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CONSEQUENCES OF ISOSPIN AND OTHER CONSERVED QUANTUM NUMBERS FOR COMPOUND NUCLEUS REACTIONS*

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ABSTRACT

Consequences of the conserved isospin quantum number T are developed for compound nucleus reactions with particular emphasis on statistical cross section fluctuations. The formalism is based on the Hauser-Feshbach and Ericson theories. The appropriate autocorrelation function in case of levels of two different isospin values in the compound nucleus contains an interference term between two Lorentzian functions that are associated with the two isospins. The formalism is applied to the statistical reactions $^{54}Cr(p,p)$ and $^{54}Cr(p,\alpha)$ where the coherence width in the proton- and α-channels are markedly different. This is attributed to the decay properties of states with different isospin in

the compound nucleus ^{55}Mn. The formalism developed for the isospin quantum number T can be extended to other quantum numbers. For the case of angular momentum and parity $J\pi$ interesting consequences with respect to heavy ion reactions are discussed.

I. INTRODUCTION

The consequences of isospin conservation are not always explicit in current formulations of the theory of nuclear reactions and are rarely taken into account in the interpretation of experiments. Noticeable exceptions, are of course the theories of isobaric analogue resonances[1] and a few investigations of isospin in direct transfer reactions.[2] An outline of the role of isospin in direct and compound nuclear reactions is given in Refs. 3 and 4.

In the present paper we want to develop the consequences of the isospin quantum number for compound reactions with particular emphasis on statistical cross section fluctuations. In Sec. II the necessary formalism is presented which is based on the Hauser-Feshbach[5] and Ericson[6] theories. A difference between our results for fluctuation phenomena and the results of Ref. 4 is discussed. Section III contains the application of this formalism to the statistical reactions $^{54}Cr(p,p)$ and $^{54}Cr(p,\alpha)$. The data of Refs. 7 and 8 show a difference between the coherence widths measured in the proton and α-channels, respectively. This can now be attributed to the decay properties of states with different isospin in the compound nucleus ^{55}Mn. The results of Sec. III are discussed in Sec. IV. Finally, in Sec. V we conclude with general remarks about the effect of conserved quantum numbers on correlation functions.

II. FORMALISM OF COMPOUND NUCLEUS REACTIONS WITH CONSERVED ISOSPIN

A. Average Cross Sections

Let us begin this section by comparing the theory when the isospin T is strictly a good quantum number with the theory when T is ignored (as is the case in the standard Hauser-Feshbach theory).

The compound or fluctuating cross section is believed to arise from an S-matrix given by a Breit-Wigner expansion

$$S^{J\pi}_{cc'} = \sum \frac{g^{J\pi}_{\lambda c}\, g^{J\pi}_{\lambda c'}}{E_\lambda - E - i\Gamma_\lambda/2} \quad , \qquad (c \neq c') \tag{2.1}$$

and in Hauser-Feshbach theory one considers the $g_{\lambda c}$, $g_{\lambda c'}$ to have random phases and be statistically distributed about zero mean for each total angular momentum and parity. The channel index c as usual represents a vector coupled partial wave of definite total angular momentum J and parity π. For simplicity of presentation we suppress all angular momentum recoupling coefficients which arise in cross section formulas. In this case, the simplest form of the statistical compound nucleus theory, the energy averaged fluctuation cross section is given by

$$\langle \sigma^{J\pi}_{cc'} \rangle \propto \langle |S^{J\pi}_{cc'}|^2 \rangle = \frac{2\pi}{D^{J\pi}} \left\langle \frac{|g^{J\pi}_{\lambda c}|^2 \; |g^{J\pi}_{\lambda c'}|^2}{\Gamma^{J\pi}_\lambda} \right\rangle_\lambda$$

$$= \frac{2\pi}{D^{J\pi}} \frac{\langle |g^{J\pi}_{\lambda c}|^2 \rangle_\lambda \langle |g^{J\pi}_{\lambda c'}|^2 \rangle_\lambda}{\langle \Gamma^{J\pi}_\lambda \rangle_\lambda} W_{cc'} \tag{2.2}$$

wherein the factor $W_{cc'}$ is the width correlation factor and the brackets $\langle\ \rangle$ denote averages over the energy, unless otherwise specified. In terms of transmission coefficients the above expression becomes (we drop the $J\pi$ notation for convenience)

$$\langle |S_{cc'}|^2 \rangle = \frac{\mathcal{T}_c \mathcal{T}_{c'}}{\sum_{c''} \mathcal{T}_{c''}} W_{cc'} \quad , \tag{2.3}$$

with $W_{cc'} \simeq 1$ for $\Gamma \ll D$ (few channels) and $W_{cc'} \simeq 1 + \delta_{cc'}$ for $\Gamma \gg D$ (many channels), where Γ and D are the mean level width and mean level spacing, respectively. The $\mathcal{T}_c$ are usually calculated from the conventional optical potential in channel c i.e.

