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Nonregular fractional factorial designs such as Plackett-Burman designs are widely used in industrial experiments for run size economy and flexibility. A novel criterion, called moment aberration projection, is proposed to rank and classify nonregular designs. It measures the goodness of a design through moments of the number of coincidences between the rows of its projection designs. The new criterion is applied to rank and classify designs of 16, 20 and 27 runs. Examples are given to illustrate that the ranking of designs is supported by other design criteria.

KEY WORDS: Generalized minimum aberration; Hadamard matrix; Hamming distance; Orthogonal array; Plackett-Burman design; Projection property.

1 Introduction

Consider a door panel stamping experiment of investigating the effects of the following seven factors on the formability of a panel: (A) concentration of lubricant, (B) panel thickness, (C) force on the outer portion of the door, (D) force on the inner portion of the door, (E) punch speed, (F) thickness of lubrication, and (G) manufacturers of lubricant. Each factor has two levels. The engineer plans to conduct a 20-run experiment due to budget limitation. He could use a 20-run orthogonal design by choosing seven columns from the 20-run Plackett-Burman design. Since there are many choices, he faces a practical question: How to select a design with good design properties?

The selection of fractional factorial designs, such as the problem described in the previous paragraph, has always been an important issue in the area of experimental design. Two-level and three-level fractional factorial designs are frequently used in various scientific investigations. A *regular* design is determined by some defining relations and hence has a simple aliasing structure in that any two effects are either orthogonal or fully aliased. The run size is always a power of 2 or 3, and thus the "gaps" are getting wider as the power increases. For a regular fractional factorial design, Box and Hunter (1961) first introduced the concept of *resolution*. To further differentiate designs with the same resolution, Fries and Hunter (1980) proposed a refined criterion called *minimum aberration*. There are many additional works on minimum aberration designs; see Wu and Hamada (2000) for further references.

Nonregular factorials are commonly selected from Plackett-Burman designs (Plackett and Burman 1946) or Hadamard matrices in general. A Hadamard matrix of order N is an $N \times N$ matrix with the elements ± 1 whose columns (and rows) are orthogonal to each other. One can always "normalize" a Hadamard matrix so that its first column consists of all 1's. Removing the first column, one obtains a saturated two-level *orthogonal array* with N runs and $N-1$ columns, which is a nonregular design if N is not a power of 2. Other widely used nonregular designs are three-level and mixed-level orthogonal arrays, as described in Dey and Mukerjee (1999), Hedayat, Sloane, and Stufken (1999), and Wu and Hamada (2000). Nonregular designs are useful for factor screening and they fill the gaps between regular designs in terms of various run sizes. Unlike regular designs, nonregular designs exhibit some complex aliasing structure, i.e., there exist two effects that are neither orthogonal nor fully aliased. One can argue that this property is not a drawback. For regular designs, once the effects are confounded they cannot be separated. For nonregular designs, if the effects are partially confounded, they may be jointly estimable. Hamada and Wu (1992) proposed an analysis strategy to turn the liability of complex aliasing structure into the virtue of model estimability. See Wu and Hamada (2000, chap. 8) for further references.

Nonregular factorial designs had not received enough attention due to their complex aliasing structure until recently. In the last decade, a number of authors have studied the projection properties of two-level nonregular designs; see, e.g., Lin and Draper (1992), Wang and Wu (1995), Cheng (1995, 1998), and Box and Tyssedal (1996). More recently, Deng and Tang (1999, 2002) proposed generalized resolution and *generalized minimum aberration* for ranking nonregular two-level designs in a systematic way. Their work shows a promising direction in the study of nonregular designs. However, their approach works only for two-level designs.

In this paper, we propose a novel criterion, called *moment aberration projection*, to rank and classify nonregular designs (including multi-level designs). The key innovation is to investigate the relationship between the runs (i.e., rows), instead of the relationship between the factors (i.e., columns). For each design, we consider its projection designs and compute the power moments of the number of coincidences between the rows of its projection designs. The new criterion then assesses the goodness of designs by comparing the distribution of the power moments.

The number of non-coincidences is called *Hamming distance* which has been studied extensively in the area of coding theory. Recently, it is becoming popular in the area of design of experiment. Clark and Dean (2001) and Ma, Fang, and Lin (2001) studied the isomorphism of fractional factorial designs by means of the Hamming distances between the rows of their projection designs. Two designs are *isomorphic* (or *equivalent*) if one can be obtained from the other by permuting the rows, the columns and the levels of each column. Clark and Dean (2001) provided a necessary and sufficient condition for equivalence of two designs and an algorithm to verify the equivalence of two-level designs. Using a simple function of Hamming distances, Ma et al. (2001) proposed an efficient algorithm to verify nonequivalence of two-level designs. However, their methods can not be used to rank designs. While it is important to know whether two designs are nonequivalent, it is more important to rank them. Our primary goal here is ranking designs and classification is secondary.

Section 2 describes the moment aberration projection criterion and connections with other familiar criteria. Section 3 applies moment aberration projection to search for good designs for 16, 20, and 27 runs. For 16- and 20-run designs, the rankings by moment aberration projection and generalized minimum aberration are fairly consistent, but not identical. Nevertheless, the former has a better classification power than the latter. In particular, moment aberration projection can completely classify all 16-run orthogonal arrays. For 27 runs, many nonregular designs are found to have better projection properties than regular designs. Section 4 studies the design selection for the door panel stamping experiment. Other criteria such as estimation capacity and projection properties are used to judge the designs chosen by moment aberration projection. Concluding remarks are given in Section 5.

2 Moment Aberration Projection

2.1 Power Moments and Moment Aberration

A design of N runs and m factors is represented by an $N \times m$ matrix where each row corresponds to a run (i.e., treatment) and each column a factor. A design has s levels if each column takes on

s different values. For an $N \times m$ design $\mathbf{d}$ and a positive integer t, define the tth power moment to be

$$K_t(\mathbf{d}) = \sum_{1 \le i < j \le N} [\delta_{ij}(\mathbf{d})]^t \,, \tag{1}$$

where $\delta_{ij}(\mathbf{d})$ is the number of coincidences between the ith and jth rows.

The power moments measure the similarity among runs (i.e., rows). The first and second power moments measure the average and variance of the similarity among runs. Minimizing the power moments makes runs be as dissimilar as possible. Therefore, good designs should have small power moments. This leads to the *moment aberration* criterion (Xu 2003) that is to sequentially minimize the power moments K_1, K_2, ...

While the computation of the power moments involves the number of coincidences between rows, the power moments also measure the orthogonality among columns. This important observation was first pointed out by Xu (2003). Specifically, for an $N \times m$ design $\mathbf{d}$ with s levels, he showed that

$$K_1(\mathbf{d}) \ge Nm(N-s)/(2s), \tag{2}$$

with equality if and only if every column of $\mathbf{d}$ is balanced; and

$$K_2(\mathbf{d}) \ge Nm(N(m+s-1)-ms^2)/(2s^2), \tag{3}$$

with equality if and only if $\mathbf{d}$ is an orthogonal array (OA). Note that K_t defined in (1) is $N(N-1)/2$ times that in Xu (2003). Based on this observation, Xu (2002) developed an efficient algorithm to construct orthogonal arrays and nearly-orthogonal arrays with small runs.

