Title
WEAK INTERACTIONS AT THE SSC

Author
Chanowitz, M.S.

Publication Date
1986-03-01

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Physics Division

Presented at the Workshop on Standard
Model Physics at the SSC, Los Angeles, CA,
January 15-24, 1986; and to be published
in the Proceedings

WEAK INTERACTIONS AT THE SSC

M.S. Chanowitz

March 1986

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

DISCLAIMER

March 1985 LBL-21290

Weak Interactions at the SSC*

Michael S. Chanowitz

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

Invited talk presented at the Workshop on Standard Model Physics at the SSC, held at UCLA, January 15-24, 1986. To be published in the proceedings.

Abstract

Prospects for the study of standard model weak interactions at the SSC are reviewed, with emphasis on the unique capability of the SSC to study the mechanism of electroweak symmetry breaking, whether the associated new quanta are at the TeV scale or higher. Symmetry breaking by the minimal Higgs mechanism and by related strong interaction dynamical variants is summarized. A set of measurements is outlined that would calibrate the proton stucture functions and the backgrounds to new physics. The ability to measure the three weak gauge boson vertex is found to complement LEP II, with measurements extending to larger Q^2 at a comparable statistical level in detectable decays. B factory physics is briefly reviewed as one example of a possible broad program of high statistics studies of sub-TeV scale phenomena. The largest section of the talk is devoted to the possible manifestations of symmetry breaking in the WW and ZZ production cross sections. Some new results are presented bearing on the ability to detect high mass WW and ZZ pairs. The principal conclusion is that although nonstandard model scenarios are typically more forgiving, the capability to study symmetry breaking in the standard model (and in related strong interaction dynamical variants) requires achieving the SSC design goals of $\sqrt{s}, \mathcal{L} = 40\mathrm{TeV}, 10^{33}\mathrm{cm.}^{-2}\mathrm{sec.}^{-1}$

---

*Work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

I. Introduction

Most high energy physics accelerators and storage rings have been proposed chiefly as the next step to new, unknown energy regions. Less often a new facility is proposed with a very definite physics target in mind, calling for specific energy and luminosity: recent examples are the SPS collider, SLC, and LEP. For the SSC we have both motivations. First, the SSC will be as big a step in center of mass energy as we have ever taken with a high intensity device capable of a wide range of detailed investigations. Since it will provide both the first look at TeV scale physics and unprecedented event rates for the physics of lower energy scales, the SSC potentially commands enormous phase space for unanticipated fundamental discoveries. Second, the SSC also has a definite physics target, though less precisely known that the Z boson: to discover the mechanism of $SU(2)_L \times U(1)$ symmetry breaking. While all nonstandard scenarios that have been studied are more forgiving, in the minimal standard model, which is the topic of this workshop, this goal imposes the most severe demands on SSC design parameters. If the SSC operates at $\sqrt{s} = 40$ TeV and $\mathcal{L} = 10^{33}\mathrm{cm.}^{-2}\mathrm{sec.}^{-1}$, it will be capable of studying the symmetry breaking mechanism whether the scale of the associated new quanta is $0.2 \lesssim m \lesssim 1$ TeV or even if it is so much larger than 1 TeV that the new quanta cannot be directly produced. It is unlikely that either of these statements would hold if the SSC center of mass energy were halved or the luminosity decreased by an order of magnitude. (I am hedging a little because the conclusion depends, as discussed below, on how well we will be able to detect W and Z pairs in SSC experiments.)

The sections of this talk are organized as follows:

- II. Survey of the physics of the symmetry breaking sector
- III. Calibration studies
- IV. Gauge sector physics
- V. SSC as B factory – an example of using SSC to study sub-TeV physics

VI. Detecting the symmetry breaking sector

VII. Conclusions

II. Survey of Symmetry Breaking Sector

In the minimal standard model,[1] symmetry breaking is accomplished by adding a quartet of scalar particles, the triplet $\vec{\phi} = (w^+, z, w^-)$ and the Higgs boson H. Together $\vec{\phi}$ and H form a complex doublet of the $SU(2)_L$ symmetry. They are assumed to interact according to a potential $V(\vec{\phi}, H)$

$$V(\vec{\phi}, H) = \frac{\lambda}{4}(\vec{\phi}^2 + H^2 - v^2)^2 \tag{2.1}$$

which develops a classical minimum at $\vec{\phi}^2 + H^2 = v^2$. We choose the orientation of the fields so that at the minimum $H = v$ and $\vec{\phi} = 0$, and we say that H acquires a vacuum expectation value or vev. If we redefine H to have vanishing vev, $H \rightarrow H + v$, we can rewrite the potential as

$$V(\vec{\phi}, H) = \frac{\lambda}{4}(\vec{\phi}^2 + H^2)^2 + \lambda v H(\vec{\phi}^2 + H^2) + \frac{m_H^2}{2}H^2 \qquad (2.2))$$

where the Higgs mass is $m_H^2 = 2\lambda v^2$ and the triplet $\vec{\phi}$ remains massless, being the three Goldstone bosons corresponding to the three generators of $SU(2)_L \times U(1)$ that are broken at the classical minimum. Because of their gauge interactions the triplet $\vec{\phi}$ is transmogrified to become the longitudinal modes of the W and Z gauge bosons that acquire masses, $M_W = \frac{1}{2}gv$, where g is the $SU(2)_L$ gauge coupling constant. To agree with the measured value of G_F we fix

$$v = \frac{1}{4}\,TeV \tag{2.3}$$

leaving

$$\lambda = \frac{m_H^2}{2v^2} \tag{2.4}$$

as a free parameter. The strength of the constant λ then depends on the unknown mass m_H.

The experimental situation depends critically on whether m_H is less or greater than $2M_W$. Suppose first that $m_H < 2M_W$. Then from (2.4) the potential (2.2) defines a weakly coupled theory

$$\frac{\lambda}{4\pi^2} < 5.10^{-3} \tag{2.5}$$

and the dominant production mechanism for H is gluon–gluon fusion,[2] via a quark–antiquark loop. If there are only the three known quark generations and if $m_t \cong$ 30 GeV , then for 100 GeV $\lesssim m_H < 2M_W$ there are about 10^6 H's produced[3] in an SSC experimental year. (Here and elsewhere I set $\sqrt{s} = 40$ TeV and define an SSC year by an integrated luminosity of 10^{40}cm.$^{-2}$, corresponding to $\sim 1/3$ year by the clock at $\mathcal{L} = 10^{33}$cm.$^{-2}$sec.$^{-1}$.) The yield is greater if $m_t > 30$ GeV or if there are new quarks heavier then top. Assuming only three generations, the dominant decay is $H \rightarrow \bar{t}t$. As discussed in Section VI, noone knows at present how to use these 10^6 events to detect the Higgs boson.

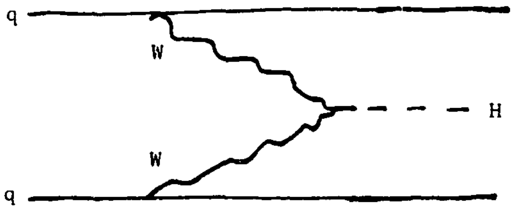

Figure 2.1, H production by WW fusion.