$$\mathcal{T}_c = 1 - |S_c^{opt}|^2 \quad . \tag{2.4}$$

Let us now suppose that there are levels of two different isospin values T called $T_>$ and $T_<$, that T is strictly conserved in S and that there are two level distributions appropriate to each T-value. This corresponds to the case where a target nucleus with isospin $T_o \neq 0$ is bombarded with protons so that $T_> = T_o + 1/2$ and $T_< = T_o - 1/2$. In this event

$$S_{cc'} = \sum_\nu \frac{g_{\nu c}^{T_<} g_{\nu c'}^{T_<}}{E_\nu^{T_<} - E - i\Gamma_\nu^{T_<}/2} + \sum_\omega \frac{g_{\omega c}^{T_>} g_{\omega c'}^{T_>}}{E_\omega^{T_>} - E - i\Gamma_\omega^{T_>}/2} \tag{2.5}$$

We now calculate $\langle \sigma_{cc'} \rangle$ and obtain (assuming two <u>uncorrelated</u> random distributions)

$$\langle \sigma_{cc'} \rangle \propto \frac{2\pi}{D^{T_<}} \frac{\langle |g_{\nu c}^{T_<}|^2 \rangle_\nu \langle |g_{\nu c'}^{T_<}|^2 \rangle_\nu}{\langle \Gamma_\nu^{T_<} \rangle_\nu} W_{cc'}^{T_<} + \frac{2\pi}{D^{T_>}} \frac{\langle |g_{\omega c}^{T_>}|^2 \rangle_\omega \langle |g_{\omega c'}^{T_>}|^2 \rangle_\omega}{\langle \Gamma_\omega^{T_>} \rangle_\omega} W_{cc'}^{T_>} \tag{2.6}$$

$$= \frac{\mathcal{T}_c^{T_<} \mathcal{T}_{c'}^{T_<}}{\sum_{c''} \mathcal{T}_{c''}^{T_<}} W_{cc'}^{T_<} + \frac{\mathcal{T}_c^{T_>} \mathcal{T}_{c'}^{T_>}}{\sum_{c''} \mathcal{T}_{c''}^{T_>}} W_{cc'}^{T_>} \quad .$$

For convenience, we introduce the obvious short hand notation

$$\langle \sigma_{cc'} \rangle = \langle \sigma_< \rangle + \langle \sigma_> \rangle \quad . \tag{2.7}$$

In this extreme limit we obtain significant differences from the usual Hauser-Feshbach theory. These are as follows:

1) Since the total transmission in channel c is given by unitarity as $\mathcal{T}_c = \mathcal{T}_c^{T_<} + \mathcal{T}_c^{T_>}$, we see by equating eqs. (2.3) and (2.7) that conservation of isospin leads to a width correlation factor $W_{cc'}$ different from those conventionally used. This is not surprising because c and c' are correlated by the requirement that the channels couple to the same T-state. More explicitly

$$W_{cc'} = \left(\frac{\mathcal{T}_c^{T_<} \mathcal{T}_{c'}^{T_<}}{\sum_{c''} \mathcal{T}_{c''}^{T_<}} W_{cc'}^{T_<} + \frac{\mathcal{T}_c^{T_>} \mathcal{T}_{c'}^{T_>}}{\sum_{c''} \mathcal{T}_{c''}^{T_>}} W_{cc'}^{T_>} \right) \left[\frac{\left(\mathcal{T}_c^{T_<} + \mathcal{T}_c^{T_>}\right)\left(\mathcal{T}_{c'}^{T_<} + \mathcal{T}_{c'}^{T_>}\right)}{\sum_{c''} \left(\mathcal{T}_{c''}^{T_<} + \mathcal{T}_{c''}^{T_>}\right)} \right]^{-1} \tag{2.8}$$

which is not typically unity because of terms like $\mathcal{T}_c^{T_<} \mathcal{T}_{c'}^{T_>}$ occurring in the denominator but not in the numerator.

2) The isospin dependent width correlation factors $W_{cc'}^T$ will in the case of charge exchange (i.e. isospin "elastic" scattering) be equal to two (when $\Gamma \gg D$) as in the familiar case of compound-elastic scattering.

3) Transmission coefficients $\mathcal{T}_c^T$ are to be calculated from an isospin optical potential for a given total T. In the limit of pure isospin the proton and its associated charge exchange neutron channel are isobarically equivalent so that a total isospin channel basis c_T can be conveniently adopted. The corresponding transmission coefficients are then obtainable from an isospin conserving nuclear potential V(T) using the relation:

$$\mathcal{T}_c^T = (c|c_T)^2 \; (1 - |S_{c_T}^{opt}|^2) \quad , \tag{2.9}$$

where $(c|c_T)$ is an isospin vector coupling coefficient in channel c and c_T is the resulting "isospin-channel". The quantity $S_{c_T}^{opt}$ is the optical model S-matrix element for a nucleon in the isospin conserving potential.[10] For target nuclei with neutron excess, one cannot have in general $\mathcal{T}_c^{T_<} = \mathcal{T}_c^{T_>}$ so that the conventional Hauser-Feshbach theory is really restricted to situations where T is single-valued. Even then, Eq. (2.3) need not be correct if there are any channels open which have dual isospin since the denominator should express the decay into all channels of the appropriate isospin only. The foregoing is true in the case of strict isospin conservation but can be properly extended[3] to include isospin mixing in the compound nucleus states.

