{pc}}^2 + \kappa^2} \sin(2kz_{\text{com}}). \quad (4.19)$$

From Eq. (4.19), we see that for small z_{com} and $\Delta_{\text{pc}} < 0$, the force provided by the cavity interaction cancels the confining force of the tweezers, resulting in a softened trap frequency. The critical point occurs when the tweezer harmonic potentially is completely canceled out.

Critical point of the mechanical Dicke model

We are now set up to solve for the critical point of our mechanical Dicke model. For this, we shall appeal to bifurcation theory [93]. We inspect how the stability of the normal phase ($p_{\text{com}} = 0, z_{\text{com}} = 0$) changes as we vary the system parameters. To learn about the stability of the normal phase, we inspect the Jacobian matrix $\mathbf{J}$ evaluated at $(p_{\text{com}} = 0, z_{\text{com}} = 0)$:

$$\mathbf{J}(p_{\text{com}}^*, z_{\text{com}}^*) = \begin{pmatrix} \partial \dot{p}_{\text{com}} / \partial p_{\text{com}} & \partial \dot{p}_{\text{com}} / \partial z_{\text{com}} \\ \partial \dot{z}_{\text{com}} / \partial p_{\text{com}} & \partial \dot{z}_{\text{com}} / \partial z_{\text{com}} \end{pmatrix} \bigg|_{(p_{\text{com}}^*, z_{\text{com}}^*)} \quad (4.20)$$

$$\mathbf{J}(0, 0) = \begin{pmatrix} 0 & -M\omega_z^2 - \frac{\hbar N^2 \Omega^2 g_0^2 k^2 \Delta_{\text{pc}}}{2M\Delta_{\text{pa}}^2 (\Delta_{\text{pc}}^2 + \kappa^2)} \\ \frac{1}{M} & 0 \end{pmatrix}. \quad (4.21)$$

The eigenvalues of $\mathbf{J}(0, 0)$ tell us about the stability of the equilibrium point $(0, 0)$. Any eigenvalue with positive real part indicates instability, as that corresponds to an exponentially growing solution. If all eigenvalues have a negative real part, the equilibrium point is asymptotically stable. Purely imaginary eigenvalues correspond to oscillatory solutions. A bifurcation occurs when an eigenvalue crosses 0, indicating a change in stability. This critical point occurs when

$$\det(\mathbf{J}(0, 0)) = \omega_z^2 + \frac{\hbar N^2 \Omega^2 g_0^2 k^2 \Delta_{\text{pc}}}{2M\Delta_{\text{pa}}^2 (\Delta_{\text{pc}}^2 + \kappa^2)} = 0 \quad (4.22)$$

Note that this condition is equivalent to the complete cancellation of the harmonic forces in Eq. (4.19). We see that there are many parameters we can scan that would take us to the critical point. In E6's work, we primarily focused on the critical pump strength,

$$\Omega_c = \sqrt{-\frac{2m\omega_z^2 \Delta_{\text{pa}}^2 (\Delta_{\text{pc}}^2 + \kappa^2)}{\hbar N g_0^2 k^2 \Delta_{\text{pc}}}}. \quad (4.23)$$

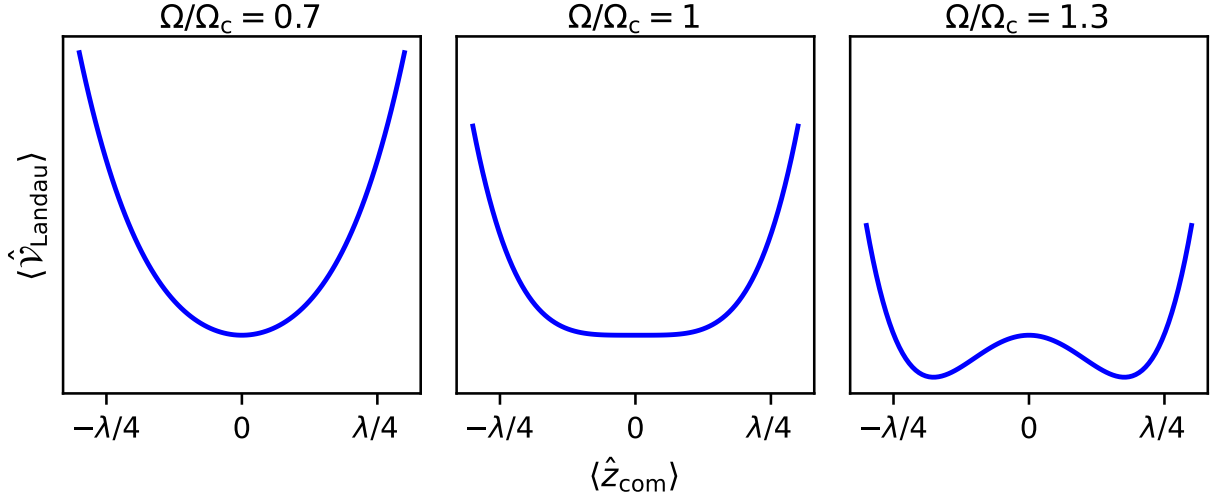


Figure 4.1: Landau potentials for the optomechanical phase transition. As a function of Ω , the Landau potential adjusts from being approximately harmonic ($\Omega < \Omega_c$) to being quartic ($\Omega = \Omega_c$) to forming a double well ($\Omega > \Omega_c$).

With the expression for Ω_c in hand, we make a few observations. Ω_c only satisfies our prior assumption that $\Omega \in \mathbb{R}$ when $\Delta_{\text{pc}} < 0$. From Eq. (4.19), we see that for small z_{com} , having $\Delta_{\text{pc}} > 0$ generates a harmonic force that further confines the atoms, stiffening the trapping potential rather than softening it. On the other hand, if $\Delta_{\text{pc}} = 0$, the cavity-induced term in Eq. (4.19) is zero. When the pump is on resonance with the cavity, the atoms emit a field that is $\pi/2$ out of phase with the pump, thus there is no interference.

4.1.3 Landau theory

We shall now see how the theory we have developed relates to Landau's theory for second order phase transitions [94]. Plugging Eq. (4.18) into Eq. (4.12), we obtain a purely mechanical effective Hamiltonian $\hat{\mathcal{H}}_{\text{mech}}$

$$\hat{\mathcal{H}}_{\text{mech}} = \frac{\hat{p}_{\text{com}}^2}{2M} + \frac{1}{2}M\omega_z^2\hat{z}_{\text{com}}^2 + \hbar \frac{N^2 g_0^2 \Omega^2}{4\Delta_{\text{pa}}^2} \frac{\Delta_{\text{pc}}}{\Delta_{\text{pc}}^2 + \kappa^2} \sin^2(k\hat{z}_{\text{com}}) \quad (4.24)$$

$$= \frac{\hat{p}_{\text{com}}^2}{2M} + \underbrace{\frac{1}{2}M\omega_z^2\hat{z}_{\text{com}}^2 - \frac{M\omega_z^2 \Omega^2}{2k^2 \Omega_c^2} \sin^2(k\hat{z}_{\text{com}})}_{\hat{\mathcal{V}}} \quad (4.25)$$

Here, we have identified the potential energy $\hat{\mathcal{V}}$ in terms of the ratio Ω/Ω_c . Expanding $\hat{\mathcal{V}}$ to fourth order in $\hat{z}_{\text{com}}$, we obtain

$$\hat{\mathcal{V}} = \underbrace{\frac{1}{2}M\omega_z^2\left(1 - \frac{\Omega^2}{\Omega_c^2}\right)\hat{z}_{\text{com}}^2 + \frac{1}{6}\frac{\Omega^2}{\Omega_c^2}M\omega_z^2k^2\hat{z}_{\text{com}}^4}_{\hat{\mathcal{V}}_{\text{Landau}}} + \mathcal{O}(\hat{z}_{\text{com}}^6). \quad (4.26)$$

The picture for how the phase transition occurs is depicted in Fig. 4.1. The atoms, initially situated at $\langle \hat{z}_{\text{com}} \rangle = 0$, see only the harmonic trap of the tweezers when $\Omega = 0$. As Ω increases, the harmonic trap softens due to the potential from the cavity canceling that of the tweezers. At $\Omega = \Omega_c$, the harmonic potential vanishes and the atoms are only confined by the quartic potential that arises from the nonlinearity of the sinusoidal standing wave. Finally, for $\Omega > \Omega_c$, the curvature at $\langle \hat{z}_{\text{com}} \rangle = 0$ becomes negative and $\hat{\mathcal{V}}_{\text{Landau}}$ deforms into a double-well potential with minima displaced from $\langle \hat{z}_{\text{com}} \rangle = 0$. These minima correspond to symmetry-broken solutions where the atoms have moved towards cavity field antinodes of the same sign. This potential $\hat{\mathcal{V}}_{\text{Landau}}$ has a form analogous to that of a generic, thermodynamic potential (such as free energy) for a system exhibiting a continuous phase transition [94]. However, it should be noted that $\hat{\mathcal{V}}_{\text{Landau}}$ is purely an optomechanical potential without notions of temperature or entropy; it is not equivalent to a Landau free energy. Nonetheless, we draw inspiration from Landau's theory in our analysis of the critical point Ω_c .

4.1.4 Temperature alters the critical point

Relating back to the discussion in Sec. 4.1.1, one might ask how the atoms' temperature affects the phase transition in this driven-dissipative system. In 2024, when we were tackling this problem, it was not clear what the approach to this question should be. Given that the system is out-of-equilibrium, temperature is not well-defined, and one could not simply use free energy as one does in equilibrium statistical mechanics. However, comparing our experimentally obtained critical point to Eq. (4.23), there was a clear discrepancy—the experimentally obtained Ω_c was systematically higher than the theoretical prediction. From mean field Monte Carlo simulations, it appeared that non-zero initial motional temperature raised Ω_c . We set out to understand the origin of this behavior.

We closely inspected the approximations we made to arrive at Eq. (4.23), and realized that the answer lay in the leap from Eq. (4.11) to Eq. (4.12), where we ignore the contributions of the normal modes which do not couple to the cavity field. To gain some intuition for what these other modes do, we examine the two-atom case in detail below.

Two-atom system

The physical system we treat here is described by the following Hamiltonian:

$$\hat{\mathcal{H}} = \frac{\hat{p}_1^2}{2m} + \frac{\hat{p}_2^2}{2m} + \frac{1}{2}m\omega_z^2\hat{z}_1^2 + \frac{1}{2}m\omega_z^2\hat{z}_2^2 + \frac{\hbar\Omega g_0}{2\Delta_{\text{pa}}}(\hat{c} + \hat{c}^\dagger)(\sin(kz_1) + \sin(kz_2)) - \hbar\Delta_{\text{pc}}\hat{c}^\dagger\hat{c}. \quad (4.27)$$

Here, as before, we have adiabatically eliminated the atomic excited state and ignored the dispersive shift of the atoms on the cavity resonance.

We can write down equations of motion for the operators according to the Lindblad master equation:

$$\dot{\hat{c}} = (i\Delta_{\text{pc}} - \kappa)\hat{c} - i\frac{\Omega g_0}{2\Delta_{\text{pa}}}(\sin(k\hat{z}_1) + \sin(k\hat{z}_2)), \quad (4.28)$$

$$\dot{\hat{p}}_1 = -m\omega_z^2\hat{z}_1 - \frac{\hbar\Omega g_0 k}{2\Delta_{\text{pa}}}(\hat{c} + \hat{c}^\dagger)\cos(k\hat{z}_1), \quad (4.29)$$

$$\dot{\hat{p}}_2 = -m\omega_z^2\hat{z}_2 - \frac{\hbar\Omega g_0 k}{2\Delta_{\text{pa}}}(\hat{c} + \hat{c}^\dagger)\cos(k\hat{z}_2), \quad (4.30)$$

$$\dot{\hat{z}}_1 = \frac{\hat{p}_1}{m}, \quad (4.31)$$

$$\dot{\hat{z}}_2 = \frac{\hat{p}_2}{m}. \quad (4.32)$$

From this point forward, we will adopt mean field theory and replace all operators with their expectation values. We see that when the cavity field is in its steady state, we have

$$c = \frac{i\Omega g_0}{2\Delta_{\text{pa}}} \frac{1}{i\Delta_{\text{pc}} - \kappa} (\sin(kz_1) + \sin(kz_2)), \quad (4.33)$$

$$c + c^* = \frac{\Omega g_0}{\Delta_{\text{pa}}} \frac{\Delta_{\text{pc}}}{\Delta_{\text{pc}}^2 + \kappa^2} (\sin(kz_1) + \sin(kz_2)). \quad (4.34)$$

Using the steady-state field, we now obtain purely mechanical equations of motion:

$$\dot{p}_1 = -m\omega_z^2 z_1 - \frac{\hbar\Omega^2 g_0^2 k}{2\Delta_{\text{pa}}^2} \frac{\Delta_{\text{pc}}}{\Delta_{\text{pc}}^2 + \kappa^2} (\sin(kz_1) + \sin(kz_2)) \cos(kz_1), \quad (4.35)$$

$$\dot{p}_2 = -m\omega_z^2 z_2 - \frac{\hbar\Omega^2 g_0^2 k}{2\Delta_{\text{pa}}^2} \frac{\Delta_{\text{pc}}}{\Delta_{\text{pc}}^2 + \kappa^2} (\sin(kz_1) + \sin(kz_2)) \cos(kz_2), \quad (4.36)$$

$$\dot{z}_1 = \frac{p_1}{m}, \quad (4.37)$$

$$\dot{z}_2 = \frac{p_2}{m}. \quad (4.38)$$

These equations are the starting point for the rest of our dynamics analysis.

Linearized system

Perhaps the simplest thing we can do to try to understand this system is to linearize it. That means we rewrite the system, currently in the form $\vec{x} = \mathbf{F}(\vec{x})$, in the form $\vec{x} = \mathbf{J}\vec{x}$ where the Jacobian matrix $\mathbf{J}$ has elements $J_{ij} = \partial F_i / \partial x_j$ evaluated at the equilibrium point of interest.

Here, $\vec{x} = (p_1, p_2, z_1, z_2)^T$. Linearizing around $\vec{x} = \vec{0}$ gives

$$\begin{pmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -Dk - m\omega_z^2 & -Dk \\ 0 & 0 & -Dk & -Dk - m\omega_z^2 \\ 1/m & 0 & 0 & 0 \\ 0 & 1/m & 0 & 0 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ z_1 \\ z_2 \end{pmatrix} \quad (4.39)$$

where I have defined $D = \frac{\hbar\Omega^2 g_0^2 k}{2\Delta_{\text{pa}}^2} \frac{\Delta_{\text{pc}}}{\Delta_{\text{pc}}^2 + \kappa^2}$ for convenience. Note that $D < 0$ since we choose $\Delta_{\text{pc}} < 0$ for the phase transition to exist.

The eigenvalues of Eq. (4.39) are given by

$$\pm i\omega_z, \pm \frac{\sqrt{-m(2Dk + m\omega_z^2)}}{m}. \quad (4.40)$$

From this, it is apparent that when $2D + m\omega_z^2 > 0$, all the eigenvalues are purely imaginary. This corresponds to the $\vec{0}$ solution being stable. However, once $2D + m\omega_z^2 < 0$, the $\vec{0}$ solution becomes unstable (a saddle point) because we get an eigenvalue with positive real part. Therefore, the critical point occurs at $2D + m\omega_z^2 = 0$. This is exactly the zero-temperature, mean field critical point that we already know. This is all to say that in the linear regime, we do not learn anything new: The critical point of the center-of-mass mode is unaffected by the presence of the relative motion mode. To see new behavior, we have to include nonlinearity.

Nonlinear coupling

To include nonlinear coupling, we return to Eqs. (4.35-4.38). Naively, we could just start expanding the $\sin(kz)$ and $\cos(kz)$ terms to nonlinear order and see where that takes us. A better approach is to rewrite the system in terms of the “sum” and “difference” modes $z_s \equiv z_1 + z_2$, $z_d \equiv z_1 - z_2$, $p_s = m\dot{z}_s$, and $p_d = m\dot{z}_d$. We can do so by adding and subtracting Eqs.(4.35-4.36), and similarly for Eqs.(4.37-4.38). This yields

$$\dot{p}_s = -m\omega_z^2 z_s - D(\sin(kz_1) + \sin(kz_2))(\cos(kz_1) + \cos(kz_2)) \quad (4.41)$$

$$\dot{p}_d = -m\omega_z^2 z_d - D(\sin(kz_1) + \sin(kz_2))(\cos(kz_1) - \cos(kz_2)) \quad (4.42)$$

$$\dot{z}_s = \frac{p_s}{m} \quad (4.43)$$

$$\dot{z}_d = \frac{p_d}{m} \quad (4.44)$$

These equations still have z_1 and z_2 in them. However, it turns out that by expanding them to third order in z_1, z_2 , we can actually write the equations solely in terms of z_s and z_d :

$$(\sin(kz_1) + \sin(kz_2))(\cos(kz_1) + \cos(kz_2)) \quad (4.45)$$

$$\begin{aligned} &= \frac{1}{6}(kz_1 + kz_2)(12 - 3k^2(z_1 - z_2)^2 - 2k^2(z_1 + z_2)^2) + \mathcal{O}(z^4) \\ &\approx \frac{1}{6}kz_s(12 - 3k^2z_d^2 - 2k^2z_s^2), \end{aligned} \quad (4.46)$$

$$(\sin(kz_1) + \sin(kz_2))(\cos(kz_1) - \cos(kz_2)) = -\frac{1}{2}k^3(z_1 - z_2)(z_1 + z_2)^2 + \mathcal{O}(z^4) \quad (4.47)$$

$$\approx -\frac{1}{2}k^3z_dz_s^2. \quad (4.48)$$

Now, the equations of motion become

$$\dot{p}_s = -m\omega_z^2z_s - \frac{Dk}{6}z_s(12 - 3k^2z_d^2 - 2k^2z_s^2), \quad (4.49)$$

$$\dot{p}_d = -m\omega_z^2z_d + \frac{Dk^3}{2}z_dz_s^2, \quad (4.50)$$

$$\dot{z}_s = \frac{p_s}{m}, \quad (4.51)$$

$$\dot{z}_d = \frac{p_d}{m}. \quad (4.52)$$

Incidentally, Eqs. (4.49-4.52) have a very simple stationary state solution in the bifurcated regime if we only consider center-of-mass motion: $z_d = 0$ and $z_s = \pm(\frac{3}{Dk^3}(m\omega_z^2 + 2Dk))^{1/2}$.

Solving the dynamics of nonlinear differential equations is generally non-trivial. However, if the system is governed by Hamiltonian evolution, then we can gain some insight easily: the phase space trajectory is simply given by the constant energy level sets of the Hamiltonian. It turns out that there is an effective Hamiltonian that produces Eqs. (4.49-4.52):

$$\mathcal{H}_{\text{eff}} = \frac{p_s^2}{2m} + \frac{p_d^2}{2m} + \frac{1}{2}(m\omega_z^2 + 2Dk)z_s^2 + \frac{1}{2}m\omega_z^2z_d^2 - \frac{Dk^3}{12}z_s^4 - \frac{Dk^3}{4}z_d^2z_s^2. \quad (4.53)$$

$\mathcal{H}_{\text{eff}}$ satisfies $\dot{p}_i = -\partial\mathcal{H}_{\text{eff}}/\partial z_i$ and $\dot{z}_i = \partial\mathcal{H}_{\text{eff}}/\partial p_i$ for $i \in \{s, d\}$.

We can rewrite $\mathcal{H}_{\text{eff}}$ in a more illuminating way:

$$\mathcal{H}_{\text{eff}} = \frac{p_s^2}{2m} + \frac{p_d^2}{2m} + \frac{1}{2}(m\omega_z^2 + 2Dk - \frac{Dk^3}{2}z_d^2)z_s^2 + \frac{1}{2}m\omega_z^2z_d^2 - \frac{Dk^3}{12}z_s^4. \quad (4.54)$$

Inspecting the z_s^2 term, we see that a non-zero z_d^2 effectively raises the transition point, which is given by $m\omega_z^2 + 2Dk - \frac{Dk^3}{2}z_d^2 = 0$. If $z_d = 0$, then we just recover the same result we got from linearization.

If we assume that the z_d mode is in thermal equilibrium and invoke the equipartition theorem, we find $\langle z_d^2 \rangle = 2k_B T / (m\omega_z^2)$, where the factor of 2 accounts for normalization of

the z_d mode. Replacing $z_d^2 \rightarrow \langle z_d^2 \rangle$ in the z_s^2 term gives us the bifurcation condition

$$m\omega_z^2 + 2Dk - Dk^3 \frac{k_B T}{m\omega_z^2} = 0. \quad (4.55)$$

We have thus obtained a critical point that depends on the temperature of the relative motion.

From this analysis of two atoms, we gain the intuition that the modes orthogonal to the center-of-mass mode affect the critical point through nonlinear coupling to the center-of-mass mode. These “other modes” effectively form a thermal bath to which the center-of-mass mode equilibrates. This result can be generalized to N atoms and all orders of nonlinear coupling to obtain the temperature-dependent critical point

$$\Omega_c(T) = \sqrt{-\frac{2m\omega_z^2 \Delta_{\text{pa}}^2 (\Delta_{\text{pc}}^2 + \kappa^2)}{\hbar N g_0^2 k^2 \Delta_{\text{pc}} \epsilon(T)}}, \text{ where} \quad (4.56)$$

$$\epsilon(T) = e^{-\eta_T^2} \frac{\sinh(\eta_T^2)}{\eta_T^2}. \quad (4.57)$$

Here, we have used the thermal Lamb-Dicke parameter $\eta_T \equiv \sqrt{\frac{\hbar k^2}{2m\omega_z} \coth(\hbar\omega_z/(2k_B T))}$. This calculation was performed by our collaborator Cosimo Rusconi; details can be found in the Supplementary Information of Ref. [55].

4.2 Experimental realization of the tweezer Dicke model

With the above theoretical background on the Dicke model and its tweezer-based variant, we are now poised to discuss its experimental realization on the E6 apparatus.

The experimental setup is represented schematically in Fig. 4.2. As described previously, we use an array of optical tweezers to place atoms at the nodes of our cavity’s standing wave. A retro-reflected, circularly polarized pump illuminates the atoms transverse to the cavity. The atoms scatter light into the cavity, and that light leaks out through the cavity mirror with higher transmission. That emission is sent to a heterodyne detector, which measures both quadratures of the y -polarized cavity electric field. The measured cavity field serves as a monitor of the atomic motion, since $c \propto \sum_{j=1}^N \sin(kz_j)$. I shall now give some details about technical choices made while performing these experiments.

4.2.1 Tweezer configuration

We chose the tweezer array to have a spacing of 5.5λ . As such, the motional mode that couples to the cavity is no longer the center-of-mass mode, but rather the “anti-phase” mode, in which neighboring atoms move in opposite directions. The choice of a half-integer-wavelength spacing was made for experimental reasons: we wished to avoid biasing of the

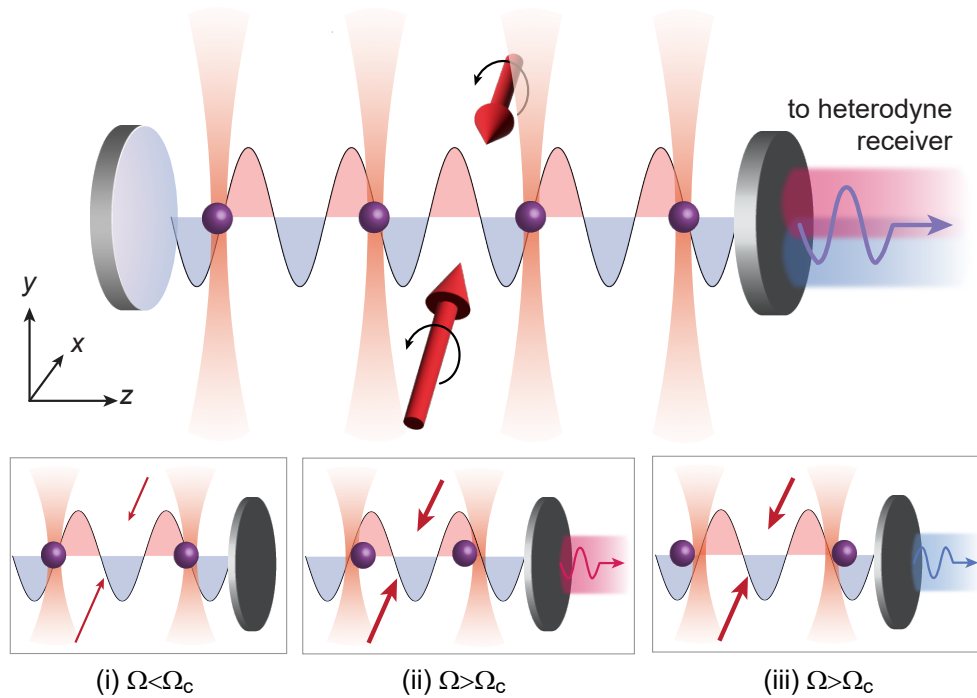


Figure 4.2: Schematic of the experimental setup for cavity-induced self-organization. Atoms are trapped in optical tweezers and placed at the nodes of the cavity field. The atom array is pumped from the side with circularly polarized light. Light emitted by the atoms into the cavity leaks out of one mirror and is measured on a heterodyne detector. Insets show the mechanism for the superradiant phase transition. (i) Below the critical point ($\Omega < \Omega_c$), the atoms remain centered in the tweezers at the nodes and only emit light incoherently. (ii-iii) Above the critical point ($\Omega > \Omega_c$), the atoms self-organize by moving towards cavity field antinodes of the same sign. Doing so, they emit light into the cavity in-phase with one another, leading to a superradiant cavity field whose phase is determined by the sign of the antinodes chosen by the atoms. Thus, the superradiant phase transition has a $\mathbb{Z}_2$ symmetry. Figure adapted from Ref. [55].

phase transition due to center-of-mass drift of the tweezer array relative to the cavity. Center-of-mass drifts occur in E6 because the tweezer AOD and other tweezer optics are mounted on a different breadboard from the science cavity. As we noticed in our previous superradiance work [53]), drifts of the tweezers relative to the standing waves (both cavity and pump) occur on the scale of minutes, with the drift of the pump being more severe than that of the cavity. This motivated us to scan the atoms along the pump standing wave on each iteration of the Cicero sequence, so that we could always monitor the drift in our data analysis. This method was eventually replaced with shot-to-shot feedback through a Red Pitaya FPGA that was set up by Leon Lu. In each iteration of the Cicero sequence, we scan the atoms along both the cavity and pump directions while illuminating them with pump and molasses light. The Red Pitaya fits a sinusoid to each scan and determines the phase. It feeds a voltage proportional to the phase offset back to the tweezer piezo mirror such that a specific control voltage from Cicero always corresponds to the same phase offset of the tweezer array from the standing wave. Although the tweezer position feedback took care of shot-to-shot variations, it was not adequate for us to be confident that the symmetry breaking we observed with an integer-wavelength-spaced array was truly spontaneous (a result of thermal and quantum fluctuations) or explicitly broken by the relative drift of the breadboards. This issue was resolved by going to half-integer-wavelength spacing. Since we believe there is no systematic drift in the experiment that biases the anti-phase mode for even atom numbers, we were confident that we were truly observing spontaneous symmetry breaking in the half-integer spacing configuration.

4.2.2 Pump polarization

We also chose the pump polarization to be circular. The idea was that the circularly polarized pump would optically pump the atoms to the stretched m_F state, with the quantization axis being set by the vector Stark shift. Once the atoms have reached the stretched state, they are only excited on the stretched transition and no polarization dynamics interfere with the self-organization process. In this configuration, we could extract a critical point that did not depend on the spin state of the atoms, which was a concern we had about other pump polarizations. For instance, we saw that with a y -polarized pump, the emitted cavity field would break symmetry in the handedness of the emitted circular polarization. This made it convoluted to determine whether the bifurcation we observed in the cavity field was due to mechanics or optical pumping of the spins. A circular polarized pump circumvented this issue, at the cost of reducing $g_0 \rightarrow g_0/\sqrt{2}$. As a result of our polarization choice, we only see signal of bifurcation occurring in the y -polarized cavity field.

4.2.3 Unexpected observations

An interesting observation we made in the process of data-taking was that having a non-zero magnetic field seemed to heat up the atoms, raising the critical point. We had taken a lot of data at a field of 4 G along the pump axis, only to later find that the symmetry breaking

was more stable at lower magnetic field. Reasoning that the vector Stark shift is sufficient to set the quantization axis, we proceeded to take data without any external magnetic field applied.

We made another accidental observation when we returned to the optomechanical Dicke transition after a hiatus during which we rebuilt the heterodyne setup (described in the following section). In that process, we had unknowingly changed the sign of the probe laser EOM sideband that we lock to on our ULE cavity, resulting in us using $\Delta_{\text{pa}} > 0$ when we tried the experiments. To our dismay, we did not see any signs of bifurcation in our heterodyne signal. After we realized the error with the pump frequency, we went back to $\Delta_{\text{pa}} = -2\pi \times 80$ MHz and saw the bifurcation return. This left an interesting puzzle—naively, our effective, purely mechanical Hamiltonian (Eq. 4.24) only depends on Δ_{pa}^2 and thus does not depend on the sign of Δ_{pa} . We reasoned that the suppression of the bifurcation actually comes from the atoms being trapped in the *nodes* of the blue-detuned pump lattice. Our Dicke model theory ignored the effect of the pump ac Stark shift (see Eq. (3.3)), since it can be taken as a constant when the atoms are localized on the pump antinodes. However, for a blue-detuned pump, the trapping effect of the pump cannot be ignored, and is known to induce dynamical behavior beyond what the Dicke model predicts [95].

A final unexpected observation made in hindsight has to do with the lifetime of the self-organized state. In Ref. [55], we only analyzed cavity field time traces over a period of $\sim 200 \mu\text{s}$; we found that probing the atoms for longer resulted in significant atom loss from heating due to the pump light. What we had reasoned to be inescapable heating may have just been the result of a beam size mismatch between the incident and retro-reflected pump beams, causing a net force on the atoms. In a subsequent re-alignment of the pump, we checked the sizes of the incident and retro-reflected pump beams at the same location outside the vacuum chamber using a beam profiler, and tuned the distance between the pump telescope lenses to make them equal (the position of the retro-reflecting mirror was fixed by equalizing the sizes of the incident and retro-reflected molasses beams). Previously, the retro-reflected beam was roughly a factor of 2 larger than the incident beam. With the beam balance fixed, we now observe that the atoms survive for > 1 s with the pump on. Perhaps the side pump creates some sort of polarization-gradient cooling when combined with the cavity field. This will be a topic of future investigation.

4.2.4 Heterodyne detection

A major technical upgrade we had to make for our Dicke model work was to integrate a heterodyne receiver into our cavity detection path. E6’s previous works relied on photon counting using an SPCM, but for the Dicke work we needed to measure both the amplitude and phase of the cavity field. More details about our heterodyne setup can be found in Sec. 2.6.2.

