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Liquidity and Liquidation

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Abstract

The manager of a firm that is selling an illiquid asset has discretion as to the sale price: if he chooses a high (low) selling price, early sale is unlikely (likely). If the manager has the option to default on the debt that is collateralized by the illiquid asset, the optimal selling price depends on whether the manager acts in the interests of owners or creditors. We model the former case. In the preferred equilibrium, the owner will always offer the illiquid asset for sale at a strictly higher price than he paid, and he will default if he fails to sell. As a result, the illiquid asset changes hands at successively higher prices; the price inflation terminates upon the first failure to sell, which results in a default chain.

1 Introduction

Decision-making authority in a corporation lies ultimately with the board of directors, at least nominally. In principle the board represents all the major stakeholders in the corporation: the owners (that is, the stockholders), the creditors, the employees and the general public. What the corporation does reflects which of these constituencies plays the dominant role on the board, and depends also on the extent to which the other groups can alter the outcome. Of these four groups, the owners and the creditors are usually the most important in practice, and we will restrict attention to them.

At least three major patterns of control may be distinguished (there are, of course, other possibilities). First, the creditors may dominate the board, in which case the chief executive officer who wants to retain his job will be solicitous primarily of their interests. This pattern is most likely to prevail for corporations in difficulty; when

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default is unlikely the creditors have no reason not to let the owners retain control. Second, stockholders' representatives may dominate, in which case they will direct the chief executive officer to maximize stock value without regard to the value of the liabilities. Third, the chief executive officer may be powerful enough to install board members who will reliably follow his or her lead. Neglecting agency issues in the relation between the chief executive and the owners (as we do in the model below), a chief executive who is not constrained by the board will also be motivated to maximize stock value so as to minimize the threat of hostile takeover, so the second and third cases do not differ materially.

In the second and third cases the owners do not have unlimited ability to transfer value from the creditors to themselves. Their ability to do so is constrained by bond indenture provisions that protect the interests of the creditors.

1.1 Owner-Creditor Conflict and Liquidation

These considerations are important because the interests of the various players are not fully aligned. Because of conflicts of interest, the action the corporation takes when it becomes unable to make scheduled payments to its creditors depends on which of the patterns of control just listed prevails. If representatives of the creditors or owners control the board and they have lost confidence in the chief executive officer, they will replace him or her. In contrast, under other control patterns—for example, if the chief executive officer controls the board—or if the assets of the corporation in question are highly valued by other corporations, the board may leave the management in place and instead elect to liquidate some or all of the assets.

As the name implies, liquidation involves selling illiquid assets, which amounts to replacing them with liquid assets (or reducing liabilities). In the usage of this paper, illiquid assets are assets that are heterogeneous, and for which it is costly to ascertain the relevant measure of asset quality. Therefore disposing of illiquid assets involves search and matching. The attribute of illiquid assets that is relevant here is that their heterogeneity (plus the fact that search is costly) partly insulates each buyer-seller pair from other buyers and sellers. Therefore markets are noncompetitive: sellers have control over prices, subject to the constraint that the probability of sale depends on the chosen price. General analyses of illiquidity based on this characterization are found in Krainer [5] and Krainer and LeRoy [6].

The theme of this paper is that optimal liquidation strategy is different depending on whether management is conducting the liquidation in the interests of owners or in the interests of creditors. The reason is that owners and creditors are affected differently by changes in the value of the corporations's assets: owners do, but creditors do not, benefit from increases in the corporation's value above the level that is required to generate cash for the bond payments. Correspondingly, creditors do, but owners do not, suffer losses when the corporation's value drops, given that default has occurred or will occur.

1.2 Debt and Default

The conflict of interest between owners and creditors is typically analyzed in the context of risky investment: owners have an incentive to undertake excessively risky investment projects because part of the risk is passed on to creditors (the savings and loan crisis in the US is the obvious example). The conflict of interest between owners and creditors that managers face in disposing of illiquid assets, although less familiar, is similar to that they face in their role as investment planners. Just as managers making capital budgeting decisions must trade off risk and return in choosing an investment strategy, managers in disposing of illiquid assets must trade off high sale proceeds (if they choose a high sale price) against high probability of early sale (if they choose a low price). As with capital budgeting decisions, the costs and benefits of different liquidation strategies are borne differently by owners and creditors. Therefore in both cases if management represents the interest of either alone, the outcome will not be that which maximizes overall corporation value.

We show below that the existence of the default option (plus the assumption that default is sometimes optimal) decreases the cost to the owners of failing to sell the assets, relative to the case in which there is no default option. If managers are acting in the interests of owners, they are induced to choose a higher sales price than they would in the absence of the default option, implying that early sale of the assets is less likely and default more likely. The higher likelihood of default adversely affects the value of the debt, thereby impairing the position of the creditors.

We argue that the conflict of interest between owners and creditors is likely to be more important in the context of liquidating assets—the setting analyzed in this paper—than in that of undertaking new investment projects, the setting customarily assumed. Typically (although, of course, not always) the corporations undertaking large and risky new investment projects are successful corporations, corporations for which default in the foreseeable future is unlikely under any of the investment options being considered. That being the case, the distortion that occurs when managers, acting in the interests of owners, ignore the costs to creditors of default is minor. However, corporations that are liquidating assets are likely to be those that have done badly, and for which therefore default is likely to occur. Often, in fact, the corporation is liquidating assets precisely in order to avoid or postpone default.

The distortion that occurs when managers ignore the costs to creditors of default is likely to be more important quantitatively for such corporations than for corporations undertaking new investment projects. In this paper we present a simple model in which a corporation's assets are illiquid and their purchase is financed by defaultable debt. The management is assumed to act on behalf of owners (without agency problems, we unrealistically assume), and its only role is to choose the strategy used to dispose of the assets upon liquidation. Inasmuch as we will not be distinguishing between owners and managers in the model, we will refer to "owner-managers" below. As suggested above, in this setting the management adopts a liquidation strategy that, besides being socially inefficient, is detrimental to the interests of creditors.

At a minimum, the model to be presented makes clear how owners will exploit creditors when they are able to do so. However, we claim also that the stylized facts that our model leads us to expect are descriptively accurate, at least in some qualitative sense. Inasmuch as it is notoriously difficult to draft and enforce bond indentures or other corporate control provisions that completely protect the creditors against the owners, it would be surprising if our model were completely without empirical relevance. Finally, we believe that because our model gives some insight on how the default option alters asset pricing decisions, it is of independent theoretical interest.

