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Abstract

In response to Assembly Bill 32, the State of California is considering three types of carbon emissions trading programs for the electric power sector: load-based, source-based, and first-seller. They differ in terms of their point-of-regulation, and in whether in-state-to-out-of-state and out-of-state-to-in-state electricity sales are regulated. In this paper, we formulate a market equilibrium model for each of the three approaches, considering power markets, transmission limitations, and emissions trading, and making the simplifying assumption of pure bilateral markets. We analyze the properties of their solutions and show the equivalence of load-based, first-seller and source-based approaches, when power sales in both directions are regulated under the cap. A numeric example illustrates the emissions and economic implications of the models. In the simulated cases, "leakage" eliminates most of the emissions reductions that the regulations attempt to impose. Further, "contract reshuffling" occurs to such an extent that all the apparent emissions reductions resulting from changes in sources of imported power are illusory.

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In reality, the three systems would not be equivalent, because there will also be pool-type markets under the California Market Redesign and Technology Upgrade (MRTU), and the three systems provide different incentives for participating in those markets. However, the equivalence results under our simplifying assumptions show that load-based trading has no inherent advantage compared to other systems in terms of costs to consumers, contrary to claims elsewhere.

Keywords: Emissions Trading, Electric Market, CO_2 Emissions, Load-based, First-seller, Source-based

1 Introduction

Due to lack of federal leadership, a number of states in the U.S. have taken actions to control greenhouse gas (GHG) emissions. The efforts by the western and the eastern states are called the Western Climate Initiative (WCI) and Regional Greenhouse Gas Initiative (RGGI), respectively. Among the WCI states, the State of California is the first state to adopt legislation limiting GHGs . On September 27 2006, the state of California passed a comprehensive bill – AB32, "The Global Warming Solutions Act" – that aims at reducing in-state GHG emissions from various sectors to 1990 levels by 2020, which is equivalent to a decrease of approximately 174 $MMTCO_2E$ (million metric tons CO_2 equivalent) per year. AB32 is the first climate change legislation in the U.S. that would regulate most polluting sectors in an economy. Led by the California Energy Commission (CEC) and the California Air Resources Board (CARB) in consultation with other agencies, a state-wide emission cap is expected to be in effect by 2012. This is expected to be accomplished with a suite of instruments such as a low carbon fuel standard for vehicles that would reduce GHGs of transportation fuels by at least 10% by 2020 [7].

California has historically been among the states with the lowest GHG emissions per capita in US, roughly 50% below the national average. If apportioned by sectors, approximately 40% of GHG emissions is from transportation, followed by 14% apiece from in-state electricity and imported electricity. Others include industrial and manufacturing for 10%, refining and other industries for 8%, and agriculture, forestry and waste for 6%. The remaining 9% is attributed to commercial, residential and other sources [6]. However, while imported electricity accounts for half of electric sector emissions, such imports represent only 22% of electricity demand in California in 2006 [5]. Thus, imported electricity is significantly dirtier than in-state generation since a significant portion is generated by coal plants located in Arizona, New Mexico, Montana and other neighboring states [8].

While AB32 does not explicitly require the implementation of emissions trading programs, the CEC and CARB have been seriously considering using that approach for the electricity sector. A central issue in the recent debate over implementation of AB32 has been over where along the electricity sector supply chain (i.e., fuel to generation to consumption) limits should be imposed; this is the "point-of-regulation" issue. Currently, there are essentially three basic proposals on the table: source-based, load-based and first-seller approaches.

Source-based systems are most popular elsewhere. The conventional point-of-regulation in electric sector cap-and-trade systems (e.g., under the SO_2 Acid Rain Program under the 1990 Clean Air Act Amendments and the USEPA NO_x SIP Call) is at the generator. In source-based systems, a fixed number of emissions allowances are either distributed or sold to generators, who can then trade among themselves. Total emissions across all the regulated entities cannot exceed the number of allowances.

However, the alternative of load-based systems has received much attention in the California debate. Such systems can be viewed as being analogous to tradable renewable energy systems that have been used in other states and Europe, in which the seller of electricity to consumers is required to have a minimum portion of renewable electricity in its portfolio, either as contracts with renewable sources or unbundled "renewable energy credits." The basic load-based program requires load-serving entities (LSEs) to track the emissions associated with the electricity they purchase from generators and third parties, and to ensure that the sales-weighted carbon emissions rate of its purchases does not exceed a target established by the regulator. A variant, which we do not consider here, unbundles emissions certificates and power, and requires the LSE to purchase an amount of emissions certificates that would be equivalent to meeting the carbon emissions target in the basic load-based program.[1,2]

The final of the three approaches is the first-seller proposal [9]. A first-seller is defined as the entity that first contracts to sell electricity in California. Consistent with this, we define generators as the point-of-regulation for in-state sources, while LSEs are designated as the "first-seller" for out-of-state to in-state sales. Although the first-seller proposal is not explicitly defined by CEC at the moment, the schedule coordinators (SCs) that contract with generators in other states to import electricity into California could be designated as the first-seller instead of the LSE [3]; however, the conclusions of the analyses in this paper would not change in that case.

Table 1 summarizes the jurisdictional coverage of the programs analyzed in this paper. Two versions each of the source- and load-based programs are considered, differing in their treatment of power imports or exports. Together with the first-seller system, this means that we analyze five systems. We refer to the two source-based systems as the "pure" and "modified" systems. The pure source-based system is equivalent to traditional cap-and-trade programs and applies to only generation by California power plants. Meanwhile, the modified source-based approach expands coverage to include imports (i.e., out-of-state to in-state sales). Similarly, the pure load-based approach imposes an emissions cap on California LSEs, including the

[1] This variant is represented by two proposals: the CO_2RC proposal [19] and the "Tradable Emission Attribute Certificates" (TEAC) proposal [13]. Under the CO_2RC proposal, clean generators who sell in the California market would, in effect, be given emissions allowances in proportion to the difference between their CO_2 emissions and a high default emissions rate. LSEs would then buy sufficient credits from the generators so that the default rate times their load minus their allowances equals the target emissions rate times their load. A related, but distinct proposal, is the TEAC proposal, in which generators sell TEACs (denominated in MWh) and energy separately, and LSEs need to buy a quantity of TEACs equal to their load. Each TEAC has an associated emissions rate, equal to the generator's emission rate. The emissions-weighted sum of TEACs that each LSE purchases can be no more than the target emissions rate times its load. Objectives of these proposals include the elimination of the need to track emissions associated with energy transactions, which lessens conflicts with pool-type markets (such as under MRTU), and the encouragement of clean out-of-state generators to voluntarily participate in emissions trading. Elsewhere, under a set of simplifying assumptions (including zero price elasticity, no transmission constraints, and no power imports), it is shown that the CO_2RC and TEAC systems are economically equivalent to source-based systems in which allowances are freely allocated to generators in proportion to their sales [15, 25].

[2] Among WCI states, Oregon is also actively investigating a load-based emissions trading program [20]. However, instead of being a net importer, the Oregon could be a net exporter of electricity during wet years due to its abundant hydropower. But, overall, it relies heavily on imports of electricity produced by the coal-dominated states of Utah, Wyoming and Montana [21].

emissions of power they import, whereas the modified load-based approach, in addition, regulates in-state to out-of-state electricity sales. The modified rather than pure approaches are of most interest here because they both attempt to account for emissions from out-of-state power production, and are therefore the main focus of our analyses. Finally, the first-seller approach is a hybrid of source- and load-based approaches, where the emissions associated with the first-seller's sales come under the cap, with California generators being responsible for their emissions and the LSEs being responsible for emissions associated with imports.

Table 1: Summary of Programs' Jurisdiction Coverage

Programs	Instate to Instate	Instate to Outstate	Outstate to Instate
First-Seller	√	√	√
Modified Load-based	√	√	√
Modified Source-based	√	√	√
Pure Load-based	√		√
Pure Source-based	√	√	

The biggest drawback of the pure source-based approach is its inability to count emissions from imported electricity (i.e., carbon leakage). This "leakage" of emissions turns out to be crucial in regulating regional GHG emissions given that California is a net importer and its generators are relatively clean compared with other western states. The first-seller and modified source-based approaches represent attempts to correct that problem. However, as our simulations show later in this paper, these approaches are not necessarily effective in preventing leakage. In contrast, while either the modified load-based or pure load-based approaches might address CO_2 leakage, other potential problems with load-based problems have been identified. One issue is whether implementation of a program that places responsibility for compliance upon LSEs would interfere with the operation of the MRTU day-ahead and real-time pool-based markets by providing incentives for clean generation to avoid those markets [25]. The other concern is whether such an approach would lead to so-called contract shuffling that would eventually result in little or no actual emissions reduction [3][4]. To the extent that there are large amounts of clean energy sources that could compete for contracts to sell into California, the likelihood of shuffling increases. On the other hand, an often cited advantage for load-based approach is that it might increase the incentive for LSEs to invest in demand side management and improved energy efficiency [3]. Another claimed advantage is that by paying only a premium for clean power sources, consumer costs would be less than in a source-based system in which the opportunity cost of allowances for marginal power sources would raise the price of all power supply [11]. Finally, whether any proposal that would regulate imported power (load-based, modified source-based, and first-seller) would survive a challenge based upon the Interstate Commerce Clause is an important question.[3]

While there is no general consensus about which approach is most appropriate for California, a lack of careful analysis of their economic implications makes the choice even harder. This paper focuses on analyzing the three proposed approaches that include power imports and exports (i.e., modified load-based, modified source-based and first-seller programs), but also considers the pure load- and source-based approaches. We address the following questions assuming that emissions associated with California's electricity consumption are required to meet the same CO_2 emission cap under three approaches. These questions are, first, do the approaches lead to different emissions allowances prices and electricity prices? The second question concerns income distribution: are profits and consumer costs the same among the proposals? Third, how do the proposals compare in terms of contract-shuffling and CO_2 leakage?

To answer these questions, we conduct a general theoretical analysis of the three approaches that regulate emissions of imports and exports and show their equivalence. An earlier analysis that drew the same conclusion for load- and source-based programs, without considering imports, is due to Southern California Edison [12]. We also conduct numerical simulations of six cases using a three zone market, with one capped zone (representing California) and two zones that export to California. Cases 1–3 are the modified source-based, modified load-based and first-seller approaches. Cases 4–6 correspond to pure source-based, pure load-based approaches, and no cap (baseline), respectively. The pure cases show how emissions leakage and contract reshuffling can change relative to the modified cases. We report results concerning profits, consumer costs, social welfare, electricity and allowances prices, electricity sales, and CO_2 emissions.

The remainder of the paper is organized as follows. In Section 2, the mathematical formulations of the market equilibrium problems for the modified source-based,

[3]Further discussion of the three approaches in terms of potential legal challenges, ability to prevent leakage, and potential interference with electricity market operations can be found in Bushnell [3].

modified load-based and first-seller approaches will be introduced. These are complementarity formulations in which each market participant solves an optimization problem (profit-maximization for generators, consumer net benefit maximization for LSEs, and maximization of transmission value for the Independent System Operator (ISO)), and market clearing conditions are imposed. In Section 3, the properties of the solutions of the three models will be discussed. In particular, we show equivalence of the three proposals. We discuss the results of the numerical example in Section 4, focusing on economic and emissions outcomes as well as the issues of CO_2 leakage and contract shuffling. Conclusions and policy implications are addressed in Section 5. The theoretical proofs and data used in the economic and emissions analysis are included in the Appendix.

2 Models

This section introduces the mathematical formulation of the three main proposals that regulate emissions of imports (modified load-based, modified source-based, and first-seller). Within each proposal, optimization problems faced by LSEs, power producers, and an ISO are presented. Market clearing conditions will also be specified. The formulations are a generalization of Hobbs [14], accounting for allowances markets and differences in points of regulation among the proposals. Examples of previous studies that also analyze electricity markets using complementarity models include [1, 2, 24].

We first summarize the notation to be used in the models.

