Conversion of poloidal flows into toroidal flows by phase space structures in trapped ion resonance driven turbulence

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Abstract.

A theory to describe the conversion of poloidal momentum into toroidal momentum by phase space structures in trapped ion resonance driven turbulence is presented. In trapped ion resonance driven turbulence, phase space structures are expected to form and can contribute to transport by exerting dynamical friction. Toroidal momentum flux by dynamical friction is calculated. It is shown that dynamical friction exerted on trapped ion granulations can mediate momentum transfer between poloidal and toroidal flows. The conversion coefficient is calculated as measurable that can be validated in present devices.

1. Introduction

The interplay among turbulence and flows is an important problem for magnetic confinement fusion, since turbulence driven flows can reduce transport by exerting $E\times B$ shear and can stabilize harmful MHD instability[1, 2, 3, 4, 5, 6, 7, 8, 9]. In torus plasmas, there can be two distinct flows in the poloidal and toroidal direction due to doubly connected topology. These two flows can interact one another, where drift wave turbulence can mediate the conversion of momentum. An example of the conversion process may be the generation of toroidal intrinsic rotation[5, 6, 7, 8, 9], where poloidal flows can set $E \times B$ shear to break the parallel symmetry[10] in drift wave turbulence and can lead to intrinsic torque to spin-up toroidal flows. The overall process may be viewed as a conversion of poloidal flows into toroidal flows. A similar process can be found in magnetic dynamo, where poloidal magnetic field can be converted to toroidal magnetic field by the Ω-effect[11].

In a recent study[12], the theory that describes the conversion process was developed by accounting for acoustic coupling in drift wave turbulence. The momentum balance theorem[13] for zonal flow evolution was extended to include acoustic waves. Based on the theorem, a coupling of perpendicular and parallel flows was discussed. However, while that study reveals a root for converting poloidal and toroidal momentum to one another, there can be a distinct mechanism in trapped ion resonance driven turbulence[14, 15, 16]. In trapped ion modes, acoustic coupling is averaged out due to the bouncing motion in the magnetic mirror. Thus acoustic coupling is not available for converting poloidal and toroidal flows. More importantly, in trapped ion resonance driven turbulence, phase space structures are expected to form due to strong wave particle resonance[17, 18, 19, 20, 21]. Once formed, phase space structure dynamics can cause several important effects, including subcritical instability[17, 18, 22], transport by dynamical friction[23, 24], zonal flow acceleration[25, 26, 27, 28], etc. Since phase space structure can drive phase space density transport by dynamical friction, we then simply expect that phase space structures can cause toroidal momentum transport. In particular, while structures would be dynamically coupled to toroidal momentum transport on one hand, structure dynamics is coupled to zonal flow acceleration on the other hand[25, 26, 27, 28]. Thus phase space structures would be a plausible candidate for connecting poloidal and toroidal flows dynamically, and for converting poloidal and toroidal momentum in trapped ion resonance driven turbulence.

In this paper, by explicitly retaining phase space structures as a component of trapped ion resonance driven turbulence, we show that plasma turbulence can convert poloidal flows into toroidal flows via plasma phase space (Fig.2, which is explained in detail later). To show this, we first discuss the conversion process by a single structure located in phase space (i.e. a phase space density hole). We calculate toroidal momentum flux associated with the growth of the structure. By relating the structure growth to poloidal flow evolution, we argue that phase space structures can connect toroidal flow evolution and poloidal flow evolution. Then, we turn to the case of an ensemble of structures, i.e. granulations. We discuss that when turbulent plasma granulations can form in phase space, mean field evolution of phase space density is described not by a quasilinear diffusion process, but by a Lenard-Balescu type evolution with dynamical friction[23, 24, 25, 26, 28]. We derive toroidal momentum evolution equation by taking toroidal precession velocity moment of the mean field transport equation of phase space density. Based on the toroidal momentum transport equation, we argue that turbulent granulations can mediate momentum transfer between two flows (poloidal and toroidal) inherent in torus plasmas. Based on the theory developed here, we give a qualitative estimate of the conversion process. The result shows that in the present devices the acceleration of toroidal flows have the same order of magnitude as the acceleration of poloidal zonal flows.

The remaining of the paper is organized as follows. In section 2, we introduce a model for trapped ion resonance driven turbulence and discuss the relation of the dynamics of a single phase space structure to toroidal momentum transport. In section 3, we turn to the case of an ensemble of multi-structures, i.e. phase space density granulations. We calculate toroidal momentum transport driven by granulations and show that granulations can mediate momentum transfer from poloidal flows to toroidal flows. Quantitative estimate for the toroidal flow acceleration is given, and a possible experiment to probe the effect of granulations is discussed. Section 4 is conclusion and discussion.

2. Conversion of poloidal flows into toroidal flows by a single structure

As a specific example of plasma turbulence, we consider trapped ion resonance driven ion temperature gradient (ITG) turbulence[14, 15, 16]. While trapped ion mode (TIM) considered here has many similarities with typical ITG, we note that they do have differences as well. For example, they have different frequency ordering, as $\omega > k_\parallel v_{thi}$ for ITG while $\omega < k_\parallel v_{thi}$ for TIM. The dependence of frequency on mode number is different as well. ITG frequency depends on ion inertia through finite Larmor radius effect $k_\perp \rho_i$, while TIM frequency depends on ion inertia through finite Banana radius effect $k_\perp \rho_b$. In order to describe the dynamics of trapped ion resonance driven turbulence, we use the model developed in literature[14, 15, 16]. The model [14, 15, 16] used here consists of the bounce averaged kinetic equation for ions and the gyrokinetic Poisson equation to account for quasi-neutrality:

$$\partial_t f + v_d(E)\partial_y f + \frac{c}{B}\{\phi, f\} = 0, \tag{1}$$

$$\frac{e\tilde{\phi}}{T_e} - \rho^2 \nabla_\perp^2 \frac{e\tilde{\phi}}{T_e} = \frac{2}{n_0\sqrt{\pi}} \int_0^\infty dE\sqrt{E}\delta f. \tag{2}$$