Example 1. Consider two 4×3 designs given below:

$$\mathbf{d}_1 = \begin{pmatrix} + & + & + \\ + & - & - \\ - & + & - \\ - & - & + \end{pmatrix} \text{ and } \mathbf{d}_2 = \begin{pmatrix} + & + & + \\ - & + & + \\ - & - & + \\ - & - & - \end{pmatrix}.$$

The first design $\mathbf{d}_1$ is a 2^{3-1} regular fractional factorial design and the second design $\mathbf{d}_2$ is a one-factor-at-a-time design. Their coincidence matrices (δ_{ij}) are

$$\begin{pmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{pmatrix} \text{ and } \begin{pmatrix} 3 & 2 & 1 & 0 \\ 2 & 3 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 3 \end{pmatrix}.$$

The tth power moment K_t is the sum of the tth power of the elements above the diagonal. It is easy to verify that $K_t(\mathbf{d}_1) = 6$ for all $t \geq 1$ and $K_1(\mathbf{d}_2) = 8$, $K_2(\mathbf{d}_2) = 14$, $K_3(\mathbf{d}_2) = 26$, etc. It is evident that $K_1(\mathbf{d}_1) < K_2(\mathbf{d}_2)$ for all $t \geq 1$. Therefore, $\mathbf{d}_1$ has less moment aberration than $\mathbf{d}_2$ and is preferred. This agrees with the well known fact that fractional factorial designs are superior to one-factor-at-a-time designs. Note that the lower bounds in (2) and (3) are archived for $\mathbf{d}_1$ since it is an OA.

It is important to note the connection between designs in statistics and codes in coding theory. For an introduction and applications on OAs, see Hedayat et al. (1999, chap. 4). The *Hamming distance* between a pair of rows is the number of places where they differ. Let $h_{ij}(\mathbf{d})$ be the Hamming distance between the ith and jth rows. Evidently, $h_{ij}(\mathbf{d}) = m - \delta_{ij}(\mathbf{d})$. Let $B_i(\mathbf{d})$ be the number of pairs of rows of $\mathbf{d}$ such that their Hamming distance is equal to i. The vector $(B_0(\mathbf{d}), B_1(\mathbf{d}), \ldots, B_m(\mathbf{d}))$ is known as the distance distribution in coding theory. It is evident that

$$K_t(\mathbf{d}) = \sum_{i=0}^{m} (m - i)^t B_i(\mathbf{d}). \qquad (4)$$

This equation implies that the first m power moments uniquely determine the rest; therefore, at most m comparisons are necessary in the moment aberration criterion.

By applying MacWilliams identities and Pless power moment identities, two fundamental results in coding theory, Xu (2003) showed that moment aberration is equivalent to minimum aberration (Fries and Hunter 1980) for regular designs and generalized minimum aberration (Xu and Wu 2001) for nonregular designs.

The moment aberration criterion can also be applied to supersaturated designs. A design is supersaturated if it does not have enough degrees of freedom to estimate all main effects. A number of criteria have been proposed by many authors. Xu (2003) showed that the minimization of K_2 is equivalent to the minimization of $E(s^2)$ (Booth and Cox 1962) for two-level designs and the minimization of ave χ^2 (Yamada and Lin 1999) for three-level designs.

2.2 Moment Aberration Projection

For an $N \times m$ design $\mathbf{d}$, consider its projections designs. For each projection design, we can compute the power moments as in (1) for any t. Instead of using all the power moments, we propose to simply use one moment for each projection because it is cheaper to compute and compare a number than a vector. Specifically, we use the pth moment K_p for a p-factor projection, i.e., let $t = p$ in

(1). For given p ($1 \leq p \leq m$), there are $\binom{m}{p}$ p-factor projections. The frequency distribution of K_p-values of these projections is called the *p-dimensional K-value distribution* and denoted by $F_p(\mathbf{d})$.

Let $F = (f(x_1), \cdots, f(x_c))$ and $G = (g(x_1), \cdots, g(x_c))$ be two frequency distributions taking c possible values (in decreasing order) on $x_1 > \cdots > x_c$. We would sequentially compare the frequency of the components and prefer one that minimizes the frequency of the largest value. For convenience, we write $F < G$ if there exists an index i ($1 \leq i \leq c$) such that $f(x_i) < g(x_i)$ and $f(x_j) = g(x_j)$ for all $j < i$.

For two $N \times m$ designs $\mathbf{d}_1$ and $\mathbf{d}_2$, suppose p is the smallest integer such that the p-dimensional K-value distributions are different, i.e., $F_p(\mathbf{d}_1) \neq F_p(\mathbf{d}_2)$. We call that $\mathbf{d}_1$ has less *moment aberration projection* (MAP) than $\mathbf{d}_2$ if $F_p(\mathbf{d}_1) < F_p(\mathbf{d}_2)$.

Example 2. (Continued from Example 1) For each design, there are three one-factor projections, three two-factor projections and one three-factor projection. The K-value distributions are as follows:

Design	$F_1 : (3, 2)$	$F_2 : (6, 5, 4)$	$F_3 : (26, 6)$
$\mathbf{d}_1$	(0,3)	(0,0,3)	(0,1)
$\mathbf{d}_2$	(2,1)	(1,2,0)	(1,0)

For $\mathbf{d}_1$, all three one-factor projections have $K_1 = 2$, all three two-factor projections have $K_2 = 4$, and one three-factor projection (i.e., $\mathbf{d}_1$ itself) has $K_3 = 6$. For $\mathbf{d}_2$, two one-factor projections have $K_1 = 3$ and one one-factor projection has $K_1 = 2$; one two-factor projection has $K_2 = 6$ and two two-factor projections have $K_2 = 5$; and one three-factor projection (i.e., $\mathbf{d}_2$ itself) has $K_3 = 26$. It is evident that $F_p(\mathbf{d}_1) < F_p(\mathbf{d}_2)$ for $p = 1, 2, 3$. Therefore, according to MAP, $\mathbf{d}_1$ is better than $\mathbf{d}_2$.

Example 3. Consider the 12-run Plackett-Burman design given in Table 1. It is constructed by cyclically shifting the first row $(+, +, -, +, +, +, -, -, -, +, -)$ to the right 10 times and adding a row of $-$'s (Plackett and Burman 1946). According to Lin and Draper (1992), there are two nonisomorphic 12×5 projection designs: design 5.1 and design 5.2. For example, columns 1–4, and 10 form design 5.1 and columns 1–5 form design 5.2. The former has two repeated runs while the latter has two mirror image runs. From (2) and (3), any one-factor projection must have $K_1 = 30$ and any two-factor projection must have $K_2 = 84$ since both designs are OAs. The 3-, 4- and 5-dimensional K-value distributions are as follows:

Design	$F_3 : 330$	$F_4 : 1728$	$F_5 : (11070, 10950)$
5.1	10	5	(0,1)
5.2	10	5	(1,0)

We see that for both designs, all ten three-factor projections have $K_3 = 330$, and all five four-factor projections have $K_4 = 1728$. This is because all three-factor projections are equivalent and so are all four-factor projections (Lin and Draper 1992). Therefore, they are not distinguishable when projected onto 3 and 4 factors. Nevertheless, they are distinguishable when projected onto 5 factors because $K_5 = 10950$ for design 5.1 and $K_5 = 11070$ for design 5.2. Therefore, according to MAP, design 5.1 is better than design 5.2, which is consistent with the conclusion by Wang and Wu (1995) and Deng and Tang (1999).