2 The Model: Informal Description [1]

There exists a single indivisible productive asset which we call a factory. There is an infinite number of potential owner-managers of this factory. These agents have common endowment of a background good, common discount factor and are risk neutral. One of these owner-managers, whom we will call the initial owner-manager, has the factory as part of his endowment. The others will buy the factory from him, each in turn having the opportunity to become the current owner-manager. The current owner-manager of the factory has a *match*, with *fit* ε, if he is able to operate the factory with operating profit of ε per period. The fit continues period after period, implying constant profit of ε per period, until the match is broken, an event that occurs with probability that is constant and independent over time. When the match is broken the owner-manager can no longer operate the factory profitably, and will sell it.

The initial owner-manager does not have a match. Therefore he immediately offers the factory for sale. The other potential owner-managers can operate the factory with varying degrees of profitability. Determining profitability is a costly search-and-matching process. In order to model this process expeditiously, we assume that each period one and only one potential owner-manager has the opportunity to determine costlessly the profit potential of the factory under his management.[2] The seller must sell to this agent immediately or forego forever the opportunity to sell to him.

Potential buyers know their fit at the time they decide whether or not to buy the factory, but the seller does not know the fit and cannot compel or induce the buyer

[1] The model described below is similar in essential respects to that of Krainer and LeRoy [6], to which readers should refer for general discussion. See also Krainer [5].

[2] An alternative specification would be to allow n potential buyers at each date. Then, for example, the factory might be assumed to be sold via a sealed-bid auction, with the highest bidder receiving the factory at the price bid by the second-highest bidder. The seller's role would be to set a reservation price. In this setting the optimal reservation price is independent of the number of bidders, therefore equaling the sale price derived below in the case of $n = 1$. This invariance of the reservation price to the number of bidders is standard in the auctions literature (Riley and Samuelson [12]). With the reservation price independent of n, it is evident that taking $n > 1$ would not materially alter the analysis.

to reveal it to him. The seller does know that the fit ε is distributed uniformly on $[0, 1]$. The seller posts a take-it-or-leave-it price; this price is chosen to balance high revenue if the factory is sold against a high probability of selling the factory.

The endowments of potential owner-managers are not sufficient to allow them to pay the purchase price of the factory in cash (here we depart from Krainer and LeRoy [6]). Therefore the contract between the buyer and the seller consists of an annuity that is secured by the factory: the buyer pays the seller an agreed-upon sum each period, constant over time, as long as he owns the factory. There is no down payment, so the purchase is 100% financed (we consider below the consequences of relaxing this assumption).

The initial owner-manager offers the factory for sale until, sooner or later, one of the potential owner-managers determines that he has a fit that is high enough to justify buying the factory, which he therefore does. The new owner-manager operates the factory for as long as his match persists, paying the initial owner-manager a fixed payment each period. Eventually the new owner-manager loses his match and offers the factory for sale. Unless he defaults, the current seller must continue to make annuity payments until the factory is sold. If the current seller sells the factory for a higher price than he paid, then he keeps the difference. Because both prices are paid in the form of annuities, the difference is an annuity as well. If the seller determines that the annuity payments are higher than the expected proceeds of the sale, he exercises his option to default on the annuity. In that case the factory, along with the annuity, reverts to its preceding owner-manager. The preceding owner-manager also has the option to default immediately, returning the factory and the annuity to a still earlier owner-manager, and so on.

For concreteness, we will think of each owner of the factory as paying his entire annuity to the preceding owner, who in turn pays the preceding owner a smaller annuity payment, and so on back to the initial owner. Alternatively, one could think of the current owner as paying annuities to all the preceding owners. Since there are no transactions costs in our model, the two specifications are obviously equivalent.

The conventions on timing—in particular, when the annuity payment is made—are important in understanding the equilibrium. Figure 1 is a flow chart depicting the sequence of events from the viewpoint of the current owner-manager of the factory.

As noted, the owner-manager with a match receives profits each period at the same rate, ε, as he received at each of the dates since he bought the factory. He learns each period whether the match continues for another period. If it does, he makes an annuity payment, receives his profit and does nothing else until the next date, at which time the cycle is repeated. If the match does not continue, the owner-manager immediately puts the factory up for sale; doing so dominates defaulting immediately because the convention on timing is such that he can make one attempt to sell the factory before the next annuity payment is due. If the first prospective buyer purchases the factory, then the current owner-manager goes into the next period without the factory. He profits insofar as the annuity he receives from the next owner-manager exceeds what he pays the preceding owner-manager. If the first prospective buyer does not purchase the factory, the current owner-manager must decide whether to default without making the annuity payment or to make the annuity payment and attempt again to sell the factory next period.

Default puts the factory in the hands of the preceding owner-manager. Since the preceding owner-manager has a lower annuity payment, he may decide to take possession of the factory, make that payment and offer the factory for sale. Alternatively, he may default in turn, in which case the factory immediately reverts to a still earlier owner-manager, who faces a similar decision problem, but with a smaller annuity payment. Sooner or later one of the owner-managers chooses to offer the factory for sale rather than continue the default chain (it turns out that in equilibrium this is always the initial owner-manager, who has no debt). When that happens the cycle repeats.

3 A Continuum of Nash Equilibria

Let p be the per-period payment on the annuity that the agent is considering paying for the factory in his role as buyer, and the annuity he is paying in his role as owner-manager or ex-owner-manager; optimal values of all decision variables depend on p. Each agent's strategy set consists of (1) a reservation fit function $\varepsilon^*(p)$, (2) a sales price function $\phi(p)$, (3) a function of p that determines whether the agent, upon failing to sell the factory at the first try, defaults or continues trying to sell it in subsequent periods, and (4) a function of p that determines whether the agent, receiving the factory upon the default of a subsequent owner-manager, continues the default chain or pays the annuity and continues trying to sell it in subsequent periods.[3] Actions (3) and (4) are essentially the same: an agent will default in one case if and only if he defaults in the other, the calculation being the same in the two cases. Hereafter we will not distinguish between the two.

The reservation fit is such that the agent in his role as buyer will buy the factory for the asking price p if his fit exceeds the reservation fit (which depends on p), and not otherwise. If the agent's fit exceeds the reservation fit, by definition his expected discounted profits from operating the factory are more than sufficient to justify the purchase price. When the owner-manager loses his fit and sells the factory, the price he asks depends on his annuity payments (and, we assume, on nothing else). The sale price $\phi(p)$ depends on the annuity payment p—specifically, always exceeds the annuity payment—because otherwise immediate default dominates selling the factory. Finally, each agent's strategy includes a determination as to what to do if he receives the factory when forfeited by default of a subsequent owner-manager. His choices are either to continue the default chain so that he never takes title to the factory, or

[3] Note that we are deleting the weakly dominated strategy of defaulting immediately upon losing the match.

to terminate the default chain. In the latter case he resumes annuity payments at the same level as when he owned the factory, meanwhile continuing to try to sell it.

