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Journal

Journal of Vision, 12(10)

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Publication Date

2012-12-03

DOI

10.1167/12.10.12

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A unified formula for light-adapted pupil size

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The size of the pupil has a large effect on visual function, and pupil size depends mainly on the adapting luminance, modulated by other factors. Over the last century, a number of formulas have been proposed to describe this dependence. Here we review seven published formulas and develop a new unified formula that incorporates the effects of luminance, size of the adapting field, age of the observer, and whether one or both eyes are adapted. We provide interactive [demonstrations](#) and software implementations of the unified formula.

Keywords: pupil diameter, pupil reflex, light adaptation, age, contrast sensitivity function, senile miosis, retinal illuminance

Citation: Watson, A. B., & Yellott, J. I. (2012). A unified formula for light-adapted pupil size. *Journal of Vision*, 12(10)12, 1–16, <http://www.journalofvision.org/content/12/10/12>, doi:10.1167/12.10.12.

Introduction

In the human eye, pupil diameter ranges between approximately 2 and 8 mm. Pupil diameter has a large effect on the optical transfer function of the eye, as illustrated in [Figure 1](#). Pupil size also has a direct effect on depth of field, as well as on retinal illuminance, which in turn influences contrast sensitivity. For these and other reasons, it is often desirable to estimate the pupil size for a given set of conditions when actual measurements are unavailable. Based on available data, various formulas have been proposed to predict pupil diameter. However, none of the existing formulas incorporate the combined effects of the observer's age, the size of the adapting field, and monocular versus binocular stimulation, all of which are known to have a significant impact on pupil size. The purpose of this note is to review these formulas, convert them all to a common format and common units, and propose a unified formula that includes the effects of luminance, age, monocular adaptation, and field size.

We review the proposed formulas in chronological order, but reserve the studies of age and monocular viewing for the end. We note that many investigators have reported large individual differences (Spring & Stiles, 1948), which may in part account for the discrepancies among formulas. Another basis for disagreements is that in some earlier studies, the roles of age, field size, and monocular adaptation appear not to have been understood and may not have been reported.

Studies generally report the size of the eye's entrance pupil, i.e., the virtual image of the physical pupil as seen through the cornea. We follow that

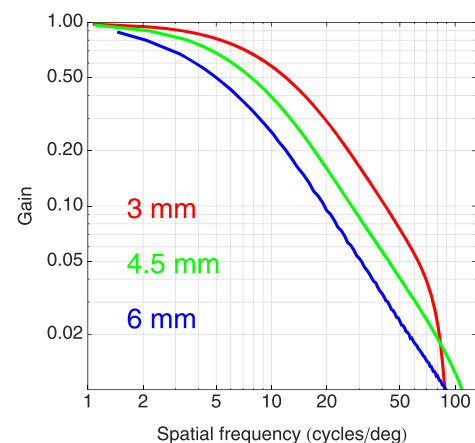


Figure 1. Effect of pupil diameter on optical filtering by the eye. Curves show mean modulation transfer functions (MTFs) for three pupil sizes computed from wavefront aberrations for 200 normal corrected healthy eyes. As the pupil enlarges, the diffraction-limited optical transfer function expands, but the amount of wavefront aberration also increases. Best optical quality generally occurs at smaller pupil sizes. Monochromatic wavefront aberrations were collected by the Indiana Aberration Study (Thibos, Hong, Bradley, & Cheng, 2002). We computed each curve by first calculating the MTF for each individual eye, assuming white light, using the method described by Ravikumar and Thibos (2008). We used 10 nm wavelength increments and assumed focus at a wavelength of 555 nm (Watson & Ahumada, 2008, 2012, in press). Each MTF was then converted to a radial mean MTF, and the mean of those means was then computed.

practice here. Retinal illuminance is proportional to the area of the entrance pupil (Atchison & Smith, 2000). We also follow the standard practice of specifying stimulus intensities by photopic luminance, even at light levels in the scotopic range. The studies we cite used a variety of luminance units; we use cd m^{-2} . Table A1 in the Appendix 4 shows conversion factors for other units.

Formulas

Holladay (1926)

Holladay (1926) collected data from three observers of unknown ages binocularly viewing a large adapting area (the interior of an adaptation chamber). He summarized the results with a formula “for a probable average healthy young eye,” (Holladay, 1926, pp. 309–310):

$$D = 7 \exp(-0.16 M^{0.4}) \quad (1)$$

where M is luminance in millilamberts. Converting to cd m^{-2} , we have

$$D_H(L) = 7 \exp(-0.1007 L^{0.4}) \quad (2)$$

This formula is shown in Figure 2. Clearly it fails for high luminances, but Holladay’s data did not go beyond about 600 cd m^{-2} .

Crawford (1936)

Crawford (1936) collected data from 10 subjects binocularly viewing a 55° diameter adapting field. No information is given regarding the age of the observers. He summarized his data with the equation

$$D = 5 - 2.2 \tanh[0.447(2.4 + \log B)] \quad (3)$$

where B is luminance in cd/ft^2 and \log indicates the logarithm of base 10. Converting to luminance in cd m^{-2} , we have

$$D_c(L) = 5 - 2.2 \tanh[0.61151 + 0.447 \log L] \quad (4)$$

This formula is pictured in Figure 3. Crawford notes that his formula is as much as 2 mm below the results of Holladay (1926), Reeves (1918), and others and ascribes this to individual differences.

Moon and Spencer (1944)

Moon and Spencer (1944) considered the data of various authors, but settled on Blanchard (1918),

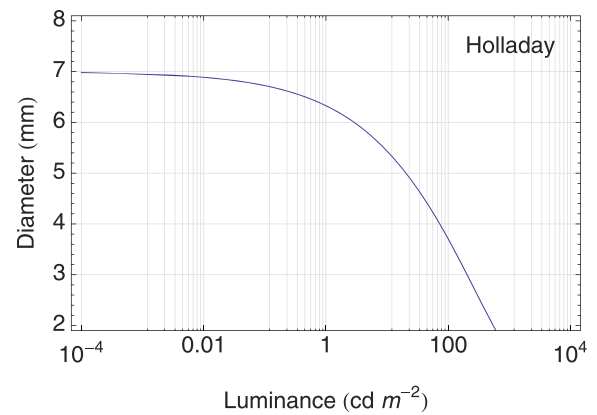


Figure 2. Pupil diameter formula of Holladay (1926).

Reeves (1918), and Crawford (1936) as the most reliable. They acknowledged the prior formula of Crawford (but misquoted it), and offered a modification which they asserted provided “an approximate average of all the data,” (Crawford, 1936, p. 321):

$$D = 4.9 - 3 \tanh[0.4(0.5 + \log M)] \quad (5)$$

where M is in millilamberts. Converting to cd m^{-2} , we get

$$D(L) = 4.9 - 3 \tanh[0.4 \log L - 0.00114] \quad (6)$$

which we approximate by

$$D_{MS}(L) = 4.9 - 3 \tanh[0.4 \log L] \quad (7)$$

This formula is shown in Figure 4. It was also adopted without attribution by Le Grand (1968) (who replaces 4.9 with 5), and appears to be given incorrectly in Wyszecki and Stiles (1982) [Equation 1(2.4.5), p. 106].

De Groot and Gebhard (1952)

De Groot and Gebhard (1952) computed a mean of data sets from eight authors, weighting each set by the

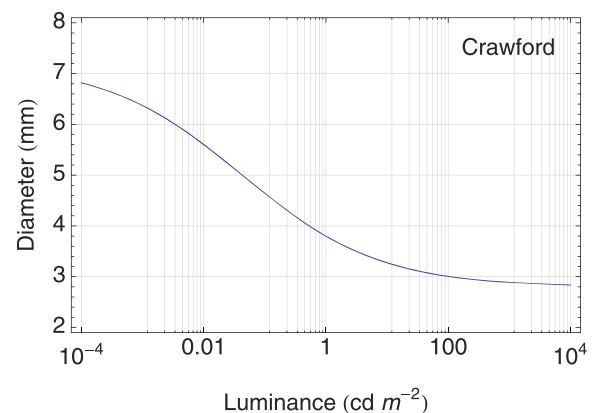


Figure 3. Pupil diameter formula of Crawford (1936).