Table 1: The 12-Run Plackett-Burman Design

Run	1	2	3	4	5	6	7	8	9	10	11
1	+	+	−	+	+	+	−	−	−	+	−
2	−	+	+	−	+	+	+	−	−	−	+
3	+	−	+	+	−	+	+	+	−	−	−
4	−	+	−	+	+	−	+	+	+	−	−
5	−	−	+	−	+	+	−	+	+	+	−
6	−	−	−	+	−	+	+	−	+	+	+
7	+	−	−	−	+	−	+	+	−	+	+
8	+	+	−	−	−	+	−	+	+	−	+
9	+	+	+	−	−	−	+	−	+	+	−
10	−	+	+	+	−	−	−	+	−	+	+
11	+	−	+	+	+	−	−	−	+	−	+
12	−	−	−	−	−	−	−	−	−	−	−

The MAP is closely related to the *generalized minimum aberration* (GMA) criterion proposed by Deng and Tang (1999, 2002). For an $N \times m$ design $\mathbf{d} = (x_{ij})$ with entries -1 or $+1$, define

$$J_m(\mathbf{d}) = \left| \sum_{i=1}^{N} x_{i1} x_{x2} \cdots x_{im} \right| . \tag{5}$$

The quantity $J_m(\mathbf{d})$ measures the correlations among the columns for two-level designs. The p-dimensional J-value distribution, called *confounding frequency vector* by Deng and Tang, is the frequency distribution of J_p-values of the p-factor projections. The GMA criterion is to sequentially

Table 2: An 18-Run Orthogonal Array

Run	1	2	3	4	5	6	7
1	0	0	0	0	0	0	0
2	0	1	1	1	1	1	1
3	0	2	2	2	2	2	2
4	1	0	0	1	1	2	2
5	1	1	1	2	2	0	0
6	1	2	2	0	0	1	1
7	2	0	1	0	2	1	2
8	2	1	2	1	0	2	0
9	2	2	0	2	1	0	1
10	0	0	2	2	1	1	0
11	0	1	0	0	2	2	1
12	0	2	1	1	0	0	2
13	1	0	1	2	0	2	1
14	1	1	2	0	1	0	2
15	1	2	0	1	2	1	0
16	2	0	2	1	2	0	1
17	2	1	0	2	0	1	2
18	2	2	1	0	1	2	0

minimize the confounding frequency vectors as MAP does. Since both GMA and MAP consider first lower dimensional projections and then proceed to higher dimensional projections, we expect that they would produce consistent results as demonstrated in our study for designs of 12, 16, and 20 runs.

One important advantage of MAP over GMA is that the former works for both two-level and multi-level designs while the latter works only for two-level designs. Here is an example for three-level designs.

Example 4. Consider the commonly used $OA(18, 3^7)$ given in Table 2. Wang and Wu (1995) showed that there are four nonisomorphic four-factor projections. Let design 4.1 consist of columns 2–5, design 4.2 columns 1–3, and 6, design 4.3 columns 1–4, and design 4.4 columns 1, 2, 5, and 7, respectively. From (2) and (3), any one-factor projection must have $K_1 = 45$ and any two-factor projection must have $K_2 = 108$ since all designs are OAs. The 3- and 4-dimensional K-value distributions are as follows:

Design	$F_3 : (351, 315, 297)$	$F_4 : (1260, 1044, 936)$
4.1	(0,0,4)	(0,0,1)
4.2	(0,1,3)	(0,1,0)
4.3	(1,0,3)	(1,0,0)
4.4	(1,3,0)	(1,0,0)

When projected onto 3 factors, all four projections of design 4.1 have $K_3 = 297$; three projections of design 4.2 have $K_3 = 297$ and one projection has $K_3 = 315$; three projections of design 4.3 have $K_3 = 297$ and one projection has $K_3 = 351$; and three projections of design 4.4 have $K_3 = 315$ and one projection has $K_3 = 351$. Therefore, according to MAP, design 4.1 is better than design 4.2, which in turn is better than design 4.3 and design 4.4. The ranking is consistent with that of Wang and Wu (1995). It is important to note that the moment aberration criterion is not able to distinguish between design 4.3 and design 4.4. In contrast, according to MAP, design 4.3 is better than design 4.4, which is supported by Wang and Wu (1995) in terms of their hidden projection properties.

As the previous example shows, the frequency distribution $F_p(\mathbf{d})$ provides a quick method to check whether two designs are nonisomorphic. A necessary condition for isomorphism of two designs is that they have the same p-dimensional K-value distribution $F_p(\mathbf{d})$ for all dimension p ($1 \leq p \leq m$). Whenever two designs have different K-value distributions, we can declare that they are not isomorphic. The same argument works for GMA based on the J-values defined in (5). However, MAP has a much better classification power than GMA (see the next section for examples). The reason is that K_t defined in (1) takes on much more values than J_m defined in (5). Deng and Tang (2002) showed that $J_m(\mathbf{d})$ only takes on a few values between 0 to N.

Although MAP provides only a necessary condition, it can be used as a preliminary step to identify nonisomorphic designs. As mentioned in the introduction, Clark and Dean's (2001) algorithm can identify isomorphism of two designs. In the worst case, their algorithm requires $m(m!)^2$ comparisons. On the other hand, MAP requires $N^2 m 2^m$ comparisons since there are $\binom{m}{p}$ p-factor projections and each K_p requires $N^2 p$ operations. The two numbers are quite different, especially when m is large. For example, when $N = 16, m = 15$ and $s = 2$, $m(m!)^2 \approx 2.6 \times 10^{25}$ and $N^2 m 2^m \approx 1.3 \times 10^8$; when $N = 32, m = 31$ and $s = 2$, $m(m!)^2 \approx 2.1 \times 10^{69}$ and $N^2 m 2^m \approx 7.8 \times 10^{13}$. In this regard, MAP is efficient and useful in identifying nonisomorphic designs.

Table 3: The Number of 16-Run Nonisomorphic Designs

Method	3	4	5	6	7	8	9	10	11	12	13	14	15
MAP	3	5	11	27	55	80	87	78	58	36	18	10	5
GMA	3	5	11	26	53	74	78	75	56	32	18	10	4
Complete Search	3	5	11	27	55	80	87	78	58	36	18	10	5

When restricted to regular designs, GMA and moment aberration are equivalent to minimum aberration. It is interesting to note that MAP is not equivalent to minimum aberration. For example, consider two regular 2^{12-3} designs given in Draper and Mitchell (1968), labeled as designs 3.4 and 3.5 in their Table 1. These two designs are not isomorphic but share the same wordlength pattern; therefore, they are equivalent under the minimum aberration criterion. On the other hand, it is straightforward to verify that their 9-, 10-, and 11-dimensional K-value distributions are different; therefore, they are not equivalent under MAP.

3 Ranking and Classification with Moment Aberration Projection

3.1 Designs of 16 Runs

According to Hall (1961), there are precisely five nonisomorphic Hadamard matrices, labeled as I, II, III, IV, and V, respectively. In particular, type I is a regular design and is equivalent to the 16-run Plackett-Burman design, whose cyclic generator is $(+,+,+,+,-,+,-,+,+,-,-,+,-,-,-)$. For convenience, we use the Plackett-Burman design for type I. Deng and Tang (2002) used GMA to rank and classify designs from the five Hadamard designs. Here we use MAP to reexamine these designs.

For each m, $3 \leq m \leq 15$, we use MAP to rank and classify all nonequivalent designs from the five Hadamard matrices by searching over all m-factor projections. Table 7 lists a few top designs ranked by MAP, their 3- and 4-dimensional K-value distributions, GMA rankings from Deng and Tang (2002), types, and the corresponding columns. As we can see from Table 7, the rankings by MAP and GMA are quite consistent. Both criteria identify the same top designs except for two cases where MAP further distinguishes between the top designs ranked by GMA, see designs 16.12.1 and 16.12.2, and 16.15.1 and 16.15.2.

As a by-product, we investigate the equivalence of projection designs from the five Hadamard matrices. Table 3 shows the number of nonisomorphic projection designs identified by MAP, GMA (Deng and Tang 2002), and a complete search of all possible permutations (Sun 1993). The number of nonisomorphic designs found by MAP matches exactly with the complete search while GMA found less designs for 6–12, and 15 columns. The conclusion is that MAP gives a complete classification for 16-run designs without a complete search of all possible permutations. In fact, only projection dimension up to 7 is needed to classify all 16-run designs.

