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Treated Wastewater Reuse: An Efficient and Sustainable Solution for Water Resource Scarcity

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Abstract
Wastewater has become a valuable resource in many regions of the world that face increased level of freshwater scarcity. Reuse of treated wastewater has high economic benefit, but it can also lead to environmental pollution. As such, explicit conditions must be defined to determine the optimality of wastewater reuse for society. In this paper, we develop a regional multi-sectoral model of water quantity–quality interaction among the urban, agricultural, and environmental sectors. Our interest lies in the feasibility of reuse, rather than the stability of the regional arrangements, therefore we apply a social planner's approach to this regional problem. We formally construct sufficient conditions that support the superiority of infrastructure development and conveyance of treated wastewater for irrigation, when measured against other common disposal alternatives. Using a numerical illustrative example, which relies on data and results from existing literature, we were able to replicate our theoretical findings, as well as to examine their robustness, when supporting assumptions are relaxed.

Keywords Wastewater reuse · Optimal allocation · Social planner · Quantity–quality management · Externalities

JEL Classification C61 · Q15 · Q25 · Q53

Abbreviations

WWTP	Wastewater treatment plant
CPR	Common pool resource
FOC	First-order conditions

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1 Introduction

Water resources are becoming scarce and their supply more volatile in numerous regions throughout the world. At the same time urban populations are growing rapidly. In 1960, urban dwellers accounted for a third of the global population, and by 2014 that number rose to nearly 55% (United Nations, Department of Economic and Social Affairs 2006; *Demographia World Urban Areas*, 12th Annual Edition 2016). Such increase in urban population added about 2.5 billion people to cities over the past 55 years. These growing urban centers produce sewage that needs to be treated and disposed of at high cost to the society. Alternatively it bears an opportunity cost associated with health concerns, which also result in high expenses to society. A common practice in developed countries is that the urban centers follow state regulations for treatment level, and transport the wastewater for disposal in a river, or the ocean—a costly operation involving energy, infrastructure, and environmental damage.

It has been argued that while wastewater has great potential (e.g., year-round availability, fertilizer cost-saving) in food, feed, and fish production at different scales, not all countries treat urban sewage, and even fewer countries re-use treated wastewater (Sato et al. 2013). While the global number of wastewater treatment plants and their capacity increased in 1990–1998 and 1999–2013, from 18,062 to 72,007 (FAO 2016), in many countries and over time, wastewater has remained a source of pollution. An estimated 80% of all wastewater is being discharged untreated into the world's waterways. Indeed, treating urban sewage is costly, but discharging it untreated is also costly (Hernández-Sancho et al. 2015).

Several alternatives could prove to be beneficial for society, such as the use of treated wastewater locally for irrigation. Reznik et al. (2017) show that the adoption of treated wastewater irrigation strategy benefits society through two subsequent routes. It's decreasing the competition over natural freshwater resources, and subsequently delays (or even eliminates) the need for investment in expensive water supply projects (e.g., reservoirs, sea water desalination). However, their analysis, which adopts a central planner approach, ignores environmental consequences of treated wastewater reuse, and focuses on the case of Israel—a unique economy in terms of its water institutions and competing sectors for treated wastewater. Other previous work evaluating the economic benefits and costs of wastewater reuse in irrigated agriculture (e.g., Dinar and Yaron 1986; Dinar et al. 1986; Hussain et al. 2001; Winpenny et al. 2010; Kanyoka and Eshtawl 2012) focused on maximizing the welfare of the agricultural sector subject to physical and regulatory constraints of wastewater treatment. The models used in these works assumed a given quantity and cost of treatment per volume unit of treated wastewater. Some works included the environment as a subsector, but with a priori imposed quality standards to be met by society in order to minimize environmental damage. Feinerman et al. (2001) address the issue of who should pay for the disposal of wastewater in their effort to resolve the cost burden allocation among wastewater producers (i.e., the city) and consumers (i.e., farmers). Using a conceptual regional model that facilitates negotiations, and an illustrative example from a coastline region in Israel, the authors reach the conclusion that the "polluter pays" principle could not be supported. Goldfarb and Kislev (2007) reached a similar conclusion using a steady-state analysis of a sustainable salt regime for the coastal aquifer in Israel.

In this paper we employ an optimal control framework in order to investigate the long-term economic role of treated wastewater reuse in a regional setting. We depart from the studies cited above, in several major aspects: (1) we endogenize both effluent quantity and quality in our model; (2) we introduce a dependency between the farmers and the city in the form of a shared groundwater source, therefore allowing a greater flexibility in finding a solution, as now tradeoffs between the two types of water (treated wastewater and groundwater) can be accounted for; and (3) our analysis is dynamic and considers environmental quality implications explicitly through the modeling of groundwater aquifer responses to natural changing conditions and outcomes of economic agents' behavior. By doing so, we enable the internalization of externalities for all agents involved.

The main purpose of this paper is to address the role of treated wastewater reuse in an economy that is characterized by growing scarcity of natural freshwater resources. We develop a conceptual model that addresses most of the omitted issues that were mentioned earlier. The model is a simplified representation of a region composed of decision-makers: a city manager and an agricultural grower, and their impact on the environment. The environment could be subject to negative impacts on the part of the city or the agricultural sector. In our model, the environment is represented by a waterway (e.g., dry riverbed, a flowing river or the immediate ocean coastline) which could be subject to direct disposal of treated wastewater, and by a groundwater basin (shared by the agricultural sector and the city—Common Pool Resource, CPR) which can be indirectly affected by the use of treated wastewater in irrigated agriculture, due to deep percolation (the term "environment" is used interchangeably for these two representations throughout the article). It is important to emphasize at this stage that while we focus on the agricultural sector as the potential consumer for recycled treated wastewater, the framework is general enough and relevant for alternative uses, such as golf courses, irrigation of municipal areas, and natural habitats, which all share similar challenges as discussed in this paper.

We use a social planner's approach to the regional problem of water quantity–quality effects on urban net income, agricultural productivity, and the environment. Of the various options facing the regional decision-maker, our results of the social planner's approach demonstrate that construction of conveyance infrastructure and the use of treated wastewater for irrigation is the superior alternative for the region, as it maximizes the net regional benefits.

The paper proceeds as follows: next, we develop the model framework and individual components, and demonstrate how they are linked; we derive several general results to be tested in the section dealing with the illustrative application in Sect. 3; and in Sect. 4 we conclude and introduce several regional policy implications of treated wastewater use in irrigated agriculture.

2 Setting the Framework

We start by developing a dynamic modeling framework of a regional setting that integrates the demand for and supply of treated wastewater with the physical medium of its application, accounting for the regulatory constraints associated with it. This, in turn, allows us to calculate the social benefits and costs associated with different decisions regarding the development of disposal options, and the production and use of treated wastewater. The model components are depicted in Fig. 1.

Under this setting, groundwater stock level G, in each period t, can be extracted to be consumed in the city ($Q^u(t)$) or in agriculture ($Q^a(t)$). The city can also consume water from an outside surface water source, the amount of water consumed from that source is notated $S(t)$ (which is constrained at an upper fixed level $\bar{S}$ over time). Out of the total amount of fresh water consumed in the city ($Q^u(t) + S(t)$), a share which is constant over

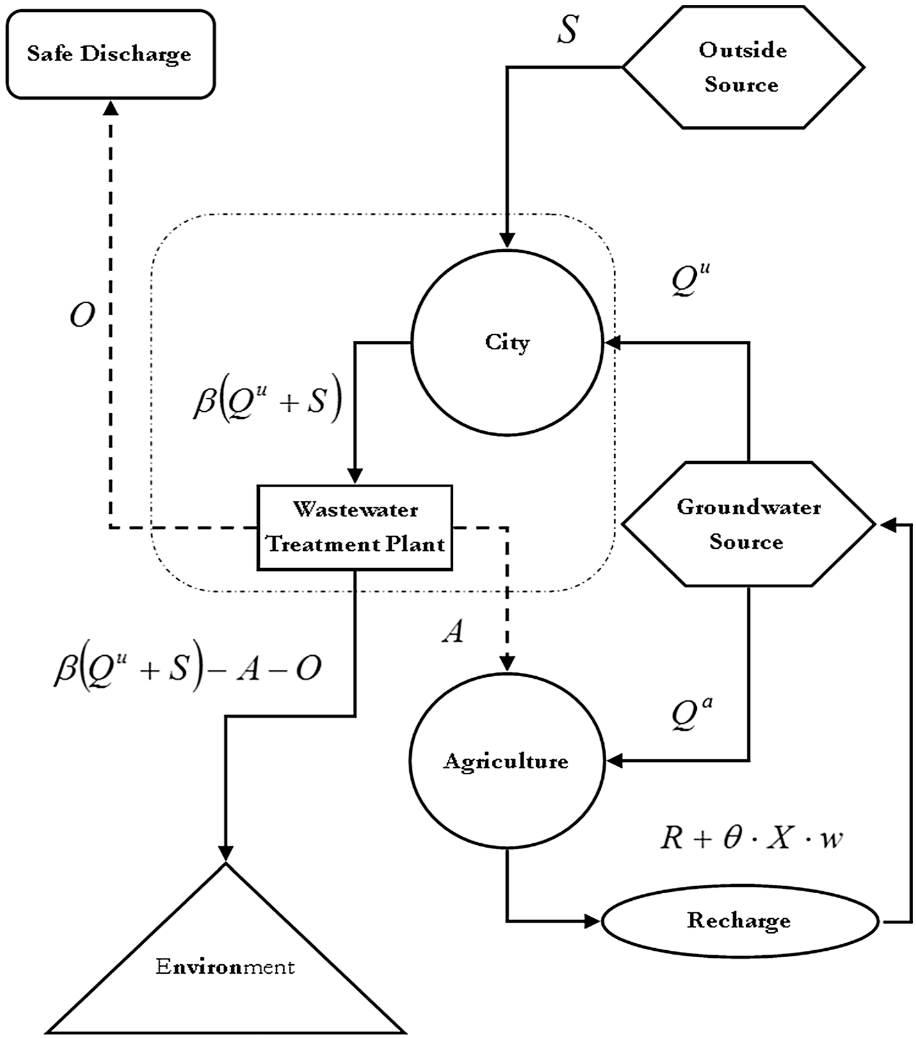

Fig. 1 Schematic regional setting

time β (where $0 < \beta < 1$) results as sewage, which has to be treated in a wastewater treatment plant (WWTP) and disposed of in order to meet assumed regulatory requirements.

For purposes of generality, we consider three options of effluent disposal sites in our framework. The first is the "zero alternative" (alternative A), which is the default option. It implies, since initially no other alternatives are available, that treated wastewater must be discharged to a nearby water body (e.g., a dry riverbed, a flowing river, or the ocean's immediate coastline). This is done at a negligible cost, but with an opportunity cost represented by an environmental pollution damage function (to be depicted later). The second option (alternative B) is to safely discharge the effluent to a remote location (far into the ocean, for example) so that the environmental pollution can be decreased (or even eliminated if all effluents are diverted away from the zero-alternative), but the construction and therefore an investment (F_B) in conveyance infrastructure are necessary. The underlying assumption is that due to ecological and health concerns the effluent should be carried away as far as possible from urban centers or from sensitive environments in order to minimize possible negative impacts. The third option (alternative C) is to convey the effluent to a nearby agricultural district where, again, conveyance infrastructure is required, bearing investment F_C, but differs from alternative B described above. In this final option the effluent is no longer considered just as a source of pollution, but it becomes an intermediate input in agricultural production. The quantities of effluents diverted to each of these three alternatives, at every period, are indicated as $E(t)$, $O(t)$ and $A(t)$ for alternative A, alternative B, and alternative C, respectively, where $E(t)$ equals $\beta \cdot [Q^u(t) + S(t)] - O(t) - A(t)$.

Groundwater storage receives inflows from natural recharge $R(t)$, which originates from rainfall, and from deep percolations produced by the farming sector $\theta \cdot X(t) \cdot w(t)$. Where θ is a constant percolation rate ($0 < \theta < 1$) per unit of applied water $w(t)$, and $X(t)$ stands for cultivated (and irrigated) land. Outflows from the groundwater source are extractions by the city and by the agricultural district, as depicted earlier.

As noted, water quality is also explicitly accounted for in our framework. We notate by $g(t)$, $\varphi(t)$, and $\psi(\cdot)$, the water qualities of groundwater, treated wastewater, and applied irrigation for agriculture, respectively. The first two quality variables are endogeneous decisions, and the third is a function of the other two. We avoid assigning $\psi(\cdot)$ with a specific functional form at this stage, and postpone the formal definition to the illustration section.[1] However, we do assume that it is positively and linearly correlated with quality of each source used independently (i.e., $\psi_g, \psi_\varphi > 0; \psi_{gg}, \psi_{\varphi\varphi} = 0$). We also implicitly assume that the level of constituents in wastewater (or sewage) is higher than that of the water consumed in the city. We assume that a representative constituent exists for the overall quality of the water, and follow common definitions, such that higher constituent level implies lower quality.[2]

We also noted earlier that the framework includes exogenous regulatory constraints associated with quality management. With respect to domestic consumption, we assume that a drinking water standard is imposed on water deliveries from all sources. For groundwater quality that maximal level is notated $\bar{g}$. Regarding the outside surface water source, we assume that deliveries to the city, $S(t)$, always adhere to the regulatory standard, and therefore quality of that source is not explicitly represented. Effluent quality $\varphi(t)$ is also assumed to be regulated, with the maximum level imposed by regulation indicated as $\bar{\varphi}$. Groundwater quality evolution is determined according to the ratios between water inflows and outflows qualities, as well as natural dilution processes in the soils, all to be formally depicted in the next section.

[1] For simplicity, one can assume that the quality of water applied is composed of a constant (representing the use of contaminating inputs, such as fertilizer and others, the introduction of which through drip irrigation is becoming a common practice in modern agriculture), plus a weighted average of the qualities according to water consumption from the different sources. Notice though, that such assumption imposes some other characteristics of $\psi(\cdot)$; we discuss these in Appendix 1.

[2] Water quality, in general, and effluent quality, specifically, should obviously be considered as a vector of quality components (e.g., salinity, nutrients, BOD, COD, SS, phosphorus, boron, and others); however, for simplicity and convenience of presentation, we assume that $g(t)$, $\varphi(t)$, and $\psi(\cdot)$ represent only one quality component. The hypothetical highest quality would be 0.