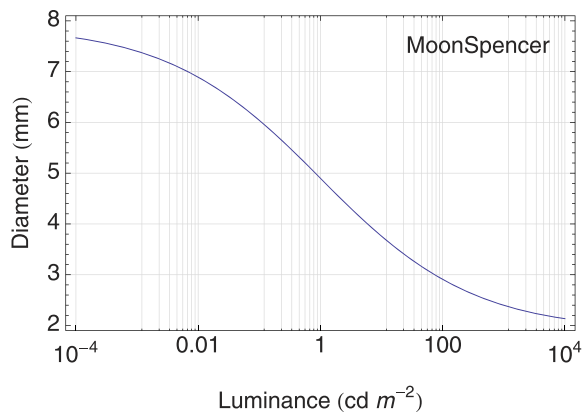


Figure 4. Pupil diameter formula of Moon and Spencer (1944).

number of observers. They noted that previous formulas tended to assume a lower asymptote of 2 mm, not necessarily demanded by the data or physiological constraints. They provided a formula which does not asymptote at 2 mm and was a better fit to their mean than the formula of Moon and Spencer (1944),

$$\log D = 0.8558 - 0.000401(8.1 + \log M)^3 \quad (8)$$

where M is in millilamberts. Converting to cd m^{-2} , and expressing as a formula for D, we have

$$D_{DG}(L) = 7.175 \exp\left[-0.00092(7.597 + \log L)^3\right] \quad (9)$$

as shown in Figure 5.

Stanley and Davies (1995)

Crawford (1936) and Bouma (1965) demonstrated that pupil size was dependent not on luminance alone, but approximately on the product of luminance and adapting field size. This observation appears as early as

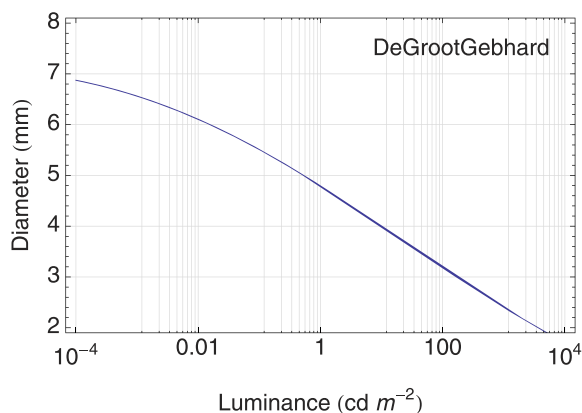


Figure 5. Pupil diameter formula of De Groot and Gebhard (1952).

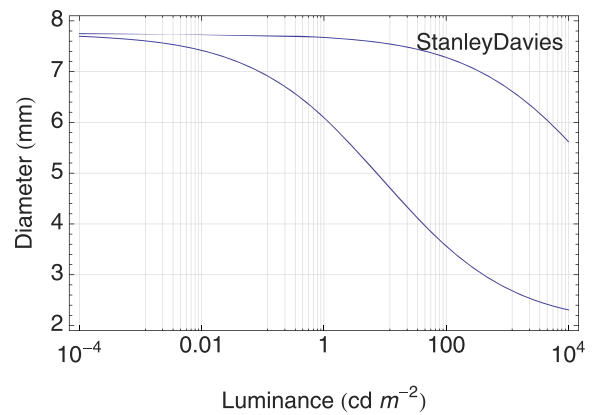


Figure 6. Pupil diameter formula of Stanley and Davies (1995). Upper curve is for a field diameter of 0.4° , lower curve is for 25.4° .

Aubert (1876) [quoted in Schweitzer (1956)], but it did not appear in formulas for pupil size until the work of Stanley and Davies (1995), who measured pupil diameter for six observers as a function of luminance and adapting field sizes ranging in diameter from 0.4 to 25.4° . They observed a strong effect of field size, which they suggested might account for the large discrepancies in earlier published results. Evidently unaware of the earlier observations of Crawford and Bouma, they noted that data for all sizes superimposed if the results were plotted as a function of the product of luminance and adapting area in degrees squared (*corneal flux density*, $\text{cd m}^{-2} \text{deg}^2$). They fit the combined data with the following function

$$D_{SD}(L, a) = 7.75 - 5.75 \left(\frac{(La/846)^{0.41}}{(La/846)^{0.41} + 2} \right) \quad (10)$$

where a is the area in deg^2 . Two examples of the function, for circular fields with diameters of 0.4 and 25.4° , are shown in Figure 6. Those two sizes are the limits of the range tested by Stanley and Davies, who caution against extrapolation outside those bounds. However Bouma (1965) appears to show integration out to much larger diameters, so we have not restricted the diameter in the formula. Atchison et al. (2011) provide a more recent confirmation of the dependence of pupil diameter on corneal flux density.

Barten (1999)

Barten (1999) adopted the formula of Le Grand (1968), itself borrowed from the formula of Moon and Spencer (1944), but inserted a term to modulate luminance by the field area, as in the formula of Stanley and Davies (1995), but here based on the work of Bouma (1965). Barten’s formula is given by

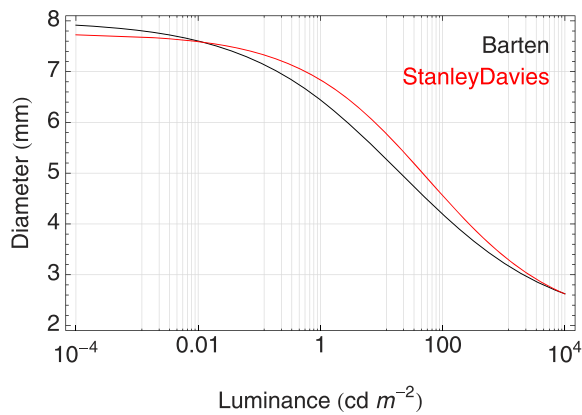


Figure 7. Pupil diameter formula of Barten (1999), for a field diameter of 10°. We also show the formula of Stanley and Davies for comparison.

$$D_B(L, a) = 5 - 3 \tanh\left(0.4 \log \frac{La}{40^2}\right) \quad (11)$$

Barten’s formula is plotted in Figure 7, along with that of Stanley and Davies for the same area, to show the similarity.

Blackie and Howland (1999)

As part of a larger project of modeling the emmetropization of the eye, Blackie and Howland (1999) fit the pupil size data of Flamant (1948) with a formula defined by

$$D = 1.945 - 1.218R - 0.07 R^2 \quad (12)$$

where R is the log of luminance in cd cm^{-2} . Converting to L in cd m^{-2} , and simplifying, we have the formula given below and plotted in Figure 8.

$$D_{BH}(L) = 5.697 - 0.658 \log L + 0.07(\log L)^2 \quad (13)$$

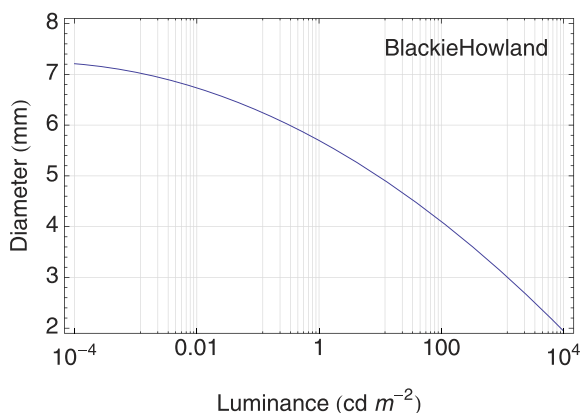


Figure 8. Pupil diameter formula of Blackie and Howland (1999).