3.2 Designs of 20 Runs

Hall (1965) showed that there are three nonisomorphic Hadamard matrices, labeled as Q, P, and N, respectively. Listings of these designs can be found in Hall (1965) as well as in Deng and Tang (2002). As noted by Wang and Wu (1995), type Q is equivalent to the 20-run Plackett-Burman design, whose cyclic generator is $(+, +, -, -, +, +, +, +, -, +, -, +, -, -, -, -, +, +, -)$. For convenience, we use the Plackett-Burman design for type Q.

As before, for each m, $3 \leq m \leq 19$, we use MAP to rank and classify all nonequivalent designs from the three Hadamard matrices by searching over all m-factor projections. Table 8 lists a few top and bottom designs ranked by MAP, their 3- and 4-dimensional K-value distributions, GMA rankings from Deng and Tang (2002), types, and the corresponding columns. As we can see from Table 8, the rankings by MAP and GMA are still consistent, but the degree of consistency is less than the 16-run case. For each m the best design 16.m.1 ranked by MAP is also ranked as the best by GMA. However, there are several cases where GMA fails to separate one or more of its top designs with 6–8, and 12–15 columns. In particular, MAP ranks three top GMA designs as 20.7.1, 20.7.2, and 20.7.8. In the next section we will show that the MAP rankings are supported by other criteria.

Observe that the bottom design in Table 8 is from type P for 5–19 columns. Since our search goes through Q first, P next and N last, this means that type Q does not contain the worst projection. This finding partially explains the superiority of type Q, a result also supported by Wang and Wu (1995).

We also study the isomorphism of projection designs from the three Hadamard matrices. Previously, Lin and Draper (1992) studied the isomorphism of projection up to 5 dimensions, and Wang and Wu (1995) extended to 6 dimensions. Deng and Tang (2002) used GMA to rank and classify all projection designs. Table 4 shows the numbers of nonisomorphic designs identified by MAP

Table 4: The Number of 20-Run Nonisomorphic Designs

Method	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
MAP	2	3	10	59	388	1265	2089	2282	1899	1300	730	328	124	40	11	6	3
GMA	2	3	10	34	51	80	125	125	80	51	34	10	3	2	1	1	1

and GMA. It is evident that MAP identifies much more designs than GMA as reported in Deng and Tang (2002), especially when the number of columns is large. For example, for 6 columns, GMA identifies only 34 designs whereas MAP identifies 59 designs, which is exactly the number of nonisomorphic designs reported by Wang and Wu (1995). We should point out that Deng and Tang (2002) considered only projection dimensions up to five (or MA-5 in their notation). Nevertheless, the number of nonisomorphic designs found by GMA would be still much less than MAP even if all dimensions are used.

3.3 Designs of 27 Runs

Commonly used 27-run designs are regular designs and minimum aberration is often used to select the best design for a given number of columns (see Wu and Hamada (2000) for details). The 27-run Plackett-Burman design, generated by cyclically permuting the *column* vector $(0,0,1,0,1,2,1,1,2,0,1,1,1,0,0,2,0,2,1,2,2,1,0,2,2,2)$ 12 times and adding a row of zeros, is equivalent to a saturated regular 3^{13-10} design. Lam and Tonchev (1996) showed that there are 68 nonisomorphic saturated 27-run OAs with 3 levels and 13 columns, denoted as $OA(27,3^{13})$. Among them, only one design is regular and other 67 designs are nonregular.

Here we use MAP to rank and classify all projection designs from the 68 saturated arrays. For simplicity, we label the 68 $OA(27,3^{13})$ arrays as types 1–68 according to their MAP rankings. MAP can separate all but one of the 68 arrays: Two nonisomorphic arrays are ranked as the 24th by MAP. Table 9 lists the top designs and all regular designs identified by MAP. A design is regular if it is from type 68. Table 9 shows clearly that all regular designs (except for 27.3.1 and 27.4.1) are ranked at the bottom. In other words, according to MAP, nonregular designs are better than regular designs. This agrees with the observation from Cheng and Wu (2001), who studied the projection properties of three-level designs for a second-order model and concluded that nonregular designs have better projection properties than regular designs.

Table 5: The Number of 27-Run Nonisomorphic Designs Identified by MAP

Type	3	4	5	6	7	8	9	10	11	12	13
Total	8	139	1833	6230	10300	10273	6977	3305	1176	331	67
Regular	2	3	3	4	4	3	3	2	1	1	1

In Table 9, most of the top designs can be found from three designs: types 1, 5, and 68. Type 68 is equivalent to a regular design and the Plackett-Burman design is used here for convenience. Table 10 lists the other two designs for easy reference.

Table 5 shows the the number of nonisomorphic designs identified by MAP and the number of regular designs. Chen, Sun, and Wu (1993) presented a complete catalogue of 27-run regular designs with an exhaustive search. It is interesting to note that their catalogue is not complete in the sense that they missed a regular design, namely 27.4.139.

4 Connection With Other Design Criteria

4.1 Estimation Capacity and Hidden Projection Properties

Cheng, Steinberg, and Sun (1999) first studied the model robustness of regular minimum aberration designs in terms of estimation capacity. Following their approach, consider models containing all main effects and f two-factor interactions (2fi's for short). Since we do not know in advance which f 2fi's are significant, we consider all possible combinations. Let E_f be the number of estimable models and D_f be the average D efficiency of all models, where the D efficiency is calculated as in Wang and Wu (1995). One would prefer designs with high estimation capacity E_f and large average efficiency D_f.

It is known that nonregular designs have some hidden projection properties. Box and Tyssedal (1996) showed that if N is not a multiple of 8, any saturated OA with N runs and two levels has projectivity 3, i.e., any 3-factor projection contains a complete 2^3 factorial design, possibly with some points replicated. Therefore, any 3-factor projection can entertain all three main effects and all 2fi's among them. Cheng (1995) further showed that if N is not a multiple of 8, any OA with N runs and two levels has the following hidden projection property: Any 4-factor projection can entertain all four main effects and all 2fi's among them. Let P_f be the number of f-factor projections

Table 6: Design Efficiency and Estimation Capacity

Design	D_1	D_2	D_3	D_4	D_5	D_6	D_7	N_1	N_2	N_3	N_4	N_5	N_6	N_7	P_5
20.7.1	.97	.94	.92	.89	.86	.82	.79	0	0	0	0	0	2	34	21
20.7.2	.97	.94	.92	.89	.86	.82	.79	0	0	0	0	0	6	96	20
20.7.3	.97	.94	.92	.89	.86	.82	.79	0	0	0	0	0	13	197	20
20.7.4	.97	.95	.92	.89	.86	.83	.79	0	0	0	0	1	22	215	20
20.7.5	.97	.95	.92	.89	.86	.83	.79	0	0	0	0	0	6	91	19
20.7.6	.97	.95	.92	.89	.86	.83	.79	0	0	0	0	0	9	138	19
20.7.7	.97	.95	.92	.89	.86	.83	.79	0	0	0	0	0	5	76	18
20.7.8	.97	.95	.92	.89	.86	.83	.79	0	0	0	0	0	5	75	17
20.7.9	.97	.94	.91	.88	.85	.82	.78	0	0	0	0	0	6	93	19
20.7.10	.97	.94	.92	.89	.85	.82	.78	0	0	0	0	1	22	213	19
20.7.384	.94	.88	.83	.79	.74	.68	.61	0	0	0	9	156	1317	7252	4
20.7.385	.94	.88	.84	.79	.74	.68	.62	0	0	0	9	153	1264	6798	2
20.7.386	.93	.87	.82	.77	.71	.65	.58	0	0	0	13	231	1986	10825	2
20.7.387	.93	.87	.81	.76	.71	.65	.57	0	0	0	15	255	2095	10995	0
20.7.388	.91	.84	.76	.68	.57	.37	0	0	0	0	105	1890	18335	116280	0

that can entertain all f main effects and all 2fi's among them. One would prefer designs with large P_f values.