We seek a symmetric Nash equilibrium: each agent's chosen strategy is a best response to the same strategy when chosen by other agents.[4]

It is apparent that this is a complicated game. There exists an infinite number of players, and each of an infinite subset of these agents plays an infinite number of times with probability one. Further, there exists an infinite number of plays by nature, the outcomes of which are not observed symmetrically. We are not aware of any general results on games like this. Therefore instead of characterizing the equilibrium set we will display one class of Nash equilibria in this section and two equilibria in the two following sections.

The equilibria analyzed in this section and the next are not subgame perfect, but the equilibrium of Section 5 is.

Consider any $\overline{p}$ such that $0 < \overline{p} < 1$. Associated with this value of $\overline{p}$ is a Nash equilibrium for which the decision rules are as follows:

- Buy rule: Buy if $p = \overline{p}$ and $\varepsilon \geq \overline{p}$, and not otherwise.
- Sell rule: Sell for $p = \overline{p}$.
- Default rule: for the non-initial owners, always default as soon as the match fails.

The buy rule is optimal because the agent knows that when he loses his match he will be unable to sell the factory for more than he paid. This is so because subsequent owner-operators will buy only if $p = \overline{p}$, in which case the current owner has a net cash flow of zero since the annuity he is paying also equals $\overline{p}$. Therefore he will buy only if net cash flow is positive. The sell rule is optimal because the buyer will buy only if $p = \overline{p}$, so the seller has no alternative to setting $p = \overline{p}$. The default rule is optimal because default weakly dominates continuing trying to sell since, again, there is no prospect of generating a positive cash flow from sale.

The game just described involves sequential play, implying that every node of the game tree is the initial node of a subgame. The equilibrium just described is not subgame perfect and, in fact, is not very plausible: being presented with a price not equal to $\overline{p}$, it makes no sense for the buyer to refuse to buy the factory regardless of his fit. Yet if he deviates from this rule, it is no longer generally optimal for the seller to sell for $p = \overline{p}$.

[4] In Section 4 below it is convenient to treat the initial owner and non-initial owners of the factory separately. This amounts merely to anticipating part of the equilibrium; adopting notation that represents the symmetric nature of the equilibrium would be unnecessarily cumbersome.

4 A Nash Equilibrium with Increasing Prices

In this section we derive a Nash equilibrium that is more interesting than that of the preceding section, as well as more plausible. The equilibrium to be derived involves each owner of the factory who has lost his match offering it for sale at a price strictly higher than that which he paid. The analytical procedure for solving the model involves (1) conjecturing the decision rule for default, and (2) computing buy and sell rules that are best responses to each other and to the assumed default rule.

We begin by conjecturing that any owner-manager other than the initial owner-manager (who has no debt) will always default upon failing to sell at the first opportunity, as in the equilibria described in the preceding section. Correspondingly, we conjecture that such owner-managers will always elect to continue the default chain upon the default of a subsequent owner rather than terminate it. Thus, according to the conjecture, in equilibrium the factory will always be returned immediately to the original owner-manager upon the failure of a subsequent owner-manager to sell at his first opportunity.

4.1 Equilibrium Behavior of Non-Initial Owner-Managers

We begin by determining the behavior of the non-initial owner-managers. The behavior of the initial owner-manager is considered in the following subsection.

All potential owner-managers are risk neutral and have discount rate β. Let $v(\varepsilon, p)$ be the net value to its owner-manager of a factory with fit ε and annuity payment of p per period. Risk neutrality implies that $v(\varepsilon, p)$ is given by

$$v(\varepsilon, p) = \beta(\varepsilon - p) + \beta\pi v(\varepsilon, p) + \beta(1 - \pi)z(p), \tag{1}$$

where π is the probability that the match will continue into the next period, assumed given and constant, and $z(p)$ is the value of a factory with annuity payment p for which the owner-manager does not have a match. The owner-manager who has just lost his match could default immediately, but as noted above that option is dominated by offering the factory for sale and defaulting thereafter if the factory does not sell to the first prospective buyer.

If the owner-manager offers the factory for sale at price y, then its value is

$$\mu(y)\lambda(y)(y - p), \tag{2}$$

where $\mu(y)$ is the probability that the factory will be sold to the first prospective buyer. The coefficient $\lambda(y)$ is the value per unit of an annuity secured by the factory. The unit value $\lambda(y)$ of the annuity depends on y because y affects the probability of a subsequent default. With probability $1 - \mu(y)$ the owner-manager will fail to sell the factory, in which case by assumption he will default, which has zero value.

The seller will set y to maximize (2), and $z(p)$ equals the resulting maximum:

$$z(p) = \max_y [\mu(y)\lambda(y)(y - p)]. \tag{3}$$

Define $p' = \phi(p)$ as the maximizing value of y:

$$p' = \arg\max_y [\mu(y)\lambda(y)(y - p)] \equiv \phi(p). \tag{4}$$

From the definition of p', $z(p)$ is given by

$$z(p) = \mu(p')\lambda(p')(p' - p). \tag{5}$$

Finally, we need to evaluate $\lambda(p')$, the unit value of the annuity payment. The current owner-manager (with annuity payment p) sells the factory in exchange for an annuity payment of $p' = \phi(p)$ per period. The new owner-manager will eventually lose his match and try to sell the factory for p'', where $p'' = \phi(p')$. If he succeeds, which occurs with probability $\mu(p'')$, the annuity will have value $\lambda(p'')$ per unit of payment. If he fails he will default, in which case the annuities terminate and the factory is returned to the original owner-manager. It follows that $\lambda(p')$ is given by

$$\lambda(p') = \beta + \beta\pi\lambda(p') + \beta(1 - \pi)\mu(p'')\lambda(p''). \tag{6}$$

4.2 Equilibrium Behavior of the Initial Owner-Manager

The initial owner-manager of the factory has no annuity payment, so p does not enter the initial owner-manager's decision problem. Also, because the initial owner-manager will continue trying to sell the factory until he succeeds in doing so (rather than default, as the non-initial owner-managers do), his problem is

$$\max_y [\mu(y)y(\lambda(y) + c(y)) + \beta(1 - \mu(y))m] \tag{7}$$

$$c(y) = m \sum_{n=1}^{\infty} \beta^n \left\{ \sum_{j=1}^{n} \begin{pmatrix} n-1 \\ j-1 \end{pmatrix} \pi^{n-j}(1-\pi)^j \left(\prod_{k=1}^{n-1} \mu(\phi^k(y)) \right) (1 - \mu(\phi^j(y))) \right\}. \tag{8}$$

Here $\phi^k(y) \equiv \phi(\phi(...\phi(y)))$, and m equals the maximized value of (7). The function $c(y)$ is the component of the current value of the factory accounted for by the eventual default of a non-initial owner. It represents $m\beta^n$, the discounted value of the factory conditional on the event that the j-th owner of the factory is the first to default, and that this default occurs n periods in the future, multiplied by the relevant probability—the term in braces—summed over j and n. The term $c(y)$ is a determinant of the maximized value of the initial sale price because different sale prices affect the probability distribution of subsequent defaults: the higher the initial sale price the earlier default will occur, and therefore the higher is the expected present value of the discounted collateral. At the optimum this effect just offsets at the margin the effect of a higher sale price on (1) the size of the annuity payment, (2) the value of the annuity, and (3) the probability of sale.

