Title

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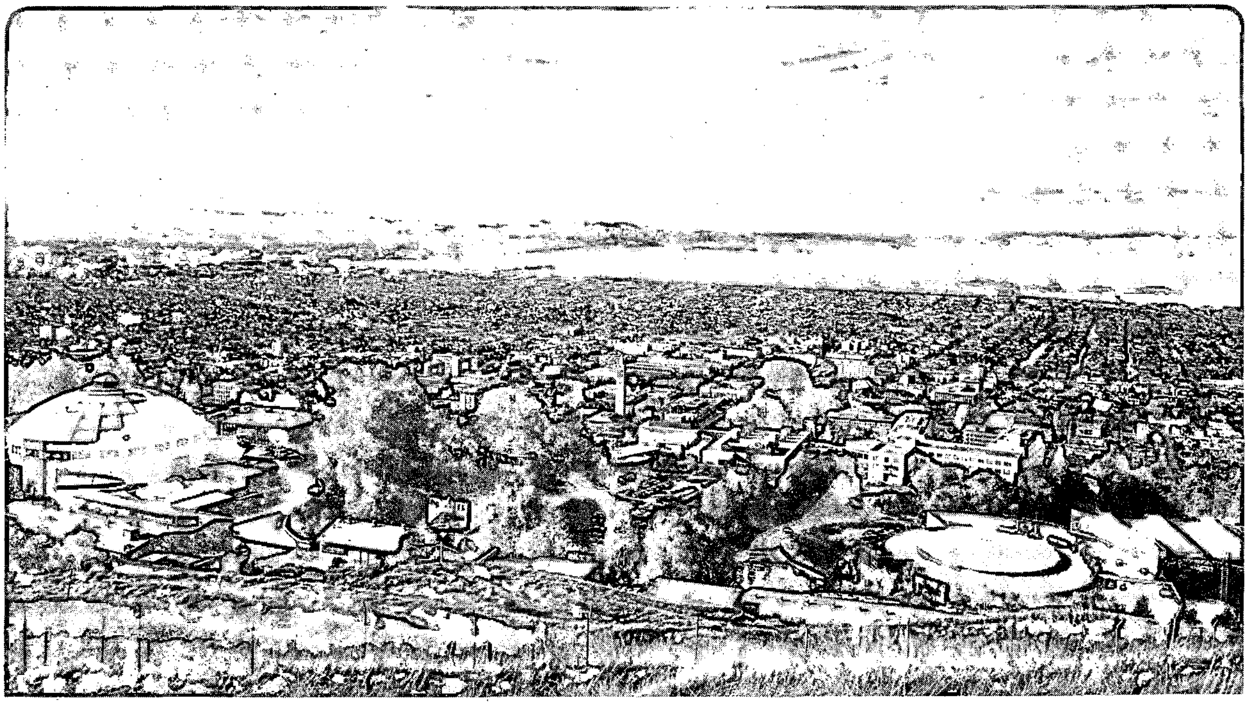

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A Simple Procedure for Estimating the Effective Hydraulic Conductivity of a Two-Dimensional Saturated or Partly-Saturated Fracture Network

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ABSTRACT

The effective medium theory developed by Kirkpatrick has been used as the basis for a procedure for finding the effective gridblock-scale conductivity of a two-dimensional fracture network. In contrast to computationally-intensive flow simulations, the proposed procedure requires only the solution of a single nonlinear equation. The method has been tested against some previously published numerical simulations of flow through saturated and unsaturated networks, and in both cases yields reasonably accurate predictions of the macroscopic hydraulic conductivity.

INTRODUCTION

Many of the sites that have been proposed as potential locations of underground radioactive waste repositories contain fractured rocks. For example, both the saturated zone and the unsaturated zone at Yucca Mountain, Nevada contain many hydrogeologic units that are extensively fractured[1]. When modeling the hydrological behavior of these sites, for either the purpose of site characterization or performance assessment, computational gridblocks are often used that contain large numbers of individual fractures[2]. In order to treat these gridblocks as equivalent continua, it is necessary to develop a procedure for relating the hydraulic properties of the individual fractures, and the topology of the fracture network, to the overall gridblock-scale permeability[3]. The purpose of this paper is to describe a simple upscaling procedure, and demonstrate its use in cases of both saturated and unsaturated flow.