4.2 Door Panel Stamping Experiment

Return to the door panel stamping experiment mentioned in the introduction. The goal is to choose a 20×7 design with good design properties. Here we evaluate the top ten and bottom five designs given in Table 8 in terms of estimation capacity and hidden projection properties.

First consider estimation capacity and design efficiency. There are 21 2fi's and $\binom{21}{f}$ different models with all 7 main effects and f 2fi's. Table 6 shows the results in terms of D_f and N_f for $f = 1, \ldots, 7$, where $N_f = \binom{21}{f} - E_f$ is the number of non-estimable models. All top ten designs can entertain up to four 2fi's while all five bottom designs can not entertain some four 2fi's. All top ten designs have larger average efficiency than the bottom five designs. It is evident that the top ten designs are better than the bottom five designs in terms of both design efficiency and estimation capacity. Among the top ten designs, eight (except 20.7.4 and 20.7.10) can entertain any five 2fi's. Their average efficiencies are pretty close to each other. Note that design 20.7.1, ranked as the first by MAP, is indeed the best in terms of estimation capacity. It has only two non-estimable models with six 2fi's while all other designs have at least five non-estimable models with six 2fi's.

Next consider the hidden projection properties of the 20×7 designs. As discussed earlier, for any OA with 20 runs and 7 columns, any four-factor projection can entertain all four main effects and all 2fi's among them. It is interesting to consider five-factor projections here. In Table 6, the last column, P_5, shows the number of five-factor projections that can entertain all five main effects and ten 2fi's among them. Note that the number of estimable models is fairly consistent with the MAP ranking. The top ten designs can estimate more models than the bottom five designs. Recall that designs 20.7.1, 20.7.2, and 20.7.8 are ranked as the first by GMA. They can estimate 21, 20, and 17 models, respectively, which are consistent with their MAP rankings. The top design 20.7.1 is the only design that can estimate all 21 models; therefore, it is also the best in terms of hidden projection properties.

In conclusion, MAP ranking is supported by estimation capacity and hidden projection properties. For an experiment with 20 runs and 7 factors such as the door panel stamping experiment, the top design 20.7.1 is recommended.

5 Concluding Remarks

By an exhaustive computer search, Beder (1998) observed that all 16-run OAs can be embedded in some Hadamard matrices. Therefore, all the 16-run OAs can be found from five Hadamard matrices of order 16. However, it is well known that not every OA can be embedded in Hadamard matrices. One can easily find many such examples from 20-run designs. Our goal is to look for new nonisomorphic designs which have good MAP rankings.

For each given number of columns, we use an efficient algorithm due to Xu (2002) to construct as many as 5,000 OAs. For each OA, we compute its K-value distributions and compare them with all K-value distributions of projection designs from Hadamard matrices. We find 10, 79, 338, 386, 107, and 15 new OAs for 6,7,8,9,10, and 11 columns, respectively. In particular, we find two new top designs with 6 columns and two new top designs with 7 columns that are not found from the three Hadamard matrices. The four new 20-run designs are listed in Table 11. We should mention that a similar effort was done previously by Li (2000), who searched for new designs by using both GMA and estimation capacity. However, our approach is more straightforward and much easier than his.

Similar to 20-run designs, there are many 27-run OAs that are not part of any saturated $OA(27, 3^{13})$. In particular, the 27-run OAs with 5–10 columns given in Xu (2002) have less moment aberration projection than the top designs given in Table 9.

In conclusion, the moment aberration projection is a simple and yet powerful tool for ranking and classifying designs. It also provides a nice blueprint to search for good designs.

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Table 7: Some 16-Run Designs Ranked by MAP

Design	F_3	F_4	GMA	Type	Columns
16.3.1	(0,0,1)	–	1	I	1 2 3
16.3.2	(0,1,0)	–	2	II	4 8 12
16.3.3	(1,0,0)	–	3	I	1 2 13
16.4.1	(0,0,4)	(0,0,0,0,1)	1	I	1 2 3 4
16.4.2	(0,0,4)	(0,0,0,1,0)	2	I	1 2 3 8
16.4.3	(0,1,3)	(0,0,1,0,0)	3	II	1 4 8 12
16.5.1	(0,0,10)	(0,0,0,0,5)	1	I	1 2 3 4 7
16.5.2	(0,0,10)	(0,0,0,1,4)	2	I	1 2 3 4 6
16.5.3	(0,1,9)	(0,0,2,0,3)	3	II	1 2 4 8 12
16.6.1	(0,0,20)	(0,0,0,3,12)	1	I	1 2 3 4 6 8
16.6.2	(0,2,18)	(0,1,4,1,9)	2	II	1 2 4 7 8 12
16.6.3	(0,4,16)	(0,3,6,0,6)	4	III	1 2 4 8 10 12
16.6.4	(0,4,16)	(0,4,4,0,7)	3	II	1 4 6 8 11 12
16.7.1	(0,0,35)	(0,0,0,7,28)	1	I	1 2 3 4 6 8 9
16.7.2	(0,4,31)	(0,4,8,3,20)	2	II	1 2 4 7 8 11 12
16.7.3	(0,6,29)	(0,6,12,1,16)	3	III	1 2 4 7 8 10 12
16.8.1	(0,0,56)	(0,0,0,14,56)	1	I	1 2 3 4 6 8 9 12
16.8.2	(0,8,48)	(0,12,16,6,36)	2	II	1 2 4 7 8 11 12 15
16.8.3	(0,12,44)	(0,18,24,1,27)	3	V	1 2 4 7 8 10 12 15
16.8.4	(0,12,44)	(0,18,24,1,27)	3	III	1 2 4 7 8 10 12 15
16.9.1	(0,16,68)	(0,48,0,14,64)	1	II	4 5 6 7 8 9 10 11 12
16.9.2	(0,20,64)	(0,48,24,6,48)	2	III	2 3 4 5 8 9 10 11 12
16.9.3	(0,22,62)	(0,48,36,2,40)	3	V	1 2 4 7 8 9 10 12 14
16.10.1	(0,32,88)	(0,96,32,10,72)	1	III	2 3 4 5 8 9 10 11 12 13
16.10.2	(0,32,88)	(0,104,16,14,76)	2	III	2 4 8 9 10 11 12 13 14 15
16.10.3	(0,32,88)	(0,112,0,18,80)	3	II	4 5 6 7 8 9 10 11 12 13
16.11.1	(0,48,117)	(0,156,72,8,94)	1	V	1 2 4 7 8 9 10 11 12 13 14
16.11.2	(0,48,117)	(0,160,64,10,96)	2	IV	2 3 4 5 6 7 8 9 10 11 12
16.11.3	(0,48,117)	(0,168,48,14,100)	3	III	2 4 7 8 9 10 11 12 13 14 15
16.11.4	(0,48,117)	(0,168,48,14,100)	3	V	1 2 4 8 9 10 11 12 13 14 15
16.11.5	(0,48,117)	(0,168,48,14,100)	3	III	2 3 4 5 8 9 10 11 12 13 14
16.12.1	(0,64,156)	(0,240,96,15,144)	1	V	1 2 4 7 8 9 10 11 12 13 14 15
16.12.2	(0,64,156)	(0,240,96,15,144)	1	IV	2 3 4 5 6 7 8 9 10 11 12 13
16.12.3	(0,64,156)	(0,256,64,23,152)	2	III	2 3 4 5 8 9 10 11 12 13 14 15
16.13.1	(0,88,198)	(0,360,160,15,180)	1	IV	2 3 4 5 6 7 8 9 10 11 12 13 14
16.13.2	(2,80,204)	(20,320,160,15,200)	2	V	1 2 3 4 5 8 9 10 11 12 13 14 15
16.13.3	(2,80,204)	(20,336,128,23,208)	3	III	2 3 4 5 6 8 9 10 11 12 13 14 15
16.14.1	(0,112,252)	(0,504,224,21,252)	1	IV	2 3 4 5 6 7 8 9 10 11 12 13 14 15
16.14.2	(4,96,264)	(44,408,240,17,292)	2	V	1 2 3 4 5 6 8 9 10 11 12 13 14 15
16.14.3	(4,96,264)	(44,432,192,29,304)	3	III	2 3 4 5 6 7 8 9 10 11 12 13 14 15
16.15.1	(7,112,336)	(84,504,336,21,420)	1	IV	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
16.15.2	(7,112,336)	(84,504,336,21,420)	1	V	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
16.15.3	(11,96,348)	(132,432,288,33,480)	2	III	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
16.15.4	(19,64,372)	(228,288,192,57,600)	3	II	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
16.15.5	(35,0,420)	(420,0,0,105,840)	4	I	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