Electrons are dissipative in the frequency range of interest $\nu_{e,eff} \gg \omega$. Thus electrons are treated as laminar. Here, $v_d(E) = v_{d,0}E/T_i$ is an energy dependent magnetic precession drift velocity, and ρ^2 includes both the classical and neo-classical polarization effects. In the case of long wave length, $\rho^2 = \rho_s^2(1 + 1.6q^2/\sqrt{\epsilon_0})$[29]. ($q$ is the safety factor and ϵ_0 is the inverse of the aspect ratio of plasmas) . For this model, the Kubo number[30], which measures coherence of the turbulence, is

$$K \sim \frac{\tilde{v}}{(|d\omega/dk_\theta - \omega/k_\theta|\Delta k_\theta)\Delta_c}. \tag{3}$$

Here $\tilde{v}$ is the typical fluctuation velocity and Δ_c is the correlation length. Given the weakly dispersive nature of long wavelength drift wave turbulence, it is very easy to have long $\tau_{ac} \equiv |(d\omega/dk_\theta - \omega/k_\theta)\Delta k_\theta|^{-1}$, and thus Kubo number $K = \tau_{ac}\tilde{v}/\Delta_c > 1$ is likely. Hence phase space structures due to strong wave-particle resonance, such as granulations, can form in this model. The model used in the paper for trapped ion turbulence is arguably the minimal model that can capture the role of granulations on drift wave dynamics. This is since the fast bouncing motion of trapped ions average out the parallel heating, as $dE/dt = \overline{v_\parallel E_\parallel} \to 0$ where $\overline{(...)}$ is the bounce average. This fact is manifested in the absence of energy scattering term in the bounce kinetic equation. This allows us to focus on the 2D spatial nonlinear dynamics of drift waves, while keeping the effect of velocity space resonance (toroidal ion precession resonance).

Before starting a detailed analysis, we start by considering a simple case of a single structure in phase space density (Fig.1). Here, we consider a hole structure[18, 21] (a localized deficit) located at (x_0, E_0) in phase space. x_0 is the radial position of the structure and $E_0/T_i \cong \omega/\omega_{Di} \sim \sqrt{\epsilon_0}v_{*i}/v_{Di} \sim 1/\sqrt{\epsilon}_0$ is the energy of resonating trapped ions. Once formed, a phase space structure can grow by extracting free energy, as shown in Fig.1. In this process, the structure can climb against the gradient by scattering off electrons and polarization charge. Since the total f is conserved during the displacement (i.e. $df/dt = 0$), the depth must grow. The growing process can be formulated by following analysis in literature[18, 25] as:

$$\partial_t \langle \delta f^2 \rangle = -2\langle \tilde{v}_x \delta f \rangle \left. \frac{\partial \langle f \rangle}{\partial x} \right|_0 \tag{4}$$

where δf is the increment due to the displacement, $\langle ... \rangle$ is the zonal average, and $(...)|_0$ denotes the location (x_0, E_0). Note scattering in energy does not enter here, since it is averaged out by the bouncing motion in the magnetic mirror. As clearly seen in Eq.4, structure growth is tied to the flux of phase space density, $\langle \tilde{v}_x \delta f \rangle$.

The structure dynamics can be related to toroidal momentum transport as follows. To demonstrate the connection, we multiply the toroidal precession velocity $\bar{v}_{Di}\bar{E}(B_\zeta/B_\theta)$ of the structure and take velocity moment of Eq.4, to obtain toroidal momentum flux associate with the growing process:

$$\begin{aligned}\langle \tilde{v}_x \tilde{v}_\zeta \rangle &= \int d^3v v_{Di} \bar{E}_0 \frac{B_\zeta}{B_\theta} \langle \tilde{v}_r \delta f \rangle \\ &\cong -v_{Di} \frac{1}{\sqrt{\epsilon_0}} \frac{B_\zeta}{B_\theta} \frac{\partial}{\partial t} \int d^3v \frac{\langle \delta f^2 \rangle}{2\partial \langle f \rangle / \partial x|_0}.\end{aligned} \tag{5}$$

This relates toroidal momentum flux to dynamics of a phase space structure. Importantly, Eq.5 describes the conversion of momentum between toroidal and poloidal flows, mediated by a phase space structure. This can be seen by noting that the righthand side is proportional to the evolution of *poloidal* pseudomomentum of the structure[25, 26]. We can explicitly see the link of this term to poloidal momentum of fluctuation by considering non-resonant limit, as $\int d^3v \langle \delta f^2 \rangle / \partial \langle f \rangle / \partial x \to -k_\theta N_\mathbf{k}$ where $N_\mathbf{k}$ is the wave action density. By exchanging the poloidal pseudomomentum, a single