B. Cross Section Fluctuations

As in the previous subsection we compare the theory when isospin T is strictly conserved with the theory when T is ignored (as is the case in the standard theory of Ericson fluctuations).

The two central quantities in fluctuation theory are the variance of cross sections and the mean level width of the compound nucleus. Both quantities are obtained from the autocorrelation function

$$C(\varepsilon) = \frac{\langle \sigma(E+\varepsilon)\sigma(E) \rangle}{\langle \sigma(E) \rangle^2} - 1 \quad . \tag{2.10}$$

The normalized variance is equal to $C(\varepsilon=0)$. In order to evaluate $C(\varepsilon)$ we introduce following Brink and Stephen[11] the correlation function $c(\varepsilon)$ between the

S-matrix elements by

$$\langle S_{cc'}(E+\varepsilon)S^{*}_{cc'}(E)\rangle = c(\varepsilon)\langle|S_{cc'}(E)|\rangle^{2} \quad . \qquad (2.11)$$

From the particular form of the S-matrix (Eq. 2.1) follows

$$c(\varepsilon) = \frac{\Gamma}{\Gamma-i\varepsilon} \quad . \qquad (2.12)$$

Eqs. (2.10) - (2.12) and a general theorem on joint normal distributions[11] yield the Lorentzian shape of the autocorrelation function

$$C(\varepsilon) = \frac{\Gamma^2}{\Gamma^2+\varepsilon^2} \quad . \qquad (2.13)$$

Note, that C(0) = 1, which is true for the case that the number N_{eff} of statistically independent channels[12,13] is equal to one. In general the right hand side of Eq. (2.13) would have to be multiplied by $1/N_{eff}$.

Now we suppose that there are compound nucleus levels with two different isospin values denoted as in the previous subsection as $T_>$ and $T_<$. In general the number of open decay channels will be different for the $T_>$ and $T_<$ states. Hence the ratios $\Gamma_>/D_>$ and $\Gamma_</D_<$ will be different for the two isospins. The meaning of the subscripts at the mean level width and the mean level spacing is obvious. In order to apply fluctuation theory we have to require[9,15]

$$\Gamma_</D_< > 2 \quad \text{and} \quad \Gamma_>/D_> > 2 \quad . \qquad (2.14)$$

Since not only the Γ/D ratios but also the level densities are different for the two isospins the mean level widths $\Gamma_<$ and $\Gamma_>$ will not be the same, in general.

In order to derive the appropriate correlation function in this case we obtain from Eq. (2.5) using the definitions (2.7) the modified version of Eq. (2.11), i.e.,

$$\langle S_{cc'}(E+\varepsilon)S^{*}_{cc'}(E)\rangle = \frac{\Gamma_<}{\Gamma_<-i\varepsilon}\langle\sigma_<\rangle + \frac{\Gamma_>}{\Gamma_>-i\varepsilon}\langle\sigma_>\rangle \quad . \tag{2.15}$$

Inserting Eq. (2.15) into Eq. (2.10) then leads to

$$C(\varepsilon) = \frac{\langle\sigma_<\rangle^2\Gamma_<^2}{\langle\sigma_{cc'}\rangle^2(\Gamma_<^2+\varepsilon^2)} + \frac{\langle\sigma_>\rangle^2\Gamma_>^2}{\langle\sigma_{cc'}\rangle^2(\Gamma_>^2+\varepsilon^2)} + \frac{2\langle\sigma_<\rangle\langle\sigma_>\rangle\Gamma_<\Gamma_>(\Gamma_<\Gamma_>+\varepsilon^2)}{\langle\sigma_{cc'}\rangle^2(\Gamma_<^2+\varepsilon^2)(\Gamma_>^2+\varepsilon^2)} \quad . \tag{2.16}$$

We notice in Eq. (2.16) the existence of an "interference term" between the two Lorentzian functions that are associated with the two isospins. This term guarantees that the variance C(0) remains unity whatever values the quantities $\sigma_<$, $\sigma_>$, $\Gamma_<$ and $\Gamma_>$ take. This result is contrary to Eq. (5.13) of Ref. 4, which would predict a damping of the fluctuations (i.e. $C(0) < 1$) due to isospin conservation. This seems to be physically inconsistent since isospin T is summed coherently and should therefore be treated very much like the other coherently summed conserved quantum numbers J and π. In the latter case it is known that the variance is not decreasing with increasing numbers of J^{π} values unless the cross section is integrated over angle so that the sum over J^{π} becomes incoherent. The sum over T is coherent even if the cross section is integrated over angle; an incoherent sum is obtained only when a sum over projectile charge states is performed.

Using the above method, correlation functions involving correlations between different channels and angles can be calculated as well. It is also possible to generalize these results to the case of isospin mixing (Ref. 3).

