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Author

Good, Myron L.

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October 23, 1956

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Radiation Laboratory
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ABSTRACT

The expressions for the θ_1 and θ_2 amplitudes in a beam of neutral θ mesons traversing an absorber are put in terms of forward-scattering amplitudes. It is discovered that a phase-shift term as well as an absorption term is needed to describe the regeneration of θ_1 's in the unscattered beam.

A simple relation is derived between the intensities of the above process and of the θ_1 's regenerated by scattering. Experimental verification of the relation may give additional information about the nature of the neutral K's.

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INTRODUCTION

The "particle-mixture" hypothesis of Gell-Mann and Pais¹ led to the prediction by Pais and Piccioni² of a startling phenomenon, the regeneration of short-lived θ_1 mesons in an absorber placed in a beam of long-lived θ_2 mesons. There seem to be two mechanisms of regeneration: (a) regeneration by scattering, in which the θ^0 and $\bar{\theta}^0$ components of the θ_2 scatter differently, thus giving rise to a θ_1 component in the scattered beam; and (b) regeneration by absorption, in which the composition of the unscattered beam changes with depth in the absorber, so that an unscattered θ_1 component develops. This note is a discussion of the interrelation of these two phenomena.

It is found that there is a simple relation between them, at least for small absorber thicknesses. The concept of the complex index of refraction of the absorber for the θ^0 and for the $\bar{\theta}^0$ proves useful. (This is the index of the absorber as a whole, in the sense of slow-neutron optics or light optics, rather than the index of the interior of the nucleus, as in the "optical model" of nuclear scattering.)

DESCRIPTION OF UNSCATTERED BEAM

First, we need a description of the composition of the unscattered beam.

A recent paper by K. Case³ has developed the equations governing the time dependence of the amplitudes a_1 and a_2 of the θ_1 and θ_2 states in traversing an absorber. We refer to Case's paper for the details of the derivation; we will

¹ M. Gell-Mann and A. Pais, Phys. Rev. 97, 1387 (1955).

² A. Pais and O. Piccioni, Phys. Rev. 100, 1487 (1955).

³ K. Case, Phys. Rev. 103, 1449 (1956).

deal only with the differences between his treatment and ours.

In treatment of strong interactions, the $\theta^0, \theta^{\bar{0}}$ representation is the more useful. If $a_0, a_{\bar{0}}$ are the amplitudes for $\theta^0, \theta^{\bar{0}}$, i. e.,

$$\psi = a_0 \theta^0 + a_{\bar{0}} \theta^{\bar{0}},$$

then the spatial behavior of these amplitudes in the absorber is determined, as far as the strong interactions are concerned, by their forward-scattering amplitudes, just as the amplitude of the electric field of a plane wave in matter is determined in ordinary optics:⁴

$$\frac{\partial a_0}{\partial x} = i \left(\frac{2\pi N}{k^2} A^0(0) + 1 \right) k a_0, \quad (1)$$

$$\frac{\partial a_{\bar{0}}}{\partial x} = i \left(\frac{2\pi N}{k^2} A^{\bar{0}}(0) + 1 \right) k a_{\bar{0}},$$

where x = coordinate along the beam,

N = number of nuclei per cm^3 ,

k = wave number in free space, and

$A^0(\phi), A^{\bar{0}}(\phi)$ = complex scattering amplitude for $\theta^0, \theta^{\bar{0}}$, scattering through angle ϕ ($\phi = 0$ for forward scattering).

These equations are equivalent to saying that there is a complex index of refraction for θ^0 and for $\theta^{\bar{0}}$:

$$n_0 = 1 + \frac{2\pi N}{k^2} A^0(0), \quad (2)$$

$$n_{\bar{0}} = 1 + \frac{2\pi N}{k^2} A^{\bar{0}}(0)$$

⁴ See, for instance, Lax, *Revs. Modern Phys.* **23**, 287 (1951). The scattering amplitudes involved in Eq. (1) are those for elastic scattering only.

so that we have

$$\begin{aligned}\frac{\partial a_0}{\partial x} &= i n_0 k a_0, \text{ and} \\ \frac{\partial a_{\bar{0}}}{\partial x} &= i n_{\bar{0}} k a_{\bar{0}}.\end{aligned}\tag{3}$$

The imaginary part of the forward-scattering amplitude is related to the total cross section by the "optical theorem,"

$$\begin{aligned}\sigma_{\text{total}}^0 &= \frac{4\pi}{k} \operatorname{Im} \left(A^0(0) \right), \\ \sigma_{\text{total}}^{\bar{0}} &= \frac{4\pi}{k} \operatorname{Im} \left(A^{\bar{0}}(0) \right).\end{aligned}\tag{4}$$

Thus the imaginary part of n describes the attenuation of the beam, as governed by the total cross section (not the absorption cross section).