UPSCALING PROCEDURE

A two-dimensional fracture network is usually modeled as a network of conductive elements that are connected to each other at "volumeless" nodes. The conductance C_i of the ith fracture segment depends on geometrical features such as the mean aperture, the fracture wall roughness, the amount of contact area of the two rock faces, and the length of the segment[4]; if the rock is only partially saturated, the conductance also depends on the hydraulic potential[5]. If the values of all of the conductances were known, and the topology of the network were also known, evaluation of the overall macroscopic conductance of the network would require the solution of a system of algebraic equations that arise by applying the equation $Q = C\Delta P$ to each conductor, and invoking the fact the the sum of the fluxes into each node must be zero. Numerical solution of the flow equations is typically inconvenient, mainly due to the need for constructing a computational grid for the entire fracture network. It would therefore be useful to devise a computationally simple method for estimating the macroscopic transmissivity of a block that contains a network of fractures.

The first step in our proposed upscaling procedure is to replace each conductance C_i in the network by some suitable effective conductance, $\bar{C}$. According to the effective medium approximation of Kirkpatrick[6], a suitable value of $\bar{C}$ can be found by solving the following implicit equation:

$$\sum_{i=1}^{N} \frac{\bar{C} - C_i}{(z/2-1)\bar{C} + C_i} = 0 \,, \tag{1}$$

where the mean coordination number z is defined as the average number of conductors that meet at each node, and the sum is taken over each fracture segment in the network. After finding $\bar{C}$, we first assume that the conductors are arranged on a square lattice, which itself is located within a rectangular computational gridblock. If a potential difference ΔP were imposed from top to bottom across this block, the total flow through the block would be given by $Q = N_x \bar{C} \Delta P$, where N_x is the number of vertical fracture channels in the lattice. According to Darcy's law, however, $Q = kL_x L_y \Delta P / L_z$, where L_y is thickness of the block in the third (transverse) direction, and L_z is the length of the block in the flow direction. Hence, the permeability of the block will be given by $k = N_x \bar{C} L_z / L_x L_y$.

However, we must also account for the fact that the actual mean coordination number z will typically be less than four, in which case the "equivalent" square lattice would have only $z/4$ of its bonds intact. This can be accounted for by using using eq. (1) on a square lattice for which a fraction $z/4$ of its bonds have conductivity $C_i = \bar{C}$, and a fraction $1-(z/4)$ of the bonds have $C_i = 0$. In this case eq. (1) yields the result $\bar{\bar{C}} = (z/2-1)\bar{C}$, which shows that the expression given in the previous paragraph for k must be multiplied by the factor $(z/2-1)$.

SATURATED FRACTURE NETWORK

The procedure described above has been used to predict the overall gridblock conductivity of the fracture network that was analyzed by Priest[7], under conditions of saturated flow (Fig.1). After removing dead-end fracture segments that do not contribute to the flow, the mean coordination number was found to be $z = 3.40$. Eq. (1) was used to find $\overline{C}$, based on the individual fracture segment transmissivities, which varied by a factor of 290 from smallest to largest. This value was was then corrected by the multiplicative factor $(z/2) - 1 = 0.70$. As there are eight intersections of the fracture network with the four sides of the square outer boundary of the grid-block, the actual irregular network was replaced with a square network containing two horizontal and two vertical fractures (i.e., 8 intersections /4 sides = 2 intersections/side). After using Darcy's law and taking into account the imposed potential gradient across the gridblock, we found a predicted flowrate (per unit length in the third direction) of $0.929 \times 10^{-6}\,\mathrm{m}^3/\mathrm{s}$, which compares very well with the value of $0.947 \times 10^{-6}\,\mathrm{m}^3/\mathrm{s}$ that Priest found by solving the full set of twenty simultaneous algebraic equations.