III. REANALYSIS OF $^{54}Cr(p,p)$ AND $^{54}Cr(p,\alpha)$ FLUCTUATION EXPERIMENTS

A. General Remarks

In the present section we want to reanalyze the statistical fluctuations observed in the reactions $^{54}Cr(p,p)$ (Ref. 7) and $^{54}Cr(p,\alpha)$ ^{51}V (Refs. 7 and 8). The target nucleus ^{54}Cr has isospin $T_o = 3$ so that in the compound nucleus ^{55}Mn states with isospin $T_< = 5/2$ and $T_> = 7/2$ are formed. Both types of states can decay through proton emission while due to the isospin zero of the α-particle only the $T_<$ states can decay to low lying states in ^{51}V. This simple picture is of course justified only under neglect of isospin mixing. For medium mass compound nuclei there is some evidence that supports this assumption[16]. Among other, the most relevant information with respect to the problems discussed in Sec. II is that the mean coherence width Γ_p determined from the proton decay channels is significantly larger than the width Γ_α, found from the α-decay channel. This points to an isospin dependence of the reaction where $\Gamma_> > \Gamma_<$.

In part B of this section, the experimental results are summarized. We then, in subsection C, determine the quantities $\Gamma_<$ and $D_<$ from the (p,α)-experiment. In subsection D we derive the ratio $\Gamma_>/D_>$ as well as $\sigma_>$ and $\sigma_<$ from the compound nucleus cross sections in the (p,p)-experiment. Finally, in subsection E the coherence width $\Gamma_>$ is derived by application of Eq. (2.16).

B. Experimental Results

In the reactions $^{54}Cr(p,p)$ and $^{54}Cr(p,\alpha)$ the mean excitation energy in the compound nucleus ^{55}Mn was $E_x = 17.6$ MeV. The mean coherence width was found to be $\Gamma_p = 13.9 \pm 0.7$ keV. Note that in Table 11 of Ref. 7 the results from the Fourier analysis of the excitation functions are quoted. Here, we rather refer to the results obtained from the autocorrelation function which are given in Tables 5a and 5b of Ref. 7. No errors are quoted there. We estimated them using the formula given under "FRD errors of correlation functions" in Table 2 of Ref. 7. Some results of the $^{54}Cr(p,\alpha)^{51}V$ reaction are reported in Ref. 7. An extensive investigation of this reaction is published in Ref. 8. In Ref. 7 the mean coherence width was found to be $\Gamma_\alpha = 8.9 \pm 0.8$ MeV (see Table 5c of Ref. 7 and the above remarks concerning Γ_p). This is in fair agreement with Ref. 8, where after linear extrapolation to the mean excitation energy of 17.6 MeV in ^{55}Mn we obtain the value of $\Gamma_\alpha = 8.2 \pm 0.6$ keV. (Again there is no error given in Ref. 4. We estimated it as above.) In the following, we adopt the value $\Gamma_\alpha = 8.2 \pm 0.6$ keV of Ref. 8 since it is the more complete investigation of this reaction.

The total level density $\rho = 1/D$ from the (p,α) experiment of Ref. 8 has also to be reduced to the excitation energy $E_x = 17.6$ MeV. We can do this by estimating $d\rho/dE_x$ from Fig. 9 of Ref. 8. The result is

$$\rho(E_x = 17.6\ \text{MeV}) = 4.06 \times 10^5\ \text{MeV}^{-1}\ {}^{+\,100\%}_{-\ \ 20\%} \quad .$$

The high upper limit of the error is somewhat arbitrary, but has been introduced because in Ref. 8 there is no determination of the direct reaction contribution to the fluctuating cross sections. However, Table 5c of Ref. 7

indicates that there is some direct (p,α) reaction competing with the fluctuating one. This is in agreement with similar findings in the $^{45}Sc(p,\alpha)$ reaction[17]. The lower error limit on ρ results from the uncertainties in the absolute (p,α) cross section and the mean level width of Ref. 8.

From the total level density ρ we have to calculate the density of spin 1/2 levels $\rho_{1/2}$ in order to make the comparison with the results of Ref. 7 possible. Using the usual spin dependence of nuclear level densities (see e.g. Ref. 18) we may write

$$\rho = (1/2)\rho_{1/2} \exp\left(3/(8\sigma_C^2)\right) \sum_J (2J+1) \exp\left(-J(J+1)/2\sigma_C^2\right). \tag{3.1}$$

Here σ_c is the spin cut off factor of the compound nucleus. The sum in Eq. (3.1) may be approximated by

$$\sum_J (2J+1) \exp\left(-J(J+1)/2\sigma_C^2\right) \cong 2\sigma_C^2 \tag{3.2}$$

so that

$$\rho \cong \rho_{1/2} \exp\left(3/(8\sigma_C^2)\right)\sigma_C^2 \quad . \tag{3.3}$$

With $\sigma_C^2 = 20$ from Ref. 8 it follows that

$$\rho_{1/2} = 1/D_{1/2} = 2.0 \times 10^4 \ \mathrm{MeV}^{-1} \begin{matrix} +100\% \\ -\ 20\% \end{matrix} \quad .$$

C. Mean Level Width $\Gamma_<$ and Mean Level Spacing $D_<$ from the (p,α)-Experiment

The autocorrelation function in the (p,α)-experiment determines $\Gamma_\alpha = \Gamma_<$ since the second and the third terms in Eq. (2.16) vanish.