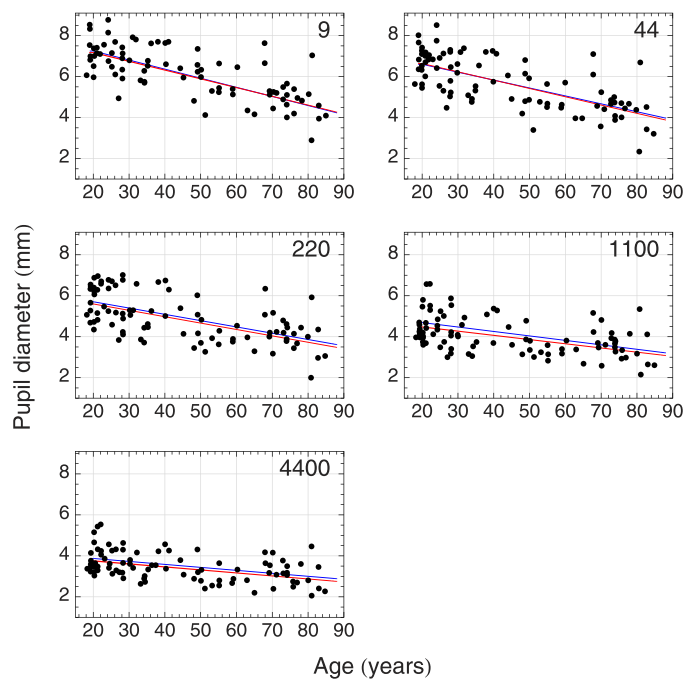


Figure 9. Effect of age on pupil diameter. We extracted and replotted 435 of the 455 data points from Figure 2 of Winn et al. (1994) (the remaining 20 points may have been obscured by other points). The linear fits of Winn are shown by the blue lines. The red lines show the predictions of the unified formula proposed here (Equation 21 below). The adapting luminance is shown in the upper right of each panel.

Winn, Whitaker, Elliott, and Phillips (1994)

Winn et al. (1994) measured pupil sizes for 91 subjects ranging in age from 17 to 83 years with normal healthy eyes. Adaptation was monocular, and the adaptation field was a 10° diameter circular disk. Accommodation was controlled and minimized. Five luminance levels between 9 and 4400 cd m^{-2} were used. For each luminance, pupil diameter was plotted against age, and a linear regression was fit to each set, as shown in Figure 9. The estimated slopes and intercepts are shown in Table 1, along with r^2 values.

The slope values are plotted in Figure 10 as a function of log luminance. In order to create a formula based on the Winn results, we have first fit the slope

cd m^{-2}	Slope	Intercept	r^2
9	-0.043	8.046	0.557
44	-0.04	7.413	0.486
220	-0.032	6.275	0.377
1100	-0.02	4.854	0.226
4400	-0.015	4.07	0.214

Table 1. Slopes and intercepts estimated by Winn et al. (1994) for the relation between pupil diameter and age at various adapting luminances.

values as a function of log luminance with a cubic polynomial. This is then used to construct a slope function $W_S(L)$. It specifies the proportional variation of pupil diameter with age at a given luminance. It is given by

$$W_S(L) = \sum_{k=0}^3 s_k \left(\log(\min(4400, \max(9, L))) \right)^k$$

$$s_0 = -0.024501$$

$$s_1 = -0.0368073$$

$$s_2 = 0.0210892$$

$$s_3 = 0.00281557. \tag{14}$$

In this expression, we restrict the luminance to the range explored by Winn et al., and as a result the function is flat at high and low luminances. This function is plotted as the red curve in Figure 10.

We also fit the intercepts as a function of log luminance with a cubic polynomial. We use this to construct a function $W_I(L)$ that returns an intercept for a given luminance,

$$W_I(L) = \sum_{k=0}^3 b_k \left(\log(\min(4400, \max(9, L))) \right)^k$$

$$b_0 = 6.9039$$

$$b_1 = 2.7765$$

$$b_2 = -1.909$$

$$b_3 = 0.25599 \tag{15}$$

This function is plotted along with the data in Figure 11.

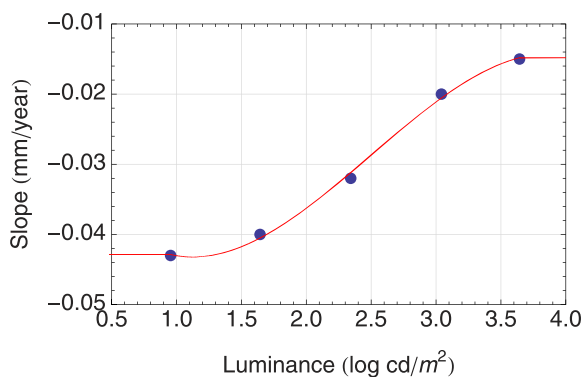


Figure 10. Slopes estimated by Winn et al. (1994) for the linear relation between pupil diameter and age at various adapting luminances, along with a fitted polynomial (Equation 14).

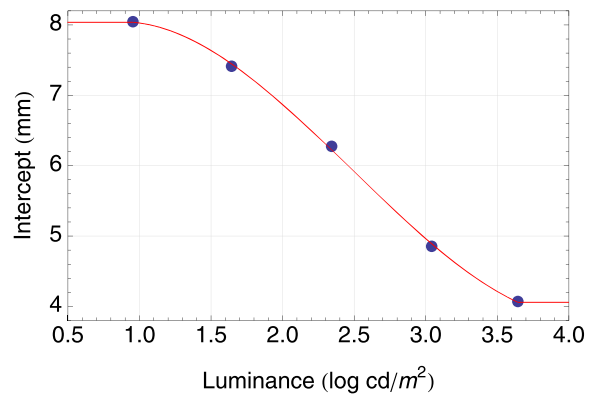


Figure 11. Intercepts estimated by Winn et al. (1994) for the relation between pupil diameter and age at various adapting luminances, along with a fitted polynomial (Equation 15).

We combine these two functions to produce a formula to approximate the data of Winn et al. (1994), given a luminance L and an age y in years,

$$D_W(L, y) = W_S(L)y + W_I(L) \tag{16}$$

This function is pictured in Figure 12.

Developing the unified formula

Monocular effect

In pupil experiments it is common to stimulate only one eye and measure pupil size in the other. The effect will generally be different than if both eyes view the same adapting light; a useful pupil formula should predict the difference. Blanchard (1918) and Reeves (1918) provide data comparing monocular and binocular adaptation in a single observer (both present identical figures). ten Doesschate and Alpern (1967)

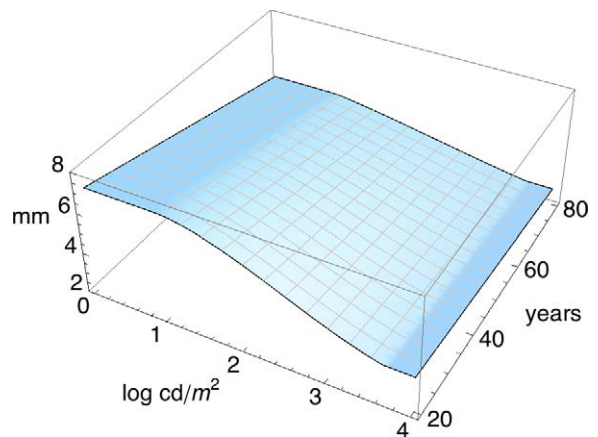


Figure 12. Pupil diameter according to the formula (Equation 16) developed to approximate the data of Winn et al. (1994).