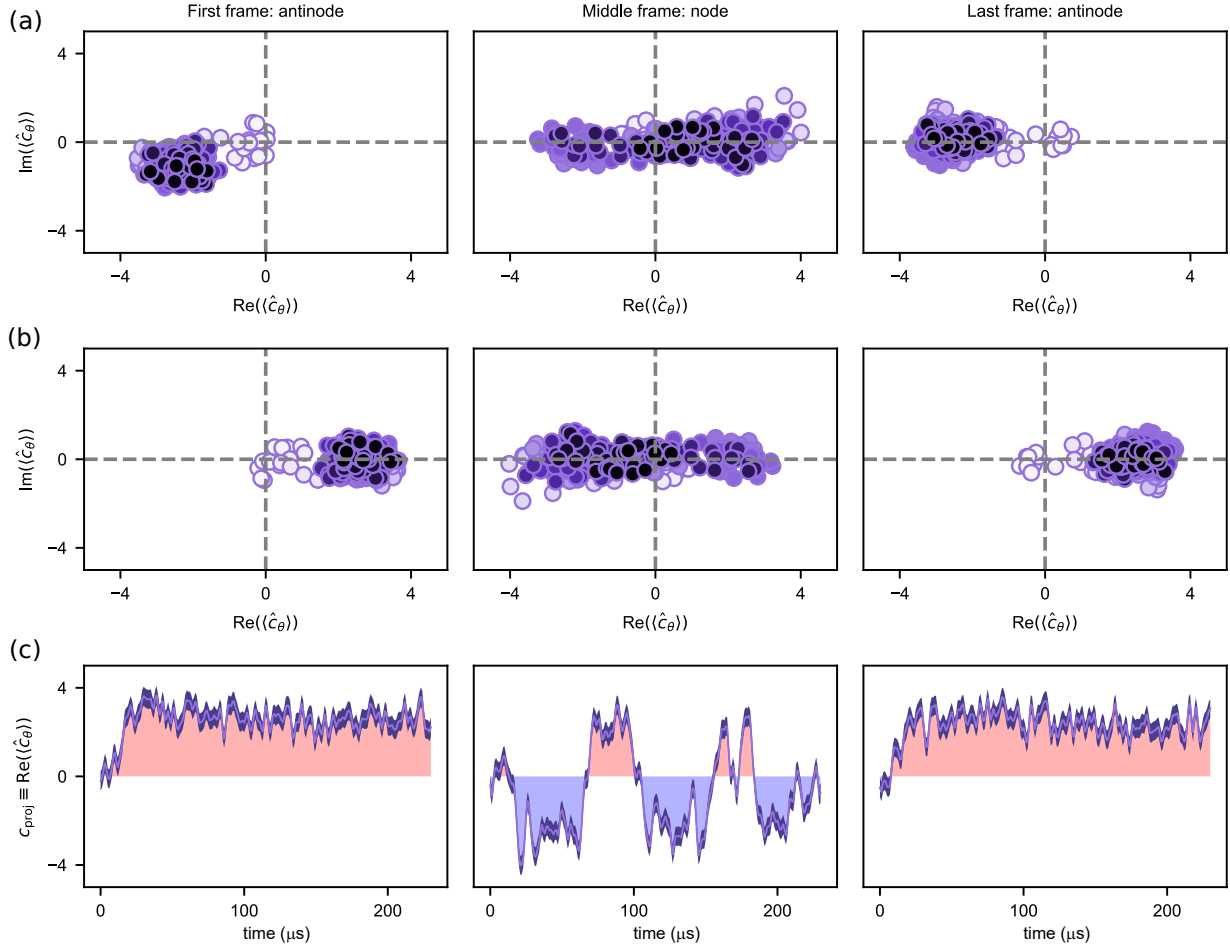


Figure 4.3: Heterodyne phase calibration. (a) Scatter plots of the complex cavity field before phase correction. The real ($\text{Re}(\langle \hat{c}_\vartheta \rangle)$) and imaginary ($\text{Im}(\langle \hat{c}_\vartheta \rangle)$) quadratures of the detected field with respect to the average phase of the atoms' emission into the cavity, ϑ , are plotted. Lighter points indicate earlier times and darker points indicate later times. From these data, the angle ϕ of the major axis in each frame is determined by PCA. (b) Detected field after rotation by ϕ . Because PCA only determines the angle of the major axis up to a π rotation, in the antinode frames, we define the average projection of the field onto the axis defined by ϑ to be positive. The sign of the node frame is determined by comparing ϑ_{node} to the angle linearly interpolated from $\vartheta_{\text{first frame}}$ and $\vartheta_{\text{last frame}}$ and assuming that the two angles are within $\pi/2$ of each other. (c) Time traces of c_{proj} . Values of c_{proj} are obtained by projecting the phase-corrected measurements of $\hat{c}$ (shown in (b)) onto the ϑ -axis. The shaded dark purple regions indicate the shot noise level. Data shown in figure were taken with $N = 18$, $\Delta_{\text{pa}} = -2\pi \times 80 \text{ MHz}$, $\Delta_{\text{pc}} = -2\pi \times 2.15 \text{ MHz}$, and $\Omega = 2\pi \times 63.9 \text{ MHz}$. Figure adapted from Ref. [55].

Phase calibration

To account for relative phase drifts ϕ in the pump and LO paths from fluctuating fiber lengths, we implement a calibration sequence affectionately called “ANA” where A=antinode and N=node (see Fig. 4.3). The logic is as follows. When probing the Dicke phase transition, we place the atoms at the cavity field nodes, and we wish to know whether the atoms have broken symmetry by moving towards the positively signed or negatively signed antinodes. The sign of the symmetry breaking is reflected in the phase of the atoms’ emission ϑ_{nodes} , measured relative to the phase $\vartheta_{\text{antinodes}}$ of the field emitted when atoms are placed at the (positive) antinodes (relative to the pump phase). However, the measured phase ϕ_{measured} also contains the pump-LO phase drift ϕ : $\phi_{\text{measured}} = \phi + \vartheta_{\text{antinodes}} + \vartheta_{\text{nodes}}$. To extract the phase ϑ_{nodes} , we must subtract off $\phi + \vartheta_{\text{antinodes}}$. We may calibrate $\phi + \vartheta_{\text{antinodes}}$ by explicitly placing a 5λ -spaced array of atoms at the antinodes and measuring the phase of the major axis of the emission, determined through principal component analysis (PCA) and with its sign defined such that the average projection of the data onto that axis is positive. We do this antinode calibration before and after the measurement at the nodes. Because the pump-LO relative phase drifts over time, in the antinode frames we measure $\vartheta_{\text{first frame}} = \phi_{\text{first}} + \vartheta_{\text{antinodes}}$ and $\vartheta_{\text{last frame}} = \phi_{\text{last}} + \vartheta_{\text{antinodes}}$, and linearly interpolate between these two phases to estimate the phase ϕ during the node frames. In the node frames, we similarly use PCA to determine the angle of the major axis, and compare this angle to $\phi + \vartheta_{\text{antinodes}}$. We project the data onto the major axis and determine their overall sign by assuming the phase of the data does not drift by more than $\pi/2$ from $\phi + \vartheta_{\text{antinodes}}$. In between each frame, we performed ~ 3 ms of molasses cooling.

Converting heterodyne signal to photon number

To get an absolute conversion between the measured voltage on Gagescope and $\langle \hat{c} \rangle$, we sent probe light into the bare cavity and recorded the demodulated amplitude obtained from the heterodyne detector. At the time, the LO power right before the second PBS was measured to be 0.57 mW. We then compared the demodulated amplitude with the photon count rate measured from the optical power. We equated the measured photon count rate to $2\bar{n}\kappa\epsilon$, where $\bar{n}$ is the intra-cavity photon number and $\epsilon \approx 30\%$ is the optical path efficiency. Setting $\langle \hat{c} \rangle = \sqrt{\bar{n}}$, we calculated a conversion factor between the heterodyne signal and the photon number.

4.3 Future directions

4.3.1 Hysteresis and bistability from quadratic optomechanics

In Eq. (4.11), we saw that there are two contributions of the cavity to the Hamiltonian: the Dicke interaction (proportional to $\hat{c} + \hat{c}^\dagger$) and the dispersive shift (proportional to $\hat{c}^\dagger \hat{c}$). All the analyses done above ignored the dispersive shift, which is negligible for large $|\Delta_{\text{pa}}|$.

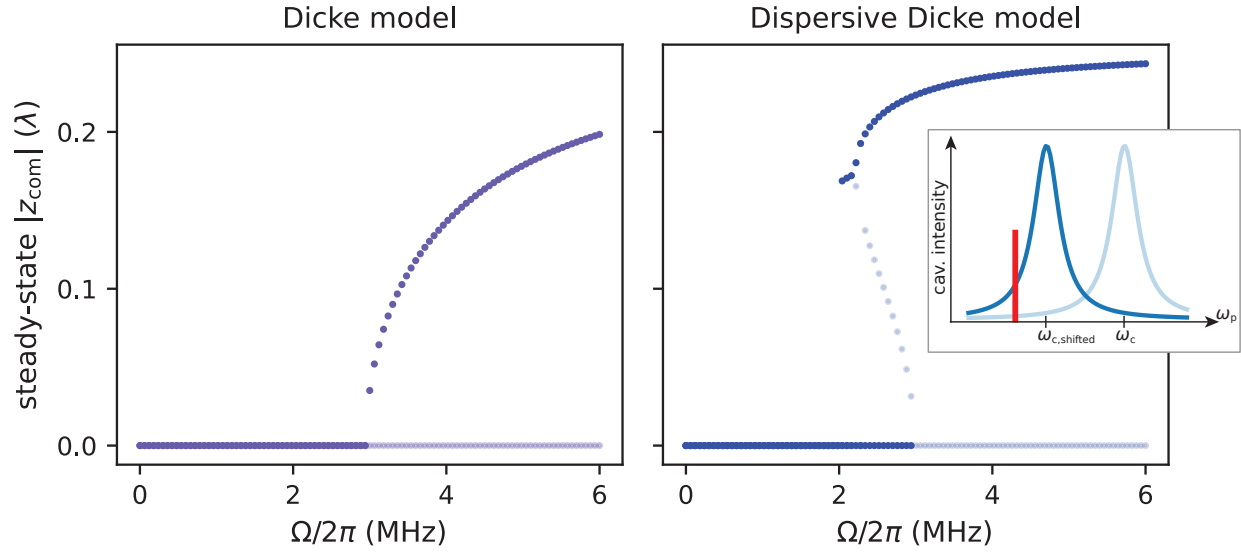


Figure 4.4: Dispersive shift induces bistability. Including the dispersive shift of the cavity resonance from the atoms, relevant at small $|\Delta_{\text{pa}}|$, introduces a Duffing bistability to the Dicke model. The left panel shows the steady state center-of-mass position as a function of Ω in the vanilla Dicke model. Dark points correspond to stable steady states and light points correspond to unstable steady states. The right panel shows the steady states in the presence of the dispersive shift. We see an unstable branch connecting the superradiant ($|z_{\text{com}}| > 0$) phase to the normal phase, corresponding to a bistable region. Inset shows that the dispersive shift brings the cavity resonance closer to the pump frequency, marked by the red line. Simulation uses the parameters $\Delta_{\text{pc}} = -2\pi \times 6$ MHz and $N = 10$.

Now, we consider what the dispersive shift adds to our model. The $\hat{c}^\dagger \hat{c}$ contribution to the Hamiltonian can be written as

$$-\hbar \underbrace{\left(\Delta_{\text{pc}} - \sum_{j=1}^N \frac{g_0^2}{\Delta_{\text{pa}}} \sin^2(k\hat{z}_j) \right)}_{\tilde{\Delta}_{\text{pc}}} \hat{c}^\dagger \hat{c}. \quad (4.58)$$

This makes it apparent that the cavity frequency depends on the displacement of the atoms from the cavity nodes. This is reminiscent of the Duffing (or Kerr) nonlinearity often observed in nanomechanical systems [96, 97], which leads to bistability for a range of Ω when $\Delta_{\text{pc}} < 0$ and $\Delta_{\text{pa}} < 0$. This region corresponds to the range of Ω for which $\Omega < \Omega_c(\Delta_{\text{pc}})$ but $\Omega > \Omega_c(\tilde{\Delta}_{\text{pc}})$. In this range of Ω , with the atoms situated at the nodes and the pump detuned from the cavity by $|\Delta_{\text{pc}}| \gg \kappa$, the pump is sufficiently detuned such that the system does not bifurcate and the atoms remain at the nodes. However, if the atomic center-of-mass were displaced from the nodes, the dispersive shift brings the cavity closer to resonance with

the pump, reducing $|\tilde{\Delta}_{\text{pc}}|$ such that the self-organized phase is stable and self-consistent with the displaced atom positions. Connecting these two branches of stable solutions is an unstable branch, shown in Fig. 4.4. This implies that one should observe hysteresis in the bistable region: the steady-state positions should depend on whether one approaches the critical point from $\Omega > \Omega_c$ or from $\Omega < \Omega_c$.

The dispersive shift modifies the mechanical force equation for the center-of-mass:

$$\dot{p}_{\text{com}} = -M\omega_z^2 z_{\text{com}} - \frac{\hbar N^2 \Omega^2 g_0^2 k}{4\Delta_{\text{pa}}^2} \frac{\Delta_{\text{pc}}}{\left(\Delta_{\text{pc}} - \frac{Ng_0^2}{\Delta_{\text{pa}}} \sin^2(kz_{\text{com}})\right)^2 + \kappa^2} \sin(2kz_{\text{com}}). \quad (4.59)$$

A comparison of the steady-state solutions to Eq. (4.19) and Eq. (4.59) is shown in Fig. 4.4. Though we have not observed hysteresis or bistability in E6 as of yet, this could be a neat direction to pursue, as it incorporates the rich, paradigmatic phenomena of nonlinear systems into the Dicke model.

4.3.2 Direct imaging of symmetry-broken states

A question we often received about our work on the optomechanical Dicke phase transition was whether we could directly image the atom positions to verify their correlation with the cavity field phase. After all, we have seen that our fluorescence imaging is capable of resolving displacements in a single atom's center-of-mass position of ~ 150 nm. Previously, however, we thought this measurement would be very difficult due to the short atom lifetime of $\sim 200 \mu\text{s}$ with the pump light on. Fluorescence imaging in E6 requires ~ 100 ms of exposure time to distinguish a with-atom signal from a without-atom signal with $> 99\%$ fidelity. Furthermore, the symmetry-broken states have a finite lifetime, a key mesoscopic feature observed in Ref. [55]. While the cavity time traces provide time-resolved information, fluorescence imaging does not, so observing the atoms having moved to one side of the cavity field nodes would require real-time gating of the imaging camera based on the sign of the cavity field measured at our heterodyne detector. At the time, we felt that such a gated camera was not worth our investment. However, with our newfound discovery that the atom lifetime is much extended after proper beam-balancing of the pump light (see Sec. 4.2.3), perhaps it will be possible to observe the displacements in the atom positions. Without gating, we still might not be able to resolve symmetry-broken states individually, but perhaps we could resolve a smearing out of the atoms' density distribution when in the superradiant phase. The correlation between atom spatial ordering and superradiance has been confirmed experimentally in recent work [98].

4.3.3 Test of the fluctuation-dissipation theorem

Second-order phase transitions in macroscopic systems are characterized by a diverging susceptibility at the critical point [94, 99, 100]. In Ref. [55], one of our key observations was the finite optomechanical susceptibility χ at the critical point for mesoscopic systems. We

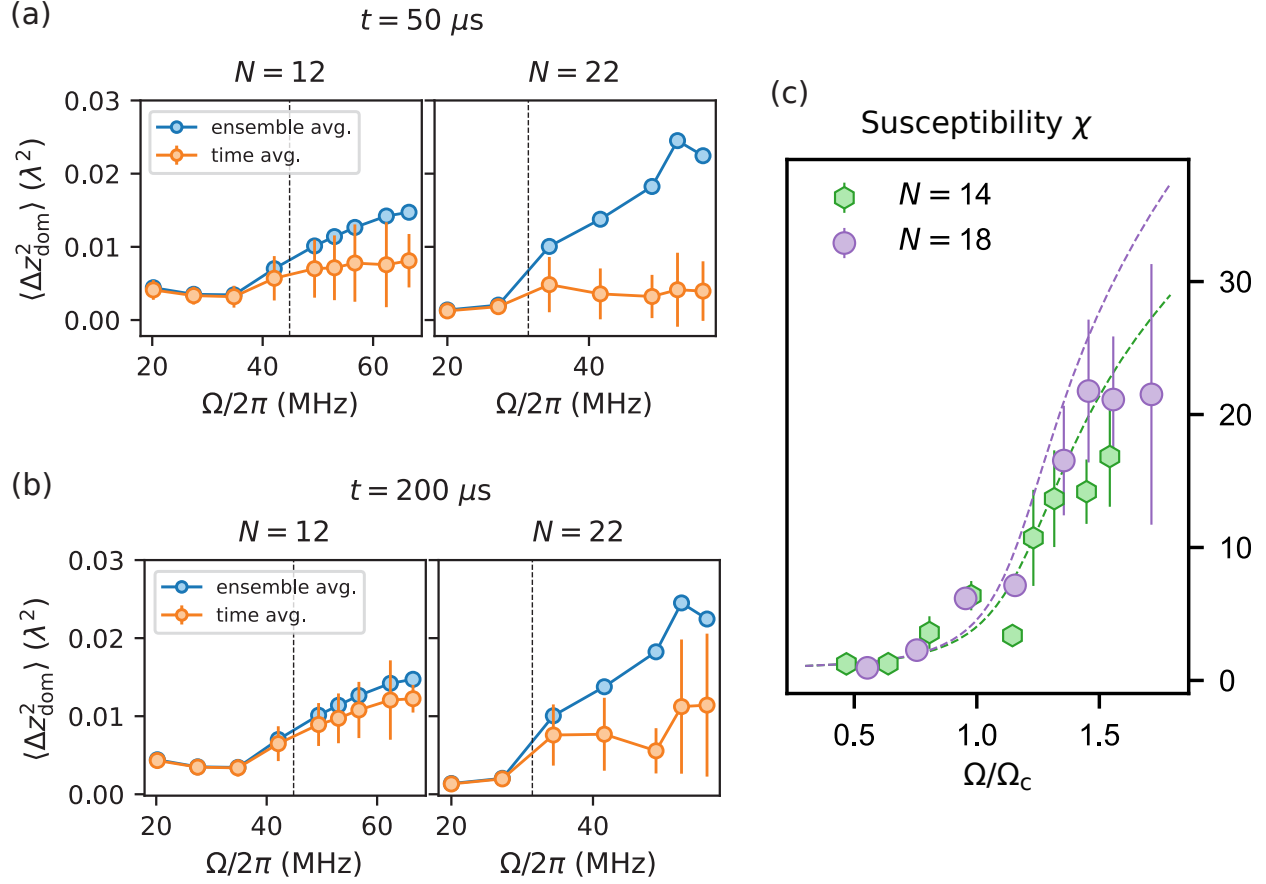


Figure 4.5: Fluctuations and susceptibility to dc forces. (a-b) Variance $\langle \Delta z_{\text{dom}}^2 \rangle$ measured using either an ensemble average (blue) or time average (orange) for $N = 12$ and $N = 22$ over a time duration of (a) $50\ \mu\text{s}$ or (b) $200\ \mu\text{s}$. Dashed lines indicate the critical point Ω_c . Note that the time average approaches the ensemble average when averaging over longer times; this effect is more pronounced at lower N , showing that this is the result of mesoscopic fluctuations. (c) The fluctuations shown in (a-b) may be compared to the susceptibility χ measured in Ref. [55], which measures the response of the atomic center-of-mass position to a dc external force (created by displacing the tweezers). Panel (c) is adapted from Ref. [55].

performed this measurement by displacing the tweezers by a varying amount δz to explicitly bias the dominant mode (in this case, the anti-phase mode) and measuring the response of the cavity field. By normalizing the measured field by the field amplitude when the atoms are explicitly placed at antinodes of the same sign, we obtained a proxy for the dominant mode displacement z_{dom} . Then, $\chi = \left. \frac{\partial \langle z_{\text{dom}} \rangle}{\partial (\delta z)} \right|_{\delta z=0}$, i.e. χ is the slope of the response of $\langle z_{\text{dom}} \rangle$ with respect to δz . Here, the angle brackets refer to ensemble averages. With this definition, we saw that the value of χ increases with N , in accordance with our intuition about macroscopic systems. Intriguingly, we also saw that χ continues to increase as Ω surpasses Ω_c , which was also in agreement with our equilibrium theory (see Fig. 4.5(c)). However, the interpretation of this result is not so straightforward.

Another equilibrium statistical mechanical result is that the susceptibility is proportional to the variance of the order parameter. In our system, $\chi = Nm\omega_z^2 \langle \Delta z_{\text{dom}}^2 \rangle / (k_B T)$, where $\langle \Delta z_{\text{dom}}^2 \rangle$ is measured at $\delta z = 0$. This standard result is known as the fluctuation-dissipation theorem (FDT) [101, 102, 103, 104, 105]. Defining the partition function $\mathcal{Z} = \int \int \exp(-\beta(\mathcal{H}_{\text{mech}} - Nm\omega_z^2 z_{\text{dom}} \Delta z)) dz_{\text{dom}} dp_{\text{dom}}$, where $\beta = 1/(k_B T)$, we can derive this result through classical statistical mechanics:

$$\chi = \frac{\partial \langle z_{\text{dom}} \rangle}{\partial \Delta z} = \frac{\partial}{\partial \Delta z} \frac{\int \int z_{\text{dom}} \exp(-\beta(\mathcal{H}_{\text{mech}} - Nm\omega_z^2 z_{\text{dom}} \Delta z)) dz_{\text{dom}} dp_{\text{dom}}}{\mathcal{Z}} \quad (4.60)$$

$$= \beta Nm\omega_z^2 \left(\frac{\int \int z_{\text{dom}}^2 \exp(-\beta(\mathcal{H}_{\text{mech}} - Nm\omega_z^2 z_{\text{dom}} \Delta z)) dz_{\text{dom}} dp_{\text{dom}}}{\mathcal{Z}} - \frac{(\int \int z_{\text{dom}} \exp(-\beta(\mathcal{H}_{\text{mech}} - Nm\omega_z^2 z_{\text{dom}} \Delta z)) dz_{\text{dom}} dp_{\text{dom}})^2}{\mathcal{Z}^2} \right) \quad (4.61)$$

$$= \frac{Nm\omega_z^2}{k_B T} (\langle z_{\text{dom}}^2 \rangle - \langle z_{\text{dom}} \rangle^2). \quad (4.62)$$

Through the FDT, we find that a classical equilibrium statistical mechanical treatment predicts $\chi \propto \sqrt{N}$ at the critical point in our system:

$$\langle z_{\text{dom}}^2 \rangle|_{\Omega=\Omega_c} = \frac{\int z_{\text{dom}}^2 \exp\left(-\frac{Nm\omega_z^2 k^2 z_{\text{dom}}^4}{6k_B T}\right) dz_{\text{dom}}}{\int \exp\left(-\frac{Nm\omega_z^2 k^2 z_{\text{dom}}^4}{6k_B T}\right) dz_{\text{dom}}} \quad (4.63)$$

$$= \frac{\Gamma(3/4)}{4\Gamma(5/4)} \sqrt{\frac{6k_B T}{Nm\omega_z^2 k^2}} \propto \frac{1}{\sqrt{N}} \quad (4.64)$$

$$\Rightarrow \chi \propto \sqrt{N}. \quad (4.65)$$

However, our data quality at the time of Ref. [55] was not good enough to resolve this scaling. If we are able to improve our statistics in the future, it could be interesting to try to resolve the $\sqrt{N}$ -scaling.

In our experiments thus far, we have only considered the effects of a dc force originating from a static tweezer displacement. However, the fluctuation-dissipation theorem generally

relates the susceptibility of a system driven by a fluctuating force to the power spectral density $S_{zz}(\omega)$. The classical and quantum forms of the FDT are given by

$$\text{classical: } S_{zz}(\omega) = -\frac{2k_B T}{\omega} \text{Im}(\chi(\omega)), \quad (4.66)$$

$$\text{quantum: } S_{zz}(\omega) = 2\hbar(\bar{n}_{\text{th}}(\omega) + 1)\text{Im}(\chi(\omega)), \text{ where} \quad (4.67)$$

$$S_{zz}(\omega) = \int e^{i\omega\Delta t} \langle z(t)z(t+\Delta t) \rangle d(\Delta t). \quad (4.68)$$

In the quantum version, $\bar{n}_{\text{th}}(\omega)$ is the Bose-Einstein distribution for the thermal occupation of phonons.

From the FDT, we immediately see that χ is related to $S_{zz}(\omega)$, which is the Fourier transform of the autocorrelation function. That is, $S_{zz}(\omega)$ measures the spectrum of fluctuations *over time*. On the other hand, χ is taken to be an ensemble-averaged quantity. Thus, the FDT assumes that the system is ergodic; ergodicity is baked into the assumption of thermal equilibrium in deriving the FDT [101]. However, this assumption breaks down in the presence of symmetry breaking, making phase transitions an interesting setting for studying violations of the FDT [106, 107, 108]. Returning to the static version of the FDT, we see that we get differing results for the variance $\langle \Delta z_{\text{dom}}^2 \rangle$ depending on whether we take a time average or ensemble average (Fig. 4.5). In a macroscopic system, we expect χ to drop to zero for $\Omega > \Omega_c$ because only symmetry-broken solutions occur. However, in a mesoscopic system, we expect χ to increase because of the finite time it takes to switch between symmetry-broken states (this was a key finding in Ref. [55]). Indeed, in Fig. 4.5, we see that if we only examine time averages of $\langle \Delta z_{\text{dom}}^2 \rangle$ for $50 \mu\text{s}$, $\langle \Delta z_{\text{dom}}^2 \rangle$ peaks near Ω_c and then drops as Ω is increased beyond Ω_c . In contrast, the ensemble averaged variance steadily rises as Ω increases, in concurrence with our ensemble-averaged measurements of χ . As we increase the amount of time over which we average, the time-averaged variance starts to approach the ensemble-averaged variance, and this convergence is much more pronounced for $N = 12$ than it is for $N = 22$. In the future, it could be very interesting to study how the “ergodic time” varies with atom number, as this provides another way of quantifying mesoscopic fluctuations. We may also find violations of the FDT due to the mesoscopic and driven-dissipative nature of our system.

It would also be interesting to experimentally probe the FDT when the system is driven by an oscillating force. This requires measuring the ac susceptibility, $\chi(\omega)$, by jiggling the tweezers in anti-phase motion. E6 does not currently have this capability because it requires an AOM to be placed before the AOD. Since the speed of sound in the AOD crystal is much slower than that in an AOM crystal ($c_{s,\text{AOD}} = 650 \text{ m/s}$ vs. $c_{s,\text{AOM}} = 4200 \text{ m/s}$), the AOD alone cannot achieve a modulation frequency near the tweezer trap frequency ($\sim 90 \text{ kHz}$). Planning this AOM upgrade is currently in the works. The design shown in Fig. 4.6 is being tested on the side. To modulate the tweezers in anti-phase motion, we plan to drive the AOM with two tones, modulated π out-of-phase with each other. The AOM aperture is imaged onto the AOD aperture. To create an array of N tweezers spaced by 5.5λ , we will

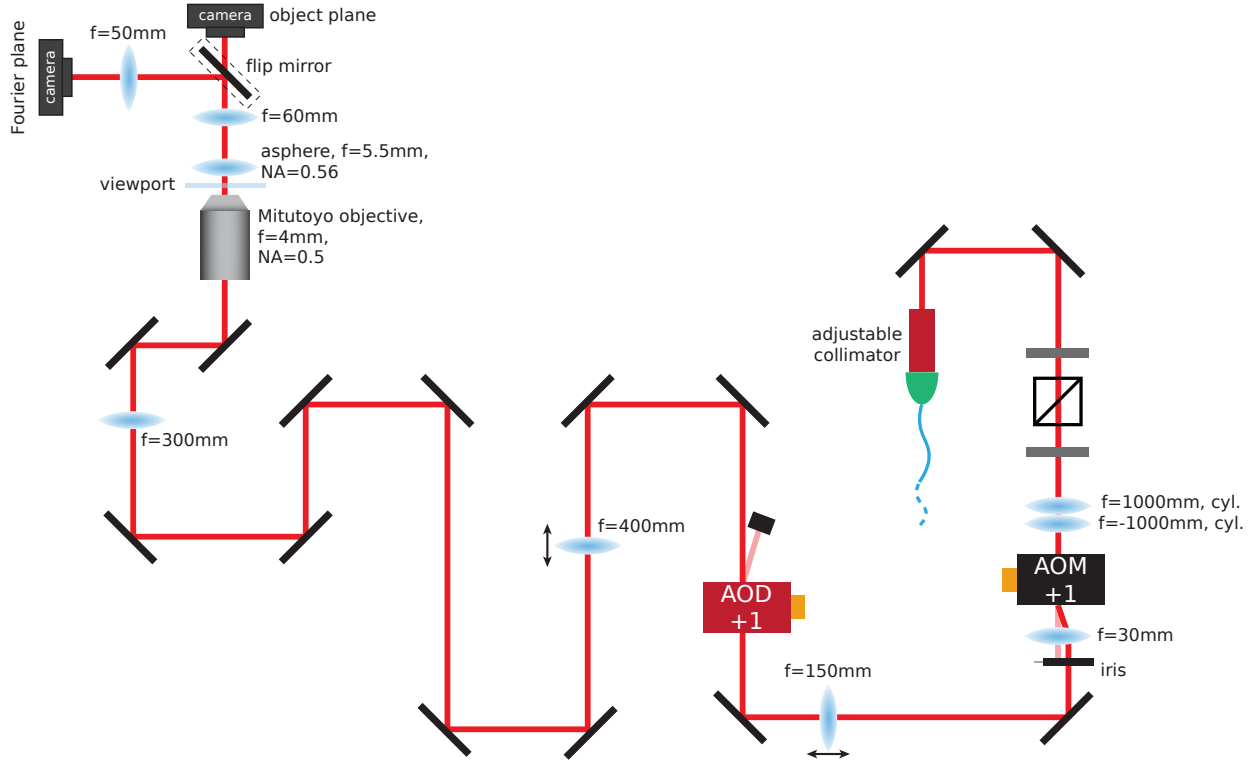


Figure 4.6: Tweezer optics upgrade design. This is the layout tested on the side, to be implemented on the tweezer upper breadboard. The AOM aperture is imaged onto the AOD aperture to allow for fast modulation of the tweezer positions. The design includes a cylindrical lens pair for beam shaping, as well as lenses on translation stages (double arrows) for precise alignment. An adjustable collimator will also allow us to tune the size of the tweezer waist, a degree of freedom we previously did not have.

choose the two AOM tones to correspond to a tweezer spacing of 5.5λ in the atom plane and drive the AOD with $N/2$ tones whose frequency difference corresponds to 11λ spacing in the atom plane.

Chapter 5

The n -phase Dicke model

As we have seen in the previous chapter, the Dicke model is paradigmatic for studies of driven-dissipative phase transitions, playing a pivotal role in interfacing quantum optics with many-body physics. Driven-dissipative versions of the Dicke model have been realized experimentally, in mesoscopic samples of atoms such as those created in E6, as well as in bulk gases placed within high-finesse optical cavities [56, 109]. The atoms are driven optically, providing an energy input to the system, while the cavity emits light, providing dissipation. The two-level quantum emitter of the original Dicke model is replaced by optically driven atoms undergoing low-energy Bragg or Raman transitions, between different mechanical or hyperfine-spin states, respectively. Under certain conditions, the driven-dissipative system reaches a steady state that shows a normal-to-superradiant phase transition with $\mathbb{Z}_2$ symmetry.

The Dicke model considers spins that are all symmetrically coupled to the cavity; after all, Dicke's original proposal considered emitters all packed within a fraction of a wavelength of light. Inspired by the capability of local control afforded by tweezer arrays in macroscopic cavities, we were motivated to investigate how the phenomenology of the Dicke model changes when the atoms' couplings to the electromagnetic field are inhomogeneous. This modification of the Dicke model has been studied in the context of disorder, where different spins have different energy-level spacings and can couple to the cavity with different strengths [110, 111, 112, 113] and with a continuum of phases [113]. Studies have further utilized disordered couplings among atoms placed in a multimode cavity to realize glassy systems [114, 115]. In contrast, there has been limited exploration of what instead happens when the complex spin-cavity coupling varies *discretely* from spin to spin as $\sum_{i=1}^N (\chi_i^* \hat{c} + \hat{c}^\dagger \chi_i) \hat{\sigma}_i^x$, where $\hat{c}$ is the electromagnetic mode field operator, σ_i is the pseudo-spin $1/2$ vector for emitter i , and χ_i is a complex-valued emitter-mode coupling strength.