The real part of n describes the De Broglie oscillations of the waves. For $n = 1$, this gives the free-space wave number, so that the real part of $n-1$, or the real part of $A(0)$, describes the phase shift of the wave relative to its behavior in free space.

In general, the real and imaginary parts of $A(0)$ are of the same order of magnitude. Therefore, the phase shifts involved are on the order of radians per nuclear mean free path, or radians per inch. Ordinarily, one would not be interested in such a small phase shift; however, we are here concerned with a coherent linear combination of two states. This small phase shift affects the coherence, and therefore is important.

Including the effect of the weak interactions, and changing to the θ_1, θ_2 representation in the same manner as Case, we obtain, for the equations of motion of a_1, a_2 ,

$$\begin{aligned} \frac{d}{dt} a_1 &= i\beta c k \left[\left(\frac{n_0 + n_0}{2} \right) a_1 - i \left(\frac{n_0 - n_0}{2} \right) a_2 \right] - \left(i\omega_1 + \frac{1}{2\gamma\tau_1} \right) a_1, \\ \frac{d}{dt} a_2 &= i\beta c k \left[\left(\frac{n_0 + n_0}{2} \right) a_2 + i \left(\frac{n_0 - n_0}{2} \right) a_1 \right] - \left(i\omega_2 + \frac{1}{2\gamma\tau_2} \right) a_2, \end{aligned} \quad (5)$$

where βc = velocity of particle,

t = laboratory time

$\frac{d}{dt} = \frac{\partial}{\partial t} + \beta c \frac{\partial}{\partial x}$ = total time derivative evaluated at a moving point

$$\gamma = (1 - \beta^2)^{-1/2},$$

$\omega_{1,2}$ = De Broglie frequencies of θ_1, θ_2 particles,

$$(\omega_{1,2})^2 = (kc)^2 + (m_{1,2}c^2)^2, \text{ and}$$

$\tau_{1,2}$ = proper mean life for decay of θ_1, θ_2 .

The cross terms linking the a_1 and a_2 equations depend on the difference in index of refraction (or difference in forward-scattering amplitude) for the $\theta^0, \bar{\theta}^0$ components.

This treatment makes it clear that the regeneration phenomenon depends on the difference in total cross section and on the difference in phase shift of the $\theta^0, \bar{\theta}^0$ waves, rather than on the difference in absorption cross section.

The solution of these equations, in Case's notation, is

$$a(t) = \begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix} = \begin{pmatrix} a_1(0) - R a_2(0) \\ 1 - R^2 \end{pmatrix} e^{-\lambda_1 t} \begin{pmatrix} 1 \\ R \end{pmatrix} + \begin{pmatrix} a_2(0) - R a_1(0) \\ 1 - R^2 \end{pmatrix} e^{-\lambda_2 t} \begin{pmatrix} R \\ 1 \end{pmatrix}, \quad (6)$$

where

$$\lambda_1 = \omega + \Delta,$$

$$\lambda_2 = \omega - \Delta,$$

$$R = \frac{\beta c k (n_0 - n_{\bar{0}})}{-i \left(\frac{\omega_2^0 - \omega_1^0}{\gamma} \right) - \frac{1}{2\gamma} \left(\frac{1}{\tau_2} - \frac{1}{\tau_1} \right) + 2\Delta} = - \left(\frac{i \left(\frac{\omega_2^0 - \omega_1^0}{\gamma} \right) + \frac{1}{2\gamma} \left(\frac{1}{\tau_2} - \frac{1}{\tau_1} \right) + 2\Delta}{\beta c k (n_0 - n_{\bar{0}})} \right)$$

$$\omega = \frac{1}{2} \left(i(\omega_2 + \omega_1) + \frac{1}{2\gamma} \left(\frac{1}{\tau_2} + \frac{1}{\tau_1} \right) - i\beta c k (n_0 + n_{\bar{0}}) \right),$$

$$\Delta = + \frac{1}{2} \left[\left(\frac{i(\omega_1^0 - \omega_2^0)}{\gamma} + \frac{1}{2\gamma} \left(\frac{1}{\tau_1} - \frac{1}{\tau_2} \right) \right)^2 + \left(i\beta c k (n_0 - n_{\bar{0}}) \right)^2 \right]^{1/2}$$

These expressions are the same as given by Case,³ with the following exceptions:

1. The relativistic time dilation, which "slows down" the θ_1 decay and the $\theta_1 - \theta_2$ mass-difference frequency, has been included.⁵ It has the effect of increasing Case's parameter β^6 by a factor γ . Because the smallness of β makes the Pais-Piccioni experiment difficult, this factor can be important in practical situations.