Sets and Indices

$\mathcal{N}$	Set of zones in a network, including both power producers and LSEs
$\mathcal{N}^C \subset \mathcal{N}$	Set of zones that are under the emissions cap
$\mathcal{N}^N \subset \mathcal{N}$	Set of zones that are not under the emissions cap; $\mathcal{N}^N = \mathcal{N} \backslash \mathcal{N}^C$
$\mathcal{A}$	Set of (directional) links in the transmission network $\mathcal{N}$
$\mathcal{F}$	Set of power producers
$\mathcal{F}^C \subset \mathcal{F}$	Set of power producers under the emissions cap
$\mathcal{H}$	Set of generating units
$i, j \in \mathcal{N}$	Zones i, j are in the power generation network
$f \in \mathcal{F}$	Power producer f
$h \in \mathcal{H}$	Generator h
$h \in H_{i,f} \subset \mathcal{H}$	Generator h belongs to power firm f and is in zone i.

Dimension

N	Number of zones in the network $\mathcal{N}$
I	Number of electricity production zones that are under CO_2 regulation
J	Number of demand zones that are under CO_2 regulation
L	Number of directional links in the network $\mathcal{N}$; $\lvert\mathcal{A}\rvert = 2L$
F	Number of power producers.

Parameters

P_j^0	Price intercept of the inverse demand function at zone j [\$/MW(h)]
Q_j^0	Quantity intercept of the inverse demand function at zone j [\$/MW(h)]
K_f^P	Initial free allocation of CO_2 allowances to power producer f [tons]
K_j^{LSE}	Initial free allocation of CO_2 allowances to LSE j [tons]
E_{fih}	Emission rate of firm f's generator h, in zone i [tons/MW(h)]
$\overline{E}$	System-wide CO_2 cap [tons]
X_{fih}	Capacity of firm f's generator h, in zone i [MW]
$C_{fih}(x)$	firm f's (convex) production cost function for generator h, in zone i [\$/MW(h)]
$PTDF_{ki}$	The (k, i)-th element of the power transmission-distribution factor matrix
T_k	Transmission upper bound on link k.

Variables

z_{fihj}	LSE j's purchase of power from firm f's generator h in zone i [MW(h)]. (Note that there is assumed to be just one LSE per zone; more general assumptions are possible, but would not change the conclusions of our analysis.)
s_{fj}	Power firm f's sale of electricity to LSE in zone j [MW(h)]
x_{fihj}	Electricity sold to LSE j, produced by firm f with generator h at zone i [MW(h)]
y_i	MWs transmitted from the hub to zone i [MW]
w_i	Fees charged for transmitting electricity from the hub to zone i [$/MW]
p_j	The price of electricity at zone j [$/MW(h)]
p_{fihj}	The price of electricity produced by generator (f, i, h) sold to LSE j [$/MW(h)]
p^{CO_2}	CO_2 price [$/ton].

2.1 Modified Source-based Emissions Trading Program

Under the modified source-based program, the compliance liability is with generators as long as a producer sells electricity to in-state consumers. In-state sources that export also must comply. This is contrast to our pure source-based system, in which out-of-state sources are exempted from regulation when selling electricity to in-state customers. In what follows, we summarize the optimization problems faced by LSEs, producers and the ISO, followed by their optimality conditions and market clearing conditions.

Under a source-based approach within a perfectly competitive market, an LSE is simply a price taker. The LSE is assumed to act to maximize the net benefits received by its consumers; if demand was perfectly inelastic, this would be the same as minimizing consumer costs. The benefit function is the integral of the consumers' demand curve, assumed to equal their willingness-to-pay for electricity. The demand curve is:

$$p_j = P_j^0 - \frac{P_j^0}{Q_j^0} \sum_f s_{fj}. \tag{1}$$

In a source-based system, the LSE's costs include only the expense of power. Power is procured through bilateral contracts.

Under the modified source-based program, all producers selling in the regulated zones have to buy allowances. Since imports are also subject to the emissions cap, out-of-state imports need to buy CO_2 allowances from the allowances market to cover the electricity they deliver to California.[4] Here, we only consider a scenario in which in-state power producers have an initial fixed endowment of allowances (obtained either

[4] An important policy issue under this approach would be whether the CO_2 allowances for in-state

by auctions or other means), while imports have to purchase allowances. It turns out the initial fixed endowments do not affect power producers decisions, given that the producers behave competitively in both the electricity and allowances markets.

Producer f solves the following optimization problem:[5]

$$\begin{aligned}
\underset{s_{fj},\ x_{fihj}}{\text{maximize}} \quad & \sum_{j\in\mathcal{N}} p_j s_{fj} - \sum_{i\in\mathcal{N},h\in H_{i,f}} C_{fih}\left(\sum_{j\in\mathcal{N}} x_{fihj}\right) - \sum_{j\in\mathcal{N}}\left[w_j s_{fj} - \left(\sum_{i\in\mathcal{N},h\in H_{i,f}} w_i x_{fihj}\right)\right] \\
& - p^{CO_2}\left(\sum_{i\in\mathcal{N}^C,h\in H_{i,f},j\in\mathcal{N}} x_{fihj}E_{fih} + \sum_{i\in\mathcal{N}^N,h\in H_{i,f},j\in\mathcal{N}^C} x_{fihj}E_{fih} - K_f^P\right) \\
\text{subject to} \quad & \sum_{j\in\mathcal{N}} x_{fihj} \le X_{fih},\ \forall\, i\in\mathcal{N}, h\in H_{i,f},\ (\rho_{fih}), \\
& \sum_{i\in\mathcal{N},h\in H_{i,f}} x_{fihj} = s_{fj}, \forall\, j\in\mathcal{N},\ (\theta_{fj}), \\
& x_{fihj} \ge 0\ , \forall\, i\in\mathcal{N}, h\in H_{i,f}.
\end{aligned} \tag{2}$$

The first term within the last parenthesis (i.e., $\sum_{i\in\mathcal{N}^C,h\in H_{i,f},j\in\mathcal{N}} x_{fihj}E_{fih}$) are the emissions associated with in-state sources, no matter where the electricity is sold; whereas the second second term (i.e., $\sum_{i\in\mathcal{N}^N,h\in H_{i,f},j\in\mathcal{N}^C} x_{fihj}E_{fih}$) are the emissions of imported electricity. The two constraints are capacity and sales balance constraints. Within this paper, we make the blanket assumption that the cost function $C_{fih}(\cdot)$ is convex and differentiable. It follows that Problem (2) is convex in its own variables (s, x) for each $f \in \mathcal{F}$. As a result, any local optimum to the problem is also a global optimum.[6] In addition, since the constraints in a firm's profit optimization problem are all linear, the linear constraint qualification holds at any feasible point. Hence,

sources should be auctioned. It is likely to be argued on discrimination grounds that if allowances are allocated for free to in-state sources, then imports should also receive allowances. A system that allocates allowances for free only to in-state generation would be subject to a commerce clause-based challenge. Avoiding this by allocating allowances to imports, however, would be difficult because CO_2 emissions associated with imports cannot be unambiguously determined. How to allocate allowances and how to distribute economic rents associated with those allowances are issues that are beyond the scope of this paper.

[5]If the free emissions allocation is proportional to the power sold rather than a fixed amount, the last parenthesis in the objective function of Problem (2) can be changed to $\sum_{i\in\mathcal{N}^C,h\in H_{i,f},j\in\mathcal{N}} x_{fihj}(E_{fih} - R^S) + \sum_{i\in\mathcal{N}^N,h\in H_{i,f},j\in\mathcal{N}^C} x_{fihj}(E_{fih} - R^S)$, where R^S is the target or default emissions rate at which allowances are awarded to generators.

[6]Here we assume that producers absorb all costs incurred by transmission congestion, and add

a necessary and sufficient condition for an optimal solution to satisfy is given by the following first-order conditions:

$$
\begin{aligned}
&0 \le x_{fihj} \quad \perp \quad \nabla_x C_{fih}\left(\sum_{j' \in \mathcal{N}} x_{fihj'}\right) + p^{CO_2} E_{fih} - w_i + \rho_{fih} - \theta_{fj} \ge 0, \\
&\qquad\qquad\qquad \forall\ i \in \mathcal{N}, h \in H_{i,f}, j \in \mathcal{N}^C, \\
&0 \le x_{fihj} \quad \perp \quad \nabla_x C_{fih}\left(\sum_{j' \in \mathcal{N}} x_{fihj'}\right) - w_i + \rho_{fih} - \theta_{fj} \ge 0,\ \forall\ i \in \mathcal{N}^N, h \in H_{i,f}, j \in \mathcal{N}^N, \\
&0 \le s_{fj} \quad \perp \quad -p_j + w_j + \theta_{fj} \ge 0,\ \forall\ j \in \mathcal{N}, \\
&0 \le \rho_{fih} \quad \perp \quad X_{fih} - \sum_j x_{fihj} \ge 0,\ \forall\ i \in \mathcal{N}, h \in H_{i,f}, \\
&\theta_{fj} \text{ free}, \qquad \sum_{i,h \in H_{i,f}} x_{fihj} - s_{fj} = 0, \forall\ j \in \mathcal{N}.
\end{aligned}
\tag{3}
$$

If x_{fihj} is greater than zero (therefore, $s_{fj} > 0$) and subject to emissions cap, the above conditions indicate that the power price is equal to the sum of marginal cost ($\nabla C_{fih}(\sum_{j \in \mathcal{N}} x_{fihj})$), the transmission charge (w_i), the CO_2 cost ($p^{CO_2} E_{fih}$), and a scarcity rent if the output is at its upper bound (θ_{fj}). The term $p^{CO_2} E_{fih}$ is omitted when x_{fihj} is not subject to the emissions cap. Notice that the allowance position (i.e., $\bar{K}_f^P$), a firm's initial possession of allowances, does not enter into the optimality conditions. Hence, as long as firms treat their grant of allowances as exogenous and their allocations are not linked to firms' future generation, how allowances are allocated does not affect the firms' behavior. That said, allocation of the allowances does affect the distribution of economic rents.

The system operator is assumed to maximize the value of transmission network, without strategically manipulating power flows to earn profits. We use the same

the costs to the market price of electricity. Alternative assumptions could be made, but would not affect our fundamental equivalence results.

formulation as in Metzler et al. [17]:[7]

$$\begin{aligned} \underset{y_i}{\text{maximize}} \quad & \sum_{i \in \mathcal{N}} w_i y_i \\ \text{subject to} \quad & \sum_{i \in \mathcal{N}} y_i = 0, \quad (\eta) \\ & \sum_{i \in \mathcal{N}} PTDF_{ki} y_i \leq T_k, \ \forall \ k \in \mathcal{A}, \quad (\lambda_k). \end{aligned} \tag{4}$$

ISO's first-order conditions are as below.

$$\begin{aligned} y_i \text{ free}, \qquad & w_i - \sum_{k \in \mathcal{A}} PTDF_{ki} \lambda_k + \eta = 0, \ \forall \ i \in \mathcal{N}, \\ \eta \text{ free}, \qquad & \sum_{i \in \mathcal{N}} y_i = 0, \\ 0 \leq \lambda_k \quad \perp \quad & T_k - \sum_i PTDF_{ki} y_i - T_k \geq 0, \ \forall \ k \in \mathcal{A}. \end{aligned} \tag{5}$$

Two sets of market clearing conditions are associated with this problem. Respectively, they determine (1) the charges for transmission by, in a sense, clearing the market for scarce transmission capacity, and (2) the allowances prices:[8]

$$\begin{aligned} w_i \text{ free}, \qquad & y_i = \sum_f s_{fi} \ - \sum_{f,j,h \in H_{f,i}} x_{fihj}, \ \forall \ i \in \mathcal{N}, \\ 0 \leq p^{CO_2} \quad \perp \quad & \overline{E} \ - \sum_{f,h,j,i \in \mathcal{N}^C} x_{fihj} E_{fih} \ - \sum_{f,h,j \in \mathcal{N}^C, i \in \mathcal{N}^N} x_{fihj} E_{fih} \geq 0. \end{aligned} \tag{6}$$

Note that the first equation in (6) implies that $\sum_{i \in \mathcal{N}} y_i = 0$. Hence, the corresponding equation in formulation (5) is redundant. We refer the system consisting of formulation (1), (3), (5) and (6) as the modified source-based model, or Model SB.