In our work [116], we theoretically investigate this scenario in the pumped, optomechanical context described in the previous chapter. We find that it is possible to generate systems with discrete symmetries higher than $\mathbb{Z}_2$ with a simple modification. We consider that N emitters are divided into n groups indexed by j , and the phase of the spin-cavity coupling for each group is set to $2\pi j/n$. Similar forms of this discrete symmetry have been

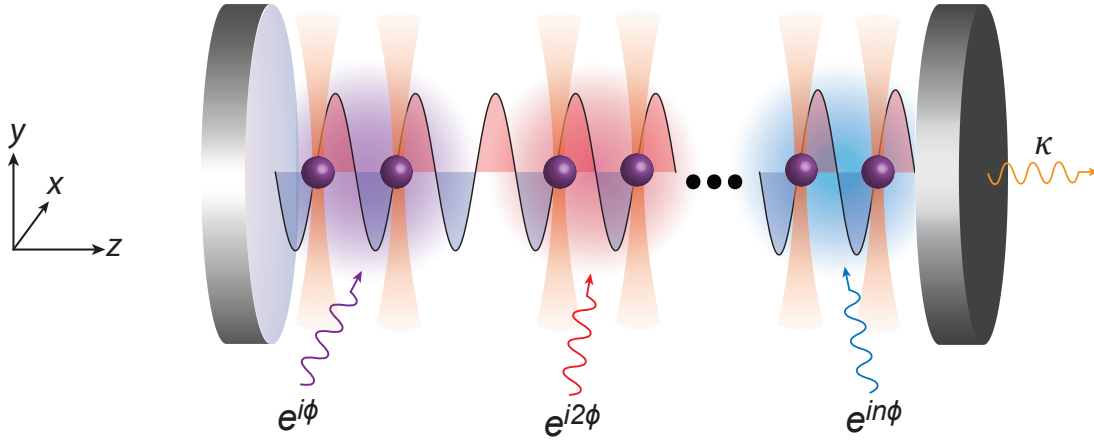


Figure 5.1: Schematic of the setup for the n -phase Dicke model. An optical tweezer array is used to place N atoms at the nodes of the cavity field. The atoms are divided into n groups, each containing N/n atoms. Each group is then driven with a pump laser with a different optical phase, with the phase between groups incrementing by $\phi \equiv 2\pi/n$. Figure adapted from Ref. [116]

previously explored in Ising-like systems such as the Potts model [117] and the q -state clock model [118]. Generalizing from the $\mathbb{Z}_2$ parity symmetry of the canonical Dicke model, the system we consider now has $\mathbb{Z}_n$ symmetry for even n and $\mathbb{Z}_{2n}$ symmetry for odd n . We call this the n -phase Dicke model, which is a specific variant of the more general multi-species Dicke model [119, 120].

The n -phase Dicke model possesses intriguing features beyond superradiant steady states that break higher symmetries. In our numerical analysis, we found that the superradiant phase is separated from the normal phase by a first-order phase transition that asymptotes to being second-order when the pump is very far red-detuned from cavity resonance. In addition, the normal phase is unstable in the presence of non-zero cavity dissipation. This instability has its origins in non-reciprocal interactions. Analyzing the non-reciprocal coupling in a bosonic formalism, we find that one can tune the cavity-coupling phase difference between two groups of emitters to achieve ideal non-reciprocity.

5.1 Theoretical model

The setup of the model is depicted in Fig. 5.1. In the context of the E6 apparatus, we consider a tweezer array of N atoms situated at the nodes of the cavity field. For ease of description, we take the atoms to be spaced by an integer-number of wavelengths, rather than half-integer, though this should not affect the physics. The array is divided into n groups, with each group containing $\nu \equiv N/n$ atoms. The atoms are then illuminated with

a pump field whose phase advances stepwise by $\phi \equiv 2\pi/n$ between groups; thus the pump field at the group with index j is given by $\Omega e^{i\phi j}$. As before, we assume that the pump is far-detuned from atomic resonance ($|\Delta_{\text{pa}}| \gg \gamma$) so that we can adiabatically eliminate the atomic excited state. Going into the rotating frame of the pump (frequency ω_{p}), we are left with the Hamiltonian

$$\hat{H}_n = -\hbar\Delta_{\text{pc}}\hat{c}^\dagger\hat{c} + \sum_{j=1}^n \left[\frac{\hat{p}_j^2}{2\mu} + \frac{\mu\omega_z^2\hat{z}_j^2}{2} + \frac{\hbar\Omega\nu}{\Delta_{\text{pa}}}g_0\sin(k\hat{z}_j)(e^{-i\phi j}\hat{c} + e^{i\phi j}\hat{c}^\dagger) \right], \quad (5.1)$$

where we have defined $\mu \equiv \nu m$ as the total mass of each group of atoms. Note that in Eq. (5.1), we have defined the pump Rabi frequency Ω with a factor of 2 difference from the definition used in the previous chapters. For the sake of consistency with our published work [116], we use the convention in Eq. (5.1) for the rest of this chapter. Eq. (5.1) includes approximations. We ignore the dispersive shift of the cavity resonance by the atoms, which is negligible for large $|\Delta_{\text{pa}}|$. We also include only the center-of-mass mode $\hat{z}_j$ of each group and ignore other modes of motion, similar to Eq. (4.12); in fact, the case of $n = 1$ reduces to Eq. (4.12).

From Eq. (5.1), we obtain the following equations of motion for the expectation values of operators under the master equation (ignoring stochastic noise on the cavity field):

$$\dot{c} = (i\Delta_{\text{pc}} - \kappa)c - i \sum_{l=1}^n \frac{\nu\Omega}{\Delta_{\text{pa}}} e^{i\phi l} g_0 \sin(kz_l), \quad (5.2)$$

$$\dot{p}_j = -\mu\omega_z^2 z_j - \frac{\nu\Omega}{\Delta_{\text{pa}}} \hbar k g_0 \cos(kz_j) (e^{i\phi j} c^* + e^{-i\phi j} c), \quad (5.3)$$

$$\dot{z}_j = \frac{p_j}{\mu}. \quad (5.4)$$

5.1.1 System symmetries

We consider the symmetries of this Hamiltonian. $\hat{H}_n$ preserves $\mathbb{Z}_n$ symmetry, since $\hat{H}_n$ is invariant under performing the gauge transformation $c \rightarrow e^{i\phi}c$ and simultaneous permutation of the group index j . The system also has a $\mathbb{Z}_2$ symmetry: simultaneous transformations $\{z, p\} \rightarrow \{-z, -p\}$ and $c \rightarrow -c$ also lead to the same Hamiltonian. Together, these symmetries result in $\mathbb{Z}_{2n}$ symmetry for odd n and $\mathbb{Z}_n$ symmetry for even n (Fig. 5.2(a)). In the even n case, the $\mathbb{Z}_2$ symmetry is redundant because it is equivalent to permuting the group index $n/2$ times while advancing the cavity field phase by $n\phi/2 = \pi$.

Another way to understand the symmetries is to consider what configurations of atoms $\{z_j\}$ maximize the magnitude of the adiabatic cavity field, given by

$$c = \frac{i\nu\Omega}{\Delta_{\text{pa}}} \frac{1}{i\Delta_{\text{pc}} - \kappa} \sum_{j=1}^n e^{i\phi j} g_0 \sin(kz_j). \quad (5.5)$$

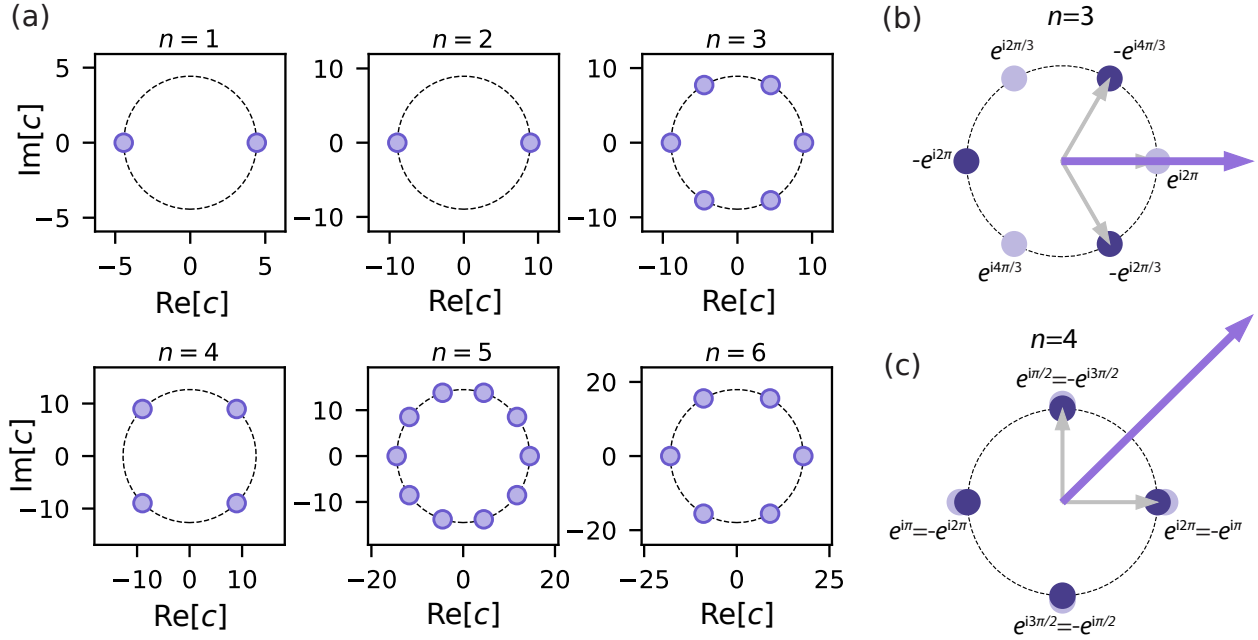


Figure 5.2: Symmetries of the n -phase Dicke model. (a) Real and imaginary quadratures of the steady-state cavity field (solid circles) calculated using the atomic center-of-mass positions found through Lyapunov function minimization, shown for $n = 1$ through $n = 6$. The results show $\mathbb{Z}_n$ symmetry for even n and $\mathbb{Z}_{2n}$ symmetry for odd n . The parameters used are $\nu = 30$, $\omega_z = 2\pi \times 70$ kHz, $\Delta_{\text{pa}} = -2\pi \times 100$ MHz, $\Delta_{\text{pc}} = -2\pi \times 4$ MHz, $\kappa = 0$, $g_0 = 2\pi \times 3$ MHz, and $\Omega = 2\pi \times 20$ MHz. (b) Example illustrations of symmetry-broken states for $n = 3$ and $n = 4$. Light (dark) circles correspond to $e^{i\phi_j}$ ($-e^{i\phi_j}$) on the complex plane. Light arrows represent the phasors $e^{i\phi_j} \sin(kz_j) \simeq \pm e^{i\phi_j}$ in a symmetry-broken solution, assuming that the atoms have self-organized by moving towards the antinodes at either $z_j = \lambda/4$ or $z_j = -\lambda/4$. The dark arrows show the sum $\sum_j e^{i\phi_j} \sin(kz_j)$, which is proportional to the cavity field, in the chosen symmetry-broken state.

The total cavity field is the sum of the fields emitted by each group, thus it is proportional to $\sum_j e^{i\phi_j} \sin(kz_j)$. Since we expect the center-of-mass position of each group to move towards the positive or negative antinodes during self-organization, we can approximate $\sin(kz_j) \approx \pm 1 \equiv a_j$. Then, $\sum_j a_j e^{i\phi_j}$ is the sum of the phasors $a_j e^{i\phi_j}$ on the unit circle in the complex plane. If $a_j = 1$ for all j , $\sum_j a_j e^{i\phi_j} = 0$ (likewise if $a_j = -1$ for all j), so we need to pick some of the a_j coefficients to be 1 and some to be -1 to have a non-zero sum. Let us first consider the case of even n . In order to maximize the magnitude of the sum, we choose $n/2$ groups to have $a_j = 1$ and the other $n/2$ groups to have $a_j = -1$ such that all $a_j e^{i\phi_j}$ lie within one half of the complex plane. Since $-e^{i\phi_j} = e^{i\phi(j+n/2)}$, this condition is satisfied by picking $a_j = 1$ for $n/2$ consecutive j (modulo n) and setting the other $a_j + n/2 = -1$. There are n unique ways of achieving this, thus leading to $\mathbb{Z}_n$ symmetry. For odd n , we similarly want to pick the a_j coefficients such that all $a_j e^{i\phi_j}$ lie in one half of the complex plane. However, in this case, the phasors $-e^{i\phi_j}$ are not degenerate with any of the phasors $e^{i\phi_j}$, so they introduce another n unique points along the unit circle for a total of $2n$. Thus, the magnitude of $\sum_j a_j e^{i\phi_j}$ is maximized by picking a set of a_j coefficients that correspond to a set of n consecutive points (out of $2n$) along the unit circle. There are $2n$ unique sets that satisfy this; in each set there are $(n-1)/2$ phasors where $a_j = \pm 1$ and $(n+1)/2$ phasors where $a_j = \mp 1$. As a result, the system has $\mathbb{Z}_{2n}$ symmetry for odd n . Phasors for $n = 3$ and $n = 4$ are schematically represented in Fig. 5.2(b)-(c).

5.2 Symmetry-broken steady states

The geometrical understanding of symmetry-broken solutions from the phasor diagrams in Fig. 5.2(b)-(c) can be numerically verified in the limit $\kappa \rightarrow 0$, in which case the system does not possess non-reciprocal interactions. First, we obtain purely mechanical equations of motion by substituting Eq. (5.5) into Eq. (5.3):

$$\dot{p}_j = -\mu\omega_z^2 z_j - \frac{2\hbar\nu^2\Omega^2}{\Delta_{\text{pa}}^2} \frac{g_0^2 k \cos(kz_j)}{\Delta_{\text{pc}}^2 + \kappa^2} \sum_{l=1}^n \left[\Delta_{\text{pc}} \cos(\phi(j-l)) - \kappa \sin(\phi(j-l)) \right] \sin(kz_l), \quad (5.6)$$

$$\dot{z}_j = \frac{p_j}{\mu}. \quad (5.7)$$

Taking $\kappa = 0$, these equations become

$$\dot{p}_j = -\mu\omega_z^2 z_j - \frac{2\hbar\nu^2\Omega^2}{\Delta_{\text{pa}}^2} \frac{g_0^2 k \cos(kz_j)}{\Delta_{\text{pc}}} \sum_{l=1}^n \cos(\phi(j-l)) \sin(kz_l), \quad (5.8)$$

$$\dot{z}_j = \frac{p_j}{\mu}. \quad (5.9)$$

To solve for the steady states of Eqs.(5.8)-(5.9), we draw an analogy to Hamiltonian systems. If we can find a function such that applying Hamilton's equations to it gives us back Eqs.(5.8)-(5.9), then we know the minima of this function are (Lyapunov) stable equilibria. This

function is known as a Lyapunov function [93]. For Hamiltonian systems, the Hamiltonian itself is a Lyapunov function. In our system, we find a Lyapunov function L :

$$L = \sum_{j=1}^n \left(\frac{p_j^2}{2\mu} + \frac{\mu\omega_z^2 z_j^2}{2} + \mathcal{C} \left[-\frac{\Delta_{\text{pc}} \cos(2kz_j)}{4k} + \sum_{l < j} \frac{\sin(kz_j)}{k} \sin(kz_l) (\Delta_{\text{pc}} \cos(\phi(j-l))) \right] \right). \quad (5.10)$$

Here, we have defined $\mathcal{C} = \frac{2\hbar\nu^2\Omega^2}{\Delta_{\text{pa}}^2} \frac{g_0^2 k}{\Delta_{\text{pc}}^2 + \kappa^2}$ for convenience. For $\kappa = 0$, we find that $\partial L / \partial p_j = \dot{z}_j$ and $-\partial L / \partial z_j = \dot{p}_j$ for all n . Note that for $n = 1$ and $n = 2$, L gives the correct equations of motion even for $\kappa \neq 0$ (as we shall see, this is because the $n \leq 2$ cases do not have non-reciprocal forces). To obtain the plots in Fig. 5.2(a), we minimize L for $n = 1, 2, \dots, 6$ using simplicial homology global optimization (SHGO) [121]. SciPy's SHGO function conveniently solves for local minima in addition to the global minimum, which is very useful for searching for degenerate, symmetry-broken equilibrium points.

In the $\kappa = 0$ limit, the steady states for all n behave analogously to $n = 1$. Just like in the canonical Dicke model, there is a stable normal phase at low pump strength, which transitions continuously into a stable superradiant phase above Ω_c . The main difference lies in the $\mathbb{Z}_n$ and $\mathbb{Z}_{2n}$ symmetries of higher n . Notably, the stability of the normal phase and the continuous phase transition result from the ‘‘cavity-dispersive’’ force terms proportional to $\Delta_{\text{pc}} \cos(\phi(j-l))$ in Eq. (5.6) as well as the absence of the terms proportional to $\kappa \sin(\phi(j-l))$ in Eq. (5.6), which we call the ‘‘cavity-reactive’’ terms.

Upon taking $\kappa > 0$, we restore the cavity-reactive terms and again ask what the steady states of the system are. We can no longer use the Lyapunov function trick, and are forced to numerically solve for the sets $\{z_j\}$ that make $\dot{p}_j = 0$ (Eq. (5.6)). Our experience with this was that the ability of the numerical solver to find symmetry-broken steady states was extremely sensitive to the initial guess that we input into the solver. For most of the parameter space we explored, using the solution from Lyapunov function minimization was sufficient. However, near the phase boundaries and especially when $|\Delta_{\text{pc}}| \sim \kappa$, we had to input more educated initial guesses to account for the cavity-reactive terms, which cause steady states to appear in configurations where some groups are displaced from the cavity nodes by distances exceeding 0.5λ . In light of the fickleness of numerical solvers, we were greatly aided by the graphical solution that the $n = 4$ case admits. Here, the pump phase difference between groups is $\pi/2$ and the system has the symmetry $z_1 = -z_3$ and $z_2 = -z_4$. Reducing the problem to two variables allows us to plot the $\dot{p}_1(z_1, z_2) = 0$ and $\dot{p}_2(z_1, z_2) = 0$ contours simultaneously; their intersection points are steady states, shown in Fig. 5.3(a).

We elucidate the phase diagram with proximity to cavity resonance and pump strength, identifying three dynamical phases: normal, dispersive broken symmetry, and reactive broken symmetry (Fig. 5.3(a)-(d)). In the normal phase, which exists at low Ω , ($z_j = 0$, $p_j = 0$) is the only steady state. At higher values of Ω , multiple steady states exist. In the dispersive regime ($|\Delta_{\text{pc}}| \gg \kappa$), there are nine steady states in the region $-\lambda/4 < \{z_1, z_2\} < \lambda/4$. We determine the stability of each steady state by evaluating the eigenvalues of the Jacobian matrix evaluated at each steady state (Fig. 5.3(e)), similar to as described in the previous

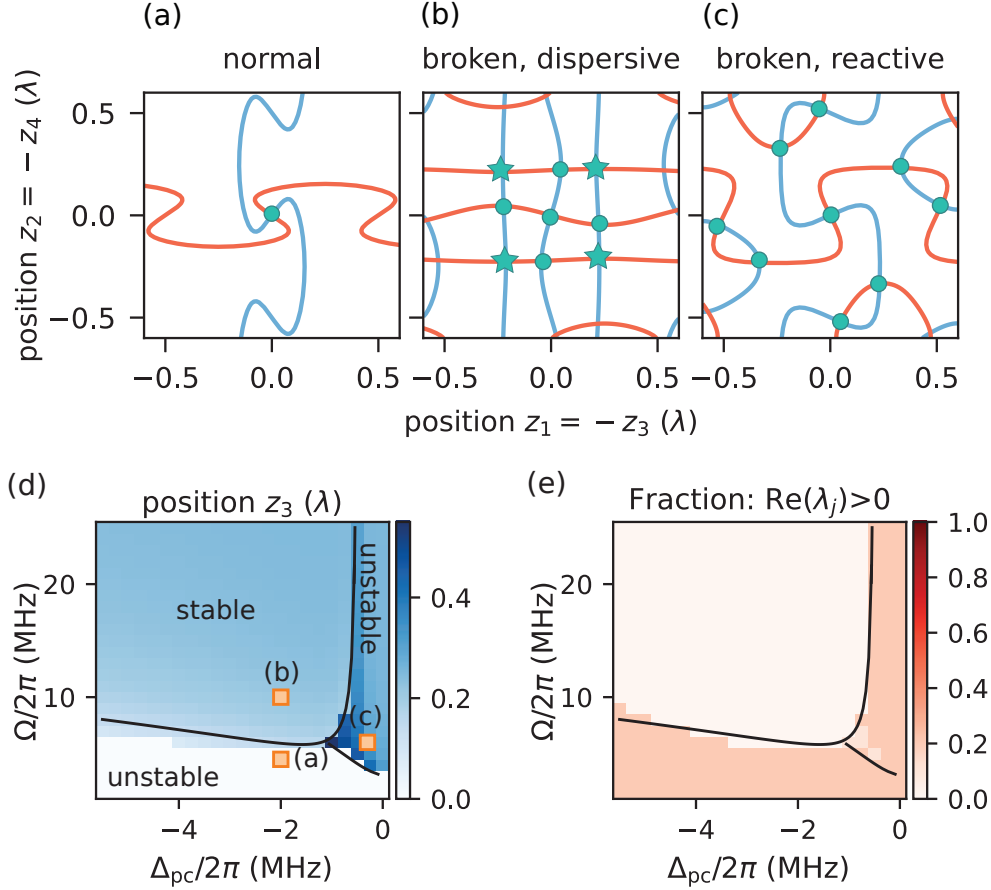


Figure 5.3: Phase diagram of the n -phase Dicke model. (a-c) Contour plots of the forces $\dot{p}_1 = 0$ (blue) and $\dot{p}_2 = 0$ (red) in three different parameter regimes for $n = 4$, where we have set $z_1 = -z_3$ and $z_2 = -z_4$. Steady states (teal markers) occur where the blue and red curves intersect. Jacobian eigenvalue analysis is used to determine whether each steady state is unstable (circles) or stable (stars). (d) Phase diagram for $n = 4$ showing the steady-state position of one group (z_3) at a particular symmetry-broken solution. Orange squares correspond to the parameters used in (a-c). Black lines are phase boundaries calculated by expanding Eq. (5.6) to third order around the symmetry-broken solutions. (e) Fraction of Jacobian eigenvalues λ_j with a positive real part, corresponding to the phase diagram shown in (d). The stable region of (c) is the parameter regime where no eigenvalues have a positive real part. Calculations in figure are done with $\Delta_{pa} = -2\pi \times 100$ MHz, $\kappa = 2\pi \times 0.5$ MHz, $\nu = 30$, $g_0 = 2\pi \times 3$ MHz, and $\omega_z = 2\pi \times 70$ kHz. Figure adapted from Ref. [116].

chapter. The presence of eigenvalues with positive real part indicates instability, as these eigenvalues correspond to exponentially growing eigenmodes. Nonetheless, we numerically find that for $n = 3$ through $n = 6$, there exists a parameter regime at larger Ω and $|\Delta_{\text{pc}}| > \kappa$ where the superradiant states are stable and $\mathbb{Z}_n$ or $\mathbb{Z}_{2n}$ symmetry breaking can be observed in the cavity field trajectories (see Ref. [116] for more details).

5.3 Non-reciprocal interactions

As mentioned earlier, non-reciprocal interactions from the cavity-reactive forces serve to destabilize the normal phase and cause the emergence of first-order phase transitions. We now show how non-reciprocity arises in the n -phase Dicke model.

In the normal phase, our equations of motion conveniently lend themselves to a treatment in terms of bosonic operators. We linearize Eq. (5.6) and rewrite it in terms of bosonic ladder operators $\hat{b}_j$, which annihilate tweezer phonons in the j th group of atoms. Defining $\mathbf{b} = (\hat{b}_1, \hat{b}_2, \dots, \hat{b}_n)^T$, this results in the equation of motion $\dot{\mathbf{b}} = -i\mathbf{H}_{\text{eff}}\mathbf{b}$. $\mathbf{H}_{\text{eff}}$ is an effective Hamiltonian whose matrix elements are given by

$$H_{\text{eff},jl} = \begin{cases} \omega_z + \frac{ck\Delta_{\text{pc}}}{\mu\omega_z}, & j = l \\ \frac{ck[\Delta_{\text{pc}}\cos(\phi(j-l)) - \kappa\sin(\phi(j-l))]}{\mu\omega_z}, & j \neq l \end{cases} \quad (5.11)$$

We see that $H_{\text{eff},jl} \neq H_{\text{eff},lj}$ unless $\kappa\sin(\phi(j-l)) = 0$ for all values of j and l . Since $\phi = 2\pi/n$, this results in $\mathbf{H}_{\text{eff}}$ being non-Hermitian when $n > 2$ and $\kappa > 0$. The fact that $H_{\text{eff},jl} \neq H_{\text{eff},lj}$ means that the groups of atoms interact non-reciprocally [122]: the amplitude with which phonons tunnel from group j to group l is not equal to the amplitude with which phonons tunnel from group l to group j .

We can introduce tuneable non-reciprocal interactions if we allow a variable phase difference φ between the groups. For simplicity, we now consider the case of just two groups. Then, we have

$$\mathbf{H}_{\text{eff}}(\varphi) = \begin{pmatrix} \omega_z + ck\Delta_{\text{pc}}/(\mu\omega_z) & ck(\Delta_{\text{pc}}\cos\varphi + \kappa\sin\varphi)/(\mu\omega_z) \\ ck(\Delta_{\text{pc}}\cos\varphi - \kappa\sin\varphi)/(\mu\omega_z) & \omega_z + ck\Delta_{\text{pc}}/(\mu\omega_z) \end{pmatrix}. \quad (5.12)$$

With this Hamiltonian in hand, we show how one can tune φ to achieve maximally non-reciprocal interactions. An example of such fully non-reciprocal interactions is an isolator, which only allows excitations to propagate in one direction. In our system, these excitations are phonons. Suppose that we would like group 1 to be completely decoupled from the motion of group 2, but not vice versa. Achieving this requires $H_{\text{eff},12}(\varphi) = 0$, which is satisfied for $\varphi = \tan^{-1}(-\Delta_{\text{pc}}/\kappa)$. We note that this is equivalent to setting $\varphi + \tan^{-1}(-\kappa/\Delta_{\text{pc}}) = \pm\pi/2$, where $\tan^{-1}(-\kappa/\Delta_{\text{pc}}) \equiv \theta$ is the phase of the cavity field. This relation has the simple physical interpretation of requiring the cavity field emitted by group 2 to be $\pi/2$ out-of-phase with the pump field at group 1. We may also interpret $\varphi + \theta$ as a synthetic flux,

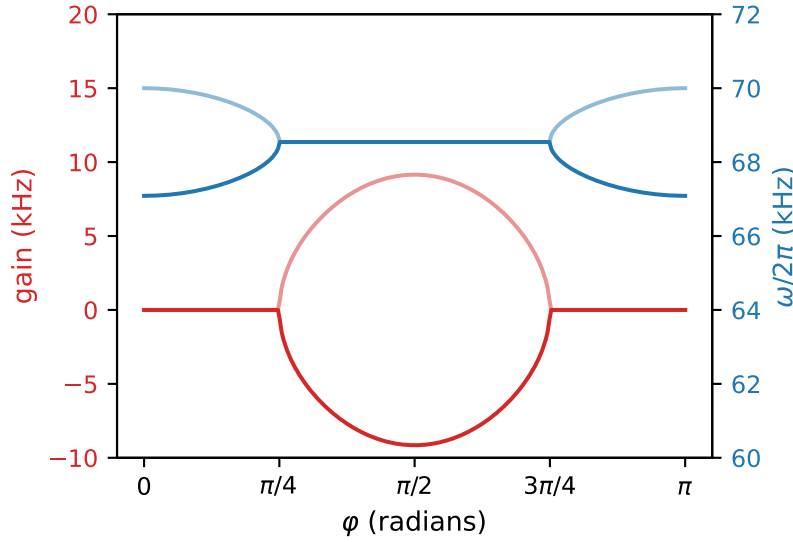


Figure 5.4: Exceptional points. We plot the real (blue) and imaginary (red) parts of the two eigenvalues (light and dark curves) of $\mathbf{H}_{\text{eff}}(\varphi)$ using the parameters $\Delta_{\text{pc}} = -\kappa$, $\Omega = 2\pi \times 0.5$ MHz. We see that exceptional points occur at $\varphi = \pi/4$ and $3\pi/4$, coinciding with the values of φ for which we have ideal non-reciprocal interactions.

analogous to the flux found in other systems with non-reciprocal interactions [123, 124, 125]. This form of non-reciprocity has been considered in systems of nanoparticles held close together in optical tweezers [126, 127, 128, 129] and a SiN membrane positioned in an optical cavity [125].

We can calculate how the phonon occupation numbers $\hat{b}_1^\dagger \hat{b}_1$ and $\hat{b}_2^\dagger \hat{b}_2$ evolve over time:

$$\frac{d}{dt} \hat{b}_1^\dagger \hat{b}_1 = \frac{d\hat{b}_1^\dagger}{dt} \hat{b}_1 + \hat{b}_1^\dagger \frac{d\hat{b}_1}{dt} = \frac{i\mathcal{C}k}{\mu\omega_z} (\Delta_{\text{pc}} \cos \varphi + \kappa \sin \varphi) (\hat{b}_2^\dagger \hat{b}_1 - \hat{b}_1^\dagger \hat{b}_2), \quad (5.13)$$

$$\frac{d}{dt} \hat{b}_2^\dagger \hat{b}_2 = \frac{d\hat{b}_2^\dagger}{dt} \hat{b}_2 + \hat{b}_2^\dagger \frac{d\hat{b}_2}{dt} = -\frac{i\mathcal{C}k}{\mu\omega_z} (\Delta_{\text{pc}} \cos \varphi - \kappa \sin \varphi) (\hat{b}_2^\dagger \hat{b}_1 - \hat{b}_1^\dagger \hat{b}_2). \quad (5.14)$$

We see that in the fully reciprocal case ($\varphi = 0$), we have $d(\hat{b}_1^\dagger \hat{b}_1)/dt = -d(\hat{b}_2^\dagger \hat{b}_2)/dt$, consistent with the total energy of the system being conserved. In the fully non-reciprocal case, e.g. $\varphi = \tan^{-1}(-\Delta_{\text{pc}}/\kappa)$, then $d(\hat{b}_1^\dagger \hat{b}_1)/dt = 0$ while $d(\hat{b}_2^\dagger \hat{b}_2)/dt \neq 0$, consistent with group 1 being decoupled from the motion of group 2.

Non-Hermitian Hamiltonians lend themselves to the formation of exceptional points [130, 131]: points where the eigenvectors coalesce. Such points do not occur in Hermitian Hamiltonian systems, where the eigenvectors are orthogonal. Upon solving for the eigenvalues and eigenvectors of Eq. (5.12), we find that two exceptional points occur under the same

conditions as ideal non-reciprocal interactions: $\varphi = \tan^{-1}(\pm\Delta_{\text{pc}}/\kappa)$. As seen in Fig. 5.4, the eigenvalues are purely real except for the region in between the exceptional points, where the system possesses degenerate eigenfrequencies ω with damping and anti-damping rates corresponding to gain. Explicitly, for the choice $\Delta_{\text{pc}} = -\kappa$ used in Fig. 5.4, at the two exceptional points we have degenerate eigenvalues $-\mathcal{C}k\kappa/(\mu\omega_z) + \omega_z$. At $\varphi = \pi/4$, the single non-trivial eigenvector is $(0, 1)^T$ and at $\varphi = 3\pi/4$ it is $(1, 0)^T$, further showing how $\hat{b}_1$ and $\hat{b}_2$ become non-reciprocally coupled.