The mean level distance $D_{1/2,<}$ of the states with isospin $T_<$ is, however, not equal to the quantity $D_{1/2}$ from the previous subsection. We derive $D_{1/2,<}$ from the averaged fluctuating (p,α) cross sections which according to Eq. (2.6) can be written as

$$\langle \sigma_{p\alpha} \rangle = \frac{\mathcal{T}_p^{T_<} \mathcal{T}_\alpha^{T_<}}{\sum_{c''} \mathcal{T}_{c''}^{T_<}} , \tag{3.4}$$

where we have made use of the facts that the reaction proceeds only through the isospin $T_<$ states and that the width correlation factor $W_{p\alpha}^{T_<}$ is unity. The transmission coefficients $\mathcal{T}_p^{T_<}$ can be expressed as (Eq. (2.9))

$$\mathcal{T}_p^{T_<} = (T_o, 1/2, T_o, -1/2 | T_o-1/2, T_o-1/2)^2 (1 - |S_{pT_<}^{opt}|^2 . \tag{3.5}$$

We have

$$(T_o, 1/2, T_o, -1/2 | T_o-1/2, T_o-1/2)^2 = \frac{2T_o}{2T_o+1} . \tag{3.6}$$

In the α-channel, the vector coupling coefficient is unity, hence

$$\mathcal{T}_\alpha^{T_<} = 1 - |S_{\alpha T_<}^{opt}|^2 . \tag{3.7}$$

Substituting as usual[19]

$$\frac{2\pi \Gamma_<}{D_<} = \sum_{c''} T_{c''}^{<} , \tag{3.8}$$

we obtain from Eq. (3.4)

$$\langle \sigma_{p\alpha} \rangle = \frac{2T_o}{2T_o+1} \frac{D_<}{2\pi\Gamma_<} \tau_p^{T_<} \tau_\alpha^{T_<} , \tag{3.9}$$

where

$$\tau_c^T = 1 - |S_{cT}^{opt}|^2 \tag{3.10}$$

is the transmission coefficient calculated in the usual way from optical model phase shifts. We assume that τ_c^T does not explicitly depend on the isospin T, but only on geometry and channel energy, a fact which is true e.g. in the framework of the sharp cut-off model. (For a more detailed treatment, see, however, the discussion following Eq. (2.9)). We shall therefore drop the superscript on τ that refers to the isospin. Expression (3.6) is of course schematic in that it does not exhibit angular momentum conservation. The full formula may be found in Refs. 8, 20, and 21 by help of which the expression $\sigma_{p\alpha}^{HF}$ in the following equation is defined

$$\langle \sigma_{p\alpha} \rangle = \frac{2T_o}{2T_o+1} \frac{D_{1/2,<}}{\Gamma_{1/2,<}} \sigma_{p\alpha}^{HF} \quad . \tag{3.11}$$

The quantity $\sigma_{p\alpha}^{HF}$ is apart from D/Γ the usual Hauser-Feshbach cross section. Actually there is a slight difference between the analyses of Refs. 7 and 8 in that the former allows for a dependence of Γ on total angular momentum J while the latter does not. Because the effect is small in proton induced reactions we disregard this difference here (see Subsection IV. C). We note, however, that the factor $2T_o/(2T_o+1)$ does of course not appear in the expressions of Refs. 7, 8, 20 and 21, because they do not exhibit isospin conservation. Hence, the level spacing $D_{1/2}$, which we derived in Sec. III. B from the total level density ρ given in Ref. 8 is connected with $D_{1/2,<}$ via the relation

$$D_{1/2} = \frac{2T_o}{2T_o+1} D_{1/2,<} \quad . \tag{3.12}$$

Using the result for $\rho_{1/2}$ from Sec. III. B we obtain

$$D_{1/2,<} = 0.058\ \text{keV}\ {}^{+30\%}_{-50\%} \quad .$$

From $\Gamma_{1/2,<}$ and $D_{1/2,<}$ results

$$\Gamma_{1/2,<}/D_{1/2,<} = 141\ {}^{+100\%}_{-\ 20\%} \quad .$$

D. Ratio of $\Gamma_>/D_>$ and the Cross Sections $\langle\sigma_>\rangle/\langle\sigma_<\rangle$

From Eq. (2.6) and the discussion in Sec. III. C follows that the average compound nucleus (p,p)-cross section can be written in the form of Eq. (3.11), namely

$$\begin{aligned}\langle\sigma_{pp}\rangle &= \left[\left(\frac{2T_o}{2T_o+1}\right)^2 \frac{D_{1/2,<}}{\Gamma_{1/2,<}} W_{pp}^{T_<} + \frac{1}{(2T_o+1)^2} \frac{D_{1/2,>}}{\Gamma_{1/2,>}} W_{pp}^{T_>}\right]\sigma_{pp}^{HF} , \\ &= \qquad \langle\sigma_<\rangle \qquad + \qquad \langle\sigma_>\rangle \quad .\end{aligned} \tag{3.13}$$