2.2 Modified Load-based Emissions Trading Program

A pure load-based emissions trading program assigns compliance requirements to in-state LSEs. Consequently, an in-state generation source would not be subject to

[7] This formulation is also equivalent to that in Adler et al. [1].

[8] Note that $\overline{E}$ could be equal to $\sum_f K_f^P$, but in general they could differ if some of the allowances are retained by government for other purposes, such as price control or reservation for new entrants.

any emissions regulation if it exports electricity to out-of-state load. Hence, total in-state GHG emissions may not be reduced. Such an outcome would conflict with the program's purpose to curb GHG emissions through a cap-and-trade program. One remedy is to put in-state generating sources that export to out-state load, along with in-state LSEs, under GHG regulation.[9] This is what we refer to as the "modified" load-based approach, and is the main focus of this subsection.

When considering the modified load-based approach, we partition LSEs into two groups: those under cap-and-trade programs (i.e., $j \in \mathcal{N}^C$) and those not under cap-and-trade (i.e., $j \in \mathcal{N}^N$). We follow the same assumptions as in Section 2.1 and model a market in which LSEs buy power through bilateral contracts with generators. However, a subtle difference remains regarding bilateral markets under the source-based approach versus the load-based approach. Strictly speaking, the load-based scheme requires LSEs to trace the exact source of each MWh of electricity procured, to accurately account for the emissions rate, while the source-based scheme has no such requirement. As a result, we were able to use a more general term s_{fj} to represent firm f's sales to demand zone j in the source-based approach, but under the load-based approach, we need to use variables with finer granularity, z_{fihj} and x_{fihj}, to model both the LSEs' problems and power producers' problems, respectively.

An LSE in load zone $j \in \mathcal{N}$ attempts to maximize benefits of consumption (integral of the demand curve) minus payments for power and, in addition, allowances. For a zone that is subject to CO_2 emissions cap (that is, $j \in \mathcal{N}^C$), an LSE's problem is as follows.[10]

$$\begin{aligned} \underset{z_{fihj}}{\text{maximize}} \quad & P_j^0\left(\sum_{i,f,h} z_{fihj}\right) - \frac{P_j^0}{2Q_j^0}\left(\sum_{i,f,h} z_{fihj}\right)^2 \\ & - \sum_{i,f,h\in H_{i,f}} p_{fihj} z_{fihj} - p^{CO_2}\left(\sum_{f,i,h} z_{fihj:j\in\mathcal{N}^C} E_{fih} - K_j^{LSE}\right) \qquad (7) \\ \text{subject to} \quad & z_{fihj} \geq 0, \ \forall\ f \in \mathcal{F}, i \in \mathcal{N}, h \in \mathcal{H}. \end{aligned}$$

The term, $p^{CO_2}(\sum_{f,i,h} z_{fihj:j\in\mathcal{J}^C} E_{fih} - K_j^{LSE})$, would be omitted when an LSE is not

[9] To our knowledge, there is no proposed load-based program that attempts to regulate in-state to out-of-state sales. However, it might be a precautionary requirement to prevent producers from taking advantage of programs to increase their exports or to do "round-trip" trades.

[10] In a program that instead awards allowances to LSEs in proportion to their load, the terms in last parenthesis of objective function in Problem (7) will be replaced with $\sum_{f,i,h} z_{fihj:j\in\mathcal{N}^C}(E_{fih} - R^L)$, where R^L is target or default emissions rate.

subject to emissions cap. Hence, this yields first-order conditions of two types for the LSEs as follows:

$$
\begin{aligned}
&0 \leq z_{fihj} \quad \perp \quad -P_j^0 + \frac{P_j^0}{Q_j^0}\left(\sum_{i,f,h\in H_{i,f}} z_{fihj}\right) + p_{fihj} + p^{CO_2}E_{fih} \geq 0,\\
&\qquad\qquad\qquad \forall\ j \in \mathcal{N}^C, f \in \mathcal{F}, i \in \mathcal{N}, h \in \mathcal{H},\\
&0 \leq z_{fihj} \quad \perp \quad -P_j^0 + \frac{P_j^0}{Q_j^0}\left(\sum_{i,f,h\in H_{i,f}} z_{fihj}\right) + p_{fihj} \geq 0,\ \forall\ j \in \mathcal{N}^N, f \in \mathcal{F}, i \in \mathcal{N}, h \in \mathcal{H}.
\end{aligned} \tag{8}
$$

For all $j \in \mathcal{N}^C$ with $z_{fihj} > 0$, the following condition needs to be satisfied: $p_j = P_j^0 - \frac{P_j^0}{Q_j^0}(\sum_{i,f,h\in H_{i,f}} z_{fihj}) = p_{fihj} + p^{CO_2}E_{fih}$. The term p_j is the marginal benefit (per MWh) of consumers residing in $j \in \mathcal{N}^C$ for electricity, regardless of its emissions rate.[11] The term p_{fihj} ($= p_j - p^{CO_2}E_{fih}$) is the expenditure by an LSE to purchase one MWh of electricity from a specific generator with an emission rate of E_{fih}. Thus, if a generator has a high emissions rate, it would receive less revenue per MWh compared to cleaner sources.

In the load-based approach, power producers do not face CO_2 emission regulation, except for exports from in-state sources to out-of-state consumers. Hence, in-state generators have to purchase CO_2 allowances to ship power out of the state. This ensures that all emissions within the state are covered by the program. The producer

[11] Individual consumers are assumed to view electricity as a homogeneous product, regardless of type of technology used to generate it. Therefore, their marginal benefit is always p_j. In contrast, an LSE is subject to an emissions cap and treats electricity as a differented product with regard to emission rates. However, LSEs are allowed to pass on CO_2 costs to consumers in the price of electricity.

f's profit maximization problem is as below:

$$
\begin{aligned}
\underset{x_{fihj}}{\text{maximize}} \quad & \sum_{i,j\in\mathcal{N},h\in H_{i,f}} p_{fihj}x_{fihj} - \sum_{i\in\mathcal{N},h\in H_{i,f}} C_{fih}\left(\sum_{j\in\mathcal{N}} x_{fihj}\right) \\
& - \sum_{i,j\in\mathcal{N},h\in H_{i,f}} (w_j - w_i)x_{fihj} \\
& - p^{CO_2}\left(\sum_{i\in\mathcal{N}^C,h\in H_{i,f},j\in\mathcal{N}^N} x_{fihj}E_{fih}\right) \qquad (9) \\
\text{subject to} \quad & \sum_j x_{fihj} \le X_{fih}, \ \forall\ i,j\in\mathcal{N}, h\in H_{i,f}, \ (\rho_{fih}) \\
& x_{fihj} \ge 0, \ \forall\ i,j\in\mathcal{N}, h\in H_{i,f}.
\end{aligned}
$$

The optimality conditions are given below.

$$
\begin{aligned}
0 \le x_{fihj} \quad \perp \quad & -p_{fihj} + w_j + \nabla_x C_{fih}\left(\sum_{j'\in\mathcal{N}} x_{fihj'}\right) - w_i + \rho_{fih} \ge 0, \\
& \forall\ i \notin \mathcal{N}^C \text{ or } j \notin \mathcal{N}^N, h\in H_{i,f} \\
0 \le x_{fihj} \quad \perp \quad & -p_{fihj} + w_j + \nabla_x C_{fih}\left(\sum_{j'\in\mathcal{N}} x_{fihj'}\right) - w_i + \rho_{fih} + p^{CO_2}E_{fih} \ge 0, \\
& \forall i\in\mathcal{N}^C \quad \text{and} \quad j\in\mathcal{N}^N, h\in H_{i,f} \\
0 \le \rho_{fih} \quad \perp \quad & X_{fih} - \sum_j x_{fihj} \ge 0, \ \forall\ i\in\mathcal{N}, h\in H_{i,f}.
\end{aligned}
\qquad (10)
$$

The ISO's problem is the same as in the source-based approach (4). The new set of market clearing conditions are as follows.

$$
\begin{aligned}
w_i \text{ free}, \quad & y_i = \sum_{f,i',h\in H_{i',f}} x_{fi'hi} - \sum_{f,j,h\in H_{i,f}} x_{fihj}, \ \forall\ i\in\mathcal{N}, \\
0 \le p^{CO_2} \quad \perp \quad & \overline{E} - \sum_{f,i,h,j\in\mathcal{N}^C} z_{fihj}E_{fih} - \sum_{f,h,i\in\mathcal{N}^C,j\in\mathcal{N}^N} x_{fihj}E_{fih} \ge 0, \qquad (11) \\
p_{fihj} \text{ free}, \quad & z_{fihj} = x_{fihj}, \ \forall\ f\in\mathcal{F}, i,j\in\mathcal{N}, h\in\mathcal{H}.
\end{aligned}
$$

We refer to the model consisting of equations (8), (10), (5) and (11) as the modified load-based model, or Model LB.

2.3 First-Seller Emissions Trading Scheme

In the first-seller approach we consider here, the point of regulation is the first entity that brings energy into California. Our interpretation is that for the in-state generating sources, the first-seller approach is equivalent to the source-based approach. In our model setting, we do not consider power producers buying electricity from other firms, nor do we consider power marketers. Hence, only LSEs are responsible in our model for bringing out-of-state power into California, if imports are needed or more economical. Consequently, in the first-seller approach, imports are treated the same way as in the load-based approach, where the LSEs are the point of regulation. The first-seller approach can therefore be viewed as a hybrid of the source-based and load-based approaches.[12]

Again we assume that LSEs explicitly sign bilateral contracts with generators to provide electricity to a specified zone $j \in \mathcal{N}$. Also there are in-state LSEs and out-of-state LSEs. Under the first-seller scheme, an in-state LSE (i.e., $j \in \mathcal{N}^C$) faces a profit optimization problem as follows.[13]

$$\begin{aligned}
\underset{z_{fihj}:\ j\in\mathcal{N}^C}{\text{maximize}} \quad & P_j^0 \left(\sum_{f\in\mathcal{F},i\in\mathcal{N},h\in\mathcal{H}} z_{fihj} \right) - \frac{P_j^0}{2Q_j^0} \left(\sum_{f\in\mathcal{F},i\in\mathcal{N},h\in\mathcal{H}} z_{fihj} \right)^2 \\
& - \sum_{f\in\mathcal{F},i\in\mathcal{N},h\in\mathcal{H}} (p_{fihj} z_{fihj}) - p^{CO_2} \left(\sum_{f,h,i\in\mathcal{N}^N} z_{fihj} E_{fih} - K_j^{LSE} \right) \qquad (12) \\
\text{subject to} \quad & z_{fihj} \geq 0, \ \forall f \in \mathcal{F}, i \in \mathcal{N}, h \in \mathcal{H}.
\end{aligned}$$

While for an out-of-state LSE, there is no cost or profit associated with CO_2 emissions:

$$\begin{aligned}
\underset{z_{fihj}:\ j\in\mathcal{N}^N}{\text{maximize}} \quad & P_j^0 \left(\sum_{i,f,h} z_{fihj} \right) - \frac{P_j^0}{2Q_j^0} \left(\sum_{i,f,h} z_{fihj} \right)^2 \\
& - \sum_{i,f,h} (p_{fihj} z_{fihj}) \qquad (13) \\
\text{subject to} \quad & z_{fihj} \geq 0, \ \forall f, i, h.
\end{aligned}$$

[12] It could instead be assumed, for example, that a third importing party is responsible for imports, but this would not materially affect the conclusions of our analysis.