While we refer to Eq. (5.12) as an effective Hamiltonian, it really just describes the equations of motion for the bosonic operators $\hat{b}_1$ and $\hat{b}_2$. We can go a step further and ask if there is a Hamiltonian that, through the Heisenberg equation of motion, gives rise to the equations of motion described by Eq. (5.12). Though such a Hamiltonian is not unique, one possible form is

$$\begin{aligned} \hat{H}_{\text{NH}}/\hbar = & \left(\omega_z + \frac{\mathcal{C}k\Delta_{\text{pc}}}{\mu\omega_z} \right) (\hat{b}_1^\dagger\hat{b}_1 + \hat{b}_2^\dagger\hat{b}_2 + 1) \\ & + \frac{\mathcal{C}k\Delta_{\text{pc}}}{\mu\omega_z} \cos \varphi (\hat{b}_1\hat{b}_2^\dagger + \hat{b}_2\hat{b}_1^\dagger) \\ & + \frac{\mathcal{C}k\kappa}{\mu\omega_z} \sin \varphi (\hat{b}_2\hat{b}_1^\dagger - \hat{b}_1\hat{b}_2^\dagger). \end{aligned} \quad (5.15)$$

We have obtained a non-Hermitian Hamiltonian $\hat{H}_{\text{NH}}$. The non-Hermitian term is proportional to $\kappa \sin \varphi (\hat{b}_2\hat{b}_1^\dagger - \hat{b}_1\hat{b}_2^\dagger)$. Once again, we see that Hermiticity is restored if either $\kappa = 0$ or $\sin \varphi = 0$, and tuning φ to the ideal non-reciprocity condition results in only $\hat{b}_1\hat{b}_2^\dagger$ terms or $\hat{b}_2\hat{b}_1^\dagger$ terms surviving. We can rewrite $\hat{H}_{\text{NH}}$ in terms of position and momentum operators to obtain

$$\begin{aligned} \hat{H}_{\text{NH}} = & \frac{\hat{p}_1^2}{2\mu} + \frac{\hat{p}_2^2}{2\mu} + \frac{1}{2}\mu \left(\omega_z + \frac{\mathcal{C}k\Delta_{\text{pc}}}{\mu\omega_z} \right)^2 (\hat{z}_1^2 + \hat{z}_2^2) \\ & + \mathcal{C}k\Delta_{\text{pc}} \cos \varphi \left(\hat{z}_1\hat{z}_2 + \frac{\hat{p}_1\hat{p}_2}{\mu^2\omega_z^2} \right) \\ & + i \frac{\mathcal{C}k\kappa}{\mu\omega_z} \sin \varphi (\hat{p}_2\hat{z}_1 - \hat{p}_1\hat{z}_2). \end{aligned} \quad (5.16)$$

Eq. (5.16) allows us to see that $\hat{H}_{\text{NH}}$ preserves $\mathcal{PT}$ -symmetry. The parity operator $\mathcal{P}$ inverts spatial coordinates ($\hat{z} \rightarrow -\hat{z}$, $\hat{p} \rightarrow -\hat{p}$) and the time reversal operator $\mathcal{T}$ reverses the arrow of time ($t \rightarrow -t$, $\hat{p} \rightarrow -\hat{p}$, $i \rightarrow -i$). Combined, the $\mathcal{PT}$ -operator performs the transformations $t \rightarrow -t$, $\hat{z} \rightarrow -\hat{z}$, $i \rightarrow -i$ (the reversal of momentum is canceled out). Despite the final term in Eq. (5.16) being non-Hermitian, it obeys $\mathcal{PT}$ -symmetry. Non-Hermitian Hamiltonians obeying $\mathcal{PT}$ -symmetry are known to have real eigenvalues when the eigenstates of $\hat{H}_{\text{NH}}$ are also eigenstates of $\mathcal{PT}$ [132, 126, 127, 133]. The points where this condition breaks correspond to the exceptional points, which mark the onset of complex eigenvalues. In other systems with similar forms of non-reciprocal interactions, limit cycle phases have been seen

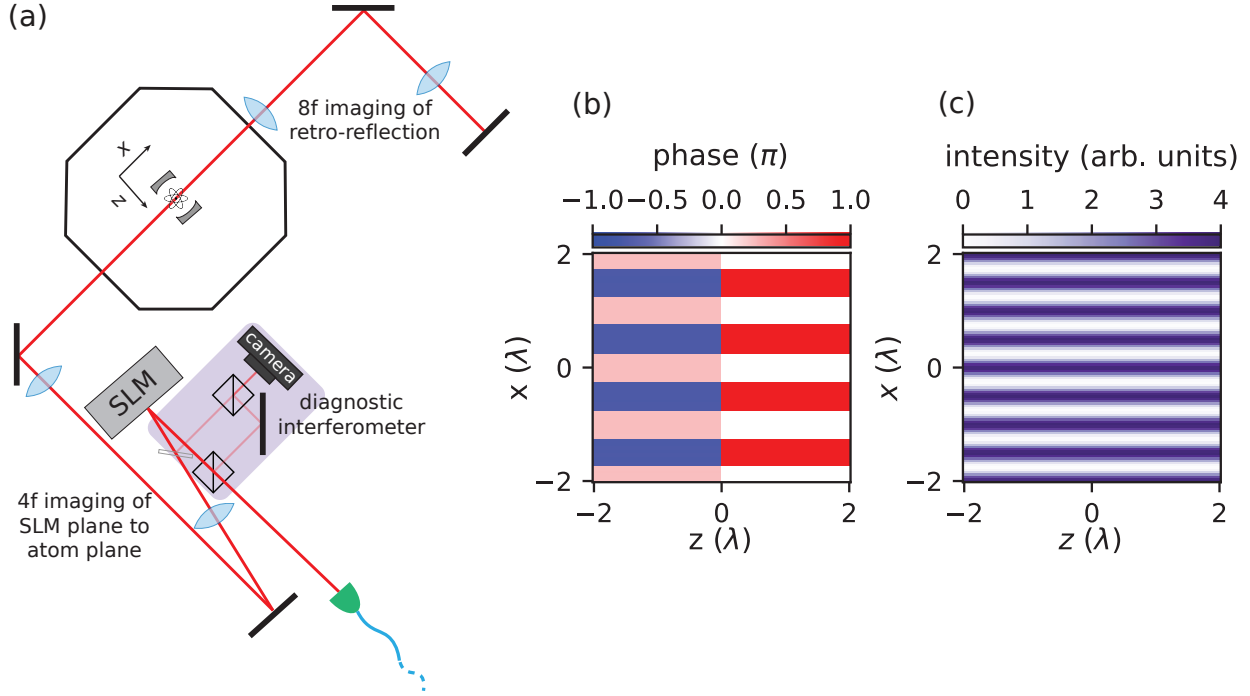


Figure 5.5: (a) Proposed pump optical path design for realizing the n -phase Dicke model and non-reciprocal interactions. The spatial light modulator (SLM) displays a phase mask which is imaged onto the atoms and retro-reflected to create a standing wave. The path design also includes a diagnostic interferometer for imaging the phase shift induced by the SLM. (b) Phase and (c) intensity profile of the pump beam where the left half of the SLM displays a $\pi/4$ phase shift.

to emerge at these exceptional points [126, 127, 134]. Perhaps similar physics can be probed in E6. It could be interesting to investigate how cavity dynamical effects (like cooling) affect non-reciprocity and the behavior at the exceptional points.

5.4 Prospects for experimental realization in E6

To realize the n -phase Dicke model in E6, we need a way to tune the pump's optical phase at different locations. There are many ways in which one can imagine doing this. The approach E6 currently plans to take is to place a liquid crystal spatial light modulator (SLM) in the pump path, and image the SLM plane onto the atom plane (Fig. 5.5(a)). The SLM naturally lends itself to creating a phase mask, which we may simply image onto the atoms. The SLM has a few advantages over an AOD or AOM for this purpose. It allows for tunable phase shifts between atoms without also introducing a frequency shift. Additionally, we wish to retro-

reflect the pump beam to reduce recoil heating of the atoms, and with the SLM approach, we only have to concern ourselves with the alignment of a single retro-reflected beam, rather than multiple, spatially separated ones.

Fitting an SLM into the pump path will also require E6 to rebuild our molasses, optical pumping, repump, and absorption imaging paths, as these optics are all rather crammed into the same space. It will also require us to replace the current $4f$ -imaging of the pump (and molasses) retro-reflection onto the atoms with $8f$ -imaging. The current $4f$ -imaging is insufficient because it inverts the spatial profile of the beam; previously this did not matter because the beam was radially symmetric. However, with the phase profile that we now plan to introduce, we need to ensure that the phase profiles of the incident and retro-reflected beams match. $8f$ -imaging achieves this by inverting the atom-plane image twice, effectively performing the identity transformation. All of these technical upgrades are in part why we have delayed carrying out this proposal. We plan to start building these upgrades soon.

In terms of data collection, we expect that our current method of heterodyne detection should be sufficient to observe $\mathbb{Z}_n$ and $\mathbb{Z}_{2n}$ symmetry breaking. We note that due to the instability of the normal phase in the n -phase Dicke model, the normal phase is no longer characterized by low cavity field amplitude but rather by spiraling trajectories, where both quadratures of the field grow exponentially over time (see Fig. S3 of Ref. [116]).

In our proposed work on the n -phase Dicke model, we are mostly interested in observing the discrete symmetry of the cavity field. On the other hand, on the tunable non-reciprocal interactions front, we wish to deduce the motional states of each of the two groups of atoms from readout of the cavity field. This problem is a bit trickier to address, since the cavity field inherently contains information about the motional state of *all* the atoms. Thus, we must find an observable that allows us to distinguish between the two groups.

One of my initial thoughts was to give each group a different tweezer trap frequency; then, the motion of each group is encoded in different sidebands in the heterodyne spectrum. Note that the conditions on φ for fully non-reciprocal interactions are unchanged when the two groups have different trap frequencies ω_1 and ω_2 . The effective Hamiltonian simply generalizes to

$$\mathbf{H}_{\text{eff}}(\varphi) = \begin{pmatrix} \omega_1 + \mathcal{C}k\Delta_{\text{pc}}/(\mu\omega_1) & \mathcal{C}k(\Delta_{\text{pc}}\cos\varphi + \kappa\sin\varphi)/(\mu\sqrt{\omega_1\omega_2}) \\ \mathcal{C}k(\Delta_{\text{pc}}\cos\varphi - \kappa\sin\varphi)/(\mu\sqrt{\omega_1\omega_2}) & \omega_2 + \mathcal{C}k\Delta_{\text{pc}}/(\mu\omega_2) \end{pmatrix}. \quad (5.17)$$

Although giving the groups different trap frequencies may lend itself to a way of extracting separately the motion of each group, this scheme is complicated by the fact that if group 1 is decoupled from group 2 (but not vice versa), group 1 behaves like an external drive for group 2. Thus, group 2's motional spectrum will contain a large component at the trap frequency of group 1. An alternative approach could be to allow the groups initially to have the same trap frequency, drive the system, and then switch to a “readout” setting where the groups have far-separated trap frequencies. From the transient, we may be able to retrodict what the initial state was [135].

A less convoluted approach is to remember that because of the pump phase difference

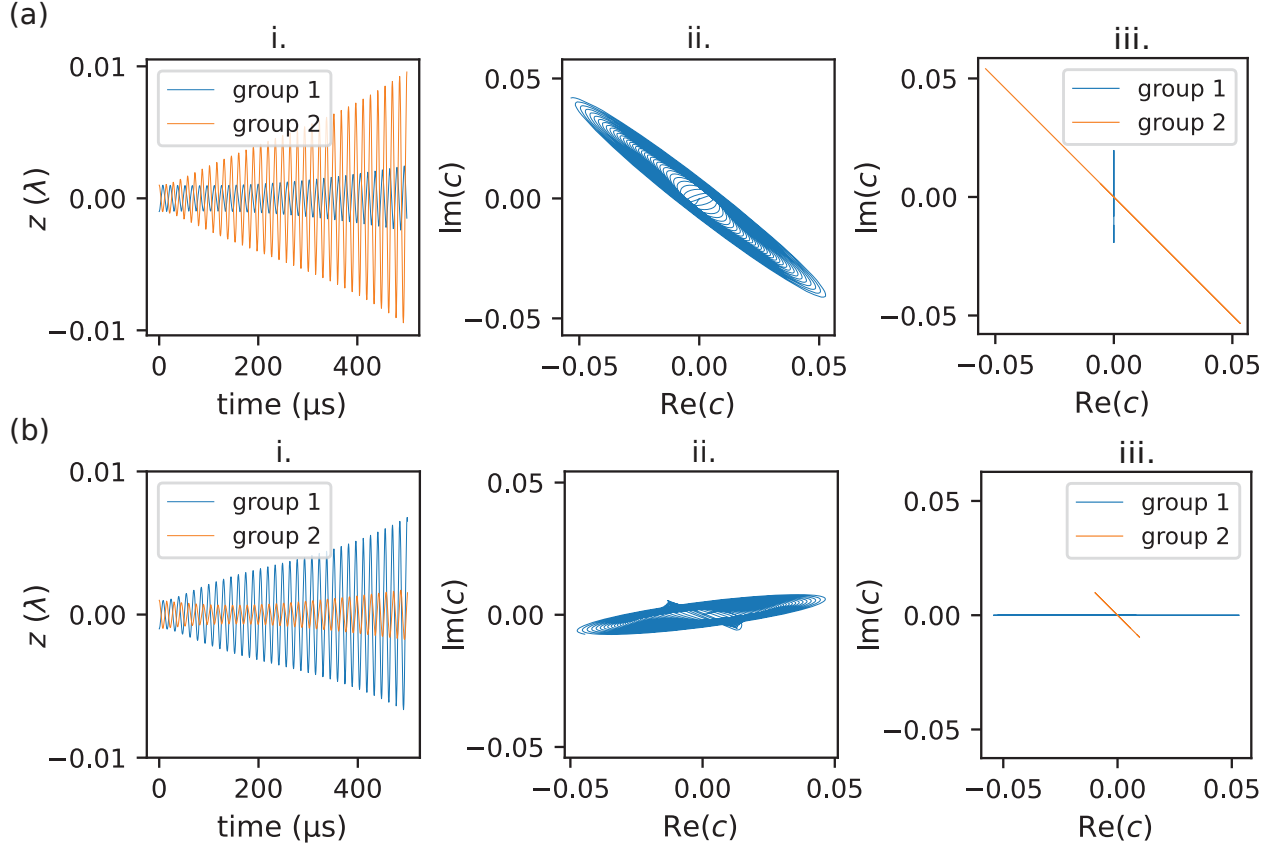


Figure 5.6: Observing non-reciprocity. We show mean-field, numerically simulated trajectories of two groups of atoms interacting non-reciprocally with (a) $\varphi = \pi/4$, $\Omega = 2\pi \times 1$ MHz, $\Delta_{\text{pc}} = -2\pi \times 0.5$ MHz and (b) $\varphi = 3\pi/4$, $\Omega = 2\pi \times 1$ MHz, $\Delta_{\text{pc}} = -2\pi \times 0.5$ MHz. In (a), we see that the motion of group 1 resonantly drives the motion of group 2 and results in the phase of the cavity field being $\pi/4$. In (b), the motion of group 2 amplifies the motion of group 1, resulting in a cavity field phase of π . Trajectories are the result of integrating the full equations of motion, including the dynamics of the cavity field. This, as well as nonlinearities from the sinusoidal profile of the cavity field, could explain the dynamics in the motional amplitude of the “decoupled” group as seen in (a)i and (b)i.

between the groups, each group emits a cavity field with a different phase: group 1 emits a field with phase $\varphi + \theta$ and group 2 emits with phase $2\varphi + \theta$. Thus, the motion of each group is separately encoded in the quadrature of the cavity field corresponding to its emitted phase. One can simply project the measured cavity field onto each quadrature to obtain the respective amplitude of the motion for each group (Fig. 5.6). In this scheme, we also set the trap frequencies of the groups to be equal, so that one group may resonantly drive the motion of the other, causing amplification of the motion (and of the corresponding quadrature of the cavity field). Examples of trajectories for the maximally nonreciprocal cases $\varphi = \pi/4$ and $\varphi = 3\pi/4$ are shown in Fig. 5.6. In Fig. 5.6(a), we see that group 1's motional amplitude stays small, while group 2's grows rapidly over time. Correspondingly, cavity field trajectory becomes elliptical, with the major axis of the ellipse close to the angle $2\varphi + \theta$. In Fig. 5.6(b) we see the opposite: group 1's motion grows rapidly while group 2's does not, and the cavity field trajectory's major axis is nearly aligned with the angle $\varphi + \theta$.

Chapter 6

The future of E6

Over the course of my time on E6, we have realized that this apparatus is capable of exploring a multitude of varied and exciting scientific directions. Many of our current pursuits had not been thought of when I started as a graduate student; it was only through careful thought, literature review, and inspiration from our many personnel that we were able to formulate new ideas and bring them to reality. As such, I have confidence that the E6 team will continue to innovate and dream up ideas for future experiments that are not on our radar at the moment.

In the time since we finished our investigation of optomechanical self-organization, E6 has rapidly built up new capabilities. In Ch. 2, I described changes made to our heterodyne detection system for our spin self-organization experiments, which were proposed by Leon Lu. To realize spin self-organization, we apply a magnetic field along the cavity axis and pump the atoms with light polarized along the cavity axis, such that the light the atoms scatter into the cavity arises from Raman scattering. The Raman field and the pump field interfere to create circularly polarized light, which imposes a fictitious magnetic field that points the atomic spins away from the cavity axis. The spins then precess in-phase with one another about the cavity axis at a frequency determined by the atom-induced circular birefringence of the cavity. As such, this model exhibits a rich phase diagram featuring a superradiant phase that breaks $U(1)$ and time-translation symmetry. It has also been interesting to study the effects of mesoscopic fluctuations and dissipation in this system. Another, separate, direction could be to also examine the quantum nature of the cavity light, for instance in the form of ponderomotive squeezing induced by the back-action of the atomic motion [136, 137].

Meanwhile, we have also rapidly built-up Rydberg atom capabilities, an effort spearheaded by Tai Xiang. One of the main complications in our Rydberg pursuits is the presence of stray electric fields from our cavity (both from static charges and the piezos beneath the mirrors). Due to the high polarizability of Rydberg atoms, these stray fields can cause huge fluctuations of the Rydberg resonance frequency. At first, we were not sure if we would be able to drive coherent excitation to the Rydberg states because of this. However, with the hard work of many, we have come up with ways to compensate for the stray electric

fields and drive Rabi oscillations on the ground-Rydberg qubit. We are currently working on integrating cavity detection with coherent Rydberg excitation. Rapid detection on the ground-Rydberg qubit could be interesting for quantum simulation experiments, for diagnosing errors in Rydberg-based entangling gates, and for high-bandwidth sensing of the Rydberg atom’s electric field environment. Eventually, we would also like to perform high-fidelity Rydberg entangling gates and operate a (small) neutral atom quantum computer contained within our cavity, using the cavity for mid-circuit readout and quantum error correction primitives.

An idea that was seeded long ago, but only came to fruition in the past couple of years, was that of mid-circuit measurement using 1529 nm-wavelength light to locally “shield” atoms from the cavity and probe light. This project is led by Tsai-Chen Lee (and formerly Zhenjie Yan), who has done much careful alignment and accounting of errors. A recent development, proposed by our new postdoc Nathaniel Vilas, was to incorporate feedforward based on our qubit measurements to improve our ability to either classify or prepare the state of a qubit whose initial state is unknown. Combined with our Rydberg efforts, this work builds towards the making of a quantum processor with rapid cavity-enabled error detection.

With the aforementioned projects wrapping up, we hope to soon pursue our ideas on testing fluctuation-dissipation relations (see Ch. 4) and for realizing the n -phase Dicke model with non-reciprocal interactions (see Ch. 5). It is my hope that these projects will open new directions for studies of non-equilibrium, driven-dissipative phenomena. Especially with the rapidly growing body of work on non-reciprocal interactions and non-reciprocal phase transitions [133, 138, 139, 119, 120, 119, 140, 125, 141], it would be very interesting to see what the atom-cavity system can contribute.

Another direction that E6 has thought about (though not made movement on) is that of using our system for quantum networking. In the quest to build networks of quantum processors over long baselines, there are proposals [142, 143, 144, 145] to use atoms in cavities as quantum repeater nodes. By entangling qubits with photons, one can use a photon signal to herald the success of entanglement between qubits separated by long distances [146, 147, 148, 149]. One could also imagine harnessing superradiance to enhance the efficiency of atom-photon interfaces [82].

In the meantime, we occasionally bring up the possibilities of upgrading our cavity and upgrading to a 2D atom array. Performing these upgrades would be rather invasive, and result in much downtime of the experiment. With all the science that we want to explore, the planning of these upgrades continues to get pushed back. Although it is true that we could accomplish more with a higher cooperativity cavity and a larger atom array, there is still a lot we can do with the current system. Perhaps these upgrades will come in the far future, or perhaps they will signal the birth of a new experiment in the Stamper-Kurn group.

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Appendix A

^{87}Rb ground state structure

In this chapter, I discuss how the ^{87}Rb ground state hyperfine structure changes with an applied magnetic field. Since in the ground state we have $\vec{J} = \vec{S}$, the Hamiltonian can be written as

$$\mathcal{H} = -\vec{\mu} \cdot \vec{B} + \mathcal{H}_{\text{hfs}} \quad (\text{A.1})$$

where the hyperfine structure Hamiltonian $\mathcal{H}_{\text{hfs}}$ is given by

$$\mathcal{H}_{\text{hfs}} = A_{\text{hfs}} \vec{I} \cdot \vec{S}. \quad (\text{A.2})$$

We have defined the magnetic dipole constant $A_{\text{hfs}} = 2\pi\hbar \times 3.417 \text{ GHz}$. We also have

$$\vec{\mu} = \mu_B (g_I \vec{I} - g_S \vec{S}), \quad (\text{A.3})$$

where g_I and g_S are the g -factors for the nuclear and electron spins respectively. Note that g_I is considerably smaller than g_S due to the gyromagnetic ratio scaling with q/m (q being the charge and m being the mass). The opposite sign of charge for the electron and the nucleus is already accounted for in our definition of $\vec{\mu}$ (Eq. (A.3)).

Let us first consider the behavior of the magnetic quantum levels under a weak magnetic field $\vec{B} = B\hat{\mathbf{z}}$. In this case, F and m_F are good quantum numbers and we treat the $-\vec{\mu} \cdot \vec{B}$ term as a perturbation. Making use of the projection theorem, we express

$$\langle F, m_F | \vec{S} | F, m_F \rangle = \frac{\langle F, m_F | \vec{S} \cdot \vec{F} | F, m_F \rangle}{\hbar^2 F(F+1)} \vec{F}, \quad (\text{A.4})$$

$$\langle F, m_F | \vec{I} | F, m_F \rangle = \frac{\langle F, m_F | \vec{I} \cdot \vec{F} | F, m_F \rangle}{\hbar^2 F(F+1)} \vec{F} \quad (\text{A.5})$$

and use $\vec{F} = \vec{I} + \vec{S}$ to write the first-order energy correction from the perturbation as

$$\Delta E_{\text{weak field}} = \mu_B g_F m_F B, \quad (\text{A.6})$$

where we define $g_F = g_S \frac{F(F+1) - I(I+1) + S(S+1)}{2F(F+1)} + g_I \frac{F(F+1) + I(I+1) - S(S+1)}{2F(F+1)}$.

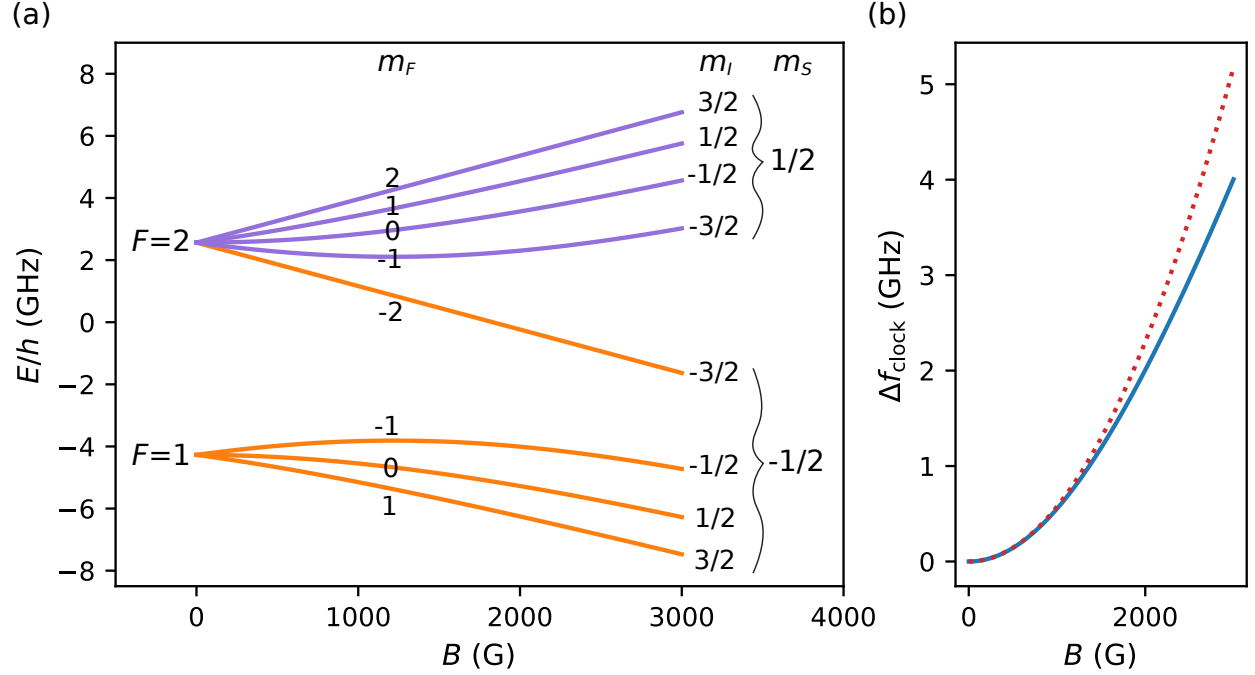


Figure A.1: ^{87}Rb ground state hyperfine structure in an external magnetic field. (a) Numerically calculated splitting of the hyperfine structure at various magnetic field strengths B . At low B , m_F is a good quantum number and the energy splittings of the $|F, m_F\rangle$ states are linear in B . At high B , m_I and m_S are good quantum numbers, and the eigenstates become split by their m_S . (b) Magnetic field-induced shift Δf_{clock} of the $|F=1, m_F=0\rangle \rightarrow |F=2, m_F=0\rangle$ clock transition for various B . Solid blue line is a numerical calculation and red dashed line shows the quadratic approximation for small fields.

Under a strong field, the magnetic interaction dominates over the hyperfine interaction, so we treat the hyperfine Hamiltonian as a perturbation on top of the fine structure eigenstates, $|S, I, m_S, m_I\rangle$. In this regime, the energy becomes

$$E_{\text{strong field}} = A_{\text{hfs}} m_I m_S + \mu_B (g_S m_S - g_I m_I) B. \quad (\text{A.7})$$

From this, we see that the magnetic field primarily splits the states according to their m_S (the contribution from m_I is much smaller). Interestingly, this means that the $|F=2, m_F=-2\rangle = |S=1/2, I=3/2, m_S=-1/2, m_I=-3/2\rangle$ stretched state, unlike the rest of the $|F=2, m_F\rangle$ states, takes on a negative slope with respect to B in this regime. This is shown in Fig. A.1(a).

In the intermediate field regime, the magnetic field mixes the non-stretched states in the $F=2$ and $F=1$ levels, leading to corrections $\mathcal{O}(B^2)$. The Hamiltonian in Eq. (A.1) can be numerically diagonalized to obtain the energy levels for all values of B , as shown in Fig. A.1(a). Explicitly, this can be done by first writing Eq. (A.1) in the $|S, I, m_S, m_I\rangle$

basis. In this basis, the magnetic interaction is purely diagonal, given by $\mu_B(g_S m_S - g_I m_I)B$. Meanwhile, the $\vec{I} \cdot \vec{S}$ term has matrix elements given by:

$$\begin{aligned} \langle S, I, m'_S, m'_I | \vec{I} \cdot \vec{S} | S, I, m_S, m_I \rangle \\ = \langle S, I, m'_S, m'_I | (I_x S_x + I_y S_y + I_z S_z) | S, I, m_S, m_I \rangle \end{aligned} \quad (\text{A.8})$$

$$= \langle S, I, m'_S, m'_I | \left(\frac{1}{2}(I_+ S_- + I_- S_+) + I_z S_z \right) | S, I, m_S, m_I \rangle \quad (\text{A.9})$$

$$= \begin{cases} m_I m_S, & \text{if } m'_S = m_S, m'_I = m_I \\ \frac{1}{2} \sqrt{(S(S+1) - m_S(m_S-1))(I(I+1) - m_I(m_I+1))}, & \text{if } m'_S = m_S - 1, m'_I = m_I + 1 \\ \frac{1}{2} \sqrt{(S(S+1) - m_S(m_S+1))(I(I+1) - m_I(m_I-1))}, & \text{if } m'_S = m_S + 1, m'_I = m_I - 1 \end{cases} \quad (\text{A.10})$$

Numerical diagonalization allows us to see the quadratic Zeeman shift to the clock transition frequency. We may compare the numerical calculation to the quadratic approximation for small fields [41, 150]:

$$\Delta f_{\text{clock}} = \frac{1}{2\pi} \frac{(g_J - g_I)^2 \mu_B^2}{2\hbar \Delta E_{\text{hfs}}} B^2. \quad (\text{A.11})$$

The result is shown in Fig. A.1(b), where the solid line shows the numerical calculation and the dashed line is Eq. (A.11). This clock transition energy shift is important to calibrate when working with the clock state hyperfine qubit, as was done in Ref. [42].

Appendix B

Resonance fluorescence in a cavity

In our mid-circuit measurement work [42], we found that when probing atom fluorescence at intensities beyond saturation, the number of photons emitted into the cavity decreases. We eventually understood this to be due to the effect known as the Mollow triplet [151]. Under strong driving, an atom (with transition frequency ω_a) not only scatters light elastically at the probe frequency ω_p , but also scatters incoherently into sidebands located at $\omega_p \pm \sqrt{\Omega^2 + \Delta_{pa}^2}$, where Ω is the probe Rabi frequency and $\Delta_{pa} = \omega_p - \omega_a$. As we increase Ω , incoherent scattering increases (at both the carrier and sideband frequencies) while coherent scattering

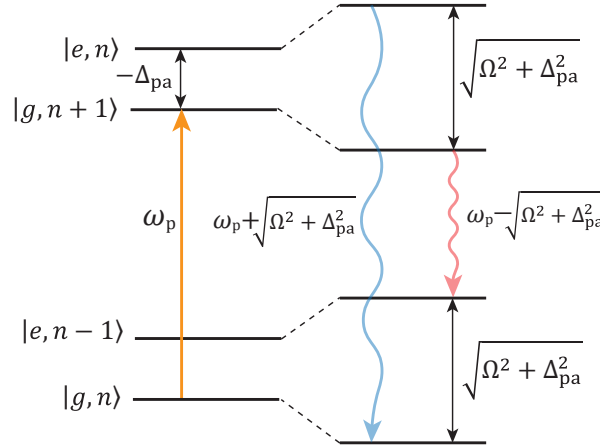


Figure B.1: Mollow triplet schematic. On the left side are the bare energy levels and on the right side are the dressed energy levels arising from the interaction between the atom and the probe laser at frequency ω_p (orange arrow). For a macroscopic number of probe photons, the energy splitting is roughly equal for the relevant Fock state occupation numbers and is given by $\sqrt{\Omega^2 + \Delta_{pa}^2}$. The atom can emit photons not only at ω_p , but also at $\omega_p \pm \sqrt{\Omega^2 + \Delta_{pa}^2}$, resulting in the Mollow triplet sidebands (blue and red wavy arrows).

