The Quantized Hall Insulator: A New Insulator in Two-Dimensions

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Quite generally, an insulator is theoretically defined by a vanishing conductivity tensor at the absolute zero of temperature T. In classical insulators, such as band insulators, vanishing conductivities lead to diverging resistivities. In other insulators, in particular when a high magnetic field B is added, it is possible that while the magnetoresistance, ρ_{xx}, diverges, the Hall resistance, ρ_{xy}, remains finite, which is known as a Hall insulator (HI)[1]. In this letter we demonstrate experimentally the existence of another, more exotic, insulator. This insulator, which terminates the quantum Hall effect (QHE) series in a two-dimensional electron system, is characterized by a ρ_{xy} which is approximately *quantized* in the quantum unit of resistance h/e^2. This insulator is termed a quantized Hall insulator (QHI)[2]. In addition we show that for the same sample, the insulating state preceding the QHE series, at low-B, is of the HI kind.

Experimentally the identification of an insulating phase is based on extrapolating the measured $\rho_{xx}(T)$ at finite T to $T = 0$. This is always an ambiguous process. However, when ρ_{xx} is exponentially increasing as $T \to 0$, the state is usually considered to be an insulator. Unfortunately, the divergent ρ_{xx} seriously hinders the determination of ρ_{xy}, since even small Hall-contact misalignment will result in a large overriding signal from the diverging ρ_{xx}. It is possible, to a certain degree, to circumvent this difficulty by symmetrizing the measurement. This can be achieved by reversing the B-field orientation, as the contribution of ρ_{xx} is symmetric in B as opposed to antisymmetric for ρ_{xy}. The effectiveness of this procedure is demonstrated in the inset of Fig. 1, where the Hall resistances obtained for the two opposite B-field directions are in dotted lines and the average, i.e., ρ_{xy} is presented with a solid line. For the remainder of this letter all Hall resistivities are obtained using this method.

We now turn to discuss our results, where our first task is to identify the different phases. The transition between insulating and quantum Hall phases can be characterized by a critical B-field value, for which ρ_{xx} is T-independent and where the derivative of the T-dependence changes sign on each side of the transition. By plotting ρ_{xx} at two different T's, we can therefore extract the transition points. In Fig. 1 we have plotted ρ_{xy} together with ρ_{xx} as a function of B. With increasing B, transition points at $B = 2.2$ T and at $B = B_C = 6.06$ T, can be identified from the crossing of the two ρ_{xx} curves obtained at different T's. In between these transitions we have the usual *QH* state, which is bordered on both sides by insulators. For clarity ρ_{xx} is normalized to $\rho_c = \rho_{xx}(B_C)$.[3]

