Title

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ABSTRACT

The angle of emission of the Cerenkov radiation is used to find the velocity of a beam of protons. Their range is also measured and we obtain points of the range energy relation for energies near 340 Mev for Be, C, Al, Cu, Sn and Pb. The data are used to evaluate the average excitation energy I for these substances. The results are summarized in Tables I and II.

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The range energy relation for protons is interesting for two types of reasons: its study has considerable intrinsic importance as a problem of physics; in addition the numerical results are extensively used by experimenters in determining energies.

For high energies (300 Mev) many of the serious difficulties besetting the very low energy part of the curve become negligible and the formula of Bethe[1]

$$-\frac{dE}{dx} = \frac{4\pi e^4}{mv^2} NZ \left(\log \frac{2mv^2}{I(1-\beta^2)} - \beta^2 - C_k/Z\right) \qquad (1)$$

can be used. Neglection of the C_k term does not introduce an appreciable error and the only empirical constant entering the formula is the average ionization energy I. Bloch has shown using the Fermi-Thomas model of the atom that I is proportional to the atomic number of the stopping substance.[2]

$$I = BZ \qquad (2)$$

The "constant" B has been determined for several substances by Bakker and Segrè[3] using the two values of I for Al and Be which have been determined in an absolute way by Wilson,[4] and Madsen and Venkateswarlu.[5]

This determination is only moderately accurate and it is clearly desirable to extend the experiment to an absolute measurement, eliminating the necessity to use the results of Wilson, and Madsen and Venkateswarlu which are obtained with light substances for which the statistical model is not well applicable. To do this a knowledge of the initial $\beta = v/c$ of the proton is necessary. Truly this can be approximately obtained from the

characteristics of the cyclotron accelerating the protons, but because of the precession of the orbits and other reasons this method is not very precise. Deflection of the beam in a known magnetic field would also give a way of measuring its energy, but although our deflected beam is very monoenergetic, as we shall see later, our deflecting magnet is not calibrated to give a precise absolute measurement of the energy.

Recently R. Mather has developed an apparatus to measure the angle of emission of the Cerenkov radiation produced by the beam in a piece of flint glass and has perfected this method to such an extent that it gives very accurate values of β. This technique affords an opportunity to measure the energy of the beam on an absolute scale and hence to determine the range of particles of known energy. Integration of (1) gives

$$R = \int_0^E \left(\frac{dE}{dx}\right)^{-1} dE \qquad (3)$$

and the range R, and the limit of the integral E being known Eq. 3 is an equation with I as the only unknown.

For the practical problem of determining the energy given the "range" of a particle we have to examine a little more carefully what we mean by range. The range given by (3) is the mean range: half of the particles travel in the material for a length larger than R and half for a length smaller than R. The length considered is the rectified trajectory and due to multiple scattering this is not the same as the distance from the entrance point in the material of a plane perpendicular to the initial direction of the beam through which half of the particles pass. Clearly the mean range given by (3) is larger than the latter "range" measured as indicated above, which we shall call R*. We can have a crude estimate of the importance of this effect by the following consideration which gives $\frac{R-R^*}{R}$. Divide the range R in small lengths ℓ; and call θ_i the angle between ℓ_i

and the direction of the incoming beam. We have

$$R - R^* = \sum l_i \cos \theta_i = \sum \tfrac{1}{2} l_i \theta_i^2 \qquad (4)$$

if θ_i is small and $\cos \theta_i$ is approximated by $1 - \frac{1}{2} \theta_i^2$. Now the average value of θ_i^2 is

$$\theta_i^2 = \frac{Z}{1836} \log \frac{E_0}{E_i} \qquad (5)$$

This formula is a crude approximation obtained from Williams formula and Eq. 1.

We replace in (4) the sum by an integral and use (5) to obtain

$$R - R^* = \frac{Z}{2 \times 1836} \int_0^R \left(\log \frac{E_0}{E(x)} \right) dx \qquad (6)$$

If we assume $R \approx E^{1.75}$ which is a good approximation of the range energy relation, we have (7)

$$R - R^* = \frac{Z R_0}{2 \times 1836 \times 1.75} \quad \text{or} \quad \frac{R - R^*}{R} = \frac{Z}{6400} \qquad (8)$$

The values of R in Table I are obtained from the values of R* directly observed, with the help of Eq. 8.