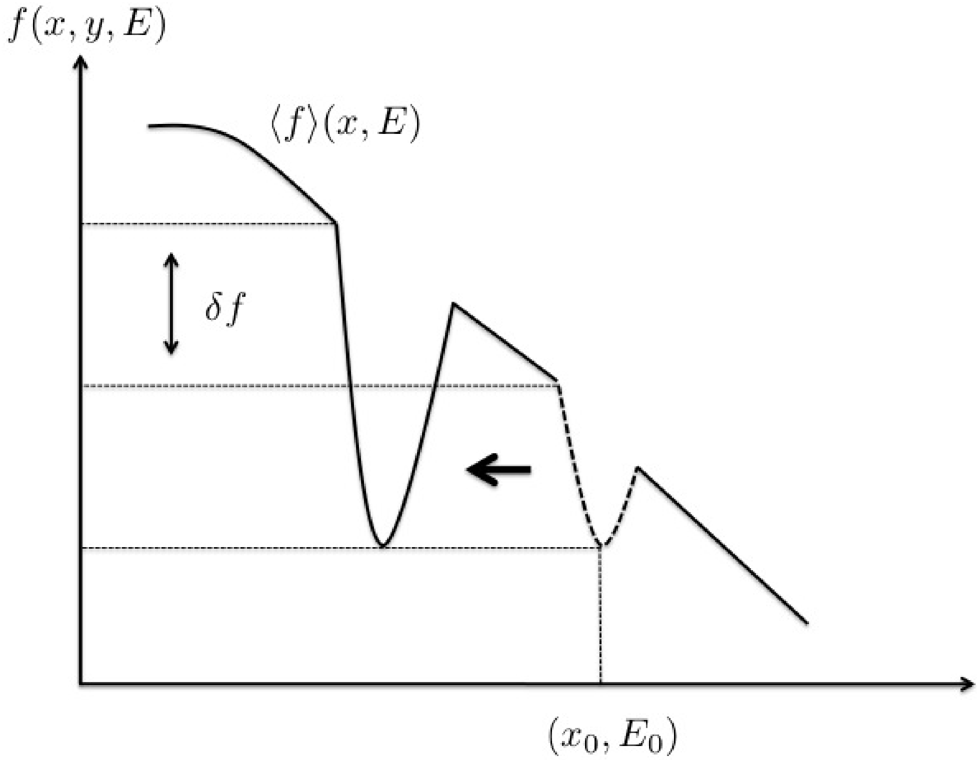

Figure 1. Illustration of a single structure in phase space density and its growth. Initially, a single structure is located at a position (x_0, E_0), $f = \langle f\rangle|_0 + f_H|_0$. By scattering off electrons and polarization charge, the structure can climb against the background gradient. Since $df/dt = 0$, the total f is conserved along the trajectory and thus the depth grows by δf.

phase space structure can drive *poloidal* flows.[25, 26, 27]. Thus the righthand side of Eq.5 is tied to the evolution of poloidal flows. Then, Eq.5 overall relates toroidal momentum transport to poloidal flow evolution, through the dynamics of a single structure in phase space. *The bottom line of this simple calculation is that a single phase space structure can convert poloidal flow momentum into toroidal flow momentum.*

3. Conversion of poloidal flows into toroidal flows by granulations

Given a single phase space structure can convert poloidal momentum into toroidal momentum as demonstrated above, now we turn to the case of an ensemble of structures, i.e. granulations, in trapped ion resonance driven turbulence. Once formed, granulations induce the effect of discreteness in plasmas[23, 24]. This effect can be modeled as incoherent fluctuation[17] in phase space density fluctuation, $\delta f = -(e\tilde{\phi}/T_i) + \delta h = -(e\tilde{\phi}/T_i) + \delta h_c + \widetilde{\delta h}$. Here, δh is a non-adiabatic part of phase space density fluctuation, $\delta h^c = R\tilde{\phi}$ is the coherent response of δh to $\tilde{\phi}$, $R = i(\omega - \omega_{Di}E + i/\tau_c)^{-1}$ is a propagator for the model used in this paper, and $\widetilde{\delta h}$ is an incoherent fluctuation due to granulations. The incoherent fluctuation cannot be written as a response to the fluctuating field $\tilde{\phi}$. $\widetilde{\delta h}$ physically describes the population of correlated resonant ions, and gives rises to discreteness in the system. This term is analogous to the incoherent fluctuation induced by particle discreteness in the test particle model near the thermodynamic equilibrium[30], albeit now the effect of discreteness is incorporated into *turbulent* plasmas with granulations. By retaining the incoherent fluctuation due to granulations, the electrostatic potential $\tilde{\phi}$ is made self-consistent by accounting for the gyrokinetic Poisson equation:

$$\hat{\epsilon}(\mathbf{k}, t)\frac{e\tilde{\phi}_{\mathbf{k}}(t)}{T_i} = \int d^3v \widetilde{\delta h}_{\mathbf{k}}(t). \tag{6}$$

where $\hat{\epsilon}(\mathbf{k}, t)$ is the plasma dielectric function[30, 26]. This suggests that the electrostatic fluctuation can be supported by two parts:

$$\frac{e\tilde{\phi}_{\mathbf{k}}(t)}{T_i} = \frac{e\tilde{\phi}_{\mathbf{k}}}{T_i}\exp(-i\omega_{\mathbf{k}}t) + \hat{\epsilon}^{-1}(\mathbf{k}, t)\int d^3v \widetilde{\delta h}_{\mathbf{k}}(t). \tag{7}$$

Here, the first term is associated with normal modes in the system with $\epsilon(\mathbf{k}, \omega_{\mathbf{k}}) = 0$, while the latter is supported by granulations. Hereafter, to focus on the effects of granulations, we only retain fluctuations supported by granulations, by assuming weakly unstable or saturated modes with $\exp(-i\omega_{\mathbf{k}}t) \to 0$ as $t \to \infty$. In this case, potential fluctuation is supported by Cerenkov emission from turbulent plasma granulations.