Note that the initial owner takes as given the functions $\phi(y)$ and $\mu(y)$ describing the behavior of the non-initial owner-managers derived in the preceding section.

4.3 Equilibrium Buyer Behavior

We next express the probability of sale $\mu(p)$ in terms of $z(p)$. From (1) the value of the factory for which the owner-manager has a match with fit ε is

$$v(\varepsilon, p) = \frac{\beta(\varepsilon - p) + \beta(1-\pi)z(p)}{1-\beta\pi}. \tag{9}$$

Now define $\varepsilon^*(p)$ as the reservation fit, so that a prospective buyer acquires the factory in exchange for an annuity payment of p if $\varepsilon > \varepsilon^*(p)$, and not otherwise. If we set the buyer's outside option equal to zero, then:

$$v(\varepsilon^*(p), p) = \frac{\beta(\varepsilon^*(p) - p) + \beta(1-\pi)z(p)}{1-\beta\pi} = 0, \tag{10}$$

implying

$$\varepsilon^*(p) = p - (1-\pi)z(p). \tag{11}$$

This equation, incidentally, implies that an agent may buy the factory even if $\varepsilon < p$, since he will have an asset worth $z(p) > 0$ when he loses the match. If $\varepsilon^*(p) < \varepsilon < p$, the prospect of eventually losing the match and selling the factory outweighs the loss incurred while the match continues, so the prospective buyer will buy the factory.[5]

As noted above, the fit ε is assumed to be distributed uniformly on the unit interval. From equation (11), the probability of sale is therefore given by

$$\mu(p) = prob(\varepsilon \geq \varepsilon^*(p)) = 1 - \varepsilon^*(p) = 1 - p + (1-\pi)z(p). \tag{12}$$

4.4 Solving the Model

The behavior of the non-initial owners is characterized by the following equilibrium decision rules:

- Buy rule: the buyer facing price p buys if $\varepsilon \geq \varepsilon^*(p)$, and not otherwise.
- Sell rule: the seller with annuity p offers the factory for sale at price $\phi(p)$.

[5] In this case the buyer would prefer to offer the factory for sale immediately rather than wait until the match is lost. However, in the interest of simplification we ruled this possibility out.

- Default rule: upon failing to sell the factory at the first try, the seller defaults for all $p > 0$.

The initial owner-manager sets the initial sale price at the value of y that maximizes (7), and continues trying to sell the factory until he succeeds.

The equilibrium behavior of the non-initial owner-managers does not depend on that of the initial owner-manager. Therefore we can characterize the behavior of non-initial owner-managers without reference to that of the initial owner-manager. Using (4) and (12), equations (3) and (6) form a vector of recursive functional equations for the functions z, λ, and ϕ:

$$z = \Psi_z(z, \lambda, \phi) \tag{13}$$
$$\lambda = \Psi_\lambda(z, \lambda, \phi) \tag{14}$$
$$\phi = \Psi_\phi(z, \lambda, \phi). \tag{15}$$

Computing these equations involves first assigning parameter values (we chose $\beta = 0.8$ and $\pi = 0.9$ for illustration) and guessing the functions z, λ, and ϕ. Then we computed functions λ_1, z_1 and ϕ_1 and repeat until convergence. The convergence criterion is $\|\phi_1(p) - \phi_0(p)\| < .0001$, $\|\lambda_1(p) - \lambda_0(p)\| < .0001$, and $\|z_1(p) - z_0(p)\| < .0001$, assuming the supremum norm throughout. In the experiments described below we found that there was no difficulty obtaining convergence.[6]

Given the functions ϕ and μ computed from the behavior of non-initial owner-managers, we then maximized (7), and set p_1 equal to the resulting maximum:

$$p_1 = \arg\max_y(\mu(y)\lambda(y)y + \beta(1 - \mu(y))m). \tag{16}$$

4.5 Equilibrium

The equilibrium path of the economy in the computed example is easily characterized using the equilibrium policy function. The initial owner of the factory, not having a match, immediately offers the factory for sale at price p_1, continuing to offer it at this price until it is sold. The second owner operates the factory, making payments of p_1 per period to the initial owner, until he loses his match. At that time he offers it for sale at price $p_2 = \phi(p_1)$. If the first prospective buyer decides to buy the factory, the initial owner will continue to receive an annuity of p_1 while the second owner will receive an annuity of $p_2 - p_1$. The third owner will operate the factory, making these payments each period, until he loses his match, at which time he offers it for sale at price $p_3 = \phi(p_2)$, and so on.

[6] We do not have a proof that Ψ_z, Ψ_λ and Ψ_ϕ define a contraction. The problem is that $z(p)$ enters multiplicatively in these functions, resulting in nonmonotonicity.

If, upon losing his match, the second owner fails to sell the factory to the first prospective buyer, he will default on the annuity. This implies returning the factory to the initial owner, who will offer the factory for sale, again at price p_1. Therefore the cycle will repeat.

Similarly, if the first $n-1$ owners of the factory succeed in selling the factory at the first try but the n-th owner fails to do so, he will default. Each of the preceding owners will reject the option to terminate the default chain and offer the factory for sale, instead passing it back to the preceding owner. Under our conventions on notation, this happens instantaneously. Therefore upon the n-th owner's failure to sell, all the annuities that the first $n-1$ owners pay the preceding owners go into default, and the factory reverts to the initial owner.

It remains to verify that, under the equilibrium values of functions $\phi(p)$ and $\varepsilon^*(p)$, each non-initial owner-manager does in fact optimally choose to default, as assumed in deriving the equilibrium. Non-initial owner-managers will default if $z(p) < p$, in which case the value of the factory is less than one annuity payment. Figure 2 plots $z(p)$ and p. It shows that the critical value of p above which default occurs is 0.34. Since $p_1 = 0.69$ and $p_{j+1} > p_j$ for all j, we see that all non-initial owner-managers will in fact behave as conjectured.