UNSATURATED FRACTURE NETWORK

We have also applied our procedure to the 2-D fracture network that was studied by Kwicklis and Healy[8] (Fig. 2). After dead-end segments were removed, this network had a mean coordination number of 3.43. For one set of simulations, 56% of the fracture segments had a mean aperture of 125 μm, and 44% of the fracture segments had a mean aperture of 25 μm. The hydraulic conductivity functions of the two different fractures types were computed by Kwicklis and Healy[8] using a variable-aperture flow model[5] (Fig. 3). As the fractures intersect the four gridblock faces in ten locations (see Fig. 2), we took our equivalent square lattice to have $10/4 = 2.5$ vertical and horizontal fractures per gridblock. The gridblock-scale conductivities computed using the procedure described above are shown in Fig. 4, where it is seen that they agree fairly well with those that were computed by Kwicklis and Healy using a numerical simulator[8]. The new method has the advantage, however, of requiring the solution of only a single equation at each value of the pressure head, as opposed to the 51 simultaneous equations required when simulating the actual flow field[8]; moreover, the proposed upscaling method does not require the construction of a numerical grid.

CONCLUSIONS

A major issue in the modeling of fluid flow in fractured rock masses is the determination of appropriate gridblock-scale conductivities. We have used Kirkpatrick's effective medium approximation[6] to develop a method for predicting the gridblock-scale hydraulic conductivity of a two-dimensional fracture network. The method requires knowledge of the mean coordination number of the fracture network, the mean fracture spacing, and the hydraulic conductivity functions of the individual fractures. However, in contrast to methods that solve for the detailed flow field through the entire network, the proposed upscaling procedure requires solution of only a single nonlinear equation. The method has been tested against numerical simulations conducted by Priest[7] on saturated flow through a fracture network, and also against the unsaturated flow simulations of Kwicklis and Healy[8]. In both cases the proposed upscaling method gave accurate predictions of the macroscopic hydraulic conductivity. Furthermore, the method is expected to be more accurate for gridblocks that contain larger numbers of fractures[6], as would typically be encountered in modeling the performance of an underground radioactive waste repository.

ACKNOWLEDGEMENTS

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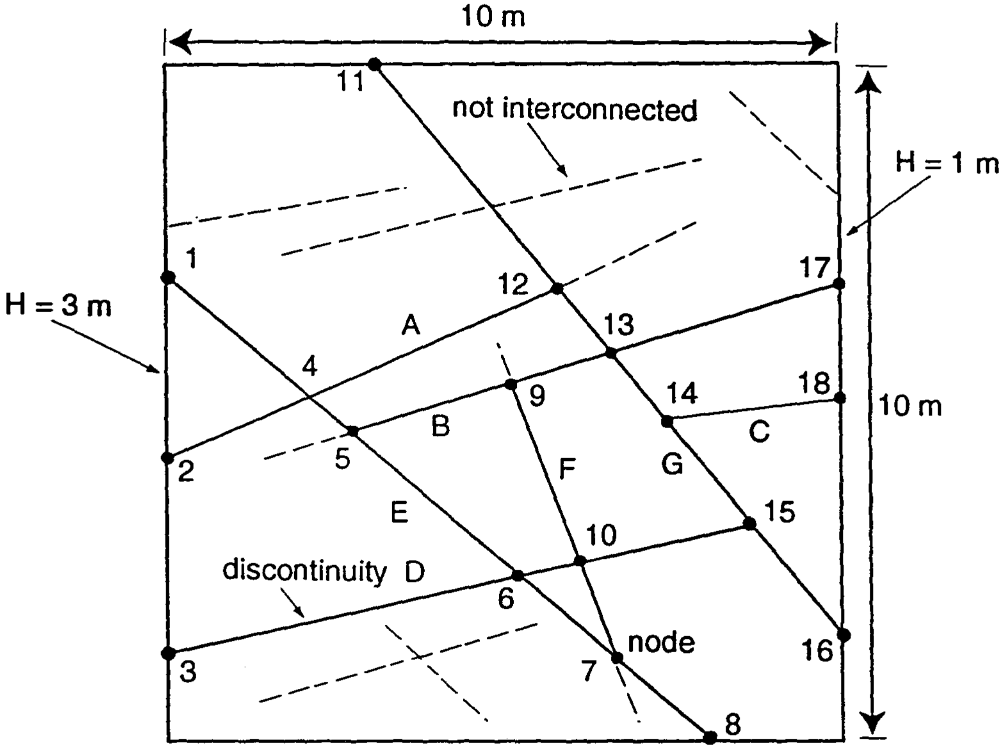