[13] The last term within the parenthesis of objective function in Problem (12) would be modified to $\sum_{i\in\mathcal{N}^C,h\in H_{i,f},j\in\mathcal{N}^N} x_{fihj}(E_{fih} - R^F)$ when facing a program that awards allowances in proportion to sales. R^F is the default or target emissions under first-seller program. The same modification should be applied to Problem (15).

The first-order conditions of the LSEs' problems are as follows.

$$
\begin{aligned}
0 \le z_{fihj} \quad &\perp \quad -P_j^0 + \frac{P_j^0}{Q_j^0}\left(\sum_{f',i',h'} z_{f'i'h'j}\right) + p_{fihj} + p^{CO_2}E_{fih} \ge 0, \\
&\forall\ f \in \mathcal{F}, h \in H_{i,f}, i \in \mathcal{N}^N, j \in \mathcal{N}^C, \\
0 \le z_{fihj} \quad &\perp \quad -P_j^0 + \frac{P_j^0}{Q_j^0}\left(\sum_{f',i',h'} z_{f'i'h'j}\right) + p_{fihj} \ge 0, \\
&\forall\ f \in \mathcal{F}, h \in H_{i,f}, i \in \mathcal{N}, j \in \mathcal{N}^N.
\end{aligned}
\tag{14}
$$

Power producer f's problem under the first-seller approach is stated below. Notice that CO_2 related costs or revenues only apply to in-state generating sources.

$$
\begin{aligned}
\underset{x_{fihj}}{\text{maximize}} \quad & \sum_{i,j\in\mathcal{N},h\in H_{i,f}} p_{fihj}x_{fihj} - \sum_{i\in\mathcal{N},h\in H_{i,f}} C_{fih}\left(\sum_{j\in\mathcal{N}} x_{fihj}\right) \\
& - \sum_{i,j\in\mathcal{N},h\in H_{i,f}} (w_j - w_i)x_{fihj} \\
& - p^{CO_2}\Big(\sum_{i\in\mathcal{N}^C,h\in H_{i,f}} x_{fihj}E_{fih} \ - \ K_f^P\Big) \qquad (15) \\
\text{subject to} \quad & \sum_j x_{fihj} \le X_{fih},\ \forall\ i \in \mathcal{N}, h \in H_{i,f}, \quad (\rho_{fih}) \\
& x_{fihj} \ge 0,\ \forall i \in \mathcal{N}, h \in H_{i,f}.
\end{aligned}
$$

The optimality conditions for a power producer $f \in \mathcal{F}$ are as below, where the term

$p^{CO_2}E_{fih}$ is omitted for out-of-state sources.

$$
\begin{aligned}
0 \le x_{fihj} \quad \perp \quad & -p_{fihj} + w_j + \nabla_x C_{fih}\left(\sum_{j' \in \mathcal{N}} x_{fihj'}\right) + p^{CO_2}E_{fih} - w_i + \rho_{fih} \ge 0, \\
& \forall\ i \in \mathcal{N}^C, h \in H_{i,f}, j \in \mathcal{N}, \\
0 \le x_{fihj} \quad \perp \quad & -p_{fihj} + w_j + \nabla_x C_{fih}\left(\sum_{j' \in \mathcal{N}} x_{fihj'}\right) - w_i + \rho_{fih} \ge 0, \\
& \forall\ i \in \mathcal{N}^N, h \in H_{i,f}, j \in \mathcal{N}, \\
0 \le \rho_{fih} \quad \perp \quad & X_{fih} \ - \ \sum_j x_{fihj} \ge 0,\ \forall\ i \in \mathcal{N}, h \in H_{i,f}.
\end{aligned}
\tag{16}
$$

The ISO's problem is exactly the same as in the source-based approach.

The market clearing conditions under the first-seller approach are given below.

$$
\begin{aligned}
w_i \text{ free}, \qquad & y_i = \sum_{f,i',h \in H_{f,j}} x_{fi'hi} \ - \sum_{f,j',h \in H_{f,i}} x_{fihj'},\ \forall\ i \in \mathcal{N}, \\
0 \le p^{CO_2} \quad \perp \quad & \overline{E} \ - \sum_{f,h,i \in \mathcal{N}^N, j \in \mathcal{N}^C} z_{fihj}E_{fih} \ - \sum_{f,h,i \in \mathcal{N}^C, j \in \mathcal{N}} x_{fihj}E_{fih} \ge 0, \\
p_{fihj} \text{ free}, \qquad & x_{fihj} = z_{fihj},\ \forall f \in \mathcal{F}, i, j \in \mathcal{N}, h \in \mathcal{H}.
\end{aligned}
\tag{17}
$$

We refer to the system consisting of (14), (16), (5) and (17) as the first-seller model, or Model FS.

3 Properties of the Models

In this section, we present some properties of the three market models (modified source- and load-based, and first-seller) introduced above. The properties show that under mild conditions, the models constructed in the previous section are well-posed in the sense that an equilibrium exists in each of the model. More importantly, the properties show that although the three approaches to implement a cap-and-trade program are structurally different, electricity producers' generation patterns and costs are the same, and so are the total GHG emissions and consumer surpluses.

The following results summarize those properties. For more succinct presentation, technical proofs are placed in the Appendix. These proofs proceed in the same manner as in Metzler et al. [17].[14]

Theorem 1 *If* $\max_{j\in\mathcal{N}}\{P_j^0 - \nabla_x C_{fi}(0)\} \geq 0,\ \forall\ f \in \mathcal{F},\ i \in \mathcal{N}$, *then there exists a solution to Model SB, Model LB and Model FS, respectively.*

Theorem 2 *Let a vector* $\mathbf{b}$ *represent the common variables shared by the three models; that is,* $\mathbf{b} = (x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2})$. *Assume that the system-wide* CO_2 *caps among the three models are the same; namely,*

$$\overline{E}^{SB} = \overline{E}^{LB} = \overline{E}^{FS} \tag{18}$$

Then the following statements are equivalent.

(a) *There exist* s *and* θ *such that* $(s,\ \theta,\ \mathbf{b})$ *solves Model SB.*

(b) *There exist* z *and* p *such that* $(z,\ \mathbf{b},\ p)$ *solves Model LB.*

(c) *There exist* z *and* p *such that* $(z,\ \mathbf{b},\ p)$ *solves Model FS.*

Some of the results to be shown in the next theorem require strictly convexity of the production cost function with respect to total generation.[15] Assume that the cost function for each firm $f \in \mathcal{F}$ at a zone $i \in \mathcal{N}$ is as follows.

$$C_{fi}\left(\sum_{j\in\mathcal{N}} x_{fij}\right) = c_{fi}\left(\sum_{j\in\mathcal{N}} x_{fij}\right) + \frac{1}{2}d_{fi}\left(\sum_{j\in\mathcal{N}} x_{fij}\right)^2. \tag{19}$$

In addition, we assume that $d_{fi} > 0$ for each $f \in \mathcal{F}$ and $i \in \mathcal{N}$.

To show the uniqueness and equivalence of consumer surpluses in the equilibria of the three models, we assume that consumers' willingness to pay is captured by a linear inverse demand function, as in Section 2. Consumer surplus is the integral

[14] In proving the existence and uniqueness results, for easier presentation, we make the assumption that $H_{i,f}$ is a singleton for each $f \in \mathcal{F}$ and $i \in \mathcal{N}$. This is not a restrictive assumption. If a zone $i \in \mathcal{N}$ has several generators, we can expand the network $\mathcal{N}$ to make each of the generator as a single zone. Denote the newly added zones as $\tilde{\mathcal{N}}(i)$, and denote the set of all the adjacent zones of i in the original network as $\mathcal{J}(i)$. Then for each $j \in \mathcal{J}(i)$, and $i' \in \tilde{\mathcal{N}}(i)$, connect them with a link with infinite capacity. Then the resulted expanded network will not alter any firms' or consumers' decisions that are made in the original network. Under this simplifying assumption, we can eliminate h indices from all variables and constraints.

[15] To be more precise, they are items (b), (c), and (e) in Theorem 3.

of the inverse demand function minus the costs for purchasing electricity. LSEs are assumed to charge consumers the marginal cost of supplying electricity, including the shadow price of allowances; in equilibrium, this is the marginal willingness to pay. The resulting expression for consumers surplus is given below:

$$CS^m \equiv \sum_{j\in\mathcal{N}} \left[P_j^0 \left(\sum_{f\in\mathcal{F}} s_{fj} \right) - \frac{P_j^0}{2Q_j^0} \left(\sum_{f\in\mathcal{F}} s_{fj} \right)^2 - \sum_{f\in\mathcal{F}} \left(P_j^m s_{fj} \right) \right], \qquad m = SB;$$

$$CS^m \equiv \sum_{j\in\mathcal{N}} \left[P_j^0 \left(\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij} \right) - \frac{P_j^0}{2Q_j^0} \left(\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij} \right)^2 - \sum_{f\in\mathcal{F},i\in\mathcal{N}} \left(P_j^m z_{fij} \right) \right], \quad m = LB \text{ or } FS, \tag{20}$$

where

$$P_j^m \equiv P_j^0 - \frac{P_j^0}{Q_j^0} \left(\sum_{f\in\mathcal{F}} s_{fj} \right), \qquad m = SB;$$

$$P_j^m \equiv P_j^0 - \frac{P_j^0}{Q_j^0} \left(\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij} \right), \quad m = LB \text{ or } FS.$$

An alternative expression for consumers surplus could be derived from the assumption that all LSE profits (e.g., resulting from free granting of allowances) are returned to consumers as a lump-sum payment. That is, LSEs are regulated entities (as the largest ones are in California), but lump sum costs or revenues from the fixed allocations of allowances are rebated to consumers in a way that does not distort consumption. Furthermore, an assumption could also be made that all allowance rents earned by producers in the SB and FS models are actually returned to consumers, for example because allowances are auctioned with the proceeds going to consumers.

Theorem 3 *Assume that the set of solutions to any of the three models are nonempty. Under the same condition (18) as in Theorem 2, together with the cost function (19), the following statements are true for the equilibrium solutions of the three models.*

(a) *The total sales to each LSE $j \in \mathcal{N}$ are unique and are the same across the three models;*

(b) *Firm f's total production in each production zone $i \in \mathcal{N}$ is unique and the same across the three models, for each $f \in \mathcal{F}$;*

(c) *Total emissions are unique and are the same across the three models;*

(d) *The consumers' marginal willingness-to-pay (price charged to consumers) is unique at each zone $j \in \mathcal{N}$ and are the same across the three models; in addition, the consumer surpluses are unique and the same across the three models;*

(e) *Total social welfare (sum of consumers, producer, ISO, and LSE surpluses) is unique and are the same across the three models.*

The definition of consumers surplus in the above theorem assumes that all allowance rents accrue to either LSEs or producers, and none to consumers. As noted, regulation of LSEs and auctioning of allowances with the proceeds going to consumers could instead result in all allowance rents being returned to consumers. Consumers surplus would then increase by the amount of the rent, but would still be identical in each of the three systems.

Thus, we have refuted the assertion that has been made [11] that load-based systems result in lower costs to consumers than source-based systems because LSEs are able to price discriminate among producers. Despite that price discrimination, the costs to consumers (more generally, their consumers surpluses) are the same in all three systems.

4 Numerical Example

In this section, we present a simple example to illustrate the equivalence of modified load-based, modified source-based, and first-seller approaches. We also compare them with three other scenarios: the pure load-based, pure source-based, and no-cap scenarios.

4.1 Assumptions

We assume there are three zones ($i = \{A, B, C\}$) connected with transmission lines with fixed capacities (Figure 1). Zone A is the regulated zone (referred to below as "**in-state**"), and zones B and C are unregulated but trade with zone A (and are referred to as "**out-of-state**"). A number of generating units ($h = \{1, 2..., 10\}$) are located in each zone owned by firms ($f = \{1, 2, 3\}$). We allow firms to own generating asset in different locations. Consumers reside in each zone with their willingness-to-pay representing by linear inverse demand curves. For simplicity, we model the markets for a single hour. A larger model with varying load over a number of hours could readily be solved, but would not provide additional insight.