Since sale on the first try is increasingly improbable for the second, third or fourth owners, it is highly unlikely that any given credit chain will involve more than three or four owners before collapsing (Table 1 shows the transition probabilities and the distribution of the length of a credit chain for the parameter values specified above). Of course, since time is infinite then a chain of any given length will occur infinitely often with probability 1.

4.6 Interpreting the Equilibrium

The policy function $\phi(p)$ is easily interpreted. As Figure 1 indicates, $\phi(p)$ is very close to $(p+1)/2$. To see why, observe that, from a Taylor expansion of (12) around $p' = 1$, we have

$$\mu(p') \cong 1 - p', \tag{17}$$

since $z(1) \cong 0$ and, since $\mu(\phi(p)) \cong 0$, also

$$\lambda(p) \cong \frac{\beta}{1-\beta\pi}, \tag{18}$$

from (6). From (3) the seller's problem is therefore approximately that of choosing y to maximize

$$\frac{(1-y)\beta(y-p)}{1-\beta\pi}, \tag{19}$$

the first-order condition for which is

$$y = \frac{p+1}{2}. \tag{20}$$

This approximation is based on the presumption that non-initial owner-managers of the factory will fail to sell on the first try, and will therefore default. This presumption is more accurate the higher the value of p, since $\mu(p)$ is close to zero when p is near 1, but as Figure 1 indicates the approximation is fairly accurate even for $p > 0$.

The initial agent chooses a selling price of 0.69. This figure is substantially higher than the limiting value of $\phi(p)$ as p approaches zero, which is approximately 0.47. The reason is that the non-initial owner-managers choose low sales prices to as to induce subsequent owner-managers to do the same, the idea being to reduce the probability of default and consequent termination of the annuity. The initial owner-manager has less motivation to reduce the probability of default because he is aware that he will recover the factory in the event of default of a subsequent owner-manager.

The optimal sale price (20), set to optimize the interests of the owner-manager, exceeds the sale price that would be preferred by creditors. It is easy to see that creditors would prefer a sale price of $y = p$: creditors continue to receive payments if a sale occurs as long as to sale price is sufficient to generate the annuity payments ($y \geq p$). Because the probability of sale is decreasing in y and the annuity payment (being equal to p) is constant in y for $y \geq p$, creditors prefer $y = p$.

The proportional markup of the price chosen by owner-managers over that preferred by creditors is thus

$$\frac{y-p}{p} \cong \frac{1-p}{2p}, \tag{21}$$

again using (20). For high p, the interest of creditors and owner-managers become more aligned, as both desire a high sale price.

The conflict of interest between owner-managers and creditors also affects the liquidity of the factory as measured by the probability of sale. Using (20), we have

$$\mu(\phi(p)) \cong 1 - y = \frac{1-p}{2}. \tag{22}$$

If the sales price were set to optimize the interest of the creditors, then $\mu(\phi(p)) \cong 1-p$. Hence if at any stage the creditors determined the sale price, the probability of sale would approximately double, relative to its value when the owner-managers determine the sale price.

The possibility of default plays a central role in generating the successively higher selling prices that occur in equilibrium. To see this, we show that if default were not possible the annuity payment would be a sunk cost, implying that all sellers would face a problem identical to that of the initial owner-manager. Because any sale results in a perpetuity if subsequent owners cannot default, (7) becomes

$$z(p) = \mu(p')\frac{\beta}{1-\beta}(p' - p) + (1 - \mu(p'))\beta(z(p) - p), \tag{23}$$

Here $z(p)$ separates into the difference between two terms: a term $\overline{z}$ reflecting the unencumbered value of the factory and a term $\beta p/(1-\beta)$ reflecting the value of the perpetuity. To verify this, substitute $\overline{z} - \beta p/(1-\beta)$ for $z(p)$ in (23). The terms in p drop out and (23) reduces to

$$\overline{z} = \mu(p')\frac{\beta p'}{1-\beta} + (1 - \mu(p'))\beta\overline{z}, \tag{24}$$

so that the value of the factory to the owner who has lost his match is a weighted average of its discounted value if sold and its value if not sold, with the relevant probabilities as weights, irrespective of the preexisting annuity payments. Hence the optimal sales price without default is a constant in p.

5 Subgame Perfect Equilibrium

The equilibrium described in the preceding section, like the equilibria of Section 3, is not subgame perfect. It was derived under the assumption that owner-managers with annuity p will default upon failing to sell the factory at the first try, for all $p > 0$. For sufficiently low p this behavior is suboptimal. It is easy to see that the optimal default rule is to default if the value of the factory based on the prospect of its future sale, $z(p)$, is less than the annuity payment p^*, and not otherwise. Since $z(p)$ is strictly decreasing in p, it follows that there exists a critical value of p, which we call p^*, defined by

$$z(p^*) = p^*, \tag{25}$$

such that the utility-maximizing default rule is

- Default if $z(p) < p$, and not otherwise.

In the example, $z(p) = p$ for $p = 0.34$, so the optimal default rule is

- Default whenever $p > 0.34$, and not otherwise.

We need to show that the optimal decision rules for buying and selling in the Nash equilibrium of Section 4, in which agents default for all $p > 0$, continue to be optimal under the modified default rule. Since in the example $p_1 = 0.69 > p^* = 0.34$ and $p_{j+1} > p_j$ for all j, we see that all non-initial owners are in the default region under both default rules. Therefore their optimal buying and selling behavior is unaffected by the change in the default rule.

To verify subgame perfection, it remains to show that the alteration of the assumed default rule of non-initial owners will not induce the initial owner to alter his sale price. It is immediate that there cannot exist an equilibrium in which the initial owner sells at a price in the no-default region. This is so because if the second owner never defaults on the perpetuity, then the perpetuity is a sunk cost. That being the case, existence of the perpetuity cannot affect the sale price chosen by the second owner. It follows that he will choose the same price as the initial owner, p_1. Offering the factory at p_1 has a return of zero since the second owner receives an annuity with zero net payments if the factory is sold and defaults if the factory is not sold. But then an offer at p_1 cannot be optimal, since the second owner can raise the offer price and get a positive return if the factory is sold and default if the factory is not sold (contradicting both that the second owner offers p_1 and that p_1 is in the no-default region).