NOTE: 1. Type I is a Plackett-Burman design, and types II – V are as in Deng and Tang (2002).

2. The indices of F_3 (possible K values) are (744, 672, 648), and the indices of F_4 are (4160, 3776, 3632, 3584, 3392), respectively.

Table 8: Some 20-Run Designs Ranked by MAP

Design	F_3	F_4	GMA	Type	Columns
20.3.1	(0,1)	–	1	Q	1 2 3
20.3.2	(1,0)	–	2	Q	1 2 9
20.4.1	(0,4)	(0,0,1)	1	Q	1 2 3 4
20.4.2	(0,4)	(0,1,0)	2	Q	1 2 3 16
20.4.3	(1,3)	(1,0,0)	3	Q	1 2 3 6
20.5.1	(0,10)	(0,0,5)	1	Q	1 2 3 4 5
20.5.2	(0,10)	(0,0,5)	2	Q	1 2 3 4 14
20.5.3	(0,10)	(0,1,4)	3	Q	1 2 3 4 16
20.5.10	(2,8)	(4,1,0)	10	P	1 2 3 4 5
20.6.1	(0,20)	(0,1,14)	1	Q	1 2 3 4 13 16
20.6.2	(0,20)	(0,1,14)	2	Q	1 2 3 4 8 16
20.6.3	(0,20)	(0,1,14)	2	Q	1 2 3 4 5 15
20.6.4	(0,20)	(0,1,14)	1	Q	1 2 3 4 5 13
20.6.59	(4,16)	(12,3,0)	34	P	1 2 3 5 10 15
20.7.1	(0,35)	(0,3,32)	1	P	1 2 4 6 7 11 17
20.7.2	(0,35)	(0,3,32)	1	Q	1 2 3 4 8 13 16
20.7.3	(0,35)	(0,3,32)	2	P	1 2 3 6 8 11 13
20.7.4	(0,35)	(0,3,32)	2	Q	1 2 3 4 8 14 16
20.7.5	(0,35)	(0,3,32)	2	Q	1 2 3 4 12 14 16
20.7.6	(0,35)	(0,3,32)	2	P	1 2 6 7 8 10 11
20.7.7	(0,35)	(0,3,32)	2	Q	1 2 3 4 5 13 15
20.7.8	(0,35)	(0,3,32)	1	Q	1 2 3 4 5 13 16
20.7.9	(0,35)	(0,4,31)	3	Q	1 2 3 4 13 16 17
20.7.10	(0,35)	(0,4,31)	4	P	1 2 4 6 11 12 17
20.7.384	(4,31)	(16,4,15)	47	P	1 2 5 6 11 14 18
20.7.385	(4,31)	(16,4,15)	48	P	1 2 3 6 9 11 16
20.7.386	(5,30)	(20,3,12)	49	P	1 2 5 6 7 13 18
20.7.387	(5,30)	(20,5,10)	50	P	1 2 5 6 7 13 19
20.7.388	(7,28)	(28,7,0)	51	P	1 2 3 4 5 10 15
20.8.1	(0,56)	(0,6,64)	1	N	1 2 3 4 6 8 16 18
20.8.2	(0,56)	(0,6,64)	1	P	1 2 6 8 10 11 13 15
20.8.3	(0,56)	(0,8,62)	2	P	1 2 3 4 6 7 11 17
20.8.1265	(8,48)	(40,10,20)	80	P	5 6 7 10 11 12 15 16
20.9.1	(0,84)	(0,18,108)	1	P	1 2 3 4 8 9 13 14 18
20.9.2	(0,84)	(0,18,108)	2	P	1 2 3 8 9 13 14 18 19
20.9.3	(1,83)	(6,14,106)	3	P	1 2 3 4 6 7 11 12 16
20.9.2089	(12,72)	(72,18,36)	125	P	5 6 7 10 11 12 15 16 17
20.10.1	(0,120)	(0,30,180)	1	P	1 2 3 4 8 9 13 14 18 19
20.10.2	(2,118)	(14,22,174)	2	P	1 2 3 4 6 7 11 12 16 17
20.10.3	(3,117)	(21,18,171)	3	Q	1 2 3 4 6 8 13 14 16 17
20.10.2282	(12,108)	(84,18,108)	125	P	1 5 6 7 10 11 12 15 16 17

Table 8: Some 20-Run Designs Ranked by MAP (continued)

Design	F_3	F_4	GMA	Type	Columns
20.11.1	(5,160)	(40,30,260)	1	P	1 2 3 4 5 8 9 13 14 18 19
20.11.2	(6,159)	(48,26,256)	2	N	1 2 3 4 6 7 8 9 11 12 13
20.11.3	(6,159)	(48,26,256)	2	P	1 2 3 4 6 7 8 11 12 16 17
20.11.1899	(13,152)	(104,26,200)	80	P	1 2 3 4 5 6 7 8 9 10 15
20.12.1	(8,212)	(72,39,384)	1	N	1 2 3 4 6 7 8 9 11 12 13 14
20.12.2	(8,212)	(72,39,384)	1	P	1 2 3 5 6 8 10 11 13 15 16 19
20.12.3	(10,210)	(90,33,372)	2	N	1 2 3 4 5 6 8 11 12 15 16 18
20.12.1300	(15,205)	(135,32,328)	51	P	1 2 3 4 5 6 7 8 9 10 11 15
20.13.1	(14,272)	(140,47,528)	1	N	1 2 3 4 6 7 8 9 11 12 13 14 16
20.13.2	(14,272)	(140,47,528)	2	N	1 2 3 4 5 6 7 8 9 11 12 13 14
20.13.3	(14,272)	(140,47,528)	1	P	1 2 3 5 6 7 8 9 10 11 12 13 14
20.13.730	(18,268)	(180,43,492)	34	P	1 2 3 4 5 6 7 10 11 12 15 16 17
20.14.1	(20,344)	(220,60,721)	1	Q	1 2 3 4 5 6 7 8 9 13 14 16 17 18
20.14.2	(20,344)	(220,60,721)	1	Q	1 2 3 4 5 6 7 8 9 10 11 13 15 18
20.14.3	(20,344)	(220,60,721)	1	Q	1 2 3 4 5 6 7 8 9 10 13 15 16 18
20.14.328	(22,342)	(242,59,700)	10	P	1 2 3 4 5 6 7 8 10 11 12 15 16 17
20.15.1	(26,429)	(312,81,972)	1	N	1 2 3 4 6 7 8 9 11 12 13 14 16 17 18
20.15.2	(26,429)	(312,81,972)	1	Q	1 2 3 4 5 6 7 8 9 10 11 12 13 17 18
20.15.3	(26,429)	(312,81,972)	1	Q	1 2 3 4 5 6 7 8 9 10 11 12 13 15 17
20.15.124	(27,428)	(324,81,960)	3	P	1 2 3 4 5 6 7 8 9 10 11 12 15 16 17
20.16.1	(32,528)	(416,108,1296)	1	N	1 2 3 4 6 7 8 9 11 12 13 14 16 17 18 19
20.16.2	(32,528)	(416,108,1296)	1	Q	1 2 3 4 5 6 7 8 9 10 11 12 13 15 17 18
20.16.3	(32,528)	(416,108,1296)	1	N	1 2 3 4 5 6 7 8 9 10 11 12 13 14 16 17
20.16.40	(33,527)	(429,107,1284)	2	P	1 2 3 4 5 6 7 8 9 10 11 12 13 15 16 17
20.17.1	(40,640)	(560,140,1680)	1	Q	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
20.17.2	(40,640)	(560,140,1680)	1	N	1 2 3 4 5 6 7 8 9 11 12 13 14 16 17 18 19
20.17.3	(40,640)	(560,140,1680)	1	N	1 2 3 4 5 6 7 8 9 10 11 12 13 14 16 17 18
20.17.11	(40,640)	(560,140,1680)	1	P	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
20.18.1	(48,768)	(720,180,2160)	1	Q	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
20.18.2	(48,768)	(720,180,2160)	1	N	1 2 3 4 5 6 7 8 9 10 11 12 13 14 16 17 18 19
20.18.3	(48,768)	(720,180,2160)	1	N	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
20.18.6	(48,768)	(720,180,2160)	1	P	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
20.19.1	(57,912)	(912,228,2736)	1	Q	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
20.19.2	(57,912)	(912,228,2736)	1	N	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
20.19.3	(57,912)	(912,228,2736)	1	P	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