⁵ The term $\omega_2 - \omega_1$, which appears in the solution, has been replaced by $\frac{\omega_2^0 - \omega_1^0}{\gamma}$ (where $\hbar\omega_{1,2}^0$ is the rest energy of $\theta_{1,2}$) for the following reason:

The De Broglie frequencies are given by

$$\left(\hbar\omega_{1,2} \right)^2 = (\hbar kc)^2 + \left(m_{1,2} c^2 \right)^2,$$

so that, to first order, we have

$$\hbar^2 \omega \delta \omega = (mc^2) \delta (mc^2),$$

$$\hbar \delta \omega = \frac{mc^2}{\hbar \omega} \delta (mc^2) = \frac{\delta (mc^2)}{\gamma} = \frac{\hbar (\omega_2^0 - \omega_1^0)}{\gamma},$$

(This result follows even if we have $k_1 \neq k_2$, so long as the difference is of order $\frac{\delta m}{m}$.)

⁶ Case's β is essentially the ratio of absorption to decay.

2. The effects of the total cross section and of the phase shift produced by the real part of the forward scattering are included; Case's model included only the θ^0 absorption cross section.

Figure 1 shows the probability $|a_1|^2$ vs x for a pure θ_2 beam incident on the absorber, and for $\omega_2^0 - \omega_1^0 = \frac{1}{\tau_1}$. $N \sigma_{\text{total}}^0 = \frac{1}{\beta \gamma c \tau_1}$. $\sigma_{\text{total}}^0 = \frac{1}{3} \sigma_{\text{total}}^0$

Curves are plotted for the real part of the forward scattering equal to zero, one-half, and one times the imaginary part. The phase-shift term is seen to have an appreciable effect.

REGENERATION BY SCATTERING

We can now discuss the relation between the regeneration of θ_1 's in the unscattered beam and the regeneration by scattering. The composition of the unscattered beam is

$$\psi = a_0(x) \theta^0 + a_{\bar{0}}(x) \theta^{\bar{0}} = a_1(x) \theta_1 + a_2(x) \theta_2$$

where the a 's are obtained from Eq. (6) with $x = \beta c t$ and $a_2(0) = 1$, $a_1(0) = 0$ (i.e., pure θ_2 beam incident on the absorber).

The probability of seeing a θ_1 in the unscattered beam is given by

$$n_{\text{un}} = |a_1(x)|^2 = \left| \frac{a_0(x) + a_{\bar{0}}(x)}{\sqrt{2}} \right|^2, \quad (7)$$

while that of seeing a scattering through angle ϕ followed by a θ_1 decay is, in thickness dx of the scatterer, and in solid angle $d\Omega$,

$$dn_s = \left| \frac{a_0(x) A^0(\phi) + a_{\bar{0}}(x) A^{\bar{0}}(\phi)}{\sqrt{2}} \right|^2 N dx d\Omega. \quad (8)$$

The unscattered regenerated θ_1 's reflect the composition of the unscattered beam, while the scattered ones reflect the composition times the scattering matrix.

However, for small thicknesses of absorber (small here means $\frac{|\Delta|x}{\beta c} \ll 1$), a_1 is small, and $a_2 \approx 1$. Thus we obtain

$$a_1(x) \approx k \left(\frac{n_0 - n_{\bar{0}}}{2} \right) x \text{ by integrating Eq. (5) for small } x, \text{ and}$$

$$a_1(x) \approx \frac{2\pi N}{k} \left(\frac{A^0(0) - A^{\bar{0}}(0)}{2} \right) x; \quad (9)$$

in these circumstances we have

$$a_{\bar{0}} \approx -a_0 = \frac{1}{\sqrt{2}}, \text{ and thus}$$

$$n_{un} \approx \frac{\pi^2 N^2}{k^2} \left| A^0(0) - A^{\bar{0}}(0) \right|^2 x^2 \quad (10)$$

$$dn_{\bar{0}} \approx \frac{1}{4} \left| A^0(\phi) - A^{\bar{0}}(\phi) \right|^2 N dx d\Omega. \quad (11)$$