Our experimental arrangement is practically the same as the one used by Bakker and Segrè in their investigation mentioned above. The deflected beam of the 184-inch cyclotron is collimated to 1-inch diameter, passes through the Cerenkov radiation apparatus and enters an ionization chamber full of argon at atmospheric pressure. The chamber is closed by foils of copper-beryllium alloy, 2 mils thick, and its interior walls are of aluminum 7 mg/cm^2 thick. The depth of the part used is 5 cm and the diameter is 10 cm. After passing through this chamber the beam goes through a variable copper absorber carried by a wheel. This absorber can be varied from 0 to 8.62 gram/cm^2 of copper in 12 equal steps. After having passed the variable absorber the beam goes through a stack of plates of the material under investigation and then passes through an ionization chamber identical to the

one described above. The ratio of the ionization current in the two chambers is plotted as a function of the absorber between the two.

We compute all thicknesses of the wheel absorbers, windows, etc., in gr cm^{-2} of equivalent stopping power as if they were composed of the same substance as the main absorber, using the results of Ref. 3. The thickness of these absorbers are in any event a small fraction of the total thickness.

As an example of the curves obtained the case of lead is shown in Fig. 1.

We must now obtain from these data the mean range. If we call i(t) the ionization per cm of argon in the ionization chamber produced by a single particle at distance t from the end of its range in the absorbing material, and assume for the distribution of ranges due to straggling the gaussian form of probability

$$P(R) = (2\pi)^{-\frac{1}{2}} e^{-\frac{(R-R^*)^2}{2\sigma^2}} \tag{9}$$

we have for the ionization measured in our chamber

$$I = k\,(2\pi)^{-\frac{1}{2}}\,\sigma^{-1}\int e^{-\frac{(x-R^*)^2}{2\sigma^2}}\; i\,(t-x)\,dx \tag{10}$$

Assuming a new variable $(t - x)/\sigma = u$ and calling $(x - R^*)/\sigma = v$, formula 10 becomes

$$I = K\int e^{-\frac{(u+v)^2}{2}}\; i\,(u)\,du \tag{11}$$

where K is a constant. i is represented accurately enough by

$$i = \text{const.}\; t^{-0.46} \tag{12}$$

and we compute numerically the integral

$$f(x) = \int e^{-\frac{(x+t)^2}{2}}\; t^{-0.46}\,dt \tag{13}$$

This is given in Fig. 2. f(x) uses a unit of length the standard deviation of the gaussian. It will be noticed that if we normalize the ordinates in such a way as to call the maximum 1, then f(0) = 0.82. This means that, no matter what the value of the standard deviation, the center of the gaussian occurs at that value of the thickness for which f(x) is equal to 0.82 times

its maximum. This is hence R*.

The experimental standard deviation of the range distribution, σ_{exp}, is obtained by comparing the experimental curves with Fig. 2. We normalize them by multiplying the ordinates by such factors as to make the maxima of the curves equal. We then multiply the abscissae of each experimental curve by such a factor that the theoretical and experimental curve may be superimposed upon each other. The thickness of material in the experimental curve corresponding to x = 1 in the theoretical curve is the experimental standard deviation.

Theoretically the straggling can be calculated with the formula of Bohr[6]

$$\sigma^2 = 4\pi NZe^4 \int \left(\frac{dE}{dR}\right)^{-3} dE \qquad (14)$$

The values of σ_{theor} of Table I are computed by numerical integration from Eq. 14 and the values of dE/dx given in the tables of Aron et al.[7] It will be noticed that they are about 0.75 times the experimental value. If we try to attribute the difference to inhomogeneity of the energy of the beam ΔE we obtain,

$$(\sigma^2_{exp} - \sigma^2_{theor})^{\frac{1}{2}} = \left(\frac{dE}{dx}\right)^{-1} \Delta E$$

Numerically ΔE is given in column 7 of Table I. It is clear that $\Delta E/E$ is about 0.5×10^{-2}, a very good definition of the beam energy.

Unfortunately there is a disagreement between these computations and experiment which is not entirely clear to us. If we examine Fig. 1, the experimental (solid curve) and theoretical results (dotted curve) agree for the region of the curve past the maximum, but not for the region preceding it. More protons have suffered a larger loss of energy than we expected. There are several possible reasons for this, the most probable being the effect of nuclear collisions, but we have been unable to account for this

effect quantitatively. We do not think however that it affects the determination of R. We estimate the standard deviations of these measurements to approximately 1 Mev for the energy and 0.2 gr for the ranges. Since dE/dx is of the order of 2 Mev/gr cm^{-2} at 340 Mev, an error of 1 Mev corresponds to an error of 0.5 gr cm^{-2} in the range and hence most of the uncertainty comes from the energy measurements. The uncertainty in energy ΔE (column 7 of Table I) if present is too small to produce an appreciable broadening of the Cerenkov line and is not detectable in this way. From figures analyzed as indicated above we have the following results.