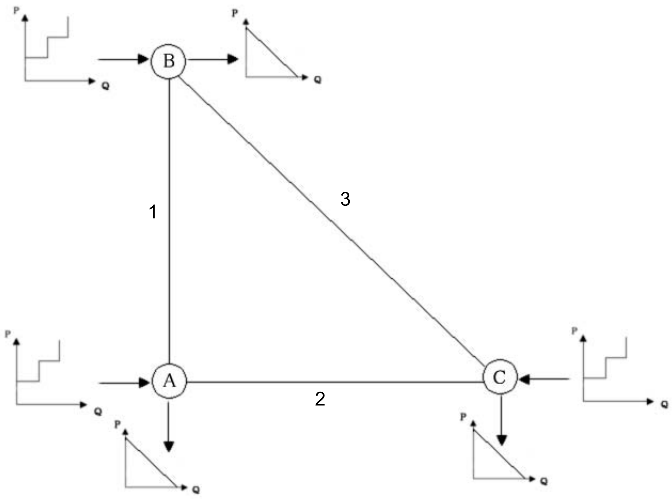

Figure 1: Network of the Numeric Example

The specific input assumptions used in this example are presented in the Appendix. Figure 2 plots marginal cost against cumulative capacity with the corresponding CO_2 emissions rates for zone A, B and C, respectively. The marginal cost for zone A ranges between 35 and 55$/MWh with a stable emissions rate around 550 kg/MWh. Zone B has two types of generating technologies, consisting of low-cost, clean hydropower with moderately expensive units with marginal cost and emission rate of approximately 25 $/MWh and 550 kgs/MWh, respectively. Zone C has units with higher emissions but lower costs. The capacity-weighted average CO_2 emissions rate is 582, 367 and 1,171 kg/MWh for zone A, B and C, respectively. Thus, whereas zone A is designed to resemble California's electricity system primarily with natural gas plants, while zones B and C represent northwest and southwest states with abundant hydropower and coal plants, respectively. In the baseline (or no-cap) case, roughly 20% and 10% of the electricity demand in zone A is met by imports from zone B and C, respectively. This is an exaggeration of the actual conditions in the western market, but it allows us to investigate possible contract shuffling and carbon leakage. We assume a demand elasticity of -0.2 at the price-quantity pair in the baseline solution.

We simulate six scenarios. In scenarios 1–3, zone A is subject to an emissions cap of 400 tons under modified load-based, modified source-based, and first-seller approaches, respectively. An additional three scenarios are designed to illustrate the cases when there are pure load- and source-based programs or no cap at all. First, in scenario 4, a 400-ton emissions cap is imposed exclusively on the in-state LSEs with no restriction on emissions associated with exported electricity (**Pure Load-Based**); second, in scenario 5, we impose a 400-ton cap on just in-state emissions sources (**Pure Source-Based**); finally, scenario 6 has no emissions cap on either in- and out-of-state sources. In next section, we report the results, including electricity and CO_2 allowances prices, sales, CO_2 emissions, CO_2 leakage and contract shuffling, and welfare outcomes. The results of 1–3 and 4-6 scenarios are displayed in Tables 2 and 3, respectively.

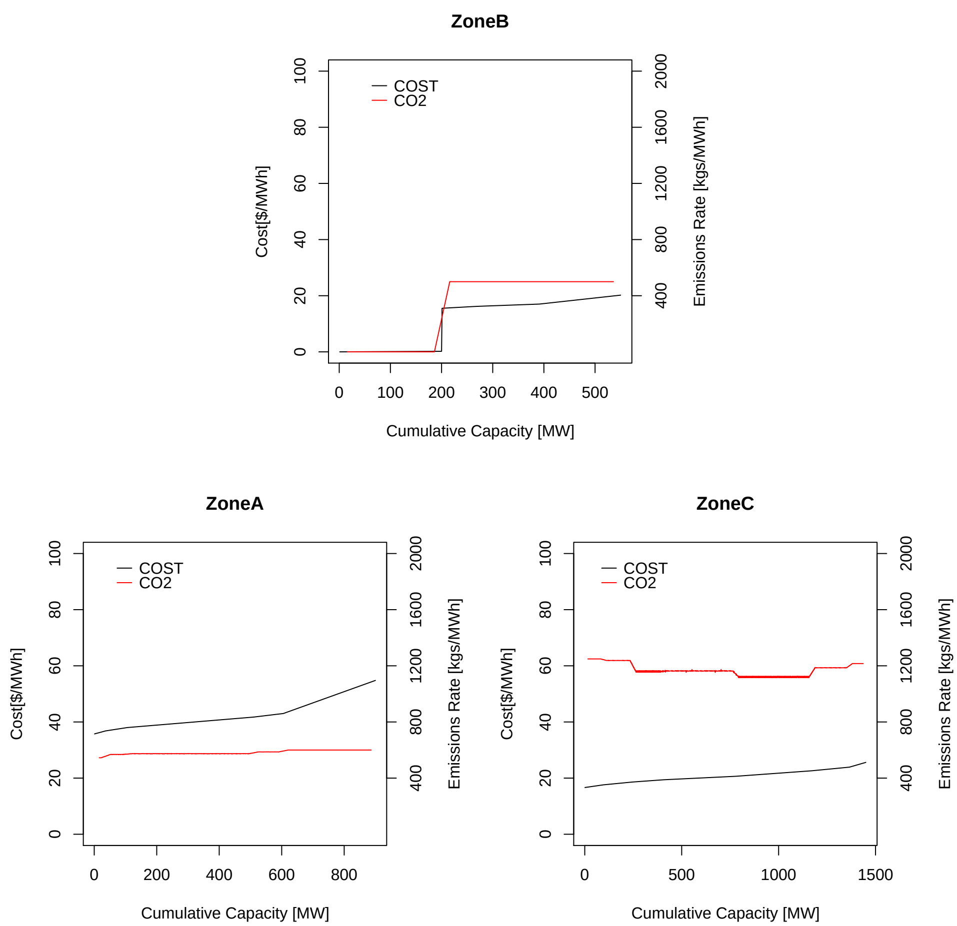

Figure 2: Plots of marginal cost and CO_2 emissions rate against cumulative capacity in zones A, B and C

4.2 Results

This section reports results of the above six scenarios. We report price and welfare results in Section 4.2.1, address the equivalence of three proposed programs in Section 4.2.2, followed by discussion of CO_2 leakage and contract shuffling in Section 4.2.3.

4.2.1 Electricity and Allowances Prices and Social Welfare Analysis

The extent to which electricity prices would increase under emissions trading depends on the level of allowances prices as well as emissions rate of the marginal unit that determines the price of electricity. As for the level of allowances prices, it depends on how programs is designed. In particular, the key parameter is whether the in-state to out-of-state and out-of-state to in-state electricity sales are regulated under the emissions programs. Overall, higher allowances prices (and, thus, higher electricity prices) occur in the three approaches that regulate imports to zone A (modified load- and source-based, and first-seller based), as shown in Table 2. Zone A's electricity price increases by \$5.6/MWh and \$24.1/MWh in contrast to the pure load-based and pure source-based scenarios, respectively, which regulate fewer transactions. This is primarily because in-state to out-of-state (e.g., California to Washington) and out-of-state to in-state (e.g., New Mexico to California) electricity sales are not regulated under the pure load- and source-based programs, respectively, while both types of transactions are regulated under the first-seller and modified load- and modified source-based approaches.

The level of allowances prices under the pure source-based approach relative to the pure load-based program depends on the relative costs of in-state and out-of-state generating sources (i.e., the direction of flow). The allowances price under first-seller (or modified source-based or load-based) approach is \$61.0/ton when cross-state electricity sales in both directions are under regulation. Under the pure load-based approach, exports from from zone A to zones B and C increase (since such exports incur no CO_2 cots); this would suppress the demand for CO_2 allowances, and thus reduce allowance prices. Yet, since zone A is a net importer, the effect on allowance prices would be minor (i.e., dropping from \$61.0/ton to \$52.3/ton). In contrast, increased out-of-state imports that result from not subjecting zone A exports to the cap under a pure source-based approach would significantly suppress demand for CO_2 allowances, and thus, allowance price drops even further (i.e., from \$61.0/ton to \$12.5/ton). In a more extreme case in which the out-of-state sources can meet a significant portion of zone A's load while transmission constraints are not binding, the allowance price under a pure source-based system that applies just to zone A could crash to zero.

The welfare analysis shows that overall social welfare (the sum of consumer, producer, LSE, and ISO surpluses) is higher in the pure load- and source-based programs than in the modified programs. This is only because emissions are **not** included in the welfare calculation, and the emissions reductions are smaller in the pure programs, which reduces fuel costs. The pure source-based program has only a slightly lower social welfare than the no cap baseline. However, this is not to say that the pure programs are superior. If the policy goal is to control CO_2 emissions in the WECC region or beyond California state territory, first-seller, modified load-based, and modified source-based approaches would be appropriate since they allow agencies to reach out and to regulate transactions in both directions.

As shown in Table 2, the total welfare gained in the first-seller, modified load-based, and modified source-based approaches is the same. This total amount is distributed differently among producers, consumers, LSEs, and the ISO because different parties receive free allowances in each of the systems. If instead all allowance rents were returned to consumers, then each market participant would have the same net benefits in each scenario, and consumer surplus would increase by the amount of that rent (from $109,000 to $133,000). We discuss this particular comparison in more detail in the next section.

4.2.2 Equivalence of First-seller, Modified Load-based and Modified Source-based Approach

As indicated in Section 2, the first-seller, modified load-based and modified source-based approaches share the following characteristic: all regulate in-state to out-of-state and out-of-state to in-state electricity sales, even though their point-of-regulation is different. Nevertheless, three approaches differ in two aspects. First, although the total sales (i.e., **Total Sales** in Table 2), and thus, retail electricity prices are the same in the three approaches, the distribution of bilateral generator-LSE electricity sales could be different as implied by Theorem 2 in Section 3. However, the pattern of generation and total production costs are unaffected.

Second, the distribution of economic rents among market parties is different even though the total social welfare is equal under three cases. This basically results because the distribution of allowance rents ($61/ton times the 400 ton cap = $24,400) differs among the three solutions in Table 2. In particular, the decision as to who should retain the LSEs' profits (i.e., $\sum_{i,f,h\in H_{i,f}} (p_{fihj}z_{fihj}) - p^{CO_2}(\sum_{f,i,h} z_{fihj:j\in\mathcal{N}^C}E_{fih} - K_j^{LSE})$ in equations (7) and (12) in the modified load-based and first-seller approaches and how producers are allocated allowances are important policy questions that affect the distribution of allowance rents. If, on one hand, producers receive all allowances for free and LSEs purchase allowances from producers in compliance with first-seller requirement, the LSE's economic rents of \$24,383 and \$5,040 under the modified load-based and first-seller program, respectively, should instead be incorporated into the producers' surplus. On the other hand, if allowances are first given to consumers, and producers and LSEs must then purchase those allowances from consumers (perhaps via a state-administered auction), the allowances rents will accrue to consumers, and will partially compensate for the pass-through of CO_2 allowances costs. Hence, those three programs will be equivalent from the perspective of welfare distribution if a consistent set of assumptions is made concerning the distribution of allowance rents.[16]

4.2.3 Contract Shuffling and CO_2 Leakage

The distribution of emissions among zones under different programs is a consequence of how the programs are designed and the degree of transmission congestion. We examine contract shuffling and emissions leakage in this section.

In electricity markets, most sales are financial transactions since actual flows in the network must follow physical laws. Contract shuffling in the current context refers to the situation in which the rearrangement of financial contracts results in no actual emissions reduction in an emissions trading program. Whether contract shuffling would occur under different cases can be analyzed by examining variables z_{fihj} in the models. Here, we sum variables z_{fihj} over subscripts f and h to get electricity sales (s_{ij}) from i to j (see **Electricity Sales** in Tables 2 and 3).

