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HIGH-FREQUENCY ANALYSIS OF A SEMI-INFINITE ARRAY OF LINE SOURCES

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1. INTRODUCTION

The electromagnetic radiation and scattering from a semi-infinite array of line currents has been widely investigated in literature [1]-[3]. This canonical problem is of importance in engineering applications involving dycois surfaces, polarizers, near field analysis of large arrays, artificially hard and soft surfaces.

In this paper, the problem of a semi-infinite array of magnetic line currents located on a perfectly conducting half-plane has been studied. This can simulate a slot array on a semi-infinite ground-plane.

The field at finite distance is evaluated by superimposing the contributions of each source, which radiates with the exact Green's function of the half-plane. Next, via a suitable application of the Poisson summation formula, the same field is represented as series of spectral integrals, that describe the radiation of the currents relevant to Floquet modes. In the spectral domain, this series is evaluated in a closed form when the currents have the same amplitude; thus, obtaining a unique spectral integral which is arranged in a suitable form for an asymptotic evaluation as that suggested in [4] [5].

The asymptotic solution, that is not reported here, includes a ray contribution that, after diffracting at the end of the array, undergoes a second diffraction at the edge of the ground plane. This double diffraction contribution ensures the required continuity to the total field when crossing the plane of the array.

2. FORMULATION

The geometry of the problem is shown in Fig. 1. A rectangular reference system is introduced, with its z axis along the lines and the y axis normal to the plane of the array. The origin is placed at the first line source, that is placed at a distance L from the edge of the ground plane. Let us assume a periodic distribution in which d denotes the distance between two contiguous elements. The amplitude of the current is $i(nd) \exp(jk_{x0}(nd))$ ($n=0, \dots$), that corresponds to the traveling feeding wave schematized in Fig. 1. By using a conventional superposition of the field radiated by each current line, the electric z field is expressed by

$$E_z = \sum_{n=0}^{\infty} f(nd+L) \quad (1)$$

where

$$f(x') = g(x', P) i(x'-L) U(x'-L) \exp(jk_{x0}(x'-L)), \quad (2)$$

U is the Heaviside unit step function, and $g(x', P)$ is the Green's function at the observation point P of the half-plane illuminated by a source placed at x' . A suitable expression for the above Green's function is

$$g(x', P) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(k_x, P) e^{-jk_x x'} dk_x \quad (3)$$

where

$$G(k_x, P) = \frac{-j}{8\pi^2} \int_{-j\infty}^{\pi+j\infty} e^{-j\rho k_x \cos\alpha - j\rho\sqrt{k^2 - k_x^2} \sin\alpha} \frac{2 \cos(\frac{\alpha}{2}) \cos(\frac{\phi}{2})}{\cos\alpha + \cos\phi} d\alpha \quad (4)$$

in which (ρ, ϕ) denotes the cylindrical coordinate of P (Fig. 1) and the branch cut in (3) is chosen in such a way that $\text{Im}(\sqrt{k^2 - k_x^2}) < 0$ in the top Riemann sheet of the k_x complex plane. The above representation explicitly single out the Fourier transform $G(k_x, P)$ of $g(x', P)$. In order to obtain the representation (3)-(4), the reciprocity principle has been applied to the usual, exact Green's function representation [6], so that no approximation has been used yet.

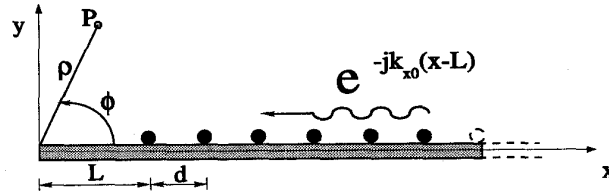


Fig. 1 Geometry of the array on a semi-infinite ground plane

The slow convergence of the series (1) suggests the use of the Poisson summation formula [1], [2], that yields

$$E_z = \sum_{n=0}^{\infty} f(nd+L) = \frac{f(L)}{2} + \sum_{m=-\infty}^{\infty} F\left(\frac{2\pi m}{d}\right) e^{-j\frac{2\pi m}{d}L} \quad (5)$$

where $F(k_x)$ is the Fourier transform (FT) of $f(x')$. This FT can be obtained by a spectral convolution between the $G(k_x, P)$ and the FT $I(k_x)$ of $i(x'-L)U(x'-L)\exp(jk_{x0}L)$, so that

$$F\left(\frac{2\pi m}{d}\right) = \int_{-\infty}^{\infty} G(k_x, P) I(k_{xm} - k_x) dk_x \quad (6)$$

where

$$k_{xm} = \frac{2m\pi}{d} - k_{x0} \quad (7)$$

is the propagation constant of the m^{th} Floquet mode.