NOTE: 1. Type Q is a Plackett-Burman design, and types P and N are as in Deng and Tang (2002).

2. The indices of F_3 (possible K values) are (1134,1086), and the indices of F_4 are (6528, 6240, 6144), respectively.

Table 9: Some 27-Run Designs Ranked by MAP

Design	F_3	Type	Columns
27.3.1	(0,0,0,0,0,0,0,1)	68	1 2 3
27.3.2	(0,0,0,0,0,0,1,0)	5	2 4 11
27.3.3	(0,0,0,0,0,1,0,0)	1	1 2 5
27.3.8	(1,0,0,0,0,0,0,0)	68	1 2 6
27.4.1	(0,0,0,0,0,0,0,4)	68	1 2 3 5
27.4.2	(0,0,0,0,0,0,2,2)	5	2 4 11 12
27.4.3	(0,0,0,0,0,0,3,1)	2	1 2 6 13
27.4.135	(1,0,0,0,0,0,0,3)	68	1 2 3 4
27.4.139	(4,0,0,0,0,0,0,0)	68	1 2 6 12
27.5.1	(0,0,0,0,0,3,4,3)	5	4 8 9 12 13
27.5.2	(0,0,0,0,0,4,4,2)	8	1 2 6 9 13
27.5.3	(0,0,0,0,0,5,2,3)	6	3 9 10 12 13
27.5.1782	(1,0,0,0,0,0,0,9)	68	1 2 3 4 10
27.5.1832	(2,0,0,0,0,0,0,8)	68	1 2 3 4 5
27.5.1833	(4,0,0,0,0,0,0,6)	68	1 2 3 4 8
27.6.1	(0,0,0,0,0,14,0,6)	37	2 5 9 10 12 13
27.6.2	(0,0,0,0,2,9,4,5)	5	2 4 8 9 11 13
27.6.3	(0,0,0,0,3,8,4,5)	6	5 6 9 10 11 12
27.6.6191	(2,0,0,0,0,0,0,18)	68	1 2 3 4 10 11
27.6.6220	(3,0,0,0,0,0,0,17)	68	1 2 3 4 5 7
27.6.6221	(4,0,0,0,0,0,0,16)	68	1 2 3 4 5 6
27.6.6230	(5,0,0,0,0,0,0,15)	68	1 2 3 4 5 8
27.7.1	(0,0,0,3,4,15,4,9)	5	2 4 6 8 9 11 13
27.7.2	(0,0,0,3,6,11,6,9)	5	2 4 5 8 9 11 13
27.7.3	(0,0,0,4,2,14,6,9)	4	2 3 5 7 8 9 10
27.7.10284	(5,0,0,0,0,0,0,30)	68	1 2 3 4 5 7 10
27.7.10298	(6,0,0,0,0,0,0,29)	68	1 2 3 4 5 6 7
27.7.10299	(7,0,0,0,0,0,0,28)	68	1 2 3 4 5 6 8
27.7.10300	(8,0,0,0,0,0,0,27)	68	1 2 3 4 5 8 9
27.8.1	(0,0,0,6,8,20,8,14)	5	2 4 5 6 8 9 11 13
27.8.2	(0,0,0,8,0,31,0,17)	1	1 2 3 5 6 9 10 11
27.8.3	(0,0,0,8,0,32,0,16)	1	1 2 3 5 6 9 10 13
27.8.10269	(8,0,0,0,0,0,0,48)	68	1 2 3 4 5 7 10 11
27.8.10272	(10,0,0,0,0,0,0,46)	68	1 2 3 4 5 6 7 8
27.8.10273	(11,0,0,0,0,0,0,45)	68	1 2 3 4 5 6 8 9
27.9.1	(0,0,0,12,0,48,0,24)	1	1 2 3 5 6 9 10 11 13
27.9.2	(0,0,0,15,0,45,0,24)	1	1 2 3 4 5 6 8 10 11
27.9.3	(0,0,0,15,0,45,0,24)	1	1 2 3 4 5 8 10 11 12
27.9.6975	(12,0,0,0,0,0,0,72)	68	1 2 3 4 5 7 10 11 12
27.9.6976	(15,0,0,0,0,0,0,69)	68	1 2 3 4 5 6 7 8 9
27.9.6977	(16,0,0,0,0,0,0,68)	68	1 2 3 4 5 6 7 8 12
27.10.1	(0,0,0,21,0,66,0,33)	1	1 2 3 4 5 6 7 11 12 13
27.10.2	(0,0,0,21,0,66,0,33)	1	1 2 3 4 5 6 7 8 9 10
27.10.3	(0,0,0,22,0,65,0,33)	1	1 2 3 4 5 6 7 8 10 11
27.10.3304	(21,0,0,0,0,0,0,99)	68	1 2 3 4 5 6 7 8 9 11
27.10.3305	(22,0,0,0,0,0,0,98)	68	1 2 3 4 5 6 7 8 9 10
27.11.1	(0,0,0,30,0,90,0,45)	1	1 2 3 4 5 6 7 8 9 11 13
27.11.2	(0,0,0,30,0,90,0,45)	1	1 2 3 4 5 6 7 8 9 10 11
27.11.3	(0,0,30,0,30,0,75,30)	2	1 2 3 4 5 6 7 8 9 10 11
27.11.1176	(30,0,0,0,0,0,0,135)	68	1 2 3 4 5 6 7 8 9 10 11
27.12.1	(0,0,0,40,0,120,0,60)	1	1 2 3 4 5 6 7 8 9 10 11 12
27.12.2	(0,0,40,0,40,0,100,40)	2	1 2 3 4 5 6 7 8 9 10 11 12
27.12.3	(2,0,0,50,0,84,18,66)	3	1 2 3 4 5 6 7 8 9 10 11 12
27.12.331	(40,0,0,0,0,0,0,180)	68	1 2 3 4 5 6 7 8 9 10 11 12
27.13.1	(0,0,0,52,0,156,0,78)	1	1 2 3 4 5 6 7 8 9 10 11 12 13
27.13.2	(0,0,52,0,52,0,130,52)	2	1 2 3 4 5 6 7 8 9 10 11 12 13
27.13.3	(3,0,0,63,0,108,27,85)	3	1 2 3 4 5 6 7 8 9 10 11 12 13
27.13.67	(52,0,0,0,0,0,0,234)	68	1 2 3 4 5 6 7 8 9 10 11 12 13

NOTE: 1. Type 68 is a Plackett-Burman design, and types 1 and 5 are given in Table 10.