2.1 The Model's Components

We now describe and formally define other relevant aspects of the different model components.

2.1.1 The City

The city is represented by an aggregated utility function $U\{I(\beta \cdot [Q^u(t) + S(t)];\varphi(t)), Q^u(t) + S(t)\}$, where the function $I(\cdot)$ represents the available income spent by the aggregated consumer to purchase a composite good. It is composed of two arguments—the first is the amount of sewage produced (and treated) in the city $\beta \cdot [Q^u(t) + S(t)]$, and the second is the effluent quality $\varphi(t)$. It is assumed that $I_1 < 0, I_{11} > 0; I_2 > 0, I_{22} < 0$; where I_i and I_{ii} stand for the first and second partial derivatives with respect to the ith argument, $i = 1, 2$. As in Feinerman et al. (2001), we assume that the cost of treating the sewage produced in the city is embedded within the available income function $I(\cdot)$, and therefore affects the level of aggregate utility, as the city is assumed to own the WWTP. Therefore, as more water is consumed by the city (and more sewage is produced and has to be treated), the income available for other goods beyond water is reduced. However, this effect diminishes with the volume of sewage, due to the WWTP economies of scale. With respect to quality: the poorer the quality of effluent produced (meaning higher $\varphi(t)$), the cheaper the treatment cost, which increases available income. As in the case of quantities, we assume that the marginal effect of effluent quality diminishes as $\varphi(t)$ rises, hence the second derivative of $I(\cdot)$ with respect to $\varphi(t)$ is negative. The cross partial derivatives I_{12}, and I_{21}, are assumed to be zero. That is, the marginal cost of treatment (and therefore the marginal available income) with respect to the quantity treated, or with respect to the quality of treated wastewater, is unaffected by changes in the other factor.

We also assume that the utility function $U(\cdot)$ is well-behaved (hereon after, the time dependency may be omitted in several places in the paper, due to presentation convenience considerations), implying positive first derivative, and negative second derivative with respect to both available income and quantity of water consumed.[3] As noted earlier, quality standards are assumed to be imposed on the water supplied to the city from all sources. Specifically, we assume that once the groundwater quality $g(t)$ exceeds the level imposed by regulation, $\bar{g}$, it needs to be treated at cost $h(g(t))$ in order to be supplied to the city, where $h(\cdot)$ is continuous and twice differentiable, such that $h'(\cdot) > 0$ and $h''(\cdot) > 0$ for $g(t) \geq \bar{g}$, otherwise $h(g(t)) = 0$.

As noted above, we assume that discharging treated wastewater to the environment is associated with a social cost in the form of a damage function $D(E(t), \varphi(t))$.[4] This means the only quantities associated with environmental (social) damage are those not diverted towards reuse or the safe disposal options. We follow Farrow et al. (2005) by assuming that the damage function is linear with respect to both arguments, and we also assume that

[3] Other standard utility characteristics for the existence of internal solutions are assumed, i.e., let Q be set equal to $Q^u + S$ then it is assumed that $\lim_{Q \to 0} {}^{dU}/_{dQ} \to \infty$; $\lim_{Q \to \infty} {}^{dU}/_{dQ} \to 0$; ${}^{dU}/_{dQ} \geq 0 \, \forall Q \geq 0$.

[4] It can be argued that although effluent quality is regulated and monitored, some contaminants, like pharmaceuticals, nitrogen, and phosphorous, are found in higher levels in the treated effluent than in other water sources, and therefore are posing health and environmental risks (Hernando et al. 2006).

$D(0, \varphi(t)) = 0$, and $D(E(t), 0) = 0$.[5] It is also assumed that $D_{E\varphi} \geq 0$ and that $D_{\varphi E} = 0$, which in turn implies that discharging lower-quality effluent to the environment means higher marginal damage for each unit of effluent discharged. However the quantities of effluent discharged do not affect the marginal damage from higher constituent concentration. The conveyance costs of the other two alternatives for effluent disposal (i.e., B and C) are characterized by variable cost curves $v(O(t))$ and $\upsilon(A(t))$ for the safe disposal (B) and for agricultural irrigation (C) alternatives, respectively. Both functions are assumed to be non-decreasing and convex (Specifically, $\lim_{O \to 0} v(O(t)) > 0$; $\lim_{A \to 0} \upsilon(A(t)) > 0$). The underlying assumption is that once a decision to build a conveyance infrastructure to either location has been made, it bears a fixed cost (which is the amortized cost of investment, and is notated by v_0, and υ_0 for alternative B, and C, respectively), and that conveyance of greater quantities of water has an increasing marginal cost.

Finally, each unit of freshwater supplied from the aquifer bears the cost of extraction $C(G(t))$, which is decreasing and convex in the groundwater stock level $G(t)$. We also assume that this unit cost approaches zero as stock reaches its maximal level $\bar{G}$ (i.e., $\lim_{G \to \bar{G}} C(G(t)) \to 0$). Without loss of generality, we assume that surface water supply is costless.

2.1.2 The Farming Sector

The farming sector grows one crop that is sensitive to both water quantity and its quality. The farming sector is a price taker and receives a payment of $P^Y(t)$ per unit of output sold at the market, and its water sources are the groundwater aquifer, which is shared with the city, (potentially) treated wastewater from the WWTP, and precipitation.[6] The per-unit area production function of the crop is $Y\{w(t), \psi(g(t), \varphi(t))\,; r(t)\}$, where $w(t)$ and $r(t)$ are the per-unit area applied water and precipitation levels, respectively. We assume also that production is increasing, both as water quantity per unit of land increases, and as water of higher quality is applied.[7] Both effects diminish with rising quantities and qualities. We also assume that there is a non-zero elasticity of substitution between water quantity and quality, and between applied water and precipitation. These assumptions are summarized below, in a notational form, where Y_i and Y_{ii} again denote the first and second partial derivatives with respect to the ith argument, this time $i = 1, 2, 3$.

$$Y_w > 0, Y_{ww} < 0; Y_\psi < 0, Y_{\psi\psi} > 0; \tag{1}$$

$$\frac{Y_w}{Y_r}, \frac{Y_w}{Y_\psi} \neq 0; \tag{2}$$

Assume for simplicity that the farming sector is not limited in its cultivable land and labor force. The only constraint the farming sector faces is water quantity. The costs

[5] Horan (2001) argues in favor of a non-decreasing convex functional form of the damage associated with water quality pollution. We choose the linearity assumption on generality considerations, and discuss the implications of each of these assumptions in detail in Appendix 1.

[6] One could also consider that the farming sector has a surface water source in the same way the city does; however, since competition over groundwater between these sectors is already accounted for, and since stochasticity is currently ignored, including a surface water source for agriculture becomes redundant.

[7] Similar to the utility function of the city, we assume internal solution properties for the per-unit land agricultural production function, with respect to water applied and its quality (see footnote 3).

associated with agricultural production are the costs of the water inputs, i.e., $C(G(t))$ for the groundwater supply $Q^a(t)$, and other costs associated with the production process (such as labor, fertilizer, management, and others), which are expressed as a function $f(X(t))$ of the cultivable land $X(t)$.

2.1.3 The Groundwater CPR

As mentioned earlier, the groundwater source is represented by two states: water stock level $G(t)$, and water quality level $g(t)$. The equations of motion defining these states at each period, t, are as follows:

$$\dot{G} = R(t) + \theta \cdot X(t) \cdot w(t) - Q^u(t) - Q^a(t) \tag{3}$$

$$\dot{g} = e(\psi(t)) - \delta(G(t)) \cdot g(t) \tag{4}$$

As described earlier, and now formally defined in Eq. (3), the groundwater table increases with recharge and deep percolations that originate from the farming sector, and decreases due to water extractions for the use of both the city and the farming sector.

Conceptually, we follow Roseta-Palma (2002, 2003) to describe the evolution of water quality in the aquifer. We assume that contaminants dissolve naturally in the ground at a given rate $\delta(G(t)) > 0$. We also assume that this decay rate is higher when groundwater stock level rises, however this effect diminishes (i.e., $\delta'(G(t)) > 0$; $\delta''(G(t)) < 0$). The function $e(\psi(t))$, which is the first component in Eq. (4), accounts for contamination caused by agricultural activity (which is assumed to be always positive), and is defined as a function of the water quality applied for irrigation.[8] It is presumed that irrigating with water high in contaminant levels increases the rate of water quality degradation in the aquifer (Candela et al. 2007; Katz et al. 2009). Specifically, we assume a non-decreasing convex function for $e(\psi(t))$. It then follows that $e'(\psi(t)) > 0$; $e''(\psi(t)) > 0$.

In the setting described thus far, the timing of construction of the infrastructure required for conveyance to either disposal alternative B or C, or both, are endogenous decisions, which considerably complicates the analysis. Given that there are two alternatives, none are developed in the initial setting, one needs to first resolve whether any of the alternatives should optimally be developed, and if both, then in what sequence? However, we argue that the modeling framework presented herein is comparable to previous models of groundwater management and conjunctive use, although it additionally includes quantity–quality tradeoffs. Yet, Roseta-Palma (2002, 2003) demonstrated that the quantity–quality optimal solution has similar characteristics to the quantity-only model, and so relying on previous contributions our claim is that optimal timing to develop the infrastructure does exist. Specifically, this argument holds with respect to reuse alternative C, as it fits the framework of a supplemental water supply project (or "backstop technology") construction (e.g., Tsur and Zemel 2000; Holland and Moore 2003, and references therein). This argument is demonstrated using a static illustrative example in Appendix 2. The sequence of investments is addressed in the following section.

[8] Roseta-Palma (2002, 2003) takes a more general approach and explicitly includes the use of contaminating inputs in agriculture to account for their effect on groundwater quality. We adopt this approach by implicitly incorporating it within the applied water quality function (see footnote 1). Since our approach focuses on the role of treated wastewater reuse in economic tradeoffs among competing sectors over water allocations, we find this solution to be a better fit for the scope of our work.

2.2 The Regional Social Planner Model

All three components described above are now integrated into one regional model, which is solved for social welfare maximization. We assume that alternative B is already in place, meaning that investment in conveyance infrastructure to carry effluent to the safe disposal location had already occurred in the past, and is now a sunk cost. The reasoning for this assumption will be given shortly. We use Appendix 1 to depict the social planner welfare maximization problems (32, 46, 52, and 65), which differ by the disposal alternatives existing in the initial conditions. For each we assume that existing conditions prevail for an infinite horizon. For problem (32) only alternative A exists. For problem (46), it is assumed that alternatives A and B exist, and for problem (52) it is assumed that alternatives A and C exist. Problem (65) includes all disposal alternatives (A, B and C) in the initial conditions.

Let T_j^* be the optimal timing for investing in alternative j, where $j \in \{B, C\}$. Let K denote the regional social welfare optimization problem (later to be defined as Problem K), in which alternative A sets the initial conditions, and a decision about T_j^* needs to be made. Let the solution space for this problem K, be Z_K, and let z_K^* be the optimal plan, such that $z_K^* \in Z_K$.

Proposition 1 *Let alternative A set the initial conditions for a regional social welfare planner facing the decision whether to develop the two alternatives for effluent discharge B and C. Then under assumptions 1 through 4,*

$$\exists T_C^* \in (0, \infty) : T_C^* < T_B^* \Rightarrow T_B^* \in \left\{ \emptyset \right\} \forall \{T_B^*(z_K^*), T_C^*(z_K^*)\} : z_K^* \in Z_K$$

The proof is provided in Appendix 1. Proposition 1 indicates that under the setting of problem K, the optimal time to invest in alternative C always preceeds the optimal time to invest in alternative B. It follows from Proposition 1 that if one wants to account for alternative B as a possible option in the social planner's solution, it must be assumed to be included in the initial setup. Proposition 1 relies on four assumptions that require some justification.

Assumption 1 The level of maximum constituents $\bar{\varphi}$ imposed by regulation is always binding in the optimal solution, regardless of the initial problem setting $i \in \{(32),(46)\}$. Such that,

$$\varphi^*(z_i^*) = \bar{\varphi}$$

Assumption 1 suggests that the maximum level of constituents permitted by regulation will be a binding constraint in the optimal solution when the problem's indefinite setting is either alternative A alone, or alternatives A and B together. It implies that an added value from treating wastewater to an inferior quality is infeasible, even when effluent is reallocated from the environment (alternative A) to the remote (safe) discharge option (alternative B). This assumption is backed up by empirical evidence regarding discharge compliance in the United States (Earnhart 2004; Shimshack and Ward 2008; Grooms 2015).

Assumption 2 Conveyance capacity to either disposal alternative, B or C, once developed, is sufficient for the system's existing and future needs, such that it is not a constraint nor a decision in the optimization problem. Let $\bar{Q}_j$ represent capacity of conveyance for alternative $j : j \in \{B, C\}$. Then,

$$\textit{The investment } F_j, \textit{ in alternative } j, \textit{ is independent from } \bar{Q}_j$$

Assumption 2 confines the decision of infrastructural conveyance development to either alternative B or C, to a timing decision only, ignoring the potential impacts of different capacity decisions. Such impacts include the possibility that safe discharge will be warranted under the optimal solution, even after developing the reuse alternative, in order to allow higher consumption in the city and avoiding the potential impacts on groundwater and agricultural productivity stemming from reuse. That strategy can indeed be optimal if the relative expansion of reuse capacity is expensive and overwhelms the aforementioned opportunity costs. We avoid the derivation of these tradeoffs analytically and discuss them in the illustrative example that follows.