We focus on a comparison of the no-cap case in Table 3 with other approaches in Tables 2 and 3. First, we consider the issue of leakage, and then turn to contract shuffling.

[16]Wolak et al. (Appendix) analyze load- and source-based programs assuming a closed system with inelastic loads, and only bilateral trading. They conclude that if allowances are owned by consumers, and producers need to purchase allowances from consumers, the two systems are economically equivalent [25]. But they also point out a number of practical and important disadvantages of load-based systems, including impacts on the efficiency of the MRTU markets.

Table 2: Summary of market equilibria under modified source-based, modified load-based and first-seller programs

	Modified Load-Based			Modified Source-Based			First-Seller		
Emission Cap [tons]			400.0			400.0			400.0
Consumer Surplus [$]			107,953			107,953			107,953
Producer Surplus [$]			25,889			50,272			45,232
ISO's Surplus [$]			4,834			4,834			4,834
LSE's Surplus [$]			24,383			N\A			5,040
Social Welfare [$]			163,059			163,059			163,059
Variable\Zone	A	B	C	A	B	C	A	B	C
Price [$/MWh]	78.1	40.9	20.8	78.1	40.9	20.8	78.1	40.9	20.8
CO_2 Emissions [tons] (from\to)									
A	317.3	0	0	317.3	0	0	317.3	0	0
B	82.7	13.4	78.0	82.7	0.8	91.5	82.7	16.8	75.6
C	0	330.7	620.0	0	368.9	581.8	0	325.2	625.5
Zonal CO_2 Emissions [ton]*	317.3	175	950.7	317.3	175	950.7	317.3	175	950.7
Associated CO_2 Emission [tons]	400	345.1	697.9	400	369.7	673.3	400.0	342	701.0
Import (+)/Export(-) Emissions [ton]	82.7	170.1	-252.79	82.7	194.7	-277.4	82.7	167	-249.7
Total CO_2 Emissions [ton]		1,443.1			1,443.1			1,443.1	
CO_2 Price [$/ton]		61.0			61.0			61.0	
Generating Cost [$]		43,358.0			43,358.0			43,358.0	
Allowances Rents [$]		24,384.0			24,384.0			24,384.0	
Electricity Sales (from\to)	A	B	C	A	B	C	A	B	C
A	555.1	0	0	555.1	0	0	555.1	0	0
B	365.4	28.7	155.9	365.4	1.6	183.0	365.4	33.5	151.1
C	0	274.1	527.9	0	301.2	500.7	0	269.3	532.6
Total Sale [MWh]	920.5	302.8	683.8	920.5	302.8	683.8	920.5	302.8	683.8

*Note: The zonal CO_2 emissions is the row sum of CO_2. For example, the zonal emissions under the modified source-based program for zone A equals $317.3 + 0 + 0 = 317.3$ tons.

Table 3: Summary of market equilibria under no-cap, pure source-based and pure load-based programs

	Pure Load-Based			Pure Source-Based			No-Cap		
Emission Cap [tons]			400			400			N\A
Consumer Surplus [$]			113,532			13,254			138,017
Producer Surplus [$]			25,635			28,369			23,464
ISO's Surplus [$]			3,986			6,659			4949
LSE's Surplus [$]			20,934			N\A			0
Social Welfare [$]			164,088			166,882			166,430
Variable\Zone	A	B	C	A	B	C	A	B	C
Price [$/MWh]	72.3	37.4	20.8	54.0	43.1	20.7	48.3	41.4	20.8
CO_2 Emission [tons] (from\to))									
A	225.0	31.2	87.8	209.2	35.7	155.1	198.8	48.0	179.1
B	175.0	0.0	0.0	96.6	7.2	71.2	78.7	8.4	87.9
C	0.0	333.3	622.9	477.7	244.6	224.9	601.3	232.8	115.6
Zonal CO_2 Emissions [ton]	344.0	175.0	956.2	400.0	175.0	947.2	425.8	175.0	950.0
Associated CO_2 Emission [tons]	400.0	364.5	710.7	783.5	287.5	451.2	878.9	290.0	381.9
Import (+)/Export(-) Emissions [ton]	56	189.5	-245.5	383.5	112.5	-496.0	453.1	115.0	-568.1
Total CO_2 Emissions [ton]		1,475.2			1,522.2			1,550.7	
CO_2 Price [$/ton]		52.3			12.5			0	
Generating Cost [$]		45,327			49,365			51,451	
Allowances Rents [$]		20,932			4,988			0	
Electricity Sales (from\to)	A	B	C	A	B	C	A	B	C
A	400.0	52.0	148.0	362.5	61.9	268.9	343.4	82.9	309.9
B	550.0	0.0	0.0	302.4	21.4	226.2	248.5	25.2	276.2
C	0.0	271.2	535.6	403.4	206.7	188.7	511.8	191.9	97.6
Total Sale [MWh]	950.0	323.2	683.6	1,068.3	290.0	683.9	1,103.7	300.1	683.8

Consistent with definitions elsewhere [23], we defined leakage as the difference between the decrease in regulated emissions and the decrease in total regional emissions. That is, increases in unregulated emissions elsewhere could, to some degree, offset the decline in regulated emissions. To obtain a gross indicator of leakage, we first calculate for each of the load-, source-, and first-seller-based proposals the decrease in emissions of generators in zone A plus imports to zone A, compared to the no-cap case (row 2 of Table 4, called E_1). These are the emissions that are subject to regulation in the modified and first-seller proposals (Table 2), and for consistency, we also consider them for the pure source- and load-based proposals (Table 3). We then calculate the total decrease in emissions in zones A+B+C relative to the no-cap case (row 4 of Table 4, called E_2). The percentage leakage is then $100\%(1 - E_2/E_1)$.

Table 4 shows that in all cases most of the apparent emission reductions in regulated emissions are lost due to leakage– i.e., increases in nonregulated emissions. For instance, for the modified- and first-seller-based proposals (Table 2), the leakage is 85%. This is because regulated emissions fall from 1,056 tons (no-cap case) to 400 tons (the cap), but total emissions fall by much less (1551 to 1443).

This leakage is in large measure due to contract shuffling. We define contract shuffling as the difference between the apparent decrease in emissions associated with power imports to A (called E_3 in Table 4) and the actual decrease in emissions in zones B and C (E_4 in Table 4). In our simulations, emissions in B and C actually unchanged, even though the emissions associated with bilateral transactions from B and C to zone A decrease. If we define the percentage of contract shuffling as $100\%(1-E_4/E_3)$, then this percentage is actually equal to 100%. Thus, the apparent emissions decreases due to changes in the composition of imports to A are completely illusory, as emissions elsewhere are actually increasing under regulation.

For example, the percentage of contract reshuffling for the modified- and first-seller-based proposals (Table 2) is 100% (last row, Table 4). This is because although emissions associated with imports by A fall from 680 tons (no-cap) to 83 tons (Table 2), emissions in zones B and C actually approximately unchanged (1,125 tons=78.7 (from B)+601.3 (from C)). This occurs because the large amounts of dirty power that used to be exported from C to A in the no-cap case (512 MWh, Table 3) diverted to B under regulation. As a result, B is then able export more of its clean power to A (compare Tables 2 and 3). Of course, these are financial transactions, and the shifts in physical power flows are much less dramatic.[17]

[17]The distribution of bilateral contracts in the no-cap model is not unique, which implies that the calculations of leakage and contract shuffling in this section are also not unique. For instance,

Table 4: Leakage and Reshuffling Results, Scenarios 1-6

	Table 2 Regulations[a]	Pure Load-Based	Pure Source-Based	No-Cap
Emissions of A + Imports to A[tons]	400	519	974	1,106
Δ Relative to No-Cap (E_1)[tons]	-706	-587	-132	
Total Emissions [tons]	1,443	1,475	1,522	1,551
Δ Relative to No-Cap (E_2)[tons]	-108	-76	-29	
Leakage[b][%]	85%	87%	78%	
Emissions Imports to A[tons]	83	175	574	680
Δ Relative to No-Cap (E_3)[tons]	-597	-505	-106	
Emissions B+C[tons]	1,125	1,131	1,122	1,125
Δ Relative to No-Cap (E_4)[tons]	0	6	-3	
Reshuffling[c][%]	100%	101%	97%	

Notes: a. Modified Source- and Load-Based, and First- Seller

b. Leakage [%] = $(1 - E2/E1) * 100\%$

c. Reshuffling [%] = $(1 - E4/E3) * 100\%$

5 Conclusion and Discussion

California is considering three proposals to regulate greenhouse gasses emitted by electric power plants under the Assembly Bill 32: source-based, load-based and first-seller approaches. These three proposals differ by their point-of-regulation and possibly the "jurisdiction" of the regulation. Our interpretation is that three proposals will regulate emissions associated with in-state and cross-state electricity transactions. However, if in-state to out-of-state sales or vice verse are not subject to cap, greater amounts of contract shuffling and CO_2 leakage with fewer real emissions reductions could possibly occur as shown in the pure load-based and pure source-based scenarios simulated in Section 4.

We have formulated equilibrium models for the three proposals. In addition to existence and uniqueness properties of the solutions, this paper shows that the three proposals are essentially equivalent in that they produce the same zonal sales, same

if a no-cap solution with more B to A bilateral contracts was instead used as the baseline, then the estimated amounts of leakage and shuffling would be smaller. However, these illustrative calculations demonstrate that large amounts of leakage and contract shuffling are very likely to occur under these regulatory proposals.

production and emission patterns, same allowance and retail electricity prices, and same social surplus. This occurs under the assumption that imports to California would be subject to the emissions cap, as well as sales from California plants to other states.

However, the proposals can differ in the distribution of social surplus; when this occurs, it is because of assumptions concerning the distribution of allowance rents. If the markets are designed so that all emissions allowance rents accrue to consumers, then the consumers surplus is the same in all three proposals.

This points out the importance of the issue of allowance allocation. The value of those allowances could represent a significant amount of wealth transfer and might also affect program's political feasibility and acceptability. For instance, it has been shown that significant windfall profits were earned by generators in the European Union Emissions Trading Scheme in 2005 given that generators received most allowances for free [22]. Additionally, for first-seller and source-based approaches to survive legal challenge, it is reasonable to conjecture that allowances need to be distributed by auctions. Otherwise, in-state sources would be given a financial windfall that their counterparts in neighboring states would not obtain if allowances are grandfathered.

Another assumption made in this paper is that CO_2 intensity of imported electricity can be unambiguously determined. This may prove to be an challenging task in reality given the complicated operations of the competitive electricity markets. A number of alternatives such as using a default or an average emissions rate have been proposed. In this modelling framework, this is equivalent to assigning a uniform emissions rate to all the out-of-state sources. However, the conclusion about the equivalence of three proposed programs will remain intact.

As California has traditionally been a leader in pioneering new environmental policies in the U.S., if the policy goal is to control regional greenhouse gas emissions in the West, a California-only emissions trading program may prove to have little impact on overall greenhouse gas emissions while possibly disrupting electricity market operations, as argued by Wolak et al.[25]. If in contrast, if the policy goal to take the lead with the hope that a comprehensive federal CO_2 cap-and-trade will follow in near future, careful consideration should be given to how the AB32 program should be designed to minimize interference with the operations of competitive electricity markets and reduce the chance that California consumers will suffer from additional electricity price increases.

Appendix

Proofs

To prove the results presented in Section 2.4, we first provide an exact definition of a mixed linear complementarity problem, which plays a central role in all the proofs.

Definition 1 *Let $M \in \Re^{n\times n}$, $N \in \Re^{n\times m}$, $q \in \Re^n$, $U \in \Re^{m\times n}$, $V \in \Re^{m\times m}$, and $r \in \Re^m$ be all given. A mixed linear complementarity problem (MLCP) is to find a vector $x \in \Re^n$ and $y \in \Re^m$ such that*

$$\begin{aligned} 0 &\leq Mx + Ny + q \geq 0; \\ 0 &= Ux + Vy + r. \end{aligned} \tag{21}$$

We denote the above problem as $MLCP(M, N, q, U, V, r)$.