Fig. 1. Fracture network used in the saturated flow simulations conducted by Priest[7]. The left and right faces are held at uniform (but different) potentials, and the top and bottom faces have a linear potential gradient imposed along their lengths.

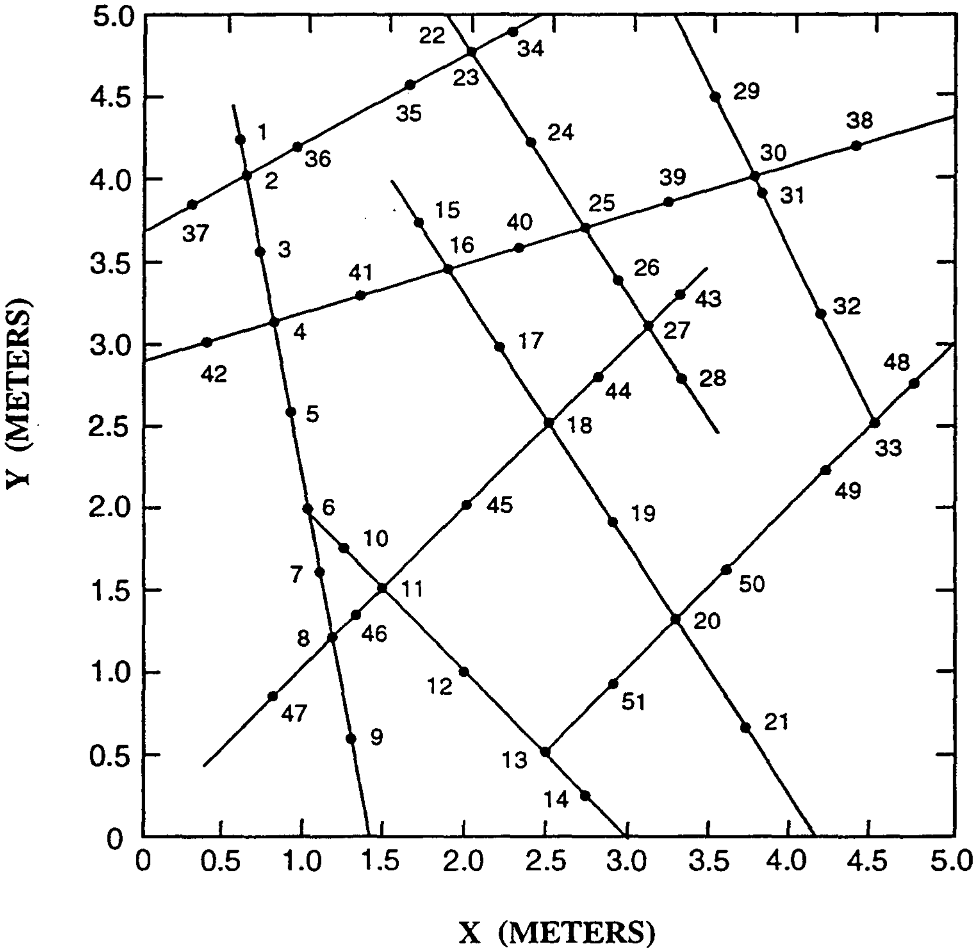

Fig. 2. Fracture network used in unsaturated flow simulations of Kwicklis and Healy[8]. The top and bottom faces are held at the same pressure, so that the flow is from top to bottom due to gravity; the lateral sides are assumed to be impermeable.