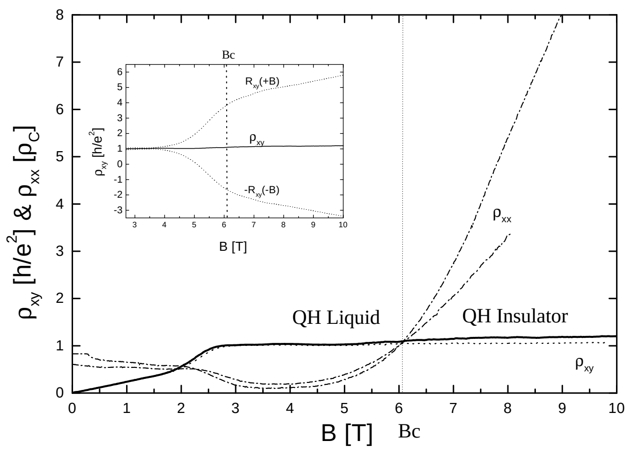

FIG. 1. The Hall and diagonal resistivities as a function of B-field. The solid line is the Hall resistivity measured at T=300 mK and a current I=200 nA, whereas the dotted line is for I=400 nA. The dash-dotted lines are ρ_{xx} at T=1.2 K (the uppermost curve) and at T=4.2 K (the lower curve). V_G=5.2 V, $\rho_c = 1.65h/e^2$ and B_C=6.06 T. The inset shows the Hall resistances for $+B$ and $-B$ in dotted lines and ρ_{xy} with a solid line. The experimental error is mainly given by $\left|\frac{\Delta R_{xy}}{R_{xy}}\right| \cdot \left(1 + \frac{(\rho_{xy} - R_{xy}(+B))^2}{\rho_{xy}^2}\right)^{1/2}$. The first term can be estimated from the fluctuations around $\nu = 1$, i.e., $\simeq 4\%$ and the second term is $\simeq 5$, leading to a total inaccuracy of 20 %. With increased current the first term is reduced to $\simeq 3\%$ and the second to $\simeq 2$, yielding a total inaccuracy of 6 %.

Focusing first on the high B-field region, at $B > B_C$, a striking observation that can be made in Fig. 1, is the large range over which ρ_{xy} remains nearly constant and close to its quantized value of h/e^2. Indeed, between 2.7 T and 10 T, the deviation of ρ_{xy} from h/e^2 is less than 20 %. When doubling the current the symmetric contribution to ρ_{xy} is further reduced and therefore the deviation is much smaller, i.e., less than 5 %. This could be due to the good contact alignment as higher currents tend to render the system more homogeneous. In terms of the Landau level filling factor ν, this means that ρ_{xy} remains approximately quantized between $\nu = 1.5$ and $\nu = 0.4$. The transition point is at $\nu_c = 0.75$.

As shown in Fig. 1, a 5-20% accuracy was obtained. In order to increase this accuracy even further but without increasing the current and to analyze the T-dependence, we used a standard low frequency lock-in technique, with 1 nA currents. In the inset of Fig. 2, we have extracted the T-dependence of ρ_{xy} from the main figure, where ρ_{xx} and ρ_{xy} are plotted as a function of B. The important result here, is the clear indication that ρ_{xy} saturates towards the quantized value, within 2%, at low enough temperatures (below 2 K).

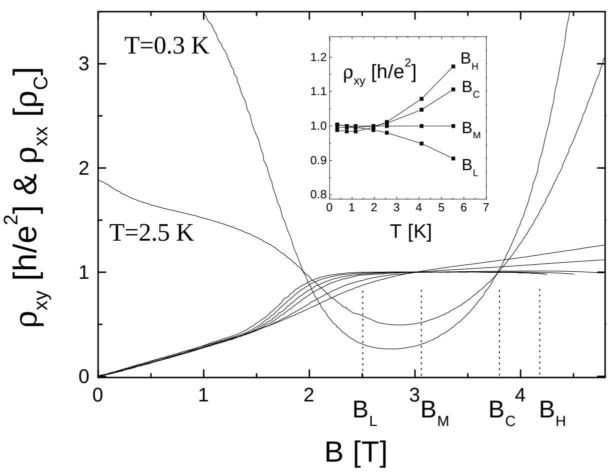

FIG. 2. The Hall and diagonal resistivities as a function of B-field for different temperatures. The T's are 0.3, 1.2, 2, 2.7, 4.2 and 5.5 K. V_G=5.4 V, $\rho_c = 1.73h/e^2$ and $B_C = 3.8$ T. In the inset we have plotted the T-dependence of ρ_{xy}. The constant curve in the inset corresponds to the T-independent ρ_{xy} at $B_M = 3.1$ T, which is close to $\nu = 1$. B_L=2.5 T corresponds to a typical B-field $\nu > 1$, where the Hall resistance decreases with T. B_C=3.8 T corresponds to the critical value defined by the T-independent ρ_{xx} and B_H=4.2 T to a typical B-field larger than B_C.