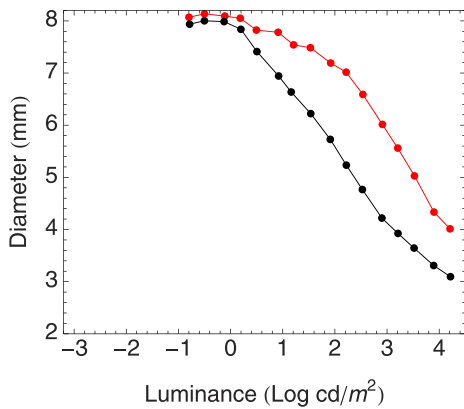


Figure 13. Pupil diameter for binocular (black) and monocular (red) adaptation. Observer G from ten Doesschate and Alpern (1967).

provide similar data for seven observers. Adaptation of one eye produces a larger pupil than adaptation of both eyes. Data from one observer are shown in Figure 13, which illustrates that the effect is as much as 1 mm, but diminishes at the extremes of high and low luminance.

We have found that monocular and binocular curves (like those in Figure 13) can be roughly superimposed by a horizontal shift on a log axis. To illustrate, we plot in Figure 14 the result of shifting the monocular data for all eight available data sets. In each case we optimized the shift by minimizing the area (shown in yellow) between the curves in the range of luminances where they overlap. The average optimal shift was -0.994 log units, or a factor of 0.1015.

In light of the success of the shifting operation, we can model the monocular effect by simply assuming an

attenuation of luminance by a factor of 0.1 when monocular viewing is used. This result is new and surprising. It also accounts almost perfectly for the results of Bartleson (1968). We formalize this with a function expressing attenuation as a function $M(e)$ of number of eyes e ,

$$M(1) = 0.1$$

$$M(2) = 1 \tag{17}$$

Effective corneal flux density

We have seen that pupil diameter is to a first approximation dependent on the product of luminance and adapting field area, or corneal flux density (Atchison et al., 2011; Aubert, 1876; Bouma, 1965; Crawford, 1936; Stanley & Davies, 1995). In the previous section we noted that corneal flux density is effectively attenuated by a factor of 10 if only one eye is adapted. We introduce the concept of *effective corneal flux density* to describe the quantity that effectively controls pupil diameter, equal to the product of luminance, area, and the monocular effect, $F = LaM(e)$.

Age slope

The data of Winn et al. (1994) show that pupil diameter varies with age, and the slope function W_S provides a means of adjusting computed values as a function of age. However, the function is deficient in two ways. First, it is limited to a modest range of

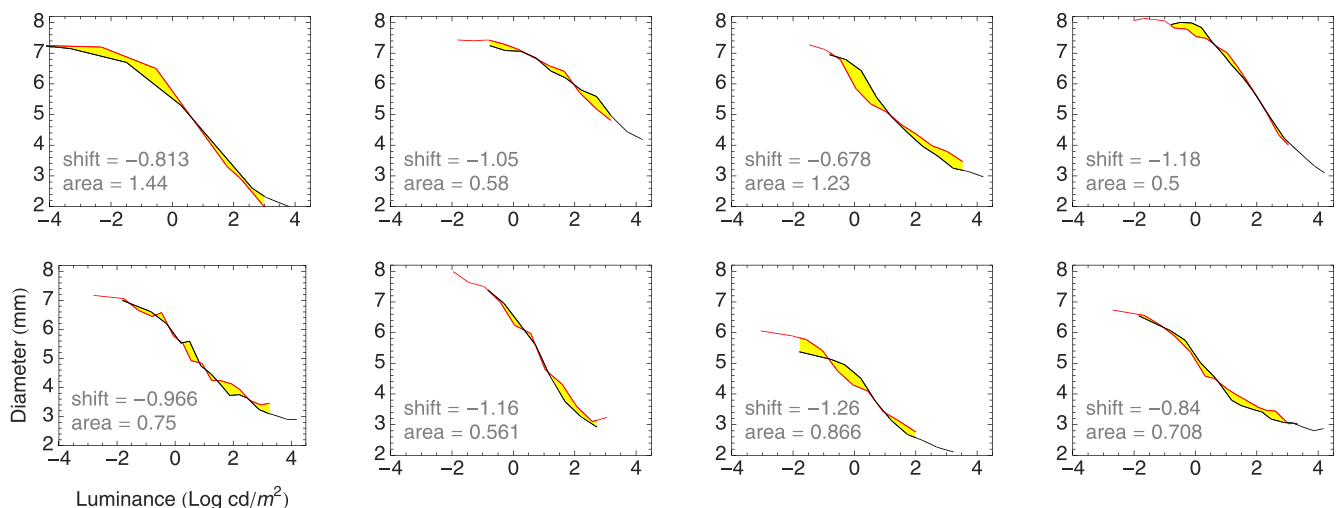


Figure 14. Monocular (red) data shifted horizontally to match binocular (black) data. First panel is Blanchard (1918) and Reeves (1918). Remaining seven panels are data from ten Doesschate and Alpern (1967). They have been converted from retinal illuminance (in Maxwellian view) to luminance by assuming a 4 mm pupil diameter. Second panel is field diameter of 0.8° , following panels are field diameter of 12° . The optimal shift and residual area of separation are shown in each panel.

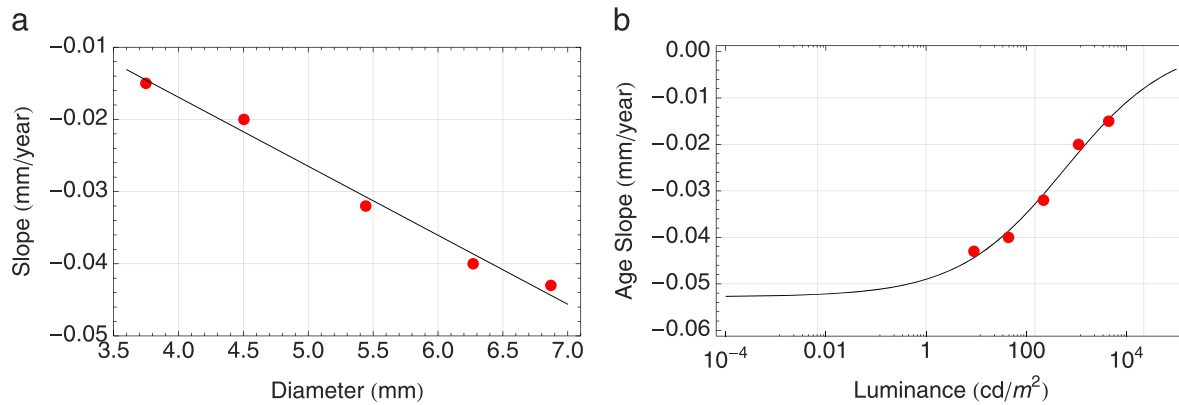


Figure 15. (a) Slope values estimated by Winn et al. (red) plotted against values of pupil diameter from Stanley and Davies for the corresponding effective corneal flux density and the best linear fit. (b) The age slope function $S(L, a, e)$ for $e = 1$, $a = 25 \delta$, and in red the slope estimates from Winn et al. (1994).

luminance (9 to 4400 cd m^{-2}), and second, it is defined only for monocular viewing and a 10° diameter adapting field. We generalize the function in the following ways. First, we assume that the slope is a function of effective corneal flux density, rather than luminance alone. Second, we observe that the reason the slope magnitude declines with age is because the maximum pupil diameter, and thus the total range of diameters, declines with age. If this is so, we should find a simple relation between the slope and the computed pupil diameter at the same luminance. In Figure 15a we show the fit of a linear model ($r^2 = 0.986$) between the slope estimates from Winn et al. and the pupil diameters computed from the formula of Stanley and Davies for the corresponding effective corneal flux density (using Winn's conditions of monocular viewing and 10° diameter adapting field).