A special case of an MLCP is $MLCP(M, N, q, -N^T, 0, r)$, which has some important properties as shown below. We summarize some known results and facts that are used to prove the main theorems.

Lemma 1 *(Metzler [18]) For an MLCP problem $MLCP(M, N, q, -N^T, 0, r)$, if M is positive semi-definite, and the problem is feasible, then a solution exists to the MLCP.*

Lemma 2 *Suppose that $MLCP(M, N, q, -N^T, 0, r)$ is feasible. If M is positive semi-definite, then for any two solutions of the MLCP, (x^1, y^1) and (x^2, y^2), the following is true.*

$$(M + M^T)x^1 = (M + M^T)x^2.$$

Proof. By Lemma 1 we know that a solution exists to $MLCP(M, N, q, -N^T, 0, r)$. The rest of the proof is almost identical to that of Theorem 3.1.7 in Cottle et al. ([10]). □

For the remaining section, we assume that the cost function is as given in Section 3 (equation (19)), with $d_{fi} > 0$ for each $f \in \mathcal{F}$ and $i \in \mathcal{N}$. Then for $m \in \{SB, LB, FS\}$, the three models presented in the previous section can be represented by an MLCP with the same structure.

$$\begin{aligned} 0 &\leq \mathbf{a}^m \perp q^m + M^m\mathbf{a}^m + \Delta^m\theta^m, \\ 0 &= \Delta^{m^T}\mathbf{a}^m. \end{aligned} \tag{22}$$

The variables in the MLCP are as follows.

$$\mathbf{a}^{SB} = \begin{pmatrix} \lambda \\ s \\ x \\ \rho \\ p^{CO_2} \end{pmatrix} \in \Re^{2K+2FN+FN^2+1},\ \mathbf{a}^{LB} = \mathbf{a}^{FS} = \begin{pmatrix} \lambda z \\ x \\ \rho \\ p^{CO_2} \end{pmatrix} \in \Re^{2K+FN+2FN^2+1};$$

$\theta^{SB} = [\theta'_{fj}] \in \Re^{FN},\ \theta^{LB} = \theta^{FS} = [p_{fij}] \in \Re^{FN^2}.$

The parameters are q^m, M^m and Δ^m, with q^m being a vector of the same dimension as $\mathbf{a}^m$, M^m being a square matrix of the appropriate dimension, and $\Delta^m \in \Re^{(2K+2FN+FN^2+1)\times FN}$ for $m = SB$, and $\Delta^m \in \Re^{(2K+FN+2FN^2+1)\times FN^2}$ for $m = LB,\ FS$. Define the following vectors: $T = [T_k] \in \Re^{2K},\ P^0 = [P^0_j] \in \Re^N,\ C = [c_{fi}] \in \Re^{FN}$, $X = [X_{fi}] \in \Re^{FN}$. Then the detailed expression of the parameters are as follows.

$$q^{SB} = \begin{pmatrix} T \\ -P^0 \\ C \\ X \\ \overline{E}^{SB} \end{pmatrix},\ q^{LB} = \begin{pmatrix} T \\ -P^0 \\ C \\ X \\ \overline{E}^{LB} \end{pmatrix},\ q^{FS} = \begin{pmatrix} T \\ -P^0 \\ C \\ X \\ \overline{E}^{FS} \end{pmatrix};$$

$$\Delta^{SB} = \begin{pmatrix} 0 \\ I_{FN\times FN} \\ -J_{FN^2\times FN} \\ 0 \\ 0 \end{pmatrix},\ \Delta^{LB} = \Delta^{FS} = \begin{pmatrix} 0 \\ I_{FN^2\times FN^2} \\ -I_{FN^2\times FN^2} \\ 0 \\ 0 \end{pmatrix},$$

where I represents the identity matrix of appropriate dimensions, and

$$J = \begin{bmatrix} \mathbf{1}_{N\times 1} & 0 & \cdots & 0 \\ 0 & \mathbf{1}_{N\times 1} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \mathbf{1}_{N\times 1} \end{bmatrix} \in \Re^{(FN^2)\times(FN)};$$

$$M^{SB} = \begin{bmatrix} 0 & M_{\lambda s} & M_{\lambda x}^{SB} & 0 & 0 \\ -M_{\lambda s}^T & M_s & 0 & 0 & 0 \\ -M_{\lambda x}^{SB^T} & 0 & M_x & M_{x\rho} & \mathcal{E}_x^{SB} \\ 0 & 0 & -M_{x\rho}^T & 0 & 0 \\ 0 & 0 & -\mathcal{E}_x^{SB^T} & 0 & 0 \end{bmatrix}$$

$$M^m = \begin{bmatrix} 0 & 0 & M_{\lambda x}^m & 0 & 0 \\ 0 & M_z & 0 & 0 & \mathcal{E}_z^m \\ -M_{\lambda x}^{m^T} & 0 & M_x & M_{x\rho} & \mathcal{E}_x^m \\ 0 & 0 & -M_{x\rho}^T & 0 & 0 \\ 0 & -\mathcal{E}_z^{m^T} & -\mathcal{E}_x^{m^T} & 0 & 0 \end{bmatrix}, \text{ where } m \in \{LB, FS\}.$$

We first provide details on the off-diagonal block matrices of the M matrices.

Let $\Pi \equiv [PTDF_{ki}] \in \Re^{2L\times N}$ denote the matrix of $PTDF$'s. Then

$$\begin{aligned} M_{\lambda s} &= -[\Pi \ \dots \ \Pi\,] && \in \Re^{2L\times FN}, \\ M_{\lambda x}^{SB} &= [\Pi \ \dots \ \Pi \ \dots \ \Pi] && \in \Re^{2L\times FN^2}. \end{aligned}$$

Assume that the vector $x = [x_{fij}] \in \Re^{FN^2}$ is arranged first by the j index, then by the i index, and then by the f index. The $M_{\lambda x}^m$ matrix, for $m = \{LB, FS\}$, is given as follows

$$M_{\lambda x}^m = [\Pi \ \dots \ \Pi \ \dots \ \Pi] - [\Pi \ \dots \ \Pi \ \dots \ \Pi]\,P \in \Re^{2L\times FN^2},$$

where $P \in \Re^{FN^2\times FN^2}$ is a permutation matrix that re-groups the matrix $[\Pi \ \dots \ \Pi \ \dots \ \Pi]$ corresponding to index i of the vector x.

$$M_{x\rho} = \begin{bmatrix} \mathbf{1}_{N\times 1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{1}_{N\times 1} \end{bmatrix} \in \Re^{FN^2\times FN}.$$

Further assume that the vector $x = [x_{fij}]$ is arranged by $j \in \mathcal{N}^C$ first, then by $j \in \mathcal{N}^N$, $i \in \mathcal{N}^C$, $i \in \mathcal{N}^N$, and $f \in \mathcal{F}$. Then E_x^m's are as follows.

$$\mathcal{E}_x^m = \left[\mathcal{E}_{x_1}^{m^T} \dots \mathcal{E}_{x_f}^{m^T} \dots \mathcal{E}_{x_F}^{m^T}\right] \in \Re^{FN^2},$$

where for each $f \in \mathcal{F}$, $\mathcal{E}_{x_f}^{m^T} = \left[E_{fi}\tilde{I}_i^m\right]_{i\in\mathcal{N}} \cdot \mathbf{1}_{N^2\times 1} \in \Re^{N^2}$. $\tilde{I}_i^m$ are diagonal matrices for each $m \in \{SB, LB, FS\}$ and $i \in \mathcal{N}$, with diagonal entries being either 0's or 1's.

There detailed forms are as follows.

$$\tilde{I}_i^{SB} = \begin{cases} \text{identity matrix of size } N, & i \in \mathcal{N}^C, \\ \text{1's for the first } J \text{ diagonal entries, 0's otherwise}, & i \in \mathcal{N}^N. \end{cases}$$

$$\tilde{I}_i^{LB} = \begin{cases} \text{0's for the first } J \text{ diagonal entries, 1's for the rest diagonal entries}, & i \in \mathcal{N}^C, \\ \text{0 matrix of size } N, & i \in \mathcal{N}^N. \end{cases}$$

$$\tilde{I}_i^{FS} = \begin{cases} \text{identity matrix of size } N, & i \in \mathcal{N}^C, \\ \text{0 matrix of size } N, & i \in \mathcal{N}^N. \end{cases}$$

$\mathcal{E}_z^m$'s are defined in a similar fashion as $\mathcal{E}_x^m$'s and are omitted here.

For the diagonal block matrices in the M matrices, first denote a diagonal matrix $Diag(\frac{P_j^0}{Q_j^0})_{j\in\mathcal{N}}$ as matrix D. Then $M_s = D \otimes \mathbf{1}_{F\times F}$, and $M_z^m = D \otimes \mathbf{1}_{FN\times FN}$, where $\mathbf{1}$ represents a matrix of all 1's with proper dimensions, and $\otimes$ denotes the Kronecker product. The M_x matrix is the same across the three models, and is a block-diagonal matrix as follows.

$$M_x = \begin{bmatrix} d_{11}\mathbf{1}_{N\times N} & & & & 0 \\ & \ddots & & & \\ & & d_{fi}\mathbf{1}_{N\times N} & & \\ & & & \ddots & \\ 0 & & & & d_{FN}\mathbf{1}_{N\times N} \end{bmatrix} \in \Re^{FN^2\times FN^2}.$$

Lemma 3 *The matrix M^m, for each $m \in \{SB, LB, FS\}$, is positive semi-definite.*

Proof. It is easy to see that M^m is positive semi-definite if only M_x, M_s and M_z are positive semi-definite matrices. M_x is trivially seen to be positive semi-definite, given the assumption that $d_{fi} > 0$ for each $f \in \mathcal{F}$, $i \in \mathcal{N}$. Since the structure of M_s and M_z^m's are the same, we need only consider one of them. Here we consider M_s. By the formulation introduced above, M_s is the Kronecker product of the matrices D and $\mathbf{1}_{F\times F}$. Since both matrices are positive semi-definite, by a known result from matrix analysis, (see, for example, Horn and Johnson [16]), M_s, as the Kronecker product of two positive semi-definite matrices, is also positive semi-definite. □

Theorem 1.

Proof. For Model SB, choose $s = x = 0$, $\lambda = 0$, $y = w = 0$, $p^{CO_2} = 0$, $\theta'_{fj} = \theta_{fj} - \eta = P_j^0$, $\forall\ f \in \mathcal{F},\ j \in \mathcal{N}$, and $\rho_{fi} = \max_{j\in\mathcal{N}}\{P_j^0 - C_{fi}\}$, $\forall\ f \in \mathcal{F}$ and $i \in \mathcal{N}$. Then the tuple $(s,\ \theta,\ x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2})$ is feasible to Model SB. By

Lemma 1 and 3, Model SB, written as $MLCP(M^{^{SB}}, \Delta, q^{^{SB}}, -\Delta^T, 0, 0)$, has a solution. Existence of a solution to Model LB and Model FS can be shown in the similar way. □

Theorem 2.

Proof. (a) ⇒ (b). Suppose that $(s,\ \theta,\ x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2})$ solves Model SB. First consider a $s_{fj} > 0$, with $f \in \mathcal{F}$ and $j \in \mathcal{N}$. By complementarity in firm f's first order condition (3), we have that $\theta_{fj} = \tilde{p}_j - w_j$, and for the particular f and j,

$$0 \le x_{fij} \quad \perp \quad c_{fi} + d_{fi} \sum_{j' \in \mathcal{N}} (x_{fij'}) + p^{CO_2} E_{fi} - w_i + \rho_{fi} - \tilde{p}_j + w_j \ge 0,\ \forall\ i \in \mathcal{N} \text{ if } j \in \mathcal{N}^C,$$
$$0 \le x_{fij} \quad \perp \quad c_{fi} + d_{fi} \sum_{j' \in \mathcal{N}} (x_{fij'}) - w_i + \rho_{fih} - \tilde{p}_j + w_j \ge 0,\ \forall\ i \in \mathcal{N}^N, \text{if } j \in \mathcal{N}^N,$$

where $\tilde{p}_j = P_j^0 - \frac{P_j^0}{Q_j^0}\left(\sum_f s_{fj}\right) = P_j^0 - \frac{P_j^0}{Q_j^0}\left(\sum_{f \in \mathcal{F}, i \in \mathcal{N}} x_{fihj}\right)$.