By retaining the effect of granulations, mean field evolution of phase space density is obtained as follows[25, 26]:

$$\partial_t \langle f \rangle + \partial_r \langle \tilde{v}_r \delta f \rangle = 0 \tag{8}$$

where $\langle ... \rangle$ denotes the zonal average. The total flux has two parts from the coherent and incoherent response $\langle \tilde{v}_r \delta f \rangle = \langle \tilde{v}_r \delta h^c \rangle + \langle \tilde{v}_r \widetilde{\delta h} \rangle$. The flux by the coherent response leads to quasilinear-like diffusive term, $\langle \tilde{v}_r \delta h^c \rangle = -D_{turb} \langle f \rangle'$ where D_{turb} is the turbulent diffusivity. The flux by the incoherent response leads to dynamical friction[25, 26, 28], $\langle \tilde{v}_r \widetilde{\delta h} \rangle = J_{i,e} + J_{i,pol}$ where

$$J_{i,e} = -\sum_{k\omega} k_\theta \rho_i v_{thi} \frac{\mathrm{Im}\epsilon_e}{|\epsilon(k,\omega)|^2} \left\langle \frac{\widetilde{\delta n}}{n_0} \widetilde{\delta h}^* \right\rangle_{k\omega} \tag{9}$$

$$J_{i,pol} = -\sum_{k\omega} k_\theta \rho_i v_{thi} \frac{(-2\rho^2 k_r)}{|\epsilon(k,\omega)|^2} \left\langle \frac{\widetilde{\delta n}}{n_0} \partial_r \widetilde{\delta h}^* \right\rangle_{k\omega} \tag{10}$$

Here, $\mathrm{Im}\epsilon$ is the imaginary part of the plasma dielectric function[25]. $\mathrm{Im}\epsilon_e \propto \nu_e^{-1}$ is due to collisional dissipation in electrons. $\mathrm{Im}\epsilon_{pol} \propto \mathrm{Im}\nabla_\perp^2 \propto k_r \partial_r$ originates from meso scale spatial variation in turbulent wake[25, 26], as it is obtained by replacing $k_r \to k_r - i\partial_r$, where ∂_r acts on the envelope of turbulent fluctuation. As these two terms are proportional to $\mathrm{Im}\epsilon$, these terms are associated with dynamical friction[23, 24]. Note that in the presence of granulations, mean field evolution takes a Lenard-Balescu form with diffusion term and dynamical friction[19, 20, 25, 26, 28]. This is in contrast to mean field evolution by plasma turbulence *without* the effect from granulations, which only involves quasilinear diffusion.

Granulations can exert dynamical friction to drive toroidal flows. To discuss the effect of dynamical friction on toroidal flows, first we note that the flow supported by trapped ions is obtained by averaging toroidal precession of each individual particles.

The averaging leads to a macroscopic flow $\langle v_\zeta \rangle = \int d^3v v_{Di}(E/T_i)(B_\zeta/B_\theta)\langle f_i \rangle = (3/2)v_{Di}(q/\epsilon_0)$. The macroscopic flow can be disturbed by granulations. This is since granulations with microscopic momentum $v_{Di}(E/T_i)(q/\epsilon_0)$ are dragged via dynamical friction. This process leads to a re-distribution of momentum and can distort the macroscopic toroidal flows. The process is formulated by taking $v_{Di}(E/T_i)(q/\epsilon_0)$ moment of the mean field evolution for $\langle f \rangle$ as:

$$\partial_t \langle v_\zeta \rangle + \partial_r \langle \tilde{v}_r \tilde{v}_\zeta \rangle = 0 \tag{11}$$

Here $\langle \tilde{v}_r \tilde{v}_\zeta \rangle \equiv \int d^3v v_{Di}(E/T_i)(q/\epsilon_0)\langle \tilde{v}_r \delta f \rangle$ is the radial flux of toroidal momentum or Reynolds stress on toroidal flow. Using the above results, we find that the Reynolds stress consists of two parts:

$$\begin{aligned} \langle \tilde{v}_r \tilde{v}_\zeta \rangle = & \int d^3v v_{Di}(E/T_i)(q/\epsilon_0)(-D_{turb}\langle f \rangle') \\ & + \int d^3v v_{Di}(E/T_i)(q/\epsilon_0)(J_{i,e} + J_{i,pol}) \end{aligned} \tag{12}$$

where D_{turb} is the turbulent diffusivity. The first piece is due to turbulent diffusion and appears in quasilinear analysis. This term can contain both the diagonal term (anomalous viscosity) and off-diagonal terms[5, 6] such as the residual stress[8], since $\langle f \rangle' = \langle f \rangle'[\nabla \langle V_\zeta \rangle, \nabla \langle n \rangle, \nabla \langle T_i \rangle]$. Since this term appears in the conventional quasilinear mean field theory with weak resonance[7], hereafter we focus on the contribution from the second piece in Eq.12. This piece ($\equiv \Pi_{r\zeta}^{gran}$) arises due to dynamical friction exerted on turbulent granulations. This effect can only be recovered by retaining the effect of granulations[23, 24]. This term is specific for collisionless plasma turbulence, where phase space dynamics plays important roles in determining behavior of the system. Clearly, $\Pi_{r\zeta}^{gran}$ shows that turbulent granulations can drive toroidal flows by exerting the Reynolds stress.

$\Pi_{r\zeta}^{gran}$ can be further calculated as follows. Since turbulent granulations are resonating with trapped ion ITG, we have $\omega_{Di}(E/T_i) \simeq \sqrt{2\epsilon_0}\omega_{*i}$. This allows us to perform the velocity integral to give

$$\Pi_{r\zeta}^{gran} \simeq \sqrt{2\epsilon_0} v_{*i} \frac{q}{\epsilon_0} \frac{\Gamma_e^{turb}}{n_0} + \sqrt{2\epsilon_0} v_{*i} \frac{q}{\epsilon_0} \frac{\partial_r \langle \tilde{v}_r \tilde{v}_\theta \rangle}{\omega_{ci}} \left(1 + 1.6 \frac{q^2}{\sqrt{\epsilon_0}}\right) \quad (13)$$