To confirm that our computed example is in fact a subgame perfect equilibrium, we must show that the optimized value of the factory $\widehat{m}$ if sold in the no-default region is less than m, its value if sold for p_1, so that the hypothesized equilibrium selling rule on the part of the initial owner is in fact optimal. In the example this turns out to be trivial: $\widehat{m}$ is clearly less than the value of the factory based on the assumption the initial owner can sell it for price p^* with probability 1. But that value is less than m:

$$\widehat{m} < \frac{\beta}{1-\beta} p^* = 1.35 < 1.59 = m. \tag{26}$$

It follows that the initial owner will not alter his selling rule under the modified default rule. Accordingly, the equilibrium price sequence $(p_1, p_2, ...)$ calculated in Section 4 is a subgame perfect equilibrium price sequence under the buy and sell rules

- Buy rule: the buyer facing price p buys if $\varepsilon \geq \varepsilon^*(p)$, and not otherwise.
- Sell rule: the seller with annuity p offers the factory for sale at price $\phi(p)$.
- Default rule: upon failing to sell the factory at the first try, the seller (except, of course, for the initial owner) defaults for all $p > p^*$, and does not default otherwise.

6 Alternative Institutional Arrangements

The financial institutions underlying the equilibrium derived above are very simple: all the preceding owners of the factory in a given credit cycle collectively finance its purchase by each subsequent owner. This specification is unrealistic. However, it should be clear that alternative more realistic institutional settings would support the same equilibrium decision rules and price sequence.

For example, one could imagine that each seller of the factory receives a lump-sum price instead of an annuity, and that a risk-neutral third-party lender finances each purchase of the factory via a 100% mortgage. The lump-sum price $p'\lambda(p')$ that corresponds to the periodic payment p' is the product of the periodic payment and the per unit value of the annuity. The payment rate on the annuity is $1/\lambda(p')$. This payment rate, of course, makes appropriate allowance for the possibility of default.

Under this setup each owner of the factory, upon losing his match, would attempt to sell to the subsequent owner for cash. If sale occurs on the first try, the current seller would receive a lump-sum payment out of which he would pay off his loan. The capital gain he receives—equal to the difference between sale price and purchase price—equals the capitalized value of the annuity $p' - p$ in the model discussed above. The new owner would take out a new loan from the bank equal to 100% of the purchase price. Since at each stage the cash price equals the expected present value of the annuity, each seller would be indifferent between receiving the annuity, as in the model of this paper, or the lump-sum payment, as under the alternative arrangement. The annuity is secured by the factory so that, upon default, under the alternative arrangement the factory would be returned to the bank rather than to the preceding owner, who is out of the loop under the alternative arrangement. The bank would then resell the factory, repeating the cycle. Since the budget sets in this setting are the same as those in the model described above, the equilibrium policy function is the same as well. Consequently the equilibrium price sequences in the two settings are also the same.

The foregoing modification of the institutional arrangement maintained the assumption of the present paper that the current owner pays for the factory using an annuity with value equal to 100% of the factory price. Suppose instead that the buyer had the option to pay cash for all or part of the purchase price. The buyer would use the financing mix as a device to precommit to a default probability, and would optimally choose the financing package that minimizes the cost of capital. This would result in an equilibrium price sequence that differs from that derived here.

7 Credit Chains

Up to now we have interpreted the model as providing an extended illustration of what would happen if agents were completely unable to find an institutional device to prevent management from acting exclusively in the interests of current owners. A less conservative interpretation would view the model as providing a substantive treatment of such phenomena as credit chains. We hesitate to endorse such a reading in the absence of an explanation of why agents are unable to devise an institutional modification that would remedy the inefficiency. However, interpreting the model in this way allows us to make an important point that must be addressed in any discussion of credit chains or, more generally, any particular pattern of financial transfers.

The preceding discussion implies that financing transfers of illiquid assets via defaultable debt may or may not be associated with credit chains: some institutional environments have them, while other substantively identical environments do not. If one specified the model so that both financing methods are available, then the equilibrium values of quantity variables would be indeterminate, agents being indifferent between the two financing methods. This indeterminacy is characteristic of models with rich financial markets and risk-neutral agents: in such models agents view all securities and portfolios as perfect substitutes (at equilibrium prices, and assuming interior consumption points; see LeRoy and Werner [9], Section 13.4). Therefore it makes no difference to the agents in the model whether sellers finance sales, as in the original setup, or whether third-party banks finance sales, as in the alternative specification just discussed.

In contrast to present setting, models of default with sparse financial markets, such as Lagunoff and Schreft [8], provide settings in which the characterization of credit chains is robust in the sense that the equilibrium with a credit chain is the only one possible. However, this robustness is achieved at the cost of drastic abstraction: indivisibilities play a central role in Lagunoff-Schreft's model (for example, Lagunoff and Schreft assumed that exactly two agents invest in each project). The contrast between Lagunoff-Schreft and the model of this paper suggests a basic question: in modeling credit chains, does one seek a robust account of why they exist, as in Lagunoff-Schreft, or only an account that is consistent with their existence, as here?

Models in which credit chains are not robust have the property that the sequence of defaults that occurs when a credit chain collapses consists merely of a transfer of the collateral from one owner to another. The initiating shock itself (in our model, the loss of a match and failure to sell the factory to the first prospective buyer) is wealth-reducing, of course, but the consequent collapse of the credit chain is not. As a result, there is no incentive for agents to devise institutions that reduce the probability of the occurrence of such collapses or diversify the attendant risk. For the same reason, such models provide no justification for regulatory intervention to prevent the collapse of credit chains. However, there is much informal evidence that collapses of credit chains are more important than this; some would argue that the collapse of the credit chain is often itself more important than the event that precipitates the collapse (for example, it could be argued that the Asian financial crises of 1997-1998 is in this class). The model presented here is not rich enough to provide a setting for such analysis.

Even though the collapse of a credit chain as modeled here is not wealth-reducing, the mechanism that produces inflation in the prices of illiquid assets does induce a misallocation of resources. In the later stages of an asset price inflation, the price set by the seller is higher than that which would be socially optimal. As a result, prospective buyers with a fairly high fit, who from the perspective of a planner should be operating the factory, will choose not to buy it.[7]

[7] Krainer and LeRoy [6] derived the reservation fit that is socially optimal in a setting that is

We observed above that the characterization in the present model of agents' assets and liabilities as constituting a credit chain is not robust. In contrast, the fact that financing transfers of illiquid assets via defaultable debt induces asset price inflations is robust. Therefore the latter and not the former should be considered the important prediction of the model.