For a $s_{fj} = 0$, by nonnegativity of x_{fij}'s, we have that $x_{fij} = 0$, $\forall\ i \in \mathcal{N}$.

Now for each $f \in \mathcal{F}$, $i, j \in \mathcal{N}$, let $z_{fij} = x_{fij}$ and define p_{fij} as follows

$$p_{fij} = \begin{cases} \tilde{p}_j, & \text{if } j \in \mathcal{N}^N; \\ \tilde{p}_j - p^{CO_2} E_{fi}, & \text{if } j \in \mathcal{N}^C. \end{cases}$$

Let $z = [z_{fij}]$ and $p = [p_{fij}]$. Then clearly $(z, x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2}, p)$ satisfies all the equality and complementarity conditions of Model LB. Hence, it solves Model LB.

(b) ⇒ (c). Assume that $(z,\ x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2},\ p)$ solves Model LB, where $p = [p_{fij}]$. For each $f \in \mathcal{F}$, $i, j \in \mathcal{N}$, define $\hat{p}_{fij}$ as follows

$$\hat{p}_{fij} = \begin{cases} p_{fij} + p^{CO_2} E_{fi}, & \text{if } i, j \in \mathcal{N}^C; \\ p_{fij}, & \text{otherwise.} \end{cases}$$

Let $\hat{p} = [\hat{p}_{fij}]$. Then it is easy to see that $(z,\ x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2},\ \hat{p})$ solves Model FS.

(c) ⇒ (a). Assume that $(z,\ x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2},\ p)$ solves Model FS, where $p = [p_{fij}]$. For each $f \in \mathcal{F}$, $i \in \mathcal{N}^N$ and $j \in \mathcal{N}^C$ with $z_{fij} > 0$, by the LSE j's first-order condition, we have that $p_{fij} + p^{CO_2} E_{fi} = P_j^0 - \frac{P_j^0}{Q_j^0}(\sum_{f,i,h \in H_{i,f}} z_{fij})$. Notice that the right-hand-side is independent of index f, i and h. Similarly, for $i \in \mathcal{N}^C$ or $j \in \mathcal{N}^N$ with $z_{fij} > 0$, $p_{fij} = P_j^0 - \frac{P_j^0}{Q_j^0}(\sum_{f,i,h \in H_{i,f}} z_{fij})$. By letting $\tilde{p}_j$, $j \in \mathcal{N}$ denote

$P_j^0 - \frac{P_j^0}{Q_j^0}(\sum_{f,i,h\in H_{i,f}} z_{fij})$, we have the following.

$$\tilde{p}_j = \begin{cases} p_{fij} + p^{CO_2}E_{fi}, & \text{if } i \in \mathcal{N}^N \text{ and } j \in \mathcal{N}^C; \\ p_{fij}, & \text{otherwise.} \end{cases}$$

Further define that for each $f \in \mathcal{F}$ and $j \in \mathcal{N}$, $s_{fj} = \sum_{i\in\mathcal{N}} x_{fij}$ and $\theta_{fj} = \tilde{p}_j - w_j$. The complementarity constraints on s variables in formulation (3) are be seen to be satisfied. It is then trivial to show that $(s,\ \theta,\ x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2})$ solves Model SB. □

Theorem 3.

Proof. (a) Let $SOL(MLCP^m)$ denote the set of solutions to the MLCP model i, where $m \in \{SB, LB, FS\}$. First consider the source-based model. Let $\mathbf{a}^{SB^1}$, $\mathbf{a}^{SB^2} \in SOL(MLCP^{SB})$. By Lemma (2), we have the following.

$$\begin{aligned} & (M^{SB} + M^{SB^T})\mathbf{a}^{SB^1} = (M^{SB} + M^{SB^T})\mathbf{a}^{SB^2} \\ \iff\ & M_s^{SB} s^1 = M_s^{SB} s^2 \\ \iff\ & \frac{P_j^0}{Q_j^0}\left(\sum_{f\in\mathcal{F}} s_{fj}^1\right) = \frac{P_j^0}{Q_j^0}\left(\sum_{f\in\mathcal{F}} s_{fj}^2\right),\ \forall j \in \mathcal{N}. \end{aligned} \tag{23}$$

The last equality constraint implies that among solutions to Model SB, total sale at zone $j \in \mathcal{N}$ in the source-based model is unique. The uniqueness of total sale $\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij}$ for each $j \in \mathcal{N}$ in solutions for Model LB and FS can be shown similarly.

Define a vector $\mathbf{b}$ the same way as in Theorem 2; namely, $\mathbf{b} = (x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2})$. By Theorem 2, we know that if $(s, \theta, \mathbf{b}) \in SOL(MLCP^{SB})$, there exist z and p such that $(z, \mathbf{b}, p) \in SOL(MLCP^{LB})$, and z' and p' such that $(z', \mathbf{b}, p') \in SOL(MLCP^{FS})$. Then we have the following.

$$\begin{array}{ccccc} \sum_{f\in\mathcal{F}} s_{fj} = & \sum_{f\in\mathcal{F},i\in\mathcal{N}} x_{fij} & = \sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij},\ \forall\ j \in \mathcal{N}. \\ & \| & \\ & \sum_{f\in\mathcal{F},i\in\mathcal{N}} z'_{fij} & \end{array}$$

It has just been shown that in a solution to Model SB, the total sale $\sum_{f\in\mathcal{F}} s_{fj}$ is unique at each zone $j \in \mathcal{N}$; similarly, the total sale $\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij}$ is unique for solutions to Model LB, and $\sum_{f\in\mathcal{F},i\in\mathcal{N}} z'_{fij}$ is unique for Model FS. Hence, we have shown that among solutions to the three models, total sale at each region $j \in \mathcal{N}$ is the same.

(b) Following similar arguments in (a), we have that

$$
\begin{aligned}
& (M^{SB} + M^{SB^T})\mathbf{a}^{SB^1} = (M^{SB} + M^{SB^T})\mathbf{a}^{SB^2} \\
\iff\ & M_x x^1 = M_x x^2 \\
\iff\ & d_{fi}\left(\sum_{j'\in\mathcal{N}} x^1_{fij'}\right) = d_{fi}\left(\sum_{j'\in\mathcal{N}} x^2_{fij'}\right),\ \forall f \in \mathcal{F}, i \in \mathcal{N}.
\end{aligned} \tag{24}
$$

Hence, $\sum_{j\in\mathcal{N}} x_{fij}$, the total production at zone i for firm f is unique in an equilibrium of Model SB. Uniqueness of the corresponding quantity in the equilibria of Model LB and Mode FS can then be shown with the same agrement. By Theorem 2, the equivalence of the equilibrium quantity across the three models follows.

(c) Total emissions in the entire system under consideration are given by $\sum_{f\in\mathcal{F},i,j\in\mathcal{N}} (x_{fij}E_{fi})$. By re-arranging the indices, we have the following relationship

$$
\sum_{f\in\mathcal{F}}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}}(x_{fij}E_{fi}) = \sum_{f\in\mathcal{F}}\sum_{i\in\mathcal{N}} E_{fi}\left(\sum_{j\in\mathcal{N}} x_{fij}\right).
$$

By the uniqueness and equivalence results of $\sum_{j\in\mathcal{N}} x_{fij}$ shown in Part (b), the same results hold true for total emissions.

(d) Consumers' surpluses, under linear inverse demand function, are given in Section 3 (20). For illustration purpose, they are written out again.

$$
\begin{aligned}
CS^m &\equiv \sum_{j\in\mathcal{N}}\left[P^0_j\left(\sum_{f\in\mathcal{F}} s_{fj}\right) - \frac{P^0_j}{2Q^0_j}\left(\sum_{f\in\mathcal{F}} s_{fj}\right)^2 - \sum_{f\in\mathcal{F}}\left(P^m_j s_{fj}\right)\right], && m = SB; \\
CS^m &\equiv \sum_{j\in\mathcal{N}}\left[P^0_j\left(\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij}\right) - \frac{P^0_j}{2Q^0_j}\left(\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij}\right)^2 - \sum_{f\in\mathcal{F},i\in\mathcal{N}}\left(P^m_j z_{fij}\right)\right], && m = LB \text{ or } FS,
\end{aligned}
$$

with

$$
\begin{aligned}
P^m_j &\equiv P^0_j - \frac{P^0_j}{Q^0_j}\left(\sum_{f\in\mathcal{F}} s_{fj}\right), && m = SB; \\
P^m_j &\equiv P^0_j - \frac{P^0_j}{Q^0_j}\left(\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij}\right), && m = LB \text{ or } FS.
\end{aligned}
$$

Then the uniqueness of P^m_j for each $j \in \mathcal{N}$ and CS^m at an equilibrium for each $m \in \{SB, LB, FS\}$, and the equivalence of P^m_j and CS^m across the models follow directly from (a).

(e) The social welfare under the modeling context is the sum of consumers, producers, ISO, and LSEs surpluses. Under perfectly competitive assumption, this is equivalent to its classical definition; namely, consumers total benefits (integral of inverse demand function) minus total production costs. Let SW denote "social welfare"; then its explicit expression is as follows.

$$SW^{SB} \equiv \sum_{j \in \mathcal{N}} \left[P_j^0 \left(\sum_{f \in \mathcal{F}} s_{fj} \right) - \frac{P_j^0}{2Q_j^0} \left(\sum_{f \in \mathcal{F}} s_{fj} \right)^2 \right] - \sum_{f \in \mathcal{F}, i \in \mathcal{N}} C_{fi}(\sum_{j \in \mathcal{N}} x_{fij}),$$

$$SW^{LB,\ \ FS} \equiv \sum_{j \in \mathcal{N}} \left[P_j^0 \left(\sum_{f \in \mathcal{F}, i \in \mathcal{N}} z_{fij} \right) - \frac{P_j^0}{2Q_j^0} \left(\sum_{f \in \mathcal{F}, i \in \mathcal{N}} z_{fij} \right)^2 \right] - \sum_{f \in \mathcal{F}, i \in \mathcal{N}} C_{fi}(\sum_{j \in \mathcal{N}} x_{fij}).$$

By the uniqueness of the total sales at a zone j and total generation at a zone i in an equilibrium to each of the three models, the uniqueness of social welfare follows. The equivalence of social welfare across the equilibria to the three models also readily follow by the results shown in (a) and (b). □

Generator Characteristics and Transmission Line Data

Table 5: Generating Data

Plant	Firm	Location	c [\$/(MWh)]	d [\$/(MWh ^{2})]	CO_2 Rate[kg/MWh]	Capacity[MW]
1	3	A	38.00	0.02	580	250
2	1	A	35.72	0.03	545	200
3	2	A	36.80	0.04	600	450
4	1	B	15.52	0.01	500	150
5	2	B	16.20	0.02	500	200
6	3	B	0.00	0.001	0	200
7	1	C	17.60	0.02	1216	400
8	1	C	16.64	0.01	1249	400
9	1	C	19.40	0.01	1171	450
10	3	C	18.60	0.02	924	200

Table 6: PTDF Values

Link\Zone	A	B	C
1	-0.3333	0.5	0
2	-0.6667	-0.5	0
3	-0.3333	-0.5	0

Table 7: Transmission Line Capacities

Link	Capacity [MW]
1	255
2	120
3	30

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