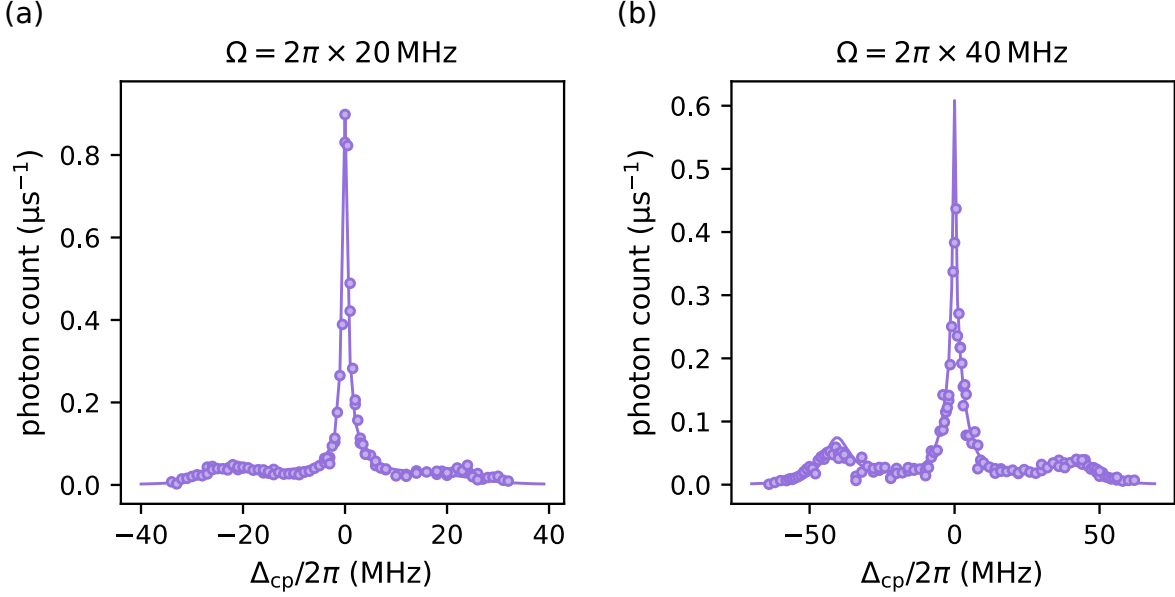


Figure B.2: Mollow triplet data. We scan Δ_{cp} and record the rate at which a single atom emits photons into the cavity for a probe strength of (a) $\Omega = 2\pi \times 20$ MHz and (b) $\Omega = 2\pi \times 40$ MHz. Solid lines are QuTiP simulation results that have been rescaled by a constant factor to account for our experimental detection efficiency.

decreases. Since incoherent scattering is filtered out by the narrow linewidth of the cavity, we see the cavity photon number fall off. The Mollow triplet spectrum can be understood schematically as shown in Fig. B.1. In analogy to the Jaynes-Cummings Hamiltonian, we consider a ladder of hybridized atom-photon eigenstates that are linear combinations of $|g, n\rangle$ and $|e, n-1\rangle$, where $|g\rangle$ is the atomic ground state, $|e\rangle$ is the atomic excited state, and $|n\rangle$ is a Fock state of photons. However, we consider a classical drive with large average photon number $\bar{n}$, such that the energy splitting of the hybridized eigenstates for $n \simeq \bar{n}$ is given by $\sqrt{\Omega^2 + \Delta_{\text{pa}}^2}$ to good approximation (notably, it does not depend on n). From this picture, we see that the dressed atom also has transitions at frequencies of $\omega_p \pm \sqrt{\Omega^2 + \Delta_{\text{pa}}^2}$. These are the frequencies of the Mollow triplet sidebands. In E6, we sought to confirm the presence of the Mollow triplet sidebands as a follow-up to our observations in Ref. [42]. Keeping the probe frequency fixed, we scanned the cavity-to-probe detuning ($\Delta_{\text{cp}} \equiv \omega_c - \omega_p$). The cavity essentially acts as a filter, measuring segments of the fluorescence power spectrum within its bandwidth.

The picture also becomes more complex when we add in the cavity. Keeping the probe frequency fixed at $\Delta_{\text{pa}} = -2\pi \times 10$ MHz, we measured the Mollow triplet at two different probe powers (Fig. B.2). Interestingly, we observed a sideband asymmetry, where we measured more cavity photons in the sideband at $\Delta_{\text{cp}} = -\sqrt{\Omega^2 + \Delta_{\text{pa}}^2}$. This concurs with

the results of simulations. We suspect that the asymmetry is a result of cavity physics and the sign of Δ_{pa} . In simulations, we see that the detuning of the taller sideband appears to have the same sign as Δ_{pa} . Our measurement of the Mollow triplet spectrum using a cavity could perhaps be of interest to those pursuing similar measurements in solid-state platforms [152, 153].

Appendix C

Tweezer sorting code

The code provided below is used to do rearrangement of the tweezer array in E6.

```
class TestEventHandler(PatternMatchingEventHandler):

    def __init__(self, Cycle_num, drop_num, AXA_num, *args, **kwargs):
        :
        super(TestEventHandler, self).__init__(*args, **kwargs)
        self.last_created = None
        self.Cycle_num = Cycle_num
        self.AXA_num = AXA_num
        self.drop_num = drop_num
        self.drop_counter=0
        self.AXA_counter=0
        self.i_counter=0

    def on_created(self, event):

        path = event.src_path
        if path != self.last_created:
            self.last_created = path
            print(f'{event.src_path}_has_been_created!')
            time.sleep(0.05)

            try:
                hf = h5py.File(f'{event.src_path}_', 'r')
            except:
                time.sleep(0.05)
                hf = h5py.File(f'{event.src_path}_', 'r')
                print('exception')
            print('read_file')
            im_array = np.array(hf['frame-00'])
```

```

hf.close()
atom_count, empty_list = analyze_image(im_array,
    tweezer_freq_list, num_tweezers)
print(atom_count, empty_list)

if 0 < atom_count:
    segment_queue_L = []
    segment_queue_R = []
    # now divide into left and right sides of the
    boundary
    mask_empty = np.diff(empty_list) > 1
    for i in range(len(mask_empty)):
        if mask_empty[i] and empty_list[0] == 0:
            empty_list_reduced = empty_list[i+1:]
            break
        else:
            empty_list_reduced = empty_list
    for i in range(len(mask_empty)):
        if mask_empty[-1-i] and empty_list[-1] ==
            num_tweezers-1:
            empty_list_reduced = empty_list_reduced[:-1-i
                ]
            break

    # now divide into left and right sides of the
    boundary
    empty_list_reduced=np.sort(empty_list_reduced)
    print('empty_list_reduced:', empty_list_reduced)
    num_empty = len(empty_list)
    boundary = empty_list[int(num_empty / 2)]
    print('boundary:', boundary)
    mask_L = empty_list_reduced <= boundary
    mask_R = empty_list_reduced > boundary
    empty_list_L = empty_list_reduced[mask_L]
    empty_list_R = empty_list_reduced[mask_R]
    for i in empty_list_L:
        if i > 0:
            segment_queue_L.append(segment_list[i - 1])
    for i in empty_list_R:
        if i < num_tweezers-1:
            segment_queue_R.append(segment_list[2*(
                num_tweezers-1)-i-1])
    segment_queue_R = np.flip(segment_queue_R)

```

```

print(f'segment_queue_L={segment_queue_L}')
print(f'segment_queue_R={segment_queue_R}')

if len(segment_queue_L) > 0:
    print('left sorting')
    for k in range(len(segment_queue_L) - 1):
        # print(segment_queue_L[k])
        lStep = k + 1 # current step is step k+1 (+1
                       # because step0 is the static config)
        llSegment = segment_queue_L[k] # associated
        data memory segment
        llLoop = 1 # pattern repeated once
        llNext = k + 2 # next step is the next sweep
        llCondition = SPCSEQ_ENDLOOPALWAYS #
        unconditionally leave current step
        llValue = (llCondition << 32) | (llLoop <<
        32) | (llNext << 16) | (llSegment)
        spcm_dwSetParam_i64(hCard,
        SPC_SEQMODE_STEPMEMO + lStep, int64(
        llValue))

    lStep = len(segment_queue_L) # current step is
    the last one in segment_queue
    llSegment = segment_queue_L[-1] # associated
    data memory segment
    llLoop = 1 # pattern repeated once
    if len(segment_queue_R) > 0:
        llNext = len(segment_queue_L) + 1 # next go
        to resorting on the right
    else:
        print('dropping')
        llNext = 2*num_tweezers + 21
        llCondition = SPCSEQ_ENDLOOPALWAYS #
        unconditionally leave current step
        llValue = (llCondition << 32) | (llLoop << 32) |
        (llNext << 16) | (llSegment)
        spcm_dwSetParam_i64(hCard, SPC_SEQMODE_STEPMEMO +
        lStep, int64(llValue))

    if len(segment_queue_R) > 0:
        print('right sorting 1')
        for k in range(len(segment_queue_R) - 1):
            lStep = len(segment_queue_L) + k + 1 #

```

```

        current step is step k+1 (+1 because
        step0 is the static config)
        llSegment = segment_queue_R[k] #
        associated data memory segment
        llLoop = 1 # pattern repeated once
        llNext = len(segment_queue_L) + k + 2 #
        next step is the next sweep
        llCondition = SPCSEQ_ENDLOOPALWAYS #
        unconditionally leave current step
        llValue = (llCondition << 32) | (llLoop
        << 32) | (llNext << 16) | (llSegment)
        spcm_dwSetParam_i64(hCard,
        SPC_SEQMODE_STEPMEMO + lStep, int64(
        llValue))

    lStep = len(segment_queue_L) + len(
        segment_queue_R) # current step is the
        last one in segment_queue
    llSegment = segment_queue_R[-1] # associated
        data memory segment
    llLoop = 1 # pattern repeated once
    llNext = 2*num_tweezers + 21 # this is sort
        of random, just want a number that is not
        called before
    llCondition = SPCSEQ_ENDLOOPALWAYS #
        unconditionally leave current step
    llValue = (llCondition << 32) | (llLoop <<
        32) | (llNext << 16) | (llSegment)
    spcm_dwSetParam_i64(hCard,
        SPC_SEQMODE_STEPMEMO + lStep, int64(
        llValue))

else:
    lStep = 2 * num_tweezers + 100
    llSegment = 2 * num_tweezers - 2 # the
        static waveform
    llLoop = 1
    llNext = 0
    llCondition = SPCSEQ_ENDLOOPALWAYS #
        unconditionally leave current step
    llValue = (llCondition << 32) | (llLoop <<
        32) | (llNext << 16) | (llSegment)
    spcm_dwSetParam_i64(hCard,
        SPC_SEQMODE_STEPMEMO + lStep, int64(

```

```

        llValue))

elif len(segment_queue_R) > 0 and len(segment_queue_L
) == 0:
    print('right_sorting_2')
    for k in range(len(segment_queue_R) - 1):
        lStep = k + 1 # current step is step k+1 (+1
            because step0 is the static config)
        llSegment = segment_queue_R[k] # associated
            data memory segment
        llLoop = 1 # pattern repeated once
        llNext = k + 2 # next step is the next sweep
        llCondition = SPCSEQ_ENDLOOPALWAYS #
            unconditionally leave current step
        llValue = (llCondition << 32) | (llLoop <<
            32) | (llNext << 16) | (llSegment)
        spcm_dwSetParam_i64(hCard,
            SPC_SEQMODE_STEPMEMO + lStep, int64(
                llValue))

        lStep = len(segment_queue_R) # current step is
            the last one in segment_queue
        llSegment = segment_queue_R[-1] # associated
            data memory segment
        llLoop = 1 # pattern repeated once
        llNext = 2 * num_tweezers + 21 # this is sort of
            random, just want a number that is not called
            before
        llCondition = SPCSEQ_ENDLOOPALWAYS #
            unconditionally leave current step
        llValue = (llCondition << 32) | (llLoop << 32) |
            (llNext << 16) | (llSegment)
        spcm_dwSetParam_i64(hCard, SPC_SEQMODE_STEPMEMO +
            lStep, int64(llValue))

else:
    lStep = 1 # current step is step 1
    llLoop = 1 # pattern repeated once
    llSegment = 2*num_tweezers-1 + self.drop_counter
        # the drop waveform
    llNext = 0
    llCondition = SPCSEQ_ENDLOOPALWAYS #

```

```

        unconditionally leave current step
        llValue = (llCondition << 32) | (llLoop << 32) |
            (llNext << 16) | (llSegment)
        spcm_dwSetParam_i64(hCard, SPC_SEQMODE_STEPMEMO +
            lStep, int64(llValue))

lStep = 2 * num_tweezers + 21
print(f"######{self.drop_counter
}#####")
llSegment = 2 * num_tweezers - 1 + self.drop_counter
    # the drop waveform
llLoop = int(10 * 0.001 * SAMP_FREQ / wf_list[
    llSegment].SampleLength) # pattern repeated once
llNext = 2 * num_tweezers + 100 # 0 # next step is
    0
llCondition = SPCSEQ_ENDLOOPALWAYS # unconditionally
    leave current step
llValue = (llCondition << 32) | (llLoop << 32) | (
    llNext << 16) | (llSegment)
spcm_dwSetParam_i64(hCard, SPC_SEQMODE_STEPMEMO +
    lStep, int64(llValue))

```

Appendix D

Superradiant and subradiant cavity scattering by atom arrays

This appendix includes the following paper [53], discussed in Ch. 3:

1. Zhenjie Yan et al. “Superradiant and Subradiant Cavity Scattering by Atom Arrays”. In: *Physical Review Letters* 131.25 (Dec. 2023), p. 253603. ISSN: 0031-9007, 1079-7114. DOI: 10.1103/PhysRevLett.131.253603. (Visited on 03/05/2024)

Superradiant and Subradiant Cavity Scattering by Atom Arrays

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We realize collective enhancement and suppression of light scattered by an array of tweezer-trapped ^{87}Rb atoms positioned within a strongly coupled Fabry-Pérot optical cavity. We illuminate the array with light directed transverse to the cavity axis, in the low saturation regime, and detect photons scattered into the cavity. For an array with integer-optical-wavelength spacing each atom scatters light into the cavity with nearly identical scattering amplitude, leading to an observed N^2 scaling of cavity photon number as the atom number increases stepwise from $N = 1$ to $N = 8$. By contrast, for an array with half-integer-wavelength spacing, destructive interference of scattering amplitudes yields a nonmonotonic, subradiant cavity intensity versus N . By analyzing the polarization of light emitted from the cavity, we find that Rayleigh scattering can be collectively enhanced or suppressed with respect to Raman scattering. We observe also that atom-induced shifts and broadenings of the cavity resonance are precisely tuned by varying the atom number and positions. Altogether, tweezer arrays provide exquisite control of atomic cavity QED spanning from the single- to the many-body regime.

DOI: 10.1103/PhysRevLett.131.253603

As highlighted by Dicke's seminal work on super- and subradiance [1], the interaction of multiple emitters with a quantum mode of light differs from that of the emitters individually. Collective “superradiant” (or “bright”) states, resulting from constructive interference, give rise to an enhanced emission rate per excitation, which grows with the number of emitters. Conversely, “subradiant” (or “dark”) states arise from destructive interference, leading to a suppression or complete cancellation of photon emission.

In the case of extended samples, with emitters distributed over distances longer than the emitted optical wavelength, the observation and control of super- and subradiance depends critically on the exact spatial distribution of the emitters. For example, the precise structure of mesoscopic samples [2–4] or periodic emitter arrays [5–7] controls whether their collective emission is enhanced or suppressed, or directed into single or multiple optical modes.

In cavity quantum electrodynamics (QED), in which each of multiple emitters couples strongly to a single-mode cavity field, the properties of (and the access to) the bright and dark manifolds depend strongly on the spatial positions of emitters within the cavity. Already for single emitters, controlling the position of atoms in a cavity advanced the field of cavity QED [8–11]. Basic effects of few-body cavity QED have been illustrated by the controlled placement of two atoms [12,13] or ions [14,15] within resonant cavities. Experiments on superconducting quantum circuits

have extended studies of collective emission into microwave cavities and waveguides to as many as 10 qubits [16–20].

In this Letter, we employ deterministically loaded atom tweezer arrays [21,22], a powerful new platform for quantum simulation [23], metrology [24,25], and information processing [26], to advance atomic cavity QED from the few- to the many-body regime while preserving precise control over the cavity interaction of each individual atom.

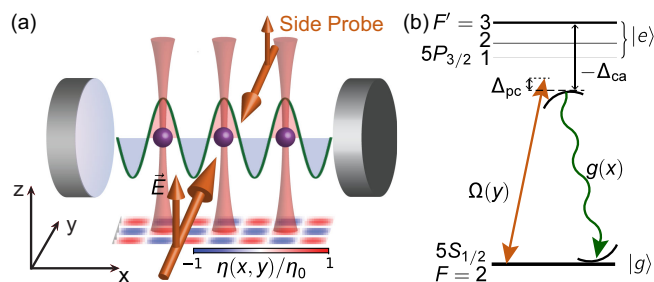


FIG. 1. Schematic of the experimental setup. (a) Atoms are driven by a pair of counter propagating side probe beams. The two-photon scattering amplitude $\eta(x, y)$ from the probe beam into the cavity, by an atom located at (x, y) , exhibits a two-dimensional checkerboard pattern. (b) The probe light (orange) and the cavity photons (green) couple the $F = 2$ hyperfine states of the $5S_{1/2}$ atomic ground state with the $5P_{3/2}$ excited states. Three hyperfine manifolds $F' = 3, 2, 1$ of the excited states contribute to the probe-cavity scattering.

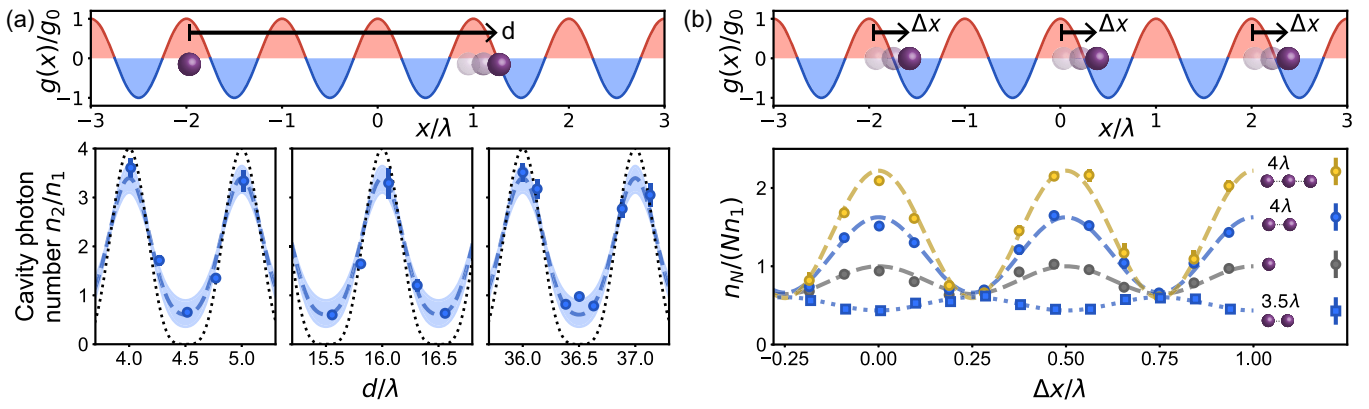


FIG. 2. (a) Cavity scattering by a two-atom array. One atom is placed at a cavity antinode, and scatters with steady-state cavity photon number n_1 on its own. Adding a second atom, at a distance d from the first, changes the cavity photon number to n_2 . Expected ratios n_2/n_1 for perfectly localized atoms (dotted black line) and atoms with $\sigma = 100(14)$ nm position fluctuations (dashed blue line, uncertainty in σ indicated by shaded area). (b) Normalized cavity photon number $n_N/(Nn_1)$ generated by N -atom arrays with fixed spacing, displaced by Δx from a cavity antinode. The array has $N = 1$ (gray), 2 (blue), 3 (yellow) atoms, and a spacing of 4λ (circles) or 3.5λ (squares). The dashed and dotted lines are cosinusoidal fits with a period of $\lambda/2$. All data are taken with $\Delta_{pc} = 0$.

With such a tweezer array, we place a fixed number of ^{87}Rb atoms at fully controlled positions along the axis of a strongly coupled Fabry-Pérot optical resonator [27,28] [Fig. 1(a)]. This method improves upon the incomplete control of atom number, position, and motion in previous approaches used in trapped-atom and -ion cavity QED experiments [29,30]. By driving this emitter array with light propagating transverse to the cavity axis while monitoring the cavity emission, we prepare both super- and subradiant (single-excitation) states. We characterize these states via the number of photons present in the cavity, which scales as N^2 for superradiant states and sublinearly for subradiant ones.

We employ an in-vacuum near-concentric Fabry-Pérot optical cavity with a TEM_{00} mode whose frequency (ω_c) lies near the transition frequency (ω_a) of the ^{87}Rb $F = 2 \rightarrow F' = 3 D_2$ transition (wavelength $\lambda = 780$ nm) [31,32]. Near the cavity center, the coupling amplitude of a single ^{87}Rb atom to this cavity mode varies as $g(x) = g_0 \cos kx$; here, $g_0 = 2\pi \times 3.1$ MHz (on the cycling transition) and $k = 2\pi/\lambda$. Given the atomic and cavity resonance half-linewidths of $\gamma = 2\pi \times 3.0$ MHz and $\kappa = 2\pi \times 0.53$ MHz, respectively, the cavity achieves the strong coupling condition with single-atom cooperativity $C = g_0^2/(2\kappa\gamma) = 3.0$ [33].

A one-dimensional array of optical tweezers is formed by laser beams with a wavelength of 808 nm sent transversely to the cavity through a high numerical-aperture imaging system. Precooled and optically trapped ^{87}Rb atoms are loaded into as many as 16 tweezers, detected through fluorescence imaging, and then sorted into regularly spaced arrays of $N = 1$ to 8 atoms [34]. The total length of these arrays is much smaller than the ~ 1 mm cavity Rayleigh range. The array is aligned to place atoms at the radial center of the cavity. Piezocontrolled mirrors

and an acousto-optical deflector are used to position the array with nanometer-scale precision along the cavity axis.

Our experiments focus on the response of this atom array when it is driven optically. For this, we illuminate the array with a transverse standing wave of monochromatic probe light, see Fig. 1(a). The probe is linearly polarized along the z axis (chosen as our quantization axis), perpendicular to the cavity, and drives each atom with a spatially dependent Rabi frequency $\Omega(y) = \Omega_0 \cos ky$. The standing-wave configuration balances photon recoil momentum, allows for positional calibration along the y axis, and reduces incoherence in photon emission caused by atomic thermal motion [34]. We employ a weak probe amplitude to ensure that the probability of having more than one atom excited simultaneously is negligible. All tweezer traps are centered at $y = 0$. The probe frequency ω_p operates at a small detuning $\Delta_{pc} = \omega_p - \omega_c$ from the cavity resonance [Fig. 1(b)]. Prior to probe illumination, the tweezer-trapped atoms are polarization-gradient cooled and prepared in the $F = 2$ ground hyperfine manifold, without control of their Zeeman state. Probe light scattered by the array into the cavity, and thence through one mirror of our one-sided cavity, is detected by a single-photon counting module.

To demonstrate the strong sensitivity of collective cavity scattering on the exact positioning of the scatterers, we first examine an array of just two atoms. As illustrated in Fig. 2(a), we first position one atom at an antinode of the cavity field. With the cavity at a large detuning $\Delta_{ca} = \omega_c - \omega_a = -2\pi \times 507$ MHz—a specific “magic” value chosen to suppress fluctuations from internal-state dynamics, as described below—we denote the steady-state cavity photon number generated by this single atom as n_1 . We then add a second atom at a variable distance d from the first, and record the cavity photon number produced by the atom pair as n_2 . At integer-wavelength separation

($d = m\lambda$ with m an integer), we observe superradiant light scattering, where the total cavity emission rate is greater than that of two individual atoms ($n_2 > 2n_1$). At half-integer-wavelength separation [$d = (m + 1/2)\lambda$], we observe subradiant light scattering ($n_2 < 2n_1$).

To account for the observed behavior, let us consider that each atom i , positioned at location (x_i, y_i) , serves as a source for light in the cavity, with elastic scattering amplitude $\eta(x_i, y_i) = g(x_i)\Omega(y_i)/2\Delta_{\text{ca}} \equiv \eta_0 \cos kx_i \cos ky_i$, which we obtain by treating the atom as a two-level emitter and adiabatically eliminating its excited state. The scattering amplitudes from all atoms add coherently. The steady-state cavity photon number scattered by N atoms is then $n_N = |\bar{a}|^2$, where the expectation value of the cavity-field amplitude is given, following a semiclassical treatment [34,36,37] in the dispersive coupling regime $|\Delta_{\text{ca}}| \gg \{\Omega_0, g_0, \gamma\}$, as

$$\bar{a} = \frac{\sum_i \eta(x_i, y_i)}{[\Delta_{\text{pc}} - \sum_i g^2(x_i)/\Delta_{\text{ca}}] + i[\kappa + \sum_i \gamma g^2(x_i)/\Delta_{\text{ca}}^2]}. \quad (1)$$

Here, we have accounted for the atom-induced dispersive shift $\sum_i g^2(x_i)/\Delta_{\text{ca}}$ and absorptive broadening $\sum_i \gamma g^2(x_i)/\Delta_{\text{ca}}^2$ of the cavity resonance.

At large $|\Delta_{\text{ca}}|$, where we can neglect atom-induced modifications of the cavity resonance, and with $\Delta_{\text{pc}} = 0$, the cavity photon number varies simply as $n_N = |\sum_i \eta(x_i, y_i)|^2/\kappa^2$. For two atoms, with the first situated exactly at the antinode and the second at an exact axial distance d , one then expects $n_2/n_1 = [1 + \cos kd]^2$, with limiting values $n_2/n_1 = 4$ from constructive interference at integer-wavelength separation, and $n_2/n_1 = 0$ from destructive interference at half-integer-wavelength separation. In our experiment, uncorrelated fluctuations in η , deriving from thermal position fluctuations of the two atoms within their tweezer traps, constrain these limiting values to $n_2/n_1 = 2(1 \pm D)$, where the ratio $D = \langle |\eta|^2 \rangle / \langle |\eta|^2 \rangle$ is the Debye-Waller factor and $\langle \rangle$ denotes an average over the position distribution of a single trapped atom. The data in Fig. 2(a) are consistent with root mean square variations of $\sigma = 100(14)$ nm in both the x and y directions of motion, with σ determined independently by measuring light scattering from a single atom [34]. In future work, σ can be reduced further through bursts of dark-state cooling [38–40]. A strong contrast between constructive and destructive interference is retained even at large atomic separations—as far as $d \simeq 30 \mu\text{m}$.

The photon scattering rate also depends on the positions of the atoms with respect to the standing-wave cavity mode. We illustrate this dependence by measuring light scattering from atom arrays with fixed integer- or half-integer-wavelength spacing while translating these arrays altogether by Δx from a cavity antinode [Fig. 2(b)]. Again, we observe either super- or subradiance when the array is aligned onto cavity antinodes. In contrast, when either type of array is

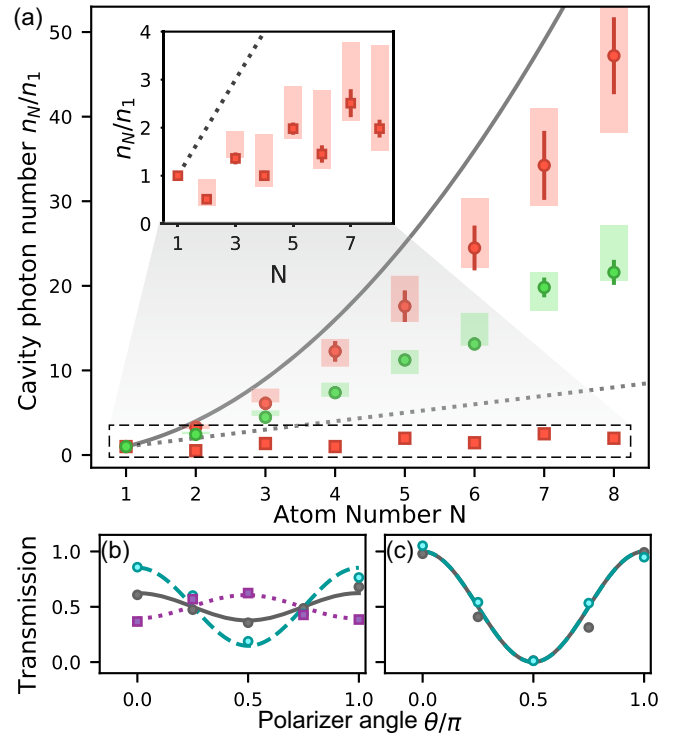


FIG. 3. (a) Normalized cavity photon number (n_N/n_1) for scattering by N -atom arrays with either integer- (circles, $d = 5.0\lambda$) or half-integer- (squares, $d = 5.5\lambda$) wavelength spacing, using cavity-atom detuning at either $\Delta_{\text{ca}} = -2\pi \times 507$ MHz (red) or $-2\pi \times 38$ MHz (green). Inset: close-up data from half-integer-wavelength arrays. The solid (dashed) black line corresponds to a quadratic (N^2 linear N) scaling. The red shaded area is calculated from Eqs. (2) and (3), while the green shaded area is the result of a Monte Carlo calculation assuming equal population among all m_F states [34]. (b),(c) Polarization analysis of cavity emission. Relative transmission through a linear polarizer at an angle θ with respect to the z axis at (b) $\Delta_{\text{ca}} = -2\pi \times 38$ MHz and (c) $\Delta_{\text{ca}} = -2\pi \times 507$ MHz. Shown for scattering by a single atom (gray circles), or $N = 8$ atoms at the constructive (cyan circles) or the destructive interference condition (purple squares). The corresponding curves show results of the Monte Carlo calculation.

aligned to cavity nodes, we observe scattering that scales linearly with N . Here, position fluctuations cause the scattering amplitude from each atom to vary between positive and negative values with equal probability, producing a cavity field with finite variance but zero average amplitude.

The superradiant emission by atoms at the constructive interference condition is further enhanced by increasing the atom number [Fig. 3(a)]. At large $|\Delta_{\text{ca}}|$ and on cavity resonance, the number of cavity photons emitted by an N -atom array is predicted to be

$$n_N = [N(\langle |\eta|^2 \rangle - |\langle \eta \rangle|^2) + N^2 \langle |\eta|^2 \rangle] / \kappa^2. \quad (2)$$

This expression consists of an incoherent part that scales linearly with N and that vanishes if atoms are fully

localized ($\sigma = 0$), and a coherent part that scales quadratically as N^2 . Our measurements with N ranging from 1 to 8 match well with this prediction, clearly exhibiting super-radiant scattering.

At the destructive interference condition, i.e., an atom array with half-integer-wavelength spacing, collective scattering is subradiant, falling below the linear scaling with N expected for an incoherent sample. Here, at similar probe and cavity settings, one expects the cavity photon number to vary as

$$n_N = \left[N(\langle |\eta|^2 \rangle - |\langle \eta \rangle|^2) + \frac{1 - (-1)^N}{2} |\langle \eta \rangle|^2 \right] / \kappa^2. \quad (3)$$

While the incoherent scattering rate remains linear in N , the coherent photon scattering by pairs of atoms with opposite phase cancel out, resulting in a total coherent contribution equal to that of either zero emitters (even N) or a single emitter (odd N). The nonmonotonic behavior observed in our experiment emerges as the coherent scattering rate exceeds the incoherent one: $|\langle \eta \rangle|^2 > \langle |\eta|^2 \rangle - |\langle \eta \rangle|^2$.

At smaller detuning $|\Delta_{\text{ca}}|$, collective light scattering is affected by two additional effects. First, coherent scattering is degraded by polarization and intensity fluctuations arising from internal state dynamics of the multilevel ^{87}Rb atom. In our setup, incoherent Raman scattering causes each atom's magnetic quantum number m_F to be distributed among all possible values. For small $\Delta_{\text{ca}} = -2\pi \times 38$ MHz, the amplitude η for emitting z -polarized light, arising primarily from the near-resonant $F = 2 \rightarrow F' = 3$ transition, varies strongly with m_F . This random variation further degrades the Debye-Waller factor, reducing the collective enhancement of light scattering as shown in Fig. 3(a).

Furthermore, at this detuning, atoms can scatter the z -polarized probe light into cavity modes of two orthogonal polarizations. Scattering into the z -polarized cavity mode, i.e., Rayleigh scattering, does not change the spin state of the ground-state atoms. The final spin state after Rayleigh scattering is independent of which atom scattered a photon. Thus, the Rayleigh scattering rate is determined by the interference of the scattering amplitudes from all atoms, allowing for enhancement or suppression. In contrast, scattering into the y -polarized cavity mode, i.e., Raman scattering, does change the spin state of the ground state. In our experiment, it is reasonable to assume the ground-state atoms are, at all times, in an incoherent mixture of spin states owing to imprecise initial state preparation and rapid state collapse caused by incoherent light scattering. Given that the final spin states produced by each atom Raman scattering a photon are orthogonal, and absent coherence among initial spin states, the Raman scattering rates in our experiment add incoherently. By operating in other regimes which allow for coherences among the ground states, collective Raman scattering can indeed be observed [41–44].