In analogy to Eq. (3.11), this last equation defines the quantity σ_{pp}^{HF}. In Eq. (3.13) we have made use of the fact that

$$(T_o,\ 1/2,\ T_o,\ -1/2|T_o+1/2,\ T_o-1/2)^2 = \frac{1}{2T_o+1} \quad . \tag{3.14}$$

Both width correlation factors are equal to 2 for elastic scattering and equal to 1 for inelastic scattering. We therefore drop the superscript of W_{pp} referring to the isospin. It might look like $\sigma_>$ is very small compared to $\sigma_<$ because of the factor $(2T_o)^2 = 36$ and thus practically impossible to determine from the last equations. This is, however, not true, since $2\pi\Gamma/D$ is related to the number of open decay channels, which is much larger for the $T_<$ levels than for the $T_>$ levels of the compound nucleus (see Subsec. IV.D). Hence, $D_>/\Gamma_> \gg D_</\Gamma_<$.

We derive the quantity $\langle \sigma_{pp} \rangle / (W_{pp} \sigma_{pp}^{HF})$ from Eq. (3.13) as an average over the results from 10 excitation functions in Tables 5a and 5b of Ref. 7. The weighted average over the last columns of these tables is

$$\frac{\langle \sigma_{pp} \rangle}{W_{pp} H_{pp}(\theta)} = (0.037 \pm 0.017) \text{ mb/sr} \quad ,$$

where the error is taken to be the rms deviation of the results of Ref. 7. Unfortunately, the quantity $H_{pp}(\theta)$ of Ref. 7 is not the same as σ_{pp}^{HF} defined here. Both quantities, however, are related by

$$\sigma_{pp}^{HF} = \lambda\!\!\!^{-}{}_p^2 \exp\left(-3/(8\sigma_C^2)\right) H_{pp}(\theta) \qquad (3.15)$$

with $\lambda\!\!\!^{-}{}_p$ being the wavelength in the proton channel. From this follows

$$\frac{\langle \sigma_{pp} \rangle}{W_{pp} \sigma_{pp}^{HF}} = 0.017 \pm 0.008 \quad .$$

Inserting this result and $\frac{D_{1/2,<}}{\Gamma_{1/2,<}}$ from Sec. III. C into Eq. (3.13) we obtain the result

$$\frac{\Gamma_{1/2,>}}{D_{1/2,>}} = 1.7 \begin{array}{l} +4.6 \\ -0.8 \end{array} \quad .$$

From Eq. (3.13) also follows the ratio

$$\frac{\langle \sigma_> \rangle}{\langle \sigma_< \rangle} = 2.3 \begin{array}{l} +6.3 \\ -1.7 \end{array} \quad .$$

The fact that this ratio is of order unity confirms that the number of open decay channels is smaller for the $T_>$ states than for the $T_<$ states. As discussed above, this effect counteracts the weighing factors resulting from isospin coupling. We need the ratio $\langle\sigma_>\rangle/\langle\sigma_<\rangle$ to calculate $\Gamma_>$ from the autocorrelation function.

E. Coherence Width $\Gamma_>$

The mean coherence width Γ_p has been obtained from an analysis of the excitation functions in the reaction ^{54}Cr(p,p) in terms of a single Lorentzian for the autocorrelation function. In order to decompose Γ_p into its components $\Gamma_<$ and $\Gamma_>$ we first derive a relation between the autocorrelation function with the single Lorentzian and the proper correlation function of Eq. (2.16), which contains two Lorentzian's associated with the two isospins $T_<$ and $T_>$ and an interference term. Equation (2.16) can be rewritten into the form

$$C(\varepsilon) = \frac{\Gamma_<^2}{\Gamma_<^2 + \varepsilon^2} \cdot \frac{\Gamma_>^2}{\Gamma_>^2 + \varepsilon^2} \left(\frac{M_{<>}^2}{M_{<>}^2 + \varepsilon^2}\right)^{-1}$$

$$= C_<(\varepsilon)\ C_>(\varepsilon)\ [C_{<>}(\varepsilon)]^{-1} \qquad (3.16)$$

where

$$M_{<>}^2 = \Gamma_<^2\ \Gamma_>^2 \left[\frac{\langle\sigma_<\rangle\ \Gamma_< + \langle\sigma_>\rangle\ \Gamma_>}{\langle\sigma_<\rangle + \langle\sigma_>\rangle}\right]^{-2} \qquad (3.17)$$

and $C_<(\varepsilon)$ and $C_>(\varepsilon)$ are the autocorrelation functions for each isospin by themselves. If $\Gamma_< \leqslant \Gamma_>$ one has the following inequalities for all ε

$$C_<(\varepsilon) \leqslant C(\varepsilon) \leqslant C_>(\varepsilon) \quad , \qquad \frac{\partial C}{\partial \varepsilon} < 0$$

and

$$\Gamma_< \leqslant M_{<>} \leqslant \Gamma_> \quad .$$

A conventional analysis uses an effective width defined in analogy to Eq. (2.13) via

$$C_{EFF}(\varepsilon) = \frac{\Gamma_{EFF}^2}{\Gamma_{EFF}^2+\varepsilon^2} \quad , \quad \frac{\partial C_{EFF}}{\partial \varepsilon} < 0 \qquad (3.18)$$