In order to write Eq.13 in terms of experimentally measurable quantities, we use simplified momentum balance relations $\partial_t \langle v_\zeta \rangle + \partial_r \Pi_{r\zeta}^{gran} = 0$ and $(1 + 1.6q^2/\sqrt{\epsilon_0})\partial_t \langle v_\theta \rangle + \partial_r \langle \tilde{v}_r \tilde{v}_\theta \rangle = 0$[29] to obtain:

$$\frac{\partial \langle v_\zeta \rangle}{\partial t} = M_{\zeta \leftarrow p}^{gran} \frac{\partial \langle v_\theta \rangle}{\partial t} \quad (14)$$

The coefficient is defined as

$$M_{\zeta \leftarrow p}^{gran} \equiv \sqrt{2} \frac{q v_{*i} \partial_r \Gamma_e^{turb}}{\sqrt{\epsilon_0} n_0 \partial_r \langle \tilde{v}_r \tilde{v}_\theta \rangle} \left(1 + 1.6 \frac{q^2}{\sqrt{\epsilon_0}}\right) + \sqrt{2} \frac{q v_{*i}}{\sqrt{\epsilon_0} L_I \omega_{ci}} \left(1 + 1.6 \frac{q^2}{\sqrt{\epsilon_0}}\right)^2 \quad (15)$$

Here L_I is the scale length of turbulent intensity, which is typically in the range of mesoscale. Eq.14 describes interference between poloidal and toroidal flows, mediated via turbulent granulations. The process is depicted in Fig.2. When poloidal momentum changes, $E \times B$ vortices in the poloidal cross section are tilted to produce $\langle \tilde{v}_r \tilde{v}_\theta \rangle$. This change in turn drags trapped ion granulations in the radial direction. Overall, the change in poloidal momentum is transferred to the change in toroidal momentum. In particular, in the absence of a boundary term, Eq.13 describes the generation of zonal flows in the toroidal direction. Thus we may alternatively say that zonal flows in the poloidal direction are converted into zonal flows in the toroidal direction.

Quantitatively, the conversion coefficient (Eq.15) may be evaluated as follows. First, we note that the typical scale of toroidal zonal flows would be the meso scale, as poloidal zonal flows have the typical scale $L_I \sim \sqrt{\rho_i a}$ where a is the minor radius of plasmas.

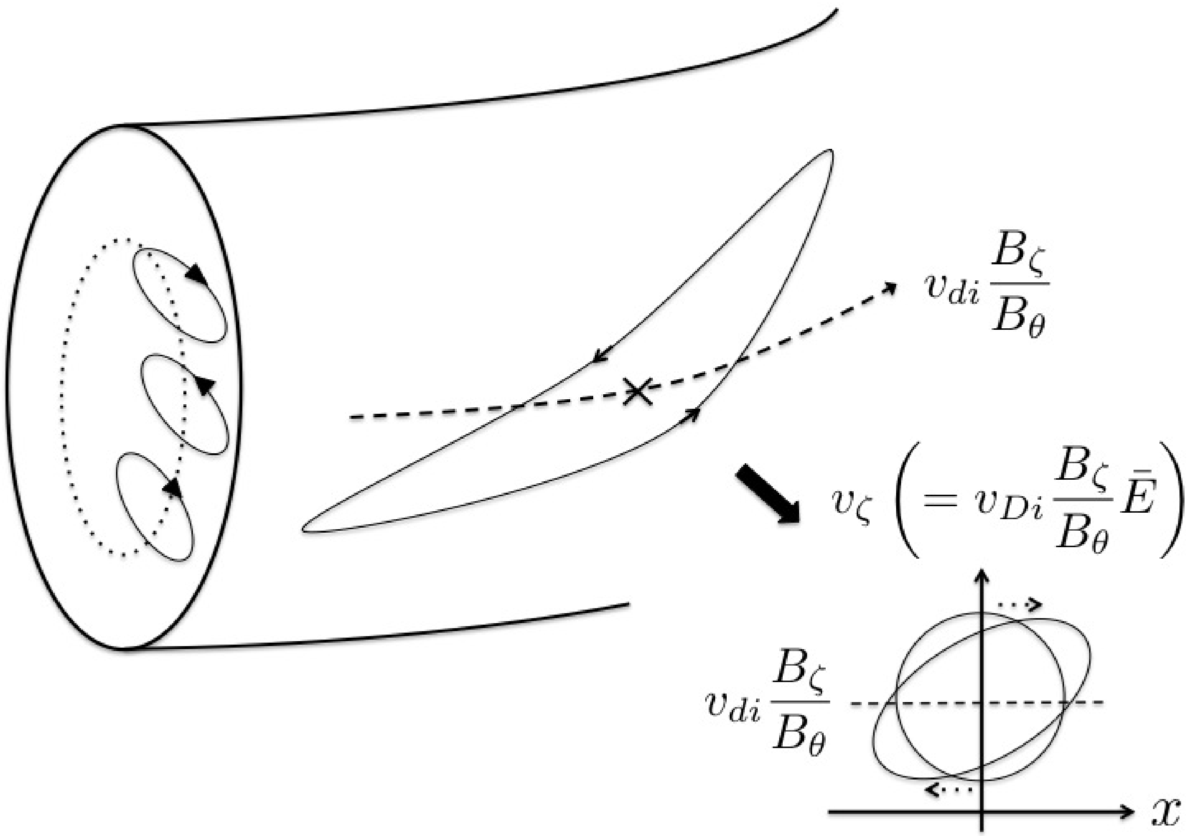

Figure 2. A schematic diagram for interference of poloidal and toroidal momenta via granulations (plasma phase space eddys). When poloidal momentum changes, $E \times B$ vortices in the poloidal plane are tilted to produce Reynolds stress. On the other hand, the Reynolds stress can drag toroidally precessing turbulent granulations (correlated resonant ions) by exerting dynamical friction $J_{i,pol}$. This can cause transport of toroidal momentum and thus can drive toroidal flows. In $(x, v_\zeta \propto E)$ phase plane, the process appears as a deformation of granulation vortex.