Assumption 3 The present value of long-term environmental damage trumps any reasonable investment and operating costs of conveyance infrastructure. Such that,

$$\int_0^\infty [e^{-\rho t} \cdot D(\cdot)]dt \geq F_j + \int_0^\infty [e^{-\rho t} \cdot l(\cdot)]dt \;:\; j \in \{B, C\}, l(\cdot) \in \{\nu(\cdot), \upsilon(\cdot)\}$$

Assumption 4 Conveyance costs functions $\upsilon(\cdot)$ and $\nu(\cdot)$, cannot be extremely different from each other. That is,

$$\frac{\upsilon''(\cdot)}{\upsilon'(\cdot)} = \kappa \frac{\nu''(\cdot)}{\nu'(\cdot)} \;:\; \kappa \in \mathbb{R}^+, \kappa \neq \{0, \infty\} \;\forall\; \{A(t), O(t)\} \in [0, \beta \cdot (Q^{u*}(t) + S^*(t))] \textit{ and}$$

$$\exists\, \{A(t), O(t)\} \in [0, \beta \cdot (Q^{u*}(t) + S^*(t))] \;:\; \frac{\upsilon'(A(t))}{\nu'(O(t))} \geq 1$$

Assumption 3 and 4 are used to guarantee that the feasible solution space includes the development of alternative B. Assumption 3 requires that long-term damage from effluent discharge to the environment will be substantial enough, such that it exceeds reasonable costs (in present value) for investing in disposal alternatives, including the costs of conveyance. It can be demonstrated that relaxing this assumption does not necessarily contradict finding optimal timing to develop conveyance infrastructure for reuse (we address this argument in the illustrative example that follows). Assumption 4 dictates that conveyance cost functions for alternative B and C will be relatively similar. This, in turn, guarantees a feasible solution for problem (65), which includes all disposal alternatives as initial conditions (Appendix 1), such that effluents could be allocated to either the safe discharge option, to the agricultural district, or both, without contradicting the conditions for optimal solution of the system.

Let us now define $V^{\tau_C}\left(z^*_{\tau_C}, G(T_C), g(T_C)\right)$ as the value function corresponding to the optimal solution $z^*_{\tau_C}$, over the period $\tau_C \equiv [T_C, \infty)$ and given the initial conditions $G(T_C)$ and $g(T_C)$, which are all quantity and quality possible states at time T_C. Following the above, the regional social planner's problem (5) is presented below.

$$\underset{Q^u,S,Q^a,A,O,\varphi,X,w,T_C}{Max} \int_0^{T_C} e^{-\rho t} \cdot \left[U\{I(\beta \cdot (Q^u(t) + S(t)), \varphi(t)), Q^u(t) + S(t)\} \right.$$
$$+P^Y(t) \cdot X(t) \cdot Y\{w(t), \psi(g(t), \varphi(t)); r(t)\} - f(X(t)) - C(G(t)) \cdot (Q^u(t) + Q^a(t))$$
$$\left. -h(g(t)) \cdot Q^u(t) - v(O(t)) - D(E(t), \varphi(t)) \right] dt + e^{-\rho T_c} \cdot \left(V^{\tau_C}\left(z^*_{\tau_C}, G(T_C), g(T_C)\right) - F_c \right) \tag{5}$$

s.t.

(a) $\dot{G} = R(t) + \theta \cdot X(t) \cdot w(t) - Q^u(t) - Q^a(t)$
(b) $\dot{g} = e(\psi(t)) - \delta(G(t)) \cdot g(t)$
(c) $\varphi(\mathrm{t}) \leq \bar{\varphi}$
(d) $X(t) \cdot w(t) \leq Q^a(t) + A(t)$
(e) $S(t) \leq \bar{S}\ S(t) \leq \bar{S}$
(f) $O(t) + A(t) \leq \beta \cdot [Q^u(t) + S(t)]$
(g) $G(0) = G_0$
(h) $g(0) = g_0$

The solution to this problem relies on a two-step procedure. First, $V^{\tau_C}\left(z^*_{\tau_C}, G(T_C), g(T_C)\right)$ needs to be characterized for any given initial conditions $G(T_C)$ and $g(T_C)$, and for every possible timing T_C. The second step is then maximizing the integer in problem (5) between time zero to T_C, with T_C being a decision variable in the optimization, and taking $V^{\tau_C}\left(z^*_{\tau_C}, G(T_C), g(T_C)\right)$ as a boundary value. The properties of the latter obviously affect the transversality conditions for the optimal solution, as discussed in detail later on. The characterization of the boundary value depicted above is equivalent to solving problem (52) (Appendix 1) for changing initial conditions of the states $G(T_C)$ and $g(T_C)$, and sub-periods τ_c. Notice that $V^{\tau_C}\left(z^*_{\tau_C}, G(T_C), g(T_C)\right)$ includes the accumulated conveyance costs over the period $\tau_C \equiv [T_C, \infty)$ (i.e., $\int_{T_C}^{\infty} e^{-\rho t} \cdot v(A(t))dt$) in terms of value at time T_C.

We define for problem (5) above its respective Lagrangian function, and derive the first-order conditions (FOC)—these are rearranged and presented below in Eqs. (6)–(19). We also denote by $\lambda_\varphi(t)$, $\lambda_W(t)$, $\lambda_S(t)$, and $\lambda_E(t)$ the Lagrangian multipliers (shadow values) associated with constraints (c) through (f), respectively. $m_1(t)$ and $m_2(t)$ are the co-states for the equations of motion in constraints (a) and (b), respectively.

$$\lambda_W(t) = C(G(t)) + m_1(t) \tag{6}$$

$$\underbrace{\underbrace{U_I I_{Q^u}}_{\text{Indirect Effect (-)}} + \underbrace{U_{Q^u}}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = C(G(t)) + h(g(t)) + \beta \cdot \left(D_E - \lambda_E(t)\right) + m_1(t) \tag{7}$$

$$\underbrace{\underbrace{U_I I_S}_{\text{Indirect Effect (-)}} + \underbrace{U_S}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = \beta \cdot \left(D_E - \lambda_E(t)\right) + \lambda_S(t) \tag{8}$$

$$\underbrace{U_I I_\varphi}_{\text{Marginal Utility (+)}} = \underbrace{D_\varphi}_{\text{Marginal Damage (+)}} + \lambda_\varphi - X(t) \cdot \underbrace{P^Y(t) \cdot Y_\psi \psi_\varphi}_{\substack{\text{VMP of Applied}\\ \text{Water Quality}\\ \text{in Agriculture (-)}}} - m_2(t) \cdot \underbrace{e_\psi \psi_\varphi}_{\substack{\text{Marginal}\\ \text{Contamination}\\ \text{Rate (+)}}} \tag{9}$$

$$P^Y(t) \cdot Y\{w(t), \psi(g(t), \varphi(t)); r(t)\} = f'(X(t)) + \left(\lambda_W(t) - \theta \cdot m_1(t)\right) \cdot w(t) \tag{10}$$

$$X(t) \cdot \left(P^Y(t) \cdot Y_w + \theta \cdot m_1(t) - \lambda_W(t)\right) = 0 \tag{11}$$

$$\dot{m}_1 - \rho \cdot m_1(t) = \underbrace{C'(G(t))}_{\substack{\text{Stock Effect on}\\ \text{Marginal Extraction}\\ \text{Cost (-)}}} \cdot (Q^u(t) + Q^a(t)) + m_2(t) \cdot \underbrace{\delta'((G(t))}_{\substack{\text{Marginal}\\ \text{Decay Rate (+)}}} \cdot g(t) \tag{12}$$

$$\dot{m}_2 + \left[\underbrace{e_\psi \psi_g}_{\substack{\text{Marginal}\\ \text{Contamination}\\ \text{Rate (+)}}} - \delta(G(t)) - r\right] \cdot m_2(t) = \underbrace{h'(g(t))}_{\substack{\text{Marginal Cost of}\\ \text{Water Treatment}\\ \text{for Urban Use (+)}}} \cdot Q^u(t) - X(t) \cdot \underbrace{P^Y(t) \cdot Y_\psi \psi_g}_{\substack{\text{VMP of Applied}\\ \text{Water Quality}\\ \text{in Agriculture (-)}}} \tag{13}$$

$$\lambda_\varphi(t) \cdot (\bar{\varphi} - \varphi(t)) = 0, \quad \lambda_\varphi(t) \geq 0 \tag{14}$$

$$\lambda_W(t) \cdot (Q^a(t) + A(t) - X(t) \cdot w(t)) = 0, \quad \lambda_W(t) \geq 0 \tag{15}$$

$$\lambda_S(t) \cdot \left(\bar{S} - S(t)\right) = 0, \quad \lambda_S(t) \geq 0 \tag{16}$$

$$\lambda_E(t) \cdot (\beta \cdot (Q^u(t) + S(t)) - O(t) - A(t)) = 0, \quad \lambda_E(t) \geq 0 \tag{17}$$

$$\lambda_E(t) = D_E - \nu'(O(t)) \tag{18}$$

$$\lambda_E(t) = \lambda_W(t) + D_E - \upsilon'(A(t)) \tag{19}$$

The following interpretations refer to the optimal solution. Equation (6) states that in the optimal solution the shadow value associated with the available water constraint for irrigation should be equal to the sum of the unit cost of extraction and the scarcity rent. Equations (7) and (8) equate the marginal utility of water consumption to the total (social) marginal cost associated with the use of each water source—groundwater and outside source, respectively. As these sources are perfect substitutes (assuming quality adheres to the drinking water standards discussed earlier), at the hypothetical case that the outside source is plentiful (i.e., if $\bar{S}$ is very large then the associated availability constraint (e) is not binding, and therefore $\lambda_S = 0$), Eqs. (7) and (8) imply that the city will only consume surface water. Since we assume that $\bar{S}$ is limited, and constraint (e) actually is binding, the optimal solution will always include positive quantities extracted from the shared groundwater CPR to complement the surface supply source, meeting unsatisfied demand. Even more so, if we assume that the surface water supply is stochastic, such that $\bar{S}(t)$ varies with time and is randomly distributed around the constant $\bar{S}$, the value of the groundwater stock will increase due to its role as supply stabilizer over time (Tsur and Graham-Tomasi 1991).

In (9) marginal utility from contamination (i.e., treatment of effluent to a lower quality) is equated with the social cost associated with it. It is worthwhile noting that in the private case of an unregulated contaminant (e.g., pharmaceuticals), and prior to investment in conveyance capacity to the farming sector (i.e., $\psi_\varphi = 0$), the right-hand side of Eq. (9) will include only the marginal damage associated with higher contamination. This means that all other components, if included in the solution, will lead to an optimal treatment which is of higher quality level. Placing Eq. (6) into (10) and (11), and given that there exists an internal solution with respect to cultivable land (i.e., $X^*(t) > 0\,\forall t$) yields the following respectively:

$$P^Y(t) \cdot Y\{w(t), \psi(g(t), \varphi(t)); r(t)\} = f'(X(t)) + \left(\underbrace{C(G(t)) + (1-\theta) \cdot m_1(t)}_{\text{VMP of Water Applied } (+)} \right) \cdot w(t) \quad (20)$$

$$P^Y(t) \cdot Y_w = C(G(t)) + (1-\theta) \cdot m_1(t) \quad (21)$$

These two equations dictate that the value of marginal product in agriculture from both inputs (water and land) will be equal to their marginal cost, accounting for scarcity and deep percolation effects. Equations (12) and (13) define the optimal paths for the co-states associated with groundwater stock level, and water quality, respectively. We express below these relationships in the form of growth rates:

$$\frac{\dot{m}_1}{m_1(t)} = \rho + \frac{\left[\overbrace{C'(G(t)) \cdot (Q^u(t) + Q^a(t))}^{(-)} + \overbrace{m_2(t) \cdot \delta'((G(t)) \cdot g(t)}^{(-)} \right]}{m_1(t)} \quad (22)$$

$$\frac{\dot{m}_2}{m_2(t)} = \rho + \delta(G(t)) - e_\psi \psi_g + \frac{\left[\overbrace{h'(g(t)) \cdot Q^u(t)}^{(+)} - \overbrace{X(t) \cdot P^Y(t) \cdot Y_\psi \psi_g}^{(-)} \right]}{m_2(t)} \quad (23)$$

As common in standard optimal control problems of renewable resources exploitation (e.g., Burt 1964; Cummings and Winkelman 1970; Tsur and Graham-Tomasi 1991; among others) groundwater stock level is depleted over time, implying that $\dot{G} < 0$ until a steady-state is reached, at which time $\dot{G} = 0$. This characteristic of the optimal solution was also proven under the assumption of population growth, which translates to increased demands over time (Tsur and Zemel 2000). However, if costs of extraction are high, and/or the impact of depletion on the accumulation of contaminants in the groundwater is substantial, then a steady-state could be reached in which the groundwater stock level is at its maximum. Recall that the pollutant level g increases with time (it is also important to notice at this stage that $m_2(t)$ should be negative, as it is associated with g—which is a pollutant), where the latter occurs due to the positive difference between the farming sector's contaminating activity and the aquifer's decreasing resilience to pollution as the water table declines. Note that scarcity is affected, both by the increasing cost of extraction and by the increasing value of higher quality water [Eq. (22)]. The shadow value of quality [Eq. (23)] accommodates the net rate of pollutant accumulation in the aquifer, and also the effect on the value of production. Equations (14) through (17) are the usual Karush–Kuhn–Tucker conditions and dictate that the shadow value of any binding constraint in the optimal solution must be non-negative. The last two equations determine the value of $\lambda_E(t)$—the shadow value associated with the effluent availability constraint, and are derived from the decision whether to allocate effluent to either (or both) of the disposal alternatives B and C.

A steady state arises when the time derivatives of the states and co-states are set at zero, which in turn translates into the following:

$$Q^u + Q^a = R + \theta \cdot X \cdot w \tag{24}$$

$$g = \frac{e(\psi)}{\delta(G)} \tag{25}$$

$$m_1 = -\frac{\left[C'(G) \cdot (Q^u + Q^a) + m_2 \cdot \delta'(G) \cdot g\right]}{\rho} \tag{26}$$

$$m_2 = \frac{\left[h'(g) \cdot Q^u - X \cdot P^Y \cdot Y_\psi \psi_g\right]}{e_\psi \psi_g - \delta(G) - \rho} \tag{27}$$

According to Eq. (24), steady-state extraction would be equal to the level of recharge (including deep percolation that originates from irrigation). Equation (25) implies that contaminant level would be set according to the ratio between the level of agricultural contamination and the pollutant decay rate in the aquifer. According to Eq. (26), scarcity rent will be higher at lower levels of groundwater stock, and as quality degrades. Groundwater quality shadow value is larger when contamination impacts on agricultural production are higher, as it reads from Eq. (27).[9] It is also noteworthy to explain that the negativity of this co-state rests on the assumption that the difference between the marginal contamination rate and the aquifer's decay rate is not too large (specifically, for this assumption to hold, that difference cannot exceed the discount rate ρ). In other words, the agriculture contamination function $e(\psi(t))$ is expanding at a moderate and constrained rate.