Consider next the case $m_H > 2M_W$ for which H decays predominantly to W^+W^- and ZZ (even, it turns out, if $m_t > M_W$).[4] The dominant production mechanism becomes WW (and ZZ) fusion,[5] figure 2.1, which dominates gg fusion for $m_H > 350$ GeV at $m_t = 40$ GeV. Production rates for the central rapidity region, $|y_{W,Z}| < 1.5$, range from $2\cdot 10^4$ H's per SSC year at $m_H = 0.4$ TeV to $7\cdot 10^3$ H's per SSC year at $m_H = 1.0$ TeV.

Referring to eq. (2.4) we see that as m_H increases above 1 TeV the interactions defined by the potential (2.2) become strong. This statement can be made precise by considering partial wave unitarily for the ww, wz, and zz scattering amplitudes (or, equivalently at high energy, the corresponding amplitudes for longitudinally polarized W and Z gauge bosons). In particular the partial wave amplitudes computed in lowest order in λ saturate the unitarily limit for[6]

$$m_H = 1.0 \text{ TeV} \qquad \text{if } \sqrt{s} \gg \mathrm{m_H} \tag{2.6}$$

and for[7]

$$\sqrt{s} = 1.8 \text{ TeV} \qquad \text{if } \mathrm{m_H} \gg \sqrt{s} \tag{2.7}$$

where in these formulas $\sqrt{s}$ denotes the ww center of mass energy. Equation (2.6) does not imply that m_H cannot be larger than 1 TeV any more than (2.7) implies that $\sqrt{s}$ cannot be larger than 1.8 TeV. Rather (2.6) and (2.7) identify domains in which higher order corrections in λ must become appreciable, that is, domains of strong coupling. Within these domains the scalars $H, w^{\pm}, z$ constitute a strongly interacting sector that is coupled weakly to ordinary matter and to the transversely polarized weak gauge bosons.

The Higgs decay width to WW and ZZ is

$$\Gamma_H = \frac{3G_F m_H^3}{16\sqrt{2}\pi} \cong \frac{1}{2}\, TeV \cdot \left(\frac{m_H}{1\mathrm{TeV}}\right)^3 \tag{2.8}$$

and therefore is so large for $m_H \gtrsim 1$ TeV that the term Higgs "particle" becomes a misnomer. Instead we must regard m_H as a parameter of the Lagrangian of a strongly coupled theory. This situation has amusing parallels with QCD. First, we cannot tell what the physical spectrum is just by looking at the Lagrangian but

must work hard to solve the strongly coupled theory. Second the theory defined by (2.1) shares a global symmetry with QCD, namely chiral SU(2) or $\mathrm{SU(2)}_L\times$ $\mathrm{SU(2)}_R$. In QCD and in (2.1) this symmetry is spontaneously broken, resulting in Goldstone bosons that are the pions in QCD or $\vec{\phi} = (w^+, z, w^-)$ in (2.1). Both theories have current algebra, PCAC, and low energy theorems that are valid to all orders in perturbation theory. In fact, eq. (2.1) is precisely the SU(2) sigma model[8] that played a crucial role in the history leading to QCD.

An intriguing possibility is that the standard Higg's sector, eq. (2.1), could play a similar role with respect to some still unknown strong interaction theory of $\mathrm{SU(2)}_L\times$ U(1) symmetry breaking, say $\mathcal{L}_{TSBT}$, the Lagrangian of the True Symmetry Breaking Theory. Schematically

$$\mathcal{L}_{Standard\ Higgs} : \mathcal{L}_{TSBT} = \mathcal{L}_{Sigma} : \mathcal{L}_{QCD} \tag{2.9}$$

That is, provided $\mathcal{L}_{TSBT}$ shares the global symmetries and the pattern of spontaneous symmetry breaking of $\mathcal{L}_{Standard\ Higgs}$, the low energy theorems deduced for the latter are also valid for the former. This is not unlikely, because in the large λ regime of the standard model the global $\mathrm{SU(2)}_L\times$ $\mathrm{SU(2)}_R$ symmetry plays a crucial role in protecting the successful relation $M_W = M_Z \cos\theta_W$ against possibly large $0(\lambda)$ corrections.[9] The program implied by "eq." (2.9) is made precise by a theorem, proved to all orders[7] in the strong coupling λ, that unitary gauge amplitudes involving W_L's and Z_L's (longitudinally polarized W and Z gauge bosons) are equal to corresponding R gauge amplitudes of w's and z's up to corrections of order M_W divided by the W and Z energies. Therefore the low energy theorems apply to W_L and Z_L amplitudes for an intermediate energy domain

$$M_W \ll E \ll m_H \tag{2.10}$$

or perhaps

$$M_W \ll E \ll \Lambda_{TSBT}. \tag{2.10'}$$

The current algebra low energy theorems are obtained in practice most simply by taking the low energy limit of the standard model amplitudes to lowest order in λ.

WW fusion is the key to measuring the W_L and Z_L scattering amplitudes and, more generally, to determining whether $\mathcal{L}_{TSBT}$ has strong interactions. The qualitative point is simple: W's and Z's created by bremstrahlung from the incident quarks are off their mass shells and must rescatter, as in figure (2.2), to appear in the final state. *Calculations discussed in section VI show that WW fusion provides an important increment to the net WW, ZZ, and WZ yields if and only if the* W_LW_L, Z_LZ_L, *and* W_LZ_L *rescattering amplitudes* (the shaded blob in figure (2.2)) *are strong.*

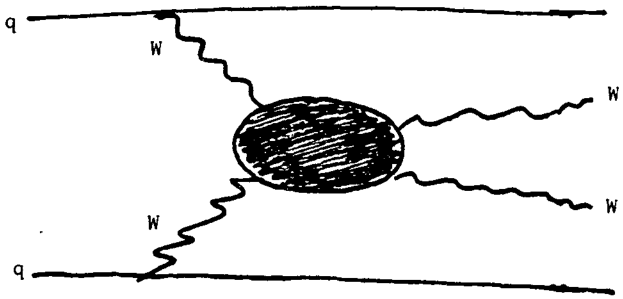

Figure 2.2, General mechanism for production of WW pairs by WW fusion.

If m_H or Λ_{TSBT} are $\gg$ 1 TeV, the domain of validity of the low energy theorems, eq. (2.10), is maximal. In this case the SSC probes the *low* energy structure of $\mathcal{L}_{TSBT}$, just as the earliest beta decay measurements probed the low energy limit of weak interactions, *i.e.*, the original Fermi theory. In fact, the new "Fermi constant" of strong W_LW_L scattering, proportional to $\lambda/m_H^2 \propto 1/v^2$, differs by just a small numerical factor from the old Fermi constant, proportional to $g^2/M_W^2 \propto 1/v^2$! At the SSC we might begin to study a second "Fermi theory", this time of strongly coupled objects, $\lambda \gg g^2$, at a scale well above the W mass, m_H or $\Lambda_{TSBT} \gg M_W$.