If one observes decays in a cloud chamber that is placed in a beam of θ_2 's behind a thickness of absorber δx that is small compared with $\beta c/|\Delta|$, he should see an angular distribution of θ_1 decays as in Fig. 2. We can now, from Eqs. (10) and (11), give the ratio of the number of events, n_{un} , in the peak, to the number of events, $\delta n_{\bar{0}}$, in the background under it in some finite angular interval $\delta\phi$ at $\phi = 0$.

(If we choose $\delta\phi$ equal essentially to the angular resolution of the apparatus, this ratio is the ratio of peak height to the background height at $\phi \approx 0$.) If we set

$$d\Omega = 2\pi(1 - \cos \delta\phi) \approx \pi (\delta\phi)^2$$

$$dx = \delta x \text{ in } dn_s,$$

$x = \delta x$ in n_{un} , then we have

$$\left[\frac{n_{un}}{\delta n_s} \approx \frac{4\pi N(\delta x)}{k^2 (\delta\phi)^2} \right]. \quad (12)$$

The scattering amplitudes cancel out, and the ratio is dependent only on the geometry and the wave length. (In order to see a large peak, one needs fairly large thicknesses, low energy, and high angular resolution.)

For $\delta\phi = 1^\circ$, 100-Mev kinetic energy θ^0 's and a 0.5-cm lead plate,

$$\frac{n_{un}}{\delta n_s} = 2.5.$$

The simple relation given by Eq. (12), which is independent of all nuclear parameters, should serve as an interesting check on our understanding of the process.

However, we need to examine carefully its limits of validity. (The analysis so far has implicitly assumed, for instance, zero spin both for θ^0 mesons and for the scattering nucleus, inasmuch as the scattering of θ^0 mesons has been described by a single scattering amplitude.)

This examination is outlined in the appendix. It is concluded that, if one performs an experiment to test the relation between scattered regenerated θ_1 's observed at small angles and unscattered regenerated θ_1 's, as given by Eq. (12),

- (a) One should use an isotopically pure element for a target;
- (b) If the spin of the target nucleus is zero, Eq. (12) should unconditionally be obeyed;
- (c) If the target nucleus spin is $1/2$, lack of agreement with Eq. (12) signifies either nonzero spin for the θ^0 or $\tau^0 \rightarrow \theta^0$ scattering that does not vanish in the forward direction.

It is possible, then, that careful observation of the two types of Pais-Piccioni regeneration may give information about the nature of the neutral K mesons beyond what was expected originally.

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APPENDIX: LIMITS OF VALIDITY OF EQUATION (12)

We need to consider the following possible complications.

1. Nonidentical Scattering Nuclei

The forward peak, n_{un} , is caused by constructive interference of large numbers of nuclei.⁷ The scattered group, δn_s , involves scattering from just one nucleus. Because of this, if there are two kinds of scatterers present, one averages their scattering amplitude, then squares, in calculating n_{un} ; whereas, in calculating δn_s , one squares first, then averages. This spoils the cancellation of scattering amplitudes that led to Eq. (12).

Equation (12), then, should in general hold only for an absorber that is a chemical element, and (in principle at least) is a single isotope of that element.

⁷ The treatment given does not make this very obvious, since the index of refraction is usually thought of as the result of interference of the individual scattered wavelets with the incident wave, rather than with one another. Suppose, however, that one does not go through the intermediate step of deriving an index of refraction for θ^0 , θ^0 , but rather describes the problem of a θ_2 wave incident on a thin scatterer directly in terms of θ^0 and θ^0 wave functions, each consisting of an incident wave and N scattered wavelets. By adding these two wave functions, one obtains the θ_1 wave function directly. The incident-wave term falls out, since there are no incident θ_1 's, and only the sum of the θ_1 scattered wavelets, $\sum_i (\theta_1)_i$, remains. Upon squaring, the terms in $i = j$ lead to the ordinary scattered intensity, δn_s , while the terms for $i \neq j$ give rise to a constructive interference in the forward direction, which is just n_{un} . One ends up with Eq. (12), but the treatment makes evident the cooperative nature of the forward peak.