In support of this description, we examine separately the cavity emission of z - and y -polarized light [Fig. 3(b)]. At small detuning, and in comparison to the light polarization emitted by a single atom, the emission of z -polarized light (relative to y -polarized light) is enhanced by superradiant scattering at integer-wavelength atomic spacing and suppressed by subradiant scattering at half-integer-wavelength atomic spacing.

For much of our Letter, we avoid these internal-state and polarization effects by operating at the aforementioned “magic” detuning of $\Delta_{\text{ca}} = -2\pi \times 507$ MHz [34,45]. Here, when accounting for transitions to all three accessible excited hyperfine states ($F' = 1, 2$, and 3), the amplitude for scattering z -polarized light is nearly identical for all Zeeman states within the ground $F = 2$ manifold. Simultaneously, the rate for Raman scattering y -polarized light is nearly extinguished, as observed in the nearly pure z polarization of light emitted from the cavity [Fig. 3(c)]. The identification of such a “magic” detuning for nearly all alkali species [34,45], which allows light scattering by an alkali atom to resemble closely that of just a two-level atom, should be beneficial to other quantum optics experiments using alkali gases.

Additionally, at small detuning, we observe significant modifications of the cavity resonance by the atomic array within. As shown in Fig. 4(a), we record n_N for various arrays as a function of the detuning Δ_{pc} of the probe from the empty-cavity resonance. From the observed emission line shapes, we extract the atom-induced cavity resonance shift and also the total cavity linewidth. The measured spectral shifts and widths show a near-linear dependence on the atom number N [Figs. 4(c) and 4(d)], and agree well with a theoretical calculation accounting for both atomic position and m_F fluctuations [34]. Owing to the $g^2(x_i)$ dependence of the cavity dispersion and absorption terms, the atom-induced cavity modification is the same for atoms at both integer- and half-integer-wavelength spacing. Notably, these modifications are reduced for atom arrays centered on the cavity field nodes, and such spatial dependence is the foundation of optomechanical coupling between the atomic motion and the cavity field [46,47]. By comparison, the atom-induced cavity modifications are all negligible for large $|\Delta_{\text{ca}}|$.

While the current work explores only the bottom of the Dicke ladder, where only one atom is excited, our platform enables study of many-excitation super- or subradiance [1,48], up to full or zero state inversion, respectively, with unprecedented control. The interplay between atomic quantum states, collective atom-cavity interaction, and optical emission properties can be fully explored [49]. Our setup would provide insight into key features of many-body decay in the presence of multiple decay channels (due to the multimode nature of the cavity), such as the scaling with atom number of the peak intensity and the time of peak

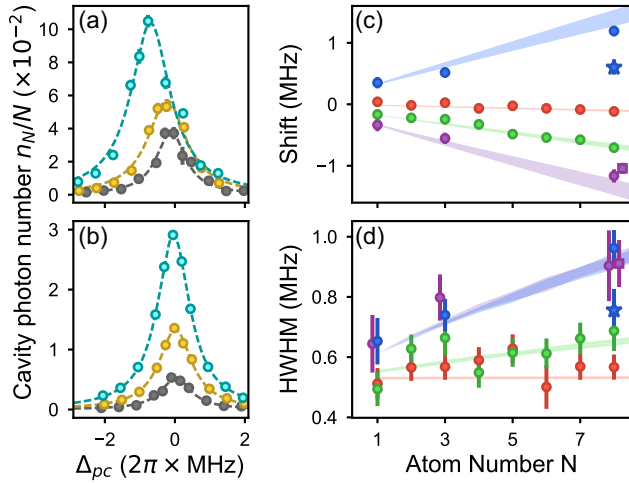


FIG. 4. Cavity spectra measured with arrays of $N = 1$ (black), 3 (yellow), and 8 (cyan) at (a) $\Delta_{ca} = -2\pi \times 38$ MHz and (b) $-2\pi \times 507$ MHz. All atoms are aligned with cavity antinodes of the same phase. Lorentzian fits (dashed lines) are used to determine the peak positions and linewidths. The cavity resonance (c) shifts and (d) half widths at half maximum (HWHM) as a function of atom number, measured with all atoms aligned with cavity antinodes of the same phase (circles), at detunings of $\Delta_{ca} = -2\pi \times 507$ MHz (red), $-2\pi \times 38$ MHz (green), $-2\pi \times 19$ MHz (purple), and $2\pi \times 19$ MHz (blue). The corresponding shaded areas are predictions from a Monte Carlo calculation [34]. Results are also shown for an array with half-integer-wavelength spacing at the cavity antinodes (squares), and an array with integer-wavelength spacing at the cavity nodes (stars). For clarity, some symbols in (c) and (d) are slightly offset in N .

emission, as well as large shot-to-shot fluctuations [50–52] in the light polarization due to self-reinforcing feedback.

Our realization of a well-controlled many-body cavity QED system, compatible with single-atom control, offers a wide range of potential quantum applications. When single-photon cavity emission is subradiantly suppressed, multiphoton emission becomes the dominant process, leading to a nonlinear photon source [13,53]. Encoding quantum information in a symmetrically excited W state or related states [54] can superradiantly speed up information exchange between matter and itinerant photons. This accelerates quantum communications and reduces the infidelity accumulated over the interaction time [14,55]. The possibility of switching between super- and subradiant states, through either phase rotation on individual atoms or moving the tweezers, allows on-demand multiphoton storage and emission [56].

Finally, coherent long-range photon-mediated momentum [30,57,58] or spin [59–63] exchanges between individual atoms can be facilitated through subsequent absorption of the cavity photons by the atoms and can be used for quantum simulation on the Dicke model [64], entanglement generation [65], or realization of quantum solvers [66]. Notably, atom arrays that combine long-range

photon-mediated interactions with short-range Rydberg interactions could benefit from the advantages of both types of interactions with distinct ranges, connectivity, and timescales [67–69].

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Appendix E

Optomechanical self-organization in a mesoscopic atom array

This appendix includes the following paper [55], discussed in Ch. 4:

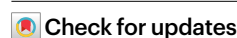
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Optomechanical self-organization in a mesoscopic atom array

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Increasing the number of particles in a system often leads to qualitative changes in its properties, such as breaking of symmetries and the appearance of phase transitions. This renders a macroscopic system fundamentally different from its individual microscopic constituents. Lying between these extremes, mesoscopic systems exhibit microscopic fluctuations that influence behaviour on longer length scales, leading to critical phenomena and dynamics. Therefore, tracing the properties of well-controlled mesoscopic systems can help bridge the gap between an exact description of few-body microscopic systems and the emergent description of many-body systems. Here we explore the mesoscopic signatures of an optomechanical self-organization phase transition using arrays of cold atoms inside an optical cavity. By precisely engineering atom–cavity interactions, we reveal how critical behaviour depends on the atom number, identify characteristic dynamical behaviours in the self-organized regime and observe a finite optomechanical susceptibility at the critical point. These findings advance our understanding of particle-number- and time-resolved properties of phase transitions in mesoscopic systems.

The scale of a physical system has a key role in determining its material properties. With increasing size comes increasing complexity, and complexity leads to the emergence of macroscopic phenomena that, often, cannot be extrapolated from microscopic behaviours¹. A well-known example of this paradigm is the appearance of phase transitions in the thermodynamic limit, that is, as a system's particle number tends to infinity. In between the microscopic and macroscopic extremes lies the mesoscopic regime. Mesoscopic systems not only exhibit the characteristics of macroscopic systems but also host a variety of rich properties themselves. In systems of intermediate size, the assumptions of local equilibrium break down and internal microstructures may affect properties on longer length scales². The fact that microscopic fluctuations have a measurable influence on mesoscopic scales also makes mesoscopic systems an interesting playground for the study of critical phenomena and phase transitions^{3–5}.

Technical advances in engineering, probing and measuring mesoscopic systems have propelled experiments in a variety of physical platforms. Notable examples include solid-state systems^{6,7}, nanostructured⁸ and nanomechanical systems^{9–12}, nanomagnets^{13,14}, and cold atoms and ions^{15–21}. Breakthroughs in the control of atoms trapped in optical tweezer arrays^{22,23} represent a leap forward in precisely constructing mesoscopic, ultracold atomic systems. We show that deterministic control over particle number and interaction strength can be achieved with an optical tweezer array inside a standing-wave optical cavity. This system uniquely lends itself to the detailed study of optomechanics in mesoscopic samples.

We focus on the mechanical self-organization generated by light-mediated forces among optically excited atoms within an optical cavity^{24,25}. When such atoms are driven by coherent light above a certain intensity, they exhibit a phase transition in which they

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spontaneously scatter light into the cavity. That light creates a spatially dependent potential on the atoms that causes them to self-organize and super-radiantly emit into the cavity with a phase that breaks $\mathbb{Z}_2$ symmetry. This self-organization phase transition maps onto the Dicke model^{26,27}. Previous experiments on the optomechanical Dicke phase transition²⁷ and other phenomena arising from cavity-mediated forces among atoms^{28–32} were conducted on bulk macroscopic samples, with particle numbers $N \gtrsim 10^5$. Here we realize the mesoscopic limit of the Dicke phase transition in deterministically prepared tweezer arrays containing between 10 and 22 atoms. We observe a critical point that depends on the atom number, a finite switching timescale between self-organized states and a collectively altered susceptibility to external forces. This setup allows us to study the crossover from the few- to many-body regime of phase transitions, advancing our knowledge of finite-size effects and dynamical behaviour in mesoscopic systems.

Cavity-mediated interactions and self-organization

To realize mesoscopic self-organization, we use the experimental setup schematically shown in Fig. 1a. It contains a one-dimensional tweezer array into which we load a deterministic number N of ^{87}Rb atoms. The array is aligned along the axis of a high-finesse Fabry–Pérot optical cavity (z axis). The cavity is placed in a vacuum and has a TEM₀₀ mode whose frequency ω_c lies near the resonance frequency ω_a of the $F=2 \rightarrow F'=3D_2$ transition of ^{87}Rb (wavelength, $\lambda = 780$ nm)^{33,34}. The atoms, prepared in the $F=2$ ground-state manifold, are exposed to a circularly polarized standing-wave pump beam of frequency ω_p (detuned from the atoms by $\Delta_{pa} = \omega_p - \omega_a$ and from the cavity by $\Delta_{pc} = \omega_p - \omega_c$) propagating along the x axis. The circularly polarized pump creates a fictitious magnetic field along x through vector light shifts and defines the atomic quantization axis. The pump drives the σ^- electric dipole transition of the atoms and its power is linearly ramped up to the desired value in 50 μs . The initial ramp optically pumps the atoms into the $|F=2, m_f=-2\rangle$ stretched state as the atoms are excited on the cycling transition, thereby ensuring that no dynamics among the $|F=2, m_f\rangle$ states interfere with the location of the critical point for self-organization. We achieve a maximum vacuum Rabi coupling of $g_0 = 2\pi \times 3.1/\sqrt{2}$ MHz between the y -polarized cavity field and the atomic σ^- transition. The cavity half-linewidth is $\kappa = 2\pi \times 0.53$ MHz, resulting in a sub-microsecond cavity response time defined by $1/\kappa$. Given that the atomic half-linewidth is $\gamma = 2\pi \times 3.0$ MHz, we achieve a single-atom cooperativity of $C = g_0^2/(2\kappa\gamma) = 1.5$ (ref. 35). Because the atom–cavity interaction strongly depends on the spatial structure of the pump and cavity standing waves^{36–39}, all the tweezers are aligned to be within the same pump antinode wavefront (so all the atoms experience the same pump Rabi frequency Ω) and are positioned at cavity nodes spaced 4.5λ apart. By spacing the tweezers by a half-integer number of wavelengths, we eliminate biasing in our system due to centre-of-mass drift of the tweezer array with respect to the cavity standing wave.

Light inside the cavity creates a potential energy landscape for the atoms through the exchange of cavity photons²⁵. For atom n at position $\hat{z}_n$, the cavity interaction potential, produced by interference between the pump and cavity fields, is proportional to $S \sin(k\hat{z}_n)(\hat{c} + \hat{c}^\dagger)$, where $\hat{c}(\hat{c}^\dagger)$ is the bosonic annihilation (creation) operator for a cavity photon, $S \simeq \Omega g_0/(2\Delta_{pa})$ is the coherent scattering rate of a pump photon into the cavity and $k = 2\pi/\lambda$. The strength of the potential, thus, depends on the atom's position inside the cavity and on the magnitude of the cavity field. When the pump is sufficiently detuned from the atomic transition and red-detuned from cavity resonance, the cavity field follows the atomic motion such that $\hat{c} + \hat{c}^\dagger \propto -S \sum_{n=1}^N \sin(k\hat{z}_n)$. In this adiabatic regime, the effective potential is minimized when $\langle \sin(k\hat{z}_n) \rangle = \pm 1$ for all n . That is, the most favourable configurations of the system are the ones in which the atoms self-organize by collectively moving to cavity field antinodes with the same sign.

The steady state of the optomechanical system is governed by competition between the cavity-mediated interaction and the tweezer potential, which, for tweezers centred at the cavity nodes, is minimized when $\langle \sin(k\hat{z}_n) \rangle = 0$ for all n . This competition gives rise to the emergence of self-organization as a function of S , which we tune through Ω , thereby defining a critical pump strength Ω_c . For $\Omega < \Omega_c$, the tweezers dominate over the cavity field and the atoms remain centred at the cavity nodes (Fig. 1a(i)). Although the atom array is uncoupled from the cavity when positioned exactly at the nodes, zero-point and thermal fluctuations shift the atom positions, causing the array to emit light incoherently into the cavity. For $\Omega > \Omega_c$, the negative curvature of the cavity potential overcomes the positive tweezer trap curvature and it becomes energetically favourable for all atoms to move towards antinodes with the same sign and for their emission to interfere constructively. The super-radiant field's phase is determined by the sign of the antinodes chosen by the atoms (Fig. 1a(ii)–(iii)). For atoms spaced by a half-integer number of wavelengths, adjacent atoms move in opposite directions to achieve self-organization (antiphase motion). We refer to the collective mode that is linearly coupled to the cavity as the dominant mode.

To confirm the self-organization experimentally, we detect the cavity field $\hat{c}$ and observe $\mathbb{Z}_2$ symmetry breaking. Assuming $\Delta_{pa} < 0$ (the case for all data shown), light scattered by the atoms affects one quadrature of the cavity field, $c_{\text{proj}} = \text{Re}(\langle \hat{c} \rangle) \cos \theta + \text{Im}(\langle \hat{c} \rangle) \sin \theta$, where $\theta = \arctan(-\kappa/(\Delta_{pc}))$, defined with respect to the phase of the pump field. One finds

$$c_{\text{proj}} = \frac{N\Omega g_0}{2|\Delta_{pa}|} \left\langle \frac{\hat{\Theta}}{\sqrt{\Delta_{pc}^2 + \kappa^2}} \right\rangle. \quad (1)$$

We define $\tilde{\Delta}_{pc} = \Delta_{pc} - \sum_{n=1}^N g_0^2 \sin^2(k\hat{z}_n)/\Delta_{pa}$ as the effective cavity detuning including the dispersive shift from the atoms. The cavity field is proportional to the operator

$$\hat{\Theta} \equiv \frac{1}{N} \sum_{n=1}^N \sin(k\hat{z}_n), \quad (2)$$

whose mean value can be interpreted as the order parameter. For atoms trapped at the nodes, $\langle \hat{\Theta} \rangle = 0$, whereas for atoms at the antinodes, $\langle \hat{\Theta} \rangle = \pm 1$. Thus, $\langle \hat{\Theta} \rangle$ takes the role of an effective magnetization for the system^{24,40}.

To detect the complex cavity field $\hat{c}$, we send y -polarized emission from one cavity mirror to a heterodyne receiver (Extended Data Fig. 1) and record the signal for a total time of 250 μs , including the 50- μs pump ramp-up time. We filter the time trace with an adjustable averaging time, ranging from 5 μs to 50 μs ; we average for a minimum of 5 μs because increased noise at shorter averaging times compromises the signal-to-noise ratio of the extracted cavity field. The values obtained at each step of the filter are treated as individual data points for $\hat{c}$. We determine the angle θ by referencing the phase of the distribution of $\hat{c}$ to that obtained with all atoms placed at antinodes of the same sign. We then project each value of $\hat{c}$ onto the axis defined by θ to obtain c_{proj} (Methods and Extended Data Fig. 2). Our measured c_{proj} is scaled such that c_{proj}^2 is the average photon number in the cavity.

To observe bifurcation in the distributions of c_{proj} , we repeat the measurement many times at different pump strengths and analyse the heterodyne data from $t = 0$ to $t = 100$ μs (where $t = 0$ is the end of the pump ramp-up) using a 5- μs averaging time. Owing to heating of the atoms from continuous exposure to pump light^{41,42}, we constrain this analysis to the 100- μs span described above. The distributions of c_{proj} show a striking bifurcation as the pump strength is increased. An example is shown in Fig. 1b. We see that once Ω surpasses a threshold near $2\pi \times 30$ MHz, the single peak at $c_{\text{proj}} = 0$ splits into two symmetric branches with a finite field amplitude.

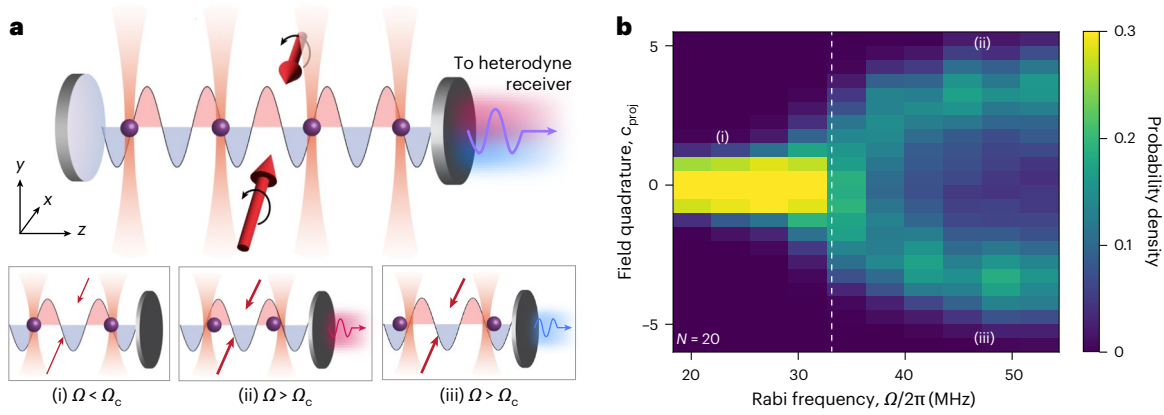


Fig. 1 | Cavity-induced self-organization. **a**, Schematic of the experimental setup. Optically tweezed atoms are placed at the nodes of the cavity field at half-integer wavelength spacing (1.5λ shown in figure for convenience). The atoms are driven by a pair of circularly polarized counterpropagating pump beams. The transmitted cavity field is detected with a heterodyne receiver. Insets (i)–(iii) illustrate self-organization. At pump strengths below the critical point, the atoms remain centred at the cavity nodes and, on average, emit zero field (i). Above the critical point, atoms self-organize by moving toward either the

positive (red) (ii) or negative (blue) (iii) antinodes. **b**, Probability distributions of cavity field projections from repeated measurements. The results shown are based on data collected with $N = 20$, $\Delta_{\text{pc}} = -2\pi \times 1.9$ MHz and $\Delta_{\text{pa}} = -2\pi \times 80$ MHz. The field distributions are plotted for different values of Ω . The data are averaged over 5- μs intervals and are sampled from the first 100 μs after the pump ramp-up has finished. The labels (i), (ii) and (iii) correspond to the drawings in **a**. The white dashed line indicates the critical point fitted from the data.

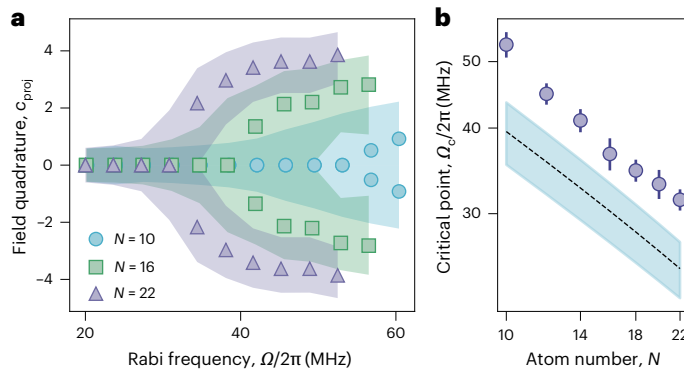


Fig. 2 | Dependence of the critical point on atom number. **a**, Bifurcation data for $N = 10$ (circles), $N = 16$ (squares) and $N = 22$ (triangles). The shaded areas show the width of the distribution at half-maximum, whereas the markers show the maxima of the fitted Boltzmann distribution. **b**, Experimentally measured Ω_c (circles) for different N values plotted on a logarithmic scale and the theoretical prediction of equation (4), based on an independent temperature measurement of $T = 35 \pm 10$ μK (black dashed line and light-blue-shaded area). The error bars represent the standard deviations resulting from statistical errors on fits of parameter B and drift of pump power over the course of the experiment, estimated to be 10%. The reported values of Ω_c account for systematic errors due to Gaussian detector noise and the Gaussian intensity profile of the pump beam. We note that there is an additional systematic error of +6.0% and -8.5% on our calibration of Ω (Methods). All data in the figure are taken at $\Delta_{\text{pa}} = -2\pi \times 80$ MHz and $\Delta_{\text{pc}} = -2\pi \times 1.9$ MHz. The value of Δ_{pa} was chosen to avoid strong atom number dependence of the atom-induced dispersive shift on the cavity resonance frequency. We also chose the resonance frequency of the bare cavity to be sufficiently larger than the pump frequency to avoid instabilities arising from the dispersive shift bringing the pump blue-detuned of the cavity.

The simplest model that captures this physics is purely mechanical and lossless, with an effective Hamiltonian

$$\hat{H}_{\text{eff}} \equiv \sum_{n=1}^N \left[\frac{\hat{p}_n^2}{2M} + \frac{M}{2} v^2 (\hat{z}_n - z_{0n})^2 \right] + \hbar \mathcal{D} N^2 \hat{\Theta}^2. \quad (3)$$

Here M is the atomic mass of ^{87}Rb , $v = 2\pi \times 93$ kHz is the tweezer trap frequency (in the harmonic approximation), $\hbar$ is the reduced Planck

constant and $\mathcal{D} \equiv \left(\frac{g_0 \Omega}{2 \Delta_{\text{pa}}} \right)^2 \tilde{\Delta}_{\text{pc}} / (\tilde{\Delta}_{\text{pc}}^2 + \kappa^2)$ is the strength of the cavity-induced mechanical potential, whose sign depends on $\tilde{\Delta}_{\text{pc}}$. The tweezer positions are $z_{0n} = 4.5 \lambda \times n$.

However, thermal fluctuations as well as motion of the atoms perpendicular to the cavity axis are relevant for a quantitative description of the experiment. By means of a more refined optomechanical model (Supplementary Information, section II), the critical pump strength is found to be

$$\Omega_c(T) = \sqrt{\frac{2 M v^2 \Delta_{\text{pa}}^2 (\Delta_{\text{pc}}^2(T) + \kappa^2)}{N g_0^2 k^2 |\Delta_{\text{pc}}(T)| \hbar \varepsilon(T)}}, \quad (4)$$

where T is the atomic motional temperature and the temperature-dependent factor $\varepsilon(T)$ is defined in Supplementary Equation (29). The dependencies of Ω_c on N and Δ_{pa} are studied in this work (Fig. 2 and Extended Data Fig. 3). Equation (4) includes two different thermal effects on the critical pump strength. First, taking an average over the thermal motional shift of the atoms yields a net shift of the cavity resonance $\Delta_{\text{pc}}(T) \equiv \langle \tilde{\Delta}_{\text{pc}} \rangle$. Second, thermal fluctuations allow the atoms to probe the nonlinearity of the effective mechanical potential in $\hat{\Theta}^2$. This leads to an increase in the critical pump strength proportional to the factor $1/\varepsilon(T)$ in equation (4). Using a temperature of 35 ± 10 μK , determined from an independent measurement³⁶, equation (4) predicts $\Omega_c = 2\pi \times (26.5 \pm 2.5)$ MHz for the parameter settings in Fig. 1b.

To determine Ω_c quantitatively from the data, we draw inspiration from Landau's theory of phase transitions⁴³. If we only consider the contribution of the dominant mode to the Hamiltonian in equation (3), the potential energy becomes $\frac{1}{2} N M v^2 z_{\text{dom}}^2 + \hbar \mathcal{D} N^2 \sin^2(k z_{\text{dom}})$, where z_{dom} is the displacement of the dominant mode with respect to the cavity nodes. If we consider the distribution of positions to be a Boltzmann distribution determined by the local potential energy, and expand the potential energy to fourth order in z_{dom} , we obtain a probability distribution $p(z_{\text{dom}})$ of the form

$$p(z_{\text{dom}}) \propto \exp(-B z_{\text{dom}}^2 - D z_{\text{dom}}^4), \quad (5)$$

where B and D are parameter-dependent constants and $z_{\text{dom}} \equiv \langle \hat{z}_{\text{dom}} \rangle$. According to Landau's theory, a second-order phase transition occurs at $B = 0$. Because c_{proj} is proportional to z_{dom} at leading order, we fit the

observed distributions of c_{proj} to equation (5) and extract the coefficient B to determine where Ω_c occurs. Analysing the distributions of c_{proj} , we observe that the fits to the Boltzmann distribution match the data well (Extended Data Fig. 4 and Methods) even though the system is driven and dissipative. From the data in Fig. 1b, we extract an experimental value of $\Omega_c = 2\pi \times (33.1^{+2.0}_{-2.8} \text{stat} \pm 1.6 \text{stat}) \text{ MHz}$. This measurement is slightly above the prediction of equation (4) for our independent measurement of the initial temperature of the trapped atoms, though within error. The slight discrepancy may be caused by heating of the trapped atoms during the experiment, owing to light scattering. In future work, it may be possible to monitor the atoms' temperature during the experiment by observing sideband asymmetry in the heterodyne power spectrum.

Mesoscopic signatures of a phase transition

We now examine how critical behaviour varies with the atom number. We first examine the dependence of Ω_c on N , and find that Ω_c diminishes as N increases. Quantitatively, for an averaging time of 5 μs , we almost recover the $\sim 1/\sqrt{N}$ dependence predicted by equation (4), as shown in Fig. 2b. Although the critical point's scaling with N has been measured before in macroscopic bosonic and fermionic realizations of the super-radiant phase transition^{41,44}, our work shows that the N -dependence is also observable in a regime far from the thermodynamic limit. In Fig. 2b, the data appear above the prediction of equation (4) if we use the temperature from our independent measurement, implying that the atoms have been heated to a temperature close to 66 μK at the time of observation.

Atomic self-organization in a cavity can be compared with the crossover between a magnetized and unmagnetized state in a spin system⁴⁰. The behaviour of $\langle \hat{\Theta} \rangle$ above the transition is analogous to that of a superparamagnet: as discussed in the seminal work⁴⁵, small ferromagnetic particles are subject to thermal fluctuations that cause their magnetization to relax on a timescale that grows with the particle volume. Averaging the magnetization over times much longer than the relaxation time results in zero net magnetization, whereas averaging over times much shorter than the relaxation time allows for the observation of quasistatic magnetic ordering⁴⁶. We similarly find that the mesoscopic self-organization of atoms has a characteristic lifetime that depends on the system size.

To reveal the dynamics of magnetization order parameter $\langle \hat{\Theta} \rangle$, we take advantage of the cavity's microsecond-scale readout capability³³. Figure 3a–c shows single-shot time traces representative of the observed dynamical behaviours in the self-organized regime: symmetry breaking, switching and rapid oscillations. For a macroscopic atom number, only symmetry breaking should occur, whereas for a mesoscopic atom number, fluctuations away from equilibrium can lead to switching between the stable states of the system⁴⁰. The rapid oscillation events are rare, occurring in only 2 out of the 180 shots obtained at the shown parameter setting; we expect they result from the energy in the dominant mode being above the energy gap separating the two symmetry-broken states.

To highlight the difference in the dynamical behaviour below and above the critical point, we calculate the first-order coherence function of the field quadrature $g^{(1)}(t_1, t_2)$, defined as $g^{(1)}(t_1, t_2) = \langle c_{\text{proj}}^*(t_1)c_{\text{proj}}(t_2) \rangle / \sqrt{\langle |c_{\text{proj}}(t_1)|^2 \rangle \langle |c_{\text{proj}}(t_2)|^2 \rangle}$, where the angle brackets denote averaging over multiple shots. In particular, $g^{(1)}(0, t)$ indicates how correlated the field at time $t_2 = t$ is to the field at time $t_1 = 0$, where $t_1 = 0$ is chosen to be at the end of the pump ramp-up. Symmetry breaking is indicated by $g^{(1)}(0, t)$ remaining high for extended t . Figure 3d shows that when $\Omega/\Omega_c = 0.8$, $g^{(1)}(0, t)$ decays rapidly, indicating the absence of symmetry breaking, and exhibits oscillations at approximately 60 kHz, corresponding to the softened trap frequency. At $\Omega/\Omega_c = 1.6$, $g^{(1)}(0, t)$ decays on a much longer timescale and oscillations are absent, indicating a prolonged lifetime of the

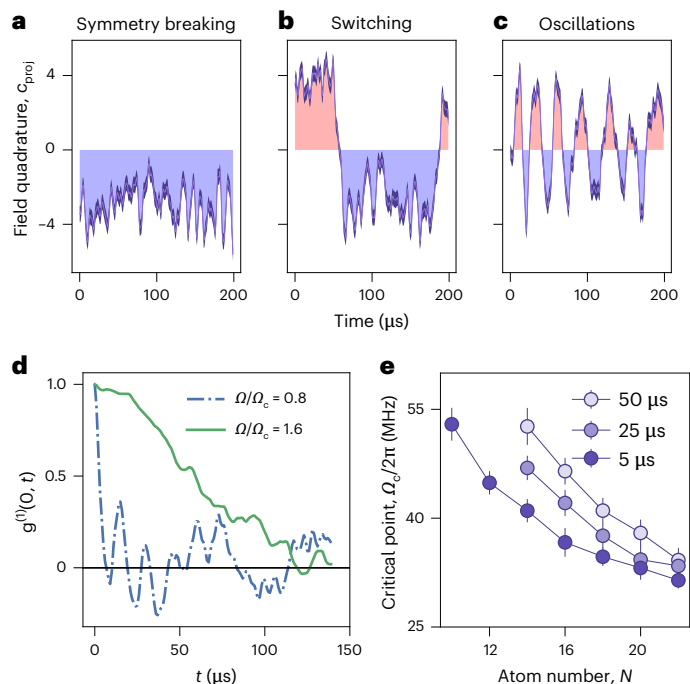


Fig. 3 | Lifetime of the self-organized state. a–c, Above the critical point, we observe three dynamical behaviours in the cavity field time traces: symmetry breaking (a), switching (b) and rapid oscillations (c). Time is measured from the end of the pump ramp-up. The shaded areas show the time spent in the $c_{\text{proj}} > 0$ state (red) and the $c_{\text{proj}} < 0$ state (blue). The purple-shaded regions indicate the shot noise level. Data are shown for $N = 20$, $\Omega = 2\pi \times 45.4 \text{ MHz}$. d, $g^{(1)}(0, t)$ for $N = 20$ at $\Omega < \Omega_c$ and $\Omega > \Omega_c$. When the system is self-organized, $g^{(1)}(0, t)$ decays on a much longer timescale. e, Dependence of the critical point on N for different averaging times. At larger N , the curves converge, indicating that self-organization persists on longer timescales. At lower N , the separation between the curves grows due to the transience of the mesoscopic system. The measurements at 25 μs and 50 μs for $N < 14$ are absent because the transition was not reached in the range of Ω values scanned. The plotted Ω_c data are subject to the same systematic errors as those described in Fig. 2b. Lines are guides to the eye. All data in the figure are taken at $\Delta_{\text{ps}} = -2\pi \times 80 \text{ MHz}$ and $\Delta_{\text{pc}} = -2\pi \times 1.9 \text{ MHz}$.

symmetry-broken states. Although we are able to observe these qualitative differences in the decay of $g^{(1)}(0, t)$ for $\Omega < \Omega_c$ and $\Omega > \Omega_c$, we note that using $g^{(1)}(0, t)$ to determine the location of Ω_c does not appear to be a robust method.