[9] There is also an effect that stems from the increase in groundwater treatment cost to meet the requirements associated with water quality that is supplied to the city; however, as explained in Appendix 1, in the social planner's problem this cost is irrelevant.

Finding the actual optimal timing T_C^* for investing in conveyance infrastructure connecting the WWTP with the farming sector involves the derivation of the transversality condition developed by Hartwick et al. (1986), which requires that the net benefits stemming from water allocations after the timing of investment would exceed the net benefits obtained prior to that investment, at least by the interest payment for the investment (Holland and Moore 2003). Holland and Moore (2003) rely on a formal proof developed by Holland (2003) to identify a continuous price path (which translates into a continuous consumption path, as well). This, in turn, enables the derivation of an optimal time rule for investing in a water import project, in which the original supply alternative is a renewable groundwater aquifer. As noted earlier, we argue that the problem presented in our paper is no different for the purposes of satisfying the same derivation, and we therefore avoid the burdensome description associated with it.

We now turn to illustrate our conceptual findings. The illustration is used to corroborate our theory, and also supplies examples for empirical analysis that could be performed to address relevant policy questions, to which the framework contributes. We use data and functional forms taken from existing literature, and without focusing on a specific region.

3 Empirical Illustrative Example

We choose salinity as the water quality parameter used in the illustration. Traditional wastewater treatment processes usually do not facilitate salinity removal. However, it has been argued that with predicted increases in the concentrations of dissolved solids (salinity) in wastewater flows, a cost-effective approach utilizing processes to remove salts could be viable (Tran et al. 2017). Recall, that there are two water-consuming sectors in the region's economy—a city, and an agricultural district. For the first, we choose to represent the utility function in the common Cobb–Douglas functional form as depicted in Eq. (28).[10]

$$\mu_u \cdot Q_u^{\eta_1+1} \cdot \varphi^{\eta_2 \cdot (\eta_3+1)} \tag{28}$$

where η_1 is the inverse price elasticity, η_3 is the inverse income elasticity, and η_2 is the effect of changes in effluent quality level φ on income, such that φ^{η_2} represents the available income function $I(\cdot)$ described earlier; μ_u is a scaling parameter that is added to maintain consistency in units used, and also facilitates annual income and population growth rate trends, which we assume to be 2.88% and 1.24%, respectively (World Bank n.d.). For both price and income elasticities, we use estimates taken from the literature (Espey et al. 1997; and Dalhuisen et al. 2003).

For the farming sector, we use calibrated production functions from Kan (2003) for two alternative crops—cotton and tomatoes, which differ at their level of salinity tolerance (Maas and Hoffman 1977). The general form is depicted in Eqs. (29) and (30) below.

$$Y = b_1 \cdot (ev - \underline{ev}) + b_2 \cdot (ev - \underline{ev})^2 \tag{29}$$

where,

$$ev = \frac{\overline{ev}}{1 + \alpha_1 \cdot (\psi + \alpha_2 \cdot w^{\alpha_3})^{\alpha_4}} \tag{30}$$

[10] While this representation is a very simplified version of the general utility function presented in the conceptual part of the paper, it still carries all the necessary qualitative characteristics assumed.

where ev(feet/year) represents periodical evapotranspiration, $\overline{ev}$ is the maximum potential evapotranspiration, and $\underline{ev}$ is the minimal evapotranspiration required for crop production; w and ψ are as defined earlier, and α_1 through α_4, b_1 and b_2 are scalars, with $b_1 \geq 0$, $b_2 \leq 0$. The set of parameters for both sectors' functions are presented in Table 1. Also presented in Table 1 are the crop prices borrowed from Kan (2003), crop production-cost function parameters (f_0 and f_1)—calibrated based on observed average size farm in the United States (USDA 2017), and per unit of land observed water applications ($\tilde{w}$)—taken from Johnson and Cody (2015).

For the groundwater source, we choose again to avoid focusing on a specific basin and adopt the characteristics of the aquifer presented in Roseta-Palma (2003).[11] Suggested by that author as a possible extension of her approach, we introduce dependency between groundwater storage level and the decay rate, according to the following relationship $\delta_0 + \delta_1 \cdot G^{\omega}$, where δ_0 and δ_1 are positive scalars, and $0 < \omega < 1$.[12] The other component in the groundwater quality equation of motion [constraint (b) in problem (5)] is depicted by $e \cdot \psi^2$, where e is a positive scalar, and ψ, the quality of applied water in agriculture is defined according to Eq. (31) as suggested earlier (footnote 1).

$$\psi = \gamma + \frac{Q_a \cdot g + A \cdot \phi}{Q_a + A} \tag{31}$$

where γ stands for salinity induced by using contaminating inputs (e.g., fertilizers and others) in crop production; the rest of the elements in Eq. (31) are decision and state variables of the optimal control problem (5) described in Sect. 2.2. The different parameters described above controlling the groundwater quantity–quality states are presented in Table 2.

3.1 Results

Using the GAMS platform, we solve the empirical application of problem (5) as it is described in the previous section, and refer to this solution as the base-scenario (using the original parameters in Tables 1 and 2). We choose a long-enough planning horizon to ensure convergence towards a steady-state. Figures 2 and 3 depict the outcomes of that base scenario with respect to the two alternative crops indicated above—cotton (C), and tomatoes (T).[13]

We found that for this base scenario all effluent volume is diverted at the steady-state towards agriculture. We perform several sensitivity analyses (to be described in detail below), and while in certain cases conveyance infrastructure for remote disposal is built and utilized, in all occasions and without exception, establishment and utilization of

[11] Two noteworthy modifications with respect to Roseta-Palma's illustration are (1) units, which we changed from metric to imperial/United States Customary System; and (2) while Roseta-Palma (2003) used pumping lift as the state variable in her illustration, we are keeping the illustration in storage level measures—the transition between the two is straightforward, and relies on basic hydrologic principals (Heath 1983, p. 28).

[12] For the calibration of this function, we require more information than the fixed decay rate reported by Roseta-Palma (2003). Unfortunately we couldn't find a reference for that information, and therefore use a range of values for the function parameters.

[13] For the neatness of presentation, the planning horizon in both figures is truncated just a few time periods after the steady-state is reached.

Table 1 Parameters for utility function in the city and agricultural production

Parameter	Value		Description/units
City			
μ_u [a]	125		
η_1 [b]	−0.51		
η_2 [b]	0.43		
	Cotton	Tomato	
Agriculture			
$\overline{ev}$	2.39	1.97	Maximum evapotranspiration
α_1	0.000013	0.0011	
α_2	47.06	21.85	
α_3	−0.99	−1.47	
α_4	3.14	2.37	
b_1	0.6	37.38	
b_2	−0.12	0	
$\underline{ev}$	0.47	0.66	Minimal required evapotranspiration for crop production
P^Y	1586.2	43.2	Crop price ($/ton)
$\tilde{w}$ [c]	2.9	2.7	Observed average water application (acre-feet/acre)
f_0	631.58	76.36	$/acre
f_1	0.29	1.33	$/acre^2

[a]This is the value for the base period

[b]Reported mean value in Espey et al. (1997) and Dalhuisen et al. (2003)

[c]*Source*: USDA, 2013 Farm and Ranch Irrigation Survey(FRIS), Table 36, http://www.agcensus.usda.gov/Publications/2012/Online_Resources/Farm_and_Ranch_Irrigation_Survey/

conveyance capacity towards the farming sector is found optimal, supporting our Proposition 1 above.

Figure 2 shows the quantities allocated to both sectors in the region. For both crop alternatives (cotton and tomatoes), as time progresses the expansion of water consumption in the city is at the expense of groundwater extractions to the agricultural district. As a result, in both cases fresh groundwater are substituted with treated wastewater to support agricultural production. However, there are differences in the timing and pattern at which this transition occurs. For cotton, treated wastewater is diverted for irrigation at the beginning of the planning horizon, and diversions gradually increase until substituting groundwater allocation completely after 52 periods; from that point onward effluent allocations continue to grow until a steady-state is reached after 128 periods.

Table 2 Groundwater parameters

Parameter	Value	Units	Description
AR	4942	Acres	Aquifer area
S	0.1		Storage coefficient/specific yield
ec	0.002	$/$10^3$ acre-feet	Per unit of volume pumping cost
θ	0.1		Irrigation return rate
δ_0	0.1		Decay rate function constant
δ_1	0.05		Decay rate slope parameter with respect to groundwater storage level change
ω	0.8		Power at which storage level is raised by in the decay rate function
γ	0.51	dS/m	Salinity level induced by input use in agriculture
e	0.7		Agriculture contamination function parameter

For tomatoes, the transition is instantaneous but occurs after 119 periods. These allocation trends result in an increasing extraction path, which stabilizes rapidly at the level of recharge from precipitation and deep percolation of agricultural irrigation, so that the aquifer's steady-state storage level is at full capacity.

As mentioned, for cotton the transition from irrigating with groundwater to utilization of treated wastewater in agriculture is faster and happens earlier. The reason is the crop's relatively high salinity-tolerance, which facilitates earlier and larger use of effluent (than tomatoes), higher in salinity content, in the farming district (Figs. 2, 3). This strategy prompts two processes generating regional benefits. The first is the increase in available income for the city, which results from treatment of effluent to a lower quality. This is possible, in the case of cotton, without inflicting significant losses to the farming sector. The second process is the expansion of water consumption in the city, which becomes plausible due to reduction of agricultural water extractions from the CPR.

Since tomatoes are more salinity-sensitive than cotton, allocations of higher saline effluent result in larger decreases in agricultural profits, and therefore the diversion of treated wastewater to the farming district under this scenario happens later in time. This also results in a lower rate of groundwater quality degradation, such that the steady-state is reached 67 periods after the steady-state in the cotton scenario (Fig. 3). The effects of the different optimal strategies between the two crop scenarios result in a regional welfare difference of $570,000 USD annually in favor of the tomato scenario (not presented). A level of cautiousness is warranted when interpreting this result. One needs not to assume that it implies that tomato (as opposed to cotton) production would be the optimal choice for the region. Rather, it means that given both crops' production and costs characteristics, a centralized management approach would yield greater regional net benefits when the farming sector specializes in a salt-sensitive crop as oppose to a salt-tolerant crop.

We turn next to validate our Proposition 1 and its supporting assumptions. For that purpose we construct six scenarios, distinguished by their initial settings and conditions, and we label them S1 through S6.[14] The first scenario (S1) represents an empirical application of problem K as it was defined earlier. Additionally, for this scenario we assume that

[14] From hereafter we assume that the farming sector grows only cotton.

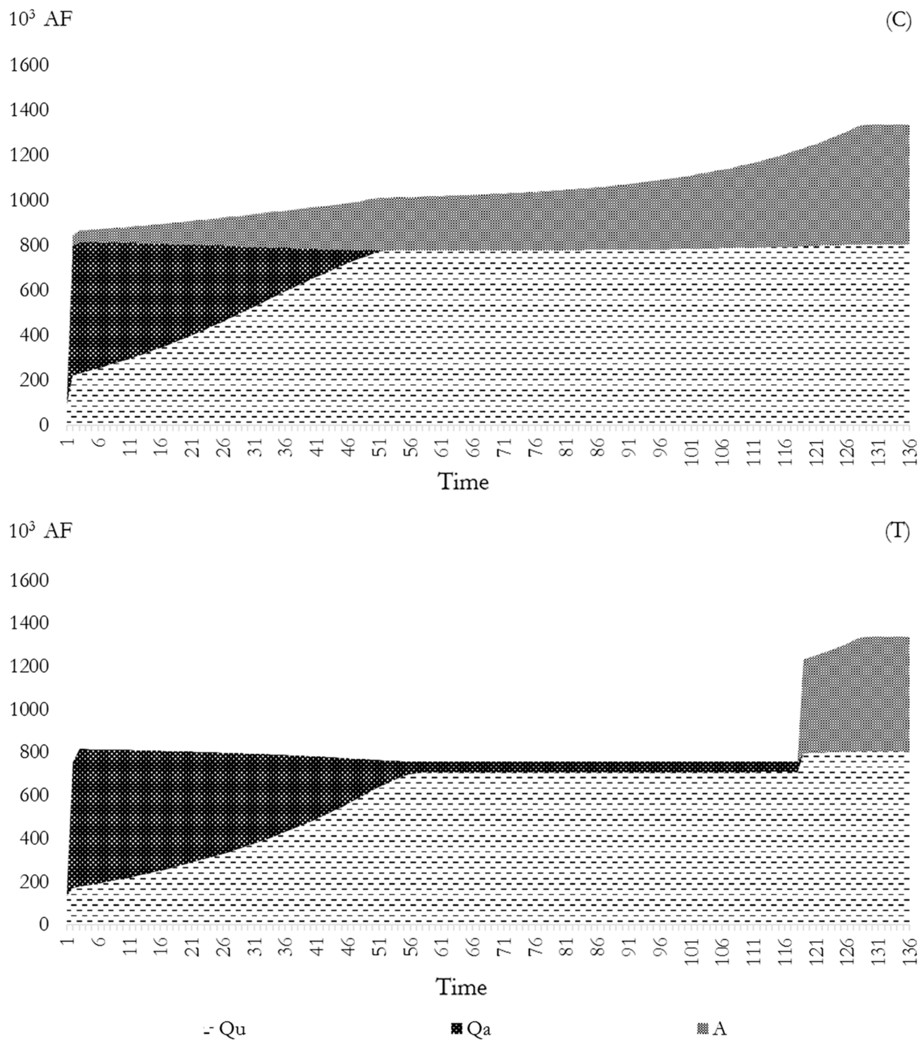

Fig. 2 Water allocation between sectors for a cotton (C) and tomatoes (T)-based farming sector (10^3 acre-feet/year). *Note*: Q_u—quantity consumed in the city and extracted from the groundwater CPR; Q_a—quantity consumed in agriculture and extracted from the groundwater CPR; A—quantity of effluent consumed in agriculture and conveyed from the WWTP

conveyance costs and investment for both disposal alternatives B and C are the same. The second scenario (S2) represents a violation of Assumption 1, in which keeping all other aspects identical with respect to scenario S1, we more than triple the maximum permitted level of constituent concentration imposed by regulation. In Scenario S3, we decrease the environmental damage function coefficient (i.e., the marginal damages D_E and D_φ) by 90% with respect to S1, violating Assumption 3. Scenarios S4 to S6 represent modifications to the conveyance cost function to the agricultural district, also with respect to scenario S1. Each of these modifications violates Assumption 4. Under scenario S4, investment in