2. Nonzero Spin of Scattering Nucleus

A detailed analysis shows that the intuitive result is correct, namely that the presence of $2s + 1$ spin substates of the scatterer is the same as having $2s + 1$ kinds of nuclei present. Equation (12) does not, in general, hold for nonzero scattering nucleus spin.

3. Nonzero Spin of the θ^0

If we neglect spin-flip processes temporarily, each spin substate of the θ^0 scatters separately, and the intensities for each substate obey Eq. (12) separately. The total intensity is the sum of the intensities of the substates, and therefore Eq. (12) is obeyed, in general, for nonzero θ^0 spin, so long as the spin of the nucleus is zero.

4. A Special Case: θ^0 Particle Spin Zero, Nuclear Spin 1/2

In this case, there are but two scattering amplitudes, and both are equal. The order in which one performs the operations of averaging and squaring is then unimportant, and therefore Eq. (12) holds, for this special case.

(If the θ^0 particle spin is not zero, and the target nucleus spin is 1/2, the scattering for target-nucleus spin parallel and antiparallel to the θ^0 spin might well be different, so that the remarks of Point 2 apply.)

5. Spin-Flip Processes

Two points are pertinent here:

- (a) Spin flip, being an intrinsically incoherent process, cannot contribute to the forward peak, but only to δn_g . Spin flip, if present, "spoils" Eq. (12).
- (b) The spin flip amplitude vanishes at 0° if either spin is zero.

Because in all the cases enumerated above, in which Eq. (12) should hold, one or the other of the spins is zero, we can conclude that spin-flip processes will not affect the question of whether or not Eq. (12) holds.

6. Parity-Exchange Scattering

If a neutral τ exists, and can turn into a neutral θ upon elastic scattering, the situation is obviously more complex than was assumed in the derivation of Eq. (12).

However, there are certain simplifications: since the intrinsic parity of the τ^0 is (by definition) different from that of the θ^0 , the orbital angular momentum must change by one unit in the process $\tau^0 \leftrightarrow \theta^0$.

To conserve total angular momentum, then, it is necessary that the component of total spin of the system change by one unit. Thus $\tau \leftrightarrow \theta$ scattering is a special kind of spin-flip process. Bearing this in mind, and restricting ourselves to these cases for which Eq. (12) should, so far, hold, we see that

- (a) For spin zero target nuclei, Eq. (12) should still hold. The argument is as follows: If the θ^0 spin is zero, there are no spins to flip, and therefore the $\tau^0 \leftrightarrow \theta^0$ scattering vanishes.⁸ If the θ^0 spin is not zero, its spin z-component can change by one unit, and $\tau^0 \leftrightarrow \theta^0$ scattering can occur. But the intensity of this contribution obeys Eq. (12) and, as in the discussion under Point 3 above, Eq. (12) then holds for the total intensity. (This result may be shown to be independent of the relative amounts and relative phases of the τ_1 's and τ_2 's postulated to be in the beam.)
- (b) For the special case of Point 4 above, the $\tau^0 \leftrightarrow \theta^0$ scattering (if it does not vanish in the forward direction) can occur, by flipping the target nucleus spin. It is therefore incoherent, and "spoils" Eq. (12).

⁸ It must be remembered that we are talking about strictly elastic scattering. If an incoming τ^0 flips the spin of a proton in a spin-zero nucleus, emerging as a θ^0 , the nucleus is left in an excited state, and the event is classified as inelastic.

LEGENDS

Fig. 1. θ_1 intensity vs distance for θ_2 beam incident on absorber.

$$\epsilon = \frac{\text{Re} (A_{(0)}^0 - A_{(0)}^{\bar{0}})}{\text{Im} (A_{(0)}^0 - A_{(0)}^{\bar{0}})} = 0, 1/2, 1$$

$$\omega_2^0 - \omega_1^0 = \frac{1}{\tau_1}$$

$$\sigma_{\text{total}}^0 = \frac{1}{3} \sigma_{\text{total}}^{\bar{0}}$$

$$N \sigma_{\text{total}}^{\bar{0}} = \frac{1}{\beta \gamma c \tau_1}$$

Fig. 2. Distribution in angle of regenerated θ_1 's near $\phi = 0$. The group δn_s refers only to elastically scattered θ 's. In general, there will also be inelastically scattered θ 's, which would have to be rejected in checking the predictions of Eq. (12).

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