In addition we were also interested in studying ρ_{xy} when ρ_{xx} is highly insulating even at low B-field, i.e., below the low B-field transition. Fig. 2 was therefore obtained by increasing the effective disorder, i.e., we reduced the density of the sample by applying a higher voltage (V_G) on the front gate of our 2D hole system (2DHS) confined in a Ge/SiGe quantum well. A more detailed description of the sample (X335) and its properties are given in ref.[4]. The low B-field transition is observed at $B = 2.2$ T in Fig. 1 and at $B = 1.9$ T in Fig. 2, where we have applied a higher V_G. For fields smaller than the value at the low B-field transition, the system is insulating and ρ_{xy} follows the classical dependence on the magnetic field, i.e., $\rho_{xy} \simeq B/nec$. We observe no temperature dependence of ρ_{xy} for low enough fields. At the same time $\rho_{xx} \simeq 7h/e^2$, at the lowest measured T, indicating that we are in the presence of a strong insulator, more precisely a *HI*. In order to measure only the diffusive part we limited the value of the maximum phase shift to be 2^o. Finally, we would like to mention that we also observed this quantization (within experimental accuracy) when we used the current contacts as Hall probes and the Hall contacts as current leads, hence a mesoscopic effect can be ruled out. The same results were obtained using a sample from a different wafer (X334), having a very similar structure.

Most previous experiments in the fractional quantum Hall regime[5–8] suggested a Hall resistance following the classical expression $\rho_{xy} \simeq B/nec$. The main difference between these samples and ours is their much higher mobilities, which can hence account for the different behavior. In the IQH regime but with higher mobilities than in our samples Shahar et al.[9,10] observed that the Hall resistivity remains close to the quantized value around the transition, but at their lowest temperatures, they observed a small deviation from the quantized value. The origin of this deviation is most likely ρ_{xx} mixing, which becomes too large at very low T and cannot be removed by symmetrizing. In our case, however, we managed to minimize this effect, which translates into an even better quantization at low T. In previous experiments, with samples of lower mobilities, ρ_{xy} either had a strong T-dependence[11] or a low accuracy[12] and others resolved only the $\nu = 2$ to insulator transition[11,13,14] along with a small B-field range in the insulating phase and therefore no generic behavior was demonstrated. In this work we presented results where we observe a Hall resistivity equal to the quantized value h/e^2, within experimental accuracy, as deep as 4 T beyond the transition point $B_C = 6.06$ T.

While most theories agree on a quantized value of ρ_{xy} at the transition, discrepancies exist for the value inside the insulator. First let us point out where the difficulties come from. In standard transport theories, such as in Boltzmann's linear response theory or in the Kubo formalism, one usually calculates conductivities. Using matrix inversion (in 2D) one obtains $\rho_{xy} = \frac{\sigma_{xx}}{\sigma_{xx}^2+\sigma_{xy}^2}$. As in an insulator the conductivities vanish, any small correction to the conductivities can alter the value of ρ_{xy} in the limit $\sigma \to 0$. In the Kubo formulation, the conductivities are expressed as a function of the frequency ω and T. The physically relevant approach, for obtaining the diffusive part of the transport coefficients, is to first take the limit $\omega \to 0$ and then $T \to 0$. In the opposite limit only the reactive part $\sigma_{xx} \sim i\omega$ is obtained[15,16], leading to a finite ρ_{xy}, for any type of insulator. In most cases the two limits do not commute.

In the framework of Kivelson, Lee and Zhang's (KLZ) global phase diagram[1], the quantum Hall liquid phases are bordered by an insulator. In the insulating phase KLZ obtained a finite ρ_{xy} in a RPA-type of approximation, by evaluating the diffusive part. Concentrating on the transition region, Shimshoni and others[17,18], considered the duality/particle-hole symmetry, leading to a quantized ρ_{xy} in the critical regime. In this context the experimental finding of an approximate quantization, throughout the transition, confirms the large region of validity of this duality. When following KLZ's analysis and considering the insulating phase outside the critical region, an exact quantization would imply the existence of an additional phase transition, i.e., from a *QHI* to *HI* behavior. This would certainly be a very interesting search to pursue in future experiments.