III. Schematic Look at High Q^2 Calibration

This section is a very brief look at what I call "calibration studies", that is, measurements at the SSC to test our understanding of the quark and gluon distribution functions. Being assured that the distribution functions are sufficiently under control, we can use them to look for new physics from the symmetry breaking sector (and elsewhere). Of course, one person's calibration is another's physics: the processes considered here are certainly of interest *per se* and could themselves be windows to surprising new physics. Most of the results quoted in this section are taken from the paper of EHLQ.[5]

The two jet cross section at large p_T is a straightforward measurement that probes the strength of quark and gluon distributions at large Q^2. From fig. 91 of EHLQ we see that the cross section is expected to be dominated by gg scattering at $M_{jj} = 1$ TeV, with a tremendous event rate of $0(10^8)$ events/SSC year in a mass interval of width $\Delta M_{jj} = 0.1$ TeV. For $M_{jj} = 3$ TeV there are $0(10^5)$ events in a 0.1 TeV M_{jj} interval, dominated equally by gg and gq scattering. For $M_{jj} > 7$ TeV, of order $0(10^4)$ events are predicted, with gq : gg : qq roughly in the ratio 3 : 1 : 1.

If the prediction for $d\sigma/dM_{jj}$ is verified, it does not verify the relative weights assigned to gg, gq , and qq scattering but only the sum. We want especially to isolate the qq component, since the qq luminosity controls the scale of WW fusion, crucial for the discussion of section VI. Unfortunately there is no kinematical region accessible at the SSC where large p_T qq scattering dominates the two jet cross section, so we will have to rely on the other measurements described here to fix the qq luminosity.

Production of high mass e^+e^- and $\mu^+\mu^-$ pairs (Drell-Yan) measures the $\bar{q}$ content of the proton. For $0.9 < M_{e^+e^-} < 1.1$ TeV and $|y_e| < 2.5$ we expect[3] of order 250 events/SSC year in each channel. For the ratio of up to down quarks, $f_u : f_d$, we consider production of $\mu^\pm\nu$ at large invariant mass. That is, we require the muon to have large p_T and large missing p_T on the opposite side. From a calculation by Gunion[10] we expect 0(500) events/SSC year for $0.45 < p_T(\mu) < 0.55$ TeV and $|y_\mu| < 2.5$. Production of $e^\pm\nu$ can also be used if it turns out to be possible to distinguish electrons from positrons at the relevant energies.

Another process probing f_u/f_d is production of $W^\pm$ + jet at large W^- jet invariant mass. For $0.9 < M_{W_j} < 1.1$ TeV and $\left|y_{W_j}\right| < 1.5$ we expect[3] $0(10^5)$ events/SSC year. The W^+j events are predominantly due to $ug \rightarrow W^+d$ and $\bar{d}g \rightarrow W^+\bar{u}$ while the W^-j events arise chiefly from the charge conjugate reactions. If only muon decays are used to measure the $W^+ : W^-$ ratio, we expect $0(10^3)$ events/SSC year. Z + jet events, produced chiefly from qg and $\bar{q}g$ scattering, occur at comparable rates.

The cross sections to produce two gauge bosons, discussed in the next section, will also be useful for "calibration" purposes. *If* the gauge sector is correctly described by the SU(2) × U(1) theory, then the ZZ, WZ, and W^+W^- cross sections allow us to calibrate various combinations of $\bar{q}q$ luminosities, except for possible new physics from other sources such as the symmetry breaking sector. In the minimal Higgs model,[1] there are no large effects in the WZ channel, while effects in the WW and ZZ channel are restricted to WW and ZZ masses around the mass of the Higgs. For instance, for $m_H = 1$ TeV there is little effect on the WW and ZZ yields for $M_{WW} < 0.5$ TeV. In more general models with strongly interacting symmetry breaking sectors, there may be measureable enhancements of WW, ZZ, *and* WZ for $M_{WW} \gtrsim 0(1 \text{ TeV})$ but little effect on the contribution from $\bar{q}q$ annihilation below $\frac{1}{2}$ TeV. The $W\gamma$ and $Z\gamma$ cross sections, also discussed in the next section, are particularly useful for calibration purposes, since they cannot be significantly affected by symmetry breaking physics.

IV. Gauge Sector

Gauge boson pairs are produced by $\bar{q}q$ annihilation,[11] $\bar{q}q \rightarrow ZZ, W^\pm Z, W^+W^-$. The $\bar{q}q \rightarrow ZZ$ amplitude is dominated by t- and u-channel quark exchanges and is therefore determined by the $\bar{q}qZ$ vertices. The $\bar{q}q \rightarrow W^\pm Z, W^+W^-$ amplitudes get contributions from t-channel quark exchange *and* from s-channel gauge boson exchanges, so that they probe the three gauge boson vertices which are uniquely determined by nonabelian gauge invariance. These measurements complement the study at LEP II of $e^+e^- \rightarrow W^+W^-$. The WW events at LEP II are cleaner than the bulk of ZZ, WZ, and WW events at SSC. However, LEP II is limited to a small range of WW invariant mass near threshold, where the fermion exchange graphs dominate over the s-channel gauge boson pole graphs that are of greatest interest.[12] At SSC the diboson mass spectrum can be measured out to much larger diboson masses. And the greater cleanliness at LEP II is almost offset by the higher yields at the SSC: at LEP II with design luminosity of 10^{32} cm.$^{-2}$ sec.$^{-1}$ we expect of order $1\frac{1}{2} \cdot 10^4$ W^+W^- events/LEP II year at the peak of the cross section. For an SSC year at design luminosity of 10^{33} cm.$^{-2}$ sec.$^{-1}$ we expect[3] of order $4 \cdot 10^5$ W^+Z events with W and Z rapidities $|y| < 2.5$. If we assume that those WZ events are only detected in the cleanest channel $Z \rightarrow e^+e^-, \mu^+\mu^-$ and $W \rightarrow e\nu, \mu\nu, \tau\nu$, we have 6000 detectable events, of order a third to a half of the total LEP II yield at the peak WW cross section.

Although $\bar{q}q \rightarrow ZZ$ does not involve the three boson vertices, it is a useful calibration channel both for theory and experiment. For $|y_Z| < 2.5$ there are[3] $2 \cdot 10^5$

ZZ events/SSC year, with ~ 750 observable in the very clean channel $e^+e^-/\mu^+\mu^- + e^+e^-/\mu^+\mu^-$. While the cross section falls rapidly with increasing diboson mass, there are a few tens of these electron/muon events above 1/2 TeV.

Bigger ZZ yields can be obtained at large ZZ mass in the channel[7,13] $ZZ \to e^+e^-/\mu^+\mu^- + \bar{\nu}\nu$ which occurs with six times the branching ratio of $Z \to e^+e^-/\mu^+\mu^- + e^+e^-/\mu^+\mu^-$. The signature for large ZZ mass is clean: one Z observed in e^+e^- or $\mu^+\mu^-$ at large p_T and comparable missing p_T on the other side (and no jet activity). The spectrum in transverse mass

$$m_T = 2\sqrt{M_Z^2 + p_T^2}$$

is shown by the dashed curve in figure 6.3, taken from ref. 13. Requiring the observed Z to have rapidity $|y| < 2.5$ we expect[13] 0(100) events with $M_T > \frac{1}{2}$ TeV and 0(10) events with $M_T > 1$ TeV (see table 6.2).