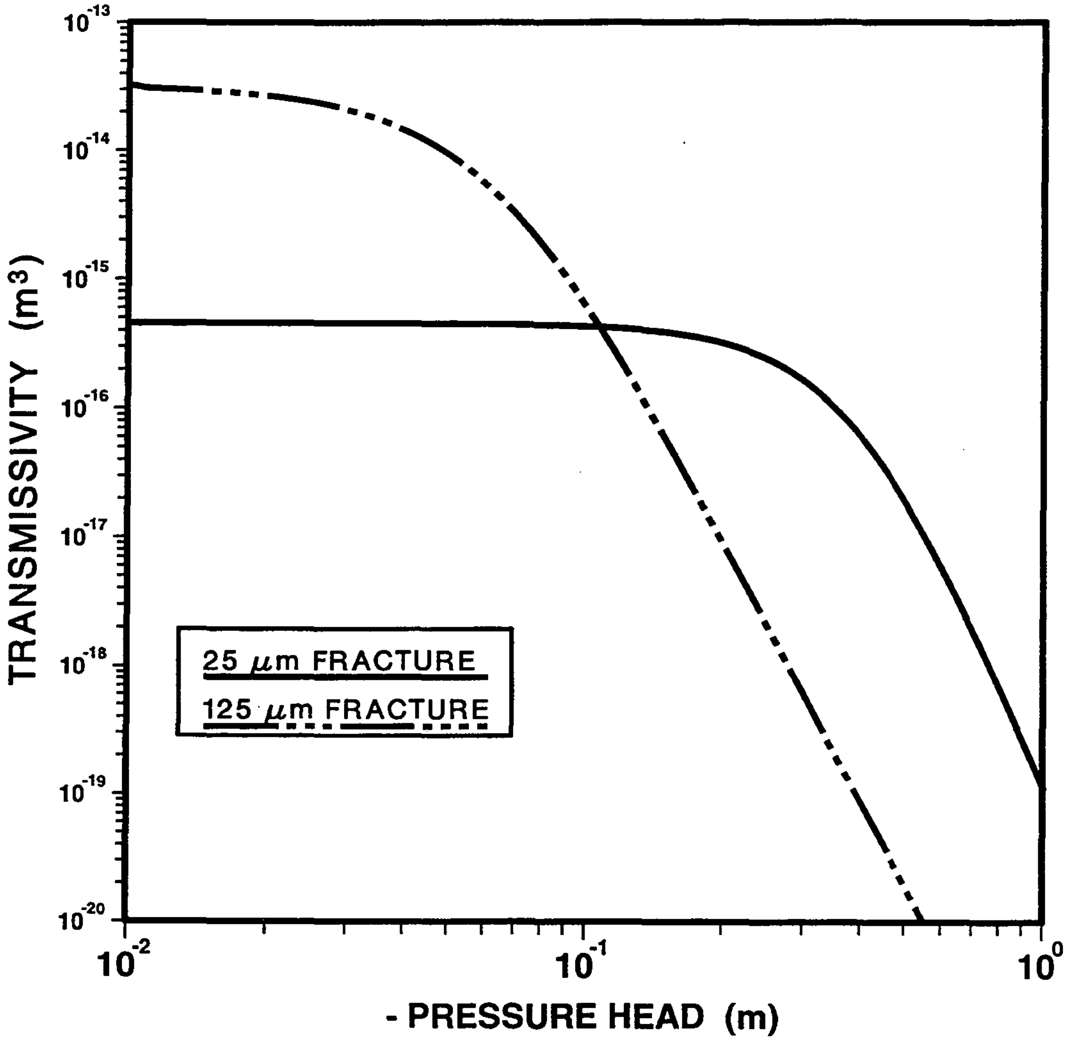

Fig. 3. Hydraulic conductivity functions for the two types of fractures used in the network shown in Fig. 2. The subvertical set of fractures have a mean aperture of 125 μm, and the subhorizontal set have a mean aperture of 25 μm.