Table I

Energy Mev	Absorber	R* gr/cm^{-2}	R gr/cm^{-2}	σ_{th} gr/cm^{-2}	σ_{exp} gr/cm^{-2}	ΔE Mev
339.7	$_4Be$	76.68	76.73	0.65	0.91	1.75
339.7	$_6C$	69.97	70.03	0.62	0.88	1.83
339.7	$_{13}Al$	79.26	79.42	0.75	1.04	1.84
338.5	$_{13}Al$	78.47	78.63	0.75	0.92	1.40
337.9	$_{29}Cu$	91.43	91.84	0.92	1.12	1.44
338.5	$_{29}Cu$	91.36	91.77	0.92	1.25	1.89
339.7	$_{29}Cu$	92.27	92.69	0.92	1.24	1.88
339.7	$_{50}Sn$	106.58	107.41	--	1.50	--
339.7	$_{82}Pb$	122.80	124.37	1.35	1.90	2.32
338.5	$_{82}Pb$	121.21	122.76	1.35	1.84	2.25

With regard to the chemical purity of the samples used we have these data:

Beryllium: 99.9 percent

Carbon: 99+ percent

Aluminum: 99.2 percent, impurities Fe, Cu

Copper: 99.9 percent, impurities O, P

Tin: 99.8 percent, impurities Pb, Sb, As

Lead: 99.85 percent, Bi 0.15 percent

In order to analyze our data further we have reported in Table II the energy, material, rectified experimental range and the range calculated by Aron et al. for the substances studied. As shown by column 5 $R_{Aron} - R$ is a small quantity showing that Aron's tables are quite accurate. It is however possible to further improve them by changing the value of I used in their calculation in such a way as to bring them in exact agreement with the experimental results. This has been performed by Aron and the values of I thus obtained are given in column 6 of Table II. Column 7 gives I/Z for the same substances.

It will be noted that I_{Al} is practically identical with the value of 150 ev found by Wilson in 1940 and I_{Be} is also in excellent agreement with the measurements of Madsen and Venkateswarlu. For the other substances our results agree quite well with the less direct measurements of Bakker and Segrè.

Table II

Energy Mev	Absorber	R gr/cm^{-2}	R Aron gr/cm^{-2}	$R_A - R$ gr/cm^{-2}	I ev	I/Z ev
339.7	$_4$Be	76.73	74.57	-2.16	59.0	14.75
339.7	$_6$C	70.03	69.40	-0.63	74.4	12.91
339.7	$_{13}$Al	79.42	79.40	-0.02	150.3	11.56
338.5	$_{13}$Al	78.63	78.95	0.32	145.5	11.19
337.9	$_{29}$Cu	91.84	97.72	0.88	312.3	10.77
338.5	$_{29}$Cu	91.77	93.01	1.24	304.0	10.48
339.7	$_{29}$Cu	92.69	93.53	0.84	313.4	10.81
339.7	$_{50}$Sn	107.41	--			
339.7	$_{82}$Pb	124.37	127.15	2.78	828.7	10.11
338.5	$_{82}$Pb	122.76	126.45	3.69	792.6	9.67

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Information Division
1/29/51 md

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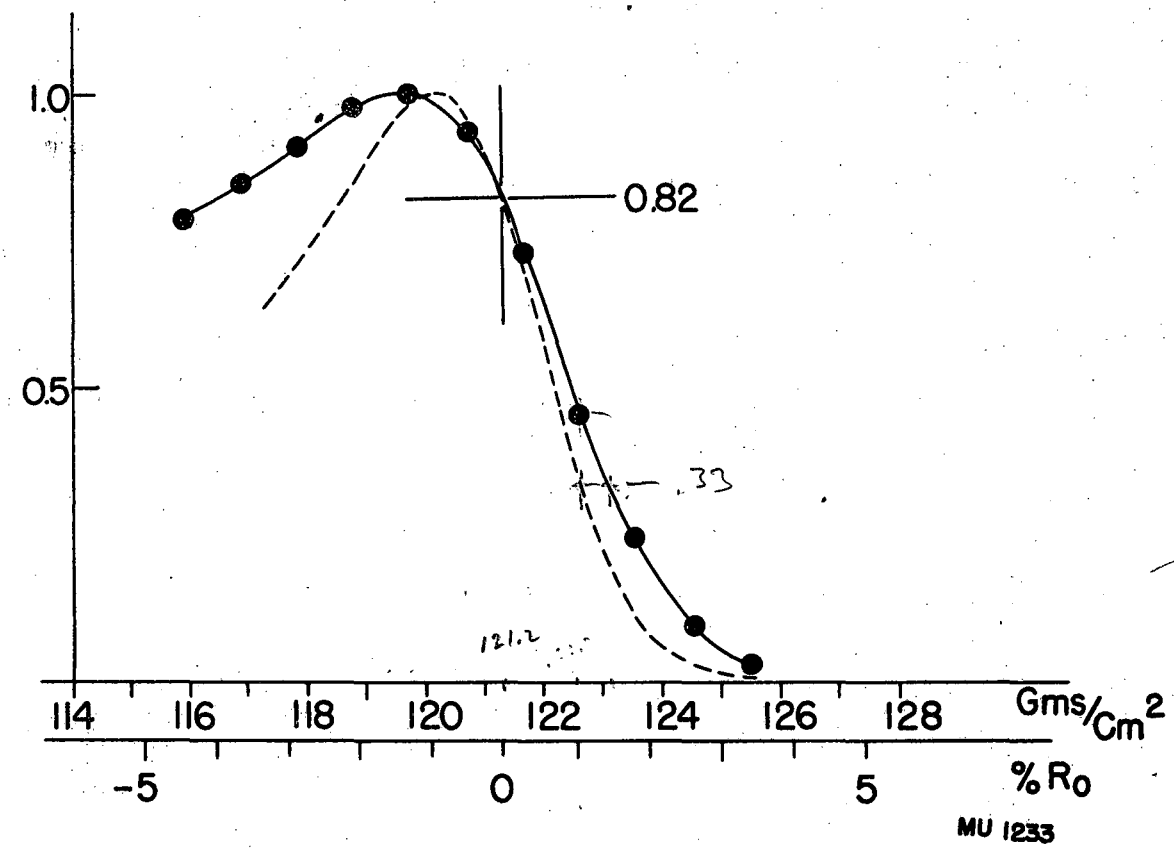
1.0
0.5
0.82
.33
121.2
114
116
118
120
122
124
126
128
Gms/Cm²
-5
0
5
% Ro
MU 1233