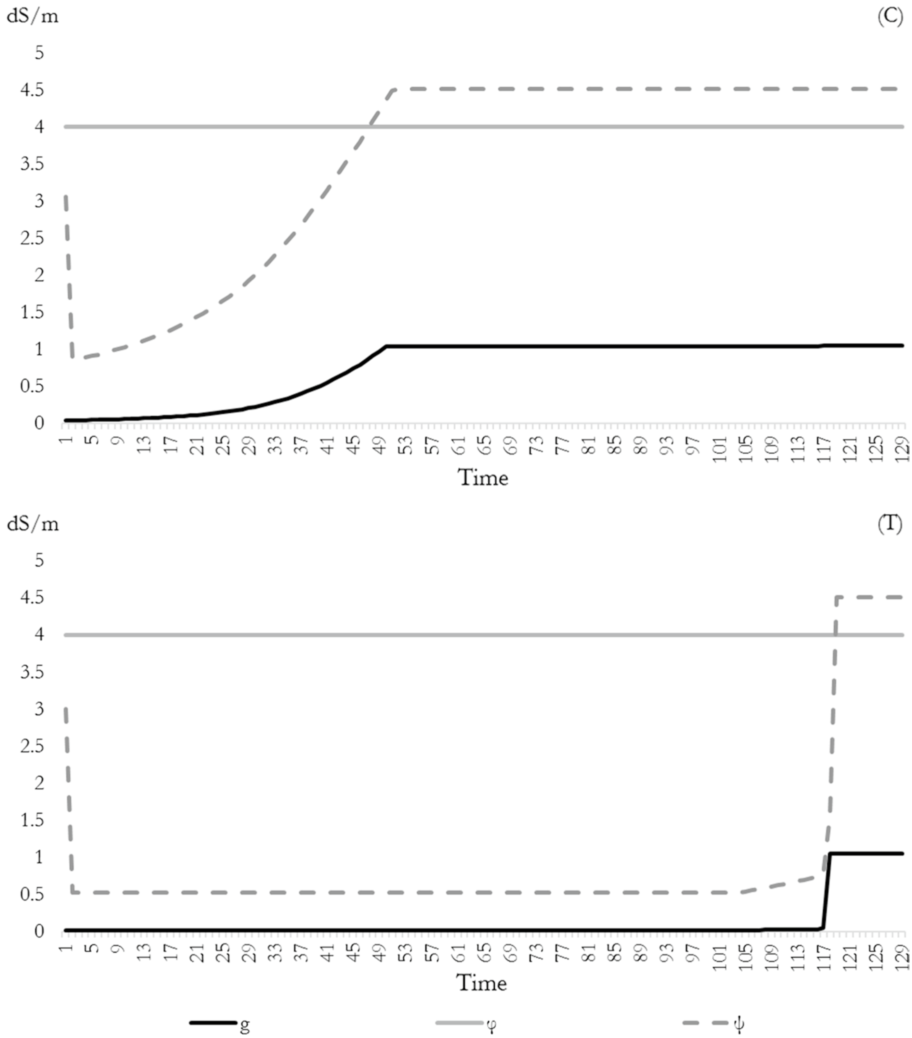

Fig. 3 Groundwater, irrigation, and WWTP effluent quality level for a cotton (C) and tomatoes (T)-based farming sector (dS/m). *Note*: g—groundwater quality level; φ—quality of treated wastewater; ψ—quality level of applied water in agriculture

alternative C is ten times the investment in alternative B, for any level of conveyance capacity developed. In scenario S5, marginal conveyance cost for agriculture is two times the marginal cost for safe discharge. In scenario S6, we modify the cost function parameters such that the second derivative (i.e., curvature) of the conveyance cost function to agriculture will be twice that of the cost function for safe discharge. The results of this analysis is reported in summary form in Table 3. The data presented in the table are the optimal timing of investment in alternatives B and C, as well as the average annual quantity of effluent allocated to each one of the disposal alternatives. Also reported in Table 3 are the results of scenarios S1 through S6 under conditions of two selected stochastic natural recharge

Table 3 Differences in optimal timing of investments and effluent allocation across scenarios

Scenarios	S1	S2	S3	S4	S5	S6
Deterministic base conditions						
Timing for investment (no. of periods)						
Timing *B*	Never	Never	Never	125	111	114
Timing *C*	111	116	123	182	111	114
Allocation of effluent (10^3 AF/year)						
Alternative *A*	317	324	450	334	317	321
Alternative *B*	0	0	0	471	139	465
Alternative *C*	527	534	208	48	393	20
Stochastic density A						
Timing for investment (no. of periods)						
Timing *B*	110	Never	Never	138	110	125
Timing *C*	110	109	122	171	110	119
Allocation of effluent (10^3 AF/year)						
Alternative *A*	261	280	354	301	281	342
Alternative *B*	151	0	0	405	96	396
Alternative *C*	283	456	166	28	356	20
Stochastic density B						
Timing for investment (no. of periods)						
Timing *B*	103	197	Never	133	102	121
Timing *C*	1	1	120	177	1	121
Allocation of effluent (10^3 AF/year)						
Alternative *A*	183	213	323	255	135	245
Alternative *B*	59	313	0	323	165	324
Alternative *C*	156	195	50	50	103	26

scenarios. In these two stochastic scenarios, long-term natural recharge events are sampled out of a set of user defined distributions, and decision and state variables' paths are determind such that they satisfy optimality conditions with respect to all sampled events. The optimal solution under these two stochastic alternatives is obtained using the Extended Mathematical Programming (EMP) language in GAMS. Both alternatives are sampled from a Gamma distribution function, but differ in the parameters that determine the distribution shape and location. The first (hereafter, Stochatic Density A), is distributed around the annual natural recharge assumed in the deterministic case (i.e., the Base Case), and the second (hereafter, Stochastic Density B), is distributed around a lower mean (by 30%) with smaller variance (by 20%) with respect to the first stochastic alternative. Both density functions are presented in Fig. 4. Table 4 presents the welfare differences (in net present values) for scenarios S2 through S6, with respect to scenario S1, for each of the natural recharge conditions assumed, as well as the welfare differences for each scenario, between the two stochastic alternatives and the deterministic recharge event.

First, it is important to note that the results of scenario S3, under all assumed recharge conditions support Proposition 1, such that only conveyance to the agricultural alternative

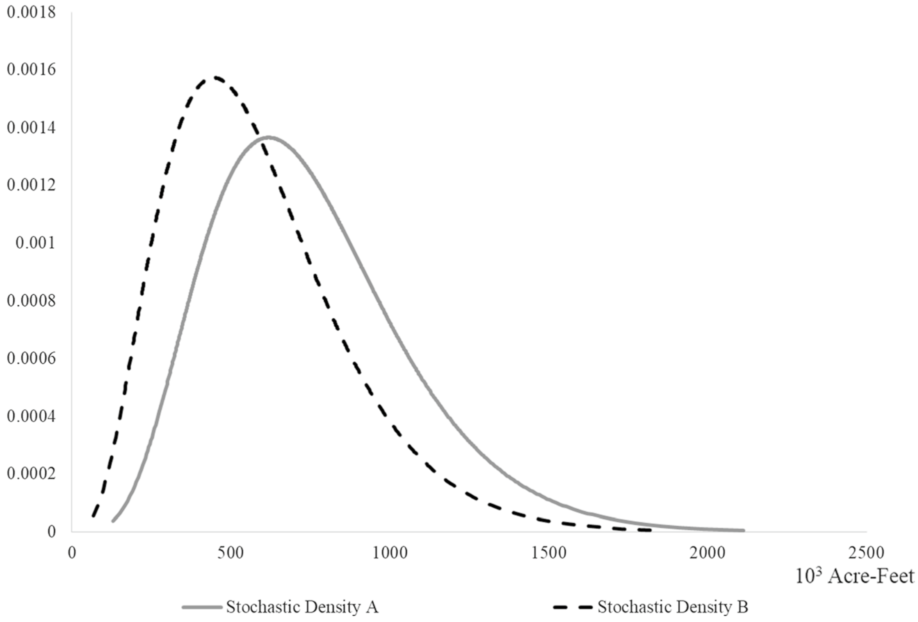

Fig. 4 Probability distribution functions for stochastic natural recharge alternatives

is developed and utilized. This is specifically interesting given that in our illustrative example we use a linear function of capacity for the investment in both effluent disposal alternatives. This, in turn, means that all the analyzed scenarios (S1 through S6) violate Assumption 2 by definition. While the results of this scenario satisfy Proposition 1, it is clear from the results of all other scenarios that relaxing our other assumptions violates Proposition 1. In these other scenarios the optimal timing for constructing alternative B, either equals, precedes or follows the optimal timing for constructing alternative C. Nevertheless, it is important to emphasize that under all scenarios, and all uncertainty conditions with respect to natural recharge the development and utilization of the reuse option (alternative C) is always found to be an optimal strategy.

A second important outcome that should be emphasized from this analysis is with respect to the sensitivity of the results to the stochastic nature of water availability. Comparing the results of all scenarios (excluding scenario S6) between the Base Case (deterministic natural recharge) and the two stochastic alternatives, it can be noticed from Table 3 that the optimal timing for constructing alternative C under uncertainty in recharge conditions always precedes the optimal timing under deterministic recharge conditions.[15] This result supports our earlier hypothesis, in which we stated that potential impacts of water availability stochastics could only contribute to the main argument in favor of the reuse

[15] For scenario S6, alternative C is developed later under uncertainty then under deterministic natural recharge conditions. We attribute this anomaly to the sensitivity of results to the number of natural recharge serieses sampled from the stochastic distributions. Due to the strong tradeoffs we found between runtime and larger sample sizes we decided to forgo our search of the most accurate solution to scenario S6. This tradeoff analysis is available from the authors upon request.

Table 4 Differences in welfare (net present value, 10^3 USD) between scenarios

Scenarios	S1	S2	S3	S4	S5	S6
With respect to S1						
Base conditions	0	64	87	−11	−1	−5
Stochastic density A	0	45	64	−37	−19	−31
Stochastic density B	0	145	132	41	0	47
Stochastic density A versus base conditions	−21	−19	−23	−26	−19	−26
Stochastic density B versus base conditions	−104	−23	−59	−53	−104	−52

alternative. As expected, the economic welfare impacts associated with uncertain recharge conditions are found to be negative for all scenarios in comparison to the (deterministic) Base Case.

4 Conclusions, Policy Implications, and Caveats

In this paper, we developed a framework that allows making optimal social decisions regarding productivity, welfare, and environmental health resulting from water quantity–quality allocation in a regional context. We demonstrated the optimality of developing and reusing treated wastewater for beneficial purposes, using a social planner's approach, in which water quantity–quality allocations affect urban net income and agricultural productivity. Our theoretical as well as empirical results suggest that of the various options facing the regional decision-maker, the development and use to capacity of treated wastewater for irrigation is the superior alternative for the region, as it maximizes the net regional benefits under a set of reasonable assumptions. The validity of these assumptions under different local conditions is an empirical question in nature, and therefore calls for future research.

One clear conclusion from our analysis is that the strong interaction between the city treatment performance, the agricultural sector resilience, and the environment affect the optimal path and preferences among investment alternatives. In terms of groundwater extraction, first order conditions suggest that in the optimal solution steady-state extraction should equal the level of recharge (both from precipitation and agricultural water deep percolation). In terms of water quality, the optimal solution requires that the contaminant level would be set taking into account the pollution created by irrigated agriculture and the pollutant decay rate in the aquifer. These two findings support the need for policy interventions to address the dual quantity–quality regulation of water resources, especially, with a possible risk of water pollution. These outcomes are in agreement with previous work addressing the optimal-combined management of both groundwater quantity and quality dimensions (Hellegers et al. 2001; Roseta-Palma 2002, 2003).

Our regional model did not address several aspects. We did not include the stochastic nature of precipitation in the analytical model and therefore, as demonstrated in the empirical illustrative section, our results might be downward biased vis-á-vis the value of the groundwater. Tsur and Graham-Tomasi (1991) showed analytically and estimated empirically the buffer value of groundwater under stochastic supply of surface water. They referred to the quantity dimension of groundwater as a source to balance scarcity effects of water availability. Considering the additional role of groundwater as a water quality-enhancing medium, would make our results even more significant. The aspects of the buffer value and the water quality-enhancing value of groundwater are left for our future research.

Another aspect that our model did not address is the extension of the social planner's solution, which is a feasibility test and maximization of regional benefits without considering the actions of individual agents. These include negotiations over the wastewater quality and the price per unit of wastewater sold between the city and the agricultural sector, and side payments among the agents (e.g., Dinar et al. 1986). Incorporating negotiated solutions into the theoretical and empirical frameworks we developed will add dimensions that are more practical in the context of multi-player groups that participate in the regional water reuse project.

Finally, our optimization model simplifies the farm-level operation. First, we consider only one agricultural decision-maker in the region. Second, we consider a farming operation with only one crop instead of a multi-crop farm, which could add more flexibility to the on-farm decisions. These aspects, which have not been part of the theoretical and empirical social planner's model, will be included in a future regional model that will be developed for the Escondido region in California.

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Appendix 1

As explained earlier in the text, we describe the social planner welfare maximization problem under different initial settings with respect to the existing disposal alternatives. For each separate problem (i.e., alternative *A* only, alternatives *A* and *B* combined, alternative *A* and *C* combined, and all alternatives together) we assume that the initial conditions prevail for an infinite horizon. We denote the problems 32, 46, 52, and 65, respectively, deriving for each its necessary first-order conditions (FOC), and the resulting optimal solution.

Optimal Plan Under the 'Zero-Alternative'

Let the regional social planner welfare maximization problem (32) associated with the existence of disposal alternative *A* only be:

$$\begin{aligned}\underset{Q^u,S,Q^a,X,w,\varphi}{Max}\int_0^\infty e^{-\rho t}\cdot\big[&U\{I(\beta\cdot(Q^u(t)+S(t)),\varphi(t)),Q^u(t)+S(t)\}\\&+P^Y(t)\cdot X(t)\cdot Y\{w(t),\psi(g(t));r(t)\}-f(X(t))\\&-C(G(t))\cdot(Q^u(t)+Q^a(t))-h(g(t))\cdot Q^u(t)-D(E(t),\varphi(t))\big]dt\end{aligned}\tag{32}$$

s.t.