Using that linear relation, we can create a new age slope function that is a linear transformation of the pupil diameter as computed by the formula of Stanley and Davies

$$S(L, a, e) = 0.021323 - 0.0095623 D_{SD}(LM(e), a) \quad (18)$$

The age slope function (for the conditions of Winn et al.) is plotted in Figure 15b, along with the values estimated by Winn. The agreement is evident. But note that the age slope function extends beyond the limits explored by Winn, as we intended.

Age effect

The age slope function describes the rate of change of pupil diameter with age (mm/year), as a function of effective corneal flux density. We construct an age effect function that describes the change in pupil

diameter as a function of luminance, area, age, a reference age, and number of eyes,

$$A(L, a, y, y_0, e) = (y - y_0)S(L, a, e), \quad 20 \leq y \leq 83 \quad (19)$$

This function will be used to adjust a reference formula for pupil diameter as a function of age and luminance. It applies only to ages between 20 and 83, the range investigated by Winn et al. (1994). In Appendix 1 we discuss an extension of the formula to ages below 20. The reference age is the age for which the reference function is defined. Ideally it would be the mean age of the group of observers on whose data the reference formula was based.

Unified formula

In addition to the well known effect of luminance, we have seen that there are systematic effects of age (Winn et al., 1994), field size, and number of eyes adapted (Blanchard, 1918; Reeves, 1918; ten Doesschate & Alpern, 1967). Here we construct a unified formula that includes all four effects

$$D_U(L, a, y, y_0, e) = D_{SD}(LM(e), a) + A(L, a, y, y_0, e) \quad (20)$$

where a is field area in deg^2 , y is age in years, y_0 is the reference age, and e is the number of eyes (one or two). Since we are using the formula of Stanley and Davies as the reference formula, the reference age should be the mean age of the population of observers used by Stanley and Davies. If we recall the definition of effective corneal flux density $F = LaM(e)$, we can rewrite the unified formula in this expanded but simplified form that makes clear the role of the Stanley Davies formula (Equation 10) and the age effect

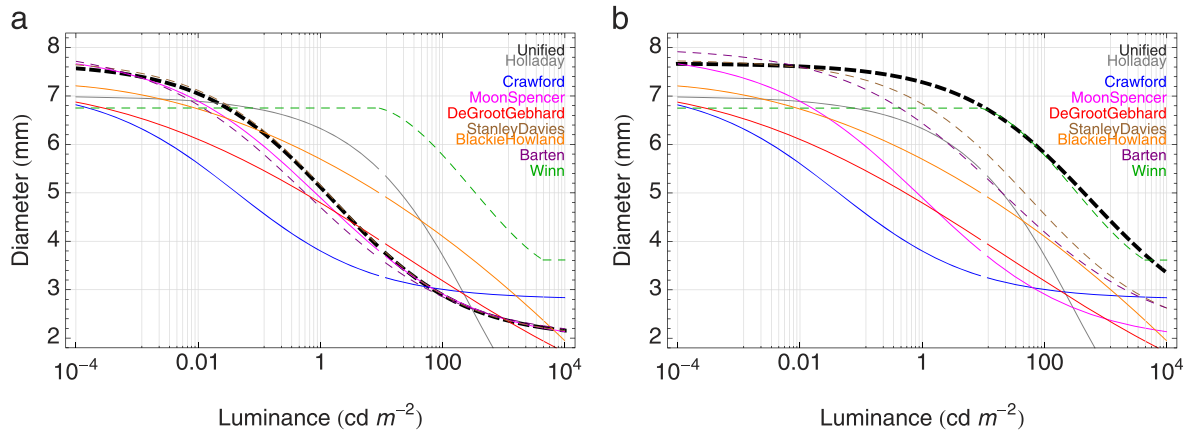


Figure 16. Pupil diameter according to several formulas: (a) field diameter 60°, binocular viewing; (b) field diameter 10°, monocular viewing. The dashed curves indicate formulas that depend upon adapting field size, observer age, or binocularity. For both figures, age $y = 30$ years.

$$D_u(L, a, y, y_0, e) = D_{SD}(F, 1) + (y - y_0)[0.02132 - 0.009562D_{SD}(F, 1)]. \quad (21)$$

Estimating the reference age

As noted above, the reference age y_0 should be the mean age of the population of observers used by Stanley and Davies. This mean age is unknown, but we can estimate it by comparing the unified formula with the data of Winn et al. (1994) as shown in Figure 9 above. We have plotted predictions from our unified formula (red) with the reference age parameter y_0 estimated by minimizing the mean squared error between the predictions and the data. The estimated value is $y_0 = 28.58$. The agreement between our unified formula (red) and the Winn estimates (blue) is striking and provides additional confirmation of the robustness of our approach.

The unified formula is shown in Figure 16 along with the other formulas developed above. We omit the formula of LeGrand, as it is nearly identical to Moon & Spencer (1944). In Figure 16a we have used parameters of field area $a = 900\pi$ (diameter = 60°), age $y = 30$, and eyes $e = 2$. This approximates the curve of Moon and Spencer, even though the unified formula is based on that of Stanley and Davies. Likewise in Figure 16b we show that when the field diameter and number of eyes are set to match those of Winn et al. (10° and monocular viewing), the unified formula provides an excellent fit to their curve over the range they measured.

Demonstration and calculator

To assist the reader in visualizing the effects of luminance, age, field size, and binocularity on pupil diameter, and to allow calculation of specific values, we

present a [demonstration](#) below in Figure 17. The [demonstration](#) makes use of the free Wolfram CDF player available at <http://www.wolfram.com/cdf-player/>. The [demonstration](#) allows the reader to set the values of the various parameters. A single value [calculator](#) is shown in Figure 18.

Retinal illuminance versus luminance

Equipped with a formula for pupil diameter as a function of luminance, we can easily compute the expected retinal illuminance corresponding to a given luminance. Of course, the relationship will depend somewhat upon age and other parameters, as shown in the [demonstration](#) in Figure 19. The near linearity of these log-log curves is striking ($r^2 > 0.99$ for all ages). If the relationship was in fact linear, it would imply a linear relationship between log luminance and log pupil diameter. This is not quite so, though the error is usually less than 1 mm.

Discussion

Unified formula

The development of our unified formula follows a logical argument that we summarize here.

1. At a given age, the pupil diameter is some function of the effective corneal flux density, given by $F = LaM(e)$. Evidence for this are the studies on field size (Atchison et al., 2011; Aubert, 1876; Bouma, 1965; Crawford, 1936; Stanley & Davies, 1995), the data on monocular adaptation (Bartleson, 1968; Blanchard, 1918; Reeves, 1918; ten Doesschate &