Secondly, by assuming a typical turbulence level of order of mixing length estimate, we have:

$$|M^{gran}_{\zeta\leftarrow p}| \sim \frac{\sqrt{2}q\rho_*}{\sqrt{\epsilon_0}}\left(1+1.6\frac{q^2}{\sqrt{\epsilon_0}}\right) \times\left(1+\sqrt{\rho_*}\left(1+1.6\frac{q^2}{\sqrt{\epsilon_0}}\right)\right) \tag{16}$$

Here $\rho_* \equiv \rho_i/a$. For example, using numerical values of $\epsilon_0 \sim 1/3$, $\rho_* \sim 1/200$, $q \sim 3$, we have $|M^{gran}_{\zeta\leftarrow p}| \sim 2.69$. Thus, when poloidal zonal flows accelerate, an acceleration with the same order of magnitude can appear in the toroidal direction as well.

The mechanism presented here can be evaluated experimentally. For example, poloidal Reynolds forcing can be measured by Langmuir probes, while toroidal acceleration can be measured by Mach probes. By taking the correlation between poloidal forcing and toroidal acceleration, we may be able to identify interference between poloidal flows and toroidal flows. In order to further clarify specific mechanisms behind the conversion process (i.e. either by granulations or by acoustic coupling[12], etc), we can evaluate a conversion coefficient of the process, $\partial_t\langle v_\zeta\rangle/\partial_r\langle\tilde{v}_r\tilde{v}_\theta\rangle$. We can compare the parameter dependence of the result with the conversion matrix derived here (Eq.16). Though indirect, such a procedure may be used to measure the effect of turbulent plasma granulations. As a caveat, we note that the error associated with probe measurements could be large. It may require more detailed, quantitative error analysis of probe measurements to determine whether the error is significant enough to make experiments inaccessible or not. This is beyond the scope of the paper and will be discussed elsewhere.

Up to this point, we discussed that turbulent granulations can drive toroidal flows by converting poloidal momentum into toroidal momentum. In this sense, the mechanism developed here is analogous to that by torque exerted by radial current, which can also convert poloidal flows into toroidal flows. This mechanism does not involve $k_\parallel$ coupling, thus can be at work in trapped ion turbulence. To distinguish the two processes, here we give a quantitative estimate of plasma parameters for the conversion by phase space structures, i.e. $\Pi^{gran}_{r\zeta}$, to be a dominant mechanism. To begin with, we note that the torque exerted by radial current can drive poloidal and toroidal flows as $\partial_t\langle v_\zeta\rangle \cong \langle J_r\rangle B_\theta/(m_i nc)$ and $(1+1.6q^2/\sqrt{\epsilon_0})\partial_t\langle v_\theta\rangle = -\langle J_r\rangle B_\zeta/(m_i nc)$. Then we have

ρ_*	1/100	1/200	1/500	1/1000
$\lvert M^{gran}_{\zeta\leftarrow p}\rvert$	6.9	2.7	0.82	0.35
$\lvert M^{gran}_{\zeta\leftarrow p}/M^{current}_{\zeta\leftarrow p}\rvert$	2.4	0.94	0.29	0.12

Table 1. The conversion matrix and its relative value for different values of ρ_*, with $\epsilon_0 = 1/3$, $q = 3$.

ρ_*	1/100	1/200	1/500	1/1000
$\lvert M^{gran}_{\zeta\leftarrow p}\rvert$	69.1	25.7	7.1	2.8
$\lvert M^{gran}_{\zeta\leftarrow p}/M^{current}_{\zeta\leftarrow p}\rvert$	14.7	5.5	1.5	0.59

Table 2. The conversion matrix and its relative value for different values of ρ_*, with $\epsilon_0 = 1/3$, $q = 5$.

$\partial_t \langle v_\zeta \rangle = M^{current}_{\zeta\leftarrow p} \partial_t \langle v_\theta \rangle$ where

$$M^{current}_{\zeta\leftarrow p} = -\left(1 + \frac{1.6q^2}{\sqrt{\epsilon_0}}\right)\frac{\epsilon_0}{q} \tag{17}$$

We can compare the two conversion mechanisms. To do so, we have

$$\frac{|M^{gran}_{\zeta\leftarrow p}|}{|M^{current}_{\zeta\leftarrow p}|} \sim \frac{\sqrt{2}q^2\rho_*}{\epsilon_0^{3/2}}\left(1 + \sqrt{\rho_*}\left(1 + 1.6\frac{q^2}{\sqrt{\epsilon_0}}\right)\right) \tag{18}$$

For example, if we use the same parameter used above, we have $|M^{current}_{\zeta\leftarrow p}| \sim 2.9$ and $|M^{gran}_{\zeta\leftarrow p}|/|M^{current}_{\zeta\leftarrow p}| \sim O(1)$. Thus the both mechanisms can be comparable. Alternatively, if we use $q \sim 5$, $\rho_* \sim 1/100$, and $\epsilon_0 \sim 1/3$, we have $|M^{gran}_{\zeta\leftarrow p}|/|M^{current}_{\zeta\leftarrow p}| \sim 14.7$, which means the conversion by granulations is more effective. Table 1 and 2 summarize the value of the conversion matrix for several q and ρ_*. As we can see, the conversion by phase space structures is more effective for larger q and larger ρ_*. This tendency is manifested in Eq.18 as well. As $\rho_* \propto a^{-1}$ is related to the size of devices, (i.e. larger machines have smaller ρ_*, and vice versa), high q region of smaller tokamaks would be more accessible to measure the conversion process by phase space structures.

