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EXTREME VALUES OF SPIN-POLARIZATION
ANALYZING POWERS IN NUCLEAR REACTIONS:
M-MATRIX CONDITIONS*

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Abstract

Sets of linear relations among elements of the transition matrix M are derived for points at which spin-1 and spin- $\frac{1}{2}$ analyzing-power components reach their extreme values.

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1. Introduction

In nuclear scattering and reactions, points in energy and angle (E_0, θ_0) at which a component of the analyzing power for spin-polarization reaches its theoretical maximum are of particular interest and importance. Experimentally, they provide valuable absolute calibrations. Theoretically, there exist at such points important conditions on, or relations among, the elements of the transition matrix M which connects the initial and final states, $\chi_f = M \chi_i$. As a consequence, other polarization observables are determined, and in reactions with a particularly simple spin structure it is possible that all such observables are determined.

The simplest case is that with the spin structure $\frac{1}{2} + 0 \rightarrow \frac{1}{2} + 0$. Here it has been shown that points exist where the analyzing power¹⁾ $A_y(E_0, \theta_0) = \pm 1$ in N-⁴He, ³He-⁴He, and ³H-⁴He elastic scattering^{2,3)}. The resulting condition on the two independent M-matrix elements $M_{f,i}$ is

$$M_{\frac{1}{2}, \frac{1}{2}} = \pm i M_{\frac{1}{2}, -\frac{1}{2}}, \quad (1)$$

and the other polarization observable, the spin rotation angle, is determined.

Similar investigations involving spin-1 projectiles have now been made. Studies of $\vec{d}$ -⁴He elastic scattering^{4,5)} ($1+0 \rightarrow 1+0$) have led to the identification of points at which the tensor analyzing-power component A_{yy} is equal to its maximum value of unity. The possibility of an extreme value A_{yy} point in a nuclear reaction, as opposed to elastic scattering, was first proposed by Seiler⁶⁾ in the ³He($\vec{d}, p$)⁴He reaction ($1+\frac{1}{2} \rightarrow \frac{1}{2}+0$). Subsequent measurements by Gruebler et al.⁷⁾ confirmed that A_{yy} does reach the value of unity, within experimental error, near the energy and angle suggested.

Finally, two extreme points of A_{yy} have been proposed⁸⁾ in the ${}^6\text{Li}(\vec{d}, \alpha){}^4\text{He}$ reaction ($1+1 \rightarrow 0+0$) near 6 and 8 MeV, respectively; and recent measurements confirm that A_{yy} again attains values very close to unity in those regions⁹⁾.

In addition, the corresponding linear relations among the M-matrix amplitudes, analogous to eq. (1), at points $A_{yy} = 1$ and $A_y = \pm 1$ have been given for processes having these particular spin structures^{5,6,8,10)}.

In the $\vec{d}-{}^4\text{He}$ study it was seen⁵⁾ that at a point of maximum vector analyzing power, $A_y = \pm 1$, three equations must be satisfied among the M-matrix elements, with the important consequence that all the remaining analyzing powers were then determined:

$$A_{yy} = 1, A_{xx} = A_{zz} = -\frac{1}{2}, A_{xz} = 0. \quad (2)$$

These necessary conditions for the existence of an $A_y = \pm 1$ point have been shown to be valid in general; that is, they are valid for all reactions with the spin structure $1 + b \rightarrow c + d$, where b, c, and d are arbitrary spins¹¹⁾.

This result follows from the requirement that the spin-1 density matrix be positive semi-definite. Correspondingly, it is of considerable interest to establish, for this general case, the conditions among the M-matrix elements that result whenever an extreme value of an analyzing-power component is reached.

2. M-Matrix Conditions

For a four-particle reaction $A+B \rightarrow C+D$ with the spin structure $a+b \rightarrow c+d$, the column vector χ_i is the direct product

$$\chi_i = \chi_a \otimes \chi_b, \text{ with } \chi_a \equiv [\chi_\alpha], \chi_b \equiv [\chi_\beta],$$

and $\chi_f = \chi_c \otimes \chi_d, \text{ with } \chi_c \equiv [\chi_\gamma], \chi_d \equiv [\chi_\delta].$

The $\alpha, \beta, \gamma, \delta$ label the respective spin magnetic substates. The M-matrix thus has the dimensions $(2c+1)(2d+1)$ by $(2a+1)(2b+1)$, and its elements $M_{\gamma\delta, \alpha\beta}$ obey the parity condition¹²⁾

$$M_{\gamma\delta, \alpha\beta} = \pi_{\text{tot}} (-1)^{\sum s_i} (-1)^{\alpha+\beta+\gamma+\delta} M_{-\gamma-\delta, -\alpha-\beta}. \quad (3)$$

Here, π_{tot} is the product of the intrinsic parities of the reaction particles, and the s_i are the spins a, b, c and d .