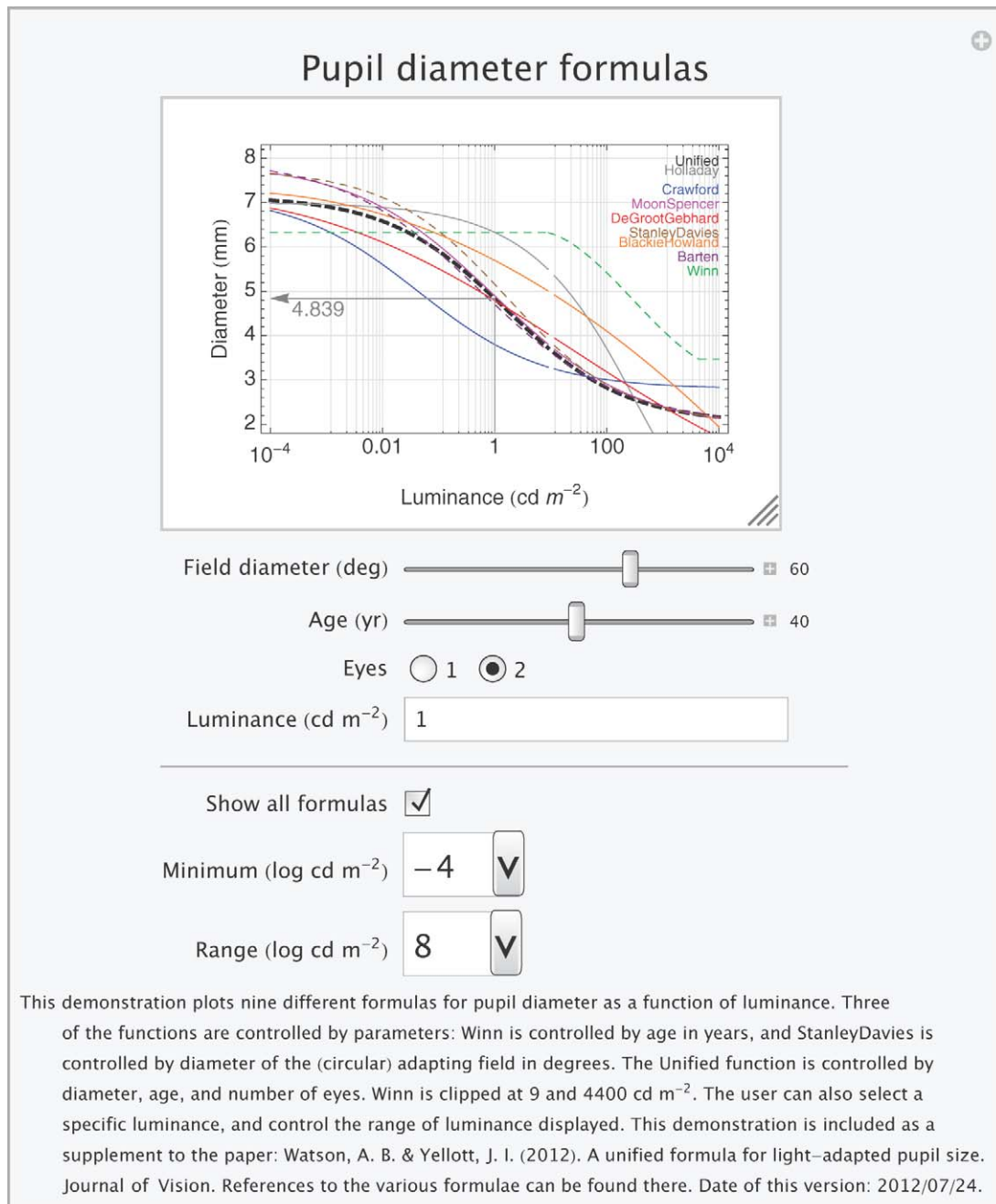
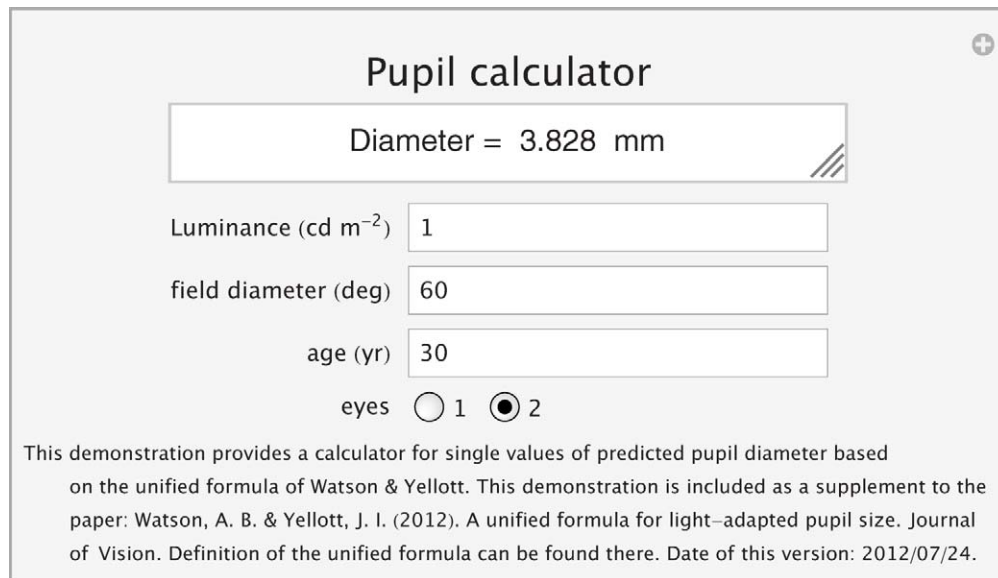


Figure 17. A [demonstration](#) of the unified formula for light-adapted pupil size.

- Alpern, 1967), and our analysis of those data (Figure 13).
- At the reference age, we take that function to be the formula of Stanley and Davies (1995). The rationale is that Stanley and Davies used relatively well-defined and modern methods and six observers, and their formula is simple and consistent with other historical formulas.
 - At any other age, pupil diameter is adjusted by a linear function of age relative to the reference age.

- Supporting this are the data of Winn et al. (1994) showing a linear variation with age at a range of luminances.
- The slope of the linear age adjustment at any luminance is itself a linear function of the reference pupil diameter (pupil diameter at the reference age for that luminance). The rationale is that the slope depends on the available range of pupil diameter variation, and that in turn is dependent upon the reference pupil diameter.



Pupil calculator

Diameter = 3.828 mm

Luminance (cd m⁻²)

field diameter (deg)

age (yr)

eyes 1 2

This demonstration provides a calculator for single values of predicted pupil diameter based on the unified formula of Watson & Yellott. This demonstration is included as a supplement to the paper: Watson, A. B. & Yellott, J. I. (2012). A unified formula for light-adapted pupil size. Journal of Vision. Definition of the unified formula can be found there. Date of this version: 2012/07/24.

Figure 18. Pupil calculator.

Variability

Our formula describes the mean pupil diameter for a given condition, but it should be understood that there may be considerable variability around that mean. We identify four possible sources of variability: between the two eyes, in a single eye over a short interval of time (seconds), in a single eye over a long interval of time (days), and between different observers.

Typically the two pupils have approximately equal diameters, regardless of whether one or both eyes are illuminated (Loewenfeld & Lowenstein, 1999). Large discrepancies (>0.4 mm) are called anisocoria, and may be diagnostic of neural pathology. Ettinger, Wyatt, and London (1991) found mean absolute differences of between 0.12 mm (at 343 cd m⁻²) and 0.36 mm (in darkness).

Careful observation shows that pupil size under prolonged constant illumination varies continuously over time. The most significant variation is generally known as *pupillary unrest*, or sometimes *hippus*. It is a spectrally broadband low frequency random fluctuation with a bandpass spectrum ranging from 0.02 to 2.0 Hz and an amplitude around 0.25 mm, somewhat dependent on the mean (Stark, Campbell, & Atwood, 1958). It is synchronous in the two eyes and thus does not contribute to anisocoria. It never occurs in total darkness, and its magnitude is greatest at the midrange of pupil diameters (Loewenfeld & Lowenstein, 1999). Pupillary unrest complicates the measurement of steady state pupil diameter that our formula is designed to compute and may contribute to some of the variability noted in the literature. Methods using still photography will produce variable results depending on the momentary phase of the waveform; much better are modern methods that

continuously monitor pupil diameter over an interval of time and then average out the unrest (Winn et al., 1994).

Over longer intervals, pupil diameter in a single eye is quite stable over time periods ranging from 3 hours (Kobashi, Kamiya, Ishikawa, Goseki, & Shimizu, 2012) up to 2 months (Robl et al., 2009).

Variability between observers is in part due to variations in age, and that component is dealt with by our unified formula. However, considerable variability remains among observers of the same age. We have estimated this variability from the data of Winn et al. (1994), extracted from their Figure 2 as described above (Figure 9). We estimated the standard deviation of the departures of the points from the best-fitting linear function of age. The result, expressed as a function of adapting luminance, is shown in Figure 20. The values range between about 1 and 0.6 mm, and are smallest at the largest luminances.