Of interest in their own right, the $\bar{q}q \to ZZ, WZ, WW$ processes also provide the background against which signals of the new physics of the symmetry breaking sector can emerge. A standard model Higgs boson below about 600 GeV produces a recognizable bump in the ZZ and WW mass spectra but for $m_H \gtrsim 1$ TeV, Γ_H is so large that we must rely on seeing an increased yield at large diboson mass, as discussed in detail in section 6. New physics of the symmetry breaking sector would also be signaled by deviations from the ratios expected for $\bar{q}q$ annihilation, which are roughly $W^+W^- : W^+Z + W^-Z : ZZ \cong 4 : 2 : 1$. Different ratios would result from a strongly interacting symmetry breaking sector, as discussed in section 6.

The polarization of the W and Z pairs is also a useful quantity, since for heavy Higgs bosons, $m_H \gg 2M_W$, or for any strongly interacting symmetry breaking sector, the additional gauge boson pairs produced by boson-boson fusion are predominantly longitudinal while from $\bar{q}q$ annihilation they are predominantly transverse,

especially at larger diboson mass.[14]

The processes $\bar{q}q \to W\gamma, Z\gamma$ may also be useful. Measurement of $\bar{q}q \to W\gamma$ is another probe of the three gauge boson vertex. Both are particularly valuable for "calibration" purposes because the $W\gamma$ and $Z\gamma$ yields from $\bar{q}q$ annihilation cannot be significantly augmented by boson-boson fusion, regardless of the nature of the symmetry breaking sector. The efficiency to detect large p_T single photons above potential backgrounds is a question requiring detailed investigation. If the efficiency turns out to be substantial, then the observable yields are potentially much larger than in the ZZ, WZ, and WW channels where only a fraction of the decays are reconstructable by presently known techniques (see section 6). The larger yield would mean that we could measure to large $W\gamma$ and $Z\gamma$ invariant mass. For instance, with $|y| < 2.5$ and $W\gamma$ or $Z\gamma$ pair mass above 200 GeV, we expect[3] 40,000 $W^\pm\gamma$ pairs and 200,000 $Z\gamma$ pairs per SSC year. Assuming for the moment photon detection efficiency of order 1, we would have ~ 10,000 $W\gamma$ pairs with the W's detected leptonically and ~ 12,000 $Z\gamma$ pairs with the Z's detected in $e^+e^- + \mu^+\mu^-$.

The three gauge boson processes, $\bar{q}q \to WWW, WWZ, WZZ$, and ZZZ, are also very interesting, since they probe both the three and four gauge boson vertices of the nonabelian theory. A calculation[15] of the total yields (with no rapidity cuts) finds for one SSC year a total of 23,000 events for $m_H = 0.2$ TeV and 11,000 events for $m_H = 0.5$ TeV. These numbers may seem substantial, but after folding in branching ratios to detectable modes and the likely effect of experimentally realistic rapidity cuts, it seems unlikely, given the present state of the art for W and Z detection, that these events can be studied experimentally.

V. B Factory

Although there is little new to say on this subject, I wish to discuss it briefly in order to keep in perspective the variety of physics that can potentially be done at the SSC. Even operating at $\mathcal{L} = 10^{33}$ cm.$^{-2}$ sec.$^{-1}$ the SSC can provide the greatest available event rates for a wide range of sub-TeV physics.[16] The problem is to learn how to do this physics in the SSC environment. In addition to the intrinsic interest of the subject, the study of rare B decays at the SSC is valuable as a prototype of this potentially broad program studying the physics of sub-TeV scales.

B physics is of special interest first of all because the long B lifetime may make it feasible to tag $\bar{B}B$ events with a high resolution vertex detector. The long lifetime also amplifies the branching ratios of rare decays. And there is the potential to study $\bar{B}B$ mixing and CP violation.

The SSC would in fact be a preeminent B factory. With the dial turned down to $\mathcal{L} = 10^{32}$ cm.$^{-2}$ sec.$^{-1}$ the raw $\bar{B}B$ production rate at the SSC surpasses[17] LEP/SLC (assuming $\mathcal{L} = 10^{31}$) by 10^5 and TeV I (assuming $\mathcal{L} = 10^{30}$) by 10^3. At Snowmass '84 the CP violations working group examined the possibility to use a high resolution vertex detector to tag $\bar{B}B$ pairs. Both central[17] and forward[18] detectors were considered, with similar results obtained in both cases. For the central detector the group concluded[17] that $\sim 5 \cdot 10^8$ $\bar{B}B$ pairs might be double-tagged in a year (as throughout this talk, 1 "year" = 10^7 sec.) of running at $\mathcal{L} = 10^{32}$. For comparison a year at LEP/SLC with $\mathcal{L} = 10^{31}$ gives a raw yield (before tagging) of $\sim 4 \cdot 10^5$ $\bar{B}B$ pairs.

The $5{\cdot}10^8$ tagged $\bar{B}B$ pairs implies a sensitivity for rare decays down to a branching ratio of order 10^{-7}. Completely reconstructable decays such as $B \to \mu^+\mu^-$ or μ^+e^- could be detected at this level. Charmless decays that are induced at the one loop level, such as $\phi K_s, K^-\pi^+, Ke^+e^-$, and $K\mu^+\mu^-$, would be detectable if they occur at the predicted levels of order a few times 10^{-5}. The '84 Snowmass study[17] concluded however, that interesting rare decays with one or more neutrinos, such as $\tau^+\tau^-$ or $\tau\bar{\nu}$, can probably not be seen above backgrounds. A more complete discussion of rare B decays is given in the talk of Deshpande[19] at this workshop.

Two methods for studying CP violation were considered by the Snowmass '84 working group.[17] Measurement of the charge asymmetry in like-sign dileptons, expected to be of order 10^{-2} to 10^{-3} in the standard model, would probably be overwhelmed by uncontrollable backgrounds that are present since the pp initial state is charge asymmetric. A more viable suggestion is to look at $B^{\pm}B^0$ where the B^0 is tagged by reconstructing its decay into a CP eigenstate such as ψK_s. CP violation is manifested by a charge asymmetry measured in the semileptonic decay of the associated $B^{\pm}$, expected to be in the range .04 to 0.1 in the standard model. The problem is statistics: for 10^{39} cm.$^{-2}$ integrated luminosity the working group estimated that $\sim$ 1000 events would pass cuts making them potentially detectable, leaving a statistically marginal asymmetry if the effect is as expected in the standard model. However, CP violation could be larger than expected in the standard model, and the event rate could be larger if the branching ratio for $B \to \psi K_s$ is bigger than the assumed value of 10^{-3}.

A third possibility for studying CP violation is discussed in Dashpande's talk.[19] The suggestion is to measure the difference $\Gamma(B^0 \to f) - \Gamma(\bar{B}^0 \to \bar{f})$ that can exist if the $B^0 \to f$ amplitude is the sum of two comparable contributions with different phase. Statistics is also problematic for this proposal.