(a) $\dot{G} = R(t) + \theta \cdot X(t) \cdot w(t) - Q^u(t) - Q^a(t)$
(b) $\dot{g} = e(\psi(g(t))) - \delta(G(t)) \cdot g(t)$
(c) $\varphi(\mathrm{t}) \leq \bar{\varphi}$
(d) $X(t) \cdot w(t) \leq Q^a(t)$
(e) $S(t) \leq \bar{S}$
(f) $G(0) = G_0$
(g) $g(0) = g_0$

Let the respective Lagrangian function be:

$$
\begin{aligned}
L = {} & U\{I(\beta \cdot [Q^u(t) + S(t)], \varphi(t)), Q^u(t) + S(t)\} \\
& + P^Y(t) \cdot X(t) \cdot Y\{w(t), \psi(g(t)); r(t)\} \\
& - f(X(t)) - C(G(t)) \cdot \left[Q^u(t) + Q^a(t)\right] - h(g(t)) \cdot Q^u(t) \\
& - D(E(t), \varphi(t)) + m_1(t) \cdot \left[R(t) + \theta \cdot X(t) \cdot w_t - Q^u(t) - Q^a(t)\right] \\
& + m_2(t) \cdot \left[e(\psi(g(t))) - \delta(G(t)) \cdot g(t)\right] + \lambda_\phi \cdot [\bar{\varphi} - \varphi(t)] \\
& + \lambda_W \cdot \left[Q^a(t) - X(t) \cdot w(t)\right] + \lambda_S \cdot \left[\bar{S} - S(t)\right]
\end{aligned}
$$

The FOC, $\frac{\partial L}{\partial z_{(32)}} = 0$ where $z_{(32)} \in \{Q^u(t), S(t), Q^a(t), \varphi(t), X(t), w(t)\}$, and $\dot{m}_i - \rho \cdot m_i(t) = -\frac{\partial L}{\partial s}$ where $i = 1, 2\,; s \in \{G(t), g(t)\}$, along with the Karush–Kuhn–Tucker conditions $\frac{\partial L}{\partial \lambda_j} \lambda_j = 0, \lambda_j \geq 0; j = \varphi, W, S$ are re-organized and presented as follows

$$\lambda_W(t) = C(G(t)) + m_1(t) \tag{33}$$

$$\underbrace{\underbrace{U_I I_{Q^u}}_{\text{Indirect Effect (-)}} + \underbrace{U_{Q^u}}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = C(G(t)) + h(g(t)) + \beta \cdot D_E + m_1(t) \tag{34}$$

$$\underbrace{\underbrace{U_I I_S}_{\text{Indirect Effect (-)}} + \underbrace{U_S}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = \beta \cdot D_E + \lambda_S(t) \tag{35}$$

$$\underbrace{U_I I_\varphi}_{\text{Marginal Utility (+)}} = \underbrace{D_\varphi}_{\text{Marginal Damage (+)}} + \lambda_\varphi \tag{36}$$

$$P^Y(t) \cdot Y\{w(t), g(t); r(t)\} = f'(X(t)) + \left(\lambda_W(t) - \theta \cdot m_1(t)\right) \cdot w(t) \tag{37}$$

$$X(t) \cdot \left(P^Y(t) \cdot Y_w + \theta \cdot m_1(t) - \lambda_W(t)\right) = 0 \tag{38}$$

$$\dot{m}_1 - \rho \cdot m_1(t) = \underbrace{C'(G(t))}_{\substack{\text{Stock Effect on}\\ \text{Marginal Extraction}\\ \text{Cost (-)}}} \cdot (Q^u + Q^a) + m_2(t) \cdot \underbrace{\delta'((G(t))}_{\substack{\text{Marginal}\\ \text{Decay Rate (+)}}} \cdot g(t) \tag{39}$$

$$\dot{m}_2 + \left[\underbrace{e_\psi \psi_g}_{\substack{\text{Marginal}\\ \text{Contamination}\\ \text{Rate (+)}}} - \delta(G(t)) - \rho \right] \cdot m_2(t) = \underbrace{h'(g(t))}_{\substack{\text{Marginal Cost of}\\ \text{Water Treatment}\\ \text{for Urban Use (+)}}} \cdot Q^u(t) - X(t) \cdot \underbrace{P^Y(t) \cdot Y_\psi \psi_g}_{\substack{\text{VMP of Applied}\\ \text{Water Quality}\\ \text{in Agriculture (-)}}} \tag{40}$$

$$\lambda_\varphi(t) \cdot (\bar{\varphi} - \varphi(t)) = 0, \quad \lambda_\varphi(t) \geq 0 \tag{41}$$

$$\lambda_W(t) \cdot (Q^a(t) - X(t) \cdot w(t)) = 0, \quad \lambda_W(t) \geq 0 \tag{42}$$

$$\lambda_S(t) \cdot \left(\bar{S} - S(t) \right) = 0, \quad \lambda_S(t) \geq 0 \tag{43}$$

Equation (33) states that the shadow value associated with the available water constraint for irrigation should be equal to the sum of the unit cost of extraction and the scarcity rent, and therefore will always be non-negative in the optimal solution. Equations (34) and (35) equate the marginal utility of water consumption to the marginal cost associated with the use of each water source—groundwater and the outside source, respectively. For the case of groundwater, according to (34), this marginal cost will be the sum of extraction cost, treatment cost (in case that the groundwater quality falls under the threshold permitted for drinking),[16] the marginal damage associated with discharging an additional unit of effluent to the environment, and the scarcity rent. In the other case, the marginal utility of consuming one more unit from the outside source should only equate to the marginal damage and the shadow value associated with that source's availability constraint. It immediately follows from these two equations that when the outside source is available in a very large amount (i.e., $\lambda_S = 0$), the optimal consumption in the city will always be based on the outside source alone. In (36) marginal utility from treating effluent to a lower quality is equated with the sum of the marginal damage associated with discharging water at a lower quality to the environment and the shadow value of the regulatory quality standard constraint. Substituting Eq. (33) into (37) and (38), and given that there exists an internal solution with respect to cultivable land (i.e., $X^*(t) > 0 \, \forall t$), yields the following respectively:

$$P^Y(t) \cdot Y\{w(t), g(t); r(t)\} = f'(X(t)) + \left[\underbrace{C(G(t)) + (1 - \theta) \cdot m_1(t)}_{\text{VMP of Water Applied (+)}} \right] \cdot w(t) \tag{44}$$

[16] It can be observed that in the social planner optimal solution, $h(g^*(t))$ will always be set to zero, that is if the initial groundwater quality g_0 is of better quality than the threshold $\bar{g}$. This is easy to show, as lower water quality doesn't have any positive effect on the objective function. It doesn't mean however that $g^*(t) \geq \bar{g}$ is not a feasible solution.

$$P^Y(t) \cdot Y_w = C(G(t)) + (1-\theta) \cdot m_1(t) \tag{45}$$

Equation (45) equates the value of marginal product in agriculture from one more unit of water applied for irrigation to its associated marginal cost, which is the sum of the unit cost of extraction and the scarcity rent, multiplied by one minus the percolation rate—accounting for that unit contribution to the groundwater stock level. Equation (44) refers to the other input used in agricultural production (in our model), which is land, and requires the identity of the marginal value of unit of land to be the marginal cost associated with its cultivation. That cost is equal to the marginal crop production cost associated with different input use (e.g., fertilizer and labor), and expressed by the function $f(\cdot)$, plus the value of water used to irrigate one additional unit of land. Equation (39) defines the optimal path for scarcity rent evolution over time. Naturally, it is dependent upon the impact of groundwater stock changes on marginal cost of extraction, multiplied by the extraction quantities, but it is also related to the opportunity costs resulting from groundwater quality degradation. The evolution of the latter is defined in Eq. (40), and is associated with the effect of groundwater quality on both sectors, i.e., the marginal cost of groundwater treatment to the city, and the value of marginal productivity of water quality in agriculture. The net discounted effect on groundwater quality, which is expressed as the sum of marginal rate of contamination from water irrigation, the decay rate, and the discount rate will also affect the optimal trajectory of this co-state. Equations (41) through (43) are the usual Karush–Kuhn–Tucker conditions, and dictate that the shadow value of any binding constraint in the optimization must be non-negative.

Safe Discharge Optimal Solution

We move on to describe the planner's problem when effluent could also be safely discharged from the WWTP remotely at a pre-determined cost. This problem is denoted (46) as follows:

$$\begin{aligned}\underset{Q^u,S,Q^a,O,X,w,\varphi}{Max} \int_0^\infty e^{-\rho t} \cdot \big[& U\{I(\beta \cdot (Q^u(t)+S(t)), \varphi(t)), Q^u(t)+S(t)\} \\ & + P^Y(t) \cdot X(t) \cdot Y\{w(t), \psi(g(t)); r(t)\} - f(X(t)) \\ & - C(G(t)) \cdot (Q^u(t)+Q^a(t)) - h(g(t)) \cdot Q^u(t) - v(O(t)) - D(E(t), \varphi(t)) \big] dt\end{aligned} \tag{46}$$

s.t.

(a) $\dot{G} = R(t) + \theta \cdot X(t) \cdot w(t) - Q^u(t) - Q^a(t)$
(b) $\dot{g} = e(\psi(g(t))) - \delta(G(t)) \cdot g(t)$
(c) $\varphi(\mathrm{t}) \le \bar{\varphi}$
(d) $X(t) \cdot w(t) \le Q^a(t)$
(e) $S(t) \le \bar{S}$
(f) $O(t) \le \beta \cdot (Q^u(t) + S(t))$
(g) $G(0) = G_0$
(h) $g(0) = g_0$

Problem (46) is slightly different than problem (32) in that it also includes the cost of conveyance to the remote location as a function of quantity, as part of the objective function.

Constraint (f) is introduced to ensure that conveyance to the remote location will be limited to the amount of available effluent. Tradeoffs between the optimal solutions of these two problems can be explained intuitively. Net benefits will only accrue to the region in the following instances: a) if the costs of conveying the effluent to the remote location are lower than the avoidable damage; b) if the city earns from treating its effluent to a lower quality, and c) if consuming more water in the city and discharging it to the remote location (avoiding the associated environmental damage) exceeds the losses to farmers from diverting shared resource water to the city. The resemblance of problems (32) and (46) implies also that their derived FOC will have some similarity. We therefore present only the FOC that differ between the two:

$$\underbrace{\underbrace{U_I I_{Q^u}}_{\text{Indirect Effect (-)}} + \underbrace{U_{Q^u}}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = C(G(t)) + h(g(t)) + \beta \cdot \left(D_E - \lambda_E(t)\right) + m_1(t) \tag{47}$$

$$\underbrace{\underbrace{U_I I_S}_{\text{Indirect Effect (-)}} + \underbrace{U_S}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = \beta \cdot \left(D_E - \lambda_E(t)\right) + \lambda_S(t) \tag{48}$$

$$\lambda_E(t) \cdot (\beta \cdot (Q^u(t) + S(t)) - O(t)) = 0, \lambda_E(t) \geq 0 \tag{49}$$

$$\lambda_E(t) = D_E - v'(O(t)) \tag{50}$$

We note the differences between Eqs. (47) and (48), and their counterparts (34) and (35) in the previous problem. These two equations equate marginal utility from water consumption in the city to their respective marginal cost, according to the source of supply. It is easy to observe that these two equations would become identical to (34) and (35), in the private case where λ_E, which is the shadow value associated with the effluent availability constraint, equals zero. Equation (50) depicts that shadow value, and defines it as equal to the difference between the marginal damage and marginal conveyance cost to the remote location. The logic is simple: diverting water from the environment to the remote location is only worthwhile as long as the costs to society are lower than the damage avoided. Equation (49) states that when that difference is positive, the constraint must be binding, which means that all effluent should be discharged at the remote location. An interesting phenomenon arises in Eq. (50) with respect to the relationship between the optimal solution of the system, and the predetermined assumptions regarding the damage function. It can be seen that once the assumption of Horan (2001) is followed (i.e., the damage function is non-decreasing and convex), the optimal solution will never suggest discharging all the effluent remotely. The reason is that when this occurs, the marginal damage avoided will be very small (since $E(t)$, the effluent diverted to the environment is small), and marginal cost of conveyance will then be higher (as $v(\cdot)$ is also assumed to be a non-decreasing and convex function, and $O(t)$, the quantity diverted to the remote location is high). That translates to a negative λ_E, which cannot be an optimal solution according to (49). When the damage function is non-decreasing and concave (or linear), the decision to divert all effluent to the remote location is always the optimal solution, and the shadow value of effluent availability constraint must be positive.

We denote by Z_i the solution space for problem i where $i \in \{(32), (46), (52), (65)\}$, and z_i^* as the optimal solution for a given empirical setting of problem i, such that $z_i^* \in Z_i$ and $V_i\left(z_i^*\right)$ is the maximum value of the objective function, given the optimal solution of the problem z_i^*.

Lemma 1 *Facing identical functional forms and sets of parameters*

$$V_{(46)}\left(z^*_{(46)}\right) \geq V_{(32)}\left(z^*_{(32)}\right)$$

Proof As described above, the optimal conditions for problem (32) can be represented as a private case of the optimal conditions solving problem (46). It immediately follows that $Z_{(32)} \subset Z_{(46)}$, and therefore $z^*_{(32)} \in Z_{(46)}$, but also that $V_{(46)}\left(z^*_{(32)}\right) = V_{(32)}\left(z^*_{(32)}\right)$. It then follows that for every given empirical setting, if $z^*_{(46)} \neq z^*_{(32)}$ then it must be that $V_{(46)}\left(z^*_{(46)}\right) \geq V_{(32)}\left(z^*_{(32)}\right)$.

Following Lemma 1 and previous definitions, the condition for investing in alternative B, such that an infrastructure for safe discharge of effluents will be developed with respect to problem (32) at an optimal time T_B^*, dividing the horizon to $[0, T_B^*]$ and $[T_B^*, \infty)$ is:

$$\begin{aligned} &V^{\tau_B}_{(46)}\left(z^*_{(46)\tau_B}\right) \geq V^{\tau_B}_{(32)}\left(z^*_{(32)\tau_B}\right) + F_B \Leftrightarrow \Omega_{(32)(46)} \geq 0 \,:\, \Omega_{(32)(46)} \\ &\quad = \Delta D(\Delta E) + \Delta U(\Delta I(\Delta Q^u, \Delta\varphi), \Delta Q^u) - \Delta Y(\Delta Q^a) - F_B - \Delta v(\Delta O) \end{aligned} \tag{51}$$

Condition (51) formally defines the intuitive justification described earlier for developing the new infrastructure for alternative B. That is, avoided damages and increased available income, as well as changes in utility and production from reallocation of shared groundwater quantities must surpass the costs of conveyance, and the investment needed to develop that infrastructure.