Other influences on pupil size

Our formula assumes that the observer fixates the center of the adapting field, consistent with the instructions given in the experiments of Winn et al. (1994). Crawford (1936) compared pupil sizes produced by a glare source located at eccentricities of 0 to 56°, finding an exponential decline of as much as 1.73 mm with eccentricity. We have found no studies other than Crawford's that systematically explore the effect of the placement of a uniform adapting field within the visual field, but his results do suggest the possibility of an effect, and suggest that our formula should be used with caution when the adapting field is not uniform or not centered on fixation.

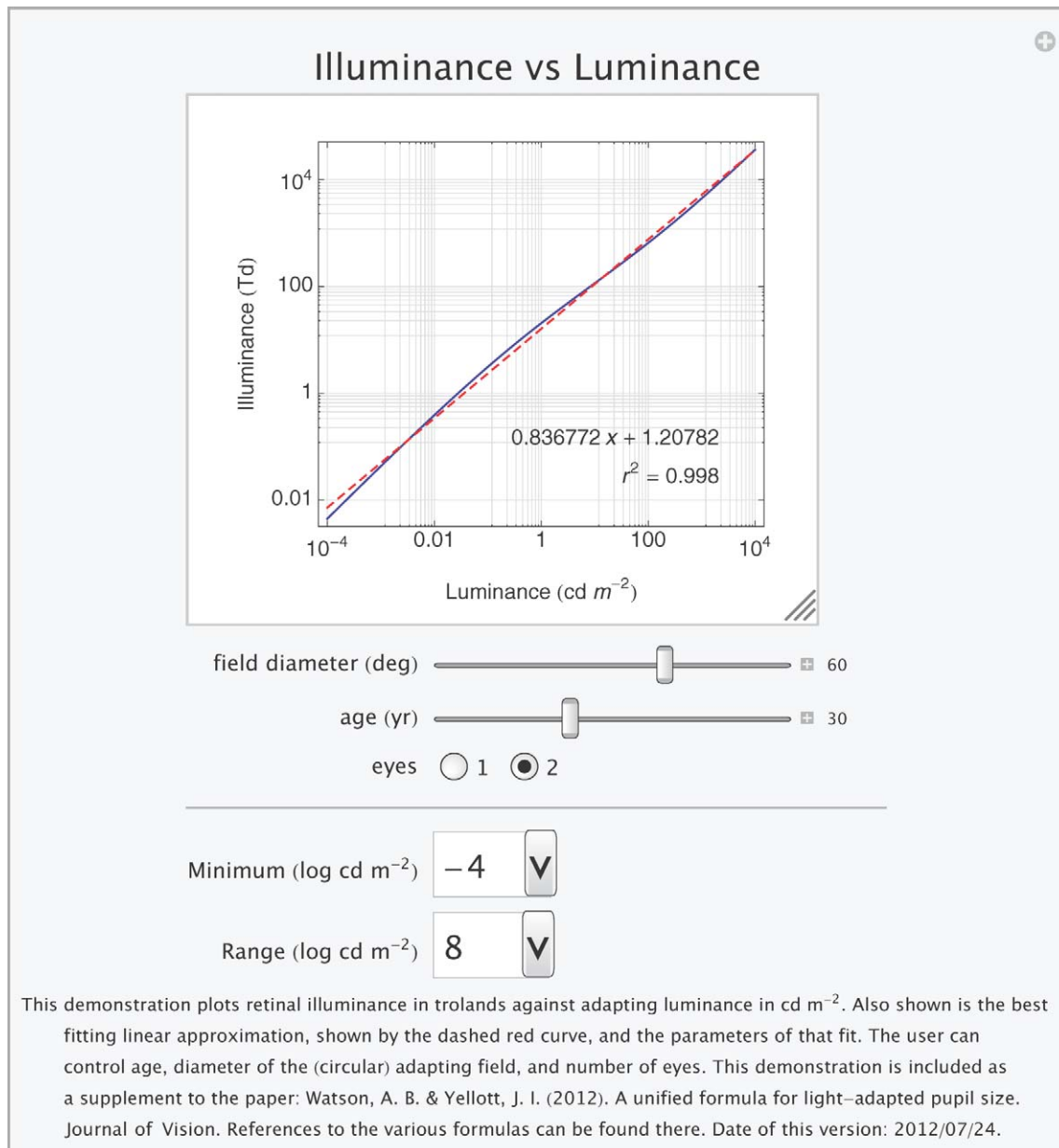


Figure 19. Retinal illuminance as a function of luminance. The [demonstration](#) allows the user to vary the parameters and the range of luminance.

While we have dwelt on the influence of adapting field luminance, it is important to acknowledge that there are many other factors that influence pupil size. Among these are accommodation (Kasthurirangan & Glasser, 2005; Marg & Morgan, 1949), mental activity (Hess & Polt, 1964), emotional arousal (Bradley, Miccoli, Escrig, & Lang, 2008), contrast (Barbur, 2004), detection (Privitera, Renninger, Carney, Klein, & Aguilar, 2010), recognition (Heaver & Hutton, 2011), and attention (Hoeks & Levelt, 1993). However, the cognitive effects are quite small, and all of these responses are largely transient, meaning the pupil diameter returns near to its prevailing value after a number of seconds.

Area summation and ipRGCs

Recently, intrinsically photosensitive retinal ganglion cells (ipRGC) have been discovered in the primate retina (Dacey et al., 2005). These cells express the photopigment melanopsin, with an action spectrum peaking at about 483 nm. These cells have been shown to exert significant control over the steady state pupil response in both macaque and human (Gamlin et al., 2007; McDougal & Gamlin, 2010). Thus while we have considered photopic luminance as the controlling variable, the action spectrum of the actual controlling quantity is likely to be a more complex mixture of rod,

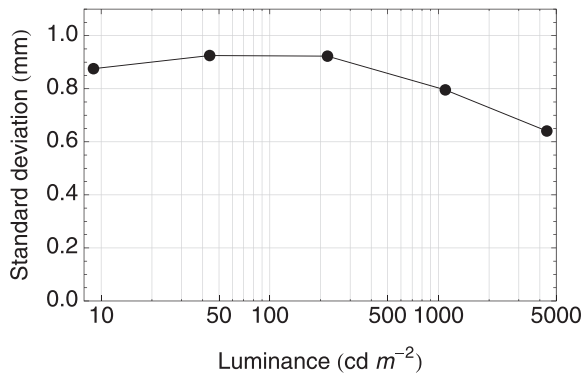


Figure 20. Standard deviation of pupil diameter as a function of adapting luminance, estimated from the data of Winn et al. (1994).

cone, and ipRGC sensitivities. The discovery of ipRGCs may also explain one mystery of the pupillary reflex. Our unified formula exhibits perfect summation of luminance over space (see formula of Stanley and Davies above), yet typical retinal ganglion cells have relatively small summation areas. In contrast, ipRGCs have very large receptive fields (Dacey et al., 2005) more consistent with the pupillary reflex.

Conclusions

Based on existing data and formulas, we have derived a unified formula for light adapted pupil diameter that includes the effects of luminance, adapting field size, age of the observer, and whether one eye or two are used. Our formula only describes the steady state in what is otherwise a dynamic reflex subject to many influences. Likewise there are large individual differences. With these caveats, we hope that our formula will be of some use in scientific and practical applications.

Acknowledgments

This work was supported by NASA Space Human Factors Engineering (WBS 466199). We thank Larry Thibos for providing the wavefront data used to create Figure 1.

Commercial relationships: none.

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Appendix 1: Extension to ages below 20 years

The unified formula developed in the body of this paper computes pupil diameter for ages between 20 and 80 years, the limits of Winn et al.'s (1994) data. Below we provide an extension to ages between 1 and 20. This extension rests on a small amount of data and some assumptions, which is why we relegate it to an appendix. But we include it in part to inspire further research to confirm, refute, or refine the formula.