and hence yields a width Γ_{EFF} at $C_{EFF}(\varepsilon=\Gamma_{EFF}) = 1/2$. Since in our experiment $\Gamma_p=\Gamma_{EFF}$ we obtain $\Gamma_>$ by introducing[22] Eq. (2.16) or (3.16) into the equation

$$C(\varepsilon=\Gamma_p) = 1/2 \quad .$$

with the known quantities $\Gamma_<$, $\langle\sigma_<\rangle$ and $\langle\sigma_>\rangle$. The result is

$$\Gamma_> = 19.1 {}^{+\,38}_{-\,1.7} \text{ keV} \quad ,$$

where the errors come mainly from the uncertainty in the ratio of $\langle\sigma_>\rangle/\langle\sigma_<\rangle$ derived in the previous subsection.

Combining the results for $\Gamma_>$ and $\Gamma_>/D_>$ one gets

$$D_{1/2,>} = 11.2 {}^{+\,9.0}_{-\,2.0} \text{ keV} \quad ,$$

where we have taken into account that the errors in $\Gamma_>$ and $\Gamma_>/D_>$ are completely correlated.

IV. DISCUSSION

A. Angular Momentum Dependence of Coherence Widths Γ_p and Γ_α

One objection may be raised against the foregoing analysis in that the difference between Γ_p and Γ_α might not only be due to isospin but also to angular momentum effects. According to Fig. 8b of Ref. 8, the angular momentum dependence of the coherence width Γ_J is negligibly small, however, the results in Fig. 20 of Ref. 7 indicate a 25% decrease of Γ_J when the spin of the compound nucleus increases from J = 1/2 to J = 9/2. The experimental quantities Γ_p and Γ_α are weighted averages of the quantities Γ_J with weighing factors given e.g. in Refs. 23 and 24. For the reaction $^{54}Cr(p,\alpha)^{51}V$ these angle-integrated weighing factors are displayed on Fig. 8a of Ref. 8. We have reproduced these results and have also calculated the appropriate weighing factors as a function of angle for both reactions $^{54}Cr(p,\alpha)$ and $^{54}Cr(p,p)$. The weighing factors center around $J \approx 5/2$ and the center changes by no more than 1 $\hbar$ as a function of both, angle and exit channel. Even the rather strong J-dependence of the coherence width given in Ref. 7 would then allow only for a 7% difference between Γ_p and Γ_α. Hence, the experimental difference between Γ_p and Γ_α must be explained as due to isospin effects.

B. Density of $T_>$ - States

In Subsec. III. E we obtain the mean distance $D_{1/2,>}$ of the $T_>$ states or equivalently their level density

$$\rho_{1/2,>} = 1/D_{1/2,>} = 89 \begin{matrix} +20 \\ -30 \end{matrix} \ \mathrm{MeV}^{-1} .$$

The $T_>$ levels in ^{55}Mn are the isobaric analogs of states in ^{55}Cr. The first $T_>$ level in ^{55}Mn is at the excitation energy

$$E_x(\Delta T = 1) = M(^{55}Cr) + \Delta E_c - \delta - M(^{55}Mn) , \quad (4.1)$$

where M designates ground state masses, ΔE_c is the Coulomb energy displacement and δ the neutron-proton mass difference. Evaluating ΔE_c with the formula in Ref. 25 we find $E_x(\Delta T = 1) = 9.96$ MeV. The isobaric analogs of the $T_>$ states in $^{55}Mn(E_x$=17.6 MeV) are found at the excitation energy $U=E_x-E_x(\Delta T=1) = 7.64$ MeV. Our result can be compared to that of Farrel et al[26] who studied s-wave neutron resonance scattering on ^{54}Cr and found 1 resonance per 47 keV between 0 and 400 keV incident energy, corresponding to the range of U = 6.25 to U = 6.65 MeV in ^{55}Cr, so that

$$\rho_{1/2}(U = 6.45 \text{ MeV}) = 2/0.047 = 43 \text{ MeV}^{-1} \quad ,$$

where the factor of 2 has been included, because in Ref. 26 only positive parity states were detected. Scaling this result to our excitation energy U = 7.64 MeV with the help of a level density formula and the parameters given in Eqs. (3) and (4) and Fig. 9 of Ref. 8 yields

$$\rho_{1/2}(U = 7.64) \approx 128 \text{ MeV}^{-1} \quad .$$

This is in satisfactory agreement with our result for $\rho_{1/2,>}$.

C. Applicability of Fluctuation Theory

We should note that our result of $\Gamma_{1/2,>}/D_{1/2,>}$ is at the limit of the applicability of fluctuation theory. Indeed, Moldauer (Ref. 27) and Dallimore and Hall (Ref. 15) have shown, that Γ/D must not be smaller than 2. As noted in Sec. IV. A, however, the most probable compound nucleus spin formed is J = 5/2 and $\Gamma_{5/2,>}/D_{5/2,>}$ is larger than $\Gamma_{1/2,>}/D_{1/2,>}$ by a factor of 2 as can be verified by the respective angular momentum dependence of Γ and D.