Optimal Plan for Treated Wastewater Reuse in Agriculture

Problem (52) addresses the case in which the planner faces the alternative to either discharge the treated wastewater to the environment or divert it for irrigation of crops in a neighboring agricultural district. It is presented as follows:

$$\begin{aligned} &\underset{Q^u,S,Q^a,A,X,w,\varphi}{Max} \int_0^\infty e^{-\rho t} \cdot \left[U\{I(\beta \cdot (Q^u(t) + S(t)), \varphi(t)), Q^u(t) + S(t)\}\right. \\ &+ P^Y(t) \cdot X(t) \cdot Y\{w(t), \psi(g(t), \varphi(t), A(t), Q^a(t)); r(t) - f(X(t))\} \\ &\left.-C(G(t)) \cdot (Q^u(t) + Q^a(t)) - h(g(t)) \cdot Q^u(t) - \upsilon(A(t)) - D(E(t), \varphi(t))\right] dt \end{aligned} \tag{52}$$

s.t.

(a) $\dot{G} = R(t) + \theta \cdot X(t) \cdot w(t) - Q^u(t) - Q^a(t)$
(b) $\dot{g} = e(\psi(g(t), \varphi(t))) - \delta(G(t)) \cdot g(t)$
(c) $\varphi(\mathrm{t}) \leq \bar{\varphi}$
(d) $X(t) \cdot w(t) \leq Q^a(t) + A(t)$

(e) $S(t) \le \bar{S}$
(f) $A(t) \le \beta \cdot (Q^u(t) + S(t))$
(g) $G(0) = G_0$
(h) $g(0) = g_0$

Problem (52) resembles problem (46) in the sense that it also includes an alternative to environmental damage from effluent discharge. However, there is a distinct difference between the two problems, as the current one facilitates effluent quality effects on agricultural productivity and groundwater quality evolution over time. We also expand the definition of the applied water quality function in order to account for the effects of water blending from the different sources, as mentioned in the text (see footnote 1). This expansion indicates that:

$$\begin{cases} \psi_{Q^a}, \psi_{Q^aQ^a} \le 0 \text{ if } g \le \varphi \\ \psi_{Q^a}, \psi_{Q^aQ^a} > 0 \text{ else} \end{cases} \tag{53}$$

$$\psi_{A\varphi} = \psi_{\varphi A} > 0; \psi_{Q^a g} = \psi_{gQ^a} > 0; \psi_{Ag} = \psi_{gA} < 0; \psi_{Q^a\varphi} = \psi_{\varphi Q^a} < 0 \tag{54}$$

$$\begin{cases} \psi_{Q^aA}, \psi_{AQ^a} \le 0 \text{ if } (g \le \varphi \cup Q^a \ge A) \text{ or } (g \ge \varphi \cup Q^a \le A); \forall Q^a, A \ge 1 \\ \psi_{Q^aA}, \psi_{AQ^a} > 0 \text{ else} \end{cases} \tag{55}$$

In addition to the assumed characteristics mentioned in the text, according to (53) the weighted average functional form indicates that the average quality is either positively, or negatively affected by the usage of each source, depending on the quality ratio between the sources. The same relationship applies for the rate at which this function increases (or decreases). Equation (54) lists the cross derivatives of quality and quantity components. In that respect, a positive cross derivative is expected when quantity and quality of the same source are considered, and a negative relationship holds when quantity (quality) of one source is derived with respect to the quality (quantity) of the other. Equation (55) implies that the cross derivative of the average quality with respect to the quantities consumed in agriculture is changing its sign, according to the proportions between effluent and groundwater quantities and qualities. As in problem (46), the costs of effluent conveyance to the agricultural district are considered explicitly in the objective function. Equation (f) is similar to its counterpart in problem (46) and limits the conveyed quantity to the irrigation district by the available effluent volume. Similar to the description following problem (46), we present the FOC that are distinct for problem (52), compared to problem (32):

$$\lambda_W(t) = C(G(t)) + m_1(t) - \underbrace{Y_\psi \psi_{Q^a}}_{(-)} \tag{56}$$

$$\underbrace{\underbrace{U_I I_{Q^u}}_{\text{Indirect Effect (-)}} + \underbrace{U_{Q^u}}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = C(G(t)) + h(g(t)) + \beta \cdot \left(D_E - \lambda_E(t)\right) + m_1(t) \tag{57}$$

$$\underbrace{\underbrace{U_I I_S}_{\text{Indirect Effect (-)}} + \underbrace{U_S}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = \beta \cdot \left(D_E - \lambda_E(t)\right) + \lambda_S(t) \tag{58}$$

$$\underbrace{U_I I_\varphi}_{\text{Marginal Utility }(+)} = \underbrace{D_\varphi}_{\text{Marginal Damage }(+)} + \lambda_\varphi - X(t) \cdot \underbrace{P^Y(t) \cdot Y_\psi \psi_\varphi}_{\substack{\text{VMP of Applied} \\ \text{Water Quality} \\ \text{in Agriculture (-)}}} - m_2(t) \cdot \underbrace{e_\psi \psi_\varphi}_{\substack{\text{Marginal} \\ \text{Contamination Rate }(+)}} \tag{59}$$

$$P^Y(t) \cdot Y\{w(t), \psi(g(t), \varphi(t)); r(t)\} = f'(X(t)) + \left(\lambda_W(t) - \theta \cdot m_1(t)\right) \cdot w(t) \tag{60}$$

$$\lambda_W(t) \cdot (Q^a(t) + A(t) - X(t) \cdot w(t)) = 0, \quad \lambda_W(t) \geq 0 \tag{61}$$

$$\lambda_E(t) \cdot (\beta \cdot (Q^u(t) + S(t)) - A(t)) = 0, \quad \lambda_E(t) \geq 0 \tag{62}$$

$$\lambda_E(t) = \lambda_W(t) + D_E + \underbrace{Y_\psi \psi_A}_{(-)} - v'(A(t)) \tag{63}$$

Similar to the argument following problem (46), it can be easily demonstrated that the optimal conditions solving problem (32) can be represented as a private case of the optimal conditions of the current problem.

Lemma 2 *Facing identical functional forms and sets of parameters*

$$V_{(52)}\left(z^*_{(52)}\right) \geq V_{(32)}\left(z^*_{(32)}\right)$$

Proof Considering our last argument, it follows that the proof of Lemma 1 applies for the relationship between problem (52) and problem (32), as well. It then follows that for every given solution $z^*_{(52)} \neq z^*_{(32)}$, since $Z_{(32)} \subset Z_{(52)}$, then $V_{(52)}\left(z^*_{(52)}\right) \geq V_{(32)}\left(z^*_{(32)}\right)$.

The differences between the previous problems (32, 46) and the current one are well summarized in Eq. (59). It is the last two components on the right-hand side of the equation that tell the story. The third component is the marginal effect of a change in effluent quality on agricultural productivity. The fourth component is the opportunity cost of groundwater quality degradation, multiplied by the marginal rate of contamination resulting from percolation of irrigation water to the groundwater aquifer. One can notice that an optimal solution of a higher quality level of effluent with respect to the previous problems (32) and (46) can be reached (e.g., $\varphi^*(t) = g(t)$), as both effects contribute to the same direction. Let us denote this set of potential solutions as $\tilde{Z}_{(52)}$ to be used in the proof that follows. Considering the expansion introduced above regarding the quality of water applied in agriculture, we note that the only effect is through the change in shadow values of the water availability constraint to the agriculture sector (56), and the one associated with the effluent availability constraint (63). Recall that $Y_{\psi\psi} > 0$ and that $\psi_{Q^aQ^a} \leq 0$, for $g \leq \varphi$, therefore Eq. (56) shows that the net benefit associated with releasing constraint (d) by one unit is higher, either when quality of water applied is better, or when supply of groundwater to the farming sector is lower—when quality of groundwater is better than that of effluent. The fact that $\psi_{Q^aQ^a}$ changes its sign as the quality ratio reverses between sources, simply implies that higher shadow value for water availability in agriculture can also prevail for higher quantities supplied from groundwater. For Eq. (63), the interpretation is rather similar. Effluent high in contaminant levels affect $\lambda_E(t)$ negatively, and therefore are less attractive for the farming sector in the optimal solution. Another point worth mentioning is related to Eq. (63). Unlike its counterpart in problem (46), the shadow value of effluent availability constraint is equated not only to the difference between marginal damage and marginal conveyance costs, but also $\lambda_W(t)$ (the shadow value of water availability for irrigation constraint) is added on the right-hand side. As already discussed previously, this shadow value is non-negative. Therefore, in the optimal solution for problem (52), as opposed to (46), even when the damage function is considered to be non-decreasing and convex, there can be a situation in which all effluent is diverted away from the environment.

Similar to condition (51), the formal definition for an optimal time T_C^* to invest in developing infrastructure for conveying treated wastewater to agriculture with respect to problem (32) is given in condition (64).

$$\begin{aligned} &V_{(52)}^{\tau_C}\left(z^*_{(52)\tau_C}\right) \geq V_{(32)}^{\tau_C}\left(z^*_{(32)\tau_C}\right) + F_C \Leftrightarrow \Omega_{(32)(52)} \geq 0 \,:\, \Omega_{(32)(52)} = \Delta D(\Delta E) \\ &\quad + \Delta m_1 \cdot G_0 \pm \Delta U(\Delta I(\Delta Q^u, \Delta\varphi), \Delta Q^u) \pm \Delta Y(\Delta Q^a, \Delta\psi) \\ &\quad - \Delta m_2 \cdot g_0 - F_C - \Delta\upsilon(\Delta A) \end{aligned} \tag{64}$$

Condition (64) defines the tradeoffs associated with potential development of the reuse system when the initial conditions are the same as in problem (32). It is composed of reduction of environmental damages, decreased scarcity rents, changes in utility and production from reallocation of groundwater extractions, and from changes to the quality of effluents and applied water in irrigation, as well as the changes to the shadow value of quality degradation and the costs of investing in the infrastructure and conveying the water.

Lemma 3 *Let* $Z_{((46)\cap(52))} \equiv Z_{(46)} \cap Z_{(52)}$. *Then,*

$$Z_{((46)\cap(52))} \neq \{\emptyset\}$$

Proof Following Lemmas 1 and 2, $Z_{(32)} \subset Z_{(46)}$, and also $Z_{(32)} \subset Z_{(52)}$. Then it immediately follows that $Z_{(32)} \subset Z_{((46)\cap(52))}$.

Lemma 4 *Consider* $Z_{(46)}$ *and* $Z_{(52)}$.

$$\begin{aligned} &\textit{either}\;\; Z_{(46)} \subset Z_{(52)} \\ &\textit{or}\;\; Z_{(46)}, Z_{(52)} \subset Z_{(46)} \cup Z_{(52)} \;:\; Z_{(52)} \not\subset Z_{(46)} \end{aligned}$$

Proof Following Lemma 3, since $Z_{((46)\cap(52))} \neq \{\emptyset\}$, it means that either $Z_{(46)} \subset Z_{(52)}$, or that $Z_{(52)} \subset Z_{(46)}$, or that both sets have a subset that is not included in the other. As noted earlier $\tilde{Z}_{(52)} \subset Z_{(52)}$, however $\tilde{Z}_{(52)} \not\subset Z_{(46)}$, it immediately follows that $Z_{(52)} \not\subset Z_{(46)}$.

Since the first relationship in Lemma 4 is a private case for the second relationship presented, it follows that every condition derived based on the latter relationship will hold for the former.

Optimal Plan Under All Alternative Disposal Locations

We now turn to analyze the social planner's problem when all disposal alternatives exist in the initial setting and throughout eternity.

$$\underset{Q^u,S,Q^a,A,O,\varphi,X,w}{Max} \int_0^\infty e^{-\rho t} \cdot \left[U\{I(\beta \cdot (Q^u(t) + S(t)), \varphi(t)), Q^u(t) + S(t)\}\right.$$
$$+ P^Y(t) \cdot X(t) \cdot Y\{w(t), \psi(g(t), \varphi(t)); r(t)\} - f(X(t))$$
$$\left. - C(G(t)) \cdot (Q^u(t) + Q^a(t)) - h(g(t)) \cdot Q^u(t) - v(O(t)) - \upsilon(A(t)) - D(E(t), \varphi(t))\right] dt \tag{65}$$

s.t.