VI. Detecting the Symmetry Breaking Sector

We begin with the bad news: the inability by any means devised to date to detect the standard model Higgs Boson at the SSC if its mass is below $2M_W$ and it decays principally to $\bar{t}t$. This is the *only* bad news for the standard model at the SSC, and, to keep it in perspective, it occurs in a particularly artificial physics scenario that isolates the Higgs boson below $2M_W$. More naturally we would expect the Higgs boson in that mass range to be accompanied by its relatives, other new quanta of similar mass, as in supersymmetry – just as the heavy Higgs boson, $m_H \sim 1$ TeV, discussed in section 2 would most naturally be accompanied by new, heavy, strongly interacting quanta.[20] Nevertheless, the light standard Higgs boson is a possibility, even if it seems unnatural to us now, and it poses an important challenge to the capability of the SSC.

As discussed in section 2, for 100 GeV < m_H < $2M_W$ of order 10^6 Higgs bosons would be produced per SSC year, but the background from $gg \to \bar{t}t$ is 10^2 larger if we assume optimistically that the $\bar{t}t$ invariant mass can be measured with 5% resolution.[21] To reduce the background consideration was given to producing the Higgs boson in association with the W boson, $pp \to WH + \cdots$, which yields 10^4 events/SSC year, with a background from $pp \to W\bar{t}t + \cdots$ that is manageably smaller than the signal.[22] However, $pp \to W\bar{t}b + \cdots$ occurs at a level two orders of magnitude greater than the signal, requiring formidable b/t separation.[22] In fact, the situation seems even bleaker: a Monte Carlo study of $H \to \bar{t}t$ showed that losses from neutrinos and from soft or backward moving fragments apparently obliterates the peak in the $\bar{t}t$ mass.[23] This is an important conclusion that very much deserves further study. The only presently known way to find the Higgs boson in this mass range is to build an e^+e^- collider with center of mass energy of order 300 GeV and a luminosity $\sim 10^{32}$ cm.$^{-2}$ sec.$^{-1}$.

While we still have more to learn about this case, it is already clear that the SSC deals handsomely with the standard model Higgs boson above the WW threshold. At this workshop Cahn[24] discussed the cases $m_H = 400$ and 600 GeV. For $m_t \lesssim 45$ GeV the dominant Higgs production mechanism is WW (and ZZ) fusion. If $m_t >$ 45 GeV, Cahn's estimates could be increased by the contribution from gluon-gluon fusion, but even for $m_t > M_W$ the modes $H \to WW$ and $H \to ZZ$ are still the dominant decays,[4] with WW : ZZ = 2 : 1. Requiring W and Z rapidities $|y| < 1.5$, the cross section from WW and ZZ fusion is 2.3 pb for $m_H = 400$ GeV and 1.4 pb

for $m_H = 600$ GeV, corresponding respectively to 23,000 and 14000 H's/SSC year. If the Higgs four-momentum can be reconstructed, it helps against the background to take advantage of the characteristic transverse momentum, $p_T(H) \cong 0(M_W)$, of the Higgs boson produced by WW fusion. Requiring $p_T(H) > 60$ GeV and $m_H - \Gamma_H \lesssim m_{WW,ZZ} \lesssim m_H + \Gamma_H$, the 400 GeV Higgs is produced with $\sigma \cong 1.2$ pb, yielding 4000 ZZ pairs/SSC year over a background of 1300 ZZ pairs from $\bar{q}q \to ZZg$, $gq \to ZZq$, and $g\bar{q} \to ZZ\bar{q}$.[24] For $m_H = 600$ GeV the same cuts give a signal of 2400 ZZ pairs over a background of 1300 ZZj events.

For $m_H = 1$ TeV we enter the strong coupling regime discussed in section 2. Perturbative calculations of the production cross section should therefore conservatively be regarded as orders of magnitude estimates. However, it is encouraging that the one loop correction to the Higgs decay width to WW and ZZ[25] is only 10% for $m_H = 1$ TeV. Since the production cross section via WW and ZZ fusion is proportional to $\Gamma(H \to WW, ZZ)$, the one loop correction to the cross section is also only 10% at $m_H = 1$ TeV.

For $m_H = 1$ TeV the production cross section at the SSC in the central region, $|y_{W,Z}| < 1.5$ is $\sim$ .7 pb. To enhance the signal over the $\bar{q}q$ annihilation background, we select the high side of the "peak" $m_{WW,ZZ} > m_H = 1.0$ TeV. Per SSC year we find[7] 1100 $H \to ZZ$ decays compared to 370 $\bar{q}q \to ZZ$ events and 2200 $H \to WW$ decays compared to 1600 $\bar{q}q \to W^+W^-$ events. Of course the "peak" has a width of 500 GeV, so to observe the signal we must rely on the excess of events, the increase in the ZZ : WW ratio, and (if statistics is sufficient) the longitudinal polarization of the W's and Z's from Higgs decay.

For $m_H > 1$ TeV or more generally for any model in which the electroweak symmetry is broken by a strongly interacting sector above 1 TeV, $\Lambda_{TSBT} > 1$ TeV, we use the low energy theorems discussed in section 2 as the basis for an estimate.[7] Recall that the theorems are valid for an intermediate energy region, larger than

M_W, so that w can be identified with W_L, but smaller than m_H orΛ_{TSBT}, so that W_L is "soft". However, for $m_H = 1$ TeV the signal in this intermediate energy region is not observable over the background from $\bar{q}q \to WW, WZ, ZZ$. Experience from hadron physics suggests that we will get a conservative estimate with the right order of magnitude by extrapolating the partial amplitudes from their low energy behavior as given by current algebra until they saturate the unitary limit.[26] For instance, the J = 0 amplitude for $w^+w^- \to zz$ obeys the low energy theorem

$$a_0(ww \to zz) = \frac{s}{16\pi v^2} \tag{6.1}$$

and we extrapolate it by

$$a_0(ww \to zz) = \frac{s}{16\pi v^2}\theta(1 - \frac{s}{16\pi v^2}) + \theta(\frac{s}{16\pi v^2 - 1)}. \tag{6.2}$$

This model[7] predicts 470 Z pairs per SSC year with $m_{ZZ} > 1$ TeV and $|y_Z| <$ 1.5, compared to 370 Z pairs from $\bar{q}q \to ZZ$. We may test the model against actual $\pi\pi$ scattering data from measured $\pi\pi$ partial wave amplitudes in the channels that contribute to the low energy theorem, (J, I) = (0, 0), (1, 1), (0, 2). The momenta are rescaled by the ratio of the electroweak vev, $v = .25$ TeV, to the QCD vev, $F_\pi = 92$ MeV, and the rescaled amplitudes are used to compute the ZZ yield. This is in fact just the prediction for SU(3) technicolor. For $1.0 < m_{ZZ} < 2.0$ TeV there are ~ 360 Z pairs from the model based on the low energy theorem and ~ 235 from the rescaled $\pi\pi$ data. For $m_{ZZ} > 2.0$ TeV the extrapolated theorem predicts 100 events while the rescaled data gives only 14. However, the model based on the low energy theorem neglects higher partial waves, $J \gtrsim 2$, which would make a large contribution above 2 TeV, so 470 Z pairs is a reasonable estimate of the total yield.