MacLachlan and Howland (2002)

MacLachlan and Howland (2002) measured pupil size in younger observers aged 1 month to 19 years. Pupils were illuminated by dim ambient illumination of 15.9 lux. Observers were grouped by gender and by age into bins about one year in width. In their Table 1 they published for each group the mean age, mean pupil diameter, standard deviation, and number of observers. They provide functions describing pupil diameter as function of age separately for male and females. However they did not describe the method by which they arrived at the functions, and we sought a single function, so we have fit our own function to the data. The differences between male and female were small and inconsistent so we ignored gender. The standard deviations varied little between groups and averaged 0.93 mm. The number of subjects per group declined strongly with age, ranging from 378 for females of mean age 0.59 to 5 at mean age 18.82; this trend was similar for males. We fit the ensemble group data (mean pupil diameter versus mean age) using a linear model,

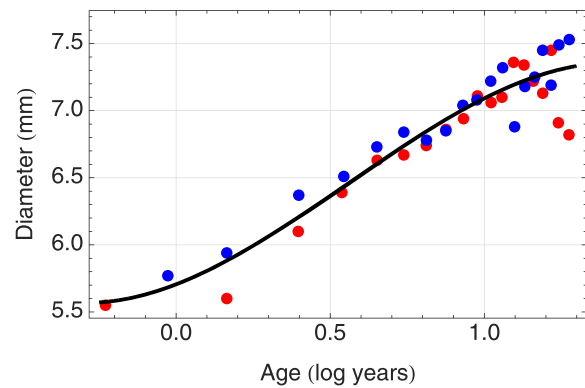


Figure A1. Polynomial fit to the data of MacLachlan and Howland (2002) for young observers (female: red, blue: male).

with weights given by the number of subjects in the group divided by the group variance.

We considered two linear models, a polynomial of age or of log age, and in each case we considered polynomial of various orders. We evaluated the fit of the model using adjusted r^2 , AIC, and BIC statistics. By all three criteria the best fit was given by a third order polynomial of log age. The resulting fit is shown in Figure A1.

We use this fit to define a function describing pupil diameter as a function of age for the conditions of MacLachlan and Howland (2002)

$$D_{MH}(y) = \sum_{k=0}^3 p_k \log(y)^k$$

$$p_0 = 5.70577$$

$$p_1 = 0.889567$$

$$p_2 = 1.22308$$

$$p_3 = -0.726731 \quad (22)$$

Equivalent age

The age slope function derived above will allow us to adjust pupil diameter for age, but only for ages greater than 20, the lower limit of Winn's data. We know that below 20, the trend with age reverses and pupils become smaller, as shown in Figure 12 (MacLachlan & Howland, 2002). Because data on younger observers were collected at only one low effective corneal flux density (F_{MH}), we do not know how young pupils react to adapting luminance. We make the assumption that younger pupils will behave in an equivalent fashion to those of an observer at an age which yields the same pupil size at F_{MH} .

First we estimate F_{MH} . We note that the pupil diameter for a 20 year old from MacLachlan and Howland is about equal to the unified result for a 20 year old with a 20° diameter binocular adapting field of 0.174 cd m^{-2}

$$D_{MH}(20) \approx D_U(0.174, 100\pi, 20, 30, 2) \approx 7.33 \dots, \tag{23}$$

and so we estimate F_{MH} as $0.174 \text{ cd m}^{-2} \times 100\pi \text{ deg}^2 = 54.66$. Next we compute, for ages y from 1 to 19, the age required by D_U at F_{MH} to yield the same pupil diameter as $D_{MH}(y)$. We discovered that those values were fit exceptionally well by an exponential (constrained to have a value of 20 at 20). The result of the fit is shown in [Figure A2](#).

This allows us to define an equivalent age transformation,

$$Y(y) = 19.291 + 44.03 \exp(-y/4.8441) \quad 1 < y < 20 \tag{24}$$

that converts an age less than 20 into an “equivalent” age greater than 20.

Unified formula for ages below 20

The equivalent age transformation allows us to extend our unified formula to ages below 20 by mapping the younger age to an older age, and then using the standard formula,

$$D_U(L, a, y, y_0, e) = D_U(L, a, Y(y), y_0, e) \quad 1 < y \leq 20. \tag{25}$$

An illustration of the effect of age from 1 to 80 years is shown in [Figure A3](#). The figure shows the change in

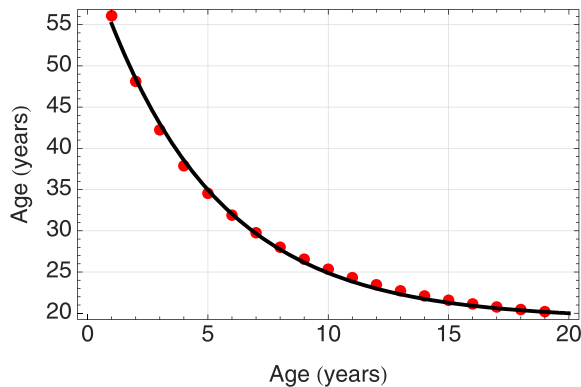


Figure A2. Equivalent age transformation. The red points are values of $\{D_U, D_{MH}\}$ and the curve is the best fitting exponential.

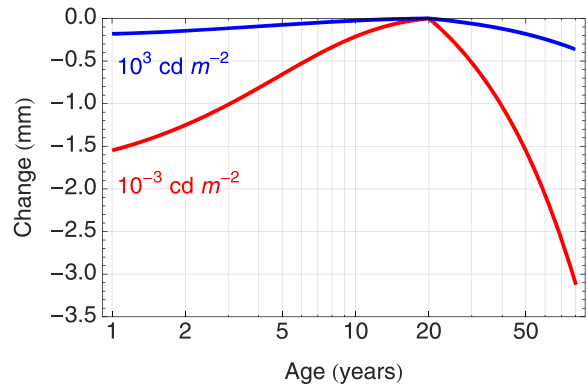


Figure A3. Change in pupil diameter from the value at age 20 for two different adapting luminances (adapting field diameter = 20° , binocular viewing).

pupil diameter from a peak at 20 years for two different adapting luminances. Pupil size declines on either side of the peak at age 20, but does so much more modestly for bright adapting fields.

We include this extension to younger ages in our demonstration and calculator (above) and our simplified formulas (below) but it should be understood that the extension is based on much less data, and many more assumptions than the formula for ages 20 to 80 years.

Appendix 2: Simplified formulas

Our unified formula is defined above through a series of nested expressions, notably [Equations 10 and 17–24](#). However for simplicity of calculation it is possible to produce simplified final expressions for the two cases of age less than and greater than or equal to 20. First we define a term that is the effective corneal flux density, raised to the power 0.41,

$$f = F^{0.41} = [LaM(e)]^{0.41} \tag{26}$$

Then

$$D_U = \frac{18.5172 + 0.122165f - 0.105569y + 0.000138645fy}{2 + 0.0630635f} \quad y \geq 20 \tag{27}$$

$$D_U = \frac{16.4674 + \exp[-0.208269y] \times (-3.96868 + 0.00521209f) + 0.124857f}{2 + 0.0630635f} \quad y < 20. \tag{28}$$

Appendix 3: Mathematica notebook

An implementation of the unified formula is provided in a Mathematica Notebook called [PupilDiameter.nb](#). It contains functions implementing all of the individual formulas described above, as well as the unified formula. A few plotting functions are also provided to illustrate the formulas. Mathematica is computer system for programming and mathematics (<http://www.wolfram.com>).

Appendix 4: Units

Many of the earlier reports use luminance units that are outside of the SI system. Here we provide some conversions to cd m^{-2} .

Unit	cd m^{-2}
Blondel	$1/\pi$
Candela/foot ²	10.7639
Millilambert	$10/\pi$
Foot Lambert (fL)	3.426

Table A1. Conversion factors for luminance units.