2. The indices of F_3 (possible K values) are (972, 900, 870, 864, 852, 846, 834, 810).

Table 10: Two 27-Run Saturated Orthogonal Arrays

(i) Type 1

Run	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	0	0	0	1	1	1	1	1	1	1	1	1
2	0	0	0	0	2	2	2	2	2	2	2	2	2
3	0	2	2	1	0	0	0	1	2	2	2	1	1
4	0	2	2	2	2	1	2	0	0	0	1	2	1
5	0	1	1	2	0	0	0	2	1	1	1	2	2
6	0	1	2	1	1	1	2	2	2	1	0	0	0
7	0	2	1	2	2	2	1	1	1	2	0	0	0
8	0	1	1	1	1	2	1	0	0	0	2	1	2
9	1	0	1	2	0	2	2	0	2	1	0	1	1
10	2	0	2	1	0	1	1	0	1	2	0	2	2
11	2	0	1	1	2	0	1	2	2	0	1	0	1
12	1	0	2	2	1	0	2	1	1	0	2	0	2
13	2	0	2	2	1	2	0	2	0	2	1	1	0
14	1	0	1	1	2	1	0	1	0	1	2	2	0
15	1	1	0	2	0	1	1	2	0	2	2	0	1
16	2	2	0	1	0	2	2	1	0	1	1	0	2
17	1	1	0	1	2	0	2	0	1	2	1	1	0
18	2	2	0	2	1	0	1	0	2	1	2	2	0
19	1	2	0	1	1	2	0	2	1	0	0	2	1
20	2	1	0	2	2	1	0	1	2	0	0	1	2
21	2	2	1	0	0	1	2	2	1	0	2	1	0
22	1	1	2	0	0	2	1	1	2	0	1	2	0
23	2	1	1	0	1	0	2	1	0	2	0	2	1
24	1	2	2	0	2	0	1	2	0	1	0	1	2
25	2	1	2	0	2	2	0	0	1	1	2	0	1
26	1	2	1	0	1	1	0	0	2	2	1	0	2
27	0	0	0	0	0	0	0	0	0	0	0	0	0

(ii) Type 5

Run	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	0	0	0	1	1	1	1	1	1	1	1	1
2	0	0	0	0	2	2	2	2	2	2	2	2	2
3	0	1	1	1	0	0	0	2	2	2	1	1	1
4	0	2	2	2	2	2	2	0	0	0	1	1	1
5	0	2	2	2	0	0	0	1	1	1	2	2	2
6	0	2	2	1	2	1	1	2	2	1	0	0	0
7	0	1	1	2	1	2	2	1	1	2	0	0	0
8	0	1	1	1	1	1	1	0	0	0	2	2	2
9	1	0	1	2	0	1	2	0	2	1	0	2	1
10	2	0	2	1	0	2	1	0	1	2	2	0	1
11	1	0	2	1	1	0	2	2	1	0	0	1	2
12	2	0	1	2	2	0	1	1	2	0	2	1	0
13	1	0	1	1	2	2	0	1	0	1	1	0	2
14	2	0	2	2	1	1	0	2	0	2	1	2	0
15	2	1	0	2	0	2	1	2	0	1	0	1	2
16	1	2	0	1	0	1	2	1	0	2	2	1	0
17	1	2	0	2	1	0	1	0	2	2	1	0	2
18	2	1	0	1	2	0	2	0	1	1	1	2	0
19	2	2	0	1	1	2	0	1	2	0	0	2	1
20	1	1	0	2	2	1	0	2	1	0	2	0	1
21	2	1	2	0	0	1	2	1	2	0	1	0	2
22	1	2	1	0	0	2	1	2	1	0	1	2	0
23	1	1	2	0	2	0	1	1	0	2	0	2	1
24	2	2	1	0	1	0	2	2	0	1	2	0	1
25	2	2	1	0	2	1	0	0	1	2	0	1	2
26	1	1	2	0	1	2	0	0	2	1	2	1	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 11: Top 20-Run Orthogonal Arrays not From Hadamard Matrices

(i) new20.6.1

Run	1	2	3	4	5	6
1	+	+	+	+	−	−
2	+	−	+	−	−	+
3	+	+	−	−	+	+
4	+	−	−	−	−	−
5	+	+	+	+	+	+
6	+	−	+	−	+	−
7	+	+	−	−	+	−
8	+	−	−	+	+	+
9	+	+	−	+	−	+
10	+	−	+	+	−	−
11	−	+	−	+	+	−
12	−	−	−	+	−	−
13	−	+	+	+	−	+
14	−	−	−	+	+	+
15	−	+	+	−	−	+
16	−	−	−	−	−	+
17	−	+	−	−	−	−
18	−	−	+	+	+	−
19	−	+	+	−	+	−
20	−	−	+	−	+	+

(ii) new20.6.2

Run	1	2	3	4	5	6
1	+	+	+	−	+	+
2	+	−	+	+	+	−
3	+	+	−	+	+	−
4	+	−	−	−	−	−
5	+	+	+	+	−	+
6	+	−	−	−	+	+
7	+	+	+	−	−	−
8	+	−	−	+	+	+
9	+	+	−	−	−	+
10	+	−	+	+	−	−
11	−	+	−	−	+	−
12	−	−	+	−	+	−
13	−	+	−	+	−	−
14	−	−	+	+	−	+
15	−	+	+	+	+	−
16	−	−	−	−	−	−
17	−	+	−	+	+	+
18	−	−	+	−	+	+
19	−	+	+	−	−	+
20	−	−	−	+	−	+

(iii) new20.7.1

Run	1	2	3	4	5	6	7
1	+	+	+	+	+	+	+
2	+	−	+	−	−	+	−
3	+	+	+	−	+	−	−
4	+	−	−	+	−	−	+
5	+	+	+	+	−	−	+
6	+	−	+	+	+	−	−
7	+	+	−	−	+	+	−
8	+	−	−	−	+	+	+
9	+	+	−	−	−	−	+
10	+	−	−	+	−	+	−
11	−	+	−	−	−	−	−
12	−	−	−	+	+	+	−
13	−	+	−	+	−	+	+
14	−	−	+	+	+	−	+
15	−	+	+	+	−	+	−
16	−	−	−	−	+	−	+
17	−	+	+	−	+	+	+
18	−	−	+	−	−	+	+
19	−	+	−	+	+	−	−
20	−	−	+	−	−	−	−

(iv) new20.7.2

Run	1	2	3	4	5	6	7
1	+	+	−	+	−	−	−
2	+	−	−	+	+	+	−
3	+	+	+	+	−	+	−
4	+	−	+	+	−	−	+
5	+	+	−	+	+	+	+
6	+	−	+	−	+	+	+
7	+	+	−	−	−	+	+
8	+	−	+	−	−	−	−
9	+	+	+	−	+	−	+
10	+	−	−	−	+	−	−
11	−	+	+	−	+	+	+
12	−	−	+	+	+	+	−
13	−	+	−	−	−	+	−
14	−	−	−	+	−	+	+
15	−	+	+	+	+	−	−
16	−	−	−	+	+	−	+
17	−	+	−	−	+	−	−
18	−	−	−	−	−	−	+
19	−	+	+	+	−	−	+
20	−	−	+	−	−	+	−