4. Conclusion and discussion

In this paper, we presented a theory to describe conversion of poloidal momentum into toroidal momentum by phase space structures in trapped ion resonance driven turbulence. Principal results of the paper are:

(i) Toroidal flows can be accelerated by phase space structures. In particular, we considered the simplified case of a single structure in phase space(Fig.1). When the structure grows, the growth can cause toroidal momentum flux, Eq.5. Since the right hand side describes evolution of poloidal pseudomomentum of the structure, and it is related to the acceleration of poloidal flow, toroidal flow can be driven by converting poloidal flow momentum via a structure in phase space.

(ii) Poloidal flows can be converted into toroidal flows via an ensemble of stochastic granulations, $\partial_t \langle v_\zeta \rangle = M^{gran}_{\zeta \leftarrow p} \partial_t \langle v_\theta \rangle$ (Fig.2). The conversion matrix is calculated as Eqs.15 and 16. For typical parameters in present devices, $|M^{gran}_{\zeta \leftarrow p}| \sim 2.69$. Thus when poloidal zonal flows accelerate a quantitatively comparable acceleration can appear in the toroidal direction. The ratio of the conversion by granulations to the conversion by the radial current is given as Eq.18. Our result indicate that it may be easier to distinguish and measure the conversion by phase space structures in higher q, smaller machines (i.e. larger ρ_*).

While we addressed the role of granulations in converting poloidal and toroidal flows with one another, the drive of toroidal flows discussed here can be understood as a part of residual stress in toroidal momentum transport in magnetic fusion community.[7, 8, 9] To elaborate this point, it is useful to recall that toroidal flow evolution equation generally takes the form:

$$\partial_t \langle v_\zeta \rangle + \frac{\partial}{\partial r} \left(-\chi_\zeta \frac{\partial \langle v_\zeta \rangle}{\partial r} + V_{pinch} \langle v_\zeta \rangle + \Pi^{res}_{r\zeta} \right) = a_\parallel \tag{19}$$

Here χ_ζ is the anomalous viscosity, V_{pinch} is the pinch[31, 32, 33], $\Pi_{r\zeta}^{res}$ is the residual stress[7, 8, 9], and $a_\parallel$ is a parallel acceleration[7, 34] which acts as a local source. $\Pi_{r\zeta}^{res}$ and $a_\parallel$ are required to spin-up plasmas from rest[8, 9]. Physically, $a_\parallel$ originates from acoustic coupling, which includes refractive force[7] or turbulent acceleration[34]. Whereas, the residual stress has several contributions[7, 8, 9]:

$$\Pi_{r\zeta}^{res} = \Pi_{r\zeta}^{RP} + \Pi_{r\parallel}^{wave} + \frac{c^2}{B^2}\langle \tilde{E}_r \tilde{E}_\parallel \rangle - \frac{B_\theta}{B_\zeta}\langle \tilde{v}_r \tilde{v}_\theta \rangle \tag{20}$$

Here, the first is the momentum flux carried by resonant particles, the second is the momentum flux carried by drift wave turbulence[7], the third term is the polarization stress[35, 36], and the fourth is the projection of perpendicular force[37], or equivalently, the $\langle \mathbf{J} \rangle \times \mathbf{B}$ torque. The first term, i.e. the momentum flux due to resonant particles can have two contributions, both from quasilinear like flux[7] and from dynamical friction, i.e. $\Pi_{r\zeta}^{RP} = (\Pi_{r\zeta}^{RP})_{QL} + (\Pi_{r\zeta}^{RP})_{DF}$. It is this $(\Pi_{r\zeta}^{RP})_{DF}$ that has been calculated in this work, i.e. $\Pi_{r\zeta}^{gran}$. In particular, we showed that $\Pi_{r\zeta}^{gran}$ can be comparable in magnitude to the projection of perpendicular force by calculating $|M_{\zeta\leftarrow p}^{gran}|/|M_{\zeta\leftarrow p}^{current}|$ in Eq.18. Thus momentum transport by phase space structures can contribute to intrinsic torque. The contribution from $(\Pi_{r\zeta}^{RP})_{DF}$ would be especially important when phase space structures can form in turbulence due to strong wave-particle resonance, such as trapped ion or trapped electron precession resonances. Relevant applications to toroidal momentum transport phenomenology, e.g. the competition between the effect developed here and other components in intrinsic torque in CTEM turbulence with acoustic coupling, implications to scaling of flows to plasma parameters, etc, will be pursued in future.

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References

[1] A. Fujisawa, K. Itoh, H. Iguchi, K. Matsuoka, S. Okamura, A. Shimizu, T. Minami, Y. Yoshimura, K. Nagaoka, C. Takahashi, M. Kojima, H. Nakano, S. Oshima, S. Nishimura, M. Isobe, C. Suzuki, T. Akiyama, K. Ida, K. Toi, S. I. Itoh, and P. H. Diamond. *Phys. Rev. Lett.*, 93:165002, 2004.

[2] P. H. Diamond, S.-I. Itoh, K. Itoh, and T.S. Hahm. *Plasma Phys. Control. Fusion*, 47:R35, 2005.

[3] H. Bilari, P. H. Diamond, and P. W. Terry. *Phys. Fluids B*, 2:1, 1990.

[4] K. H. Burrell. *Phys. Plasmas*, 4:1499, 1997.

[5] K. Ida, Y. Miura, T. Matsuda, K. Itoh, S. Hidekuma, S. I. Itoh, and JFT-2M Group. *Phys. Rev. Lett.*, 74:1990, 1995.

[6] J. E. Rice, J. W. Hughes, P. H. Diamond, Y. Kosuga, Y. A. Podpaly, M. L. Reinke, M. J. Greenwald, Ö. D. Gürcan, T. S. Hahm, A. E. Hubbard, E. S. Marmar, C. J. McDevitt, and D. G. Whyte. *Phys. Rev. Lett.*, 106:215001, 2011.