For the sake of simplicity, let us consider a uniform distribution of current amplitude $i(x'-L)=I_0$, so that

$$F\left(\frac{2\pi m}{d}\right)e^{-j\frac{2\pi m}{d}L} = \int_{-\infty}^{\infty} \frac{I_0 G(k_x, P)}{-j(k_{xm} - k_x)} e^{-jk_x L} dk_x =$$

$$= \frac{I_0}{8\pi^2} \int_{-j\infty}^{\pi + j\infty} \int_{-j\infty}^{\pi + j\infty} \frac{e^{-jk[\rho \cos(\alpha - \alpha') + L \cos\alpha']}}{(k \cos\alpha' - k_{xm})} \frac{2 \cos(\frac{\alpha}{2}) \cos(\frac{\phi}{2})}{\cos\alpha + \cos\phi} d\alpha d\alpha', \quad (8)$$

The last equality has been obtained by using the change of variable $k_x = k \cos\alpha'$ and using (4) into (8). Taking into account that

$$\sum_{-\infty}^{\infty} \frac{1}{\beta - 2\pi m} = \frac{1}{2} \cot \frac{\beta}{2} \quad (9)$$

and using (9) in (5), leads to

$$E_z = \frac{f(L)}{2} + \frac{-d}{16\pi^2} \int_{-j\infty}^{\pi + j\infty} \int_{-j\infty}^{\pi + j\infty} e^{-jk[\rho \cos(\alpha - \alpha') + L \cos\alpha']}$$

$$\times \cot\left(\frac{k d}{2}(\cos\phi' + \cos\alpha')\right) \frac{2 \cos(\frac{\alpha}{2}) \cos(\frac{\phi}{2})}{\cos\alpha + \cos\phi} d\alpha d\alpha', \quad (10)$$

where $\cos\phi' = k_{x0}/k$.

The uniform asymptotic evaluation is performed by using the technique presented in [4],[5]. In particular, the double integral contains several two-dimensional (2D) critical points that leads to the dominant asymptotic contributions;

- a) poles at the $\alpha' = \cos^{-1}(-\cos\phi' + \frac{2\pi m}{kd})$, pole at $\alpha = \pm(\pi - \phi)$,
- b) pole at $\alpha = \pm(\pi - \phi)$, saddle point at $\alpha' = \text{tg}^{-1}(\mp \rho \sin\phi / (-\rho \cos\phi + L))$,
- c) double saddle point at $(\alpha, \alpha') = (0, 0)$.

The 2D critical points of a)-type leads to the contribution of the Floquet modes (FMs) relevant to the infinite array; the 2D points of b)-type leads to diffraction contribution of each FM at the end of the array; the 2D points of c)-type yields double diffraction contributions. These represent the further diffraction at the edge of the semi-infinite ground plane of the first order (b-type) diffracted rays.

The real and the complex poles in the α' variable are relevant to homogeneous and inhomogeneous FMs, respectively. In principle, all the poles (that are explicitly apparent in (8)) should be accounted for in the asymptotic evaluation of the integral in (10). However, in most practical cases, just the extraction of the real poles plus a few number of complex poles results in a significant improvement of the asymptotic evaluation.

It is also worth noting that, due to the spectral domain approach, both the first and the second order diffracted field obtained from the above formulation show accuracy also when the two shadow boundary of each FM mode approach each other, as happens close to the cut-off condition of the same mode.

3. PRELIMINARY RESULTS

Preliminary results are obtained to test the effectiveness of the asymptotic solution. Fig. 2 shows the field radiated at $\rho=1.2\lambda$ for an array of 100 elements with $d=0.5\lambda$. The asymptotic result (continuous line) is compared with the direct calculation of the summation (1), extended at 100 lines sources. In order to simulate the effect of the array interruption at $n=100$, an additional diffraction contribution has been added to the asymptotic solution presented above. The small discrepancy between the two curves in the shadow region is due to the fact that the contribution of inhomogeneous FM are neglected in the uniform asymptotic evaluation of the double diffracted field.

Other results will be shown during the oral presentation.

4. CONCLUSIONS

A problem of a semi-infinite array of line sources located on a perfectly conducting half-plane has been studied by using a Floquet modes approach. This approach overcome the slow convergence of the direct summation of the space fields radiated by each element of the array. The formulation presented here can be extended to the 3D case.

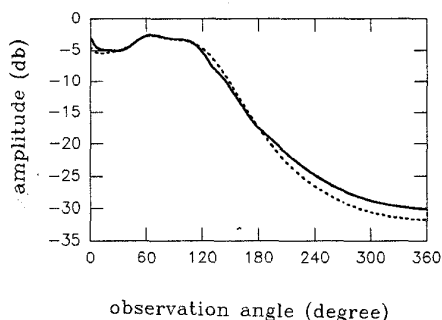


Fig. 2. Amplitude of E_z for an array of 100 lines sources. $\rho=1.2\lambda$; $\cos\phi'=30^\circ$ $d=0.5\lambda$, $L=0.8\lambda$; this solution (solid line), reference solution (dashed line).

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