$\sqrt{s} =$	10 TeV	20 TeV	30 TeV	40 TeV
ZZ	30/8	250/88	610/250	1100/470
	40	150	260	370
W^+Z	2/8	17/80	42/230	76/440
	60	180	290	390
W^-Z	0.7/3	8/36	22/110	41/230
	30	110	190	280
W^+W^-	61/12	500/120	1200/330	2200/630
	190	660	1200	1600
W^+W^+	3/12	25/110	63/300	110/560
W^-W^-	0.5/2	5/22	17/74	33/150

Table 6.1 Yields of gauge boson pairs in the central region, $|y_{W,Z} < 1.5|$, with diboson mass above 1 TeV. The first two numbers in each entry represent the yield per SSC year for the standard model with $m_H = 1$ TeV and for the extrapolated low energy theorems. The third entry is the yield from $\bar{q}q$ annihilation.

The yields[7] for all two boson channels are summarized in Table 6.1, for colliders with center of mass energies of $\sqrt{s} = 10, 20, 30, 40$ TeV. The three numbers in any entry are respectively the yield (after cuts) for a 1 TeV Higgs, for the model based on the low energy theorem, and for $\bar{q}q$ annihilation. Figure 6.1 shows the $H \to ZZ$ yields for $m_H = 1$ TeV and the four collider energies. Figure 6.2 shows the mass spectrum at 40 TeV for $\bar{q}q \to ZZ$ incremented by $H \to ZZ$ with $m_H = 1$ TeV or by the ZZ yield based on the extrapolated low energy theorem. Notice in Table

6.1 that for m_H or $\Lambda_{TSBT} \gg 1$ TeV we expect large yields in WZ and *like* sign WW pairs, reflecting large a_{11} and a_{02} amplitudes.

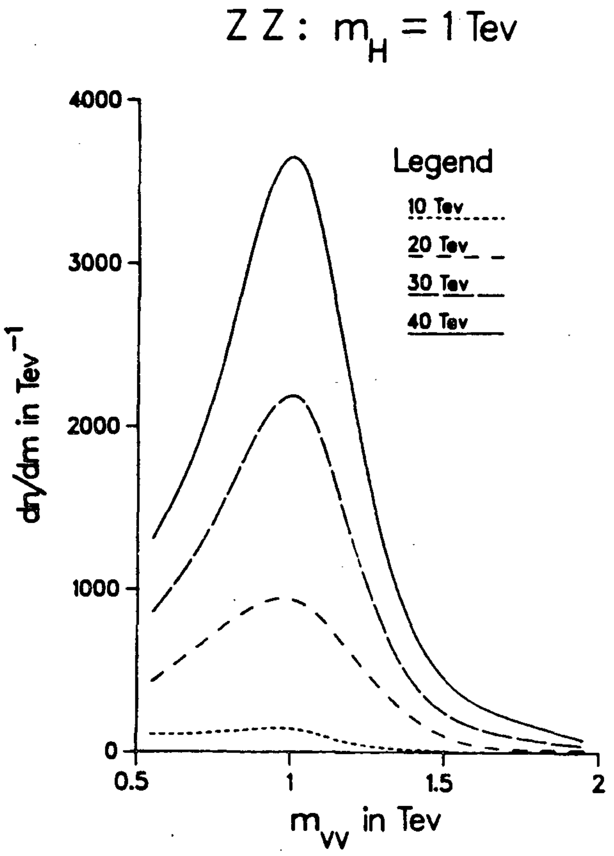

Figure 6.1, Mass distribution of ZZ pairs computed from diboson fusion in the standard model with $m_H = 1$ TeV, assuming a rapidity cut $|y_Z| < 1.5$ and integrated luminosity of 10^{40} cm.$^{-2}$.

Up to now we have not broached the question of detecting the W's and Z's. This is a difficult and critical subject that we are only beginning to give the attention it deserves. Some encouraging new results have been discussed for the first time at this workshop.

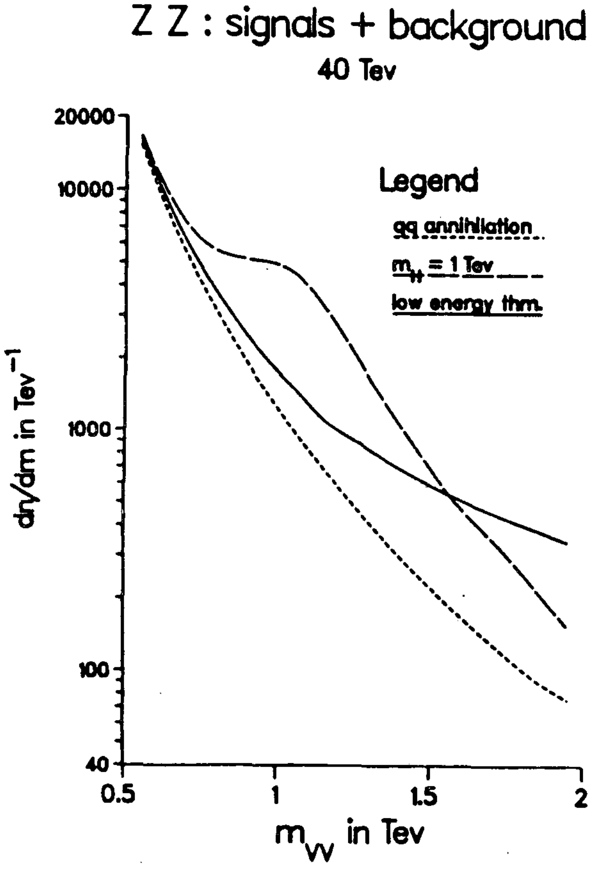

Figure 6.2, ZZ yields for $|y_Z| < 1.5$ and 10^{40} cm.$^{-2}$ integrated luminosity.

The cleanest channel, fully reconstructable and background free, is $H \to ZZ \to$

$(\bar{e}e/\bar{\mu}\mu) + (\bar{e}e/\bar{\mu}\mu)$, with a net branching ratio of $1.2 \cdot 10^{-3}$. The yields after cuts on y_Z and m_{ZZ} are modest : 14 and 9 events per SSC year at $m_H = .4$ and .6 TeV respectively over a $\bar{q}q \to ZZ$ background of 5 events.[24] For $m_H = 1$ TeV, the yield after cuts[7] is 4 above a $\bar{q}q$ background of $1\frac{1}{2}$. Even at the lower masses the statistics is marginal.