8 Comparison with the Literature

This paper belongs to a body of work that provides theoretical analysis of equilibrium when search, or search and matching, plays a major role. This literature is vast (Stigler [15], Salop [13], Salop and Stiglitz [14], Burdett and Judd [1], Burdett and Mortensen [2], Burdett, Shi and Wright [3], Butters [4], Lagos [7], Montgomery [10], and Peters [11] are a few of the major papers). At the most general level, the themes that run through all search models appear here as well. For example, a reservation wage or price—search continues until the prospective worker (for example) receives a wage offer that exceeds the expected benefit of further search—plays a prominent role universally in the search literature. The analogue here is the reservation fit: the prospective buyer buys the factory if his fit exceeds the reservation fit, so that the value of the factory to the buyer exceeds its price.

However, in most respects our model has little in common with the rest of the literature, implying that little would be gained by undertaking a detailed discussion of other papers. First, many papers on search treat the searchers, or the commodities being purchased, as homogeneous and specify that searchers search over price.[8] Inasmuch as our purpose is to apply search theory to the problem of valuing illiquid assets, and given that we have identified illiquidity with heterogeneity and costly information, we cannot avoid explicitly incorporating heterogeneity (in our case of prospective buyers of the factory). Therefore we assume that price information is costlessly available and that agents search over a quantity-related variable (the fit). As a result, it is possible to show in settings similar to that of the present paper that such measures of liquidity as the expected time to sale depend on the frequency of arrival of buyers (which is a proxy for the cost of search) in the way one would expect (Krainer and LeRoy [6]). In contrast, in most search models the number of sales at a given store depends on a purely random factor: the number of buyers who arrive at that store. There is no apparent connection to liquidity here (of course, this is not a criticism of the cited search models, inasmuch as these were not specified with

similar in essentials to the present model. However, no proof is needed to support the assertion in the text: because the sequence of prices converges to 1, the equilibrium reservation fit will necessarily eventually exceed the reservation fit that is socially optimal.

[8] As Burdett and Judd [1] noted, the assumption of homogeneity made in much of the search literature is not easy to reconcile with that of price or wage dispersion: often no attempt is made to explain why, if the commodity is homogeneous, prices or wages show dispersion.

the application to the analysis of liquidity in mind).

The specification here differs from that of most search models in other respects as well. For example, our focus of inquiry is the valuation of capital assets (the factory and the annuities secured by the factory), whereas most search models do not deal with capital goods at all. Again the reason is that we are specializing the analysis to the study of liquidity.

9 Conclusion

We have presented a model in which a corporation's management liquidates assets in the fashion that is optimal for the owners, despite the losses that this liquidation strategy imposes on creditors. Here we consider the incentives the various players have to adopt alternative possible liquidation strategies.

Ex post, the ex-owner of the factory who receives the income on an annuity collateralized by the factory has an interest in inducing the current owner of the factory to set a lower sales price upon losing his match so as to decrease the probability of default. By assumption he has no way to do so. However, *ex ante* there is no such incentive: in equilibrium the debt is priced fairly given the regime of corporate control in place, so agents in their role as creditors are indifferent as to the control regime. As always, *ex ante* it is the owners of fixed resources that have an incentive to adopt a different regime of corporate control. The owners of fixed resources are, first, the agent who has the factory as part of his initial endowment and, second, the agents who will have an early opportunity to buy the factory (we are assuming here that the agents are numbered as to the order in which they will arrive, and that this numbering is public knowledge). These agents all own equity the value of which depends on the control regime. The value of this equity is well determined, and it fluctuates over time as the random components of the model are realized.

Because equity values depend on the control regime, the agents—especially the initial owner of the factory—have an interest in adopting indentures that constrain the prices at which they and subsequent owners sell the factory. Of course, each agent, being in a different position from the others, would prefer a different such indenture (for example, the n-th potential buyer would prefer an indenture that requires the initial owner and the first $n-1$ owners to give away the factory), so it is far from obvious what indenture provisions, if any, would be adopted. As a problem in cooperative game theory, study of this question is not a proper part of the present paper.

We have interpreted the model presented above as illustrating the conflict of interest between owners and creditors. Other applications of the model are possible: we noted above that the model provides a possible explanation of credit chains. As another example, corporate takeovers require that the buyer perform "due diligence": a detailed examination of the corporation being acquired. The expenses involved in due diligence, being considerable, imply that only one or a very small number of purchasing groups will be involved in the takeover of a large firm. Corporate finance models rarely take explicit account of these expenses, despite the fact that in the absence of such expenses the takeover market could reasonably be assumed to be competitive for all but the largest takeovers. The model presented in this paper can be interpreted as providing a setting in which the expenses of due diligence are represented by assuming that only one buyer can perform due diligence per period, implying an explanation of why the takeover market is not competitive.

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10 Appendix: Figures and Tables

State:	Transition Probabilities							state
owner, match	original	1, yes	1, no	2, yes	2, no	3, yes	3, no	prob
original, no	0.682	0.318	0	0	0	0	0	0.198
buyer 1, yes	0	0.9	0.1	0	0	0	0	0.624
buyer 1, no	0.839	0	0	0.161	0	0	0	0.062
buyer 2, yes	0	0	0	0.9	0.1	0	0	0.098
buyer 2, no	0.919	0	0	0	0	0.081	0	0.010
buyer 3, yes	0	0	0	0	0	0.9	0.1	0.008
buyer 3, no	0.959	0	0	0	0	0	0	0.001

Table 1: State transition matrix and probability distribution.

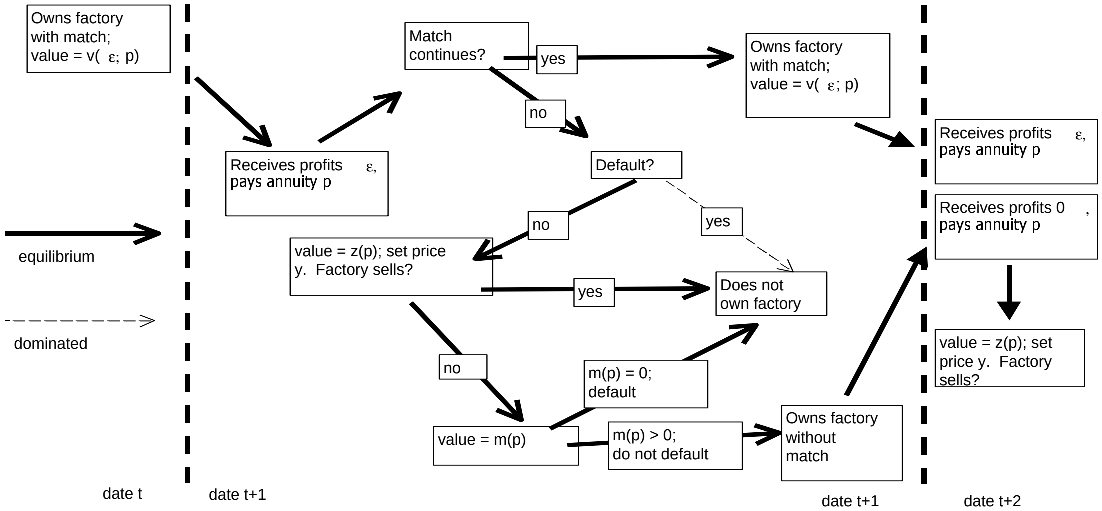

Figure 1

Time line and decision tree for first and subsequent owners.