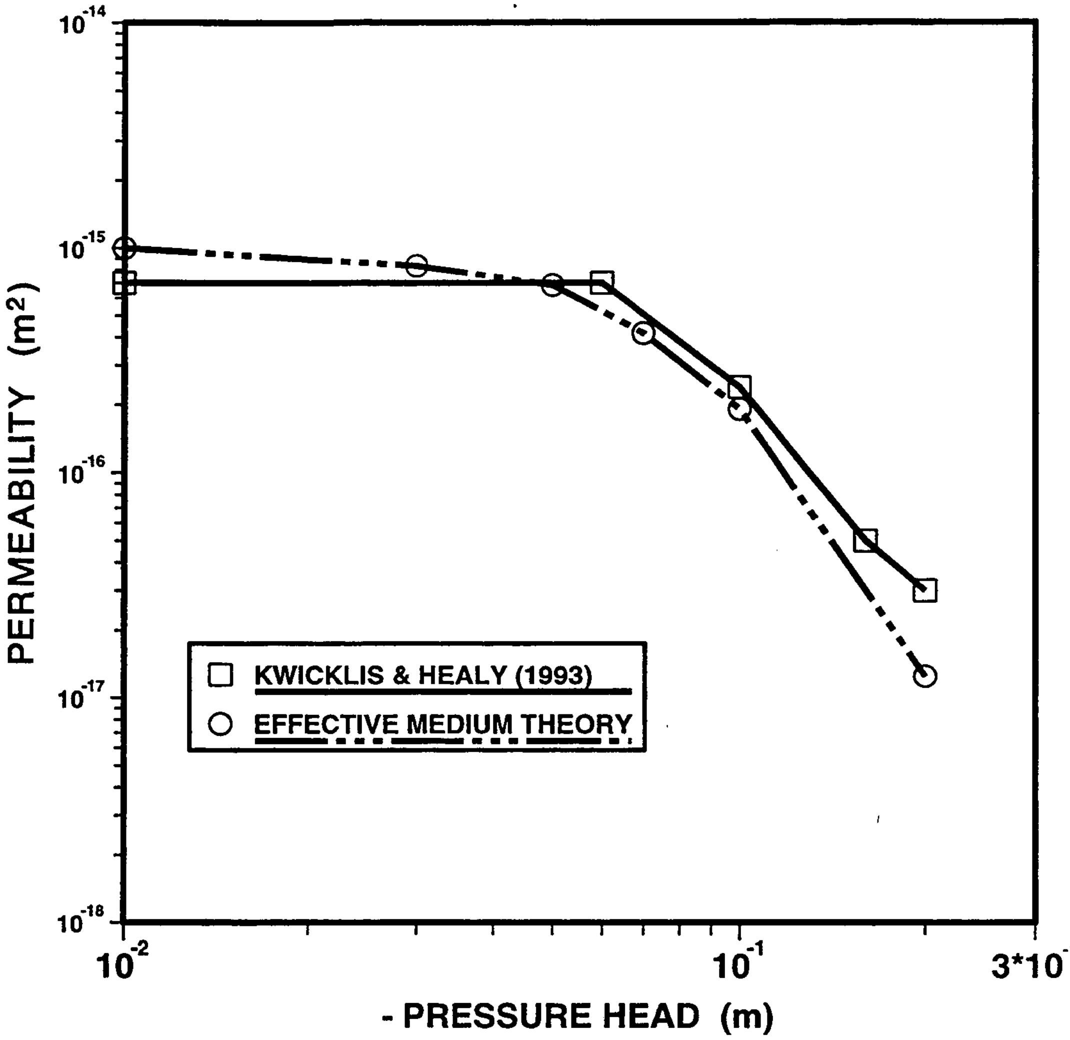

Fig. 4. Gridblock-scale hydraulic conductivity of the fracture network shown in Fig. 2, as a function of hydraulic potential.

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