In a different type of approach a semi-classical network model was considered, yielding a *QHI*[2,19]. This network model describes the insulating phase, where *QH*-liquid states exist in long range random potential minima. Transport is then obtained via a scattering matrix, parameterizing the tunneling from one *QH*-liquid to the other. For the case where phase coherence is destroyed after each scattering event, i.e., in the semi-classical limit, ρ_{xy} was shown to remain quantized.

Our main results can be stated as follows: at very low temperatures and high B-fields, $\rho_{xy} \simeq h/e^2$ deep in the insulating phase, hence forming a *QHI*. At slightly higher temperatures ρ_{xy} starts to deviate from its quantized value and approaches its classical value proportional to B. At low B-fields ρ_{xy} follows its classical value even in the strongly insulating case, i.e., forming a *HI* state. In the next paragraph we describe an interesting consequence of this quantization.

Recently Dykhne and Ruzin[20] suggested that the conductivities follow a semicircle relation, where each semicircle formed by σ_{xx} as a function of σ_{xy}, is centered around $\sigma_{xy} = (n + 1/2)e^2/h$, where n is an integer. We wanted to test this relation for the lowest Landau level, i.e., the semicircle centered at $e^2/2h$. Fig. 3 presents the result, showing a clear evidence for this relation.

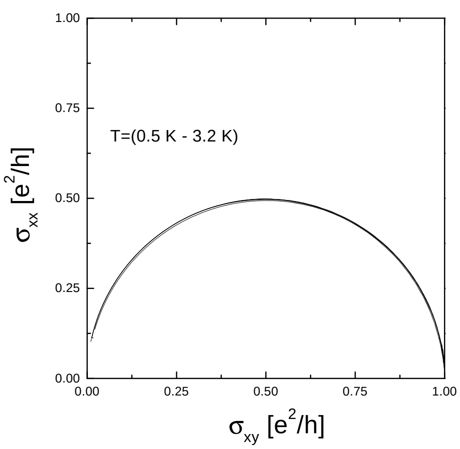

FIG. 3. σ_{xx} as a function of σ_{xy} for different temperatures. The T 's are 0.5, 0.8, 1.2, 1.8 and 3.2 K. $V_G = 5.2$ V and I=1 nA.

This result is in fact not very surprising taking into account that we have a quantized Hall resistance deep inside the insulator. Indeed,

$$\sigma_{xx}^2 + \left(\sigma_{xy} - \frac{e^2}{2h}\right)^2 = \left(\frac{e^2}{2h}\right)^2 + \frac{1 - (e^2/h)\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}, \quad (1)$$

simply using matrix inversion. As noted by Shahar et al.[10], the semicircle relation follows directly as long as $\rho_{xy} = h/e^2$. In addition it is completely independent of the value of ρ_{xx}. We emphasize that this relation holds for our entire range of T.

Ando[21] first calculated numerically the dependence of σ_{xx} on σ_{xy} and found a similar relation to the semicircle function. In the self consistent Born approximation and scaling theories a behavior of this kind is also expected[22]. A recent numerical study by R.N. Bhatt and S. Subramanian (private communication), using symmetric and nonsymmetric random potentials in a lowest Landau level model, confirmed this result. For higher Landau levels Wei et al.[23] and others[10] showed similar dependences. This is however the first clear demonstration of this relation for $\nu \leq 1$ and σ_{xx} and σ_{xy} both smaller than $e^2/2h$.

Summarizing, we have demonstrated experimentally the existence of a *HI* and a *QHI*. The *QHI* leads interestingly to the semicircle relation. The experiments presented covered a certain range of parameters, in particular in temperature and in disorder. Different behaviors occurring at even lower temperatures or higher mobilities are therefore possible and would be interesting to investigate in future work. The existence of these insulators, however, can serve as guidelines for a more complete understanding of two-dimensional systems in strong magnetic fields.

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