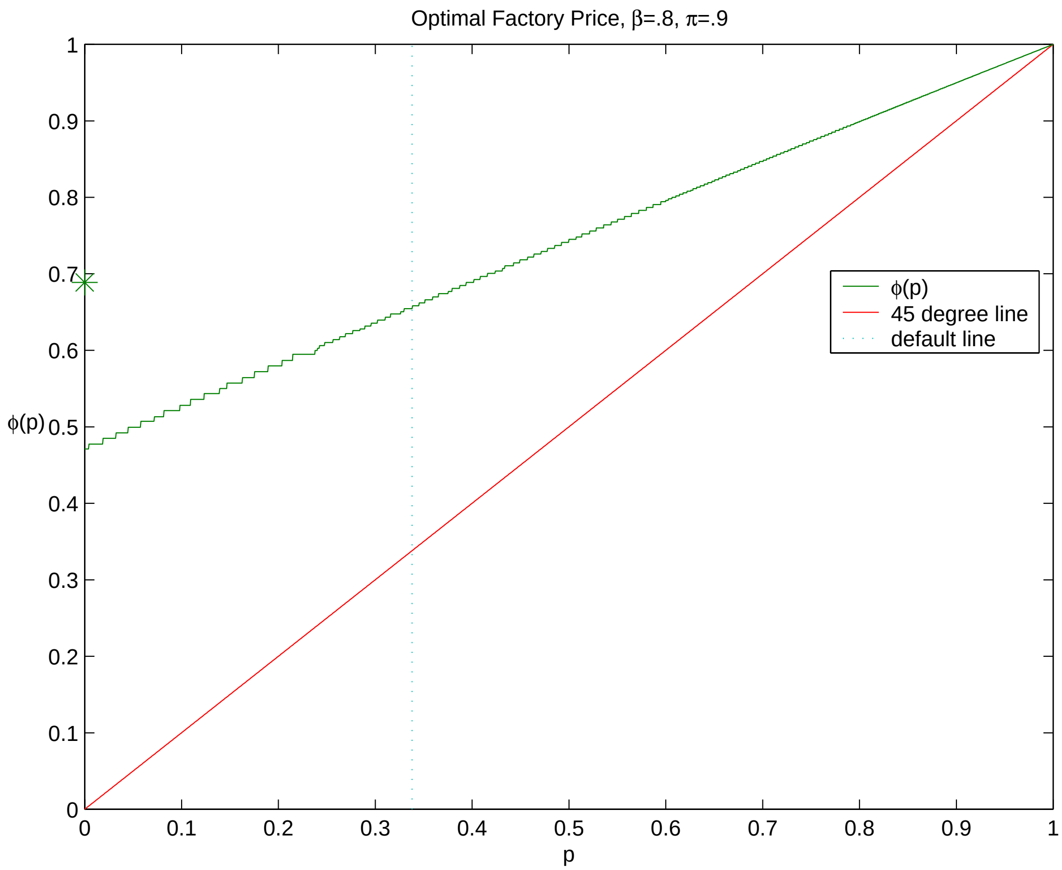

Figure 1: Optimal factory price.

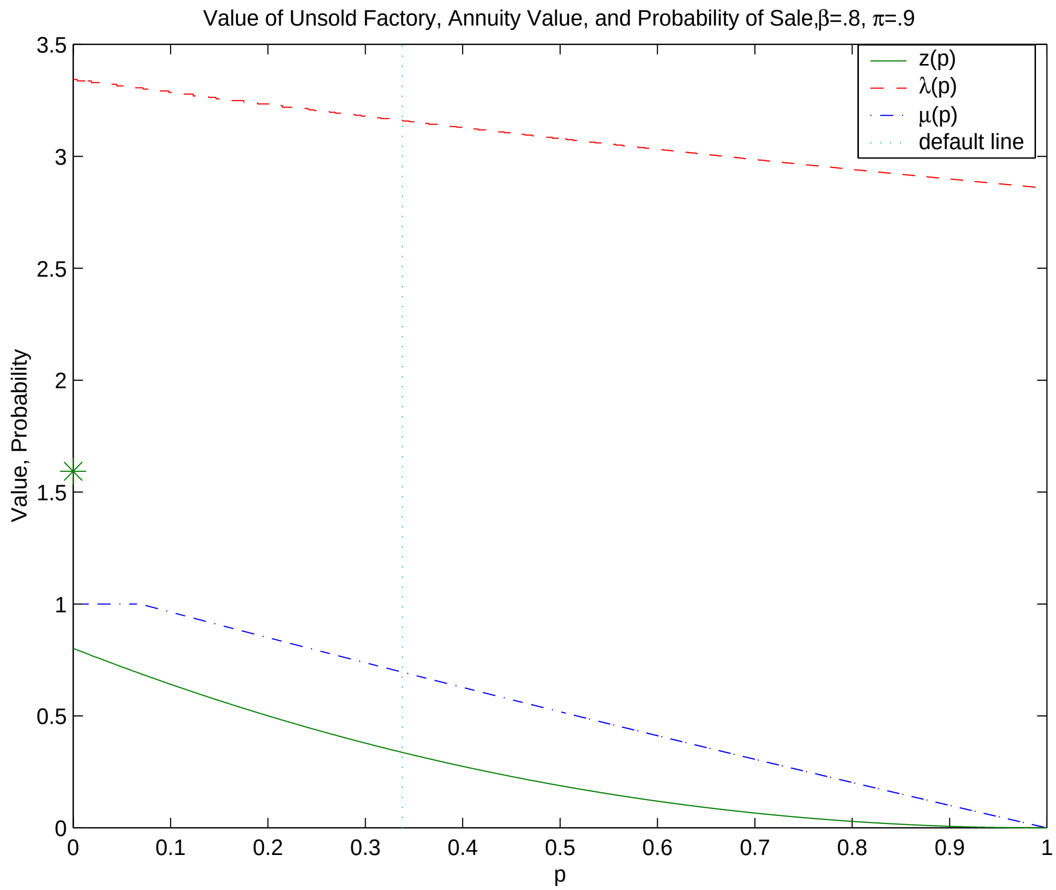

Figure 2: Value of Unsold Factory, Annuity Value, and Probability of Sale.