Fig. 1

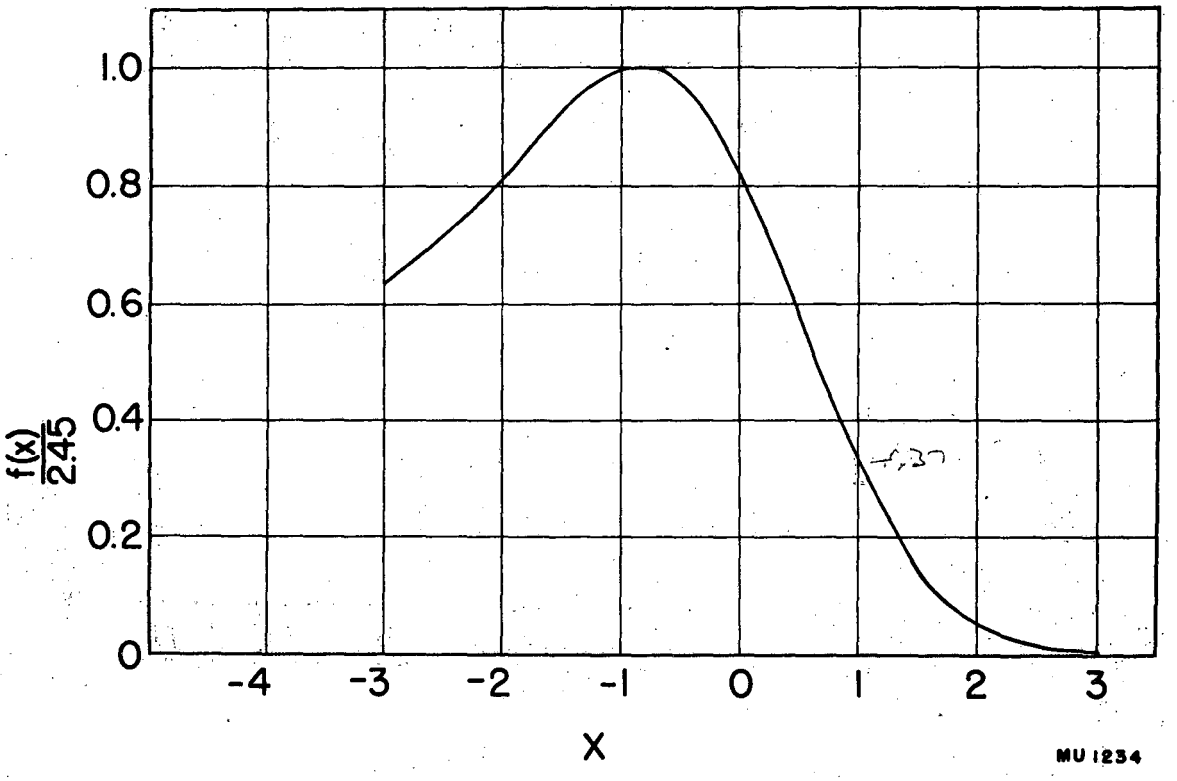

Fig. 2