However, there is a second ZZ channel[7,13] that is especially clean for the larger m_H values and with six times the branching ratio, $ZZ \to (\bar{e}e/\bar{\mu}\mu) + \bar{\nu}\nu$. The signature is one Z detected at large p_T in $\bar{e}e$ or $\bar{\mu}\mu$, large missing p_T on the opposite side, and no hot jet activity. As for $W \to e\nu$ at the CERN collider, the relevant quantity is the transverse mass, defined as

$$m_T = 2\sqrt{m_Z^2 + p_T^2} \tag{6.3}$$

where p_T is the transverse momentum of the observed Z. One smallish background, which may in fact be the largest, comes from $\bar{q}q \to WZ$ with $W \to \ell\nu$ and the charged lepton ℓ being lost, most likely if $\ell = \tau$. Detailed studies are still needed down to the level of detector simulation, but the prognosis seems good. Figure 6.3 shows the transverse mass spectrum[13] for $m_H = .6, .8, 1.0$ TeV superimposed on the rapidly falling spectrum from $\bar{q}q \to ZZ$. Table 6.2 shows the yields[13] for various m_T ranges with $m_H = .4, .6, .8, 1.0$ TeV compared to $\bar{q}q \to ZZ$. For instance, for $m_H = .6$ TeV we have 86 events in a central detector with $m_T >$ .5 TeV compared to 53 from $\bar{q}q \to ZZ$, representing a $\sim 12\sigma$ effect. For $m_H =$ 1 and $m_T > .9$ signal and background are 43 and 7, a 16σ effect. At this level of statistics it may also be possible to use the polarization of the detected Z to distinguish the $H \to ZZ$ signal from the $\bar{q}q \to ZZ$ backgrounds.[14]

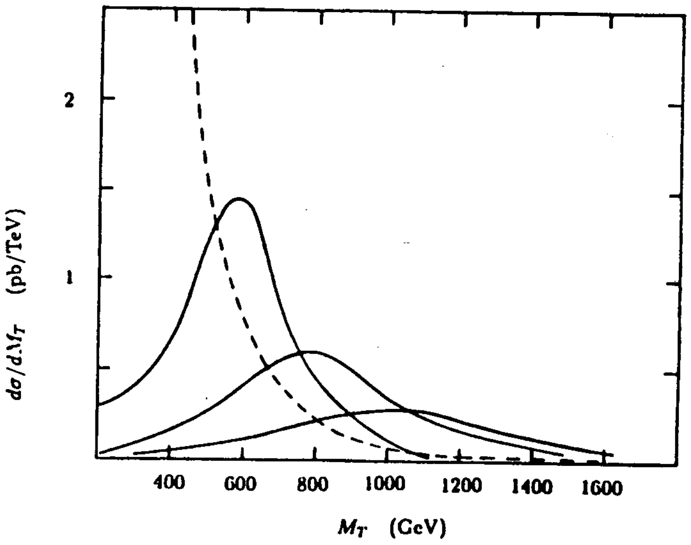

Figure 6.3 The transverse mass distribution of the background and signal for $pp \to ZZX$ for $\sqrt{s} = 40$ TeV. The transverse mass is defined in terms of the transverse momentum of the observed Z. The signals shown correspond to $M_H = 600, 800, 1000$ GeV. The dashed curve corresponds to the background from $\bar{q}q$ annihilation. The observed Z has a rapidity with magnitude less than 1.5.

These results are highly significant statistically for the canonical SSC year, but they become marginal if the luminosity is an order of magnitude smaller than the assumed $\mathcal{L} = 10^{33}$ cm^{-2}sec.$^{-1}$. At the lower luminosity it is also impossible to see the events in which both Z's are detected in e^+e^- or $\mu^+\mu^-$; though statistically marginal even at $\mathcal{L} = 10^{33}$, observation of a few of these very clean events will provide important confirmation of larger signals seen in other decay channels.

	$M_T > 400$	$M_T > 500$	$M_T > 700$	$M_T > 900$
$M_H = 400$	71/112	26/53	4/17	1/7
$M_H = 600$	107/112	86/53	26/17	8/7
$M_H = 800$	76/112	72/53	54/17	30/7
$M_H = 1000$	61/112	59/53	53/17	43/7

Table 6.2 Signal from Higgs bosons over background from $q\bar{q}$ annihilation. The observed channel is ZZ with one Z decaying to e or μ pair and the other to neutrinos. The visible Z has rapidity less than 1.5. The masses are in GeV and the events are for a standard SSC year.

It is also not possible to observe these two signals for a 1 TeV Higgs at a collider of half the energy of the SSC, $\sqrt{s} = 20$ TeV, and $\mathcal{L} = 10^{33}$. For the events $H \to ZZ \to (e^+e^-/\mu^+\mu^-) + \nu\bar{\nu}$ with $m_H = 1$ TeV and $m_T > 0.9$ TeV, the signal is 7 events per year over a background from $\bar{q}q \to ZZ$ of $2\frac{1}{2}$. Going from $\sqrt{s} = 40$ TeV to 20 TeV the signal is degraded by 6 while the background only decreases by 3. For $\sqrt{s} = 20$ TeV with $m_H = 1$ TeV only one event per year is predicted with $m_{ZZ} > 1$ TeV for the fully reconstructed mode $H \to ZZ \to \bar{e}e/\bar{\mu}\mu + \bar{e}e/\bar{\mu}\mu$.

The potentially largest yields are from the hadronic decays, that make up 3/4 of W decays and 2/3 of Z decays in the standard three generation model. However, the backgrounds from QCD jet production are enormous. Even for the mixed case, $WW \to \ell\nu + \bar{q}q$, the QCD background from $qg \to Wqg$ is 100:1 above the signal if we assume 5% resolution for the jet-jet invariant mass.[27]

During this meeting Gunion and Soldate[28] have identified a remarkable set of cuts that reduces this ratio by 0(100) while diminishing the signal by only a factor

of ~ 3. Two cuts are primarily responsible. The first is

$$p_T(j_<) > .125\, m_H$$

$$|p_T(j_<)| + |p_T(j_>)| > .35\, m_H$$

where $j_>$ denotes the jet with the greater p_T. The second cut is on θ^* of the charged lepton, defined as the angle between the lepton direction in the W rest frame and the boost axis from the W rest frame to the lab frame.[14] This cut enhances the longitudinal W's, distributed like $\sin^2\theta^*$ relative to the transverse background W's that are distributed like $1 + \cos^2\theta^*$. The W boost axis or, equivalently, the neutrino four-momentum is determined up to a two-fold ambiguity that is resolved by choosing the solution that minimizes the Higgs energy in the lab.

This procedure can only be applied to the first two generation leptons, $W \to e\nu, \mu\nu$ and the first two generation $\bar{q}q$ pairs, $W \to u\bar{d}, c\bar{s}$, since extra neutrinos make life harder in the third generation. For $m_H = 0.8$ TeV the signal is optimized with an asymmetric cut of $m_{WW} > 0.75$ TeV : the signal is 490 events over a background of 480 that is primarily QCD production of Wjj but also includes $\bar{q}q \to WW$. For $m_H = 0.3$ TeV, a symmetric interval $\Delta m_{WW} = .05\, m_H$ is chosen, controlled by the assumed 5% diboson mass resolution: 2560 events are found in the signal over a background of 4800.

These are encouraging results but they do not tell us what will actually be possible in the laboratory. First, the calculation is based on the effective W approximation[7] to WW scattering, which neglects the $0(M_W)$ transverse momentum of the produced Higgs boson. This approximation does not effect the rates, but it does mean that the reconstruction of the neutrino four-momentum, needed for the θ^* cut, is too optimistic. The neutrino transverse momentum is therefore smeared by an unknown amount of $0(M_W)$, with an effect on the final yields that remains to be studied.