D. Modification of the Formula for the Coherence Width

Equation (3.8) is the basis of calculations of the coherence width. More explicit formulations (without inclusion of isospin conservation, however) can, e.g., be found in Refs. 28, 29 and 30. The present paper shows, how isospin effects can be built into the formulae of these references. We note the following points:

i) The transmission coefficients and, hence, the partial decay widths $\Gamma_{o\nu}$ of Ref. 29 (we refer to this paper only as an example) have to be multiplied by the appropriate vector-coupling coefficients according to Eq. (2.9).

ii) The level densities ρ_ν of Ref. 29 are now isospin dependent[31] since they refer to only those levels to which isospin allowed decay is possible. For instance, the $T_<$ - levels can decay via neutron emission to the low-lying states of the residual nucleus, while the $T_>$ - levels cannot. In our example, the density $\rho_{n(T<)}$ - occuring in the neutron partial width of $\Gamma_<$ - is that of the states in ^{54}Mn. The neutron partial width of $\Gamma_>$, however, requires the density $\rho_{n(T_>)}$ of T = 3 states in ^{54}Mn since the isospin allowed neutron decay of the $T_>$ states goes to the analog states of the target. The threshold of this nA - channel, as it is usually called in the theory of isobaric analog resonances, is here at the excitation energy of 17 MeV in the compound nucleus ^{55}Mn: It is, therefore, practically closed in the present experiment. This explains why there are many more open decay channels for the $T_<$ than for the $T_>$ - states as mentioned in subsection III. D.

iii) Densities of levels with isospin one unit greater than that of the ground state (e.g. the above mentioned $\rho_{n(T_>)}$ in ^{54}Mn) can be equated to the level density in the appropriate parent nucleus (^{54}Cr in our example). The same is true for the nuclear temperatures and spin distribution parameters that occur in Eq. (14a) of Ref. 29.

V. CONSEQUENCES OF OTHER CONSERVED QUANTUM NUMBERS ON CORRELATION FUNCTIONS

Fluctuation theory in its existing form assumes that there is no dependence of the coherence width Γ on conserved quantum numbers - an assumption we know to be invalid. It is hence worthwhile noting that the formalism developed in the previous sections for the isospin quantum number T could be extended to other quantum numbers as well. For the case of the conserved quantum numbers angular momentum and parity $J\pi$ we have similar to Eqs. (2.11), (2.12) and (2.15) the correlation function $c(\varepsilon)$ defined via

$$\langle \sigma \rangle \; c(\varepsilon) = \sum_{J\pi,T} \langle \sigma^{J\pi,T} \rangle \; \frac{\Gamma^{J\pi,T}}{\Gamma^{J\pi,T} - i\varepsilon} \tag{5.1}$$

with

$$\langle \sigma \rangle = \sum_{J\pi,T} \langle \sigma^{J\pi,T} \rangle \quad . \tag{5.2}$$

Then the autocorrelation function $C(\varepsilon)$ is

$$C(\varepsilon) = |c(\varepsilon)|^2 \quad . \tag{5.3}$$

The analysis of experimental excitation functions in terms of Eqs. (5.1)-(5.3) allows us to understand why and how Γ_{EFF} defined in Eq. (3.18) for different T-distributions and now extended to include $J\pi$ as well, can be different in different reactions via the "same" compound system not only because of T but also because of different $J\pi$ distributions of $\langle \sigma^{J\pi,T} \rangle$ for the different reactions. For the reactions (p,p'), (p,α) involving mainly projectiles and ejectiles with small mass numbers the effect was found small relative to the T-dependence effect in subsection IV. C. However, the effect of different $J\pi$ distributions clearly is significant in reactions involving heavy ions.

A typical example is the behaviour of fluctuating cross sections and correlation functions for reactions like[32]

$$^{16}O + ^{12}C \longrightarrow ^{28}Si^* \longrightarrow ^{24}Mg + \alpha$$

versus[33] (5.4)

$$^{27}Al + p \longrightarrow ^{28}Si^* \longrightarrow ^{24}Mg + \alpha$$

which may be described quantitatively in terms of the formalism discussed in the present paper. Noticeably, not only are the effective coherence widths different in the $^{16}O + ^{12}C$ and in the $^{27}Al + p$ reaction, respectively, but does the effective coherence width vary slightly with scattering angle in the heavy ion reaction, because the weighing factors $\langle \sigma^{J\pi,T} \rangle$ in Eq. (5.1) vary with angle. We expect high angular momenta J to contribute relatively stronger to 0° and 180° scattering than to 90° scattering: The classical argument that the orbital angular momentum be perpendicular to the reaction plane (which leads to the well known anisotropy of Hauser-Feshbach cross sections) is more stringent for high than for low angular momenta. The two different coherence widths Γ_{EFF} from the reactions (5.4) can be understood in terms of a model.[30] A further example, where the effect of different J^π distributions is obvious, are the reactions $^{31}P(p,\alpha)^{28}Si$ and $^{16}O(^{16}O,\alpha)^{28}Si$ which has just appeared in the literature.[34] More experimental heavy ion data, particularly data with small finite range of data uncertainties, would be enlightening.

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FOOTNOTES AND REFERENCES

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