If an analyzing-power, A_a , for the spin- a particle reaches an extreme value Q ,

$$A_a = \text{tr}(\underline{M}\underline{P}\underline{M}^\dagger) / \text{tr}(\underline{M}\underline{M}^\dagger) = Q. \quad (4)$$

The corresponding operator is $\underline{P} = \underline{P}_a \otimes \underline{I}_{2b+1}$, where $\underline{I}_{2b+1}$ is the unit matrix of order $(2b+1)$, and the operator matrix $\underline{P}_a \equiv [P_{\alpha\alpha'}]$ is of order $(2a+1)$.

In order to reduce the number of terms in writing out the traces in eq. (4), it is convenient to partition the matrices $\underline{M}$ and $\underline{P}$ as follows:

$$\underline{M} \equiv [M_{\gamma, \alpha}] \quad (5)$$

of dimensions $(2c+1)$ by $(2a+1)$, with the $(2d+1)$ by $(2b+1)$ submatrices

$$M_{\gamma, \alpha} \equiv [M_{\gamma\delta, \alpha\beta}], \quad (6)$$

and

$$\underline{P} \equiv [P_{\alpha\alpha'}], \text{ with } P_{\alpha\alpha'} \equiv [P_{\alpha\alpha'}, I_{\beta\beta'}]. \quad (7)$$

Using (5) and (7), eq. (4) becomes

$$\sum_{\gamma=c}^{-c} \sum_{\alpha=a}^{-a} \left(\sum_{\alpha'=a}^{-a} M_{\gamma, \alpha} P_{\alpha\alpha'} M_{\gamma, \alpha'}^* - Q M_{\gamma, \alpha} M_{\gamma, \alpha}^* \right) = 0, \quad (8)$$

and this is the general equation to be satisfied by the M-matrix elements whenever a component of the analyzing power for the spin-a particle reaches its extreme value Q.

2.1. SPIN-1

For a polarized particle with spin $a=1$, two observables are of immediate interest, since extreme values $A_{yy}=1$ and $A_y=\pm 1$ have been observed or proposed.

$$\underline{A.} \quad A_{yy}=+1$$

At a point $A_{yy}=+1$, eq. (8) yields

$$\sum_{\gamma=c}^{-c} M_{\gamma,1} M_{\gamma,1}^* + M_{\gamma,1} M_{\gamma,-1}^* + M_{\gamma,-1} M_{\gamma,1}^* + M_{\gamma,-1} M_{\gamma,-1}^* = 0$$

when the elements of the spin-1 Cartesian tensor operator P_{yy} are inserted¹).

Thus

$$\sum_{\gamma=c}^{-c} |M_{\gamma,1} + M_{\gamma,-1}|^2 = 0 \quad (9)$$

Then with the definition (6), eq. (9) gives the set of conditions

$$M_{\gamma\delta,1\beta} + M_{\gamma\delta,-1\beta} = 0 \quad (10)$$

for all independent combinations of β, γ and δ . There are $n=(2b+1)(2c+1)(2d+1)$ such combinations, but the parity condition (3) reduces this number to $\frac{1}{2}n$ except for reactions involving four bosons. In that case, for

$$q \equiv \pi_{\text{tot}} (-1)^{\sum s_i} = +1 \quad (-1),$$

the condition $M_{00,10} + M_{00,-10} = 0$ is (is not) contained in the parity relation (3). Hence, the number of independent conditions is $\frac{1}{2}(n-1)$ or $\frac{1}{2}(n+1)$ for $q = +1$ or -1 , respectively.

B. $A_Y = \pm 1$

In the same manner, for the extreme value $A_Y = \pm 1$, eq. (8) yields

$$\sum_{Y=C}^{-C} |\sqrt{2} M_{Y,1} \mp i M_{Y,0}|^2 + |\sqrt{2} M_{Y,-1} \pm i M_{Y,0}|^2 = 0, \quad (11)$$

from which one obtains

$$M_{Y\delta,0\beta} \pm i \sqrt{2} M_{Y\delta,1\beta} = 0 \quad (12a)$$

and

$$M_{Y\delta,1\beta} + M_{Y\delta,-1\beta} = 0 \quad (12b)$$

for a total of n independent conditions. Note that eqs. (12) include the necessary and sufficient conditions (10) for the existence of the point $A_{YY} = \pm 1$, a requirement that was previously established from the positivity requirement of the density matrix¹¹).

C. $A_{YY} = -2$

For completeness, we list the M-matrix conditions for the extreme value $A_{YY} = -2$, which corresponds to the point at the apex of the Lakin cone¹¹).

For this case eq. (8) becomes

$$\sum_{Y=C}^{-C} |M_{Y,1} - M_{Y,-1}|^2 + 2|M_{Y,0}|^2 = 0, \quad (13)$$

so that

$$M_{Y\delta,1\beta} - M_{Y\delta,-1\beta} = 0 \quad (14a)$$

and

$$M_{Y\delta,0\beta} = 0. \quad (14b)$$

These, again, are n independent conditions, except for the case of four bosons for which they number $(n+1)$ or $(n-1)$ for $q = +1$ or -1 , respectively. Note that the conditions (14a) are comparable to those for $A_{YY} = +1$, given by eq. (10), in that the same matrix elements are involved. The important difference resides in the change of sign.