Second, the calculation is purely partonic. Hadronization could change the prognosis, by degrading the assumed resolutions and by causing the two jets to overlap so that they cannot be recognized as two jets. This is a greater problem for the signal than the background, since the $\sin^2\theta^*$ distribution for $W_L \to \bar{q}q$ boosts in the lab to symmetric configurations with small opening angles between the two jets. For the same reason, the angular resolution of the detector is important. Gunion and Soldate found that $\Delta r \equiv \sqrt{\Delta\phi^2 + \Delta y^2} = .15$ is sufficient to implement their cuts, which is compatible with the anticipated resolution for SSC detectors.

There is no apriori reason to expect that these problems will be fatal, but it is equally clear that they must be examined. Short of turning on the switch at the SSC, the best we can do is to use the available Monte Carlos to study the effect of hadronization, perhaps eventually including a detector simulation program. Considering the importance of WW events for SSC physics the effort would be well worthwhile.

Other possible methods of detecting W's and Z's should also be explored. One possibility under investigation[24] is to tag WW fusion events by detecting the forward jets, analagous to tagging $\gamma\gamma$ events in e^+e^- collisions by detecting the forward electrons. The very difficult problem here is to determine whether the associated QCD backgrounds can be understood and mastered. Another very difficult approach would use heavy flavor identification: among the many reasons we would like to be able to identify t's and b's is the possibility of detecting W's decaying to $t\bar{b}$.

VII. Conclusion

First, an important point though often overlooked is that even with the dial turned down to $\mathcal{L} = 10^{32}$ cm.$^{-2}$sec.$^{-1}$ the SSC would be the preeminent factory for physics phenomena involving sub-TeV energy scales.[16] B factory physics, discussed in section 5, is only one example among many. The question is whether we can find ways to do this physics at the SSC. If we can, we enlarge the phase space for major unanticipated discoveries beyond the already assured TeV-scale domain.

On the agenda of forseeable physics, the major outstanding item is the origin of electroweak symmetry breaking. Because of its probably artificial economy, the minimal standard model, with a single Higgs boson, poses the greatest challenge. By comparison nonstandard scenarios, like technicolor or supersymmetry, are a piece of cake. Except for the present (and not necessarily permanent) lacuna at $m_H < 2M_W$, the SSC is guaranteed to probe the mechanism of electroweak symmetry breaking. Even if the new physics scale is too large for the new particles to be produced, e.g., $m_H \gg 1$ TeV, the new physics will be signaled by observable effects in the WW, WZ, and ZZ cross sections. However, as discussed in detail in section 6, in order to detect these effects either for the standard model with $m_H \gtrsim 1/2$ TeV or, more generally, for any strongly interacting symmetry breaking sector at or above 1 TeV, it is essential to meet the SSC design goals $\sqrt{s}, \mathcal{L} = 40, 10^{33}$. Neither $\sqrt{s}, \mathcal{L} = 40, 10^{32}$ nor $\sqrt{s}, \mathcal{L} = 20, 10^{33}$ would be sufficient to detect the effects in the two weak boson cross sections.

Acknowledgements

I wish to thank R. Cahn, J. Gunion, and I. Hinchliffe for many helpful discussions. Work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE–AC03–76SF00098.

References

[1] S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264; A. Salam in "Elementary Particle Physics (ed. N. Svartholm, Almqvist and Wiksells, Stockholm,

---

1968), p. 367.

[2] H. Georgi, S. Glashow, M. Machacek, and D. Nanopolous, Phys. Rev. Lett. 40 (1978) 692.

[3] E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg, Rev. Mod. Phys. 56 (1984) 579.

[4] J. Gunion and M. Soldate, private communication.

[5] R. Cahn and S. Dawson, Phys. Lett. 136B (1984) 196; E 138B (1984) 464.

[6] B. W. Lee, C. Quigg and H. Thacker, Phys. Rev. D16 (1977) 1519.

[7] M. Chanowitz and M. K. Gaillard, Nuc. Phys. B261 (1985) 379; Phy. Lett. 142B (1984) 85.

[8] M. Gell Mann and M. Levy, Il Nuovo Cim. XVI (1960) 53.

[9] P. Sikivie, L. Susskind, M. Voloshin, and V. Zakharov, Nuc. Phys. B173 (1980) 189.

[10] J. Gunion in "Design and Utilization of the SSC" eds. R. Donaldson and J. Morfin, Snowmass (1984) p. 147.

[11] K. Gaemers and G. Gounaris, Z. Phys. C1 (1979) 259; R. Brown and K. Mikaelian, Phys. Rev. D19 (1979) 922; R. Brown, D. Sahdev and K. Mikaelian, Phys. Rev. D20 (1979) 1164.

[12] W. Alles, C. Boyer, A. Buras, Nuc. Phys. B119 (1979) 125.

[13] R. Cahn and M. Chanowitz, Phys. Rev. Lett. 56 (1986) 1327.

[14] M. Duncan, G. Kane and W. Repko, UMTH 85-18, 1985.

[15] M. Golden and S. Sharpe, Nuc. Phys. B261 (1985) 217.

[16] J. D. Bjorken, "An Alternating Gradient spectrometer", 9/24/85, unpublished; Fermilab-CONF-86/22, submitted to Proc. Workshop on Triggering, Fermilab Nov. 11-14, 1985.

[17] J. Cronin et al., Proc. Snowmass 1984, p. 161.

[18] J. Cronin, Proc. Snowmass 1984, p. 170.

[19] N. Deshpande, [illegible]

[20] The only dynamical model of which I am aware with an isolated heavy Higgs boson requires tuning of a parameter – see D. B. Kaplan and H. Georgi, Phys. Lett. 136B (1984) 183; D. B. Kaplan, H. Georgi, and S. Dimopolous, Phys. Lett. 136B (1984) 187.

[21] E. Eichten, I. Hinchliffe, K. Lane, C. Quigg, Proc. Snowmass 1984, p. 99.

[22] J. Gunion, P. Kalyniak, M. Soldate, and P. Galison, Proc. Snowmass 1984, p. 168.

[23] B. Cox and F. Gilman, Proc. Snowmass 1984, p. 87.

[24] R. Cahn, contribution to this workshop.

[25] J. Fleischer and F. Jegerlehner, Phys. Rev. D23 (1981) 2001.

[26] For $|a_{00}(\pi\pi \to \pi\pi)|^2$, extrapolation of the low energy theorem describes the data well for $m_{\pi\pi} < 450$ MeV and then provides a conservative underestimate of the data up to 1 GeV — see fig. 4 of N. Biswas et al., Phys. Rev. Lett. 47 (1981) 1378.

[27] W. Stirling, R. Kleiss, and S. Ellis, Phys. Lett. 163B (1985) 261; J. Gunion, A. Kunszt, and M. Soldate, Phys. Lett. 163B (1985) 389.

[28] J. Gunion and M. Soldate, contribution to this workshop, manuscript in preparation.