We next observe the N -dependence of the lifetime of the self-organized state. As discussed above, the characteristic behaviour of the system depends on the time over which $\langle \hat{\Theta} \rangle$ is averaged. On short timescales, the system displays signatures of symmetry breaking, whereas on long timescales, the system switches between the two stable states. As a result, in Fig. 3e, we see that the nominal Ω_c increases as we increase the averaging time from 5 μs to 50 μs . Remarkably, we see that Ω_c increases significantly at lower N but less so at higher N . This indicates that as the system size decreases, the characteristic timescale over which the system remains self-organized also decreases. We, thus, observe a key feature of this mesoscopic system: averaging $\langle \hat{\Theta} \rangle$ over times longer than the N -dependent lifetime erases the signatures of symmetry breaking.

Finite susceptibility at criticality

The positional control afforded by the tweezers not only allows us to observe spontaneous symmetry breaking but also to explicitly break symmetry with nanometre-scale precision and probe the system's response. By displacing adjacent tweezers in opposite directions by an amount δz , we introduce a constant force proportional to δz on the

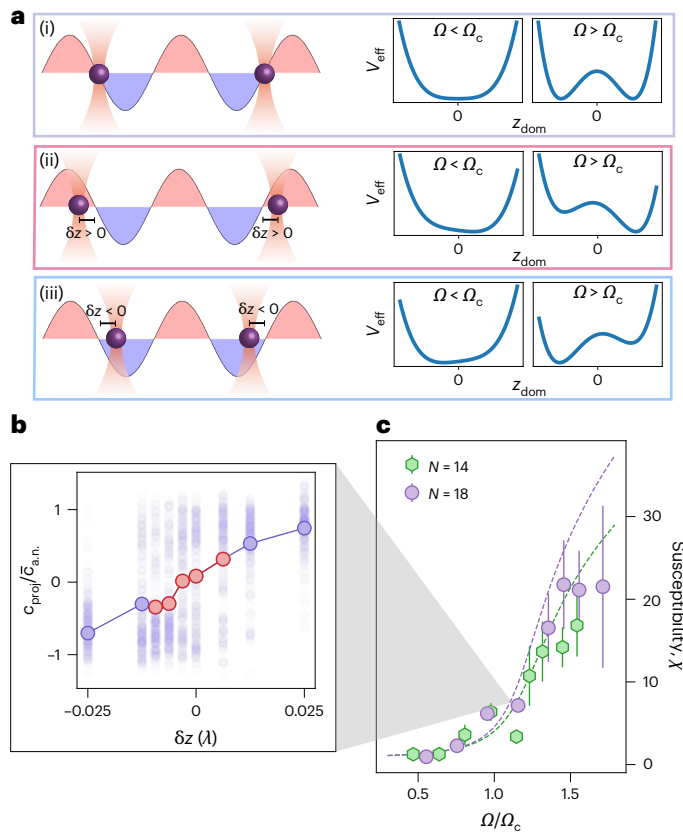


Fig. 4 | Optomechanical susceptibility. **a**, Schematic of the susceptibility measurement. The dominant mode of the array (z_{dom}) is displaced by δz . When $\delta z = 0$, the effective potential V_{eff} on the atoms is symmetric about $z_{\text{dom}} = 0$ (i). For $|\delta z| > 0$, V_{eff} tilts in the direction of the bias (ii) and (iii)). **b**, Distributions of $c_{\text{proj}}/\bar{c}_{\text{a.n.}}$ (small purple points) and their mean values (circles) at various δz values for $N = 18$ and $\Omega/\Omega_c = 1.16$. Measurement of $\partial(c_{\text{proj}}/\bar{c}_{\text{a.n.}})/\partial(\delta z)|_{\delta z=0}$ is taken by fitting the slope of $\bar{c}_{\text{proj}}/\bar{c}_{\text{a.n.}}$ for $-0.01\lambda \leq \delta z \leq 0.01\lambda$ (red circles). **c**, Susceptibility χ at various Ω/Ω_c values for $N = 14$ and $N = 18$. The data (hexagons and circles) are plotted together with the theoretical prediction based on the Boltzmann distribution for $T = 35 \mu\text{K}$ (dashed lines). All data in the figure are taken at $\Delta_{\text{pa}} = -2\pi \times 80 \text{ MHz}$ and $\Delta_{\text{pc}} = -2\pi \times 1.9 \text{ MHz}$.

atoms that biases the dominant mode (Fig. 4a). Generally, the amount a system moves in response to an applied force is characterized by its mechanical susceptibility. In our case, c_{proj} serves as a proxy for the atom positions. As such, we obtain the optomechanical susceptibility χ , which we define as

$$\chi \equiv \frac{1}{k\bar{c}_{\text{a.n.}}} \left. \frac{\partial \bar{c}_{\text{proj}}}{\partial(\delta z)} \right|_{\delta z=0}. \quad (6)$$

Here $\bar{c}_{\text{a.n.}}$ is the measurement of $\bar{c}_{\text{proj}}$ when the tweezers are positioned at the positive antinodes. The overbar indicates an average over the distribution of c_{proj} obtained from the first 50 μs of multiple shots. The quantity $\partial(\bar{c}_{\text{proj}}/\bar{c}_{\text{a.n.}})/\partial(\delta z)|_{\delta z=0}$ is measured by fitting the slope of $\bar{c}_{\text{proj}}/\bar{c}_{\text{a.n.}}$ in the range $-0.01\lambda \leq \delta z \leq 0.01\lambda$ for $N = 18$ (Fig. 4b, red circles) and $N = 14$. We note that the atoms are initialized with root-mean-square thermal fluctuations in their positions of about 100 nm, which is comparable to k^{-1} , leading to a thermal Lamb–Dicke parameter of about 0.8 (Supplementary Information, equation (18)).

From the measurement of χ for $N = 14$ and $N = 18$ at different values of Ω/Ω_c (Fig. 4c), we make two observations: (1) although χ increases with Ω/Ω_c , it does not diverge at the critical point as one would expect for a second-order phase transition in a macroscopic system; (2) the value of χ grows with the atom number for $\Omega/\Omega_c > 1$.

These observations can be explained by a model assuming $\chi \approx \partial\langle z_{\text{dom}} \rangle / \partial(\delta z)|_{\delta z=0} = NMv^2\langle \Delta z_{\text{dom}}^2 \rangle / (k_B T)|_{\delta z=0}$, where angle brackets indicate a thermal average including the tweezer bias δz (Supplementary Section IV). Thus, the susceptibility is directly related to the size of the fluctuations in the system through variance $\langle \Delta z_{\text{dom}}^2 \rangle$. In a macroscopic system, one would expect χ to diverge according to $1/(1 - (\Omega/\Omega_c)^2)$ as Ω approaches Ω_c . Instead, because of our finite atom number, our model predicts that χ should remain finite at the critical point.

Specifically, at the critical point, our model predicts that χ should be approximately proportional to $\sqrt{N}$ as a result of the variance $\langle \Delta z_{\text{dom}}^2 \rangle$ scaling as $1/\sqrt{N}$ at leading order. This has a simple interpretation: at smaller N , the fluctuations in z_{dom} are larger and the atoms sample the quartic part of the effective optomechanical potential V_{eff} , which provides the confinement that prevents the susceptibility from diverging. Although we observe finite χ at the critical point, we are unable to resolve the $\sqrt{N}$ dependence owing to the quality of our data. Nonetheless, we emphasize that this observation of finite χ is a direct measure of mesoscopic fluctuations and not just a result of non-zero temperature—a macroscopic system at non-zero temperature would still exhibit a diverging susceptibility. As Ω/Ω_c is increased beyond 1, our model predicts that χ should continue to grow and eventually scale proportionally to N , before saturating at a value of $NMv^2\lambda^2/(16k_B T)$. For $\Omega \gg \Omega_c$, the δz value required to confine $\langle z_{\text{dom}} \rangle$ effectively to one of the minima of V_{eff} scales inversely with N , leading to $\chi \propto N$. Our model assumes that the atom temperature is the same for all N ; although recoil heating may contribute to an N -dependent temperature and, in turn, affect the scaling of χ with N , the $1/\sqrt{N}$ scaling of Ω_c (Fig. 2b) belies any strong N -dependence of the temperature. We note that χ continuing to increase for $\Omega > \Omega_c$ is a consequence of us applying the tweezer shift δz to break symmetry before the cavity interaction is turned on, analogous to applying a magnetic field to a spin system before lowering its temperature below the Curie point. By contrast, if the tweezer shift were applied after the cavity interaction, one might expect χ to peak at $\Omega = \Omega_c$. Accordingly, we see the separation between the $N = 14$ and $N = 18$ data become more pronounced as we increase Ω beyond Ω_c , consistent with the prediction of χ approaching linear scaling with N . These results highlight how the response of mesoscopic systems deviates from that of macroscopic systems. Furthermore, the use of tweezers to displace the atoms provides a novel method for measuring optomechanical susceptibility, complementing experiments in which the external perturbation is introduced by injecting a weak field into the cavity^{47,48}.

Discussion

We have studied self-organization, bifurcation and symmetry breaking, as well as mesoscopic dynamics and response due to cavity-mediated interactions in an atom array, opening new directions for these systems. The realization of mesoscopic self-organization with control over individual atoms has notable implications not just for our fundamental understanding of mesoscopic systems but also for quantum simulation, optimization and metrology. On one hand, the dynamics of the cavity field provide a plethora of information that may further our understanding of finite-size scaling⁴⁹ and non-equilibrium physics in mesoscopic systems⁵⁰, as well as pave the way for time-resolved quantum simulation^{51,52}. On the other hand, mesoscopic self-organization may also open the door to programmable engineering⁵³ of exotic symmetry-breaking phases of matter⁵⁴, for instance, by illuminating groups of atoms with different phases of pump light. This system can also be used to implement a quantum numerical solver for a variety of classically hard computational problems^{55–57}, where the tweezer array allows for highly programmable initial conditions. Additionally, the optomechanical susceptibility near the critical point can be a resource for the enhanced sensing of applied forces^{58–60}.

Online content

Any methods, additional references, Nature Portfolio reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at <https://doi.org/10.1038/s41567-025-02916-7>.

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Methods

Atom array preparation

Our tweezer array is created by light at a wavelength of 808 nm that passes through a one-dimensional acousto-optic deflector, resembling a method described elsewhere⁶¹. The tweezer positions are determined by radio-frequency tones, created through an arbitrary waveform generator card, that drive the acousto-optic deflector. The atoms are initially probabilistically loaded into a tweezer array containing 40 sites. We then perform parity projection through polarization-gradient cooling in a $\text{lin}\perp\text{lin}$ configuration. The occupation of the sites is determined by fluorescence imaging. Following tweezer loading, we implement two rounds of tweezer rearrangement to bring the loaded atoms into a centred array. Unwanted atoms are dropped by diminishing the optical power in their respective tweezers to achieve the desired atom number between $N = 10$ and $N = 22$. We confirm the success of this sorting procedure with an additional fluorescence image. In the event that defects occur during the rearrangement, as noted in images taken before the pumping sequence (see the following section), we post-select our data on there being an equal number of even- and odd-indexed sites filled. We take an additional fluorescence image after the pumping sequence and retain only data in which the occupancy of the pre- and post-pumping-sequence atom array is identical. This is to ensure that there is no bias in the tweezer positions towards the positive or negative antinodes when we take the bifurcation data, for example, Figs. 1 and 2. It is possible that this post-selection of data could leave residual bias in our analysis by selecting the most favourable shots, for instance, when measuring the lifetime of the self-organized phase. However, we expect that this effect is small. To quantify, for the data shown in Figs. 1–3, the proportion of experimental runs that are kept after post-selection is 79%. We saw a single-atom loss in 12% of the total number of runs (equivalently, 57% of the discarded runs). Including these one-atom loss runs in our analysis leaves no visible effect on the results shown in the current figures.

The range of atom numbers ($10 \leq N \leq 22$) was chosen for a couple of reasons. Empirically, considering the same timescales for driving and probing the system as chosen in this work, we found that for $N < 10$, the atom array was subject to substantial atom loss already at pump strengths at which one might expect to see self-organization; thus, $N < 10$ data are not included in our analysis. The maximum atom number of $N = 22$ is mostly limited by the field of view of our objective and the available laser power for the optical tweezers.

Pumping sequence and cavity field phase calibration

We use heterodyne detection to measure the cavity field polarized in the y direction. Our heterodyne system is schematically shown in Extended Data Fig. 1. By demodulating the signal from the balanced photodiode at the pump–local oscillator beat frequency of 20 MHz, we extract both quadratures of the cavity field. Owing to the pump and local oscillator emerging from separate fibres, their relative phase (ϕ) drifts over time, thereby necessitating a calibration of the phase by referencing it to the field emitted by the atoms. To perform the phase calibration in each shot, we first demodulate the field by applying a fast Fourier transform over a 5- μs period with a Hann window applied. The window is then stepped in 1- μs increments. The average phase, up to a π rotation, is obtained by performing principal component analysis (PCA) on all the data acquired in the shot. To calibrate the cavity field phase in each shot, we use a multiframe pumping sequence (Extended Data Fig. 2) described as follows. In frame 1, we initialize all the atoms on the cavity field antinodes with 5λ spacing and pump them at a power below the critical point; we define the cavity field phase obtained here as a reference for the phase at the positive antinodes. Then, we move the tweezers into a 4.5λ -spacing configuration and simultaneously shift them to the cavity nodes. Next, in frame 2, we pump the atoms with variable power to detect self-organization and symmetry breaking. We take multiple frames in this configuration

(typically between 5 and 20 shots) to increase our data-taking rate. Last, we shift the atoms back to the antinodes with 5λ spacing and obtain another phase reference in the final frame of the sequence by the same method as in frame 1. We observe the phase drift between the first and last reference frames to be less than π . The sign of c_{proj} obtained in the middle frames is determined by comparing the phase extracted through PCA to the phase linearly interpolated between the two reference phases; we assume that the principal component analysis phase is less than $\pi/2$ shifted from the interpolated phase. Example time traces of the cavity field measurement in each frame are shown in Extended Data Fig. 2. In each frame, we linearly ramp up the pump power to the desired strength in 50 μs and then hold the power constant and simultaneously shine repump light. Before each frame, we perform 1 ms of polarization-gradient cooling to set the initial temperature of the atoms. The σ^- -polarized pump beam optically pumps the atoms into the $|F = 2, m_F = -2\rangle$ state with the quantization axis along x during the pump ramp-up. This is done so that when $\Omega < \Omega_c$, the atoms stay in $|F = 2, m_F = -2\rangle$ and no spin dynamics interfere with the spatial self-organization process.

Tweezer position calibration

As noted in the main text, a precise control of the tweezer positions with respect to the cavity and pump standing waves is crucial for our experiment. To constrain long-term drift in the position of the array along both cavity and pump axes, we perform an in-situ measurement of both positions at the end of each experimental run, and use the output of this measurement to displace the atom array onto the correct position before the start of the next experimental run. For the position measurement, after taking the post-pumping-sequence fluorescence image, we adjust the tweezer spacing (by varying the radio-frequency signal to the acousto-optic deflector) to an equal 5λ between the neighbouring tweezers. For this calibration, we use 5λ spacing so that the dominant mode is the centre-of-mass mode, which we can easily displace using a piezo-actuated mirror in the tweezer optical path (a method described elsewhere³⁶). This is in contrast to the 4.5λ spacing used in the self-organization data shots. We then expose the array to a low-intensity pump field (well below the self-organization transition) simultaneously with polarization-gradient cooling light and record the photon emission rate from the cavity. During this recording, the entire array is translated continuously first along the cavity axis and then along the pump axis using the piezo-actuated mirror. The recorded data reveal the sinusoidal variation of the cavity mode and the pump standing wave. An automated fit applied to these data determines the displacement that must be applied to the atoms to position them properly within the cavity and pump fields.

Experimental interpolation of the critical point: data and error analyses

As noted in the main text, the critical point is determined by interpolating the value of Ω that makes $B = 0$, where $k_B T \times B$ is the coefficient on the quadratic term of the mechanical potential. At each parameter setting, we fit the obtained distributions of c_{proj} to the Boltzmann distribution (equation (5)) to extract B . Examples of such distributions and their Boltzmann fits are shown in Extended Data Fig. 4a. To estimate the error in this fit of B , we implement bootstrapping: we randomly draw samples with replacement from the c_{proj} distribution at each Ω and do the fit again. This is iterated ten times. The average value and standard deviation of B are then calculated from the bootstrap samples. The sign of B determines whether the distribution is unimodal ($B > 0$) or bimodal ($B < 0$), and thus, the critical point occurs when $B = 0$. As shown in Extended Data Fig. 4b, the fitted values of B go from being positive to negative as Ω is increased.

Since we lack a complete theoretical model of how B should vary with Ω , we simply linearly interpolate the value of Ω that makes $B = 0$ and use this as Ω_c . To do so, we consider the two sampled points directly

on either side of $B = 0$; we call them by the coordinates (Ω_1, B_1) and (Ω_2, B_2) , where $B_1 > 0$ and $B_2 < 0$. Then, our experimental critical point $\Omega_c^{(\text{exp})}$ is given by the x intercept of the line that passes through these two points:

$$\Omega_c^{(\text{exp})} = \Omega_1 - \frac{B_1(\Omega_1 - \Omega_2)}{B_1 - B_2}. \quad (7)$$

The standard deviation σ_{Ω_c} on the critical point is obtained through error propagation:

$$\sigma_{\Omega_c}^2 = \left| \frac{\partial \Omega_c^{(\text{exp})}}{\partial B_1} \right|^2 \sigma_{B_1}^2 + \left| \frac{\partial \Omega_c^{(\text{exp})}}{\partial B_2} \right|^2 \sigma_{B_2}^2 + \left| \frac{\partial \Omega_c^{(\text{exp})}}{\partial \Omega_1} \right|^2 \sigma_{\Omega_1}^2 + \left| \frac{\partial \Omega_c^{(\text{exp})}}{\partial \Omega_2} \right|^2 \sigma_{\Omega_2}^2. \quad (8)$$

The standard deviations σ_{B_1} and σ_{B_2} are obtained from bootstrapping, whereas σ_{Ω_1} and σ_{Ω_2} are given by an estimate of 10% drift of the pump power over the course of the experimental runs. This estimation is based on the measurements of pump power at the start and end of data taking. We note that the linear interpolation is an approximate method for obtaining the critical point, and therefore, the error propagation applied to obtain σ_{Ω_c} may not accurately reflect the true experimental uncertainty on Ω_c .

Our extraction of $\Omega_c^{(\text{exp})}$ is also subject to systematic errors due to Gaussian noise in our detection and uncertainty in our measurement of the pump Rabi frequency. Gaussian noise, resulting from photon shot noise and electronic detector noise, causes the c_{proj} distributions to smear out and, therefore, leads us to systematically overestimate $\Omega_c^{(\text{exp})}$ when we use the method described above. We theoretically calculate how much the noise affects $\Omega_c^{(\text{exp})}$ by convolving a Gaussian distribution, whose width is determined by the fluctuations observed when no atoms are in the cavity, with the Boltzmann distribution. We then perform the same B fits and linear interpolation on the convolved distributions to estimate the percentage by which $\Omega_c^{(\text{exp})}$ changes. All Ω_c data shown in the main text and Extended Data Fig. 3 have been scaled to account for this systematic error.

The other systematic error we have identified is the error in our measurement of Ω due to uncertainty in our measurement of the pump beam size at the atoms ($173 \pm 5 \mu\text{m}$), in the positioning of the pump beam relative to the atom array, as well as in our measurement of the pump power (estimated to be $\pm 6\%$). Since the same beam size and power were applied to all data, these will cause all the Ω_c data to be shifted together up to approximately $\pm 6\%$. We also note that the average Ω value seen by the atom array depends on the atom number, since a larger array will see more of the Gaussian profile of the pump beam. To account for this, we average the pump intensity profile over the span of the atom array for each N , assuming the beam is exactly centred on the array, and factor this into our reported Ω ; this systematically shifts Ω downwards. It is also possible that the beam is not exactly centred on the atoms; we estimate the uncertainty to be about $\pm 30 \mu\text{m}$ in both the y and z directions. This off-centring would reduce Ω by up to an additional 6%. As a result, we expect that $\Omega_c^{(\text{exp})}$ could be systematically shifted upwards by up to 6% or shifted downwards by up to 8.5%, where we have added the downward errors in quadrature.

There are additional systematic error sources whose effects on the critical point have not been quantified. One such source is the tweezer trap frequency being lower at the outer edges of the array due to aberrations in the tweezer optical setup. This should push the critical point lower for larger N . Similarly, optical aberrations make the tweezer spacing uneven towards the edges of the array; we estimate the deviation to be up to 100 nm along both cavity and pump axes at

the edges of the 22-atom array. We expect this deviation to push the critical point higher for larger atom numbers.

Data availability

Data are available from the corresponding author upon reasonable request. Source data are provided with this paper.

Code availability

Code is available from the corresponding author upon reasonable request.

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Author contributions

J.H., Y.-H.L., Z.Y. and T.X. contributed to building the experimental setup, performing the experiments and analysing the data. C.C.R., S.J.M., A.A.-G., D.M.S.-K. and J.H. contributed to the theoretical model. Z.Y. and D.M.S.-K. conceived the experiments. All authors contributed to the writing of the manuscript and discussed the results. All work was supervised by D.M.S.-K. and A.A.-G.

Competing interests

The authors declare no competing interests.

Additional information

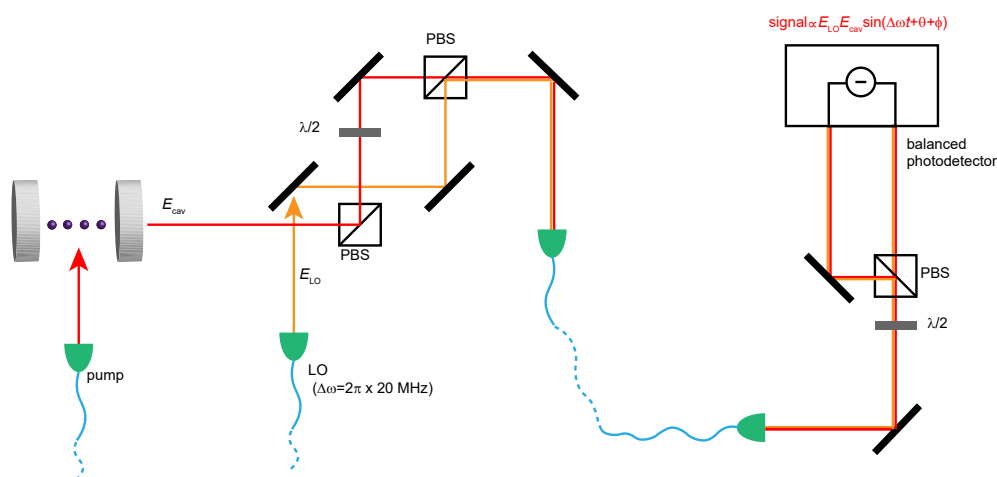
Extended data is available for this paper at <https://doi.org/10.1038/s41567-025-02916-7>.

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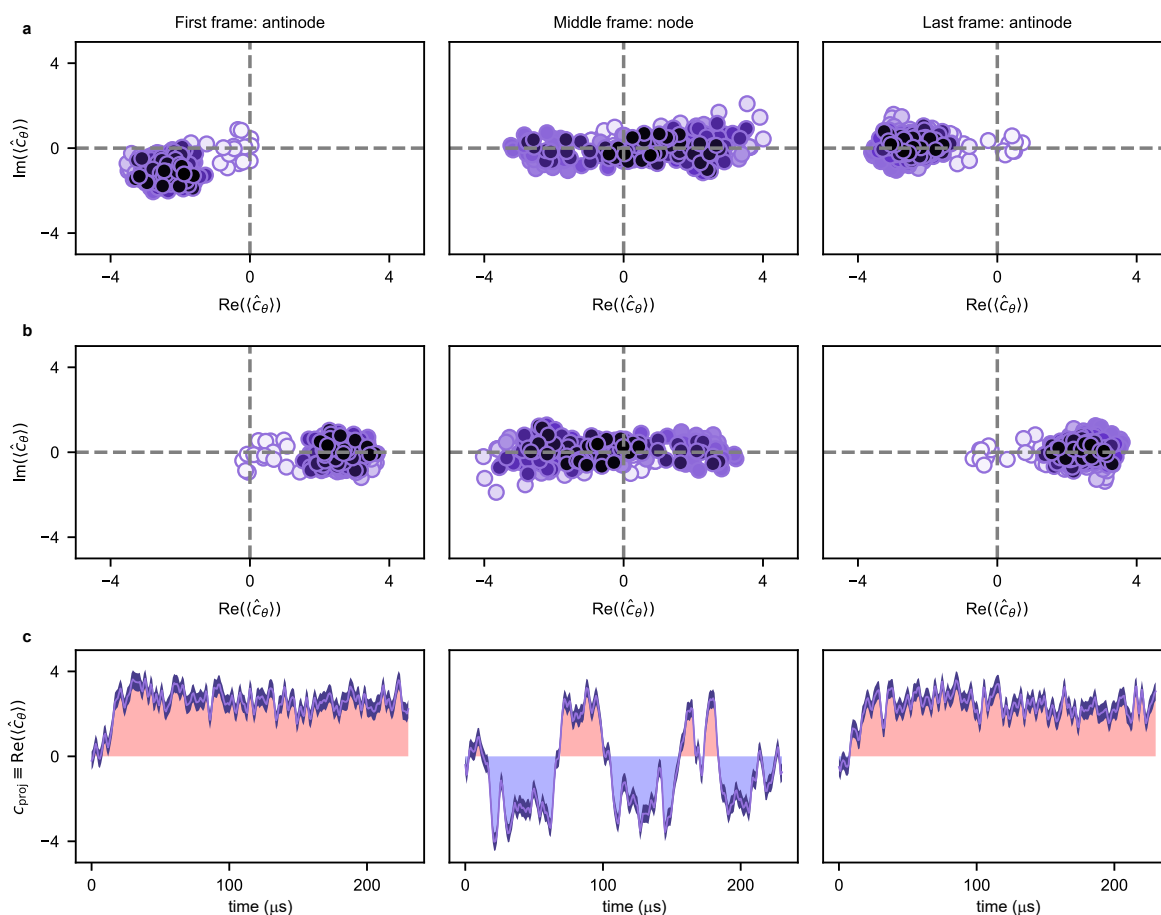
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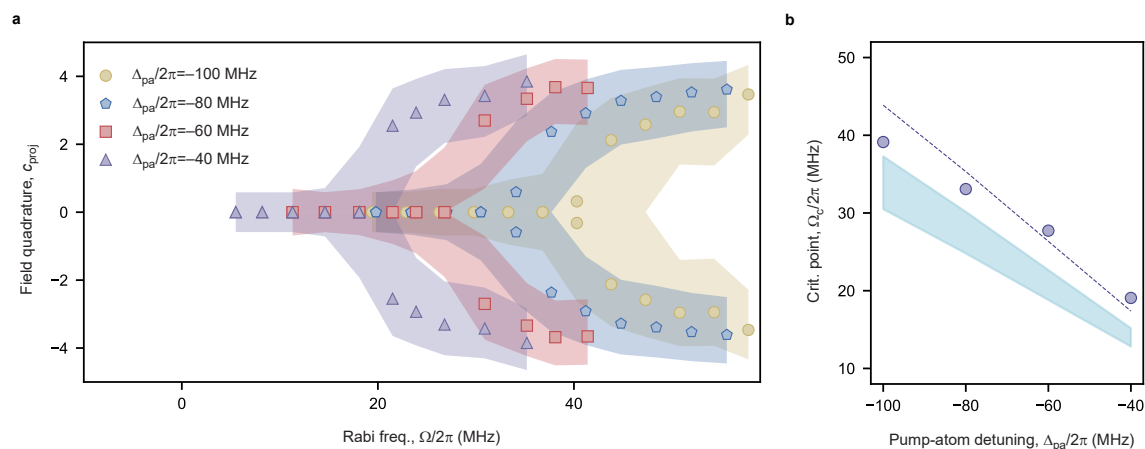
Extended Data Fig. 1 | Schematic of the heterodyne detection system. The pump and LO beams originate from separate fibres and are combined at a polarizing beamsplitter (PBS) with orthogonal polarizations. They are then both sent into a single-mode fibre, which ensures that they are mode-matched at the fibre output. Upon emerging from the fibre, they pass through a half-waveplate (labeled as $\lambda/2$) at 45° and are split at another PBS. Each output port of the last PBS

is directed to a sensor on a balanced photodetector, which subtracts the signals on the two sensors. The output voltage of the detector is proportional to $E_{\text{LO}} E_{\text{cav}} \sin(\Delta\omega t + \theta + \phi)$ where E_{LO} is the LO electric field amplitude, E_{cav} is the cavity electric field amplitude, $\Delta\omega = 2\pi \times 20$ MHz is the frequency difference between the pump and LO, ϕ is the phase of the LO relative to the pump, and θ is the phase of the cavity field relative to the pump.



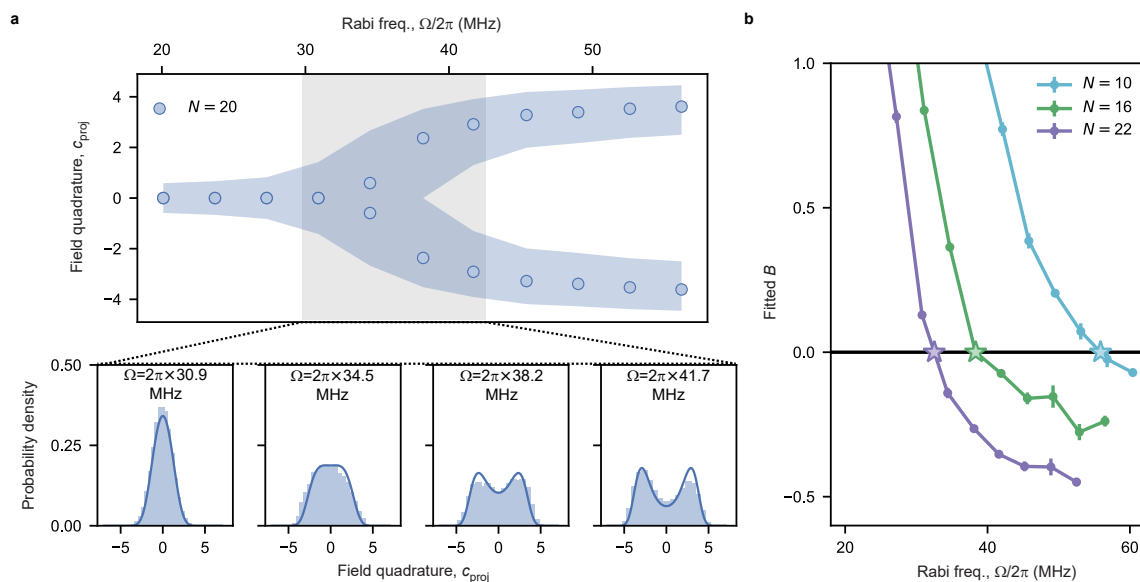
Extended Data Fig. 2 | Example time traces of the detected field in each of the three pumping frames. **a**, Scatter plots of the complex cavity field before phase correction. The real ($\text{Re}(\langle \hat{c}_\vartheta \rangle)$) and imaginary ($\text{Im}(\langle \hat{c}_\vartheta \rangle)$) quadratures of the detected field with respect to the average phase of the cavity emission, ϑ , are plotted. Lighter points indicate earlier times and darker points indicate later times. From these data, the angle ϕ of the major axis in each frame is determined by PCA. **b**, Detected field after rotation by ϕ . Because PCA only determines the angle of the major axis up to a π rotation, in the antinode frames, we define the

average projection of the field onto the axis defined by ϑ to be positive. The sign of the node frame is determined by comparing ϑ_{node} to the angle linearly interpolated from $\vartheta_{\text{first frame}}$ and $\vartheta_{\text{last frame}}$ and assuming that the two angles are within $\pi/2$ of each other. **c**, Time traces of c_{proj} . Values of c_{proj} are obtained by projecting the phase-corrected measurements of $\hat{c}$ (shown in **b**) onto the ϑ -axis. The shaded dark purple regions indicate the shot noise level. Data shown in figure were taken with $N=18$, $\Delta_{\text{pa}} = -2\pi \times 80$ MHz, $\Delta_{\text{pc}} = -2\pi \times 2.15$ MHz, and $\Omega = 2\pi \times 63.9$ MHz.