2.2. SPIN- $\frac{1}{2}$

One can also apply eq. (8) in the general case that involves a polarized particle of spin $a = \frac{1}{2}$, i.e. $\frac{1}{2} + b \rightarrow c+d$. Here A_y is the only analyzing power, and it can reach the extreme values $Q=\pm 1$. With $P_a = \sigma_y$, the Pauli spin matrix, eq. (8) yields

$$\sum_{\gamma=c}^{-c} |M_{\gamma, \frac{1}{2}} \mp i M_{\gamma, -\frac{1}{2}}|^2 = 0, \quad (15)$$

which can be satisfied only if

$$M_{\gamma\delta, \frac{1}{2}\beta} \mp i M_{\gamma\delta, -\frac{1}{2}\beta} = 0 \quad (16)$$

for all $\frac{1}{2}n$ independent combinations of the indices. There is one notable similarity among eqs. (10), (12b), (14b) and (16); in all cases the particular amplitudes with the polarized-particle spin-projection $\alpha=a$ differ only by a phase from the corresponding amplitudes with $\alpha = -a$.

For polarized particles with spin $a>1$, the relevant M-matrix conditions for an extreme value of an analyzing-power component can be derived from eq. (8) when the need arises.

3. Discussion

We have derived the sets of linear relations among the M-matrix amplitudes (6) that correspond to the existence of extreme values of spin-1 and spin- $\frac{1}{2}$ analyzing powers. These conditions, given by eqs. (10), (12), (14), and (16), are similar in form to those imposed by parity conservation, eq. (3). However, unlike the latter, which are universally valid, these hold only at points (E_0, θ_0) where a particular analyzing-power component reaches its extreme value. Thus, it is clear that at or near such points, considerably more information is available for the determination of these amplitudes and, consequently, the other observables. Discussions of such determinations, for several specific cases in which such points have been proposed and have been located, are contained in a forthcoming paper¹³).

It should be noted that the a priori probability for the existence of an extreme value of the analyzing power is certain to decrease rapidly with an increase in the number of conditions to be satisfied among the M-matrix elements. Since the matrix elements are functions of only two real variables, E and θ , increasingly strong correlations between them are required in order to fulfill the multiple conditions simultaneously. That such correlations do exist has been demonstrated recently by measurements⁷) which established the point $A_{yy} \cong 1$ in the ${}^3\text{He}(\vec{d}, p){}^4\text{He}$ reaction, and, even more conclusively, by the analytical proof of the existence of another type of maximum polarization at several points (E, θ) in $\vec{d}-{}^4\text{He}$ elastic scattering¹⁴). In both cases two relations between M-matrix elements, and thus four real conditions, are fulfilled simultaneously. Extreme values of the analyzing power can therefore be expected in a variety of processes that require a similar number of conditions to be satisfied.

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References

- 1) The Madison convention, Polarization Phenomena in Nuclear Reactions, H. H. Barschall and W. Haeberli, eds. (University of Wisconsin Press, Madison, 1971), p. xxv. We follow this convention in this paper.
- 2) G. R. Plattner and A. D. Bacher, Phys. Lett. 36B (1971) 211.
- 3) R. A. Hardekopf, G. G. Ohlsen, R. V. Poore, and Nelson Jarmie, Phys. Rev. C 13 (1976) 2127.
- 4) G. G. Ohlsen, P. A. Lovoi, G. C. Salzman, U. Meyer-Berkhout, C. K. Mitchell, and W. Grüebler, Phys. Rev. C 8 (1973) 1262.
- 5) W. Grüebler, P. A. Schmelzbach, V. König, R. Risler, B. Jenny, and D. Boerma, Nucl. Phys. A242 (1975) 285.
- 6) F. Seiler, Phys. Lett. 61B (1976) 144.
- 7) W. Grüebler, P. A. Schmelzbach, V. König, B. Jenny, R. Risler, H. R. Bürgi, and J. Nurzynski, Phys Lett. 63B (1976) 273; Nucl. Phys. A271 (1976) 29.
- 8) F. Seiler, F. N. Rad, H. E. Conzett, and R. Roy, Proc. Fourth Intl. Symposium on Polarization Phenomena in Nuclear Reactions, W. Grüebler and V. König, eds. (Birkhäuser Verlag, Basel, 1976), p. 587.
- 9) R. Risler, W. Grüebler, P. A. Schmelzbach, B. Jenny, V. König, J. Nurzynski, and H. R. Bürgi, Nucl. Phys. to be published.
- 10) F. Seiler, R. Roy, H. E. Conzett, and F. N. Rad, ref. 8, p. 550.
- 11) F. Seiler, F. N. Rad, and H. E. Conzett, ref. 8, p. 897.
- 12) M. Simonius, Lect. Notes in Physics 30 (Springer-Verlag, Heidelberg, 1974), p. 38.
- 13) F. Seiler, F. N. Rad, H. E. Conzett, and R. Roy, to be published.
- 14) F. Seiler and H. W. Roser, to be published.

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