[7] P. H. Diamond, C. J. McDevitt, Ö. D. Gürcan, T. S. Hahm, and V. Naulin. *Phys. Plasmas*, 15:012303, 2008.

[8] P. H. Diamond, C. J. McDevitt, Ö. D. Gürcan, T. S. Hahm, W. X. Wang, E. S. Yoon, I. Holod, Z. Lin, V. Naulin, and R. Singh. *Nucl. Fusion*, 49:045002, 2009.

[9] P. H. Diamond, Y. Kosuga, O". D". Gurcan, C. J. McDevitt, T. S. Hahm, N. Fedorczak, J. E. Rice, W. X. Wang, S. Ku, J. M. Kwon, G. Dif-Pradalier, J. Abiteboul, L. Wang, W. H. Ko,

Y. J. Shi, K. Ida, W. Solomon, H. Jhang, S. S. Kim, S. Yi, S. H. Ko, Y. Sarazin, R. Singh, and C. S. Chang. accepted to Nucl. Fusion, 2013.

[10] Ö. D. Gürcan, P. H. Diamond, T. S. Hahm, and R. Singh. *Phys. Plasmas*, 14:042306, 2007.

[11] H. K. Moffatt. *Magnetic field generation in electrically conducting fluids.* Cambridge University Press, Cambridge, 1978.

[12] Lu Wang, P. H. Diamond, and T. S. Hahm. *Plasma Phys. Control. Fusion*, 54:095015, 2012.

[13] P. H. Diamond, Ö. D. Gürcan, T. S. Hahm, K. Miki, Y. Kosuga, and X. Garbet. *Plasma Phys. Control. Fusion*, 50:124018, 2008.

[14] B. B. Kadomtsev and O. P. Pogutse. *Reviews of Plasma Physics*, volume 5. Consultants Bureau, 1970.

[15] W. M. Tang, J. C. Adam, and David W. Ross. *Phys. Fluids*, 20:430, 1977.

[16] G. Darmet, Ph. Ghendrih, Y. Sarazin, X. Garbet, and V. Grandgirard. *Communications in Nonlinear Science and Numerical Simulation*, 13:53–58, 2008.

[17] T. H. Dupree. *Phys. Fluids*, 15(2):334, 1972.

[18] T. H. Dupree. *Phys. Fluids*, 25(2):277, 1982.

[19] P. H. Diamond, P. L. Similon, P. W. Terry, C. W. Horton, S. M. Mahajan, J. D. Meiss, M. N. Rosenbluth, K. Swartz, T. Tajima, and R. D. Hazeltine. Theory of two-point correlation for trapped electrons and the spectrum of drift wave turbulence. In *Plasma Physics and Controlled Nuclear Fusion Research 1982*, volume 1, page 259, Vienna, 1983. IAEA.

[20] H. Biglari, P. H. Diamond, and P. W. Terry. *Phys. Fluids*, 31(9):2644, 1988.

[21] P. W. Terry, P. H. Diamond, and T. S. Hahm. *Phys. Fluids B*, 2:9, 1990.

[22] M. Lesur and P. H. Diamond. *Phys. Rev. E*, 87:031101(R), 2013.

[23] T. H. Dupree. *Phys. Rev. Lett.*, 25(12):789, September 1970.

[24] B. B. Kadomtsev and O. P. Pogutse. *Phys. Rev. Lett.*, 25(17):1155, 1970.

[25] Y. Kosuga and P. H. Diamond. *Plasma and Fusion Research*, 5:S2051, 2010.

[26] Y. Kosuga and P. H. Diamond. *Phys. Plasmas*, 18:122305, 2011.

[27] Y. Kosuga and P. H. Diamond. *Phys. Plasmas*, 19:072307, 2012.

[28] Y. Kosuga, P. H. Diamond, L. Wang, Ö. D. Gürcan, and T. S. Hahm. *Nucl. Fusion*, 53:043008, 2013.

[29] M. N. Rosenbluth and F. L. Hinton. *Phys. Rev. Lett.*, 80:724, 1998.

[30] P. H. Diamond, S.-I. Itoh, and K. Itoh. *Modern Plasma Physics Volume1: Physical Kinetics of Turbulent Plasmas.* Cambridge University Press, Cambridge, 2011.

[31] T. S. Hahm, P. H. Diamond, Ö. D̈. Gurcan, and G. Rewoldt. *Phys. Plasmas*, 14:072302, 2007.

[32] A. G. Peeters, C. Angioni, A. Bortolon, Y. Camenen, F. J. Casson, B. Duval, L. Fiederspiel, W. A. Hrnsby, Y. Idomura, T. Hein, N. Kluy, P. Mantica, F. I. Parra, A. P. Snodin, G. Szepesi, D. Strintzi, T. Tala, G. Tardini, P. de Vries, and J. Weiland. *Nucl. Fusion*, 51:094027, 2011.

[33] C. Angioni, Y. Camenen, F. J. Casson, E. Fable, R. M. McDermott, A. G. Peeters, and J. E. Rice. *Nucl. Fusion*, 52:114003, 2012.

[34] Lu Wang and P. H. Diamond. accepted to Phys. Rev. Lett, 2013.

[35] C. J. McDevitt, P. H. Diamond, O". D". Gurcan, and T. S. Hahm. *Phys. Plasmas*, 16:052302, 2009.

[36] C. J. McDevitt, P. H. Diamond, O". D". Gurcan, and T. S. Hahm. *Phys. Rev. Lett.*, 103:205003, 2009.

[37] P. H. Diamond and Y.-B. Kim. *Phys. Fluids B*, 3:1626, 1991.