Extended Data Fig. 3 | Dependence of the critical point on Δ_{pa} . **a**, Data showing the bifurcation for 20 atoms at four values of Δ_{pa} . Markers represent the maxima of the fitted Boltzmann probability distribution of c_{proj} . Shaded areas show the width of the distribution at half-maximum. Different pump-cavity detunings were chosen for each setting of Δ_{pa} to compensate for the different dispersive shifts on the cavity resonance frequency for 20 atoms placed at the nodes. The values used are $\Delta_{\text{pc}} = -2\pi \times \{1.79, 1.9, 2.02, 2.26\}$ MHz corresponding to

$\Delta_{\text{pa}} = -2\pi \times \{100, 80, 60, 40\}$ MHz. These correspond approximately to $\Delta_{\text{pc}}(T) \simeq -2\pi \times 1.6$ MHz when accounting for the shift that thermal atoms put on the cavity resonance frequency. **b**, Extracted Ω_c (circles) exhibit an approximately linear dependence on Δ_{pa} . The dashed line shows a fit to equation (4), which gives a fitted temperature of $T = 66 \pm 8 \mu\text{K}$. The shaded region shows the prediction of equation (4) for the independent temperature measurement. Error bars are smaller than the markers.



Extended Data Fig. 4 | Interpolation of Ω_c from Boltzmann fits. **a, Data showing the bifurcation for 20 atoms at $\Delta_{\text{pa}} = 2\pi \times 80$ MHz. Markers represent the maxima of the fitted Boltzmann probability distribution of c_{proj} . Blue shaded areas show the width of the distribution at half-maximum. Bottom insets show histograms (light blue) and Boltzmann fits (dark blue) for values of Ω in the gray shaded region. **b**, By fitting experimentally obtained distributions, such as those shown**

in **(a)**, to the Boltzmann distribution, we extract the parameter B at different values of Ω for various N . The N shown here correspond to the data shown in Fig. 2a of the main text. The value of Ω_c (stars) is linearly interpolated from the two points directly on either side of the $B = 0$ line. Error bars on the fitted B values are smaller than the markers. Lines are guides to the eye.

Appendix F

Higher symmetry breaking and non-reciprocity in a driven-dissipative Dicke model

This appendix includes the following paper [116], discussed in Ch. 5:

1. Jacquelyn Ho et al. “Higher Symmetry Breaking and Nonreciprocity in a Driven-Dissipative Dicke Model”. In: *Physical Review Research* 8.1 (Mar. 2026), p. 013311. ISSN: 2643-1564. DOI: 10.1103/5h1h-9sfb. (Visited on 03/24/2026)

Higher symmetry breaking and nonreciprocity in a driven-dissipative Dicke model

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Higher symmetries in interacting many-body systems often give rise to new phases and unexpected dynamical behavior. Here, we theoretically investigate a variant of the Dicke model with higher-order discrete symmetry, resulting from complex-valued coupling coefficients between quantum emitters and a bosonic mode. We propose a driven-dissipative realization of this model focusing on optomechanical response of a driven atom tweezer array comprised of n subensembles and placed within an optical cavity, with the phase of the driving field advancing stepwise between subensembles. Examining stationary points and their dynamical stability, we identify a phase diagram for $n \geq 3$ with three distinctive features: a $\mathbb{Z}_n$ ($\mathbb{Z}_{2n}$) symmetry-breaking superradiant phase for even (odd) n , a normal unbroken-symmetry phase that is dynamically unstable due to nonreciprocal forces between emitters, and a first-order phase transition separating these phases. This n -phase Dicke model may be equivalently realized in a variety of optomechanical or optomagnonic settings, where it can serve as a test bed for studying high-order symmetry breaking and nonreciprocal interactions in open systems.

DOI: [10.1103/PhysRevResearch.8.013311](https://doi.org/10.1103/PhysRevResearch.8.013311)

I. INTRODUCTION

Phase transitions of interacting many-body systems at thermal equilibrium are a long-standing topic of research. More recently, phase transitions in driven-dissipative many-body quantum systems have been explored [1–6], opening the question of whether emergent properties of such open systems differ from those found at equilibrium.

The Dicke model [7,8] is paradigmatic for the study of both equilibrium and driven-dissipative phase transitions. It has played a defining role in studies of quantum optical phenomena such as superradiance and lasing [7,9–11] as well as in quantum simulation of phase transitions in macroscopic [12–15] and mesoscopic [16] systems. The model describes a collection of N two-level quantum emitters identically coupled to a single-mode electromagnetic field. As a closed system, the Dicke model undergoes a second-order phase transition as the emitter-mode coupling strength increases, going from an unbroken-symmetry normal phase to a $\mathbb{Z}_2$ symmetry-breaking superradiant phase.

Driven-dissipative versions of the Dicke model have been realized experimentally. Prominent among these are systems involving atomic ensembles placed within a high-finesse optical cavity [12,17]. The two-level quantum emitter of the original Dicke model is replaced by optically driven atoms undergoing low-energy Bragg or Raman transitions,

between different mechanical or hyperfine-spin states, respectively. Under certain conditions, the driven-dissipative system reaches a steady state. As in the equilibrium Dicke model, the steady state shows a normal-to-superradiant phase transition.

Since the Dicke model considers spins that are all symmetrically coupled to the cavity, one might ask how the phenomenology changes when this condition is broken. This modification of the Dicke model has been studied in the context of disorder, where different spins have different energy-level spacings and can couple to the cavity with different strengths [18–21] and with a continuum of phases [21]. Disordered couplings among atoms placed in a multimode cavity have also been used to realize glassy systems [22,23]. In contrast, there has been little exploration of what instead happens when the complex spin-cavity coupling varies *discretely* from spin to spin as $\sum_{i=1}^N (\chi_i^* \hat{c} + \hat{c}^\dagger \chi_i) \hat{\sigma}_i^x$, where $\hat{c}$ is the electromagnetic mode field operator, σ_i is the pseudospin 1/2 vector for emitter i , and χ_i is a complex-valued emitter-mode coupling strength. We find that in this scenario, it is possible to generate systems with discrete symmetries higher than $\mathbb{Z}_2$ with a simple modification. Specifically, we consider that N emitters are divided into n groups indexed by j , and the phase of the spin-cavity coupling for each group is set to $2\pi j/n$. Similar forms of this discrete symmetry have been previously explored in Ising-like systems such as the Potts model [24] and the q -state clock model [25]. Generalizing from the $\mathbb{Z}_2$ parity symmetry of the canonical Dicke model, the system we consider now has $\mathbb{Z}_n$ symmetry for even n and $\mathbb{Z}_{2n}$ symmetry for odd n . We call this the n -phase Dicke model.

We propose a way to realize the n -phase Dicke model using ultracold atoms trapped in optical tweezers, though we expect the concepts to be applicable to other realizations. We specifically consider a pumped and dissipative form of the Dicke model that relies on optomechanical self-organization

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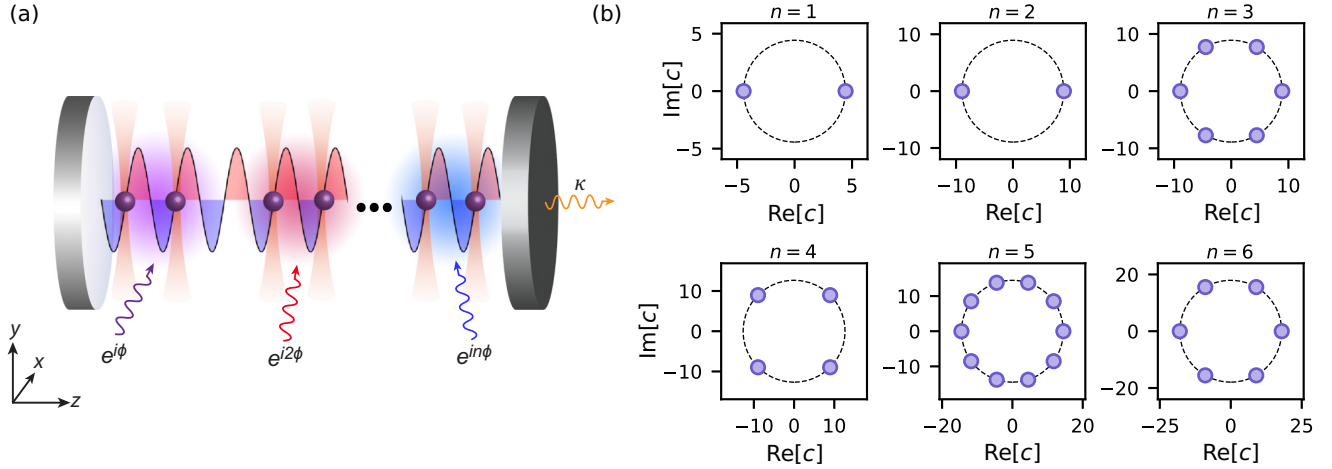


FIG. 1. (a) Schematic of the setup for the n -phase Dicke model. Atoms are held in harmonic traps at the nodes of the cavity field. The atoms are divided into n groups and each group is illuminated by a pump laser with a $\phi = 2\pi/n$ phase difference between adjacent groups. (b) Real and imaginary quadratures of the steady-state cavity field (solid circles) calculated using the atomic center-of-mass positions found through Lyapunov function minimization, shown for $n = 1$ through $n = 6$. The results show $\mathbb{Z}_n$ symmetry for even n and $\mathbb{Z}_{2n}$ symmetry for odd n . The parameters used are $\nu = 30$, $\omega_z = 2\pi \times 70$ kHz, $\Delta_{\text{pa}} = -2\pi \times 100$ MHz, $\Delta_{\text{pc}} = -2\pi \times 4$ MHz, $\kappa = 0$, $g_0 = 2\pi \times 3$ MHz, and $\Omega = 2\pi \times 20$ MHz.

of the atoms [13,14]. Here, the superradiant phase corresponds to the atoms self-organizing onto the lattice formed by the interference of the pump and cavity fields, and the atomic motion can be mapped to spins. Realization of the aforementioned $\mathbb{Z}_n$ or $\mathbb{Z}_{2n}$ symmetry occurs by adjusting separately the optical phase of the pump that drives each atom.

Analyzing this system theoretically, we characterize a phase diagram with three features. First, we find conditions at high pump strength where the cavity field and emitters together stably break the discrete $\mathbb{Z}_n$ or $\mathbb{Z}_{2n}$ symmetry, for even and odd n , respectively. Second, unlike in the canonical Dicke model, the phase transition to the stable broken symmetry state becomes first order, asymptoting to a second-order transition only when the pump light is far detuned from the cavity resonance frequency. Third, we find the normal phase to be dynamically unstable. This instability arises from nonreciprocal light-mediated interactions between the driven emitters.

II. CAVITY OPTOMECHANICAL SYSTEM

We elucidate the features of the n -phase Dicke model in the specific experimental context of cavity optomechanics with an atom tweezer array [16,26]. We consider the one-dimensional motion of a one-dimensional array of N harmonically confined ^{87}Rb atoms along the axis ($\mathbf{z}$) of a single-mode Fabry-Pérot optical cavity. The harmonic traps are centered on the cavity nodes and spaced by an integer number of wavelengths. The atoms, which have an allowed dipole transition at optical frequency ω_a , are then pumped with coherent light propagating transverse to the cavity axis (along $\mathbf{x}$) at frequency ω_p (wave number k and wavelength λ) and Rabi frequency Ω .

When Ω is uniform across all atoms, an optomechanical phase transition occurs at a critical pump strength Ω_c , related to the canonical Dicke phase transition, owing to competition

between the trapping potentials, which confine atoms to the cavity-field nodes, and the interference of the pump and cavity fields, which pulls atoms away from the nodes. A choice of gauge allows us to define Ω as being real and non-negative. With $\Omega < \Omega_c$, the atoms remain stably centered on the cavity nodes and do not scatter light coherently into the cavity; this corresponds to the normal phase of the Dicke model. With $\Omega > \Omega_c$, the atoms self-organize by moving collectively toward cavity antinodes with the same phase, where they emit coherently into the cavity. This superradiant mode breaks the system's $\mathbb{Z}_2$ symmetry, with the atoms displaced toward either the positive- or negative-signed cavity field antinodes, and the cavity light correspondingly having a phase of either 0 or π with respect to the pump field [16].

To generate higher-order symmetry, we now modify this setup as shown in Fig. 1(a). We suppose that a total of $N = n\nu$ trapped atoms, still each positioned on a cavity-field node, are divided into n groups. Each group j , containing ν atoms, is driven with a pump Rabi frequency of $\Omega e^{i\phi j}$, where $\phi = 2\pi/n$ and $j = \{1, 2, \dots, n\}$. We assume the pump is far detuned from the atomic resonance, i.e., that the absolute value $|\Delta_{\text{pa}}| \equiv |\omega_p - \omega_a|$ is much greater than both Ω and the excited-state decay rate, allowing us to adiabatically eliminate the atomic excited state and focus on dispersive atom-light interactions; this makes our treatment equivalent to that of other optomechanical systems of polarizable media. The Hamiltonian in the frame rotating at ω_p reads as

$$\hat{H} = -\hbar\Delta_{\text{pc}}\hat{c}^\dagger\hat{c} + \sum_{j=1}^n \left[\frac{\hat{p}_j^2}{2\mu} + \frac{\mu\omega_z^2\hat{z}_j^2}{2} + \frac{\hbar\Omega\nu}{\Delta_{\text{pa}}}g_0\sin(k\hat{z}_j)(e^{-i\phi j}\hat{c} + e^{i\phi j}\hat{c}^\dagger) \right], \quad (1)$$

where ω_z is the tweezer trap frequency, g_0 is the vacuum Rabi frequency, and we have defined $\mu \equiv \nu m$ (m being the

atomic mass) and $\Delta_{pc} \equiv \omega_p - \omega_c$. For simplicity, we include only the center-of-mass mode $\hat{z}_j$ of each group and ignore other modes of motion. We also ignore the dispersive shift of the cavity resonance by the atoms. Note that for $n = 1$, Eq. (1) can be mapped to the canonical Dicke model in the Holstein-Primakoff representation [27] by rewriting the position and momentum operators in terms of bosonic creation and annihilation operators for the center-of-mass mode of the array.

We highlight the symmetries of the system: $\hat{H}$ preserves $\mathbb{Z}_n$ symmetry, since performing the gauge transformation $c \rightarrow e^{i\phi} c$ and simultaneous permutation of the group index j leaves $\hat{H}$ invariant. The system also has a $\mathbb{Z}_2$ symmetry in that the simultaneous transformations $\{z, p\} \rightarrow \{-z, -p\}$ and $c \rightarrow -c$ also lead to the same Hamiltonian. These combined symmetries result in $\mathbb{Z}_{2n}$ symmetry for odd n and $\mathbb{Z}_n$ symmetry for even n . In the even n case, the $\mathbb{Z}_2$ symmetry is redundant because it is equivalent to permuting the group index $n/2$ times while advancing the cavity field phase by $n\phi/2 = \pi$.

In the following, we neglect stochastic noise on the cavity field and study the evolution of the expectation values of operators under the master equation. We obtain the following equations of motion:

$$\dot{c} = (i\Delta_{pc} - \kappa)c - i \sum_{l=1}^n \frac{\nu\Omega}{\Delta_{pa}} e^{i\phi l} g_0 \sin(kz_l), \quad (2)$$

$$\dot{p}_j = -\mu\omega_z^2 z_j - \frac{\nu\Omega}{\Delta_{pa}} \hbar k g_0 \cos(kz_j) (e^{i\phi j} c^* + e^{-i\phi j} c), \quad (3)$$

$$\dot{z}_j = \frac{p_j}{\mu}. \quad (4)$$

Here, κ is the decay rate of the cavity field. We can further reduce the equations of motion to a set of $2n$ purely mechanical equations by assuming that the cavity field is in its steady state and instantaneously follows the atomic motion. The adiabatic cavity field is described by

$$c = \frac{i\nu\Omega}{\Delta_{pa}} \frac{1}{i\Delta_{pc} - \kappa} \sum_{j=1}^n e^{i\phi j} g_0 \sin(kz_j). \quad (5)$$

From Eq. (5), one can also understand the symmetry-broken states as the sets of positions $\{z_j \simeq \pm\lambda/4\}$ that maximize the magnitude of $\sum_{j=1}^n e^{i\phi j} \sin(kz_j)$. For even n , there are n such sets and for odd n there are $2n$. We verify these $\mathbb{Z}_n$ and $\mathbb{Z}_{2n}$ symmetries by performing Lyapunov function minimization in the limit $\kappa = 0$ [Fig. 1(b)] to find all the symmetry-broken steady states [28–30]. Substituting Eq. (5) into Eq. (3) yields the force equations

$$\begin{aligned} \dot{p}_j = & -\mu\omega_z^2 z_j - \frac{2\hbar\nu^2\Omega^2}{\Delta_{pa}^2} \frac{g_0^2 k \cos(kz_j)}{\Delta_{pc}^2 + \kappa^2} \sum_{l=1}^n [\Delta_{pc} \cos(\phi(j-l)) \\ & - \kappa \sin(\phi(j-l))] \sin(kz_l). \end{aligned} \quad (6)$$

III. PHASE DIAGRAM OF THE n -PHASE DICKE MODEL

To elucidate the phase diagram of the n -phase Dicke model, we first consider the infinitely dispersive cavity limit

($\Delta_{pc}/\kappa \rightarrow -\infty$), in which the interactions are dominated by the “cavity-dispersive” terms proportional to $\Delta_{pc} \cos(\phi(j-l))$. In this limit, the “cavity-reactive” terms proportional to $\kappa \sin(\phi(j-l))$ in Eq. (6) are negligible and can be ignored. The phase diagram is now analogous to that of the Dicke model: There is a stable normal phase with unbroken symmetry corresponding to $z_j = 0$ for all j when $\Omega < \Omega_c$, which continuously transitions into a superradiant, broken-symmetry phase at $\Omega = \Omega_c$, where $\Omega_c = \sqrt{\frac{\Delta_{pa}^2 |\Delta_{pc}| m \omega_z^2}{g^2 \hbar k^2 N}}$, and the superradiant phase has $\mathbb{Z}_n$ or $\mathbb{Z}_{2n}$ symmetry for even or odd n , respectively.

However, upon restoring the cavity-reactive terms, the unbroken-symmetry state becomes unstable. By linearizing Eq. (6) around ($p_j = 0, z_j = 0$) and seeking a solution of the form $\tilde{z}_j(t) = \tilde{z}_j e^{-i\omega t}$, we find that for $n > 2$, there are two eigenfrequencies given by $\omega^2 = \omega_z^2 + \frac{N\Omega^2 \hbar k^2 g_0^2}{m\Delta_{pa}^2 \sqrt{\Delta_{pc}^2 + \kappa^2}} (\cos \theta \pm i \sin \theta)$, where $\tan \theta = -\kappa/\Delta_{pc}$ [28]. We immediately see that for noninfinite $|\Delta_{pc}|$ or nonzero κ , there are eigenfrequencies with positive imaginary part, indicating the presence of exponentially growing eigenmodes. These lead the $z_j = 0$ solution to be unstable for any $\Omega > 0$.

To understand the emergence of steady states at $z_j \neq 0$, we focus on the specific case of $n = 4$, which admits a graphical solution to the dynamically stationary states. In this case, we can reduce the system to two force equations by enforcing the symmetry $z_1 = -z_3$ and $z_2 = -z_4$. In Figs. 2(a)–2(c), we plot separately the locus of positions z_1 and z_2 where the forces $\dot{p}_1$ and $\dot{p}_2$ vanish, restricting our view to the region of position space in which symmetry-broken solutions emerge at the least distance from the cavity nodes. The intersections of these two curves denote stationary states.

Similar models of multiple atomic ensembles that couple to the cavity field with different phases have been shown to possess instabilities and dynamical solutions [31–34]. In this work, we elucidate additional aspects of the phase diagram with proximity to cavity resonance and pump strength. We identify three dynamical phases: normal, dispersive broken symmetry, and reactive broken symmetry. In the normal phase, which exists at low Ω , ($z_j = 0, p_j = 0$) is the only steady state [Fig. 2(a)]. At higher values of Ω , multiple steady states exist. In the dispersive regime ($|\Delta_{pc}| \gg \kappa$), there are nine steady states in the region $-\lambda/4 < \{z_1, z_2\} < \lambda/4$; four of these steady states break symmetry stably [stars in Fig. 2(b)], while the other five are unstable. We determine the stability by evaluating the eigenvalues of the Jacobian matrix for each steady state [28]. Closer to cavity resonance, we enter the reactive regime ($|\Delta_{pc}| \lesssim \kappa$), where there are no broken symmetry steady states for small values of $|z_1|$ and $|z_2|$, but there are near $z_1, z_2 \sim \pm 0.5\lambda$ [Fig. 2(c)]; these states are all unstable due to cavity-reactive terms dominating over cavity-dispersive terms in the equations of motion. The transitions between these phases are all discontinuous, with the exception of the transition from normal to broken symmetry in the purely dispersive limit. Numerical analyses performed for $n = 3, 5$, and 6 reveal similar phase diagrams to that shown in Fig. 2(d) [28]. Using the same parameters as in Fig. 2 while fixing $\Omega = 2\pi \times 20$ MHz and $\Delta_{pc} = -2\pi \times 4$ MHz, we numerically integrate Eqs. (2)–(4) to obtain cavity field

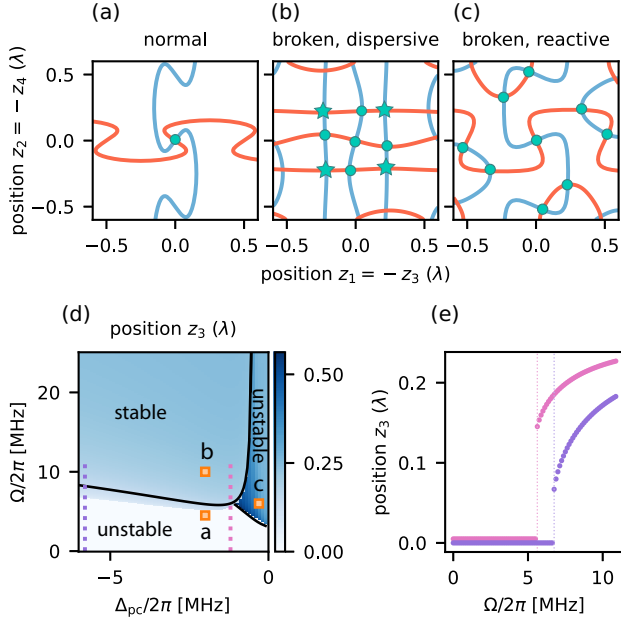


FIG. 2. (a)–(c) Contour plots of the forces $\dot{p}_1 = 0$ (blue) and $\dot{p}_2 = 0$ (red) for $n = 4$, where we have set $z_1 = -z_3$ and $z_2 = -z_4$. Steady states (teal markers) occur where the blue and red curves intersect. Jacobian eigenvalue analysis is used to determine whether each steady state is unstable (circles) or stable (stars). (d) Phase diagram for $n = 4$ showing the steady-state position of one group (z_3) at a particular symmetry-broken solution. Orange squares correspond to the parameters used in panels (a)–(c). Black lines are phase boundaries calculated by expanding Eq. (6) to third order around the symmetry-broken solutions. (e) Line cuts corresponding to the dashed lines in panel (d) showing the first-order transition as a function of Ω . Dashed lines show the location of the discontinuity for the two values of Δ_{pc} . Pink points have a slight vertical offset for visibility. In the $|\Delta_{pc}| \rightarrow \infty$ limit, the transition becomes second order (continuous). Calculations in figure are done with $\Delta_{pa} = -2\pi \times 100$ MHz, $\kappa = 2\pi \times 0.5$ MHz, $\nu = 30$, $g_0 = 2\pi \times 3$ MHz, and $\omega_z = 2\pi \times 70$ kHz.

trajectories for different initial perturbations of the atomic positions and momenta around the $z_j = 0$, $p_j = 0$ steady state. The results for $n = 3, 4, 5$ and 6 are shown in Fig. 3. The trajectories have a high density at the vertices of an n - or $2n$ -sided polygon; this confirms the $\mathbb{Z}_n$ and $\mathbb{Z}_{2n}$ symmetry breaking as well as the stability of the system's steady states for the chosen parameters. We also examine the time trajectories for $n = 4$ [Fig. 3(e)] and see that the cavity field breaks symmetry at late times. We note that the timescale for symmetry breaking appears to shorten when including the dynamics of the atomic excited state, which can also lead to instability when the cavity field is particularly strong [28].

IV. DESCRIPTION OF NONRECIPROCAL INTERACTIONS

The distinctive dynamical instabilities of the n -phase Dicke model originate from the presence of cavity reactive terms, whose effect grows when $|\Delta_{pc}|$ approaches κ . These terms represent nonreciprocal interactions in this driven-dissipative system. To elucidate the role of nonreciprocal forces, we

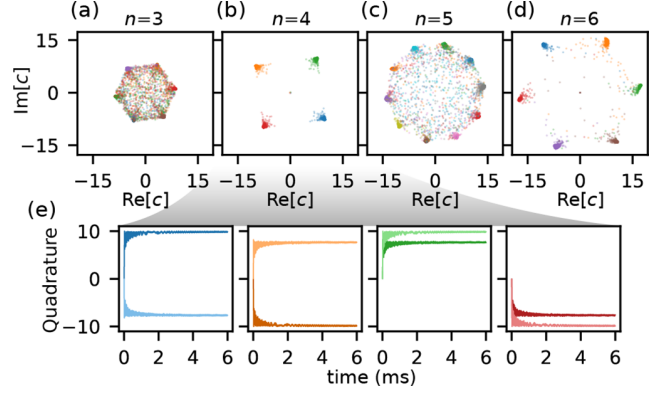


FIG. 3. Density plots of cavity field trajectories for (a) $n = 3$, (b) $n = 4$, (c) $n = 5$, and (d) $n = 6$ in a parameter regime predicted to have stable steady states. Different colors correspond to different perturbations in the initial positions of the atoms, which are the starting conditions for the numerical integration. The high concentration of single colors at the vertices of (a) a hexagon, (b) a square, (c) a decagon, and (d) a hexagon indicate that the trajectories have spontaneously broken either a $\mathbb{Z}_{2n}$ or a $\mathbb{Z}_n$ symmetry. All trajectories are integrated over a time span of 6 ms. (e) Time trajectories of the real (dark color) and imaginary (light color) quadratures of the cavity field for $n = 4$. Colors correspond to the data in panel (b). Parameters used for all data in figure are $\Omega = 2\pi \times 20$ MHz, $\Delta_{pc} = -2\pi \times 4$ MHz, $\Delta_{pa} = -2\pi \times 100$ MHz, $\kappa = 2\pi \times 0.5$ MHz, $\nu = 30$, $g_0 = 2\pi \times 3$ MHz, and $\omega_z = 2\pi \times 70$ kHz.

linearize Eq. (6) and rewrite it in terms of bosonic ladder operators $\hat{b}_j$, which annihilate tweezer phonons in the j th group of atoms. Defining $\mathbf{b} = (\hat{b}_1, \hat{b}_2, \dots, \hat{b}_n)^T$, this results in the equation of motion $\dot{\mathbf{b}} = -i\mathbf{H}_{\text{eff}}\mathbf{b}$, where $\mathbf{H}_{\text{eff}}$ is an effective Hamiltonian whose matrix elements are given by

$$H_{\text{eff},jl} = \begin{cases} \omega_z + \frac{Ck\Delta_{pc}}{\mu\omega_z}, & j = l, \\ \frac{Ck[\Delta_{pc}\cos(\phi(j-l)) - \kappa\sin(\phi(j-l))]}{\mu\omega_z}, & j \neq l, \end{cases} \quad (7)$$

where we have defined $C \equiv \frac{2\hbar\nu^2\Omega^2}{\Delta_{pa}^2} \frac{g_0^2 k}{\Delta_{pc}^2 + \kappa^2}$ for convenience. We see that $H_{\text{eff},jl} \neq H_{\text{eff},lj}$ unless $\kappa\sin(\phi(j-l)) = 0$ for all values of j and l . Since $\phi = 2\pi/n$, this results in $\mathbf{H}_{\text{eff}}$ being non-Hermitian when $n > 2$ and $\kappa > 0$. The fact that $H_{\text{eff},jl} \neq H_{\text{eff},lj}$ means that the groups of atoms interact nonreciprocally [35]: The amplitude with which phonons tunnel from group j to group l is not equal to the amplitude with which phonons tunnel from group l to group j .

Though the interactions described by H_{eff} are generically nonreciprocal, they do not realize ideal nonreciprocal interactions in the sense that it is not possible to have both $H_{\text{eff},jl} = 0$ and $H_{\text{eff},lj} \neq 0$ for $l \neq j$. However, we now show that one can achieve ideal nonreciprocity if we take $n = 2$ and no longer fix the phase difference ϕ to be $2\pi/n$. To avoid notational confusion, we define this new, variable phase difference as φ . The effective Hamiltonian has off-diagonal matrix elements $H_{\text{eff},12} = Ck(\Delta_{pc}\cos\varphi + \kappa\sin\varphi)/(\mu\omega_z)$ and $H_{\text{eff},21} = Ck(\Delta_{pc}\cos\varphi - \kappa\sin\varphi)/(\mu\omega_z)$.

To illustrate how to achieve a perfectly nonreciprocal interaction, suppose we would like group 1 to be completely decoupled from the motion of group 2, but not vice versa.

Achieving this requires $H_{\text{eff},12} = 0$, which is satisfied for $\varphi = \tan^{-1}(-\Delta_{\text{pc}}/\kappa)$. We note that this is equivalent to setting $\varphi + \theta = \pm\pi/2$, which has the simple physical interpretation of requiring the cavity field emitted by group 2 to be $\pi/2$ out of phase with the pump field at group 1. As such, $\varphi + \theta$ is analogous to a synthetic flux found in other systems with nonreciprocal interactions [36–38]. Such nonreciprocity has been considered in systems of nanoparticles held close together in optical tweezers [39–42] and an SiN membrane positioned in an optical cavity [38]. Here, we have shown that a similar type of interaction can be achieved with trapped atoms in an optical cavity.

V. CONCLUSION

We have investigated a variant of the driven-dissipative Dicke model, in which one can realize $\mathbb{Z}_n$ or $\mathbb{Z}_{2n}$ symmetry breaking by pumping an optomechanical array with n optical phases. For a strong pump, the system stably breaks symmetry in a certain parameter regime, while for a weak pump, it is described by complex eigenfrequencies and nonreciprocal interactions. Our results contribute to the growing literature on instabilities and dynamical solutions in variants of the Dicke model [31–34,43]. We have also discussed how to achieve one-way phonon propagation in this system by harnessing control over nonreciprocal interactions. Although we have examined this Dicke model variant in the context of cold atoms, we expect the concepts also to apply to cavity-coupled arrays of other types of mechanical elements, such as thin membranes [38], optomechanical crystals [44], and optically suspended nanoparticles [45]. One might also consider cavity coupling to the internal spin states of atoms, rather than

their mechanical degrees of freedom. Engineering of nonreciprocal spin excitation dynamics may have applications in quantum information science [36,46]. This model additionally has broad prospects for studies of the interplay between symmetries and nonreciprocity in the vicinity of a phase transition [32,47–49].

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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