(i) $\dot{G} = R(t) + \theta \cdot X(t) \cdot w(t) - Q^u(t) - Q^a(t)$
(j) $\dot{g} = e(\psi(t)) - \delta(G(t)) \cdot g(t)$
(k) $\varphi(\mathrm{t}) \leq \bar{\varphi}$
(l) $X(t) \cdot w(t) \leq Q^a(t) + A(t)$
(m) $S(t) \leq \bar{S}$
(n) $A(t) + O(t) \leq \beta \cdot (Q^u(t) + S(t))$
(o) $G(0) = G_0$
(p) $g(0) = g_0$

Discarding the specific functional form introduced in problem (52) for applied water quality, the FOC for problem (65) that differ from the conditions for optimal solution of problem (32) are:

$$\lambda_W(t) = C(G(t)) + m_1(t) \tag{66}$$

$$\underbrace{\underbrace{U_I I_{Q^u}}_{\text{Indirect Effect (-)}} + \underbrace{U_{Q^u}}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = C(G(t)) + h(g(t)) + \beta \cdot \left(D_E - \lambda_E(t)\right) + m_1(t) \tag{67}$$

$$\underbrace{\underbrace{U_I I_S}_{\text{Indirect Effect (-)}} + \underbrace{U_S}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = \beta \cdot \left(D_E - \lambda_E(t)\right) + \lambda_S(t) \tag{68}$$

$$\underbrace{U_I I_\varphi}_{\text{Marginal Utility (+)}} = \underbrace{D_\varphi}_{\text{Marginal Damage (+)}} + \lambda_\varphi - X(t) \cdot \underbrace{P^Y(t) \cdot Y_\psi \psi_\varphi}_{\substack{\text{VMP of Applied} \\ \text{Water Quality} \\ \text{in Agriculture (-)}}} - m_2(t) \cdot \underbrace{e_\psi \psi_\varphi}_{\substack{\text{Marginal} \\ \text{Contamination Rate (+)}}} \tag{69}$$

$$P^Y(t) \cdot Y\{w(t), \psi(g(t), \varphi(t)); r(t)\} = f'(X(t)) + \left(\lambda_W(t) - \theta \cdot m_1(t)\right) \cdot w(t) \tag{70}$$

$$\lambda_W(t) \cdot (Q^a(t) + A(t) - X(t) \cdot w(t)) = 0, \quad \lambda_W(t) \geq 0 \tag{71}$$

$$\lambda_E(t) \cdot (\beta \cdot (Q^u(t) + S(t)) - A(t)) = 0, \quad \lambda_E(t) \geq 0 \tag{72}$$

$$\lambda_E(t) = D_E - v'(O(t)) \tag{73}$$

$$\lambda_E(t) = \lambda_W(t) + D_E - \upsilon'(A(t)) \tag{74}$$

Problem (65) incorporates all disposal options, and we can now identify the required conditions to invest in alternative B or C, with respect to problems (52) or (46), as defined in Eqs. (75) and (76), respectively.

$$V_{(65)}^{\tau_B}\left(z^*_{(65)\tau_B}\right) \geq V_{(52)}^{\tau_B}\left(z^*_{(52)\tau_B}\right) + F_B \Leftrightarrow \Omega_{(52)(65)} \geq 0 : \Omega_{(52)(65)} = \Delta m_2 \cdot g_0 \\ + \Delta\upsilon(\Delta A) \pm \Delta U(\Delta I(\Delta Q^u, \Delta\varphi), \Delta Q^u) \pm \Delta Y(\Delta Q^a, \Delta\psi) - \Delta m_1 \cdot G_0 - F_B - \Delta\nu(\Delta O) \tag{75}$$

$$V_{(65)}^{\tau_C}\left(z^*_{(65)\tau_C}\right) \geq V_{(46)}^{\tau_C}\left(z^*_{(46)\tau_C}\right) + F_C \Leftrightarrow \Omega_{(46)(65)} \geq 0 : \Omega_{(46)(65)} = \Delta\nu(\Delta O) + \Delta m_1 \cdot G_0 \\ \pm \Delta U(\Delta I(\Delta Q^u, \Delta\varphi), \Delta Q^u) \pm \Delta Y(\Delta Q^a, \Delta\psi) - \Delta m_2 \cdot g_0 - F_C - \Delta\upsilon(\Delta A) \tag{76}$$

In addressing first condition (76), the only difference with respect to condition (64) is that instead of avoided damages, it is the reduction in conveyance costs to the safe discharge options that are included in condition (76). All other impacts remain the same. It immediately follows that if the avoided damages associated with conveyance of effluent away from the environment exceed the costs of discharging them safely, then an earlier investment in alternative C is preferred over investing first in alterative B and then later in alternative C. Interestingly, the same condition can be derived almost immediately from condition (51), which supports the investment in alternative B with respect to problem (32), or putting formally:

$$\textit{if } \Omega_{(32)(46)} \geq 0 : \exists T_B^* \in (0,\infty) \Rightarrow \Omega_{(32)(52)} \geq \Omega_{(46)(65)} \Rightarrow T_C^* : ((32) \rightarrow (52)) \leq T_C^* : ((46) \rightarrow (65)) \tag{77}$$

Except the difference between damage avoided and the safe discharge conveyance costs, condition (51) also includes potential utility and production changes. Recall that changes in allocation of groundwater between the city and the agricultural district are only warranted if the safe discharge conveyance costs are lower than the damage avoided, and therefore are not in contradiction to condition (77). The other potential impact is through available income, such that if effluent is discharged safely away from the environment the optimal level of effluent quality might be poorer. Hence,

Assumption 1 The level of maximum constituents $\bar{\varphi}$ imposed by regulation is always binding in the optimal solution, regardless of the initial problem setting $i \in \{(32),(46)\}$. Such that,

$$\varphi^*(z_i^*) = \bar{\varphi}.$$

Condition (75) implies that avoiding the degradation of groundwater quality and reduction in conveyance costs of effluent to the agricultural district would justify the investment in alternative B (i.e., remote and safe discharge of the effluent), as well as the reallocation of groundwater and effluent—inducing potential changes to utility and production. Obviously if this is true, then there is no justification to invest in alternative C to begin with, which means condition (75) contradicts condition (64). It then follows that,

$$\Omega_{(52)(65)} \geq 0 : \exists T_B^* \in (T_C^*,\infty) \perp \Omega_{(32)(52)} \geq 0 : \exists T_C^* \in (0,\infty) \Rightarrow T_B^* : ((52) \rightarrow (65)) \in \{\emptyset\} \tag{78}$$

This contradiction also hinges on the following assumption:

Assumption 2 Conveyance capacity to either disposal alternative B or C, once developed, is sufficient for the system's existing and future needs, such that it is not a constraint nor a decision in the optimization problem. Let $\bar{Q}_j$ represent capacity of conveyance for alternative $j : j \in \{B, C\}$. Then,

$$\textit{The investment } F_j, \textit{ in alternative } j, \textit{ is independent from } \bar{Q}_j$$

Assumption 2 implicitly states that timing is the only decision variable associated with the development of each of the disposal alternatives. It implies that since investment is fixed and not dependent upon capacity of conveyance, that expansion of wastewater treatment and consequently quantity consumed in the city are not dependent on the impact of treated wastewater disposal. Hence, the contradiction in (78) might not hold when this assumption is relaxed.

The combination of conditions (77) and (78) implies that if investment in alternative B is optimal when initially alternative A is the only option available, then an investment in alternative C, when alternative A alone sets the initial conditions, is preferred over investment in the same alternative when both alternatives A and B already exist. However, an investment in alternative B following investment in alternative C is never optimal.

Assumption 3 The present value of long-term environmental damage trumps any reasonable investment and operating costs of conveyance infrastructure. Such that,

$$\int_0^\infty [e^{-\rho t} \cdot D(\cdot)]dt \geq F_j + \int_0^\infty [e^{-\rho t} \cdot l(\cdot)]dt \; : \; j \in \{B, C\}, l(\cdot) \in \{\nu(\cdot), \upsilon(\cdot)\}$$

Assumption 3 simply implies that the damage avoided over the long-term is larger than the conveyance costs (including fixed costs) of alternatives B and C. Assumption 4 that follows, guarantees that the feasible solution space for problem (65) will not rule out any potential positive allocation of effluent to either the agricultural district or the safe discharge alternative, or both.

Assumption 4 Conveyance costs functions $\upsilon(\cdot)$ and $\nu(\cdot)$ cannot be extremely different from each other. That is,

$$\frac{\upsilon''(\cdot)}{\upsilon'(\cdot)} = \kappa \frac{\nu''(\cdot)}{\nu'(\cdot)} \; : \; \kappa \in \mathbb{R}^+, \kappa \neq \{0, \infty\} \, \forall \{A(t), O(t)\} \in [0, \beta \cdot (Q^{u*}(t) + S^*(t))] \textit{ and}$$

$$\exists \{A(t), O(t)\} \in [0, \beta \cdot (Q^{u*}(t) + S^*(t))] \; : \; \frac{\upsilon'(A(t))}{\nu'(O(t))} \geq 1.$$

This assumption is derived as follows. Equating the right-hand sides of conditions (73) and (74) we get that:

$$\lambda_W(t) = \upsilon'(A(t)) - \nu'(O(t)) \tag{79}$$

Since in the optimal solution $\lambda_W^*(t) \geq 0$, than it must be satisfied that the difference between the derivatives of the two conveyance cost functions (i.e., to the agricultural district and to the remote discharge location) is non-negative as well. Therefore, Assumption 3 dictates that the relative curvature of these functions will be confined to

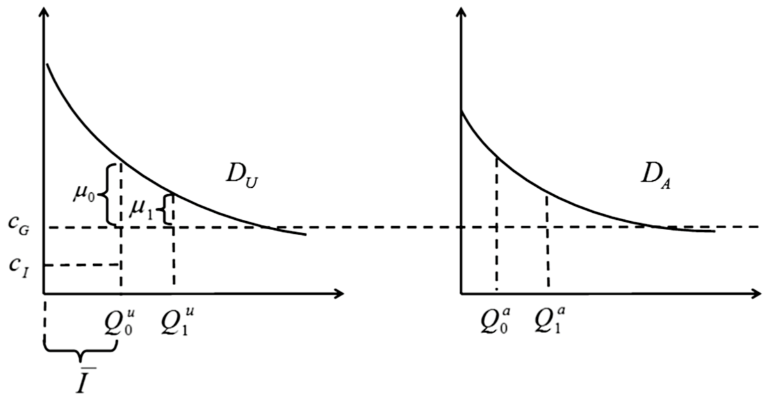

Fig. 5 Static optimal allocation illustration

a constant, such that condition (79) is satisfied for every effluent allocation between the agricultural district and the remote discharge alternatives.

Proposition 1 *Let alternative A set the initial conditions for a regional social welfare planner facing the decision whether to develop the two alternatives for effluent discharge B and C. Then under Assumptions 1 through 4,*

$$\exists T_C^* \in (0, \infty) : T_C^* < T_B^* \Rightarrow T_B^* \in \{\emptyset\} \, \forall \{T_B^*(z_K^*), T_C^*(z_K^*)\} : z_K^* \in Z_K$$

Proof Lemmas 1 through 4 suggest that both conditions (51) and (64) could potentially be satisfied, since $V_{(46)}\left(z^*_{(46)}\right) \geq V_{(32)}\left(z^*_{(32)}\right)$ as well as $V_{(52)}\left(z^*_{(52)}\right) \geq V_{(32)}\left(z^*_{(32)}\right)$, and that $Z_{(46)}, Z_{(52)} \subset Z_{(46)} \cup Z_{(52)} : Z_{(52)} \not\subset Z_{(46)}$. Assumption 1 dictates that regulation of effluent quality is binding, and so it implies that the potential income impact associated with larger contamination resulting from investment in alternative *B* is infeasible. It therefore also dictates that, for any set of optimal solutions $z^* \equiv \{z^*_{(32)}, z^*_{(46)}, z^*_{(52)}, z^*_{(65)}\}$, it must hold that $\{\varphi^*(z^*_{(52)}), \varphi^*(z^*_{(65)})\} \leq \varphi^*(z^*_{(32)}) = \varphi^*(z^*_{(46)}) = \bar{\varphi}$. This means that, from an available income perspective, and with respect to the original setting where alternative *A* alone exists, alternatives *B* and *C* are equivalently attractive. Recall that the solution space subset $\tilde{Z}_{(52)}$ includes the unique solution $z^{u^*}_{(52)} : \varphi^*(z^{u^*}_{(52)}) = g(z^{u^*}_{(52)})$, it immediately follows that the argument presented earlier, and demonstrated in Appendix 2, implies that $\exists T_C^* \in (0, \infty)$ with respect to both problems (32) and (46). We already showed that given Assumption 3 condition (51) is satisfied, such that $\exists T_B^* \in (0, \infty)$; however, in that case condition (77) also holds, and so $T_C^* : ((32) \rightarrow (52)) \leq T_C^* : ((46) \rightarrow (65))$, which according to condition (78) implies that $T_C^* < T_B^* \Rightarrow T_B^* \in \{\emptyset\}$.

Appendix 2

Our argument from the text regarding resemblance of the problem analyzed herein and previous framework of conjunctive use management can be easily illustrated using a quantity allocation optimization problem. For the sake of the illustration only, and without losing generality, it helps to consider a simplistic static example, which we depict in Fig. 5.

Holland and Moore (2003) consider a water import project to augment a renewable groundwater source. Let's notate by $\bar{Q}$ the quantity-constraining extractions prior to investing in the import project, and by $\bar{I}$ the capacity of the new project. On the demand side, let's consider the case at hand, in which supplies need not just satisfy a single sector, but two—a city, and a farming sector, both represented by demand curves that are derived from a utility function, and the value of production function, respectively. Let c_G be the cost of extraction from the aquifer and c_I the unit cost of imported water (where we assume that $c_I < c_G$, but could have equally illustrated the same for the opposite case). Let Q_0^u and Q_0^a be the optimal quantities consumed in the city and by the farming sector, respectively, from both sources. The "0" notation is used for original quantities (prior to the import project), and the notation "1" (instead) for the quantities consumed after the project's implementation. In Fig. 5, the original (optimal) allocation is dictated by the following:

$$Q_0^u + Q_0^a = \bar{Q} \tag{80}$$

$$D_U\left(Q_0^u\right) = D_A\left(Q_0^a\right) = c_G + \mu_0 \tag{81}$$

where D_U and D_A are the demand curves for the city and farming sector, respectively, and μ_0 is the shadow value of the binding constraint (31).

After the introduction of the import project, the supply constraint becomes $\bar{Q} + \bar{I}$, such that now the new optimal quantities comply with:

$$Q_1^u + Q_1^a = \bar{Q} + \bar{I} \tag{82}$$

$$D_U\left(Q_1^u\right) = D_A\left(Q_1^a\right) = c_G + \mu_1 \tag{83}$$

where μ_1 now represents the shadow value of the constraint in (15). As illustrated, let the optimal allocation dictate that the import project is used to full capacity and its water conveyed to the city, and specifically that $Q_0^u = \bar{I}$. It follows that $Q_1^u + (Q_1^a - Q_0^a) = \bar{Q}$. Returning to our original notation and requiring that $C(G)$ will be constant and equal to c_G and that $v'(A)$ is also constant and equal to c_I. Since water treated and conveyed to the farming sector from the city are equal to $\beta \cdot Q_1^u$, if we set β to be equal to $\frac{Q_0^u}{Q_1^u}$, then it immediately follows that under the same framework presented in Fig. 5, the solution for both settings would be identical, with the only difference being the income distribution between the city and the agricultural sector.

In terms of quality, as discussed in detail in Appendix 1, the necessary optimality conditions facilitate a solution in which the quality of the effluent will be equal to the quality of groundwater. Therefore any differences associated with this dimension would be mainly manifested by changes in shadow values, and should not rebut our main argument.

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